

FILTERING

M2 AI — SIGNAL PROCESSING

SIGNAL AND SYSTEMS

A signal is recorded, and distorted, by a sensor

A signal is (almost) always linked to the notion of "system"

System: Functional block that reacts to an input excitation signal and produces a response signal after applying a function to the input signal



FILTERS: LINEAR SYSTEMS INVARIANT IN TIME

A filter is a linear system that is invariant over time

Let $[h]$ be a filter with an impulse response $[h]$. Then

$$[y] = [h] * [x]$$

For finite sequences (digital signals) of size $[N]$, 2 possibilities

Zeros-padding: $[h]$ and $[x]$ $[N]$

Circular convolution: $[h]$ and $[x]$ are supposed to be periodic (different results if the period is chosen to be $[N]$ or $[N]$!)

FILTERING IN THE FOURIER DOMAIN

Let a filter with an impulse response $h[n]$:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] h[n-k]$$

then (if the Fourier transform exists):

$$Y(e^{j\omega}) = H(e^{j\omega}) X(e^{j\omega})$$

For finite sequences (digital signals), the underlying convolution is circular: Fourier transform must be done after zeros-padding !

Filtering a signal acts directly on the spectrum

IDEAL FILTERS AND REALIZABLE FILTERS

Ideal filter cut some frequencies while other are preserved. 4 types of filters: Low pass, High pass, Band pass and Band cut filter.

A filter is realizable iff its impulse response is stable and causal

Ideal filters are not realizable (not causal)

Two kind of realizable numerical filters:

Finite Impulse Response (FIR) filters: [OBJ]

Infinite Impulse Response (IIR) filters: [OBJ]

DIGITAL FILTERS

Ideal filters can be implemented in the frequency domain (but not in real time)

FIR filters are stable, but need a lot of coefficients

IIR filters can be unstable, but accurate.

Classical IIR filters: Butterworth, Tchebychev I & II, Elliptical

TO DO: FIR DELAY EFFECT

Data:

Any (short) sound file

Goal:

The FIR filter for delay effect can be implement thanks to the following input-output equation ($x[n]$ is the input and $y[n]$ is the output):

$$y[n] = \alpha x[n - \Delta]$$

Where α is the attenuation factor and Δ is the time delay

Implement the delay effect in the time domain

Determine the impulse response of the filter (numerically)

Provide the Frequency response of the filter (numerically)

TO DO: IIR DELAY EFFECT

Data:

Any (short) sound file

Goal:

The IIR filter for delay effect can be implement thanks to the following input-output equation ($x[n]$ is the input and $y[n]$ is the output):

$$y[n] = \alpha x[n] + \beta y[n - N]$$

Where α is the scaling factor, β is the attenuation factor and N is the time delay

Implement the delay effect in the time domain

Determine the impulse response of the filter (numerically)

Provide the Frequency response of the filter (numerically)

Is this implementation always stable ?

Discuss the parameters

Compare with the FIR implementation

TO DO: IMAGE FILTERING

Data:

Any image

Goal:

For a given image, implement the following filters and discuss their effects

Gradient filter

Sobel filter

Averaging filter

Gaussian filter