Semantic Web & Ontologies: Unknown Exam

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Contents

1 Exercice 0: True & False Statements

Mark the correctness of the following statements about semantics in the context of Semantic Web, using T (for true) and F (for false):

1. Sparql is the query language over RDF triples.
2. Without semantics, different tools can give different answers when querying RDF triples.
3. There is a unique way to define semantics for a language.
4. OWL and OWL2 both use model-theoretic semantics.
5. An ontology can have no models.
6. We can use Sparql to decide the satisfiability of an ontology and define "soundness" of a reasoning system without defining semantics.
7. We can define "completeness" of a reasoning system without defining semantics.
8. HermiT is a reasoner that can deal with the DL language ALC.
9. ALC contains at least one OWL2 profile.
10. An OWL ontology can have infinite interpretations.

1.1 Solution

The solutions for the True & False statements are:

- **SPARQL is the query language over RDF triples.** True

  SPARQL (SPARQL Protocol and RDF Query Language) is the W3C standard query language designed for querying RDF (Resource Description Framework) data.

- **Without semantics, different tools can give different answers when querying RDtriples.** True

  Semantics ensures that different tools interpret RDF triples consistently. Without a common semantic framework, tools may process RDF triples differently, leading to varying results.

- **There is a unique way to define semantics for a language.** False

  Different formal languages can have multiple semantic interpretations, such as model-theoretic semantics, proof-theoretic semantics, etc. This means there can be more than one way to define semantics for a language.

- **OWL and OWL2 both use model-theoretic semantics.** True

  Both OWL (Web Ontology Language) and OWL2 use model-theoretic semantics, which define meaning by associating terms with a model, providing a formal foundation for reasoning in ontologies.

- **An ontology can have no models.** True

  An ontology with no models is considered inconsistent. This means there is no interpretation that satisfies all the axioms of the ontology.

- **We can use SPARQL to decide the satisfiability of an ontology and define "soundness" of a reasoning system without defining semantics.** False

  SPARQL is a query language, not a reasoning engine. Satisfiability and soundness require reasoning and formal semantics. SPARQL cannot be used to check satisfiability of ontologies directly, nor can soundness be defined without semantics.

- **We can define "completeness" of a reasoning system without defining semantics.** False

  Completeness refers to whether a reasoning system can derive all the logically valid conclusions from a given set of axioms, which cannot be defined without semantics to establish what constitutes a valid conclusion.

- **HermiT is a reasoner that can deal with the DL language ALC.** True

  HermiT is an OWL reasoner that supports ALC (Attributive Concept Language with Complements), a foundational Description Logic (DL) used in reasoning tasks.

- **ALC contains at least one OWL2 profile.** False

  ALC is not contained within any OWL2 profiles. OWL2 profiles like OWL2EL, OWL2QL, and OWL2RL are fragments of OWL designed for specific computational properties, but ALC does not form the basis for any of them.

- **An OWL ontology can have infinite interpretations.** True

  OWL ontologies can have infinite interpretations, as the semantics allow for an infinite domain, and some concepts or roles may be interpreted over an infinite set of individuals or relationships.

2 Exercice 1: Turtle Translation

Translate the following sentences into RDF Turtle syntax or DL axioms:

Mary is a woman, Every mother is a woman,
Mary is John's wife, Mothers are women who are also parents,
At least one child of a grandparent has also a child.

2.1 Solution

The RDF Turtle syntax and Description logic axioms for the previous sentences will be added one after another as we consider each sentence:

- **Mary is a woman.** The DL Axiom is: $Woman(mary)$

```
# Turtle Syntax
:Woman rdf:type owl:Class.
:Mary rdf:type :Woman.
```

- **Every mother is a woman.** The DL Axiom is: $Mother \sqsubset Woman$

```
# Turtle Syntax
:Woman rdf:type owl:Class.
:Mother rdf:type owl:Class;
        rdfs:subClassOf :Woman.
:Mary rdf:type :Woman.
```

- **Mary is John's wife.** The DL Axiom is: $symmetric(isMarriedTo)$, $isMarriedTo(John, Mary)$

```
# Turtle Syntax
:Woman rdf:type owl:Class.
:Mother rdf:type owl:Class;
        rdfs:subClassOf :Woman.
:Man rdf:type owl:Class.
:John rdf:type :Man;
:Mary rdf:type :Woman;
:isMarriedTo rdf:type owl:ObjectProperty;
             rdf:type owl:SymmetricProperty.
# Assert the relationship
:John :isMarriedTo :Mary.
```

- **Mothers are women who are also parents.** The DL Axiom is: $Mother \equiv Woman \sqcap Parent$

```
# Turtle Syntax
:Woman rdf:type owl:Class.
:Parent rdf:type owl:Class.
:Mother rdf:type owl:Class;
        rdfs:subClassOf [rdf:type owl:Class;
                         owl:intersectionOf(:Woman, :Parent)]
:Man rdf:type owl:Class.
:John rdf:type :Man;
:Mary rdf:type :Woman;
:isMarriedTo rdf:type owl:ObjectProperty;
             rdf:type owl:SymmetricProperty.
# Assert the relationship
:John :isMarriedTo :Mary.
```

- **At least one child of a grandparent has also a child.**
  The DL Axiom is: $GrandParent \sqsubseteq (\exists hasChild.(\exists hasChild.\top))$

3 Exercice 2: NNF

Give the negation norm forms (NNF) for the following three concepts:

$$\neg(\exists r.\neg(\forall s.\forall t.(A \sqcup (B \sqcap C)))) \qquad \neg(\neg(\forall r.(A \sqcup \exists.B))) \qquad \forall r.\neg(A \sqcap \exists r.\neg B \sqcap C)$$

3.1 Solution

The basic rules to remember to obtain the NNF are:

Rules

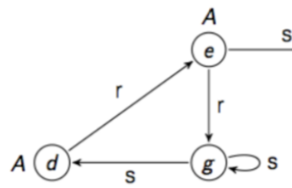
- $\boxed{\neg(\forall x.\phi) = \exists x.\neg\phi}$
- $\boxed{\neg(\exists x.\phi) = \forall x.\neg\phi}$
- $\boxed{\neg(\phi \sqcup \delta) = \neg\phi \sqcap \neg\delta}$
- $\boxed{\neg(\phi \sqcap \delta) = \neg\phi \sqcup \neg\delta}$
- $\boxed{\neg\neg\phi = \phi}$

Then, the answers are:

1. $\neg(\exists r.\neg(\forall s.\forall t.(A \sqcup (B \sqcap C)))) \underbrace{\equiv}_{\mathbf{2.}} \forall r.\neg(\neg(\forall s.\forall t.(A \sqcup (B \sqcap C)))) \underbrace{\equiv}_{\mathbf{5.}} \forall r.(\forall s.\forall t.(A \sqcup (B \sqcap C)))$
2. $\neg(\neg(\forall r.(A \sqcup \exists.B))) \underbrace{\equiv}_{\mathbf{5.}} \forall r.(A \sqcup \exists.B)$
3. $\forall r.\neg(A \sqcap \exists r.\neg B \sqcap C) \underbrace{\equiv}_{\mathbf{4.}} \forall r.(\neg A \sqcup \neg(\exists r.\neg B) \sqcup \neg C) \underbrace{\equiv}_{\mathbf{2.}} \forall r.(\neg A \sqcup (\forall r.B) \sqcup \neg C)$

4 Exercice 3: Graph

Consider the following description logic interpretation I represented in the form of a graph:



Question 1. Write down the definition of $I = (\Delta^I, ^I)$ corresponding to the graph.
Question 2. For each of the following ALC concepts C, list all the elements x of Δ^I such that $x \in C$:

$$A \sqcap B \qquad \neg(B \sqcup (\exists s.A)) \qquad \neg(\exists r.(A \sqcap B))$$

4.1 Solution

- $\boxed{\textbf{Question 1}}$

  The definition of the graph I is:

$$I = (\Delta^I, \cdot^I) = (\{d, e, f, g\}, \cdot^I) \textit{ where}$$

$$A^I = \{d, e, f\},\ B^I = \{f\},\ s^I = \{(g, d), (g, g), (e, f)\} \textit{ and } r^I = \{(d, e), (e, g)\}$$

- $\boxed{\textbf{Question 2}}$

  1. $A \sqcap B = \{f\}$.
  2. $\neg(B \sqcup (\exists s.A)) = \neg(\{f, g, e\}) = \{d\}$ where $B = \{f\}$ and $\exists s.A = \{g, e\}$.
  3. $\neg(\exists r.(A \sqcap B)) = \top$ because $A \sqcap B = \{f\}$ but $\exists r.(A \sqcap B) = \emptyset$.

5 Exercice 4: Tableau Algorithm

Using Tableau algorithm, check the satisfiability of the following concepts C with respect to the given TBoxes. If C is satisfiable, give the interpretation corresponding to your tableau construction:

- TBox is $\emptyset$ and C is $(A \sqcup B \sqcup \forall R.B) \sqcap (\neg A \sqcap \neg B \sqcap \exists R.\neg B)$
- TBox is $\emptyset$ and C is $(\forall R.\forall S.\forall R.\exists S.A) \sqcap (\exists R.\exists S.\exists R.\forall S.\neg A)$

5.1 Solution

The solutions are as follows:

1. $\boxed{\text{TBox is } \emptyset \text{ and C is } (A \sqcup B \sqcup \forall R.B) \sqcap (\neg A \sqcap \neg B \sqcap \exists R.\neg B)}$

   Let's consider the element a_0 and the concept C, such as $a_0 \in C$ and the decomposition of C as the conjunction of concepts C_1 and C_2, $C := C_1 \sqcap C_2$ where:

   - $C_1 = A \sqcup B \sqcup \forall R.B$
   - $C_2 = \neg A \sqcap \neg B \sqcap \exists R.\neg B$

As $a_0 : C$ $(a_0 \in C) \implies a_0 : C_1,\ a_0 : C_2$. Then, as $a_0 : C_2$, we have that:

$$a_0 : \neg A, a_0 : \neg B, a_0 : \exists R.\neg B$$

Currently, we have the following conditions at a_0:

$$a_0 : A \sqcup B \sqcup \forall R.B,\ a_0 : \neg A,\ a_0 : \neg B,\ a_0 : \exists R.\neg B$$

Since a_0 already satisfies $\neg A$ and $\neg B$, a_0 must satisfy, within the disjunction concept C_1, the concept $\forall R.B$, then we have:

$$a_0 : \forall R.B,\ a_0 : \neg A,\ a_0 : \neg B,\ a_0 : \exists R.\neg B$$

The condition $a_0 : \exists R.\neg B$ implies that exists some element a_1 such that $R(a_0, a_1)$ and $a_1 : \neg B$. Thus, we introduce the element a_1 and for the moment we know that the following holds:

$$R(a_0, a_1),\ a_1 : \neg B,\ a_0 : \forall R.B,\ a_0 : \neg A,\ a_0 : \neg B$$

From the formula $a_0 : \forall R.B$, we know that for all elements a_1 such that $R(a_0, a_1)$ holds, then $a_1 : B$. Thus, we face the following clash:

$$a_1 : \neg B,\ a_1 : B$$

We can conclude that the concept C is unsatisfiable.

2. $\boxed{\text{TBox is } \emptyset \text{ and C is } (\forall R.\forall S.\forall R.\exists S.A) \sqcap (\exists R.\exists S.\exists R.\forall S.\neg A)}$

Let's consider the element a_0 and the concept C, such as $a_0 \in C$ and the decomposition of C as the conjunction of concepts C_1 and C_2, $C := C_1 \sqcap C_2$ where:

- $C_1 = \forall R.\forall S.\forall R.\exists S.A$
- $C_2 = \exists R.\exists S.\exists R.\forall S.\neg A$

As $a_0 : C$, it implies that $a_0 : C_1,\ a_0 : C_2$. If $a_0 : C_1$, then for all elements a_1 such that $R(a_0, a_1)$ holds, we have that $a_1 : \forall S.\forall R.\exists S.A$. Then, repeating the reasoning, for all elements a_2 such that $S(a_1, a_2)$ holds, we have that $a_2 : \forall R.\exists S.A$. Once again, for all elements a_3 such that $R(a_2, a_3)$ holds, we have that $a_3 : \exists S.A$. Finally, if $a_3 : \exists S.A$, then it exists some element a_4 such that $S(a_3, a_4)$ holds and $a_4 : A$ (a_4 belongs to A). Let's summarize the previous information:

$$a_0 : C_1 \longleftrightarrow R(a_0, a_1),\ S(a_1, a_2),\ R(a_2, a_3),\ S(a_3, a_4),\ a_4 : A$$

Similarly, if $a_0 : C2$, then for some element a_1 such that $R(a_0, a_1)$ holds, then $a_1 : \exists S.\exists R.\forall S.\neg A$. Thus, if $a_1 : \exists S.\exists R.\forall S.\neg A$, it exists some element a_2 such that $S(a_1, a_2)$ holds and $a_2 : \exists R.\forall S.\neg A$. Once again, it exists an element a_3 such that $R(a_2, a_3)$ holds and $a_3 : \forall S.\neg A$. Finally, for all elemnents a_4 such that $S(a_3, a_4)$ holds, $a_4 : \neg A$. Let's have a summary of both concepts C_1 and C_2:

- $a_0 : C_1 \longleftrightarrow R(a_0, a_1),\ S(a_1, a_2),\ R(a_2, a_3),\ S(a_3, a_4),\ a_4 : A$
- $a_0 : C_2 \longleftrightarrow R(a_0, a_1),\ S(a_1, a_2),\ R(a_2, a_3),\ S(a_3, a_4),\ a_4 : \neg A$

Clearly, there is a clash, then the concept C is unsatisfiable.

Remark A Csat (concept satisfiability) refers to the problem of determining whether a given concept C is satisfiable with respect to a $TBox$. In other words, Csat checks if there exists an interpretation / model where the concept C can be true, meaning that the interpretation is not contradictory.

6 Exercice 5: Ontologies

- **Question 1**: Is the ontology $O = (TBox,\ ABox)$ defined as follows

$$TBox\ = \{A \sqsubseteq B, B \sqsubseteq \exists r.T, \exists r.T \sqsubseteq D, B \sqcap D \sqsubseteq \bot\}\ and\ ABox\ = \emptyset$$

  a consistent ontology ? If yes, please give a model of O. Otherwise, justify your answer.

- **Question 2**: The same question as the previous one for $O' = O \cup \{A(a)\}$.

- **Question 3**: Which of the following subsumptions are always true? Justify your answer by the semantics of ALC.

$$\neg(A \sqcap B) \sqsubseteq \neg A \sqcup \neg B \qquad \forall s.A \sqcap \forall s.B \sqsubseteq \forall s.(A \sqcap B) \qquad \exists s.(\neg A \sqcap B) \sqsubseteq (\exists s.\neg A) \sqcap (\exists s.B)$$

6.1 Solution

- $\boxed{\textbf{Question 1}}$ The ontology previously O defined, is consistent and a model that satisfies such ontology is:

$$I = (\Delta^I, \cdot^I) = (\emptyset, \cdot^I)\ where$$

$$A^I = B^I = C^I = D^I = \emptyset,\ r^I = \emptyset$$

- $\boxed{\textbf{Question 2}}$

  Let's consider this new ontology O' and check whether it's consistent or not. As $ABox = \{A(a)\}$, a must belong to A^I, and therefore, by the $TBox$ axiom $A \sqsubseteq B$, we have that $a \in B^I$. Then, as $a \in B^I$ and $B \sqsubseteq \exists r.T$, there must exist some element b such that $r(a, b)$ holds and $b \in T^I$. Then, for the $TBox$ axiom $\exists r.T \sqsubseteq D$, we know that $r(a, b)$ holds and $\exists r.T \sqsubseteq D$ means that if an individual has an r-successor in T (individual a has an r-successor $b \in T^I$, then the individual must be in D, so $a \in D^I$. Finally, $B \sqcap D = \{a\} \sqsubseteq \bot$ is false, therefore the ontology is not consistent.

7 Exercice 6

Consider the knowledge base consisting of the following axioms:

- $RRated \sqsubseteq CatMovie$, $CatMovie \sqsubseteq Movie$
- $RRated = (\exists.hasScript.ThrillerScript) \sqcup (\forall hasViolenceLevel.High)$
- $Person \sqsubseteq \neg Movie$
- $\exists hasViolenceLevel.\top \sqsubseteq Movie$

Show that $Person \sqsubseteq \bot$ is a logical consequence of this knowledge base.

7.1 Solution

Ask someone.