# Preferences

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# 1 Introduction and Motivation

This lecture introduces the **fundamental principles of consumer choice theory**, serving as the foundation for understanding the **demand curve** in microeconomics. While supply and demand models provide powerful tools for analyzing markets, we will emphasize the need to **understand the behavior underlying these curves**.

Where do demand curves originate? How do consumers make choices between different goods? This lecture explores the **key assumptions of consumer preferences**, the concept of **utility functions**, and the graphical representation of preferences through **indifference curves**.

The analysis focuses on **utility maximization under constraints** and provides insights into core economic principles, such as **diminishing marginal utility** and the **marginal rate of substitution (MRS)**.

# 2 Model Assumptions

At the core of consumer choice theory lies the **utility maximization model**, where consumers aim to maximize their satisfaction (utility) subject to a **budget constraint**. Let's start with the assumptions about **consumer preferences** that ensure choices are **rational and consistent**.

## 2.1 Preference Relations

- A consumer's **preference relation** indicates their ranking of consumption bundles:
  - $(x_1, x_2) \succ (y_1, y_2)$: The consumer **strictly prefers** the bundle $(x_1, x_2)$ to $(y_1, y_2)$.
  - $(x_1, x_2) \sim (y_1, y_2)$: The consumer is **indifferent** between the two bundles.
  - $(x_1, x_2) \succeq (y_1, y_2)$: The consumer **weakly prefers** the bundle $(x_1, x_2)$ to $(y_1, y_2)$.

## 2.2 Key Assumptions

There are 3 main assumptions, that follow common sense from the usual consumer, that need to be considered to simplify the complexity of models representing choice theory.

1. **Completeness:** Consumers can always compare two bundles of goods and express a preference for one over the other, or indicate indifference.

   *Example:* A consumer can compare Bundle A (2 pizzas, 1 cookie) with Bundle B (1 pizza, 2 cookies) and either prefer one or remain indifferent, i.e, both bundles are considered similar and therefore indifferent to the consumer.

2. **Transitivity:** If a consumer prefers Bundle A over Bundle B, and Bundle B over Bundle C, then they must also prefer Bundle A over Bundle C.

   $$\text{If } A > B \text{ and } B > C, \text{ then } A > C$$

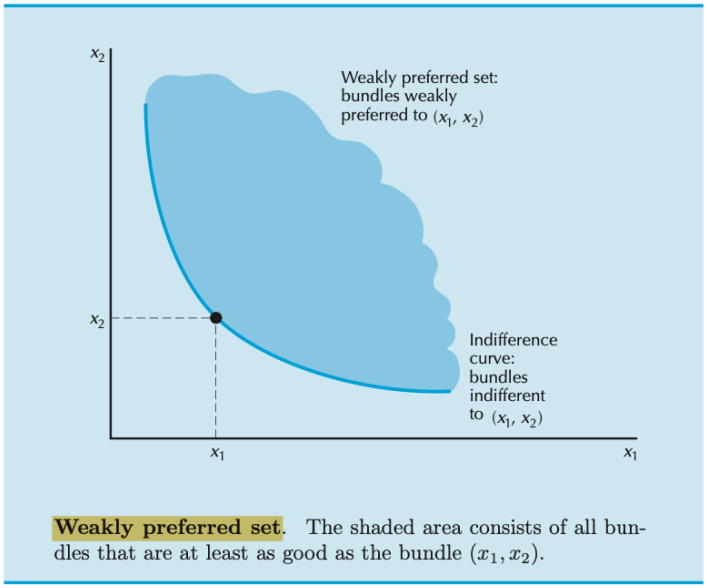
3. **Non-Satiation (More is Better):** Consumers always prefer more of a good to less, assuming no external costs (e.g., waste, storage).

   *Example:* The first slice of pizza brings immense satisfaction, while the tenth slice might not bring as much joy.

These assumptions allow economists to construct **utility functions** and **indifference curves** as reliable tools for analyzing consumer behavior.

# 3 Indifference Curves and Their Properties

Let $(x_1, x_2)$ be a given bundle. The **weakly preferred set** contains all bundles weakly preferred to $(x_1, x_2)$. Indifference curves represent all combinations of bundles (two or more) of goods that provide the same level of utility to a consumer.




**Weakly preferred set**. The shaded area consists of all bundles that are at least as good as the bundle $(x_1, x_2)$.

Figure 1: Weakly Preferred Set

## 3.1 Well-Behaved Preferences: Key Properties of Indifference Curves

We need to distinguish 4 key properties defining **Well-Behaved Preferences** and their Indifference Curves.

1. **Consumers Prefer Higher Indifference Curves:** Higher curves represent bundles with more goods, which consumers prefer due to the **more is better** assumption. For instance, in figure 2, the bundle C (2 cookies, 2 pizzas) will be considered better in terms of utility compared to bundles A (1 cookie, 2 pizzas) and C (2 cookies, 1 pizza).

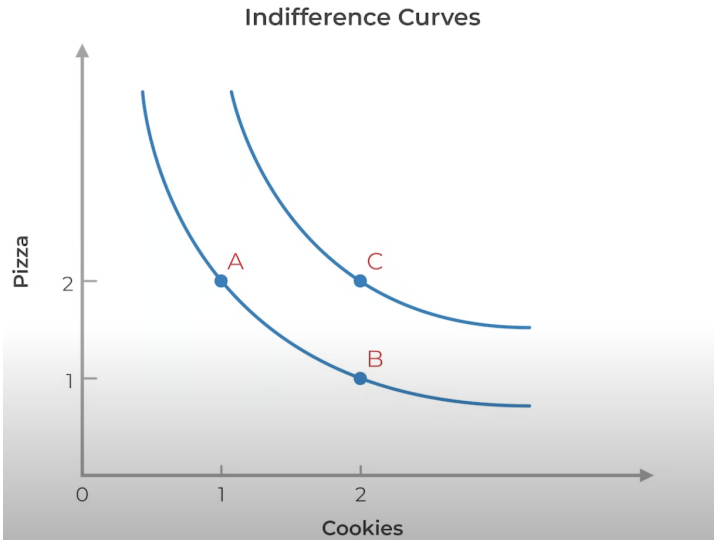



Figure 2: Indifference Curves

2. **Indifference Curves Never Cross:** If two curves crossed, it would violate **transitivity**. For instance, in figure 3, if the consumer considers indifferent bundles A and B, as well as bundles A and C, then bundles B and C are considered as well indifferent, however it's pretty clear that bundle B, provides the same quantity of cookies (2) and more pizzas than bundle C, which directly violates the the third assumption of Non-Satiation.

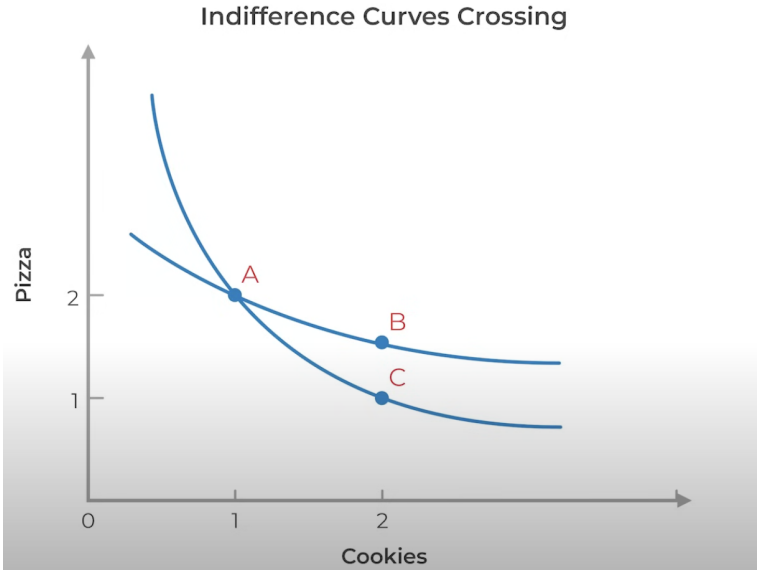



Figure 3: Intersection of Indifference Curves

3. **Indifference Curves are Downward Sloping:** To maintain the same utility, consumers must give up some of one good to gain more of another. For instance, in figure 4, once again the third assumption of Non-Satiation is violated as bundle $(2, 2)$ is obviously a better deal than bundle $(1, 1)$.

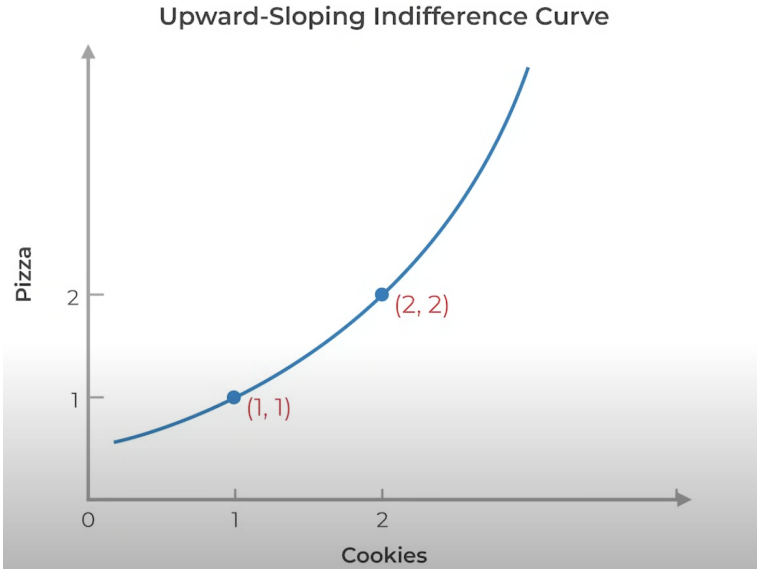
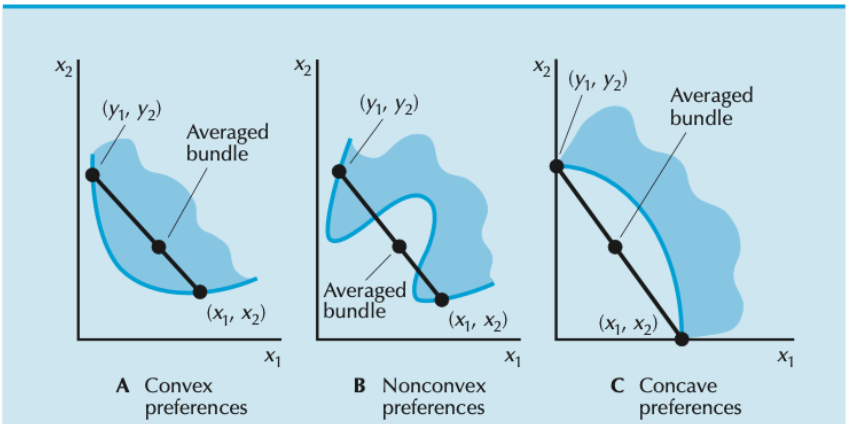



Figure 4: Upward-Slopping Indifference Curve

4. **Only One Indifference Curve Passes Through Every Bundle:** Every unique bundle corresponds to one and only one indifference curve.




**Various kinds of preferences.** Panel A depicts convex preferences, panel B depicts nonconvex preferences, and panel C depicts "concave" preferences.

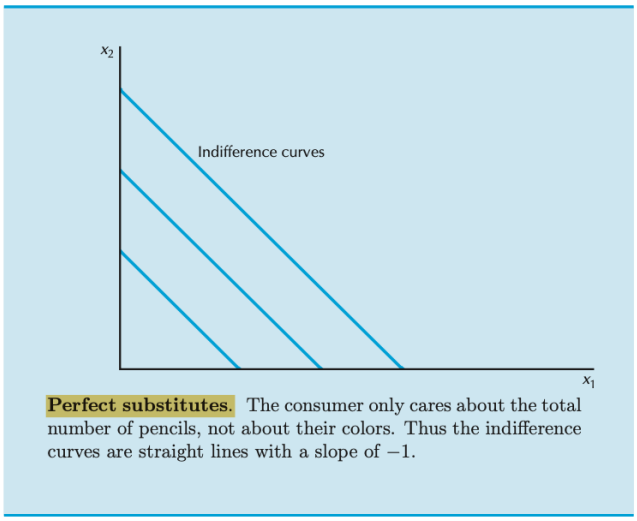
Figure 5: Well-Behaved Preference (Case A)

These properties ensure that **indifference curves are well-behaved** (Case A) and accurately represent rational consumer preferences.

## 3.2 Special Cases of Preferences

### 3.2.1 Perfect Substitutes

- Two goods are **perfect substitutes** if the consumer is willing to substitute one good for the other at a constant rate.
- **Example**: Red and blue pencils. If the consumer values only the total number of pencils, any bundle $(x_1, x_2)$ such that $x_1 + x_2 = 20$ is on the same indifference curve. If $x_1$ contributes nothing to this bundle




**Perfect substitutes.** The consumer only cares about the total number of pencils, not about their colors. Thus the indifference curves are straight lines with a slope of $-1$.

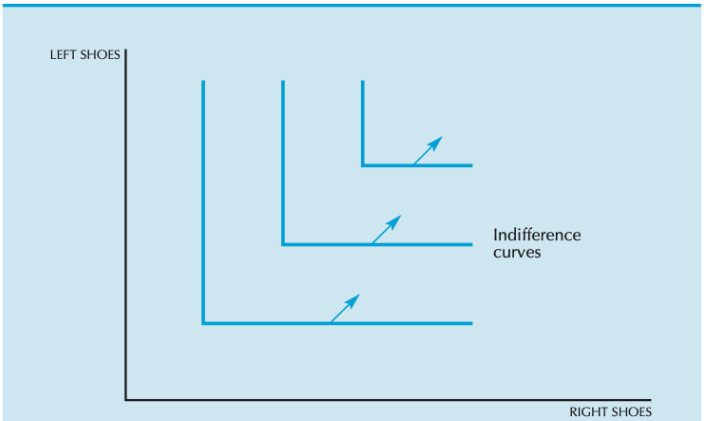
Figure 6: Perfect Substitutes Preferences

$(0, 10)$ and $x_2$ alone provides the full utility (10), the consumer is perfectly happy with $x_2$ compensating entirely for the absence of $x_1$.

### 3.2.2 Perfect Complements

- Two goods are **perfect complements** if they are always consumed together in fixed proportions.
- **Example:** Right and left shoes. The consumer's utility depends on the smaller of the two quantities and the consumer considers indifferent having (10 left shoes, 11 right shoes) and (10 left shoes, 10 right shoes), because what is he going to do with 1 extra lonely shoe?

$$U(x_1, x_2) = \min(x_1, x_2).$$

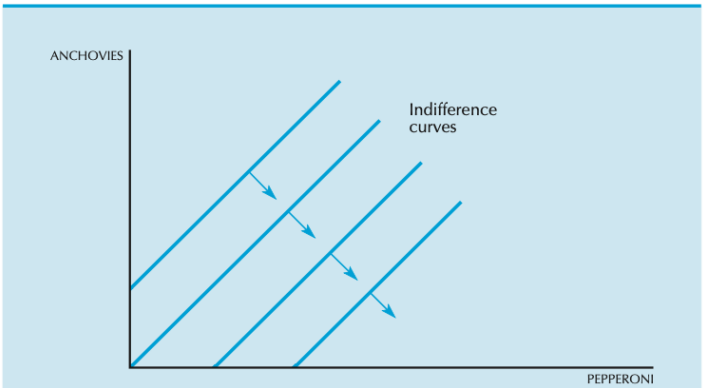



**Perfect complements.** The consumer always wants to consume the goods in fixed proportions to each other. Thus the indifference curves are L-shaped.

Figure 7: Perfect Completements Preferences

### 3.2.3 Bad Commodity

- A bad is a commodity the consumer dislikes.
- **Example:** Pepperoni (good) and anchovies (bad). Indifference curves slope upward because more of the good is required to compensate for additional units of the bad.

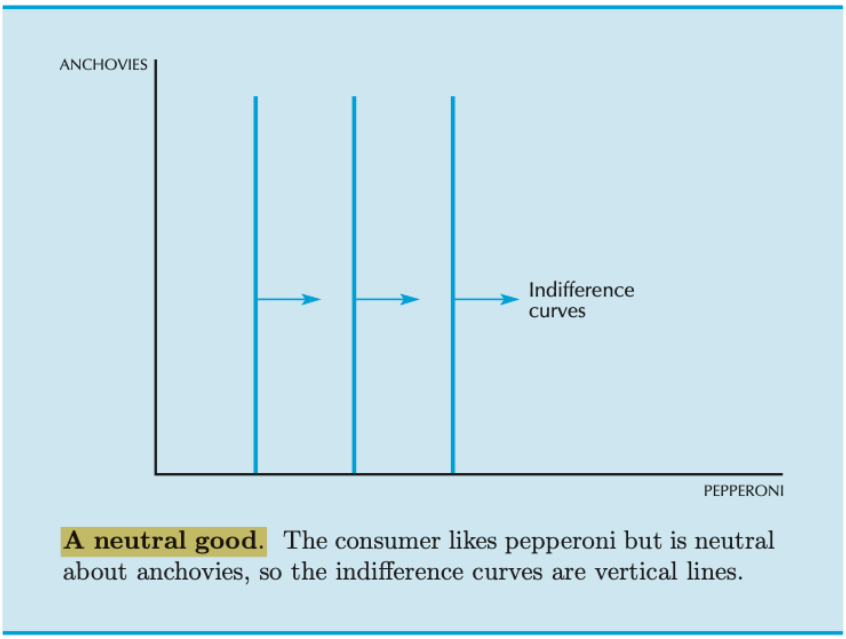



**Bads.** Here anchovies are a "bad," and pepperoni is a "good" for this consumer. Thus the indifference curves have a positive slope.

Figure 8: Bad Commodity Preferences

### 3.2.4 Neutral Commodity

- A **neutral good** is one that the consumer does not care about.
- **Example:** If the consumer values only $x_1$, any change in $x_2$ leaves the consumer indifferent.

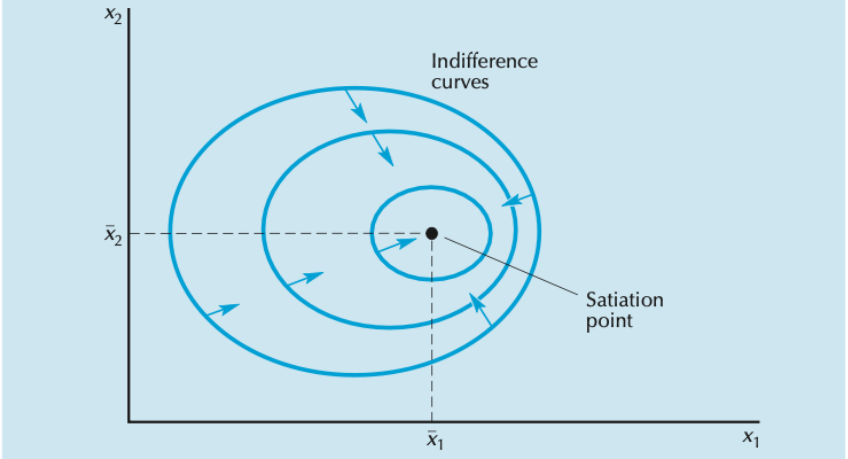



**A neutral good.** The consumer likes pepperoni but is neutral about anchovies, so the indifference curves are vertical lines.

Figure 9: Neutral Goods Preferences

### 3.2.5 Satiation

- A consumer has a **satiation point** or **bliss point** if there is a most preferred bundle $(x_1, x_2)$, and utility decreases as the consumer moves farther from this point. This is the case where we consider external costs.




**Satiated preferences.** The bundle $(\bar{x}_1, \bar{x}_2)$ is the satiation point or bliss point, and the indifference curves surround this point.

Figure 10: Satiated Preferences

# 4 Utility Functions

A utility function mathematically represents consumer preferences, assigning a numerical value to each consumption bundle based on the satisfaction it provides. The utility function translates preferences into mathematical models, enables economists to **rank consumption bundles** and serves as the foundation for constructing **indifference curves**.

## 4.1 Key Concepts

- **Ordinal vs. Cardinal Utility:** *Ordinal Utility:* Ranks bundles based on preference. *Cardinal Utility:* Attempts to assign specific numerical values to utility.
- **Example Utility Function:**

$$U = \sqrt{X \cdot Y}$$

  where $X$ is the quantity of cookies and $Y$ is the quantity of pizzas.

# 5 Marginal Utility (MU)

The Marginal Utility (MU) measures the **additional satisfaction** a consumer gains from consuming one more unit of a good.

## 5.1 Law of Diminishing Marginal Utility

As consumption of a good increases, the additional satisfaction ($MU$) from each extra unit decreases.

$$MU = \frac{\Delta U}{\Delta Q}$$

*Example:* The first slice of pizza provides a high satisfaction, however the fifth slice of pizza, even though provides satisfaction, it is a lower additional satisfaction. This principle explains pricing strategies, such as **bulk discounts** in fast-food restaurants.

# 6 Marginal Rate of Substitution (MRS)

The Marginal Rate of Substitution (MRS) represents the **rate at which a consumer is willing to trade one good for another while maintaining the same level of utility**.

$$MRS = -\frac{\Delta x}{\Delta y}$$

Where $\Delta x$ is the Marginal Utility of Cookies and $\Delta y$ the Marginal Utility of Pizza.

## 6.1 Behavior of the MRS

- **Perfect Substitutes**: MRS is constant (e.g., $-1$).
- **Perfect Complements**: MRS is either zero or infinity, depending on the proportions.
- **The MRS is always decreasing in well-behaved preferences or convex preferences**. The more you have of one good, the more willing you are to give some of it up in exchange for the other good

  *Example:* Initially, a consumer might trade **2 slices of pizza for 1 cookie**. Later, they might only trade **1 slice of pizza for 1 cookie**. This reflects the principle of **diminishing marginal utility** and explains why **indifference curves are convex to the origin**.

### 6.2 Example

We will represent two goods: good $X = Pizza$ (on the horizontal axis) and good $Y = Money$ (on the vertical axis). Money represents the amount the consumer can spend on other goods or save.

1. **Convex Case** (Normal and Rational) ✓

   A **convex curve** reflects that the consumer prefers a balance between pizza and money rather than extremes. As they consume more pizzas, the marginal utility of an additional pizza decreases. Therefore, they will be willing to give up less money to get one more pizza. For instance, if the consumer has 10€ and 1 pizza, they might be willing to give up 4€ for a second pizza. However, if they already have 4 pizzas, they might only be willing to give up (pay) 1€ for a fifth pizza.

   - **Key Insight:** The more pizzas they have, the less relative value they assign to an additional pizza, and the less money they are willing to sacrifice.
   - **Graph Shape**: The curve is convex towards the origin, reflecting a decreasing marginal rate of substitution (MRS).

2. **Concave Case** (Irrational or Anomalous) ✗

   A **concave curve** implies that as the consumer gets more pizzas, they become willing to give up more money for an additional pizza. This contradicts the law of diminishing marginal utility, as their willingness to pay increases with each additional pizza. For instance, if the consumer has 1 pizza and 10€, they might be willing to give up 1€ for a second pizza. However, if they already have 4 pizzas, they might now be willing to give up €5 for a fifth pizza.

   - **Key Insight:** As they obtain more pizzas, their willingness to pay increases instead of decreasing. This is illogical because the marginal value of a good typically decreases as consumption increases.
   - **Graph Shape**: The curve is concave outward, representing an increasing marginal rate of substitution (MRS), something that contradicts rational consumer behavior.

These concepts help explain real-world phenomena such as **pricing strategies, consumption choices, and market behavior**. Understanding these foundational principles enables better analysis of economic systems and consumer decision-making.