Technology

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1 Motivation

This chapter begins the study of firm behavior by examining the technological constraints imposed by nature on production. Just as consumer theory uses tools to understand how individuals maximize utility, production theory employs similar methods to describe how firms transform inputs into observable outputs. The key idea is that there are only certain feasible ways (technologies) to convert inputs into outputs, and these technological constraints limit the choices available to firms.

2 Technological Concepts

2.1 Inputs and Outputs

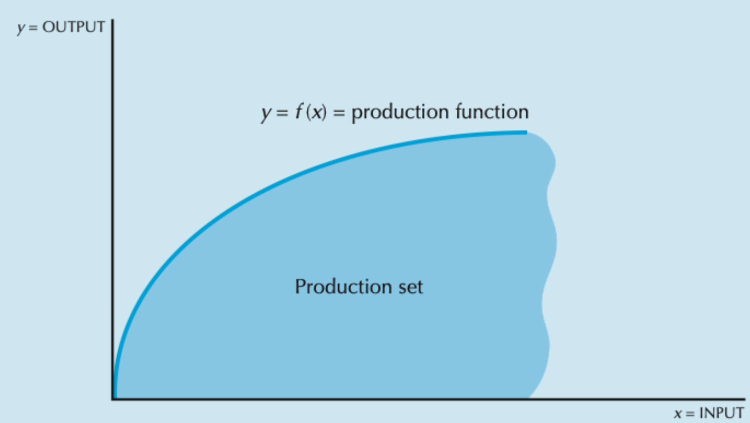
The key concepts to base the chapter are:

- Factors of Production: These are the inputs used in the production process. They are generally classified into categories such as land, labor, capital, and raw materials.
- Capital Goods: Capital goods are produced inputs that are used to produce other goods. They include items like machinery, buildings, computers, and other equipment.
  - Physical Capital Goods: These refer to tangible assets (machines, buildings, vehicles) that are used in production.
  - Financial Capital: This term refers to the money used to start up or maintain a business, distinct from physical capital goods.

For example, consider a **car manufacturing plant**. The plant uses labor (workers), capital goods (robots, assembly lines), and raw materials (steel, plastic) to produce cars.

2.2 Production Set and Function

All technologically feasible input-output combinations make up the production set. Within this set, the production function defines the boundary, showing the maximum output attainable for a given level of inputs. In cases involving multiple inputs, the production function is expressed as a function of all these inputs. An isoquant is a curve that connects all combinations of inputs that yield the same level of output, similar to an indifference curve in consumer theory.



A production set. Here is a possible shape for a production set.

Figure 1: Production Set

2.2.1 Cobb-Douglas Production Function & Car Sector

The Cobb-Douglas production function is a mathematical way to describe how firms convert inputs into output. It takes the form

$$f(x_1, x_2) = Ax_1^a x_2^b,$$

where:

- x_1 and x_2 are the inputs (such as labor and capital),
- A is a constant that represents the overall scale or productivity of the production process -essentially, how much output you get when you use one unit of each input,
- a and b are parameters that measure how sensitive the output is to changes in each input.

Unlike utility functions (where the exact numerical value isn't as important, so we often normalize A to 1 and set $a + b = 1$), the actual level of output matters in production. Therefore, A, a, and b can take on values that accurately reflect the production technology.

An important feature of the Cobb-Douglas production function is its isoquants. An isoquant is a curve that shows all the combinations of inputs that yield the same level of output. The isoquants for a Cobb-Douglas production function have a smooth, convex shape—similar to the Cobb-Douglas indifference curves used in consumer theory—which makes them well-behaved and easy to analyze.

Imagine a **car manufacturer** that uses labor (L) and machinery (capital goods) (K) to produce cars. Suppose the production function is given by

$$f(L, K) = 2L^{0.5}K^{0.5}.$$

Here, $A = 2$ indicates the overall productivity of the car production process. The exponents 0.5 on both L and K (with $0.5 + 0.5 = 1$) suggest constant returns to scale - doubling both labor and capital will double the output.

- Production Function Interpretation: This function tells us the maximum number of cars that can be produced for any given combination of labor and capital. For example, if the manufacturer uses 16 units of labor and 9 units of capital, the maximum number of cars produced is:

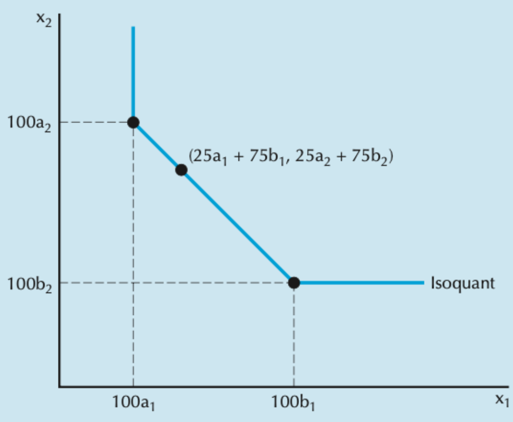
$$f(16, 9) = 2 \times 16^{0.5} \times 9^{0.5} = 2 \times 4 \times 3 = 24 \text{ cars.}$$

- Isoquant Explanation: An isoquant for this function is a curve that connects all combinations of L and K that yield, say, 24 cars. One point on this isoquant is (16, 9) as calculated above. Another might be (9, 16), if that combination also produces 24 cars. The smooth, convex nature of these isoquants makes it clear how the manufacturer can substitute labor for capital (or vice versa) while still producing the same number of cars.

2.3 Technology Properties: Monotonicity and Convexity of Production Techniques

It is common to assume certain properties about technology:

- **Monotonicity (Free Disposal)**: This property states that if you increase the amount of any input (while keeping others constant), you can produce at least as much output as before. In other words, having extra inputs can't hurt production because any surplus can be discarded without cost.
- **Convexity of Production Techniques**: This property means that if you have two different production techniques (input combinations) that each yield the same output, then any weighted average (blend) of these techniques will also produce at least that same output. This also shows that isoquants will have a convex shape. For example, if **Technique A** and **Technique B** each produce 1 unit of output using different amounts of inputs, then **mixing these techniques proportionally will still yield at least 1 unit of output**, showing that diverse, blended production plans are feasible and efficient.



Convexity. If you can operate production activities independently, then weighted averages of production plans will also be feasible. Thus the isoquants will have a convex shape.

Figure 2: Convexity of Production Techniques

3 Marginal Product of Factors and Technical Rate of Substitution

3.1 Marginal Product

The marginal product of an input or factor is the additional output produced by using one more unit of that input while holding other inputs constant. Mathematically, for an input x_1, it is defined as:

$$MP_1(x_1, x_2) = \frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_2} \approx \frac{\partial f(x_1, x_2)}{\partial x_1}.$$

3.1.1 Law of Diminishing Marginal Product

The marginal product diminishes because when you add more of one input while keeping the others fixed, the fixed inputs become a bottleneck. In other words, the additional units of the variable input have less fixed input to work with, so each extra unit contributes less to total output than the previous one.

- For example, in farming, one worker on an acre might produce 100 bushels of corn. Adding another worker might initially double output, but as you continue adding workers, they start to crowd the same piece of land, share tools, and interfere with each other's work. Thus, each new worker produces fewer additional bushels than the previous one.

- The same principle applies to the car manufacturing example. If a factory has a fixed number of machines (capital) and you add more workers (labor) while keeping the machinery constant, each additional worker has less equipment or workspace available. This results in a lower marginal increase in output per additional worker, demonstrating diminishing marginal productivity.

3.1.2 Marginal Product of Cobb-Douglas Car Sector

Recall the Cobb-Douglas production function for a car manufacturer:

$$f(L, K) = 2L^{0.5}K^{0.5},$$

where L represents labor (e.g., hours or number of workers) and K represents capital (e.g., machine hours or capital goods). To calculate the marginal product of labor (MP_L), we take the partial derivative of $f(L, K)$ with respect to L:

$$MP_L = \frac{\partial f(L,K)}{\partial L} = 2 \times 0.5 \times L^{-0.5} \times K^{0.5} = \sqrt{\frac{K}{L}}.$$

For example, if the manufacturer employs $L = 16$ units of labor and uses $K = 9$ units of capital, then

$$MP_L = \sqrt{\frac{9}{16}} = \frac{3}{4} = 0.75.$$

This means that, holding capital constant at 9 units, adding one additional unit of labor increases output by approximately 0.75 cars. Similarly, the marginal product of capital (MP_K) is given by:

$$MP_K = \frac{\partial f(L,K)}{\partial K} = 2 \times 0.5 \times L^{0.5} \times K^{-0.5} = \sqrt{\frac{L}{K}}.$$

Thus, the marginal products indicate how sensitive the production is to changes in each input, providing valuable insight into the efficiency of the production process in the car sector.

3.2 Technical Rate of Substitution

The technical rate of substitution (TRS) tells us how much of one input a firm must add to compensate for a reduction in another input while keeping output/production constant; it is the slope of the isoquant. Mathematically, when output remains fixed, the change in output is given by

$$TRS(x_1, x_2) = \frac{\Delta x_2}{\Delta x_1} = -\frac{MP_1}{MP_2}.$$

3.2.1 Diminishing Technical Rate of Substitution

As previously mentioned, the technical rate of substitution (TRS) is defined as

$$TRS = -\frac{\Delta x_2}{\Delta x_1}.$$

Usually, when you move along an isoquant in the direction of increasing x_1 (i.e., Δx_1 is positive), if the amount of x_2 you can reduce (Δx_2, which is negative) becomes smaller in absolute terms relative to the increase in x_1, then the ratio $|\Delta x_2/\Delta x_1|$ decreases. This means that the TRS diminishes in absolute value.

Conversely, if you decrease x_1 (so Δx_1 is negative) and, to keep output constant, you have to increase x_2 by a larger amount (larger positive Δx_2), then the absolute value of $\Delta x_2/\Delta x_1$ increases, and thus the TRS increases (in absolute value).

3.2.2 TRS of Cobb-Douglas Car Sector

For the Cobb-Douglas production function we discussed in the car sector, assume the production function is

$$f(L,K) = 2L^{0.5}K^{0.5},$$

where L represents labor and K represents capital. The marginal products are:

- Marginal product of labor (MP$_L$): $MP_L(L,K) = \sqrt{\frac{K}{L}}$.
- Marginal product of capital (MP$_K$): $MP_K(L,K) = \sqrt{\frac{L}{K}}$.

Thus, the TRS is:

$$TRS(L,K) = -\frac{MP_L}{MP_K} = -\frac{\sqrt{\frac{K}{L}}}{\sqrt{\frac{L}{K}}} = -\sqrt{\frac{K}{L} \cdot \frac{K}{L}} = -\sqrt{\frac{K^2}{L^2}} = -\frac{K}{L}.$$

The TRS of $-\frac{K}{L}$ tells us that to maintain the same level of output, if the firm reduces labor by one unit, it must increase capital by $\frac{K}{L}$ units. For example, if the firm is currently using $L = 16$ units of labor and $K = 9$ units of capital, then

$$TRS = -\frac{9}{16} \approx -0.5625.$$

This means that for each one unit of labor given up, the firm must add approximately 0.5625 units of capital to keep output unchanged. Essentially, the TRS provides a measure of how easily one input can be substituted for another in the production process without affecting the overall output.

4 The Long Run and the Short Run

4.1 The Short Run

In the short run, at least one factor of production is fixed and cannot be changed. Firms can only adjust the variable inputs.

- A farmer has a fixed amount of land in the short run. He can hire more workers to boost production, but his land remains constant, limiting how much additional output he can generate.
- A car manufacturer may have a fixed plant size or a set number of machines in the short run. While the firm can vary labor or work shifts to increase production, its production capacity is constrained by the existing factory and equipment.

4.2 The Long Run

In the long run, all factors of production are variable. Firms can adjust every input to optimize production and respond to market conditions.

- In the long run, the farmer can purchase additional land or invest in better equipment. This flexibility allows him to expand production beyond the limits imposed by his original, fixed plot of land.
- In the long run, the car manufacturer can build new factories, upgrade machinery, or reconfigure the production process. This complete flexibility lets the firm adjust all inputs to meet demand and improve efficiency.

4.3 Returns to Scale

- Constant Returns to Scale: When a firm experiences constant returns to scale, increasing all inputs by a certain factor results in output increasing by exactly that same factor. For example, if a car manufacturer doubles both labor and capital (e.g., workers and machinery), then the total number of cars produced also doubles.
$$t \cdot f(x_1, x_2) = f(t \cdot x_1, t \cdot x_2) \quad t > 1$$
- Increasing Returns to Scale: Increasing returns to scale occur when a proportional increase in all inputs results in a more than proportional increase in output. In the car manufacturing context, if the company doubles its labor and capital and finds that it produces more than twice the number of cars, it is experiencing increasing returns to scale. This might happen due to efficiencies such as better specialization, improved coordination, or economies of scale.
$$t \cdot f(x_1, x_2) < f(t \cdot x_1, t \cdot x_2) \quad t > 1$$
- Decreasing Returns to Scale: Decreasing returns to scale happen when a proportional increase in all inputs leads to a less than proportional increase in output. For a car manufacturer, this would mean that if the firm doubles its labor and capital, the number of cars produced increases by less than double. This might occur because of management inefficiencies, coordination problems, or limitations in technology when scaling up production.
$$t \cdot f(x_1, x_2) > f(t \cdot x_1, t \cdot x_2) \quad t > 1$$