Firm Supply

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1 Motivation

In this chapter we will see how to derive the supply curve of a competitive firm from its cost function using the model of profit maximization.

2 Market Environments

When a firm decides how much to produce and at what price to sell, it must operate under several layers of constraints, in particular the technological and economic constraints:

- Technological Constraints: These are the physical and production-based limitations a firm faces, dictated by its production function. The production function outlines which combinations of inputs (like labor, capital, and raw materials) can produce certain levels of output. No matter how eager a firm is to maximize profit, it must adhere to the physical realities of what can be produced.
- Economic Constraints: These constraints emerge from the technological constraints and are represented by the cost function. While technological constraints tell us what is physically possible, the cost function shows the economic feasibility by mapping production levels to the associated costs. Even if a certain level of output is physically attainable, high costs might render it economically unviable.

Besides the previous reminders, we need to define the following terms:

- Market Constraints: Market constraints are limitations imposed by the behavior of consumers—specifically, how much of a good people are willing to buy at different prices. Regardless of a firm's ability to produce or set a price, it can only sell as much as consumers are willing to purchase. This consumer behavior is captured by the demand curve.
- Demand Curve Facing the Firm: This curve represents the relationship between the price a firm sets and the quantity of the product that will be sold.
  - For a **monopolist** (a firm with a market all to itself): The demand curve facing the firm is simply the market demand curve, which shows the overall consumer willingness to pay for various quantities.
  - For **firms in a competitive market**: The situation becomes more complex, as each firm must consider how other firms' pricing and output decisions will affect its own sales.
- Market Environment: This term refers to the overall setting in which firms make their pricing and production decisions, including how they interact and respond to one another. The market environment encapsulates the idea that a firm's actions are not made in isolation but are influenced by competitors. Firms must anticipate and react to the behavior of others when deciding on price and output.
- Pure Competition: Pure competition is the simplest type of market environment where many small firms compete, and each firm is a price taker. In pure competition, no single firm has enough market power to influence the market price; instead, the **market price is determined by overall supply and demand**. The demand curve facing an individual firm in this environment is perfectly elastic (i.e., horizontal) at the market price because any attempt to charge a higher price would result in losing all its customers.

2.1 Pure Competition in Markets

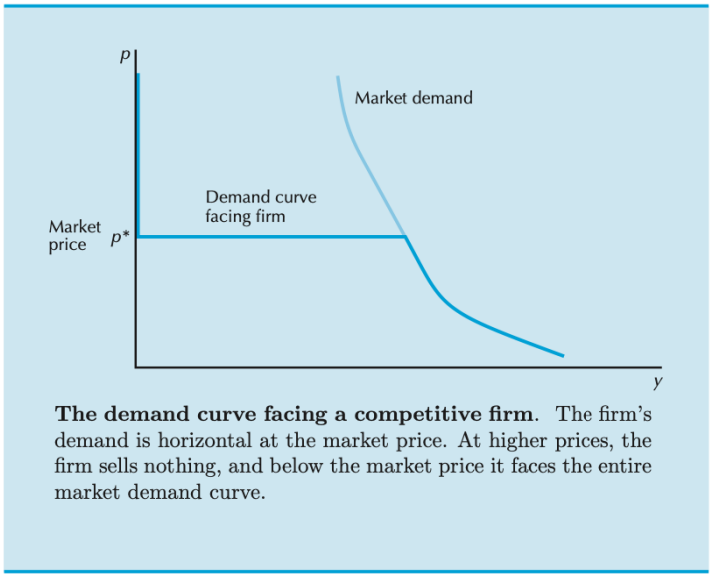
The following characteristics can be described under a purely competitive market scenario:

- Price Taker: In pure competition, each firm is a price taker, meaning it accepts the market price as given—its own production level cannot affect this price.
- **Market Demand Curve vs. Demand Curve Facing a Firm:**
  - **Market Demand Curve:** Shows the relationship between the overall market price and the total quantity demanded by all consumers.

- Demand Curve Facing a Firm: For a competitive firm, this curve is perfectly horizontal at the market price. It indicates that the firm can sell any quantity it produces at that fixed price (up to its production or stock limit).

- **Firm's Pricing Choices Relative to the Market Price:**
  - Increasing the Price: If a firm charges more than the market price, it sells nothing because buyers will choose competitors.
  - Following the Market Price: Selling at the market price allows the firm to sell as much as it produces without affecting the price.
  - Decreasing the Price: Lowering the price below the market level is unnecessary and counterproductive because the firm is already able to sell all its output at the market price, and reducing the price only cuts revenue.

In a purely competitive market, **the firm's only strategic decision is how much to produce**. Since it is a price taker, the market price is fixed by overall supply and demand, and its own actions do not change that price.



The demand curve facing a competitive firm. The firm's demand is horizontal at the market price. At higher prices, the firm sells nothing, and below the market price it faces the entire market demand curve.

Figure 1: Pure Competition

2.2 The Supply Decision of a Competitive Firm

A competitive firm's supply decision is all about choosing the output level that maximizes its profit. The firm asks itself, how much should I produce?

- **Profit Maximization:** The firm wants to maximize its profit, which is total revenue minus total costs:

$$\text{Profit} = py - c(y)$$

  where p is the market price (which the firm takes as given), y is the quantity produced, and $c(y)$ is the cost of producing y.

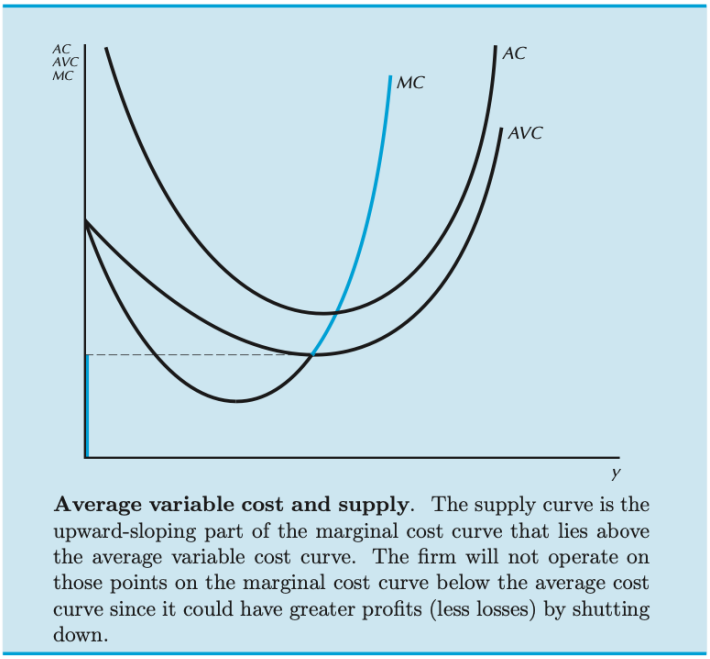
- **Marginal Analysis:** To find the optimal output, the firm considers what happens when it produces one more unit. The extra revenue from selling one more unit is the market price p (since $\Delta R = p\,\Delta y$). The extra cost of producing one more unit is called the marginal cost, $MC(y)$.

- **Equating Marginal Revenue and Marginal Cost:** Profit is maximized when the additional revenue equals the additional cost, i.e., when:

$$p = MC(y)$$

- If $p > MC(y)$, the firm can increase profit by producing more because the revenue from selling one more unit exceeds the cost of producing it.
- If $p < MC(y)$, the firm should produce less because the cost of producing an additional unit is higher than the revenue it would bring in.

- **Supply Curve Identity:** In this setup, the firm's marginal cost curve represents its supply curve. For any given market price p, the optimal production level y is found where $p = MC(y)$.



Average variable cost and supply. The supply curve is the upward-sloping part of the marginal cost curve that lies above the average variable cost curve. The firm will not operate on those points on the marginal cost curve below the average cost curve since it could have greater profits (less losses) by shutting down.

Figure 2: Supply Curve

2.2.1 Short Scenario

Let's use the following cost function with **fixed costs** (100, which in the short run are high and will lead to losses at first for the firm) and **variable costs** ($5y + y^2$):

$$c(y) = 100 + 5y + y^2$$

which gives us the marginal cost

$$MC(y) = \frac{\partial c(y)}{\partial y} = 5 + 2y.$$

Recall that when the market price is $p = \$15$, the profit-maximizing output is determined by

$$p = MC(y) \quad \Rightarrow \quad 15 = 5 + 2y \iff y = 5.$$

Now, let's see what happens if the firm produces a different output:

- **Case 1:** Producing $y = 4$ (Below the Optimal Level)

  The revenue would be $R(4) = 15 \times 4 = \$60$, the cost $c(4) = 100 + 5(4) + 4^2 = 100 + 20 + 16 = \$136$ and the profit made $\pi(4) = R(4) - c(4) = 60 - 136 = -\$76$. At $y = 4$, the firm is producing less, specifically not producing the unit where the marginal cost equals the market price. Notice that the marginal cost of the 4th unit is

$$MC(4) = 5 + 2(4) = 13$$

- **Case 2:** Producing $y = 6$ (Above the Optimal Level)

  The revenue would be $R(6) = 15 \times 6 = \$90$, the cost $c(6) = 100 + 5(6) + 6^2 = 100 + 30 + 36 = \$166$ and the profit made $\pi(6) = R(6) - c(6) = 90 - 166 = -\$76$. At $y = 6$, the cost of producing the extra (6th) unit is

  $$MC(6) = 5 + 2(6) = 17,$$

  which exceeds the market price of \$15. This means that the 6th unit adds more to cost than to revenue, reducing overall profit.

- **Case 3:** Optimal Production at $y = 5$:

  The revenue would be $R(5) = 15 \times 5 = \$75$, the cost $c(5) = 100 + 5(5) + 5^2 = 100 + 25 + 25 = \$150$ and the profit made $\pi(5) = R(5) - c(5) = 75 - 150 = -\$75$. At $y = 5$, the firm produces exactly where $MC(5) = 15$. This is the point where any additional unit would cost more than what it earns (if produced) and any less means the firm misses out on a profitable opportunity. Even though the firm might be incurring a loss overall (due to fixed costs), producing 5 units minimizes the loss relative to producing either 4 or 6 units.

Price equals marginal cost is a necessary condition for profit maximization.
It is not in general a sufficient condition.

2.3 Shutdown Condition in Competitive Firms

Recall that the cost function is:

$$c(y) = 100 + 5y + y^2, \quad \text{with fixed cost } F = 100 \text{ and variable cost } cv(y) = 5y + y^2.$$

Then, the Average Variable Cost (AVC):

$$AVC(y) = \frac{c_v(y)}{y} = \frac{5y + y^2}{y} = 5 + y.$$

The Shutdown Rule states that if the market price p is less than the minimum AVC, the firm should produce zero output. In this case, the minimum value for the AVC is 5 as the firm can decide to produce $y = 0$.

- **Case 1: Price Below AVC** Let $p = 4$. Even at $y = 1$, $AVC(1) = 5 + 1 = 6$, which is greater than $p = 4$. This means that producing any positive amount (even $y = 1$) yields revenue 4 per unit, which doesn't cover the variable cost of 6 per unit. Then, the sensible decision for the firm is to shut down (produce $y = 0$) and only lose its fixed cost (\$100).

- **Case 2: Price Above AVC** Let $p = 15$. The optimal output was $y = 5$, and has an AVC of $AVC(5) = 5 + 5 = 10$ which is below $p = 15$, so production covers variable costs. Then, the sensible decision for the firm is to produce $y = 5$ units. where $p = MC(y)$ (here, $y = 5$).

2.4 Profits and Producer's Surplus

2.4.1 Total Revenue (TR)

The total revenue is the total amount of money received from selling the output, determined as follows:

$$TR = p \times y,$$

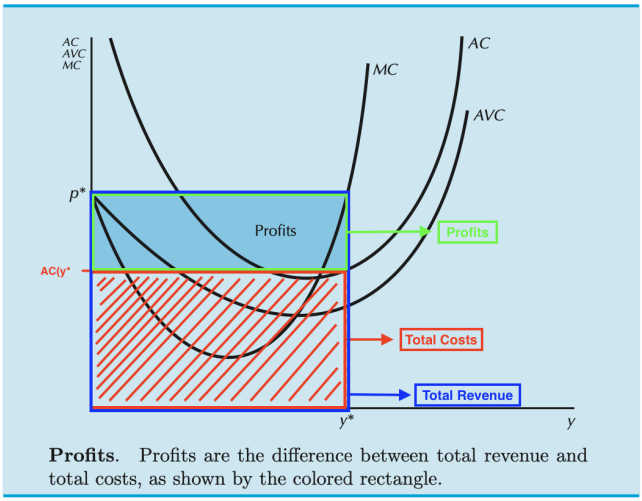
where p is the market price and y is the quantity produced.

2.4.2 Profits

The profits are defined as the difference between total revenue and total costs.

$$\text{Profits} = p \cdot y - c(y) = p \cdot y - [c_v(y) + F],$$

where $c_v(y)$ is the variable cost and F is the fixed cost.



Profits. Profits are the difference between total revenue and total costs, as shown by the colored rectangle.

Figure 3: Profits

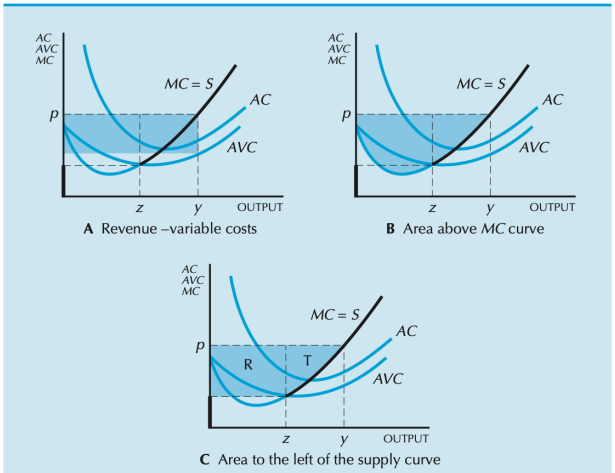
2.4.3 Producer's Surplus (PS)

The producer's surplus is defined as the extra benefit a firm gets by selling at a market price that exceeds its variable cost. Equivalently, it is the firm's profits plus its fixed costs.

$$PS = p \cdot y - c_v(y).$$

There are 3 ways to measure producer's surplus:

- **Method 1:** The difference between the "revenue box" (total revenue) and the "variable cost box" (total variable cost).
- **Method 2:** The area above the marginal cost (MC) curve up to the chosen output y.
- **Method 3:** By combining the area of a box up to the output z (where the AVC curve intersects the MC curve) with the area above the MC curve beyond z.

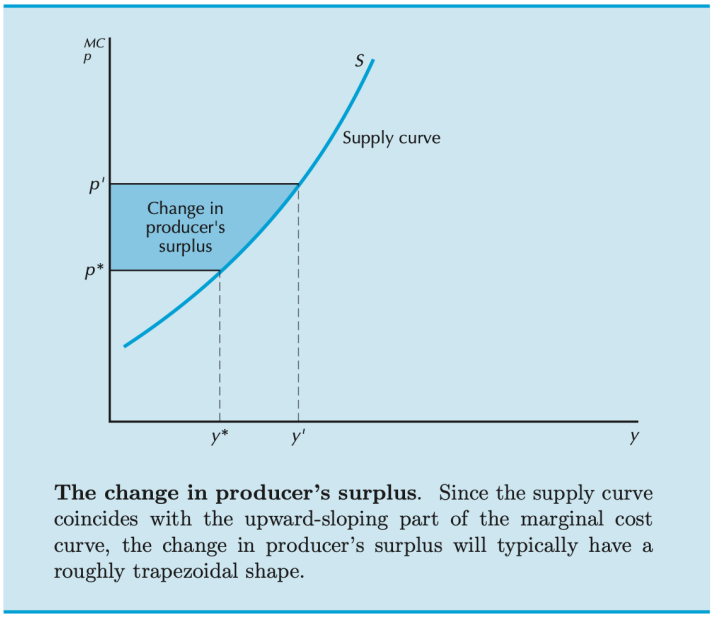


Producer's surplus. Three equivalent ways to measure producer's surplus. Panel A depicts a box measuring revenue minus variable cost. Panel B depicts the area above the marginal cost curve. Panel C uses the box up until output z (area R) and then uses the area above the marginal cost curve (area T).

Figure 4: Producer's Surplus Methods

2.4.4 Change in Producer's Surplus

When a firm changes its output from y^* to y', the change in producer's surplus is the additional area (usually a trapezoidal region) that represents the extra revenue (over variable costs) earned from the additional units produced.



The change in producer's surplus. Since the supply curve coincides with the upward-sloping part of the marginal cost curve, the change in producer's surplus will typically have a roughly trapezoidal shape.

Figure 5: Change in Producer's Surplus

2.4.5 Short Scenario

Coming back to our previous example:

- **Cost Function:** $c(y) = 100 + 5y + y^2$
- **Fixed Cost:** $F = 100$
- **Variable Cost:** $cv(y) = 5y + y^2$
- **Given Market Price:** $p = 15$
- **Optimal Output** (using $p = MC(y)$): The **Marginal Cost** is $MC(y) = 5+2y$, then $15 = 5+2y \iff y^* = 5$
- **Total Revenue (Optimum):** $TR = 15 \times 5 = 75$
- **Variable Cost:** $c_v(5) = 5(5) + 5^2 = 25 + 25 = 50$
- **Total Cost:** $c(5) = c_v(5) + F = 50 + 100 = 150$
- **Profits** $= TR - c(5) = 75 - 150 = -75$. (The loss here is due to the high fixed cost.)
- Producer's Surplus: $PS = TR - c_v(5) = 75 - 50 = 25$

Even though the firm makes a loss (because the fixed cost isn't covered), its producer's surplus is positive. This surplus represents the benefit from selling at a price above the variable cost. If the firm were to change its output from $y^* = 5$ to another level y', the change in producer's surplus would be shown by the additional area (trapezoidal in shape) between the two output levels on the graph.

2.5 Competitive Firm: Long Run vs. Short Run

2.5.1 Long-Run vs. Short-Run Supply

The Short-Run Supply Curve is determined by setting price equal to marginal cost when some inputs (e.g., plant size) are fixed.

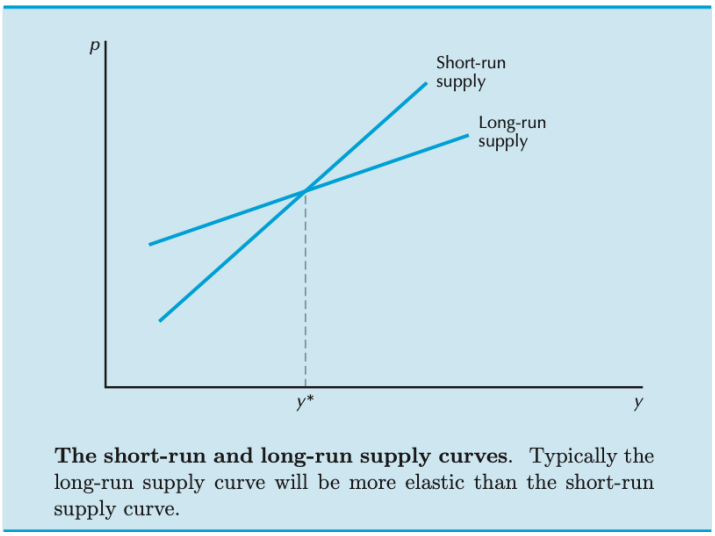
$$p = MC(y, k) \quad \text{(with } k \text{ fixed)}$$

The Long-Run Supply Curve is determined by setting price equal to marginal cost when all inputs can be adjusted optimally.

$$p = MC_\ell(y) = MC(y, k(y)) \quad \text{(where } k(y) \text{ is chosen to minimize cost)}$$

2.5.2 Coincidence at Optimal Output (y^*)

In the short run, the firm's chosen fixed factor (k^*) might be exactly the one it would choose in the long run. At that output (y^*), the short-run and long-run marginal costs (and thus supply curves) coincide.



The short-run and long-run supply curves. Typically the long-run supply curve will be more elastic than the short-run supply curve.

Figure 6: Intersection of Long & Short Run Curves

2.5.3 Elasticity Difference

Because the firm can adjust more inputs in the long run, the long-run supply curve is typically more elastic than the short-run supply curve. A change in price allows for more adjustments (e.g., changing plant size), leading to a bigger response in output.

2.5.4 Zero-Profit Condition in the Long Run

In the long run, a firm can exit the market, so long-run profits cannot be negative. The price must at least cover average cost:

$$p \geq \frac{c(y)}{y}.$$

Therefore, the relevant portion of the long-run supply curve is the upward-sloping part of the marginal cost curve above the long-run average cost curve (i.e., the portion where the firm can cover all costs, including fixed costs).