Université Paris Saclay	MSc AI & MSc Data Science.
TC2-Optimization for Machine Learning	Faïcel Chamroukhi

Mid-Term Exam. November 28th. 2024. Duration: 45'.

Instructions:

- **Only pens are allowed:** Documents, calculators, phones, headphones, computers, tablets, are forbidden.
- It is forbidden to write with a pencil or a **red pen**.
- Your double exam sheet must include, in the designated area, your last name, first name, and signature.
- This designated area must be concealed by gluing.
- All your supplementary sheets must be numbered.
- The grading scale is provided for reference only.

Exercice 1. 4pts (2; 2)

1. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a differentiable function. What are the first order optimality conditions (FOC) of optimizing f without constraints ✓
2. Let $f : \mathbb{R}^n \to \mathbb{R}$ be a twice-differentiable function. What are the second order optimality conditions (SOC) of optimizing f without constraints. ✓

Exercice 2. 8pts (2; 3; 3)

1. Let $f(x) = x^4 - 4x^2 + 1$. Is $f(x)$ convex? Provide justification. ✓
2. Let $f(x) = \max(x, 0)$. Is $f(x)$ convex? Provide justification. ✓
3. Show that the function $\sigma(x) = \frac{1}{1+e^{-x}}$ is convex when $x \leq 0$ by proving that its second derivative is non-negative. Not finished

Exercice 3. 8pts (4;2;2)
Consider the quadratic function $f(x) = \frac{1}{2}x^\top Ax + b^\top x + c$, where $A \in \mathbb{R}^{n \times n}$ is a symmetric matrix, x and $b \in \mathbb{R}^n$, and $c \in \mathbb{R}$.

1. Calculate the gradient and the Hessian of $f(x)$. ✓
2. Discuss how the convexity properties of $f(x)$ change depending on A. ✓
3. Given $A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, determine if $f(x)$ is convex. ✓