Budget Constraints

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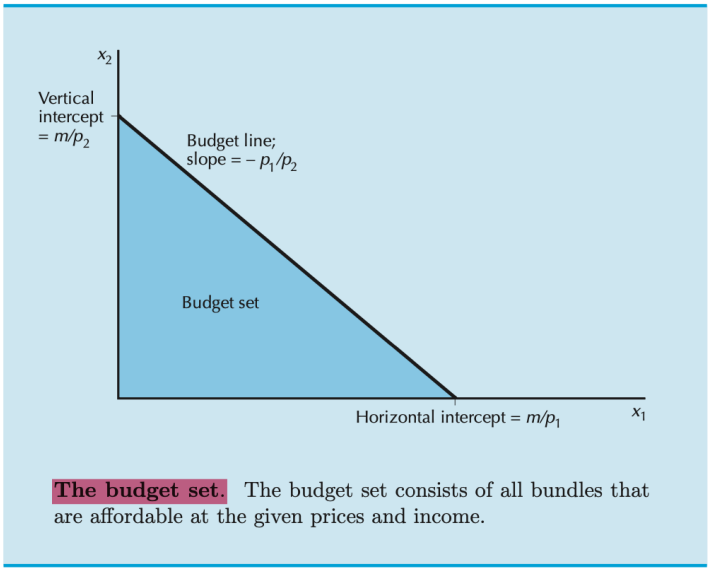
1 The Budget Constraint

1.1 Two-Good Model

The budget constraint considers two goods, x_1 and x_2, with prices p_1 and p_2 respectively.

- **Consumption Bundle:** A pair (x_1, x_2) showing quantities of each good consumed. The cost of a bundle is $p_1x_1 + p_2x_2$.
- **Total Expenditure:** $TE(x_1, x_2) = p_1x_1 + p_2x_2$.
- **Budget Constraint Formula:** $p_1x_1 + p_2x_2 \leq m$, where m is the consumer's income.
- **Budget Set:** The set of all consumption bundles (x_1, x_2) that satisfy $p_1x_1 + p_2x_2 \leq m$.
- **Budget Line:** The line $p_1x_1 + p_2x_2 = m \longleftrightarrow x_2 = \frac{m}{p_2} - \frac{p_1}{p_2}x_1$, representing all bundles where the consumer spends their entire income.



The budget set. The budget set consists of all bundles that are affordable at the given prices and income.

Figure 1: Budget Line

- **Rate of Substitution:** The slope of the budget line, $-\frac{p_1}{p_2}$, which measures the trade-off rate between x_1 and x_2 is defined as the rate of substitution $\frac{\Delta x_2}{\Delta x_1} = -\frac{p_1}{p_2}$.
- **Opportunity Cost:** The cost of foregone alternatives, quantified by the slope of the budget line. Economists sometimes say that the slope of the budget line measures the opportunity cost of consuming good 1. In order to consume more of good 1 you have to give up some consumption of good 2. Giving up the opportunity to consume good 2 is the true economic cost of more good 1 consumption; and that cost is measured by the slope of the budget line.

2 Budget Line Variations

2.1 Income Changes

- **Increase in m:** Parallel outward shift of the budget line.
- **Decrease in m:** Parallel inward shift of the budget line.

2.2 Price Changes

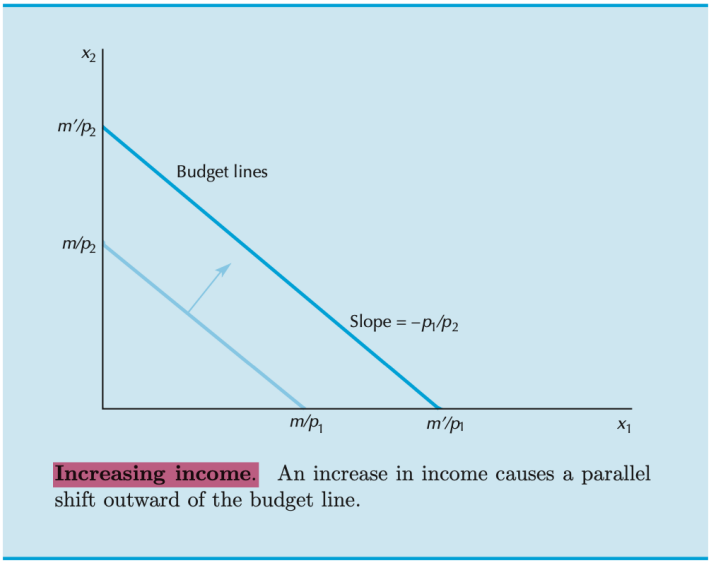
- **Increase in p_1:** Rotates the budget line inward (steeper slope), holding p_2 constant.
- **Increase in p_2:** Rotates the budget line inward (flatter slope), holding p_1 constant.

2.3 Simultaneous Increase in p_1 and p_2

Changes the slope depending on relative price changes.

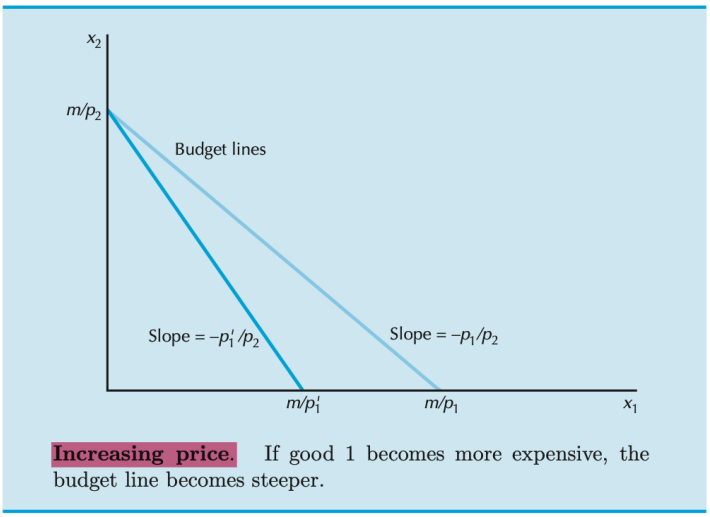
2.4 Increase in p_1, p_2, and m proportionally

The budget line remains unchanged.



Increasing income. An increase in income causes a parallel shift outward of the budget line.

Figure 2: Increase Income



Increasing price. If good 1 becomes more expensive, the budget line becomes steeper.

Figure 3: Increase Prices

3 Taxes, Subsidies, and Rationing

3.1 Taxes

- **Quantity Tax:** Adds t per unit, changing p_1 to $p_1 + t$, steepening the budget line.
- **Value Tax:** Adds a proportionate tax τ, making the price $p_1(1+\tau)$, which also makes it steeper.

3.2 Subsidies

- **Quantity Subsidy:** Reduces p_1 to $p_1 - s$, flattening the budget line.
- **Value Subsidy:** Reduces p_1 to $p_1(1-\sigma)$, flattening the budget line.

3.3 Lump-Sum Tax/Subsidy

In the case of a tax, this means that the government takes away some fixed amount of money, regardless of the individual's behavior.

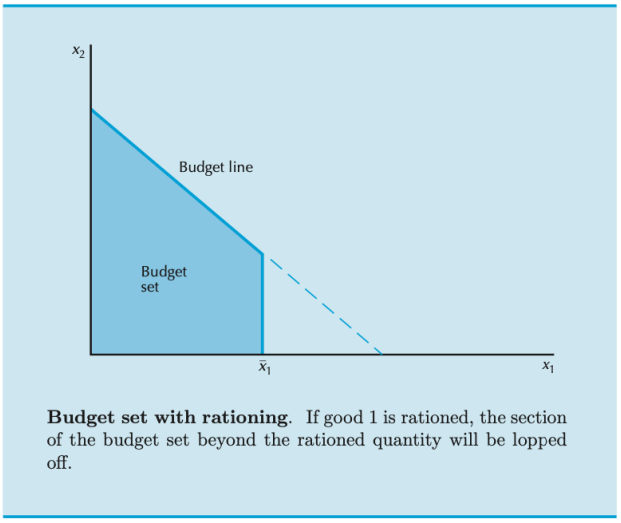
- **Tax:** Reduces m, shifting the budget line inward.
- **Subsidy:** Increases m, shifting the budget line outward.

3.4 Rationing

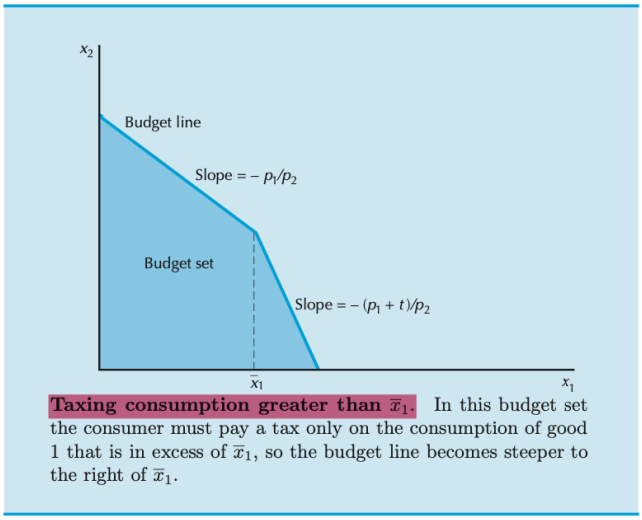
Imposes a maximum on x_1, truncating the budget set.

3.5 Combinations

Sometimes taxes, subsidies, and rationing are combined. For example, we could consider a situation where a consumer could consume good 1 at a price of p_1 up to some level x_1, and then had to pay a tax t on all consumption in excess of x_1. Here the budget line has a slope of $\frac{-p_1}{p_2}$ to the left of x_1, and a slope of $\frac{-(p_1+t)}{p_2}$ to the right of x_1.



Budget set with rationing. If good 1 is rationed, the section of the budget set beyond the rationed quantity will be lopped off.



Taxing consumption greater than $\bar{x}_1$. In this budget set the consumer must pay a tax only on the consumption of good 1 that is in excess of $\bar{x}_1$, so the budget line becomes steeper to the right of $\bar{x}_1$.

Figure 4: Rationing & Combination

4 The Food Stamp Program

4.1 Pre-1979 Food Stamp Program

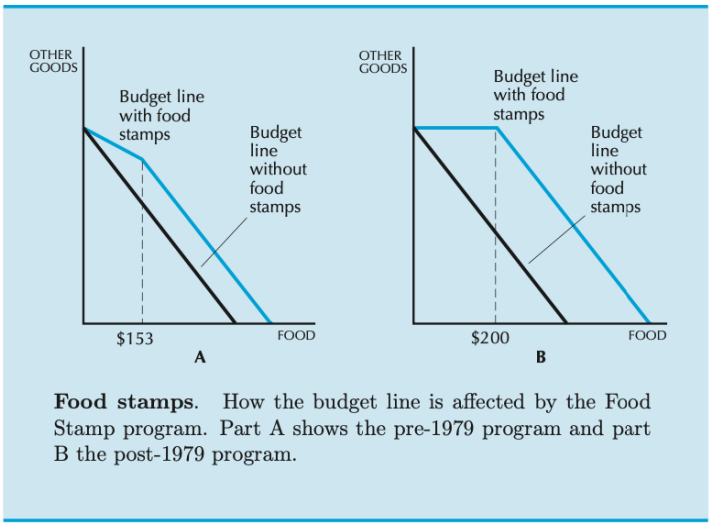
The Food Stamp program before 1979 functioned as an ad valorem subsidy on food purchases. The key features and economic effects of this system are as follows:

4.1.1 Key Features

- **Eligibility and Allocation:** Households meeting specific income thresholds could purchase food stamps, which were redeemable for food at retail outlets. **Example:** A family of four could receive up to \$153 in food coupons per month.
- **Cost of Food Stamps Based on Income:** The cost of food stamps to the household depended on its income:
  - A family with an income of \$300 paid \$83 for the \$153 food stamp allotment.
  - A family with an income of \$100 paid \$25 for the \$153 allotment.
- **Subsidy Rates:** Subsidy rates varied with income:
  - The family paying \$83 received \$1.84 ($= \frac{153}{83}$) in food value for every dollar spent on food stamps.
  - The family paying \$25 received \$6.12 ($= \frac{153}{25}$) in food value for every dollar spent on food stamps.

4.1.2 Economic Implications

- **Budget Line Analysis:** The household's consumption choices are depicted with expenditures on food on the horizontal axis and expenditures on all other goods on the vertical axis. The price of each good is normalized to 1, making the initial slope of the budget line −1 (each dollar spent on food reduces the amount available for other goods by 1).



Food stamps. How the budget line is affected by the Food Stamp program. Part A shows the pre-1979 program and part B the post-1979 program.

Figure 5: Food Stamp Program Pre-1979 vs Post-1979

- **Effect of the Subsidy:** Households could purchase food at a subsidized rate up to the maximum allotment:
  - A household that paid \$25 for \$153 in food stamps effectively experienced a slope of approximately −0.16 (= 25/153).
  - For each dollar spent on food, the household sacrificed only \$0.16 worth of other goods.
- **Budget Line Segmentation:**
  1. **Up to \$153 in food spending:** Subsidized Slope: Budget line slope = −0.16.
  2. **Beyond \$153 in food spending:** Normal Slope: Budget line returns to −1 as additional food purchases are made without subsidy.
  3. **Kink in Budget Line:** The transition from the subsidized slope to the normal slope introduces a "kink," reflecting the maximum amount of subsidized food.
- **Impact of Household Income:** Higher-income households paid more for food stamps, receiving a smaller effective subsidy. This made the slope of the budget line steeper as income increased.

4.2 Post-1979 Food Stamp Program

The 1979 reform fundamentally altered the program by directly granting food stamps to eligible households without requiring purchase. This created a lump-sum subsidy effect.

- **Direct Grants:** Eligible households received a monthly allotment of food stamps (e.g., 200) without any cost.
- **No Resale:** Food stamps could not be legally sold, ensuring the subsidy was used exclusively for food purchases.

4.2.1 Economic Implications

1. **Budget Line Analysis:** The lump-sum grant shifts the budget line outward by the value of the food stamps (e.g., \$200), increasing the household's purchasing power. The slope remains -1, as each dollar spent on food or other goods reduces the amount available for the alternative by 1.

2. **Budget Set Expansion:** The new budget set allows households to consume more food without sacrificing spending on other goods, within the constraints of the food stamp grant.

3. **Differences from Pre-1979 Program:** The kink in the budget line is removed, as the lump-sum grant applies uniformly to all eligible households regardless of their food expenditure. No effective price reduction per unit of food occurs; instead, the grant increases the household's effective income allocated to food.

4.3 Mathematical Summary

Pre-1979 Budget Line

$$\text{For food expenditure } F \leq 153: \quad m - G = 1 \cdot F + (25/153) \cdot F$$

$$\text{For } F > 153: \quad m - G = 1 \cdot F$$

Post-1979 Budget Line

$$\text{New Budget Line (with food stamps value } S): \quad m + S = F + G$$

Where F is food expenditure, G is expenditure on other goods, and S is the food stamp grant (e.g., 200). The budget line is shifted outward by S, with no change in slope (-1).