Demand

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Contents

1 Introduction

In this chapter, we examine how the demand for a good changes as prices and income change. This analysis is referred to as **comparative statics**, which studies how choices adjust before and after a change in the economic environment. For the consumer, the main factors affecting optimal choice are prices and income.

2 Normal and Inferior Goods

We analyze how the consumer's optimal choice changes with income:

- **Normal Goods:** Demand for a good increases when income increases: $\frac{\Delta x_1}{\Delta m} > 0$

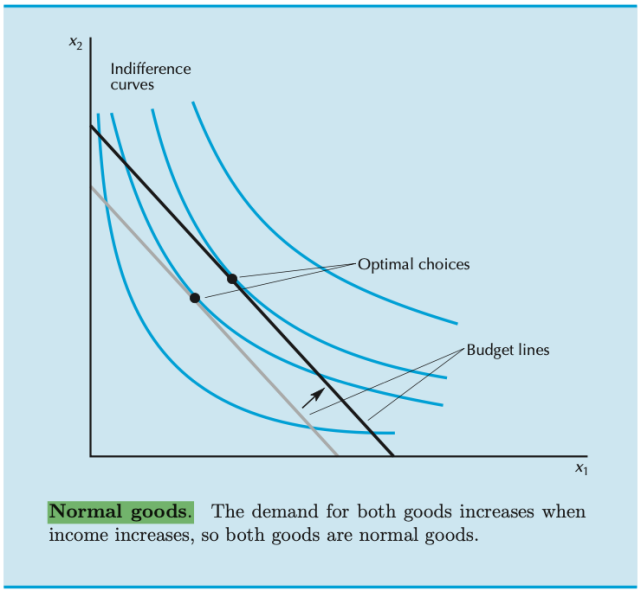


Figure 1: Normal Good

- **Inferior Goods:** Demand for a good decreases when income increases: $\frac{\Delta x_1}{\Delta m} < 0$

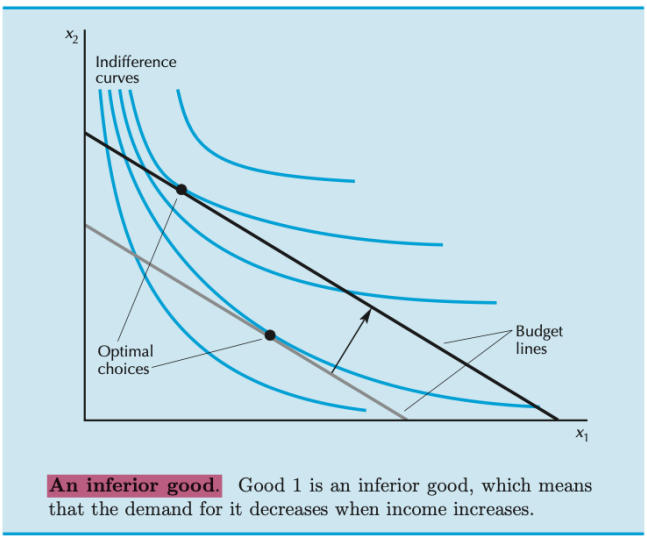
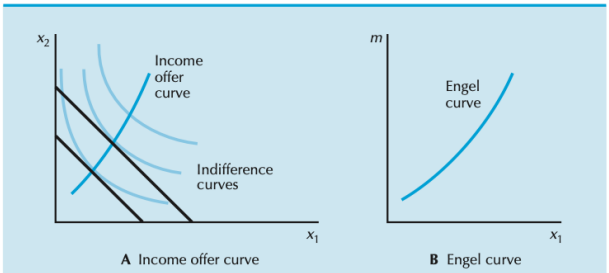


Figure 2: Inferior Good

In real life, some goods may be normal up to a certain level of income and inferior afterward.

3 Income Offer Curves and Engel Curves

- **Income Offer Curve:** The income offer curve (or income expansion path) shows how the optimal bundle (x_1, x_2) changes as income changes, with prices fixed. For normal goods, the income expansion path will have a positive slope.
- **Engel Curve:** The Engel curve plots the demand for one good (e.g., x_1) as a function of income, holding all prices constant.



How demand changes as income changes. The income offer curve (or income expansion path) shown in panel A depicts the optimal choice at different levels of income and constant prices. When we plot the optimal choice of good 1 against income, m, we get the Engel curve, depicted in panel B.

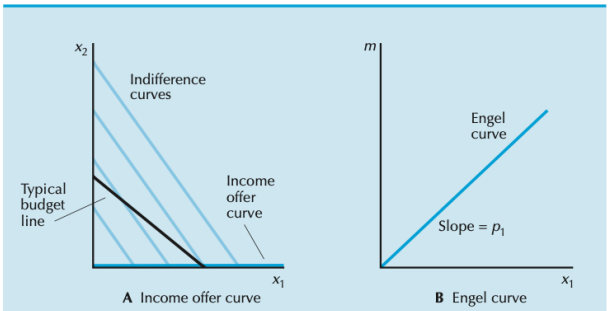
Figure 3: Income Offer Curve & Engel Curve

3.1 Some Examples

3.1.1 Perfect Substitutes

For $U(x_1, x_2) = ax_1 + bx_2$, we have the following curves:

- **Income Offer Curve:**
  - If $p_1 < p_2$, consume only x_1: $x_1 = \frac{m}{p_1}$, $x_2 = 0$ (Figure 4)
  - If $p_1 > p_2$, consume only x_2: $x_1 = 0$, $x_2 = \frac{m}{p_2}$.
  - If $p_1 = p_2$, any combination on the budget line is optimal.
- **Engel Curve:** For $p_1 < p_2$, the Engel curve for x_1 is: $x_1 = \frac{m}{p_1}$



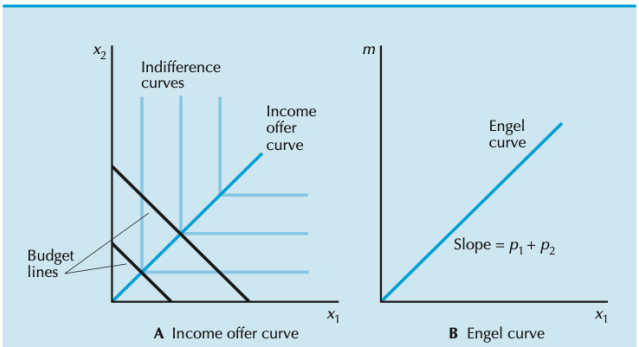
Perfect substitutes. The income offer curve (A) and an Engel curve (B) in the case of perfect substitutes.

Figure 4: Perfect Subsitutes: Income Offer Curve & Engel Curve

3.1.2 Perfect Complements

For $U(x_1, x_2) = \min\{ax_1, bx_2\}$ and $a = b = 1$.

- **Income Offer Curve:** We know that this consumer is purchasing the same amount of good 1 and good 2, no matter what the prices. Let this amount be denoted by $x = x_1 = x_2$. Then $p_1x_1 + p_2x_2 = m \longleftrightarrow x = \frac{m}{p_1+p_2}$. The income offer curve is a diagonal line through the origin, where m varies and the prices p_1, p_2 remain constant, i.e the Income Offer Curve: $(x_1, x_2) = (\frac{m}{p_1+p_2}, \frac{m}{p_1+p_2})$
- **Engel Curve:** The demand for good 1 is: $x_1 = x_2 = \frac{m}{p_1+p_2} \longleftrightarrow x_1(p_1 + p_2) = m$.



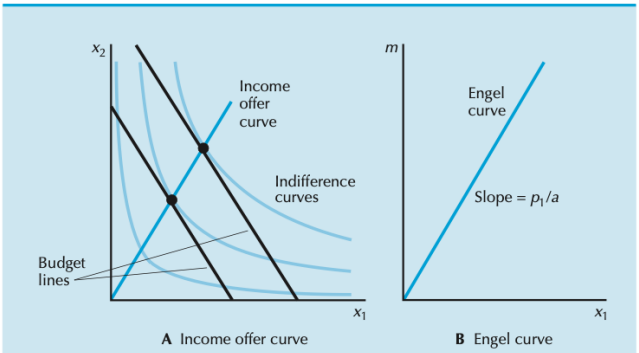
Perfect complements. The income offer curve (A) and an Engel curve (B) in the case of perfect complements.

Figure 5: Perfect Complements: Income Offer Curve & Engel Curve

3.1.3 Cobb-Douglas Preferences

For $U(x_1, x_2) = x_1^a x_2^{1-a}$:

- **Demand Functions:** $x_1 = \frac{am}{p_1}, \quad x_2 = \frac{(1-a)m}{p_2}$
- **Income Offer Curve:** $(x_1, x_2) = (\frac{am}{p_1}, \frac{(1-a)m}{p_2})$
- **Engel Curve:** For x_1, the Engel curve is: $x_1 = \frac{am}{p_1}$ where m varies and prices p_1, p_2 are fixed. Linear relationships for both goods.



Cobb-Douglas. An income offer curve (A) and an Engel curve (B) for Cobb-Douglas utility.

Figure 6: Cobb-Douglas Preferences: Income Offer Curve & Engel Curve

3.1.4 Homothetic Preferences

Preferences are homothetic if they can be represented by a utility function homogeneous of degree 1:

$$u(cx_1, cx_2) = c^1 \cdot u(x_1, x_2) \quad \text{for } c > 0.$$

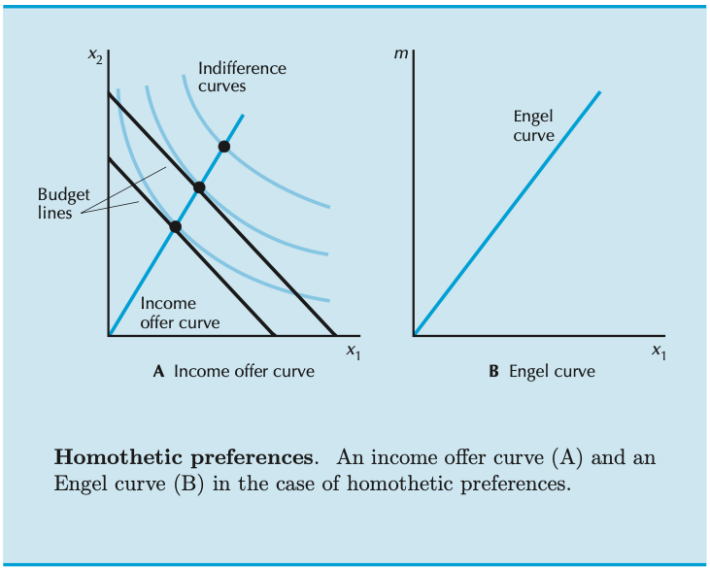


Figure 7: Homothetic Preferences: Income Offer Curve & Engel Curve

- **Luxury Good:** If demand for the good increases more than proportionally with income ($\Delta x_1/\Delta m > 1$).
- **Necessary Good:** If demand increases less than proportionally with income ($\Delta x_1/\Delta m < 1$).
- **Proportion of Income spend on a good** 1: $p_1 \cdot \frac{x_1(p_1,p_2,m)}{m}$

- Example:
  - Cobb-Douglas utility function: $u(x_1, x_2) = x_1^a x_2^b$
  - $x_1(p_1, p_2, m) = \frac{a}{a+b}\frac{m}{p_1}$
  - Proportion of income spent on good 1: $\frac{a}{a+b}$
  - Does not depend on m so neither luxury nor necessary
- Example:
  - $x_1(p_1, p_2, m) = (8m - m^2)/10p_1$ for $0 \leq m < 8$.
  - Proportion of income spent on good 1: $0.8 - 0.1m$ so necessary

Figure 8: Example of Luxury and Necessary Goods

3.1.5 Quasilinear Preferences

For $U(x_1, x_2) = v(x_1) + x_2$:

- **Optimal Choice:** The consumption of x_1 is fixed at x_1^* regardless of income, and all additional income is spent on x_2.
- **Example:** For $v(x_1) = \sqrt{x_1}$, the derivative $v'(x_1) = \frac{1}{2\sqrt{x_1}}$ and by setting $MRS = -\frac{v'(x_1)}{1} = -\frac{p_1}{p_2} \longleftrightarrow \frac{1}{2\sqrt{x_1}} = \frac{p_1}{p_2}$, we find $x_1^* = (\frac{p_2}{2p_1})^2$. Once x_1^* is determined, x_2^* can be found using the budget constraint and we obtain the optimal choices which define the income offer curve:

$$x_1^* = (\frac{p_2}{2p_1})^2, \quad x_2^* = \frac{m - p_1 x_1^*}{p_2}$$

Therefore, the Engel Curve would be:

$$x_1(m) = \begin{cases} \frac{m}{p_1}, & \text{if } m < p_1 \cdot \left(\frac{p_2}{2p_1}\right)^2, \\ \left(\frac{p_2}{2p_1}\right)^2, & \text{if } m \geq p_1 \cdot \left(\frac{p_2}{2p_1}\right)^2. \end{cases}$$

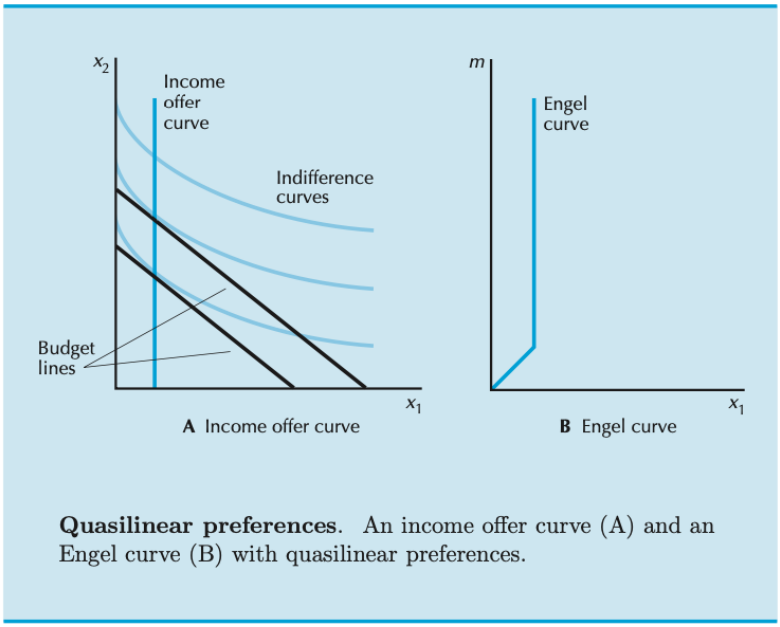


Figure 9: Quasilinear Preferences: Income Offer Curve & Engel Curve

4 Ordinary, Giffen, and Veblen Goods

- **Ordinary Goods:** Demand increases as the price decreases ($\Delta x_1/\Delta p_1 < 0$).

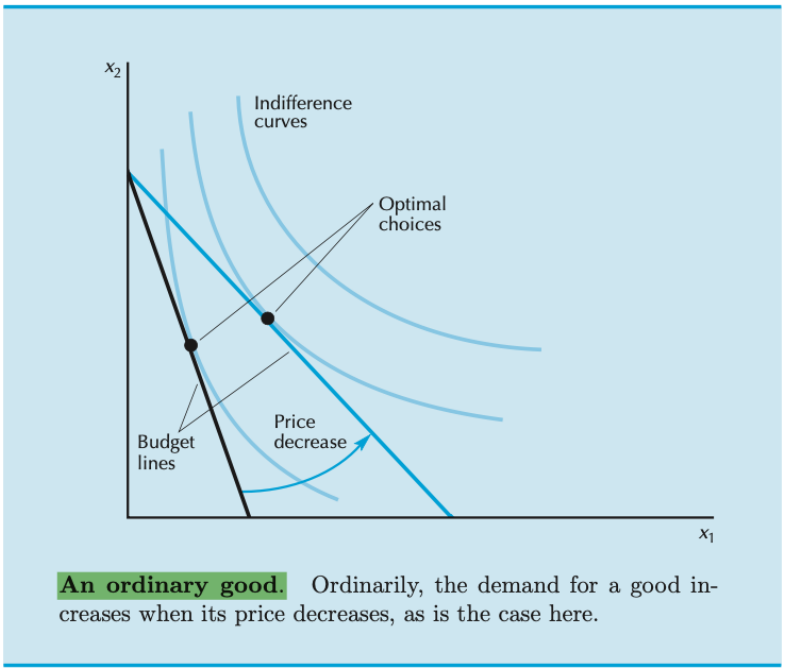
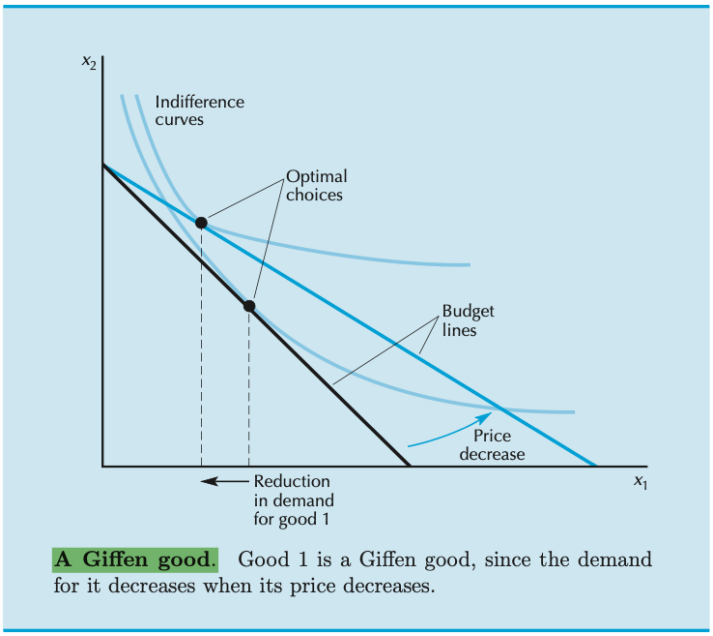


Figure 10: Ordinary Good

- **Giffen Goods:** Demand increases as the price increases, violating the law of demand ($\Delta x_1/\Delta p_1 > 0$). Or, equivalently, the demand for the good decreases when its price falls.
- **Veblen Goods:** Demand increases with price due to status or prestige. When the demand increases as the price increases, in apparent contradiction of the law of demand, it resuls in an upward-sloping demand curve. The higher prices of Veblen goods may make them desirable as a status symbol in the practices of conspicuous consumption, or speculation.



A Giffen good. Good 1 is a Giffen good, since the demand for it decreases when its price decreases.

Figure 11: Giffen (~Veblen) Good

5 The Price Offer Curve and the Demand Curve

- **Demand Function:** Relates quantities of goods (x_1, x_2) demanded to prices (p_1, p_2) and income m.
- **Demand Curve:** Isolates the relationship between quantity and price, fixing other factors. The only interest is to study demanded quantity vs price.
- **Price Offer Curve:** Shows the locus of optimal bundles (x_1, x_2) as p_1 changes, keeping p_2 and m constant.

5.1 Examples

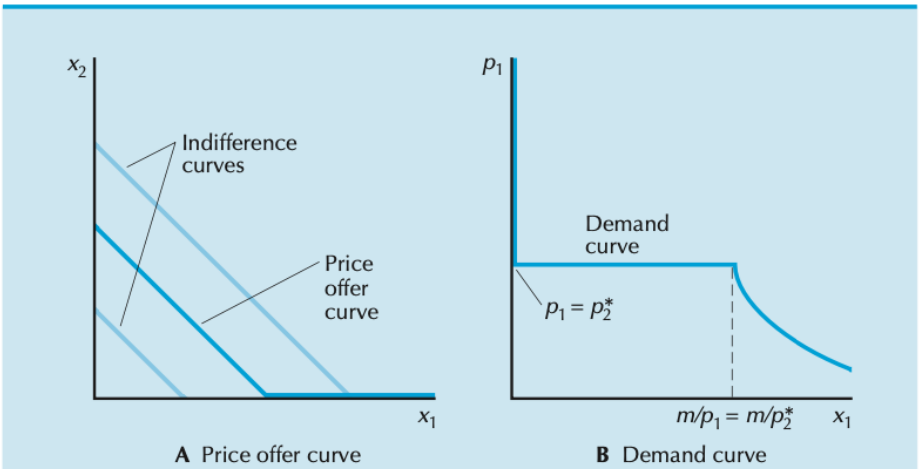
5.1.1 Perfect Substitutes

- **Price Offer Curve:** The price offer curve is defined as follows:

$$\begin{cases} (x_1, x_2) = (\frac{m}{p_1}, 0), & \text{if } p_1 < p_2, \\ (x_1, x_2) = (0, \frac{m}{p_2}), & \text{if } p_1 > p_2, \\ (x_1, x_2) = (\frac{m}{2p_1}, \frac{m}{2p_2}) & \text{if } p_1 = p_2 \end{cases}$$

- **Demand Curve:** For the good x_1 would be:

$$x_1(m) = \begin{cases} \frac{m}{p_1} & \text{if } p_1 < p_2, \\ 0 & \text{if } p_1 > p_2, \\ \frac{m}{2p_1} & \text{if } p_1 = p_2 \end{cases}$$

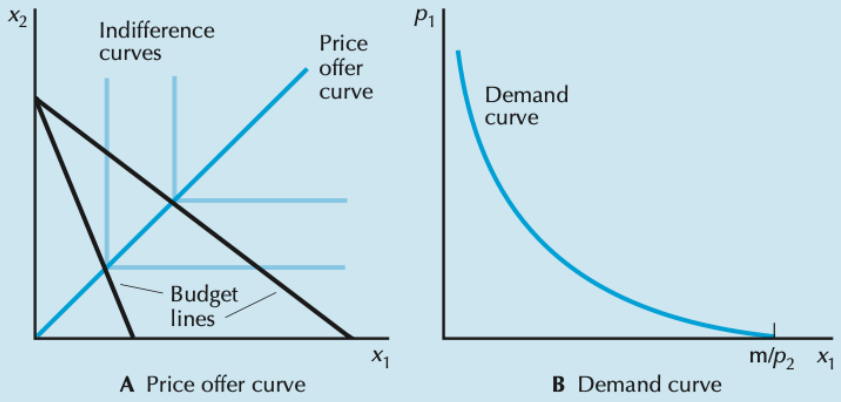


Perfect substitutes. Price offer curve (A) and demand curve (B) in the case of perfect substitutes.

Figure 12: Perfect Substitues Preferences: Price Offer Curve & Demand Curve

5.1.2 Perfect Complements

- **Price Offer Curve:** A diagonal line (x_1, x_2) where $x_1 = x_2 = \frac{m}{p_1+p_2}$.
- **Demand Curve:** $x_1 = \frac{m}{p_1+p_2}$.

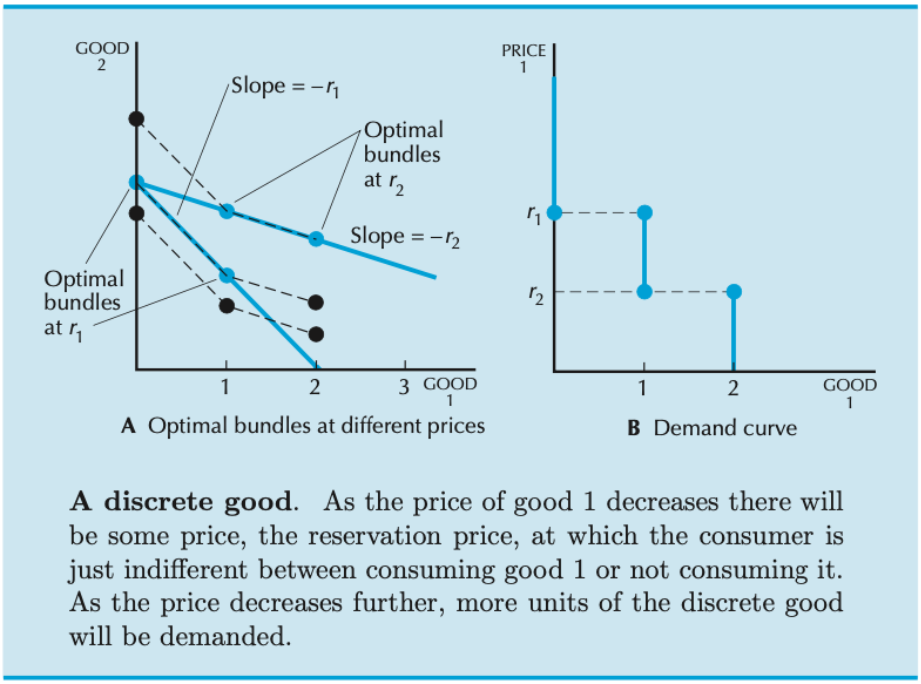


Perfect complements. Price offer curve (A) and demand curve (B) in the case of perfect complements.

Figure 13: Perfect Complements Preferences: Price Offer Curve & Demand Curve

5.1.3 Discrete Goods

For reservation prices $r_1, r_2, \ldots$, the demand curve is stepwise, reflecting the price at which the consumer is indifferent to purchasing the next unit.



A discrete good. As the price of good 1 decreases there will be some price, the reservation price, at which the consumer is just indifferent between consuming good 1 or not consuming it. As the price decreases further, more units of the discrete good will be demanded.

Figure 14: Discrete Good Preferences: Price Offer Curve & Demand Curve

5.1.4 Quasilinear Preferences

For $U(x_1, x_2) = v(x_1) + x_2$, the price offer curve is:

$$(x_1, x_2) = (x_1^*, m - p_1 x_1^*),$$

where x_1^* is fixed.

6 Substitutes and Complements

Since the first chapters, we have been using these 2 preferences as examples to study but we didn't properly define them:

- **Substitutes:** If $\frac{\Delta x_1}{\Delta p_2} > 0$, good 1 is a substitute for good 2.
- **Complements:** If $\frac{\Delta x_1}{\Delta p_2} < 0$, good 1 is a complement to good 2.