

TIME-SCALE (WAVELET) ANALYSIS

M2 AI — SIGNAL PROCESSING

CONTINUOUS WAVELET TRANSFORM

Idea: be sensitive to irregularities instead of oscillations

Let ψ be an admissible wavelet and its dilated and translated versions

$$\psi_{a,b}$$

The continuous wavelet transform is given by:

$$CWT_{\psi}(f)(a,b)$$

$|CWT_{\psi}(f)(a,b)|^2$ is called the magnitude scalogram

Properties

It is a time-scale transform

It is invertible: ψ

We have energy preservation

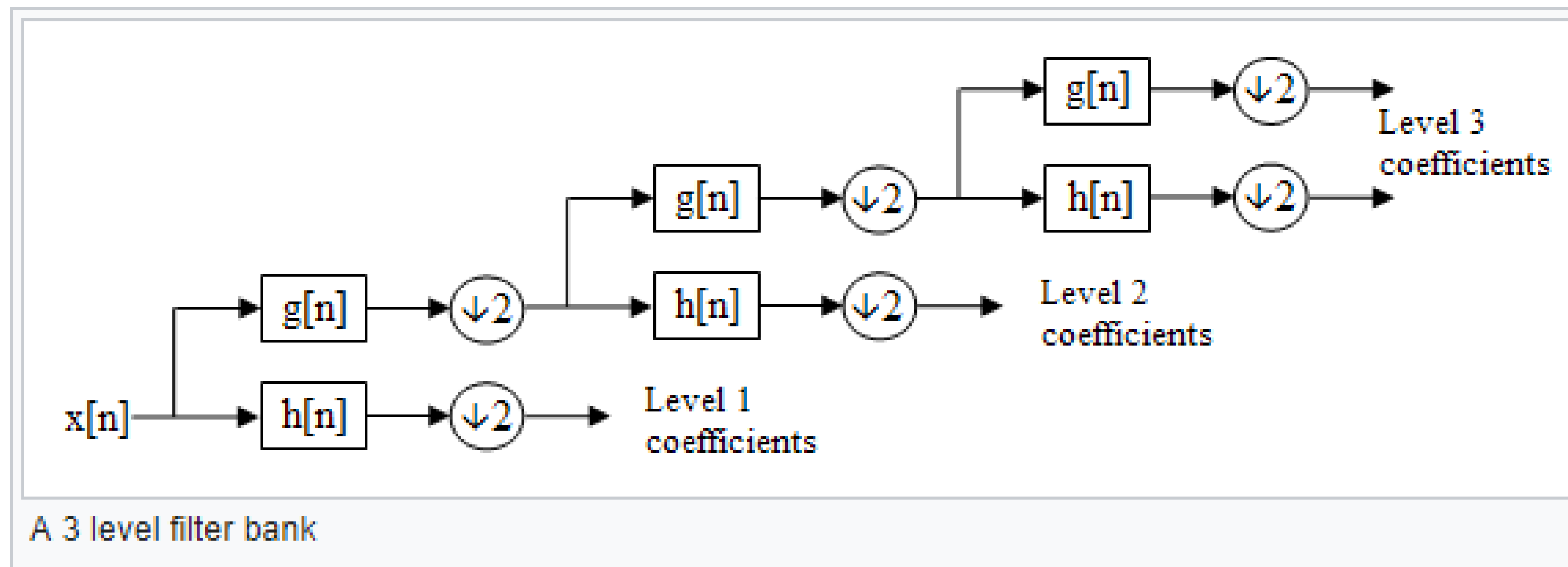
ORTHOGONAL WAVELET TRANSFORM

Comes from “multiresolution” analysis on a dyadic scale

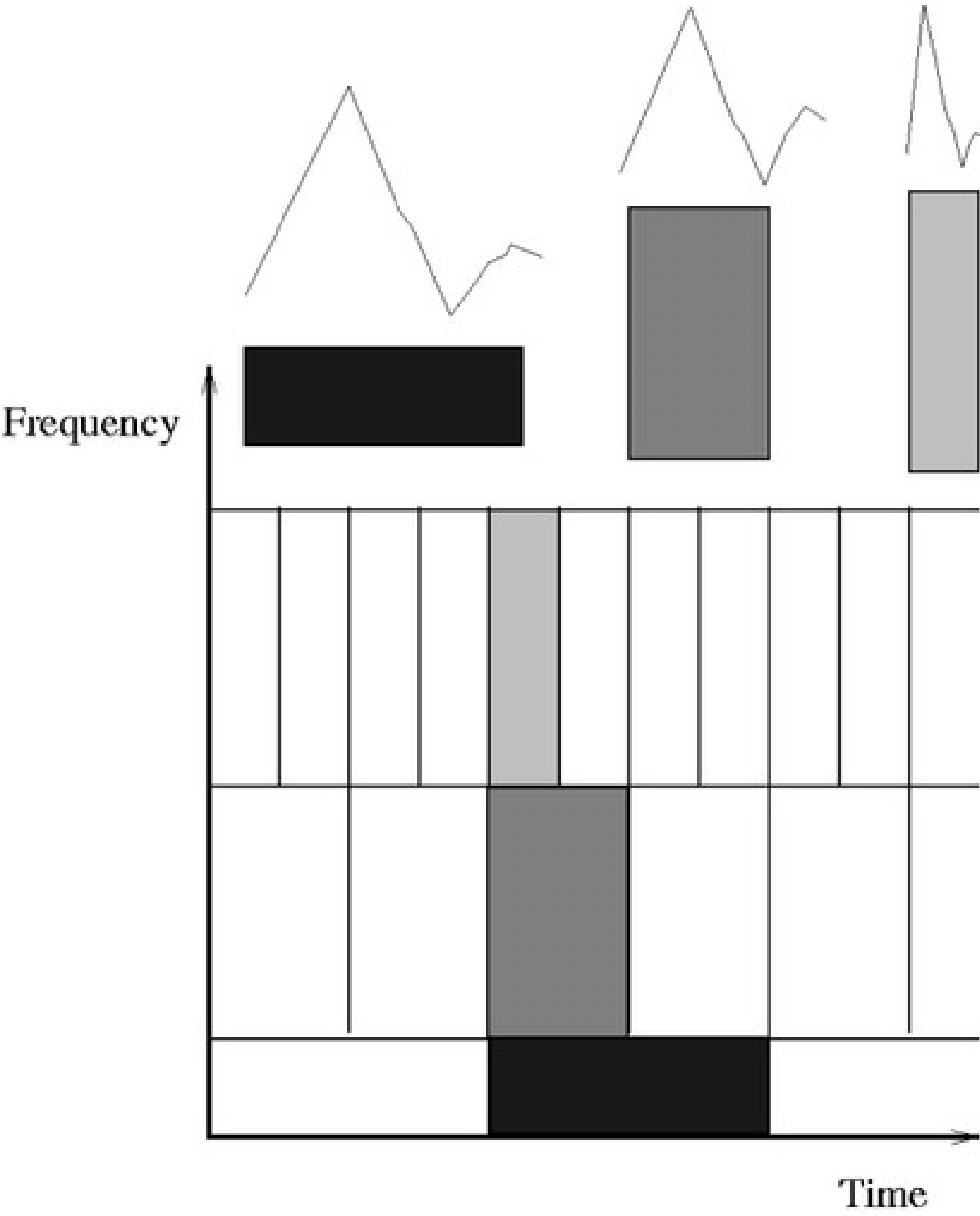
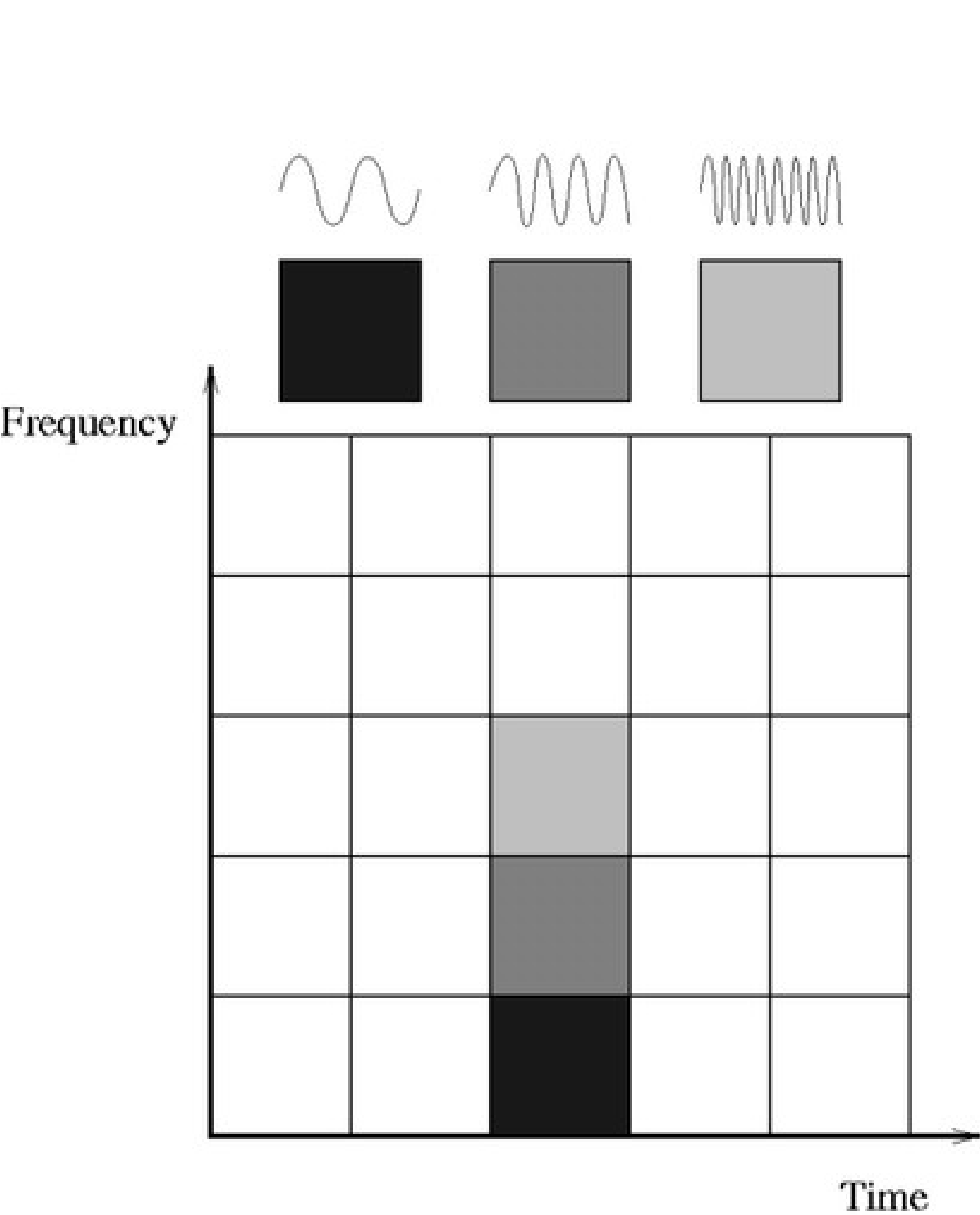
In practice, a Fast (orthogonal) Wavelet Transform can be computed thanks to filter bank and subsampling

The same filter bank can be used to reconstruct the signal from the wavelet coefficients

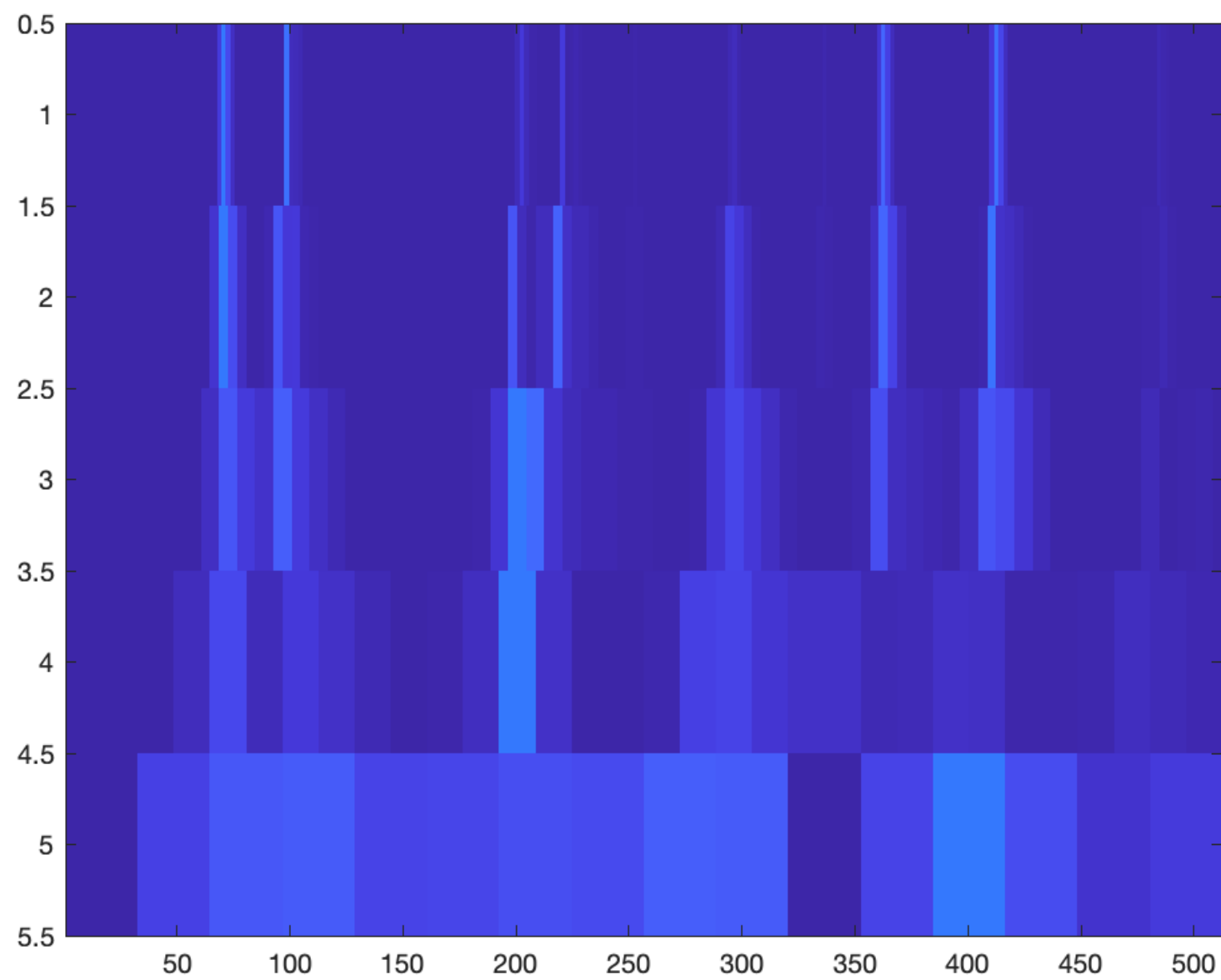
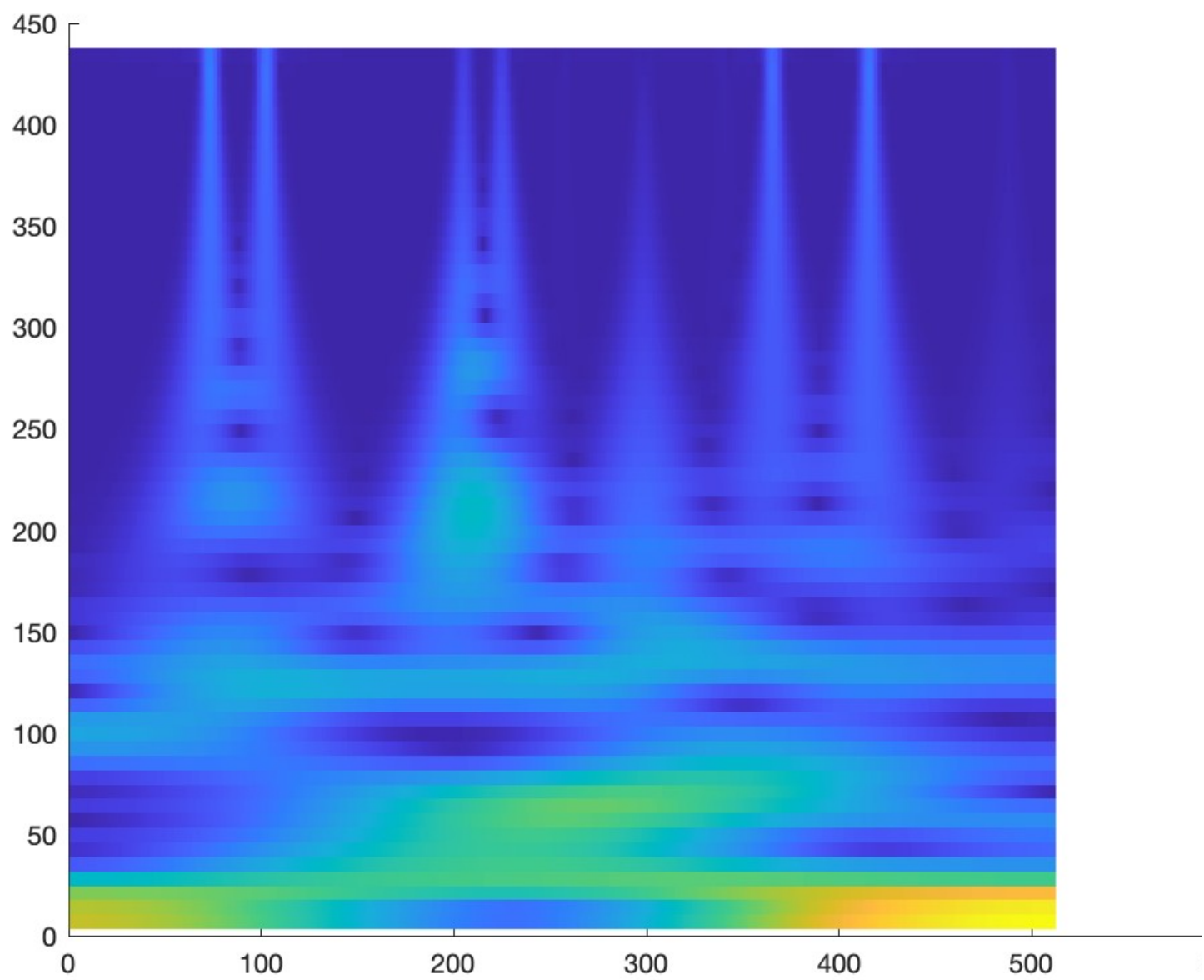
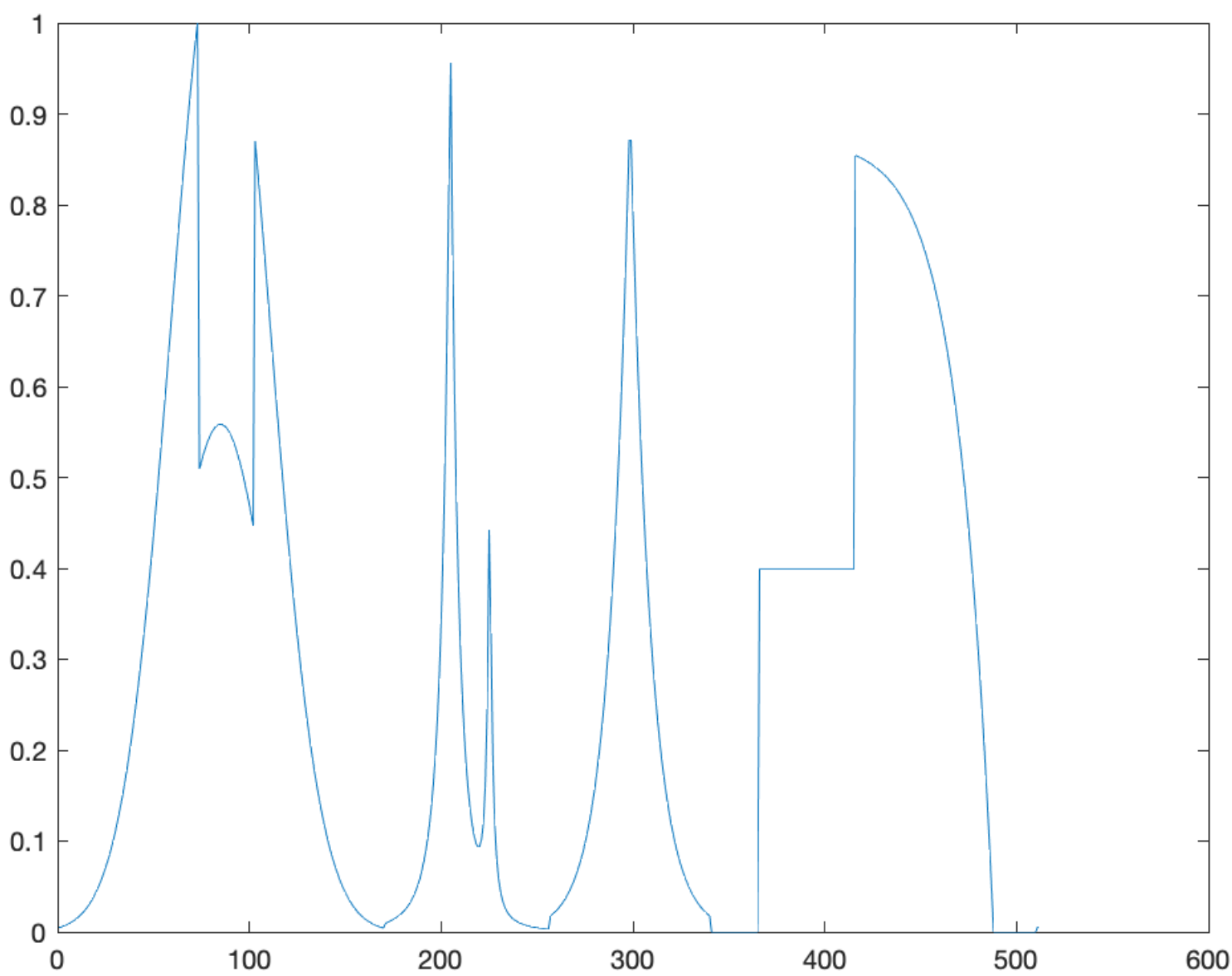
A wavelet is then fully determined by two filters (which must fulfill certain conditions): a low pass filter $g[n]$ and a high pass filter $h[n]$



TIME-FREQUENCY TILING



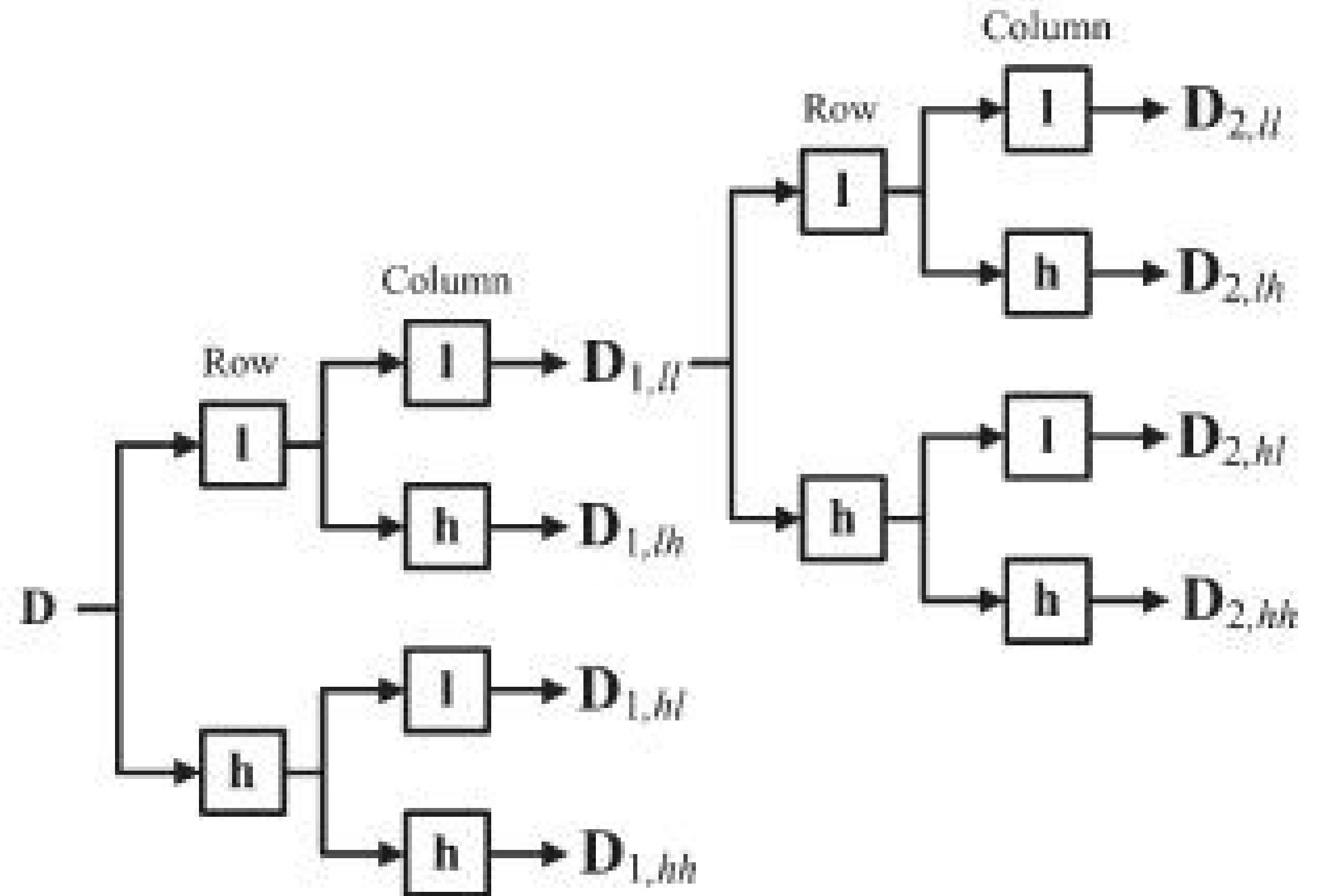
SCALOGRAM: CONTINUOUS VS ORTHOGONAL



2D FWT

For images, we need a 2D FWT

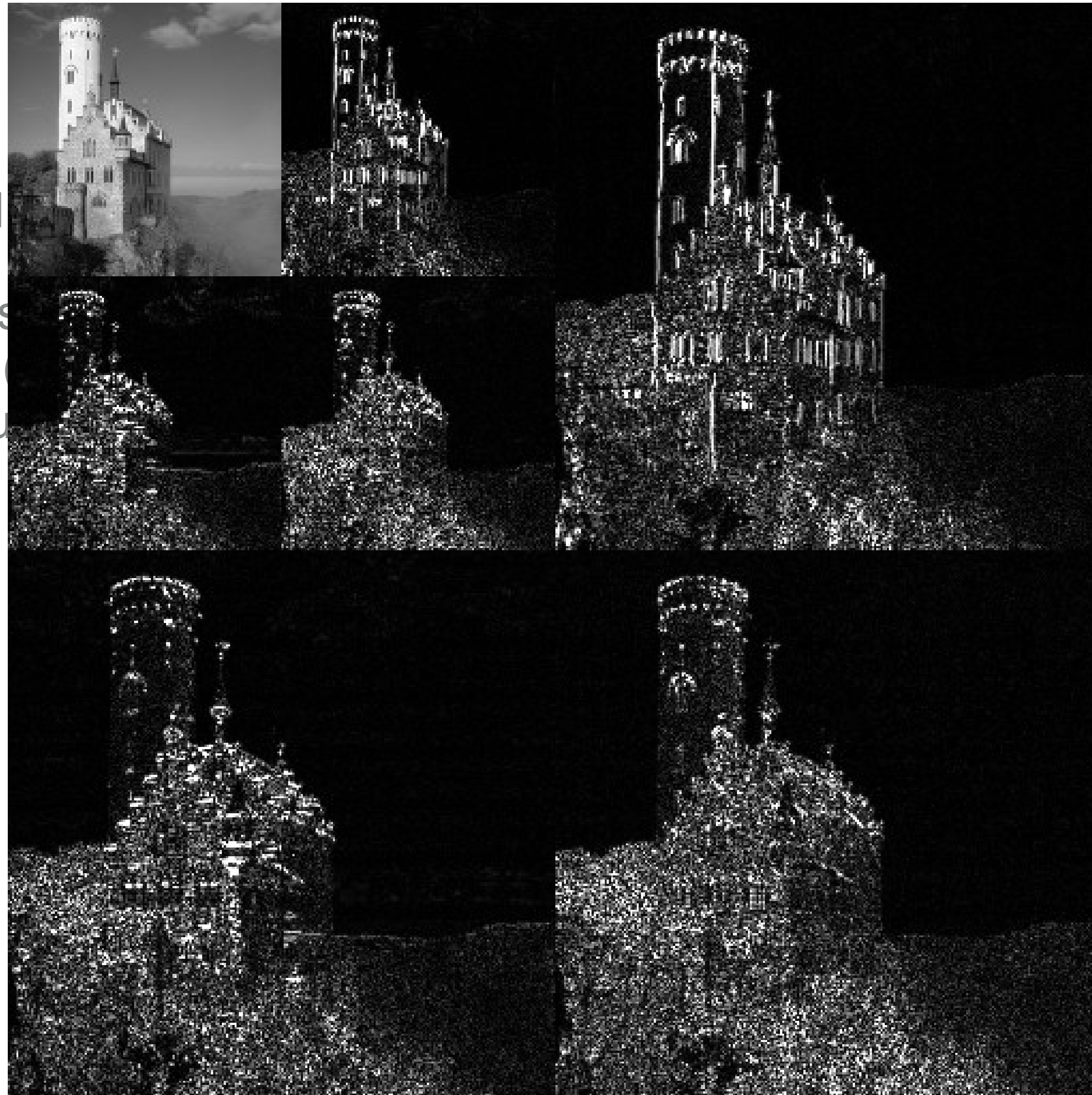
It is constructed by using a separable filter bank (we first filter the row, then the column with the same filters)



2D FWT

For images, we need

It is constructed by using a separable filter bank (first the row, then the column, using the same filters)



TO DO: DENOISING IN THE WAVELET DOMAIN

Data

Image you want

Todo

Simulate a noisy version of the image using the noises at various SNR Level (0 db, 10 dB and 20 dB)

Denoise the image by performing hard thresholding and soft thresholding in the wavelet domain

Discuss the parameters (wavelet type, number of level, type of thresholding)

Denoise the image by implementing translation invariant wavelet transform (see the numerical tour on image wavelet denoising)