Market Demand

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1 Aggregate/Market Demand Function in Continuous Scenario

The aggregate (or market) demand for good 1 is obtained by summing up the individual demand functions of all consumers. Specifically, if $x_i^1(p_1, p_2, m_i)$ is consumer i's demand for good 1 (depending on prices p_1 and p_2 and individual income m_i, generally m and p_2 are fixed), then the aggregate demand is given by:

$$X^1(p_1, p_2, m_1, \ldots, m_n) = \sum_{i=1}^{n} x_i^1(p_1, p_2, m_i)$$

This aggregate demand function generally depends on both the prices of the goods and the distribution of incomes across consumers. Under the representative consumer assumption, where we summarize the individual incomes by their total $M = \sum_{i=1}^{n} m_i$, the aggregate demand simplifies to a function of prices and total income:

$$X^1(p_1, p_2, M)$$

1.1 Adding Up "Linear" Demand Curves

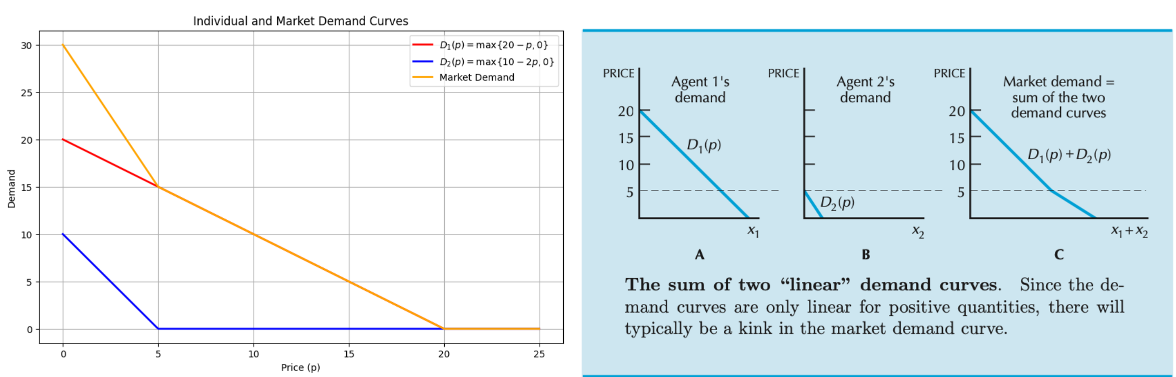
Suppose that one individual's demand curve is $D_1(p) = 20 - p$ and another individual's is $D_2(p) = 10 - 2p$. What is the market demand function? We have to be a little careful here about what we mean by "linear" demand functions since a negative amount of a good usually has no meaning, we really mean that the individual demand functions have the form:

$$D_1(p) = max\{20 - p, 0\} \qquad\qquad D_2(p) = max\{10 - 2p, 0\}$$

When aggregating individual demand curves, we must ensure that the demand is non-negative. The market (aggregate) demand function is the sum of the individual demands:

$$D_{\text{market}}(p) = D_1(p) + D_2(p) = \max\{20 - p,\, 0\} + \max\{10 - 2p,\, 0\}.$$

To understand how this sum behaves, we break the analysis into different ranges of p:



The sum of two "linear" demand curves. Since the demand curves are only linear for positive quantities, there will typically be a kink in the market demand curve.

Figure 1: Continuous Market Demand Example

- **For $p < 5$:** We have that $20 - p > 0$ (since $p < 20$) and $10 - 2p > 0$ (since $p < 5$). So, $D_1(p) = 20 - p$ and $D_2(p) = 10 - 2p$. The aggregate Market demand for this portion is:

$$D_{\text{market}}(p) = (20 - p) + (10 - 2p) = 30 - 3p.$$

- **For $5 \le p < 20$:** In this case $20 - p > 0$ still, so $D_1(p) = 20 - p$, and $10 - 2p \le 0$ (since $p \ge 5$), so $D_2(p) = 0$. Therefore the Market demand:

$$D_{\text{market}}(p) = (20 - p) + 0 = 20 - p.$$

- **For $p \ge 20$:** Finally, both $20 - p \le 0$ and $10 - 2p \le 0$, so both demands are zero and the Market demand is:

$$D_{\text{market}}(p) = 0.$$

Thus, the market demand function is defined as follows and it's plot depicted in Figure 1:

$$D_{\text{market}}(p) = \begin{cases} 30 - 3p, & \text{if } p < 5, \\ 20 - p, & \text{if } 5 \le p < 20, \\ 0, & \text{if } p \ge 20. \end{cases}$$

2 Aggregate/Market Demand Function in Discreet Scenario

When a good is available only in discrete units (zero or one), each consumer's demand is determined solely by their reservation price — the highest price at which they are willing to buy one unit. The market demand in this case is the sum of the individual demand curves represented by these reservation prices. As the market price decreases, more consumers are willing to buy the good, ensuring that the market demand curve slopes downward, as shown in Figure 2

2.1 Adding Up Discreet Demand Curves

Assume there are two consumers with the following reservation prices:

- **Consumer A:** Reservation price = \$15 (Buys the good if the price is at most \$15.)
- **Consumer B:** Reservation price = \$10 (Buys the good if the price is at most \$10.)

Each consumer's demand function is defined as:

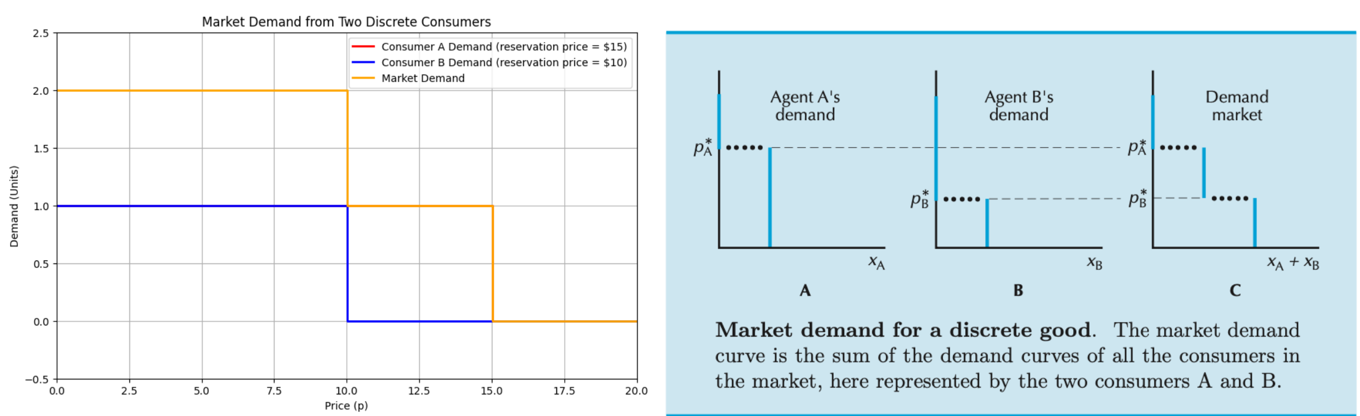
$$D_A(p) = \begin{cases} 1, & \text{if } p \le 15, \\ 0, & \text{if } p > 15, \end{cases} \qquad D_B(p) = \begin{cases} 1, & \text{if } p \le 10, \\ 0, & \text{if } p > 10. \end{cases}$$

The market demand is the sum of individual demands:

$$D_{\text{market}}(p) = D_A(p) + D_B(p).$$

Thus, the market demand behaves as follows:

- For $p \le 10$: Both consumers are willing to buy, so $D_{\text{market}}(p) = 1 + 1 = 2$.
- For $10 < p \le 15$: Only Consumer A is willing to buy, so $D_{\text{market}}(p) = 1$.
- For $p > 15$: Neither consumer is willing to buy, so $D_{\text{market}}(p) = 0$.



Market demand for a discrete good. The market demand curve is the sum of the demand curves of all the consumers in the market, here represented by the two consumers A and B.

Figure 2: Discreet Market Demand Example

3 Elasticity

3.1 Demand Responsiveness

While the slope of a demand function ($\Delta q/\Delta p$) might seem like a natural measure of responsiveness (how demand change changes with respect to income or price) , it is problematic because it depends on the units of measurement. To overcome this, economists use a unit-free measure called elasticity. The price elasticity of demand (ε) is defined as the percent change in quantity demanded divided by the percent change in price:

$$\varepsilon = \frac{\Delta q/q}{\Delta p/p} = \frac{p}{q} \cdot \frac{\partial q}{\partial p}.$$

This unit-free measure allows for consistent comparisons across different contexts regardless of the units used for price or quantity. The sign of the elasticity of demand is generally negative, since demand curves invariably have a negative slope. It is common in verbal discussion to refer to elasticities of 2 or 3, rather than −2 or −3. We will try to keep the signs straight in the text by referring to the absolute value of elasticity, but you should be aware that verbal treatments tend to drop the minus sign.

3.2 Example: Computing Elasticity

Let's consider a linear demand curve given by $q = a - bp$, with a constant slope of $-b$. Using the price elasticity of demand formula

$$\varepsilon = \frac{p}{q} \cdot \frac{\partial q}{\partial p} = -\frac{bp}{a - bp}.$$

Key observations from this analysis include:

- **At $p = 0$:** The elasticity is 0, indicating that demand is perfectly inelastic when the price is zero. This means that a change in price leads to a relatively small change in the quantity demanded
- **At $q = 0$ (i.e., $p = \frac{a}{b}$):** The elasticity approaches $-\infty$, showing that demand becomes extremely elastic as quantity demanded falls to zero.
- **Unit Elasticity:** Setting elasticity to 1 (in absolute value) means that the percentage change in quantity demanded exactly equals the percentage change in price - this is called unit elastic demand. At this point, total revenue is maximized and remains unchanged for small changes in price, marking the transition between elastic and inelastic demand. To find the price where the elasticity is −1 (unit elastic demand), we set and solve por p:

  $$\frac{bp}{a - bp} = 1 \quad \longleftrightarrow \quad bp = a - bp \quad \longleftrightarrow \quad 2bp = a \quad \longleftrightarrow \quad p = \frac{a}{2b}.$$

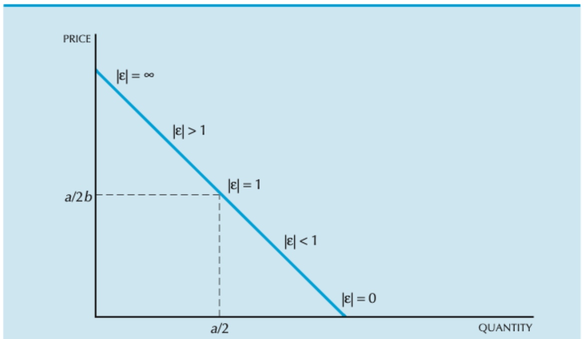
  Thus, the elasticity of demand equals −1 when $p = \frac{a}{2b}$.

3.3 Elasticity Properties

Let's define properly what we obtained in the previous example:

- Elastic Demand: When the absolute value of elasticity is greater than 1, a 1% increase in price leads to more than a 1% decrease in quantity demanded.
- Inelastic Demand: When the absolute value of elasticity is less than 1, a 1% increase in price results in less than a 1% decrease in quantity demanded.
- Unit Elastic Demand: When elasticity is exactly –1, the percentage change in quantity demanded equals the percentage change in price

Take as example the perfect substitutes of red pencil and blue pencil. The passage highlights that a good's demand elasticity largely depends on the availability of substitutes. When perfect substitutes exist, like red versus blue pencils - a small increase in price for one leads consumers to switch entirely to the other, resulting in highly elastic demand. Conversely, if there are few substitutes, the demand tends to be inelastic, as consumers have fewer alternatives despite price changes.



The elasticity of a linear demand curve. Elasticity is infinite at the vertical intercept, one halfway down the curve, and zero at the horizontal intercept.

Figure 3: Price of Elasticity

3.4 Revenue and Elasticity

Revenue (R) is defined as the product of the price p of a good and the quantity sold q: $R = p \cdot q$.

How Revenue Is Influenced by Elasticity:

- **Change in Revenue:** When the price changes by Δp and the quantity changes by Δq, the new revenue becomes:

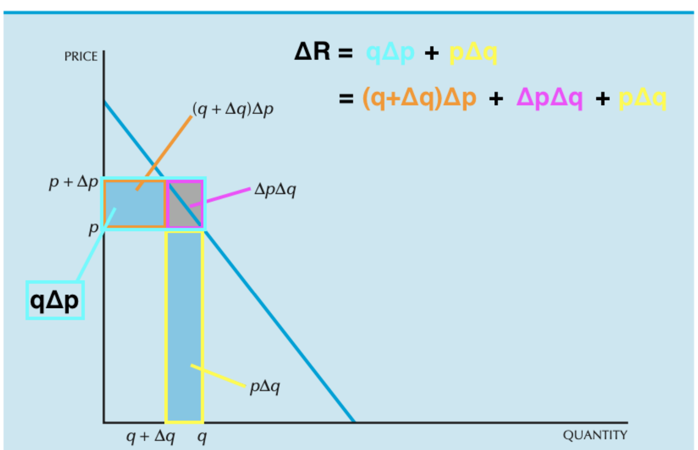
$$R' = (p + \Delta p)(q + \Delta q) = pq + q\,\Delta p + p\,\Delta q + \Delta p\,\Delta q.$$

For small changes, the term $\Delta p\,\Delta q$ can be neglected, yielding:

$$\Delta R \approx q\,\Delta p + p\,\Delta q.$$

Dividing by Δp gives the rate of change of revenue with respect to price:

$$\frac{\Delta R}{\Delta p} \approx q + p\,\frac{\Delta q}{\Delta p}.$$



How revenue changes when price changes. The change in revenue is the sum of the box on the top minus the box on the side.

Figure 4: Revenue and Price Change

- **Relating to Elasticity:** The price elasticity of demand, $\varepsilon(p)$, is defined as:

$$\varepsilon(p) = \frac{p}{q}\frac{\Delta q}{\Delta p}.$$

Rearranging the revenue change rate equation by dividing the second term by q and substituting the elasticity, we have:

$$\frac{\Delta R}{\Delta p} \approx q\left(1 + \frac{p}{q}\frac{\Delta q}{\Delta p}\right) = q\left(1 + \varepsilon(p)\right).$$

- **Revenue Increase/Decrease:** The sign of $\Delta R/\Delta p$ determines whether revenue increases or decreases with a price increase:

  - **Revenue Increases:** If $1 + \varepsilon(p) > 0$. Since $\varepsilon(p)$ is negative (due to the inverse relation between price and quantity), this condition becomes:

  $$\varepsilon(p) > -1 \quad \text{or equivalently} \quad |\varepsilon(p)| < 1.$$

  This situation corresponds to inelastic demand, meaning that the percentage drop in quantity is less than the percentage increase in price, leading to higher revenue.

  - **Revenue Decreases:** If $1 + \varepsilon(p) < 0$, which means:

  $$\varepsilon(p) < -1 \quad \text{or equivalently} \quad |\varepsilon(p)| > 1.$$

  This is elastic demand, where the quantity demanded falls more than proportionately compared to the price increase, resulting in lower revenue.

Important

- **Revenue increases** when price increases if demand is inelastic ($|\varepsilon(p)| < 1$).
- **Revenue decreases** when price increases if demand is elastic ($|\varepsilon(p)| > 1$).

From a **seller's perspective, having an inelastic demand is generally preferable** because of mainly 3 reasons:

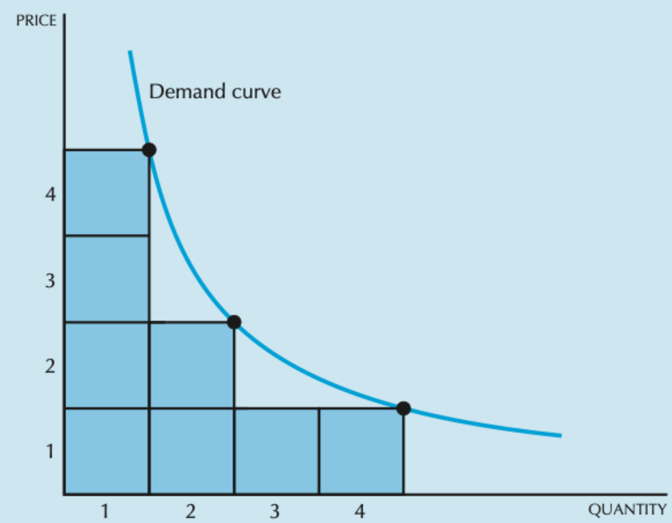
1. **Revenue Stability:** With inelastic demand, consumers don't reduce their quantity demanded significantly when prices increase. This means that if you raise your price, the loss in sales volume is proportionally smaller than the gain from the higher price, potentially increasing overall revenue.
2. **Pricing Power:** Inelastic demand gives firms more pricing power because consumers are less sensitive to price changes. This allows businesses to raise prices without fearing a steep drop in sales.
3. **Profit Margins:** Higher prices with only minor reductions in quantity sold typically lead to improved profit margins.

In contrast, if demand is elastic, small increases in price can lead to significant decreases in quantity demanded, potentially reducing revenue.

3.5 Unit Elastic Demand

We know that if the elasticity is 1 (in fact is -1) at price p, then the revenue will not change when the price changes by a small amount. A constant elasticity demand or unit elastic demand curve is useful because:

- **Uniform Responsiveness:** The percentage change in quantity demanded is the same for any percentage change in price, making analysis simpler and predictions more consistent.
- **Revenue Analysis:** For example, when elasticity is -1 everywhere, revenue (price × quantity) remains constant regardless of small price changes, which is useful for understanding revenue stability.
- **Analytical Simplicity:** The demand function $q = Ap^{\varepsilon}$ (or equivalently, $\ln q = \ln A + \varepsilon \ln p$) makes it easier to work with and estimate in empirical studies, due to its simple, log-linear form.



Unit elastic demand. For this demand curve price times quantity is constant at every point. Thus the demand curve has a constant elasticity of -1.

Figure 5: Unit Elastic Demand

For instance, consider a constant elasticity demand curve with elasticity $\varepsilon = -1$. A common form is:

$$q = A \cdot p^{\varepsilon} = \frac{A}{p},$$

where A is a constant. Let's choose $A = 100$. Then the demand function is:

$$q = \frac{100}{p}.$$

Let's try different price values and see how the market demand woulld be affected:

1. If $p = 5$, then $q = \frac{100}{5} = 20 \implies$ **Revenue:** $R = p \cdot q = 5 \times 20 = 100$.
2. If $p = 10$, then $q = \frac{100}{10} = 10 \implies$ **Revenue:** $R = 10 \times 10 = 100$.
3. If $p = 20$, then $q = \frac{100}{20} = 5 \implies$ **Revenue:** $R = 20 \times 5 = 100$.

This example shows that with a constant elasticity of -1, revenue remains constant regardless of the price, as long as the small changes hold. This stability in revenue can be very useful for pricing strategies, since any price change (with the corresponding change in quantity demanded) will not alter total revenue. Examples of strategies that could be applied considering that elasticity remains constant for a given good could be:

- **Cost-Responsive Pricing:**
  - Scenario: Suppose a firm experiences a change in production costs (either an increase or a decrease).
  - Strategy: With constant elasticity ($\varepsilon = -1$), the firm can adjust its prices without worrying about revenue fluctuations. If production costs rise, the firm can raise prices to maintain profit margins; if costs fall, lowering prices might help capture more market share without reducing revenue.
  - Outcome: The firm can pass cost changes on to consumers while keeping revenue stable, allowing it to focus on controlling costs and margins rather than revenue variability.
- **Market Segmentation and Price Discrimination:**
  - Scenario: A company may wish to differentiate its pricing across different customer segments or regions.
  - Strategy: With a constant elasticity demand curve, the firm knows that any price change will not alter total revenue. This predictability enables it to design targeted pricing strategies. For instance, the firm might charge a higher price in markets where consumers are less price-sensitive (or where there's less competition) and a lower price in more competitive segments, while overall revenue stays stable.
  - Outcome: The company can optimize its profit margins and market coverage by tailoring prices to different consumer groups, knowing that the overall revenue impact of these adjustments is predictable.

In both strategies, the key advantage of constant elasticity is the predictability in revenue behavior, which helps firms plan price adjustments with a clear understanding of their impact on overall revenue.

4 Marginal Revenue

4.1 Inverse of Elasticity

It is the additional revenue generated from selling one more unit of a good. For small changes in output, if revenue $R = p \cdot q$ then:

$$MR = \frac{\Delta R}{\Delta q} \approx p + q\frac{\Delta p}{\Delta q}.$$

Notice how the only factor changing is differentiating by q compared to elasticity. In fact, the definition is highly related to elasticity ε:

$$\varepsilon = \frac{p}{q}\frac{\Delta q}{\Delta p} \quad\longleftrightarrow\quad \frac{1}{\varepsilon} = \frac{q}{p}\frac{\Delta p}{\Delta q},$$

which leads to the formula:

$$MR = p\left(1 + \frac{1}{\varepsilon}\right).$$

Marginal revenue tells us how total revenue changes as supply increases. With the formula $MR = p\left(1 + \frac{1}{\varepsilon}\right)$.

How Revenue Is Influenced by Marginal Revenue:

- **Unit Elastic Demand** ($\varepsilon = -1$): When elasticity is -1, the expression becomes:

  $$MR = p\left(1 + \frac{1}{-1}\right) = p(1-1) = 0.$$

  This indicates that revenue is maximized; increasing supply further does not change total revenue.

- **Elastic Demand** ($|\varepsilon| > 1$): Here, $\frac{1}{\varepsilon}$ is a small negative number (since ε is negative and its absolute value is greater than 1), so:

  $$MR > 0.$$

  This means that increasing supply increases revenue.

- **Inelastic Demand** ($|\varepsilon| < 1$): In this case, $\frac{1}{\varepsilon}$ is a large negative number (in absolute value greater than 1), so:

$$MR < 0.$$

This implies that increasing supply reduces revenue.

This relationship is crucial for firms when deciding how much to produce and at what price, as it directly connects consumer responsiveness to changes in price with the firm's revenue outcomes.

5 Income Elasticity and Expenditure Share

The income elasticity of demand measures the responsiveness of the quantity demanded of a good to a change in consumer income. Mathematically, the income elasticity of demand can be expressed as:

$$\text{Income Elasticity of Demand} = \frac{\text{\% Change in Quantity Demanded}}{\text{\% Change in Income}} = \frac{\partial q}{\partial m}\frac{m}{q}.$$

where:

- q is the quantity demanded,
- m is income,
- $\frac{\Delta q}{q}$ is the percentage change in quantity, and
- $\frac{\Delta m}{m}$ is the percentage change in income.

What is the change of income elasticity for different types of goods?

- **Normal Goods:** Goods for which an increase in income leads to an increase in quantity demanded. Therefore, they have a positive income elasticity.
- **Inferior Goods:** Goods for which an increase in income leads to a decrease in quantity demanded. Therefore, they have a negative income elasticity.
- **Luxury Goods:** A subset of normal goods that are highly responsive to income changes. They have an income elasticity greater than 1, meaning a 1% increase in income results in more than a 1% increase in demand.

Besides, we define the concept of expenditure share. The expenditure share of good i, denoted by s_i, is defined as the fraction of total income spent on that good. It is given by:

$$s_i = \frac{p_i x_i}{m},$$

where p_i is the price of good i, x_i is the quantity of good i purchased, and m is the total income of the consumer.