Buying and Selling

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January 30, 2025

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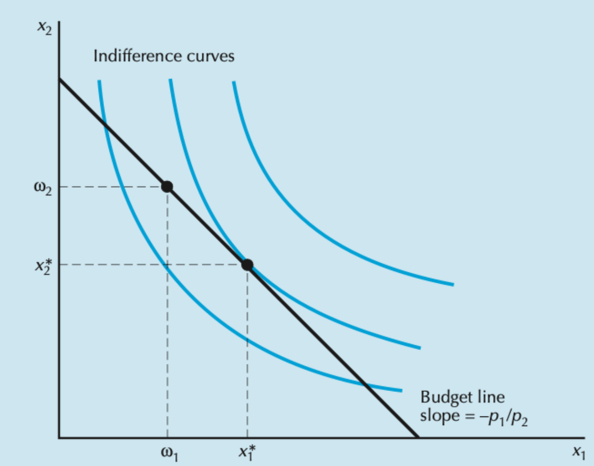
1 Endowment, Net and Gross Demands

In consumer theory, we often begin by considering the endowment of two goods, denoted as (ω_1, ω_2). This represents the quantity of each good that the consumer possesses before entering the market. Once the consumer engages in trade, we distinguish between two important concepts: gross demand and net demand. The gross demand for a good is the total amount the consumer ultimately consumes, reflecting the quantity they take home after trading. On the other hand, the net demand is the difference between the consumer's gross demand and their initial endowment. Net demand indicates how much of a good the consumer buys (if positive) or sells (if negative) in the market. These concepts are key to understanding the consumer's behavior in response to price and income changes.

2 Budget Constraint and Endowment

When the consumer goes to the market, 3 things can happen, either he loses satisfaction by an inefficient trade (downgrading his indifference curve), either he stays in the same indifference curve, or improves his current situation by obtaining a bundle of goods more valuable. The budget constraint ensures that the value of the bundle a consumer ends up with after trading must equal the value of their initial endowment. This reflects the idea that the consumer can only trade within the resources provided by their endowment, ensuring no borrowing or gifting. Mathematically, this is expressed as:

- **Original Budget Constraint:** $p_1 x_1 + p_2 x_2 = m$
- **Endowment Constraint:** $m = p_1 \omega_1 + p_2 \omega_2$
- **Combining these gives:** $p_1 x_1 + p_2 x_2 = p_1 \omega_1 + p_2 \omega_2$



The budget line. The budget line passes through the endowment and has a slope of $-p_1/p_2$.

Figure 1: Indifference Curves and Endowment

This equation ensures that the value of the bundle consumed equals the value of the initial endowment.

2.1 Example

Imagine a consumer has an initial endowment of goods: 5 apples ($\omega_1 = 5$) and 3 oranges ($\omega_2 = 3$). The consumer enters the market and chooses to end up with 8 apples and 1 orange.

- **Gross Demands** (x_1, x_2): These are the total amounts of goods the consumer consumes. In this case:

- $x_1 = 8$ apples
- $x_2 = 1$ orange

- **Net Demands** $(x_1 - \omega_1, x_2 - \omega_2)$: These are the differences between the consumed amounts (gross demands) and the initial endowments. Here:
  - $x_1 - \omega_1 = 8 - 5 = 3$: The consumer buys 3 more apples than they originally had.
  - $x_2 - \omega_2 = 1 - 3 = -2$: The consumer sells 2 oranges, as the net demand is negative.

Thus the consumer ends up consuming 8 apples and 1 orange and he purchases 3 apples and sell 2 oranges in the market. Besides, we would say that the consumer is a net buyer of 3 apples and a net seller of 2 oranges.

3 Price Changes: In favour and against Preferences

3.1 Against Preferences

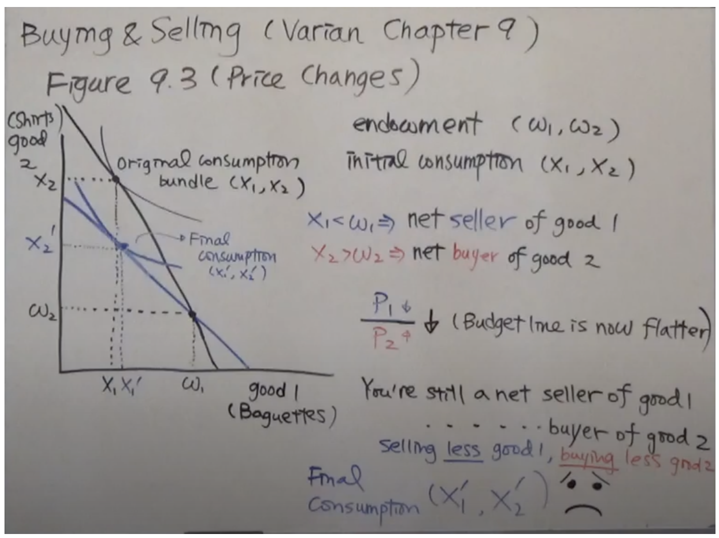


Figure 2: Against Preferences Scenario

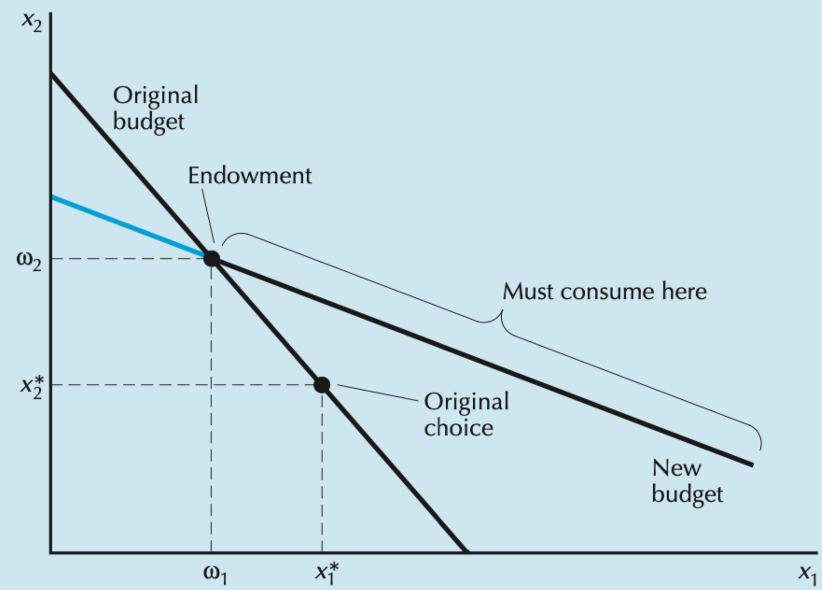
In this scenario, imagine a world with two goods: baguettes (good 1) and t-shirts (good 2). Suppose your initial endowment (ω_1, ω_2) consists of many baguettes and only a few t-shirts. This isn't ideal, as you'd prefer to trade some of your baguettes for t-shirts to achieve a better balance. Naturally, you'd hope for a higher price for baguettes (p_1) to maximize your earnings from selling them and a lower price for t-shirts (p_2) to make buying them more affordable.

The initial endowment determines your initial budget line, which has a slope of $-p_1/p_2$ and corresponds to the black line. Suppose your initial consumption bundle is such that $x_1 < \omega_1$, meaning you consume fewer baguettes than your endowment. This makes you a net seller of baguettes and at the same time, $x_2 > \omega_2$, so you consume more t-shirts than your endowment, making you a net buyer of t-shirts.

Now imagine an unfortunate situation where prices move against your preferences: the price of baguettes falls $p_1 \downarrow$, and the price of t-shirts rises $p_2 \uparrow$. The items you're selling (baguettes) lose value, and the items you wish to buy (t-shirts) become more expensive. As a result, your budget line becomes flatter (because the slope shrinks $-\frac{p_1\downarrow}{p_2\uparrow} = \downarrow$) while still passing through the initial endowment. Your new consumption bundle shifts to x_1' and x_2', reflecting reduced consumption of both goods. Although you remain a net seller of baguettes and a net buyer of t-shirts, you sell fewer baguettes and buy fewer t-shirts than before. Consequently, your utility decreases, leaving you worse off and less satisfied than before.

This situation illustrates how adverse price changes negatively impact your ability to trade and consume. In the next image, we'll discuss a more favorable scenario, where the price of baguettes rises, and the price of t-shirts falls, leaving you in a much happier position.

3.2 In favour of Preferences



Decreasing the price of good 1. If a person is a buyer and the price of what she is buying decreases, she remains a buyer.

Figure 3: In favour of Preferences Scenario

As previously, the situation begins with a consumer who has an endowment of two goods: good 1 (baguettes) and good 2 (t-shirts). The consumer starts with relatively few baguettes but plenty of t-shirts. Because of this, he chooses to sell some of their t-shirts and use the money to buy more baguettes. His initial budget line reflects these choices and passes through his endowment point. The fact that the consumer ends up with more baguettes $(x *_1^{>} \omega_1)$ means they are a net buyer of good 1, while consuming fewer t-shirts than their initial endowment $(x*_2 < \omega_2)$ indicates they are a net seller of good 2.

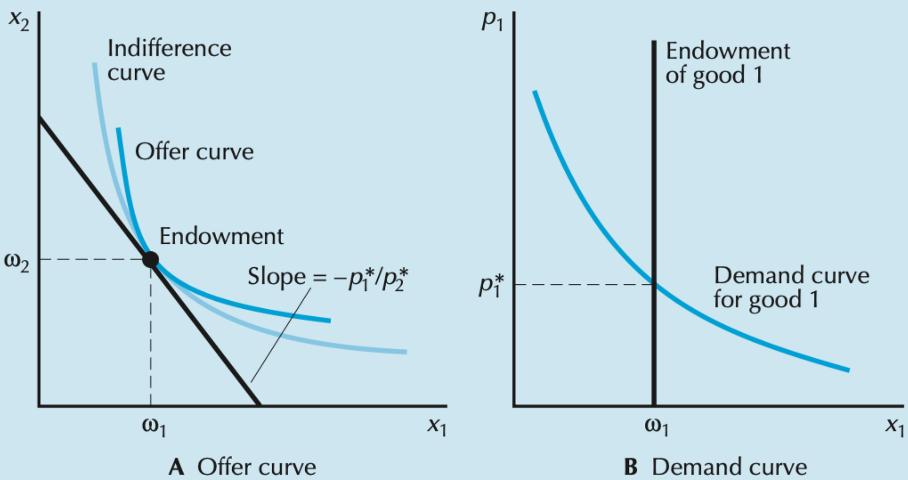
As someone who buys baguettes and sells t-shirts, the consumer naturally prefers a lower price for baguettes (so they can buy more) and a higher price for t-shirts (to sell at a better rate). Suppose the prices change: the price of baguettes drops, and the price of t-shirts rises. The new budget line reflects these changes and becomes flatter (once again due to $-\frac{p_1\downarrow}{p_2\uparrow} = \downarrow \implies$ flatter slope) while still passing through the endowment point.

The new optimal consumption point must lie on the upper part of the new budget line, above the original budget line, where it is written "Must consume here". This outcome is supported by two arguments. First, by the weak axiom of revealed preference, the consumer chose ("Original Choice") a lower point in the budget line and discarded the higher portion of the original budget line (blue segment) before the price change, so he would not rationally choose a point below it now. Second, from an economic perspective, a lower price for baguettes makes it even more attractive to continue buying them, while the higher price of t-shirts provides a stronger incentive to keep selling them. It would be irrational for the consumer to switch from selling t-shirts to buying them after the price increase, as this would contradict their economic interests and reduce their welfare.

Ultimately, after the price changes, the consumer remains a buyer of baguettes and a seller of t-shirts. This behavior aligns with rational utility maximization, ensuring the consumer makes choices that improve or maintain their welfare.

4 Price Offer Curves & Demand Curves

4.1 Price Offer Curve



The offer curve and the demand curve. These are two ways of depicting the relationship between the demanded bundle and the prices when an endowment is present.

Figure 4: Price Offer Curve & Demand Curve

Let's explain how to derive the price offer curve, which traces the bundles of x_1 and x_2 that a consumer chooses given different income levels or relative prices. Suppose we start with an endowment (ω_1, ω_2) and the initial prices of good 1 and good 2 are p_1^* and p_2^*, respectively. Besides, let's suppose that when these are the prices, the consumer reaches a bundle where he does not want to trade, which becomes the starting point of our analysis. Now, if we fix the price of good 2 (p_2^*) and allow only the price of good 1 (p_1^*) to change, we can analyze how the consumption bundle adjusts.

- For example, if the price of good 1 decreases below its initial value p_1^*, it becomes relatively cheaper. This incentivizes the consumer to buy more of good 1, making them a net buyer of this good since x_1' exceeds ω_1. Simultaneously, good 2 becomes relatively more expensive (relatively because $\frac{p_1}{p_2^*} < \frac{p_1^*}{p_2^*}$ then this means that fewer units of good 1 are needed to trade for a unit of good 2, making good 2 relatively costlier in terms of good 1.), encouraging the consumer to sell some of good 2, as x_2' is now less than ω_2.

- Conversely, if the price of good 1 increases above p_1^*, the opposite happens. Good 1 becomes relatively more expensive, so the consumer prefers to sell it, reducing their consumption of good 1 to x_1' below ω_1. Meanwhile, good 2 becomes relatively cheaper, leading the consumer to buy more of it, increasing their consumption to x_2' above ω_2. In both cases, the new budget line still passes through the endowment point but adjusts its slope according to the new relative price.

Tracing all the consumption bundles that arise from different relative prices creates the offer curve, which reflects how the consumer's optimal consumption bundle changes with relative price variations. It's important to note that absolute prices don't matter as much as the relative price ratio (p_1/p_2), which determines the slope of the budget line. For example, if both prices double (e.g., $p_1 = 2p_1^*$ and $p_2 = 2p_2^*$), the budget line remains essentially the same because the relative price and slope are unchanged.

4.2 Demand Curve

Previously, we established that the consumer starts with an endowment of two goods, denoted as ω_1 and ω_2, and the initial prices of these goods are p_1^* and p_2^*, respectively. Once again, let's focus on what happens when the price of good 2 remains constant, but the price of good 1 changes, to understand the demand curve.

if the price of good 1 decreases to a level below its initial price, p_1^*, good 1 becomes relatively cheaper. As a result, it makes sense for the consumer to buy more of good 1 and sell some of good 2. This adjustment means the consumer's final consumption of good 1, x_1, will be greater than their initial endowment, ω_1 (that's why the curve decreases from left to right). Conversely, if the price of good 1 increases above p_1^*, good 1 becomes relatively more expensive. In this case, the consumer will sell some of their endowment of good 1 to buy more of good 2. Consequently, the consumer's final consumption of good 1, x_1, will be lower than their initial endowment, ω_1 (that's why the curve increases from right to left).

In summary, when the price of good 1 decreases, the consumer consumes more of it, while when the price of good 1 increases, the consumer consumes less of it. This relationship between price and consumption forms the foundation for deriving the demand curve for good 1. I hope this explanation clarifies the concept, and I'll see you next time!

5 Net Supply & Gross and Net Demand

In the previous discussion, we considered a situation where the consumer has an initial endowment of ω_1 and ω_2, and the prices of the two goods are p_1^* and p_2^*.

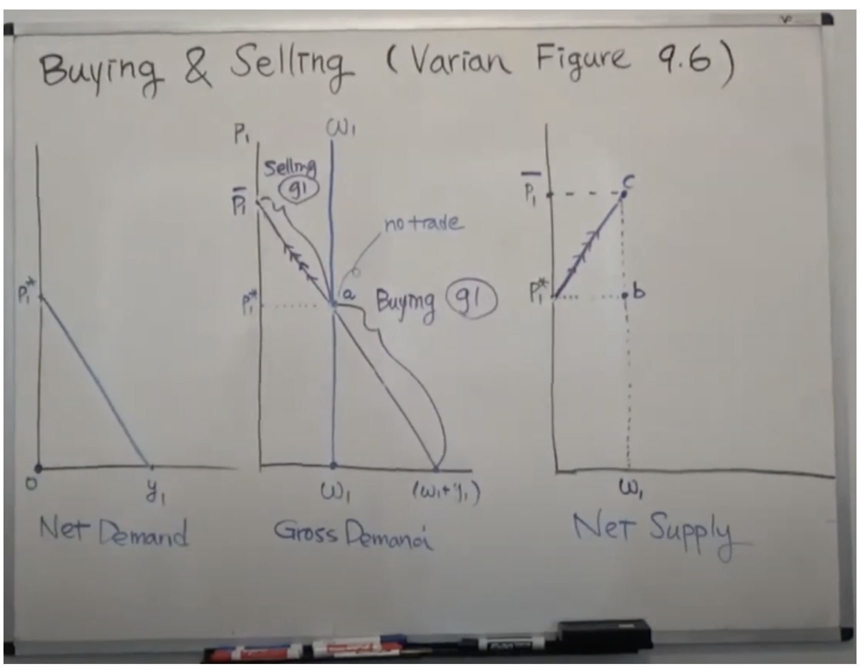


Figure 5: Net Supply, Gross Demand & Net Demand

- **Let's first focus on the middle image:** At these prices, the consumer does not engage in any trade and simply consumes their initial endowment. This "no-trade" scenario is depicted in the middle panel, where the consumption point is at a, the endowment point. Here, the consumer neither buys nor sells good 1 or good 2.

  If the price of good 1 decreases below p_1^*, it becomes relatively cheaper, incentivizing the consumer to buy more of good 1. In this case, the consumption bundle shifts to a point where $x_1 > \omega_1$, meaning the consumer becomes a net buyer of good 1. Conversely, if the price of good 1 rises above p_1^*, good 1 becomes relatively more expensive, and the consumer chooses to sell some of their endowment of good 1 to buy more of good 2, making them a net seller of good 1.

- **Let's switch to the left plot:** When the price of good 1 is exactly p_1^*, the consumer buys or sells nothing and simply consumes their initial endowment. However, when the price of good 1 drops below p_1^*, the consumer begins to purchase additional units of good 1 from the market. For example, if the price of good 1 falls to zero, the consumer will purchase enough to consume $\omega_1 + y_1$, where y_1 represents the additional units bought. Here, the gross demand for good 1 is the total consumption ($\omega_1 + y_1$), while the net demand is the additional amount purchased (y_1). The relationship between these quantities can be visualized through two identical triangles: one showing the price and net demand, and the other showing the total gross demand relative to the endowment.

- **Finally, the right plot:** If the price of good 1 increases above p_1^*, the consumer begins selling their endowment of good 1, reducing their own consumption. For instance, at a price of p_1^{bar}, the consumer sells their entire endowment (ω_1). This behavior can also be represented graphically as a mirrored relationship between the triangle formed by price and supply (right plot) and the one formed by price and gross demand (middle plot). These two triangles are identical in size and shape, reflecting the symmetry between buying and selling behaviors.

6 The Slutsky Equation revisited

6.1 The Slutsky Equation in Rate of Change Form

To derive the final equation which mixes the the Slutsky Equation in rate of change form with the concept of Endowment Income Effect (EIE) , we start with the basic decomposition of the total effect of a price change into the substitution effect and income effect:

$$\Delta x_1 = \Delta x_1^S + \Delta x_1^n$$

Where:

- Δx_1^S: Substitution effect (change in consumption due to relative price change, holding utility constant).
- Δx_1^n: Income effect (change in consumption due to the change in real income after adjusting for price changes).

Next, divide both sides of the equation by Δp_1, the change in the price of good 1:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^S}{\Delta p_1} + \frac{\Delta x_1^n}{\Delta p_1}$$

To rewrite the income effect term, recall that the change in income due to a price change, ΔM, can be expressed as:

$$\Delta m = x_1 \cdot \Delta p_1 \quad \longleftrightarrow \quad \Delta p_1 = \frac{\Delta m}{x_1}$$

Substituting this expression for Δp_1 into the income effect term $\frac{\Delta x_1^n}{\Delta p_1}$, we get:

$$\frac{\Delta x_1^n}{\Delta p_1} = \frac{\Delta x_1^n}{\Delta m} \cdot x_1$$

Now substitute back into the total effect equation and we obtain the final equation:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^S}{\Delta p_1} - \frac{\Delta x_1^n}{\Delta m} \cdot x_1$$

This equation shows how the total change in demand for good 1 due to a price change is the sum of the substitution effect and the adjusted income effect. The negative sign indicates that the income effect works in the opposite direction for normal goods.

6.2 The Slutsky Equation with the Endowment Income Effect (EIE)

The Slutsky Equation expresses the total change in demand for a good (x_1) in response to a price change (p_1) as the sum of the substitution effect, income effect, and, in the context of endowments, the endowment income effect (EIE). Starting with the equation previously proved, the total effect is:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^S}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1,$$

where:

- $\frac{\Delta x_1^S}{\Delta p_1}$ is the substitution effect.
- $\frac{\Delta x_1^m}{\Delta m} x_1$ represents the income effect.

Let's proceed step by step:

- **Step 1:** In the context of endowments, a change in the price of good 1 (p_1) impacts income (m) because income is derived from the value of the initial endowment (ω_1, ω_2). The endowment income effect (EIE) captures this impact and is defined as:

  $$\text{EIE} = \left(\frac{\Delta x_1^m}{\Delta m}\right) \cdot \left(\frac{\Delta m}{\Delta p_1}\right),$$

  where:

  - $\frac{\Delta x_1^m}{\Delta m}$ is the change in demand due to a change in income.
  - $\frac{\Delta m}{\Delta p_1}$ is the change in income due to a change in the price of good 1.

- **Step 2:** Substituting for $\frac{\Delta m}{\Delta p_1}$ Income is defined as:

  $$m = p_1\omega_1 + p_2\omega_2,$$

  so the change in income with respect to p_1 is:

  $$\frac{\Delta m}{\Delta p_1} = \omega_1.$$

  Substituting this into the expression for the endowment income effect, we get:

  $$\text{EIE} = \frac{\Delta x_1^m}{\Delta m} \cdot \omega_1.$$

- **Step 3:** Revising the Slutsky Equation The total effect can now be rewritten as:

  $$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^S}{\Delta p_1} - \frac{\Delta x_1^m}{\Delta m} x_1 + \frac{\Delta x_1^m}{\Delta m} \omega_1.$$

  Factoring out $\frac{\Delta x_1^m}{\Delta m}$, the equation becomes:

  $$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^S}{\Delta p_1} + \frac{\Delta x_1^m}{\Delta m}(\omega_1 - x_1).$$

The final equation, including the substitution effect and the endowment-adjusted income effect, is:

$$\frac{\Delta x_1}{\Delta p_1} = \frac{\Delta x_1^S}{\Delta p_1} + (\omega_1 - x_1)\frac{\Delta x_1^m}{\Delta m}.$$

This equation accounts for how the price change affects demand directly through substitution and indirectly through changes in income derived from the endowment.

6.3 Sign of the Endowment Income Effect

The equation for the rate of change in demand for good a due to a price change is:

$$\frac{\Delta x_a}{\Delta p_a} = \frac{\Delta x_a^s}{\Delta p_a} + (\omega_a - x_a)\frac{\Delta x_a^m}{\Delta m}.$$

Here, the total effect ($\frac{\Delta x_a}{\Delta p_a}$) is the sum of the substitution effect ($\frac{\Delta x_a^s}{\Delta p_a}$) and the endowment-adjusted income effect ($(\omega_a - x_a)\frac{\Delta x_a^m}{\Delta m}$). Each case below analyzes the substitution and income effects together to explain how they interact to determine the total effect, depending on whether prices rise or fall, and whether the consumer is a net buyer or seller.

- **Case 1:** Net Seller ($\omega_a - x_a > 0$) and Price Increases ($\Delta p_a > 0$)

  If the consumer is a net seller, they sell some of their endowment of good a, meaning $\omega_a > x_a$. When the price of good a increases, the substitution effect discourages consumption of good a because it becomes relatively more expensive compared to other goods. The substitution effect, $\frac{\Delta x_a^s}{\Delta p_a}$, is negative ($-$) because demand for a decreases. At the same time, the price increase raises the income from selling good a, causing a positive income effect if good a is normal ($\frac{\Delta x_a^m}{\Delta m} > 0$). The income effect works in the opposite direction of the substitution effect, partially offsetting the reduction in demand. For an inferior good ($\frac{\Delta x_a^m}{\Delta m} < 0$), the income effect reinforces the substitution effect, further decreasing demand. The total effect is:

$$\frac{\Delta x_a}{\Delta p_a} = (-) + (+)(\omega_a - x_a), \text{ for a normal good,}$$

$$\frac{\Delta x_a}{\Delta p_a} = (-) + (-)(\omega_a - x_a), \text{ for an inferior good.}$$

- **Case 2:** Net Seller ($\omega_a - x_a > 0$) and Price Decreases ($\Delta p_a < 0$)

  When the price of good a decreases, the substitution effect encourages more consumption of a because it becomes cheaper relative to other goods. The substitution effect, $\frac{\Delta x_a^s}{\Delta p_a}$, is negative ($-$), but since the price is falling, this implies consumption increases. As a seller, the consumer earns less income from selling good a, so the income effect is negative if the good is normal ($\frac{\Delta x_a^m}{\Delta m} > 0$), further reducing consumption. For an inferior good ($\frac{\Delta x_a^m}{\Delta m} < 0$), the income effect partially offsets the substitution effect because the reduced income increases demand for the inferior good. The total effect is:

$$\frac{\Delta x_a}{\Delta p_a} = (-) + (-)(\omega_a - x_a), \text{ for a normal good,}$$

$$\frac{\Delta x_a}{\Delta p_a} = (-) + (+)(\omega_a - x_a), \text{ for an inferior good.}$$

- **Case 3:** Net Buyer ($\omega_a - x_a < 0$) and Price Increases ($\Delta p_a > 0$)

  If the consumer is a net buyer, they purchase additional units of good a, meaning $\omega_a < x_a$. When the price of good a increases, the substitution effect discourages consumption of a, so $\frac{\Delta x_a^s}{\Delta p_a}$ is negative ($-$). As a buyer, the consumer has less purchasing power due to the price increase, leading to a negative income effect if the good is normal ($\frac{\Delta x_a^m}{\Delta m} > 0$), which reinforces the substitution effect. For an inferior good ($\frac{\Delta x_a^m}{\Delta m} < 0$), the income effect works in the opposite direction of the substitution effect, partially offsetting the decrease in demand. The total effect is:

$$\frac{\Delta x_a}{\Delta p_a} = (-) + (-)(\omega_a - x_a), \text{ for a normal good,}$$

$$\frac{\Delta x_a}{\Delta p_a} = (-) + (+)(\omega_a - x_a), \text{ for an inferior good.}$$

- **Case 4:** Net Buyer ($\omega_a - x_a < 0$) and Price Decreases ($\Delta p_a < 0$)

  When the price of good a decreases, the substitution effect encourages more consumption of a because it is now cheaper. The substitution effect, $\frac{\Delta x_a^s}{\Delta p_a}$, is negative (−), but since the price is falling, this implies increased demand. As a buyer, the reduced price increases the consumer's purchasing power. If the good is normal ($\frac{\Delta x_a^m}{\Delta m} > 0$), the income effect reinforces the substitution effect, leading to a larger increase in demand. For an inferior good ($\frac{\Delta x_a^m}{\Delta m} < 0$), the income effect works against the substitution effect, partially reducing the increase in demand. The total effect is:

$$\frac{\Delta x_a}{\Delta p_a} = (-) + (+)(\omega_a - x_a), \text{ for a normal good,}$$

$$\frac{\Delta x_a}{\Delta p_a} = (-) + (-)(\omega_a - x_a), \text{ for an inferior good.}$$

- **Case 5:** No Trade ($\omega_a - x_a = 0$)

  If the consumer neither buys nor sells good a, the endowment income effect vanishes because $(\omega_a - x_a) = 0$. In this situation, the total effect is purely driven by the substitution effect. When the price of good a increases, demand decreases due to the substitution effect ($\frac{\Delta x_a^s}{\Delta p_a}$), and when the price decreases, demand increases for the same reason. The total effect is:

$$\frac{\Delta x_a}{\Delta p_a} = \frac{\Delta x_a^s}{\Delta p_a}.$$

Find a numerical example to compute it