Choice

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January 30, 2025

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1 Introduction

Building on the previous chapter on utility, we now focus on how consumers make choices to maximize their utility subject to their budget constraints. This chapter introduces the concept of the optimal choice and explores how preferences and prices influence consumer demand.

2 Optimal Choice

The optimal choice (x_1^*, x_2^*) is the consumption bundle that maximizes the consumer's utility while satisfying their budget constraint. At the optimal choice:

- The indifference curve is tangent to the budget line if the solution is an **interior optimum**.
- There is no intersection between the set of affordable bundles and the set of bundles the consumer prefers to (x_1^*, x_2^*).
- The necessary condition for an optimal choice is:

$$MRS = -\frac{p_1}{p_2}.$$

- This ensures that the consumer's personal trade-off between the goods (MRS) matches the market trade-off (price ratio). If $MRS \neq -\frac{p_1}{p_2}$, the consumer can still improve their utility.

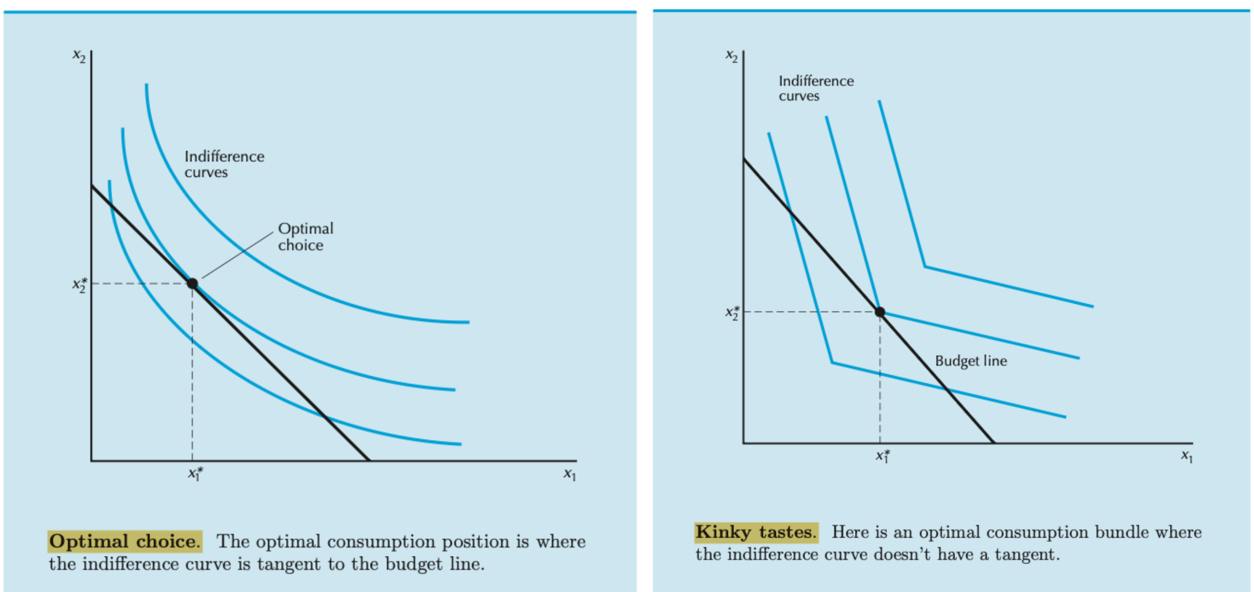


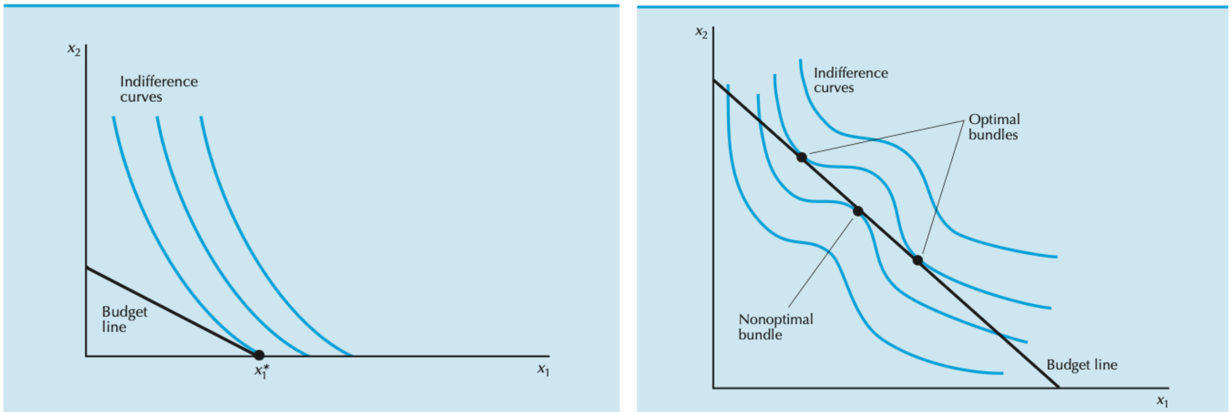
Figure 1: Well-Behaved & Kinky Tastes Optimal Choice

2.1 Boundary vs. Interior Optimum

- A **boundary optimum** occurs when the consumer consumes all of one good and none of the other (e.g., perfect substitutes or concave preferences).
- An **interior optimum** occurs when the consumer consumes some of both goods. For interior optima:

$$MRS = -\frac{p_1}{p_2}.$$

For convex preferences, satisfying this tangency condition is sufficient to ensure the choice is optimal, but for non-convex preferences, then it becomes not sufficient.



Boundary optimum. The optimal consumption involves consuming zero units of good 2. The indifference curve is not tangent to the budget line.

More than one tangency. Here there are three tangencies, but only two optimal points, so the tangency condition is necessary but not sufficient.

Figure 2: Boundary Optimum & Non-Convex Preferences

3 Consumer Demand

3.1 Demand Function

The consumer's demand bundle is the optimal combination of goods at given prices (p_1, p_2) and income m. The demand functions are:

$$x_1(p_1, p_2, m), \quad x_2(p_1, p_2, m).$$

These relate the optimal quantities of goods demanded to the consumer's income and the prices of the goods.

3.2 Examples of Consumer Demand

3.2.1 Perfect Substitutes

The utility functions is $u(x_1, x_2) = p_1 x_1 + p_2 x_2$.

- If $p_2 > p_1$, the consumer spends all their money on good 1: $x_1 = \frac{m}{p_1}, \quad x_2 = 0$.
- If $p_1 > p_2$, the consumer spends all their money on good 2: $x_1 = 0, \quad x_2 = \frac{m}{p_2}$.
- If $p_1 = p_2$, any bundle on the budget line is optimal.

3.2.2 Perfect Complements

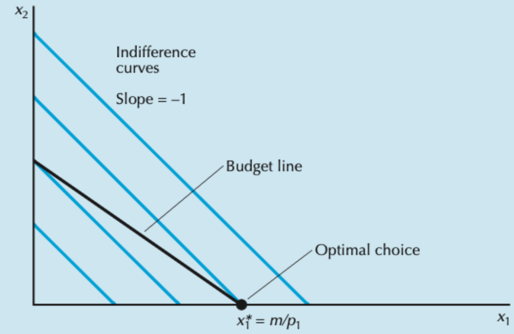
- Utility: $u(x_1, x_2) = \min\{ax_1, bx_2\}$.
- The consumer always consumes goods in fixed proportions: $x_1 = x_2 = \frac{m}{p_1 + p_2}$.

3.2.3 Neutral Goods and Bads

- A neutral good does not affect the consumer's utility.
- If good 1 is a good and good 2 is a bad, the consumer spends all their money on good 1: $x_1 = \frac{m}{p_1}, \quad x_2 = 0$.

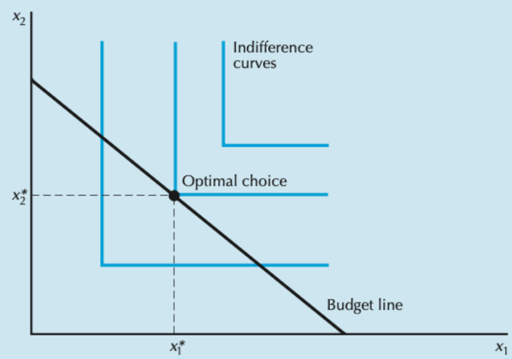
3.2.4 Discrete Goods

For discrete goods, the consumer compares the utility of each feasible bundle and chooses the one with the highest utility.



Optimal choice with perfect substitutes. If the goods are perfect substitutes, the optimal choice will usually be on the boundary.

Figure 3: Perfect Substitutes Optimal Choice



Optimal choice with perfect complements. If the goods are perfect complements, the quantities demanded will always lie on the diagonal since the optimal choice occurs where x_1 equals x_2.

Figure 4: Perfect Completements Optimal Choice

3.2.5 Concave Preferences

For concave preferences, the optimal choice is always a boundary solution, where the consumer consumes only one of the goods.

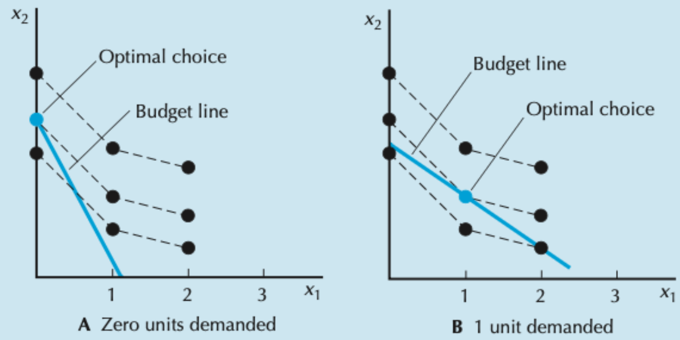
3.2.6 Cobb-Douglas Preferences

For Cobb-Douglas utility $u(x_1, x_2) = x_1^c x_2^d$, the optimal choices are $(x_1, x_2, p_1, p_2, m > 0)$:

$$x_1 = \frac{c}{c+d} \cdot \frac{m}{p_1}, \quad x_2 = \frac{d}{c+d} \cdot \frac{m}{p_2}.$$

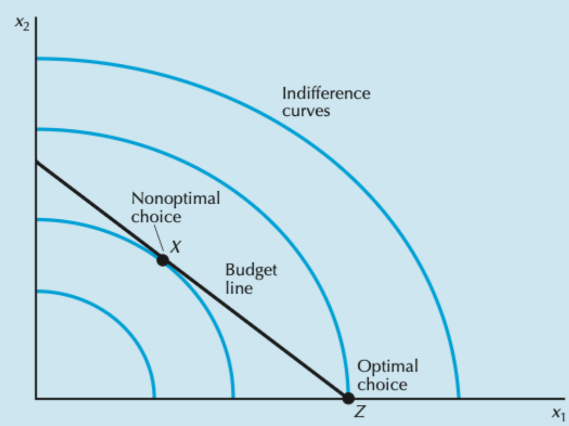
These solutions can be derived using three different approaches:

1. **Substitution Method:** Substitute x_2 (from the budget constraint) into the utility function, differentiate, and solve.
2. **MRS Method:** Set $MRS = -\frac{p_1}{p_2}$, solve with the budget constraint.



Discrete goods. In panel A the demand for good 1 is zero, while in panel B one unit will be demanded.

Figure 5: Discreet Goods Optimal Choice



Optimal choice with concave preferences. The optimal choice is the boundary point, Z, not the interior tangency point, X, because Z lies on a higher indifference curve.

Figure 6: Concave Preferences Optimal Choice

3. **Lagrange Method:** Solve using the Lagrange function: $L = x_1^c x_2^d + \lambda(m - p_1 x_1 - p_2 x_2)$.

Example: For instance, let's derive the formula for the optimal choice under Cobb-Douglas preferences using the **substitution approach**. By expressing x_2 in terms of x_1 using the budget constraint, we obtain: $x_2 = \frac{m - p_1 x_1}{p_2}$.
Now, we substitute x_2 into the utility function $U(x_1, x_2) = x_1^c \left(\frac{m-p_1x_1}{p_2}\right)^d = x_1^c \cdot \frac{(m-p_1x_1)^d}{p_2^d}$.
Then, we just need to maximize the utility with respect to x_1:

$$\frac{dU}{dx_1} = cx_1^{c-1} \cdot \frac{(m-p_1x_1)^d}{p_2^d} - dx_1^c \cdot \frac{p_1(m-p_1x_1)^{d-1}}{p_2^d} = \frac{x_1^{c-1}(m-p_1x_1)^{d-1}}{p_2^d}\left[c(m-p_1x_1) - dp_1x_1\right]$$

Setting this derivative to 0 ($x_1, x_2 > 0, p_2 > 0$ and $x_2 = \frac{m-p_1x_1}{p_2} > 0 \implies m - p_1x_1 > 0$) :

$$c(m - p_1x_1) - dp_1x_1 = 0 \longleftrightarrow x_1 = \frac{cm}{p_1(c+d)} \implies x_2 = \frac{dm}{p_2(c+d)}.$$

4 Choosing Taxes: Quantity Tax vs. Income Tax

4.1 Quantity Tax

A quantity tax increases the price of a good by t. The new budget constraint is:

$$(p_1 + t)x_1 + p_2x_2 = m.$$

The government collects revenue (the optimal choice of the consumer):

$$R = tx_1^*.$$

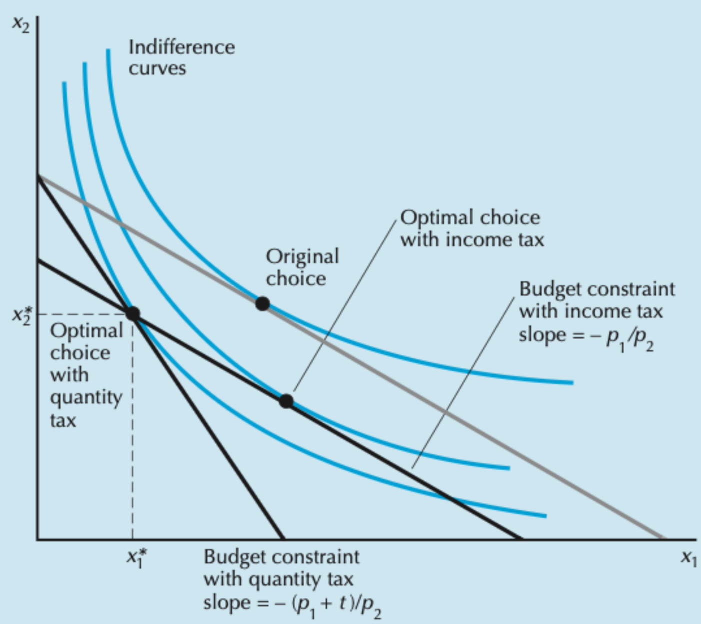
4.2 Income Tax

An income tax reduces the consumer's income by R. The new budget constraint is:

$$p_1x_1 + p_2x_2 = m - R = m - tx_1^* \quad \longleftrightarrow \quad p_1x_1 + p_2x_2 = m - tx_1^*.$$

4.3 Comparison

- A quantity tax distorts the consumer's choices because it changes the price ratio.
- An income tax is less distortionary because it reduces overall income but leaves the price ratio unchanged.
- The consumer is generally better off under an income tax, as they can choose a bundle on a higher indifference curve.



Income tax versus a quantity tax. Here we consider a quantity tax that raises revenue R^* and an income tax that raises the same revenue. The consumer will be better off under the income tax, since he can choose a point on a higher indifference curve.

Figure 7: Quantity Tax vs Income Tax