

# Profit Maximization

Pablo Mollá Chárlez

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## Contents

<b>1</b>	<b>Motivation</b>	<b>2</b>
<b>2</b>	<b>The Organization of Firms</b>	<b>2</b>
2.1	Profits . . . . .	2
2.2	Stock Market Value . . . . .	2
2.3	The Boundaries of the Firm . . . . .	3
2.4	Fixed and Variable Factors . . . . .	3
<b>3</b>	<b>Short-Run Maximization and Isoprofit Lines</b>	<b>3</b>
3.1	Short-Run Maximization . . . . .	3
3.1.1	Short Example . . . . .	4
3.2	Isoprofit Lines . . . . .	5
3.2.1	Short Example 1 . . . . .	6
3.2.2	Short Example 2 . . . . .	6
3.3	Comparative Statistics . . . . .	7
<b>4</b>	<b>Long-Run Maximization</b>	<b>8</b>
4.1	Profit Maximization and Constant Returns to Scale . . . . .	8
<b>5</b>	<b>Revealed Preferences</b>	<b>9</b>
5.1	WAPM . . . . .	9
5.2	Farmer and Subsidies Example . . . . .	10

# 1 Motivation

In the last chapter we discussed ways to describe the technological choices facing the firm. In this chapter we describe a model of how the firm chooses the amount to produce and the method of production to employ. The model we will use is the model of profit maximization: **the firm chooses a production plan so as to maximize its profits.**

In this chapter we will assume that the firm faces fixed prices for its inputs and outputs. We said earlier that economists call a market where the individual producers take the prices as outside their control a competitive market. So in this chapter we want to **study the profit maximization problem of a firm that faces competitive markets for the factors of production it uses and the output goods it produces**

## 2 The Organization of Firms

In a **capitalist economy**, firms are ultimately **owned by individuals**, even though they may take the legal form of **proprietorships**, **partnerships**, or **corporations**.

A **proprietorship** is **owned and managed by a single individual**, while a **partnership** involves **two or more individuals directly managing the business**. In contrast, a **corporation** is a **separate legal entity that differentiates between owners and managers**. Despite these organizational differences, the **common objective across these forms is typically profit maximization**, aligning the actions of managers (or owners) with the goal of benefiting the firm's owners

### 2.1 Profits

Profit is defined as the difference between **total revenue** and **total cost**. In mathematical terms, if a firm produces  $n$  outputs  $(y_1, \dots, y_n)$  priced at  $(p_1, \dots, p_n)$  and uses  $m$  inputs  $(x_1, \dots, x_m)$  at prices  $(w_1, \dots, w_m)$ , its profit  $\pi$  is given by:

$$\pi = \underbrace{\sum_{i=1}^n p_i y_i}_{\text{Total Revenue}} - \underbrace{\sum_{j=1}^m w_j x_j}_{\text{Total Cost}}.$$

Here,  $\sum_{i=1}^n p_i y_i$  represents total revenue from all outputs, and  $\sum_{j=1}^m w_j x_j$  represents the total cost of all inputs.

### 2.2 Stock Market Value

Firms often invest in assets (like factories) that **yield benefits over many periods**. Because these investments produce a flow of future revenues and incur future costs, they are valued using the concept of **present value**, which converts future amounts into today's dollars using the interest rate. In a world of perfect certainty, **the present value of a firm's future profits equals its total market value—reflected in its stock price**. Each share's price represents the present value of the expected dividends, so a firm's objective of maximizing the present value of its profit stream is equivalent to maximizing its stock market value. Even when uncertainty makes direct profit maximization hard to define, aiming to maximize stock market value remains a clear goal that aligns with the interests of the shareholders, despite the complexity of multi-period and uncertain environments.

- **Annuity Formula:** An annuity is a series of equal payments made at regular intervals for a finite period. The present value (PV) of an annuity with constant payment  $C$ , **discount rate**  $r$ , and  $n$  periods is given by:

$$PV = C \times \frac{1 - (1 + r)^{-n}}{r}.$$

- **Perpetuity Formula:** A perpetuity is a series of equal payments made at regular intervals indefinitely. The present value of a perpetuity with constant payment  $C$  and discount rate  $r$  is:

$$PV = \frac{C}{r}.$$

These formulas are used to determine the current value of future cash flows by accounting for the time value of money.

## 2.3 The Boundaries of the Firm

The boundaries of the firm refer to the decision-making process about which activities a firm should produce internally ("make") and which it should outsource ("buy"). This delineation determines the scope of the firm's operations and what functions remain under its direct control. We can distinguish 3 types of different management of functions assignments:

- **Internal Monopolists:** An internal monopolist is a service or function that is provided by a dedicated unit or division within the firm. Although this unit may operate without external competition, offering potential advantages like keeping money within the company, it may also suffer from inefficiencies, lower quality, or sluggish service since it lacks the competitive pressures that drive improvements in price and performance.
- **External Monopolist:** An external monopolist is an outside provider that is the sole supplier of a particular good or service. Because there are no competitors in this external market, the firm has little choice but to accept the high prices and/or low quality of the service offered by that provider. Usually the least desirable scenario for a firm.
- **Preferences of a Competitive Market:** Managers generally prefer to buy goods and services from a competitive market where multiple suppliers vie for business. This competition typically drives prices down and quality up, ensuring that the firm receives better value. When competitive options are available, outsourcing becomes more attractive than relying on an internal monopolist or being forced to deal with an external monopolist.

## 2.4 Fixed and Variable Factors

- **Fixed Factor:** A fixed factor is an input that a firm must pay for regardless of its level of output; it cannot be adjusted in the short run. For example, a long-term lease on a building must be paid even if the firm produces nothing.
- **Variable Factor:** A variable factor is an input that the firm can adjust freely within the time period. The amount used can vary with the level of production. For instance, labor hours or raw materials can be increased or decreased as output changes.
- **Quasi-Fixed Factor:** A quasi-fixed factor is an input that remains fixed in quantity when the firm produces any positive output, but if the firm produces nothing, the cost of that input is avoided. An example is electricity for lighting in a factory; if the factory is not operating, no electricity is needed, but once production starts, a fixed amount of electricity is required regardless of how much is produced.

# 3 Short-Run Maximization and Isoprofit Lines

## 3.1 Short-Run Maximization

Let's consider the short-run profit-maximization problem when input 2 is fixed at some level  $\bar{x}_2$ . Let  $f(x_1, x_2)$  be the production function for the firm, let  $p$  be the price of output, and let  $w_1$  and  $w_2$  be the prices of the two inputs. Then the profit-maximization problem facing the firm can be written as:

$$\max_{x_1} \underbrace{pf(x_1, \bar{x}_2) - w_1x_1 - w_2\bar{x}_2}_{\pi(x_1)}.$$

Consider the firm's profit function in the short run, where input 2 is fixed at  $\bar{x}_2$ . The profit function is:

$$\pi(x_1) = p \cdot f(x_1, \bar{x}_2) - w_1x_1 - w_2\bar{x}_2.$$

Since  $\bar{x}_2$  and  $w_2\bar{x}_2$  are constant with respect to  $x_1$ , the firm's problem reduces to maximizing:

$$\pi(x_1) = p \cdot f(x_1, \bar{x}_2) - w_1x_1 \quad (\text{ignoring the constant term}).$$

To find the optimal  $x_1^*$ , we use the first-order condition for maximization. This requires taking the derivative of  $\pi(x_1)$  with respect to  $x_1$  and setting it equal to zero:

$$\frac{\partial \pi(x_1)}{\partial x_1} = p \cdot \frac{\partial f(x_1, \bar{x}_2)}{\partial x_1} - w_1 = 0.$$

The term  $\frac{\partial f(x_1, \bar{x}_2)}{\partial x_1}$  is the marginal product of input 1, denoted as  $MP_1(x_1, \bar{x}_2)$ . Thus, the equation becomes:

$$p \cdot MP_1(x_1, \bar{x}_2) - w_1 = 0.$$

Solving for  $x_1$  at the optimum, we have:

$$p \cdot MP_1(x_1^*, \bar{x}_2) = w_1.$$

This condition states that, at the profit-maximizing level of input 1, the value of the marginal product (the extra revenue generated by one more unit of input 1, given by  $p \cdot MP_1$ ) must equal the cost of that input ( $w_1$ ). This is a standard result in microeconomics: a firm maximizes profit by employing inputs up to the point where the cost of an additional unit is exactly offset by the additional revenue it generates.

### 3.1.1 Short Example

Imagine a car manufacturer that produces cars by hiring workers. Each worker has a cost (wage) and contributes additional output (cars) to the production process. Suppose each car sells for \$10,000 and each worker is paid \$500. The additional revenue generated by hiring one more worker is determined by how many extra cars that worker helps produce - **this is called the worker's marginal product**. Let's say the **production function** is  $f(L) = 2\sqrt{L}$  (with fixed capital), where  $L$  is the number of workers. The marginal product (MP) of labor is then

$$MP(L) = \frac{d}{dL} 2\sqrt{L} = \frac{1}{\sqrt{L}}.$$

The extra revenue from hiring one more worker is given by the sale price times the marginal product:

$$\text{Additional Revenue} = 10,000 \times \frac{1}{\sqrt{L}}.$$

What does it mean the **additional revenue**?

- When  $L = 1$ ,  $MP(1) = \frac{1}{\sqrt{1}} = 1$  which means that the additional revenue is  $10,000 \times 1 = \$10,000$ . This means that hiring one worker produces enough extra output to generate \$10,000 in revenue.
- When  $L = 2$ ,  $MP(2) = \frac{1}{\sqrt{2}} \approx 0.7071$  which means that the additional revenue from the third worker is  $10,000 \times 0.7071 \approx \$7,071$ . Each additional worker now contributes about \$7,071 in extra revenue.
- When  $L = 399$ ,  $MP(399) \approx \frac{1}{\sqrt{399}} \approx 0.05006$  which means that the additional revenue for the worker number 400 would be  $10,000 \times 0.05006 \approx \$500.63$ .
- When  $L = 400$ ,  $MP(400) = \frac{1}{\sqrt{400}} = \frac{1}{20} = 0.05$ . This means that the 401 worker hired, would produce an additional revenue of  $10,000 \times 0.05 = \$500$ , meaning that his own wage would be compensated by his work.

A profit-maximizing firm will hire workers until the additional revenue exactly equals the additional cost (\$500 per worker). So we set

$$10,000 \times \frac{1}{\sqrt{L}} = 500.$$

Solving for  $L$ :

$$\frac{10,000}{\sqrt{L}} = 500 \quad \Rightarrow \quad \sqrt{L} = \frac{10,000}{500} = 20 \quad \Rightarrow \quad L = 400.$$

Therefore:

- **Before 400 workers:** If the firm has fewer than 400 workers, the extra output generated by hiring one more worker would yield more than \$500 in additional revenue, so it makes sense to hire another worker.
- **At 400 workers:** The additional revenue from the last worker is exactly \$500, which equals the cost. This is the optimal hiring point because any additional worker would bring in less revenue than their cost, reducing overall profit.
- **Beyond 400 workers:** If the firm hires more than 400 workers, the extra worker's contribution (marginal product) drops further, meaning the revenue they bring in would be less than \$500, leading to a decrease in profit.

This example shows the standard microeconomic result: **a firm maximizes profit by employing inputs up to the point where the cost of an additional unit is exactly offset by the additional revenue it generates.**

### 3.2 Isoprofit Lines

The starting point is the firm's profit function, where we just replace  $f(x_1, \bar{x}_2)$  by  $y$ :

$$\pi = p y - w_1 x_1 - w_2 x_2,$$

where:

- $p$  is the output price,
- $y$  is the output,
- $x_1$  and  $x_2$  are two inputs (for instance, labor and capital),
- $w_1$  and  $w_2$  are the corresponding input prices.

To see which combinations of inputs and outputs yield a constant profit level  $\pi$ , we solve the profit equation for  $y$ :

$$y = \frac{\pi}{p} + \frac{w_1}{p}x_1 + \frac{w_2}{p}x_2.$$

This equation is called an **isoprofit line**. It represents all the combinations of  $x_1$ ,  $x_2$ , and  $y$  that yield the same profit  $\pi$ . Each different profit level gives a different **isoprofit line**, and its slope tells you how changes in the inputs must be compensated by changes in output to keep profit constant.

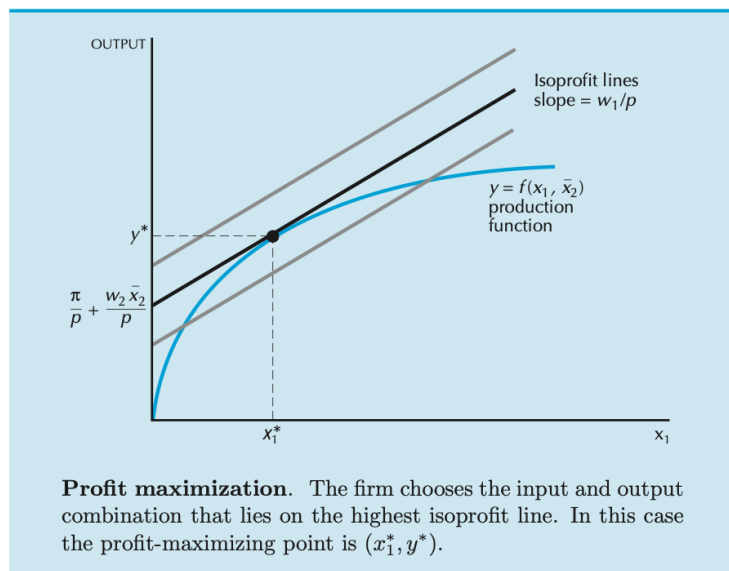


Figure 1: Isoprofit Line & Production Function

When a firm is maximizing profit, it wants to reach the highest possible **isoprofit line** that it can attain given its production technology. The production function (which tells you the maximum output that can be produced with a given combination of inputs) and the isoprofit lines interact. **The optimal decision is typically at the point where an isoprofit line is just tangent to the production function**, meaning that any movement along the production function would not yield higher profit. This point of tangency is given by

$$\underbrace{p \cdot MP_1(x_1, \bar{x}_2) = w_1}_{\text{Optimal choice when } \bar{x}_2 \text{ fixed}} \quad \& \quad \underbrace{p \cdot MP_2(\bar{x}_1, x_2) = w_2}_{\text{Optimal choice when } \bar{x}_1 \text{ fixed}}$$

### 3.2.1 Short Example 1

Imagine a car manufacturer produces cars using two inputs:

- Labor ( $x_1$ )—say, measured in worker-hours.
- Capital ( $x_2$ )—for instance, machine-hours or the use of assembly equipment.

Suppose that each car sells for  $p = \$20,000$ , the wage rate is  $w_1 = \$50$  per worker-hour, the cost of capital is  $w_2 = \$100$  per machine-hour and the firm currently earns a profit of  $\pi = \$200,000$ . The isoprofit line is then given by:

$$y = \frac{200,000}{20,000} + \frac{50}{20,000}x_1 + \frac{100}{20,000}x_2 = 10 + 0.0025x_1 + 0.005x_2.$$

This line tells the manufacturer that if they choose any combination of labor ( $x_1$ ) and capital ( $x_2$ ) such that the resulting output  $y$  satisfies this equation, they will achieve a profit of \$200,000. For instance, if the manufacturer decides to use no extra capital ( $x_2 = 0$ ) and hires 400 additional worker-hours, then:

$$y = 10 + 0.0025(400) = 10 + 1 = 11 \text{ cars.}$$

This means that with these inputs, the car manufacturer would produce 11 cars, and when sold at \$20,000 each, the revenue minus the input costs would yield the \$200,000 profit.

### 3.2.2 Short Example 2

Consider a concrete example using a **Cobb-Douglas production function** in the car sector. Suppose a firm produces cars with the production function

$$f(L, K) = 2\sqrt{LK},$$

where  $L$  is labor (e.g., worker-hours) and  $K$  is capital (e.g., machine-hours). The firm sells each car for a price  $p$ , pays a wage rate  $w_1$  for labor, and a rental rate  $w_2$  for capital. The profit function is

$$\pi = p \cdot f(L, K) - w_1L - w_2K.$$

In choosing  $L$  and  $K$  to maximize profit, one important step is to **find the point where an isoprofit line is tangent to the production function**. This tangency condition means that at the optimum the value of the marginal product of each input equals its price. In other words, we must have

$$p \cdot MP_L(L, K) = w_1 \quad \& \quad p \cdot MP_K(L, K) = w_2.$$

For the given production function:

- The marginal product of labor is

$$MP_L(L, K) = \frac{\partial f}{\partial L} = 2 \cdot \frac{1}{2} \cdot L^{-1/2} K^{1/2} = \frac{K^{1/2}}{L^{1/2}}.$$

- The marginal product of capital is

$$MP_K(L, K) = \frac{\partial f}{\partial K} = 2 \cdot \frac{1}{2} \cdot L^{1/2} K^{-1/2} = \frac{L^{1/2}}{K^{1/2}}.$$

Let's assume the following parameters:

- Output price:  $p = 20$ ,
- Wage rate:  $w_1 = 10$ ,
- Rental rate of capital:  $w_2 = 40$ .

Then the tangency conditions become:

- **For labor:**

$$20 \cdot \frac{K^{1/2}}{L^{1/2}} = 10 \iff \frac{K^{1/2}}{L^{1/2}} = 0.5 \iff \frac{K}{L} = 0.25 \iff K = 0.25L.$$

- **For capital:**

$$20 \cdot \frac{L^{1/2}}{K^{1/2}} = 40 \iff \frac{L^{1/2}}{K^{1/2}} = 2 \iff \frac{L}{K} = 4 \iff K = \frac{L}{4}.$$

Notice that both conditions yield the same relationship:  $K = 0.25L$  (or equivalently,  $L/K = 4$ ). This means that, at the optimum, for every 4 units of labor, the firm uses 1 unit of capital.

### 3.3 Comparative Statistics

We can use the geometry depicted in Figure ?? to analyze how a firm's choice of inputs and outputs varies as the prices of inputs and outputs vary. This gives us one way to analyze the comparative statics of firm behavior. Let's recall that the **isoprofit line** is described as:

$$y = \frac{\pi}{p} + \frac{w_1}{p}x_1 + \frac{w_2}{p}x_2 \quad \text{and the slope is } \frac{w_1}{p}.$$

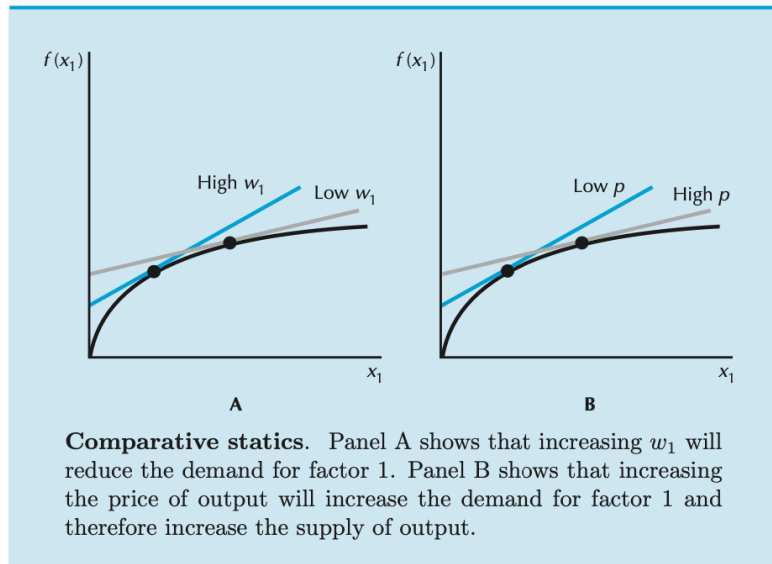


Figure 2: Comparative Statistics: Input Price & Output Price

- **Change in the Price of Factor 1 ( $w_1$ ):**
  - Increasing  $w_1$  makes the isoprofit line steeper.
  - A steeper isoprofit line forces the tangency point (and hence the optimal mix) to shift left, meaning the firm employs less of factor 1.
  - This illustrates that the demand for factor 1 slopes downward: as its price increases, the optimal quantity used decreases.

- **Change in the Output Price ( $p$ )**

- A decrease in the output price makes the isoprofit line steeper.
- This steeper line similarly shifts the tangency point to the left, leading to a reduction in the optimal use of factor 1.
- With factor 2 fixed in the short run, a reduction in factor 1 implies lower output.
- Thus, a decrease in the output price ultimately reduces the supply of output, indicating that the supply function is upward-sloping.

- **Change in the Price of Factor 2 ( $w_2$ )**

- In the short run, factor 2 is fixed; therefore, changes in  $w_2$  do not affect the slope of the isoprofit line.
- Consequently, the optimal level of factor 1 remains unchanged, and there is no impact on output supply in the short run.

## 4 Long-Run Maximization

In the long run the firm is **free to choose the level of all inputs**. Thus the long-run profit-maximization problem can be posed as

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1x_1 - w_2x_2.$$

This is basically the same as the short-run problem described above, but now both factors are free to vary. The condition describing the optimal choices is essentially the same as before, but **now we have to apply it to each factor**.

$$p \cdot MP_1(x_1^*, x_2^*) = w_1 \quad \& \quad p \cdot MP_2(x_1^*, x_2^*) = w_2 \quad \equiv \text{Factor Demand Curves}$$

### 4.1 Profit Maximization and Constant Returns to Scale

Suppose a firm chooses an optimal input mix  $(x_1^*, x_2^*)$  to produce output  $y^* = f(x_1^*, x_2^*)$  and earns an optimal profit:

$$\pi^* = py^* - w_1x_1^* - w_2x_2^*.$$

Under constant returns to scale, if the firm doubles all inputs, it doubles its output. That is, if it uses  $2x_1^*$  and  $2x_2^*$ , then the output becomes  $2y^*$  because  $f(2x_1^*, 2x_2^*) = 2f(x_1^*, x_2^*)$ . If the firm were earning positive profits with  $\pi^*$ , doubling its inputs would double profits to  $2\pi^*$ . **However, this contradicts the profit maximization hypothesis**. The fact that doubling the inputs leads to double the profits means that the **original input choice wasn't truly optimal** because the firm could have increased its profits simply by expanding.

**In a competitive market**, firms are price takers, and if one firm could scale up profitably, then other firms could do the same. **This competitive pressure forces profit margins down**. In **long-run equilibrium**, if one firm makes positive profits, new entrants (or the firm itself) will expand production until profits are driven down to zero. **The only sustainable long-run outcome for a competitive firm with constant returns to scale is zero economic profits**. This is because any positive profit would incentivize expansion, which in turn increases supply, drives down the market price, and eliminates the extra profit.

- **Does this mean that any firm is unequivocally lead to zero profits?**

Not necessarily. In a **perfectly competitive market** with constant returns to scale, the long-run equilibrium condition forces firms to earn zero economic profit—that is, they earn just enough to cover all their opportunity costs. However, this "zero profit" result is a theoretical outcome under specific assumptions (perfect competition, free entry and exit, constant returns to scale). In the real world, firms may earn positive **accounting profits** due to factors like market power, barriers to entry, innovation, or temporary imbalances. Zero economic profit simply means that no firm earns an above-normal return once all costs (including opportunity costs) are accounted for.



- What is the difference between accounting profits and economic profits?

- **Accounting Profit:** This is the firm's total revenue minus its explicit, out-of-pocket expenses (like wages, rent, materials, etc.). It's the profit figure typically reported in financial statements.
- **Economic Profit:** This takes accounting profit a step further by also subtracting the opportunity costs (the benefits the firm foregoes by choosing one option over another). Zero economic profit means the firm is just covering all its explicit costs plus its opportunity costs - earning a "normal" return—so there's no extra profit above what it could earn elsewhere. **The opportunity cost is often estimated by comparing the return you could have earned by investing in a well-known alternative, such as the S&P 500, gold, or another investment.** Essentially, it's the rate of return you forego by using your capital in the current project instead of the next best option available

In short, a firm can show positive accounting profits while having zero economic profit if its total returns only match its opportunity costs.

## 5 Revealed Preferences

### 5.1 WAPM

Suppose the following situations:

- **At Time t:**

- The firm chooses plan  $(y^t, x_1^t, x_2^t)$  when facing prices  $(p^t, w_1^t, w_2^t)$ .
- If the firm were to use the plan chosen at time s instead, its profit under time t prices would be  $p^t y^s - w_1^t x_1^s - w_2^t x_2^s$ .
- Since the firm is profit maximizing at time t, the profit from its chosen plan must be at least as high as the profit from any alternative plan, including the s plan.
- This gives the first inequality:

$$p^t y^t - w_1^t x_1^t - w_2^t x_2^t \geq p^t y^s - w_1^t x_1^s - w_2^t x_2^s.$$

- **At Time s:**

- Similarly, at time s the firm chooses plan  $(y^s, x_1^s, x_2^s)$  with prices  $(p^s, w_1^s, w_2^s)$ .
- If the firm were to use the plan chosen at time t, its profit under time s prices would be  $p^s y^t - w_1^s x_1^t - w_2^s x_2^t$ .
- The profit-maximizing choice at time s must yield at least as much profit as the alternative, yielding the second inequality:

$$p^s y^s - w_1^s x_1^s - w_2^s x_2^s \geq p^s y^t - w_1^s x_1^t - w_2^s x_2^t.$$

In short, **the first inequality** ensures that the plan chosen at time t is optimal under t's prices (it beats the s plan when evaluated at t's prices). **The second inequality** ensures that the plan chosen at time s is optimal under s's prices (it beats the t plan when evaluated at s's prices). **Both must be true** because a profit-maximizing firm will never choose a plan that yields lower profit than some alternative, given the prevailing prices in each period. These 2 inequalities are called **Weak Axiom of Profit Maximization (WAPM)**.

Let's delve into both equations. Let's **transpose the 1st inequality** and **add it to the 2nd inequality**:

$$\begin{aligned}
 & p^s y^s - w_1^s x_1^s - w_2^s x_2^s \geq p^s y^t - w_1^s x_1^t - w_2^s x_2^t \\
 \iff & -p^s y^t + w_1^s x_1^t + w_2^s x_2^t \geq -p^s y^s + w_1^s x_1^s + w_2^s x_2^s \quad (\text{Transpose 2nd}) \\
 \iff & (p^t - p^s) y^t - (w_1^t - w_1^s) x_1^t - (w_2^t - w_2^s) x_2^t \geq (p^t - p^s) y^s - (w_1^t - w_1^s) x_1^s + (w_2^t - w_2^s) x_2^s \quad (\text{Add 1st}) \\
 \iff & \underbrace{(p^t - p^s)}_{\Delta p} \underbrace{(y^t - y^s)}_{\Delta y} - \underbrace{(w_1^t - w_1^s)}_{\Delta w_1} \underbrace{(x_1^t - x_1^s)}_{\Delta x_1} - \underbrace{(w_2^t - w_2^s)}_{\Delta w_2} \underbrace{(x_2^t - x_2^s)}_{\Delta x_2} \geq 0
 \end{aligned}$$

The inequality

$$\Delta p \Delta y - \Delta w_1 \Delta x_1 - \Delta w_2 \Delta x_2 \geq 0$$

is derived from the profit maximization condition and states that the net change in revenue (from the output price and output quantity changes) must at least offset the net change in costs (from changes in input prices and input quantities). Let's compare some statistics:

- If only the output price changes (with  $\Delta w_1 = \Delta w_2 = 0$ ), the inequality reduces to  $\Delta p \Delta y \geq 0$ . This means that when the output price increases ( $\Delta p > 0$ ), the output level must not decrease ( $\Delta y \geq 0$ ), ensuring that the supply curve is upward sloping.
- If only the price of an input changes (say, factor 1 with  $\Delta w_1 > 0$  while  $\Delta p$  and  $\Delta w_2$  remain constant), then the inequality implies  $-\Delta w_1 \Delta x_1 \geq 0$ , or  $\Delta x_1 \leq 0$ . Thus, an increase in the price of an input leads to a decrease in its usage.

## 5.2 Farmer and Subsidies Example

The U.S. government currently spends between \$40 and \$60 billion a year in aid to farmers. A large fraction of this amount is used to subsidize the production of various products including milk, wheat, corn, soybeans, and cotton. If the government stops giving subsidies to farmers, the price farmers receive for products like milk, wheat, and corn will drop. Some farmers argue they could simply produce more - say, by expanding their herds - to make up for the lower price and maintain their income. However farmers are wrong, profit maximization theory tells us that if the selling price falls, a profit-maximizing firm will actually reduce its output, not increase it. In other words, large agribusiness farms, which typically focus on maximizing profits, would likely produce less when prices drop, rather than more. So, the elimination of subsidies would lead to a lower supply, contrary to the idea that farmers could offset lower prices by producing more.

For instance, imagine both situations:

- **With Subsidy:** Imagine a dairy farmer who, due to government subsidies, receives \$1.20 per gallon of milk (even though the market price might be \$1.00 per gallon). At this higher effective price, the farmer produces, say, 100 gallons of milk, as the higher revenue makes it profitable to produce that much.
- **Removal of the Subsidy:** If the subsidy is removed, the farmer now only receives the market price of \$1.00 per gallon. For a profit-maximizing farmer, this lower price means the additional revenue from each extra gallon of milk is reduced. Consequently, the farmer will adjust by reducing production—perhaps down to 80 gallons- because producing more milk is no longer as profitable given the lower price.

The removal of the subsidy lowers the effective price received by the farmer, which in turn reduces the incentive to produce as much milk, leading to a decrease in supply.

### Libertarian View:

From a libertarian perspective, government subsidies are generally seen as harmful to the market. They distort prices, creating artificial incentives that prevent resources from being allocated efficiently. Subsidies give certain companies an unfair competitive advantage, which can hurt foreign competitors who don't receive similar support. Moreover, by interfering with natural market forces, subsidies can stifle innovation, as firms may rely on government support rather than competing to improve and reduce costs. In essence, libertarians argue that no company should be subsidized because such interventions undermine the principles of free, competitive markets.