Simulating the Dynamics of Atomics Qubits Coupled to a Laser Field

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Qubits dynamics is the core of quantum computation, as it dictates the form in which different operations and algorithm can be performed. In this work I simulate the dynamics of atomic qubits under laser control field, by implementation of a Monte Carlo Algorithm and compare the results with an analytic solution.

Theory:

A physically realizable qubit can be taken to consist of the ground and first excited state of an atom. In the case of this atomic qubit, the **control field used to manipulate the unit is electromagnetic radiation**. In the case where the frequency of the radiation is equal or close to the resonant frequency of the qubit, the evolution of the system is governed by the following**, non-Hermitian Hamiltonian,**

$$\hat{H} = \frac{\hbar}{2}\begin{pmatrix} 0 & \Omega \\ \Omega & -i\Gamma \end{pmatrix}. \tag{1}$$

Where Γ is the decay rate to the ground state, and Ω is the Rabi frequency at which the population of the states oscillates. For a **pure state**, *i.e.* of the form $|\psi\rangle = a|0\rangle + b|1\rangle$, the information on the population of the states is contained in the density matrix:

$$\rho = \begin{pmatrix} |a|^2 & ab^* \\ a^*b & |b|^2 \end{pmatrix} \tag{2}$$

whose evolution under the Hamiltonian is given by [1]:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[\hat{H}, \rho] \tag{3}$$

Bloch Equations:

Bloch equations can be extracted from Eq.(3), and have the form:

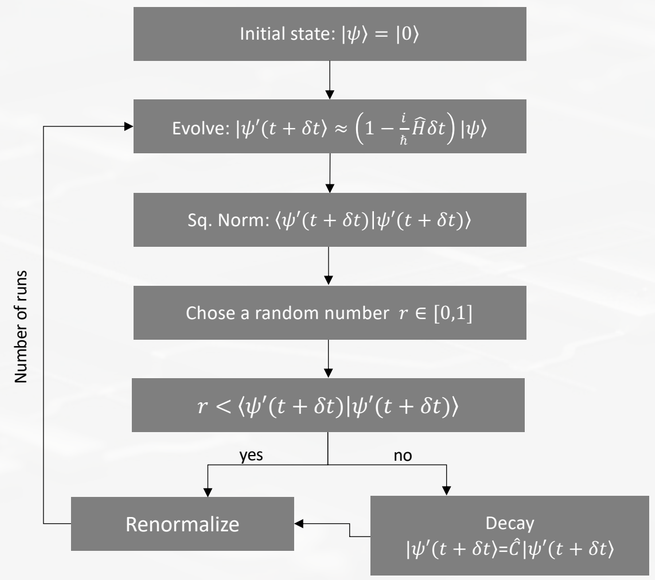
$$\begin{aligned} \frac{d\rho_{11}}{dt} &= i\frac{\Omega}{2}(\rho_{10} - \rho_{01}) - i\Gamma\rho_{11} \\ \frac{d\rho_{01}}{dt} &= i\frac{\Omega}{2}(\rho_{00} - \rho_{11}) - i\Gamma\rho_{01} \end{aligned} \tag{4}$$

where the indices indicate the matrix elements in Eq.(2), and together with $\dot{\rho_{11}} = -\dot{\rho_{00}}$ and $\dot{\rho_{10}} = \dot{\rho_{01}}^*$ which follows from the requirement that the total probability is always equal to unity, they **give a complete description of the time evolution of the density matrix** [2]. Bloch equations have straightforward analytic solutions, which **for the case of the population of the first excited state is**

$$\rho_{11} = \frac{\Omega^2}{2\Omega^2 + \Gamma^2}\left\{1 - e^{-\frac{3\Gamma t}{4}}\left[\cos\left(\sqrt{\Omega^2 - \frac{\Gamma^2}{16}}t\right) + \frac{3\Gamma}{\sqrt{16\Omega^2 - \Gamma^2}}\sin\left(\sqrt{\Omega^2 - \frac{\Gamma^2}{16}}t\right)\right]\right\} \tag{5}$$

Monte Carlo Simulation:

Computationally, the time evolution of an atomic qubit can be simulated using a Monte Carlo algorithm of the from [3]:



where,

$$\hat{C} = \sqrt{\frac{\Gamma}{2}}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{6}$$

is the **quantum jump operator**, which transfers a system partially or totally in the first excited state to the ground state .

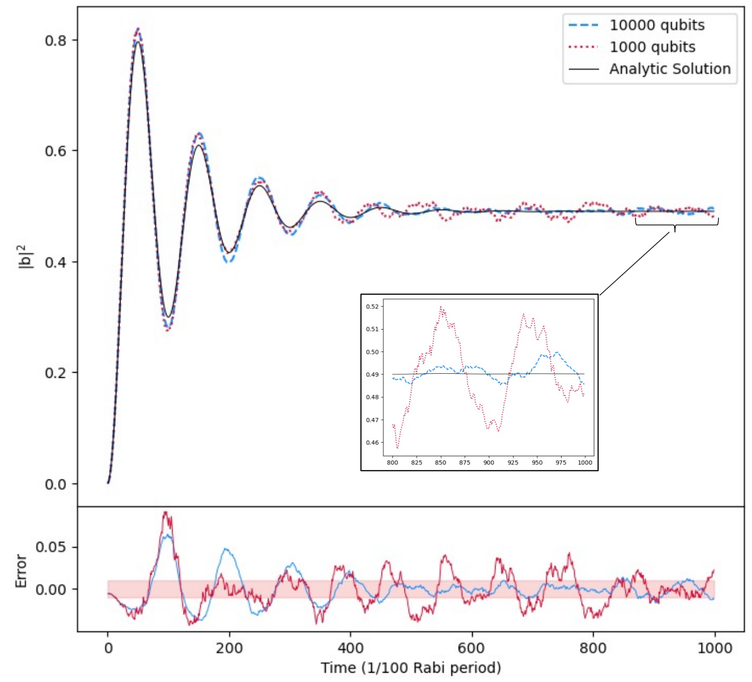


Fig1. Comparison of the analytic solution, shown in black, with the average of 1000 and 10000 quantum trajectories, simulated by a Monte Carlo Algorithm. Ten Rabi periods are showed. The bottom plot shows the normalized residuals for both cases. The semi-transparent line denotes an error of 1%.

Results:

Starting with a qubit completely in the ground state, I implement the Monte Carlo algorithm by evolving the system in time steps of size $\delta t = \Omega^{-1}/100$ and with a decay rate $\Gamma = 0.2\Omega$ for for **1000 and 10000 qubits** and compare the **average quantum trajectories** with the analytic solution given by Bloch equations. It was found that both simulations converge towards the analytic solution, additionally, when increasing the number of qubits by a factor of 10, **the error** with respect to the analytic values **decreases significantly, converging to values of ≈1% for 10000 qubits.**

Further Work:

Being able to simulate the dynamics of qubits under the control fields used to manipulate them is a fundamental first step in the study of higher-dimensional quantum units (qudits). My goal for the project is to progress to the study of such quantum systems, beginning with qutrits (three levels), **for which the methodology used to study the time evolution of qubits can be extrapolated** [4]. Subsequent stages of the project include the derivation of the effective Hamiltonian for those systems and the implementation of the algorithm simulating the time evolution of qutrits under laser control fields.

[1] B.J. Bransden and C.J. Joaquin, Quantum Mechanics, Pearson Education, Essex (2000), p.655. **[2]** R. Loudon, The Quantum Theory of Light, Oxford University Press, Oxford (2000), p. 72. **[3]** K. Mølmer, Y. Castin, and J. Dalibard, Monte Carlo wavefunction method in quantum optics, *J. Opt. Soc. Am. B,* Vol. 10, No. 3 (1993), p. 524. **[4]** G. Karpat, Correlation dynamics of qubit-qutrit systems in a classical dephasing environment, *Phys, Lett*. Vol. 375, no 47 (2011), pp. 4166-4171, **Background**: https://research.ibm.com/blog/127-qubit-quantum-processor-eagle