Utility

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1 Introduction

The theory of consumer behavior has been reformulated entirely in terms of **consumer preferences**, and **utility** is seen only as a way to describe preferences. The utility function provides a numerical representation of these preferences but does not directly measure happiness or satisfaction.

2 Utility Function

- A **utility function** assigns a number to every consumption bundle such that more-preferred bundles receive larger numbers. Mathematically: $(x_1, x_2) \succ (y_1, y_2) \iff u(x_1, x_2) > u(y_1, y_2)$. This is referred to as **ordinal utility**, as it only reflects the ranking of bundles, not the intensity of preferences.

2.1 Monotonic Transformation

- A **monotonic transformation** is a way of rescaling a utility function without altering the preference ordering. A function $f(u)$ is monotonic if its derivative is always positive: $f'(u) > 0$.
- **Example:** If $u(x_1, x_2) = x_1 + x_2$, then $f(u) = u^2$ or $f(u) = \ln(u)$ are monotonic transformations of $u(x_1, x_2)$.

2.2 Constructing a Utility Function

- Not all preferences can be represented by a utility function.
- A utility function exists only for preferences that satisfy certain mathematical properties (e.g., completeness and transitivity).

3 Examples of Utility Functions

3.1 Perfect Substitutes

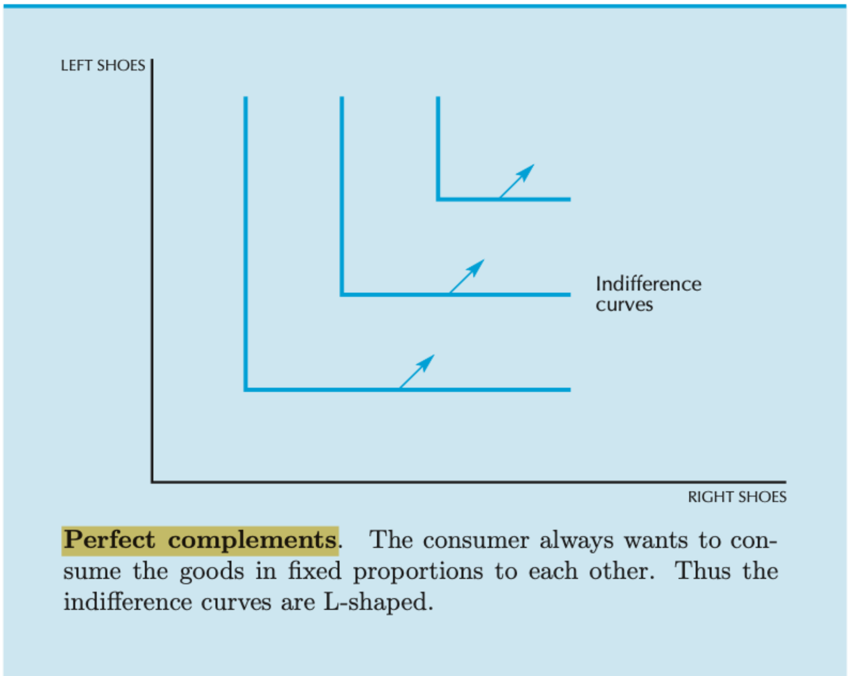
- Preferences for **perfect substitutes** are represented by: $u(x_1, x_2) = ax_1 + bx_2$, where $a, b > 0$.
- The slope of the indifference curve is: $-\frac{a}{b}$.

Perfect substitutes. The consumer only cares about the total number of pencils, not about their colors. Thus the indifference curves are straight lines with a slope of −1.

Figure 1: Perfect Substitutes Preferences

3.2 Perfect Complements

- Preferences for **perfect complements** are represented by: $u(x_1, x_2) = \min\{ax_1, bx_2\}$, where $a, b > 0$.
- Indifference curves are L-shaped.

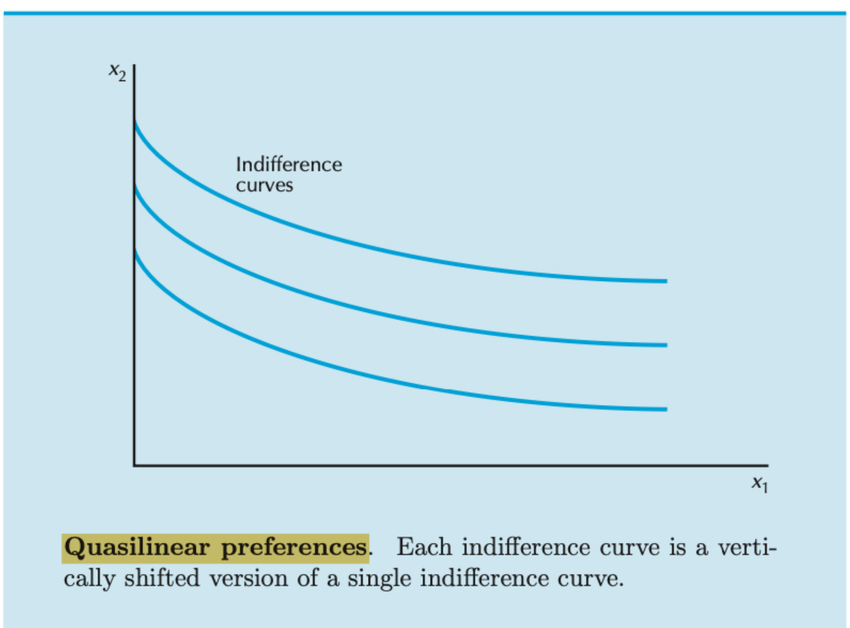


Perfect complements. The consumer always wants to consume the goods in fixed proportions to each other. Thus the indifference curves are L-shaped.

Figure 2: Perfect Complements Preferences

3.3 Quasilinear Preferences

- **Quasilinear preferences** are represented by: $u(x_1, x_2) = v(x_1) + x_2$, where $v(x_1)$ is a nonlinear function of x_1.



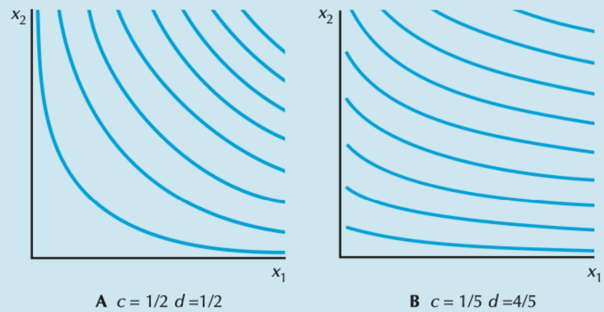
Quasilinear preferences. Each indifference curve is a vertically shifted version of a single indifference curve.

Figure 3: Quasilinear Preferences

3.4 Cobb-Douglas Preferences

- Preferences with **Cobb-Douglas utility** are represented by: $u(x_1, x_2) = x_1^c x_2^d$, where $c, d > 0$. The parameter c indicates how much the consumer values good x_1 and d indicates how much the consumer values good x_2.

- Indifference curves are smooth and convex.



Cobb-Douglas indifference curves. Panel A shows the case where $c = 1/2$, $d = 1/2$ and panel B shows the case where $c = 1/5$, $d = 4/5$.

Figure 4: Cobb-Douglas Preferences

4 Marginal Utility

- **Marginal utility** measures the rate of change of utility with respect to a good. For good 1:

$$MU_1 = \frac{\Delta U}{\Delta x_1} = \frac{u(x_1 + \Delta x_1, x_2) - u(x_1, x_2)}{\Delta x_1}.$$

5 Marginal Utility and MRS

- The **Marginal Rate of Substitution (MRS)** is the slope of the indifference curve and measures the rate at which the consumer is willing to substitute good 2 for good 1:

$$MRS = -\frac{MU_1}{MU_2}.$$

- In well-behaved preferences (monotonic (the more the better) and convex) the MRS is:

$$MRS = -\frac{MU_1}{MU_2} = -\frac{p_1}{p_2}.$$

- If $MRS = -k$, then consumer is willing to give up k units of good 2 to gain 1 unit of good 1.

6 Example: Cobb-Douglas Utility

- Consider the Cobb-Douglas utility function: $u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$.

- The marginal utilities are:

$$MU_1 = \frac{\partial u}{\partial x_1} = 0.5x_1^{-0.5}x_2^{0.5},$$

$$MU_2 = \frac{\partial u}{\partial x_2} = 0.5x_1^{0.5}x_2^{-0.5}.$$

- The MRS is:

$$MRS = -\frac{MU_1}{MU_2} = -\frac{0.5x_1^{-0.5}x_2^{0.5}}{0.5x_1^{0.5}x_2^{-0.5}} = -\frac{x_2}{x_1}.$$

7 Conclusion

Utility functions provide a powerful way to describe consumer preferences and to derive important economic concepts like marginal utilities and the MRS. Different functional forms of utility functions represent different types of preferences, such as perfect substitutes, perfect complements, or Cobb-Douglas preferences.