TIME-FREQUENCY ANALYSIS

M2 AI — SIGNAL PROCESSING

FOURIER REMINDER

Spectral analysis: frequency content of a function (Think about musical notes!)

Measure the similarity (correlation, angle) between pure (complex) sine and a signal

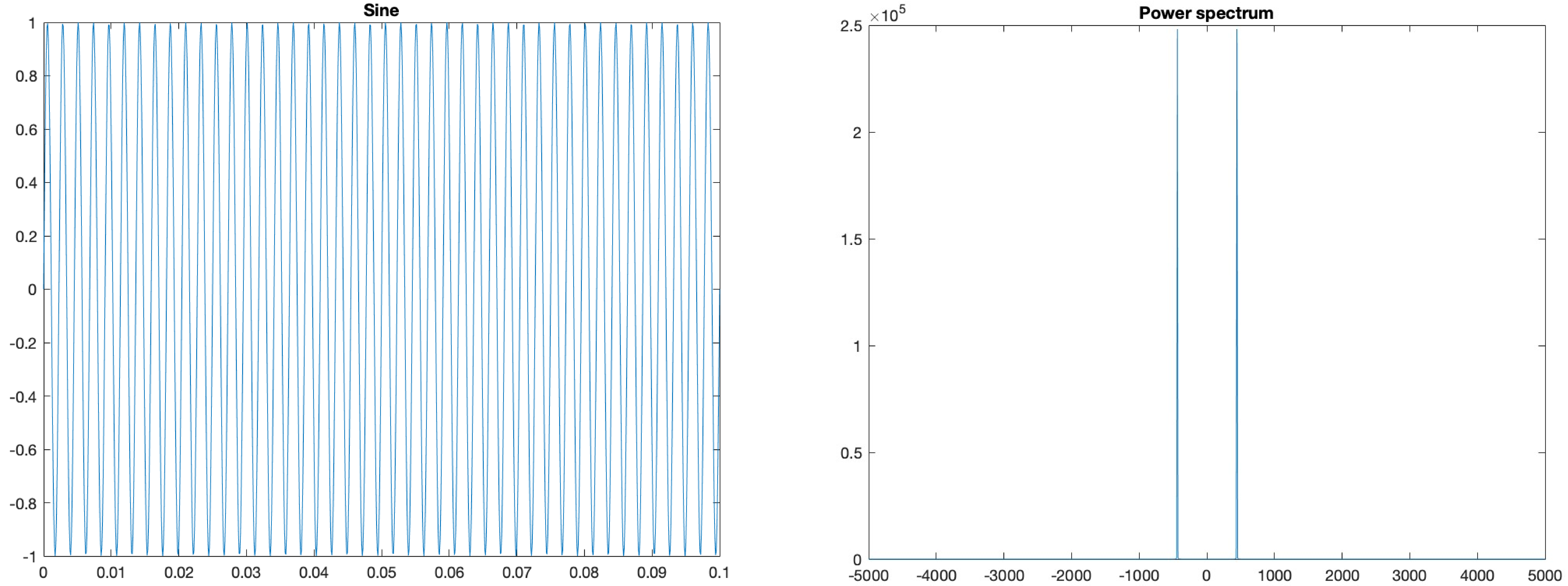
Sines are eigen signals of time-invariant linear systems (filters)

Fourier analysis computes the correlation between the signal a pure sine at various frequencies

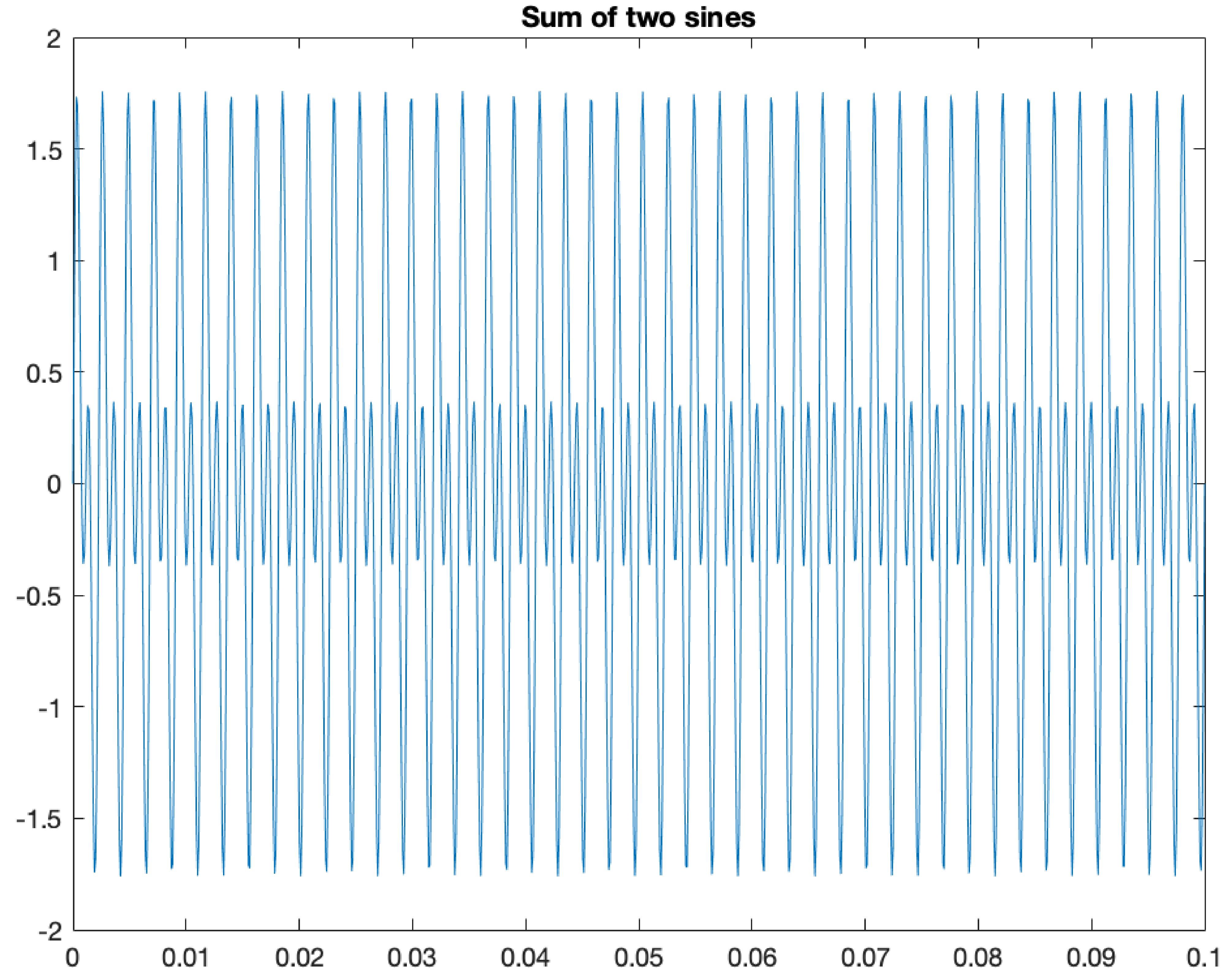
Limitation of Fourier analysis:

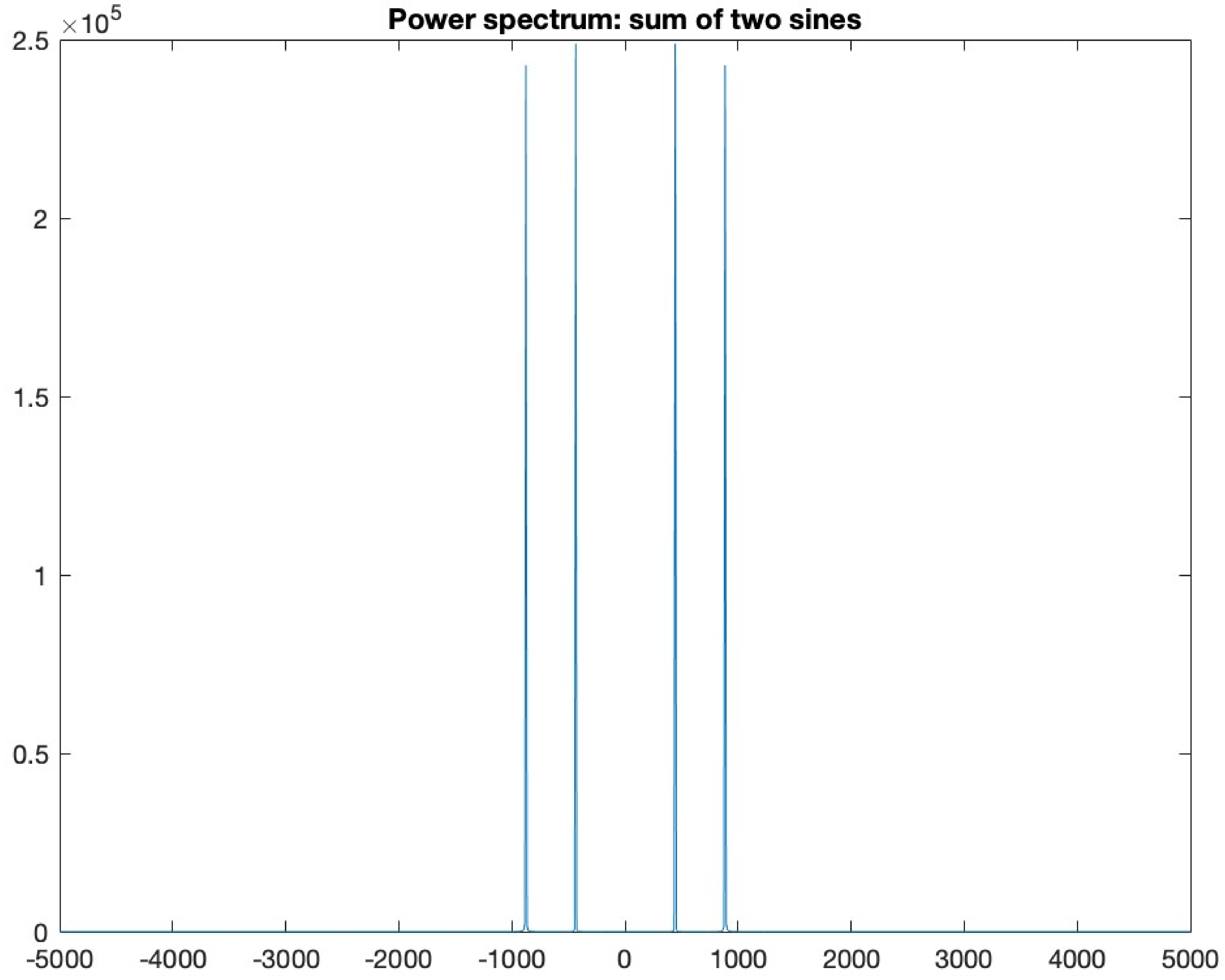
- We obtain a pure frequency content from a pure temporal content
- What is the difference between a sum of sine, and a succession of sines?

FOURIER EXAMPLE: 1 SINE

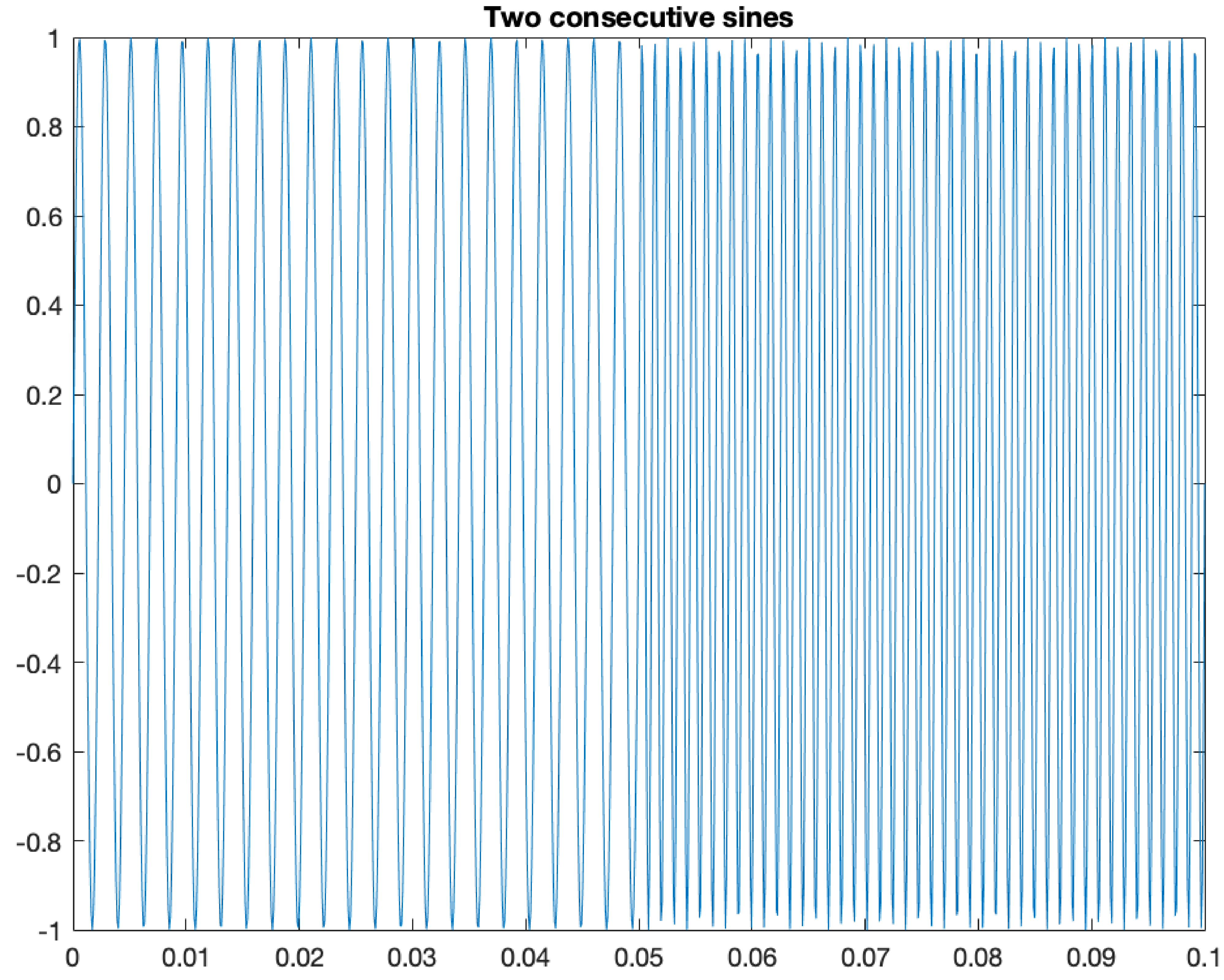


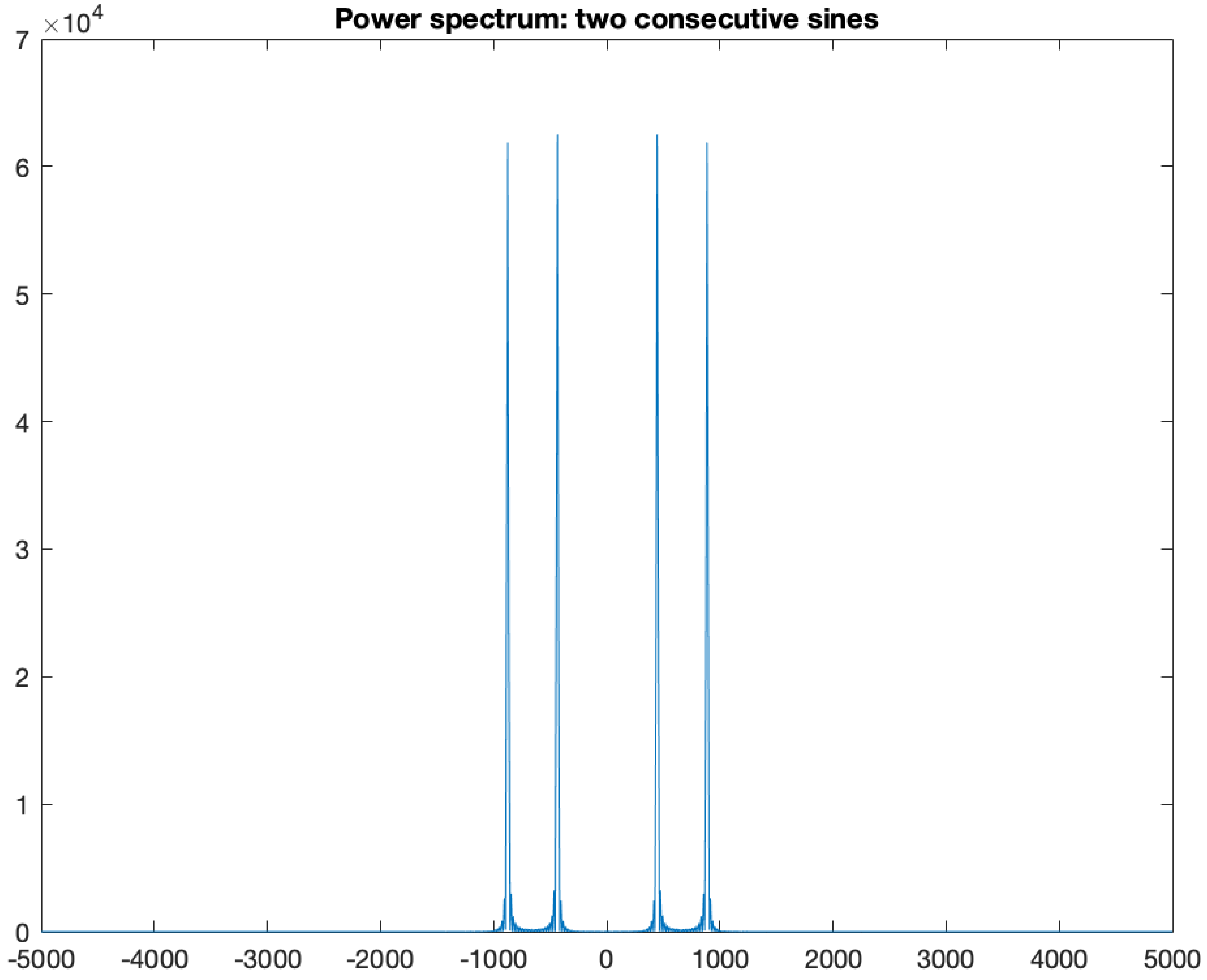
FOURIER EXAMPLE: SUM OF 2 SINES





FOURIER EXAMPLE: SUCCESSION OF 2 SINES





SHORT-TIME FOURIER TRANSFORM (STFT)

Idea: perform a local spectral analysis of the signal thanks to a sliding window

Let be a smooth window localized around . Let the time-frequency atom

The time-frequency transform of a signal computes its correlation with the time-frequency atom :

It corresponds to the Fourier transform of the windowed signal

Parameters of the STFT:

- The length (and shape) of the window
- The redundancy in time (hope size between two windows)
- The redundancy in frequency (length of the frequency transform inside one window)

DISCRETE STFT (GABOR TRANSFORM)

Let ￼ be a digital signal and let ￼ be a window. The discrete STFT is given by

￼

￼ control the redundancy in time (the hope size, in samples, between two windows)

￼ control the redundancy in frequency (usually ￼ or ￼)

Using the matrix notation, all the time-frequency coefficients ￼ can be computed by the analysis operator ￼: ￼

Each column of ￼ is one time-frequency atom.

The number ￼ of columns depends on the time-frequency redundancy,

DISCRETE INVERSE STFT

We do not have in general:

With matrix notation:

The invert of a Gabor dictionary is obtained by the canonical dual, which is also a Gabor transform constructed using a dual window

With matrix notation:

If the Gabor dictionary is a Parseval Frame (or a normalized tight frame), then and

SYNTHESIS OPERATION

We have

With , that is

However, it exists an infinity of synthesis coefficients such that

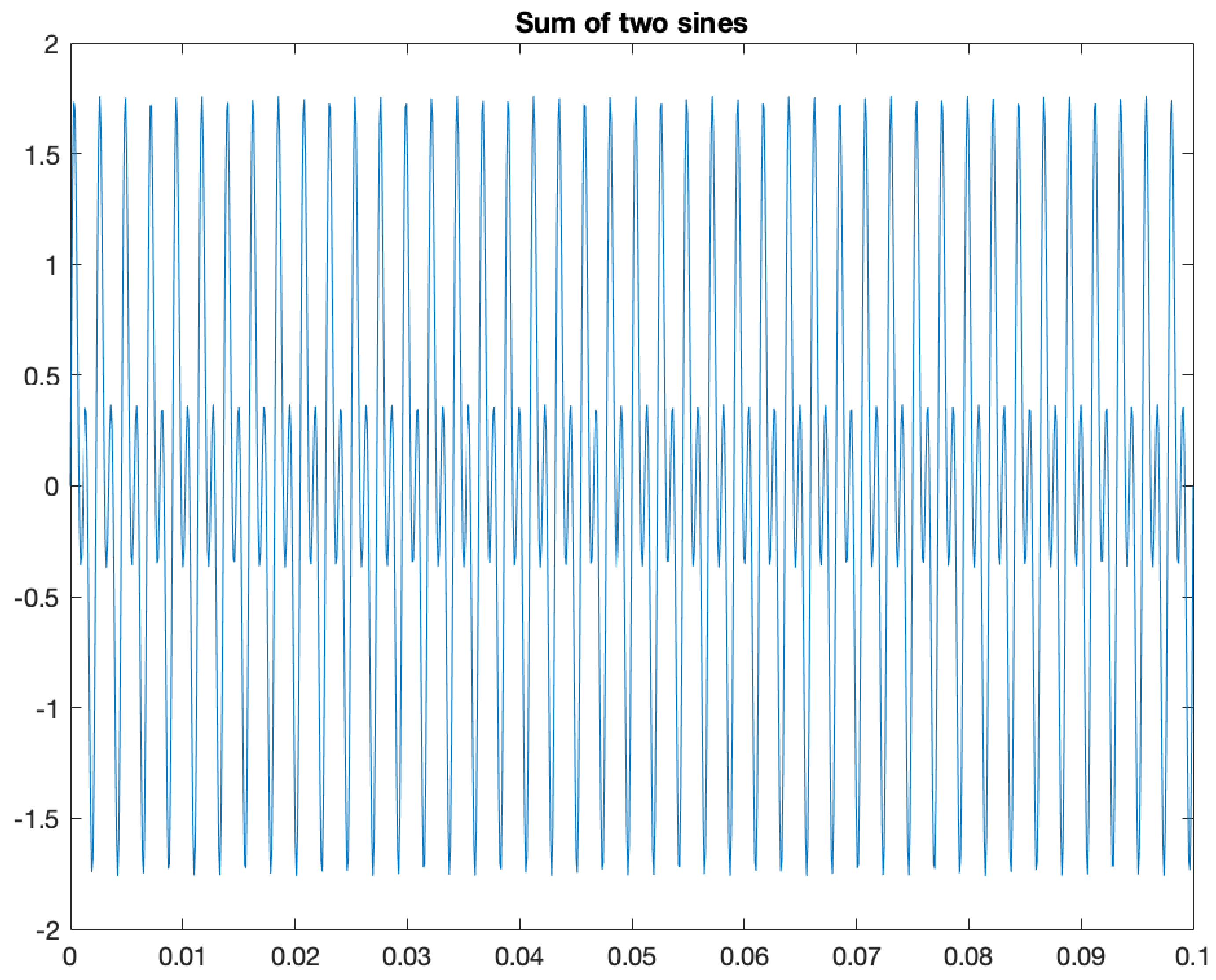
Beware:

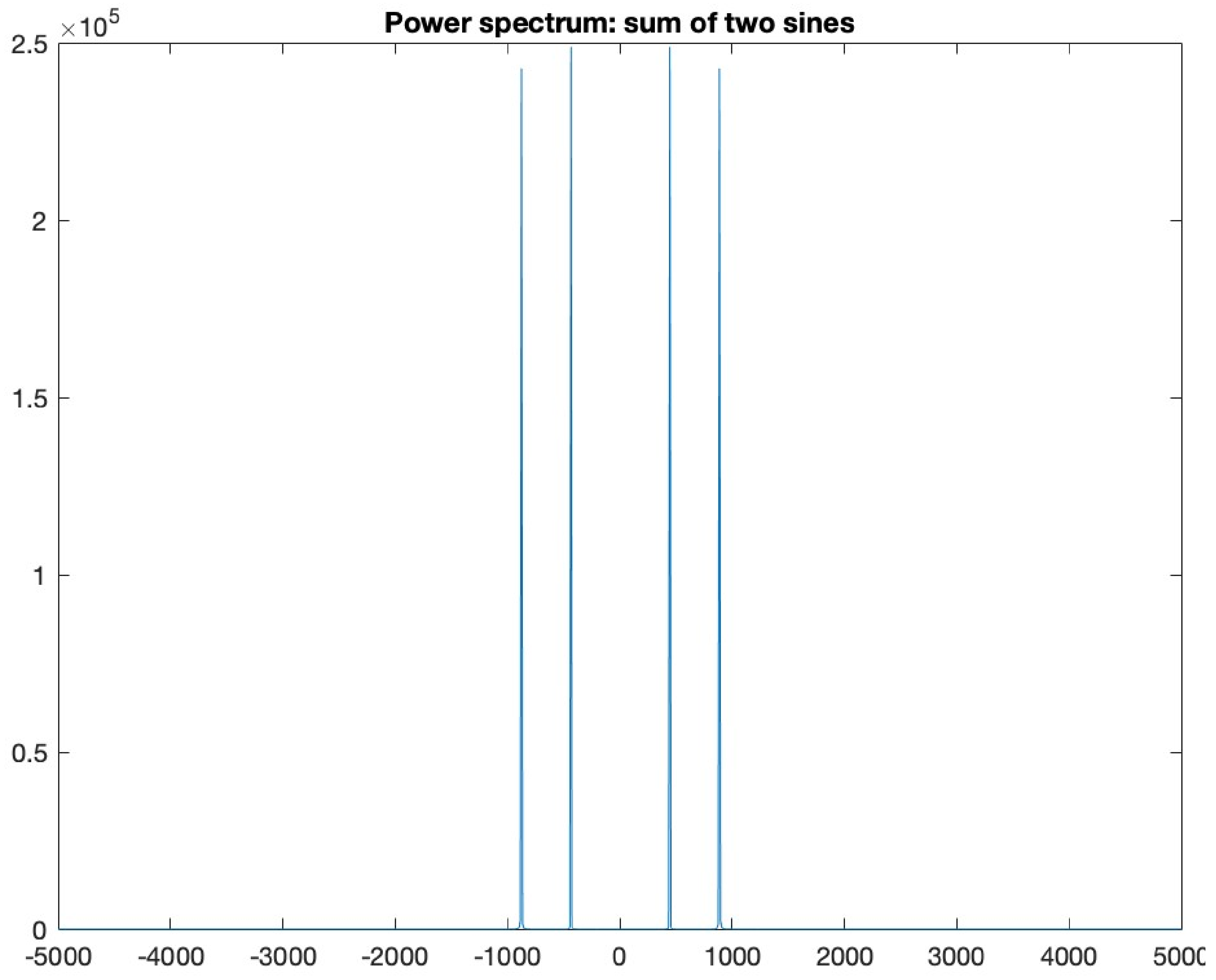
- In some implementations, the "invert" operator is the "synthesis" operation and must be performed with the appropriate dual window
- It is more useful to have access to the "synthesis" operator rather than the actual invert operator

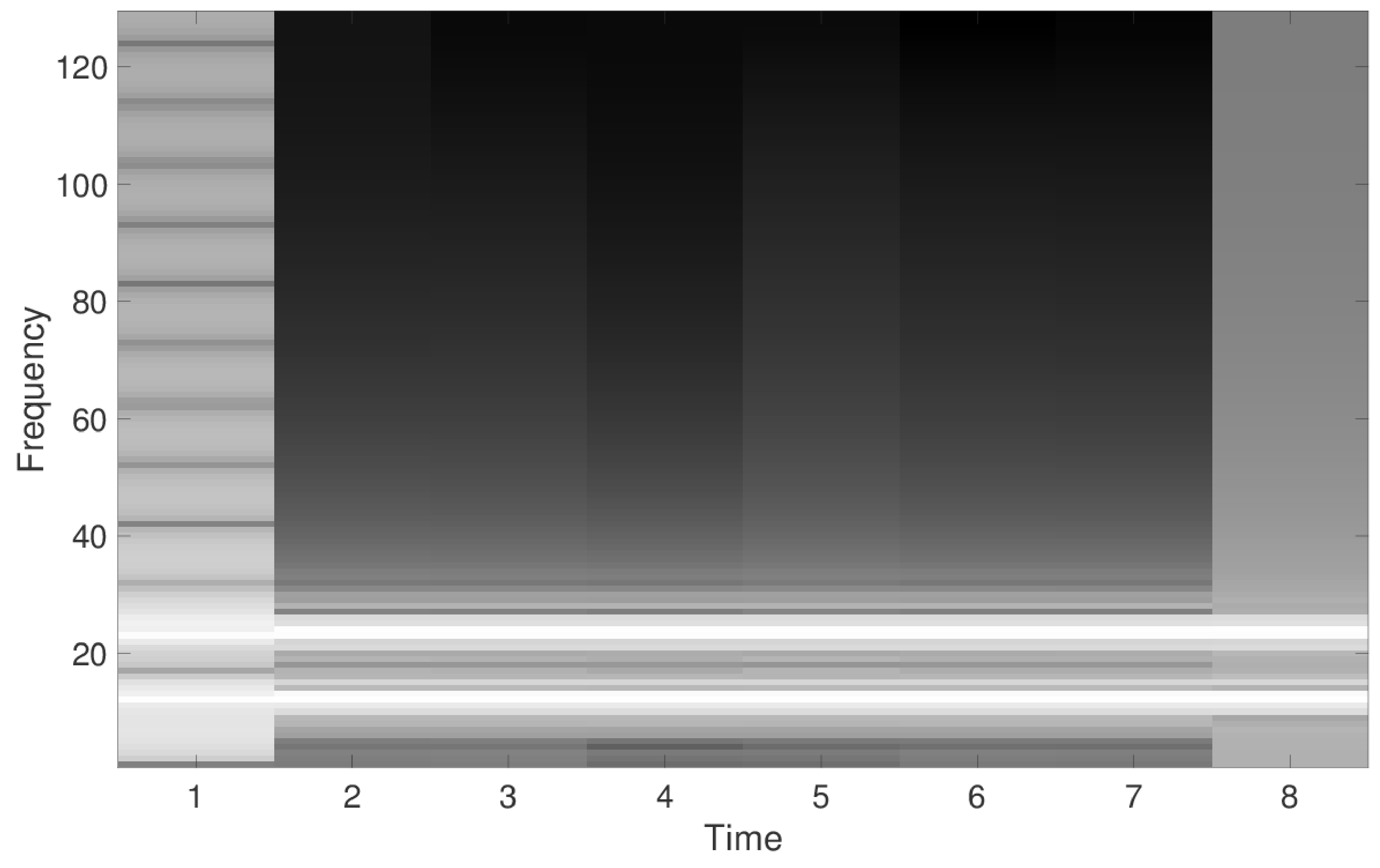
HOW TO CHOOSE THE PARAMETERS

- Heisenberg's uncertainty principle
  - A signal cannot be both well localized in time and in frequency.
  - Consequence: short windows are more adapted to "transient", and long windows to "tonal", "stationary", parts of the signal
- Common choices for a high-fidelity audio signal with a sampling frequency of 44.1 kHz with a window of size
  - Shape of the window: Hann, Hamming, Gaussian
  - Length of the window: between 256 samples to 4096 samples. Common choice: 1024 samples
  - Redundancy in time: overlap of 50% or 75% between two consecutive window
  - Redundancy in frequency: FFT of size or

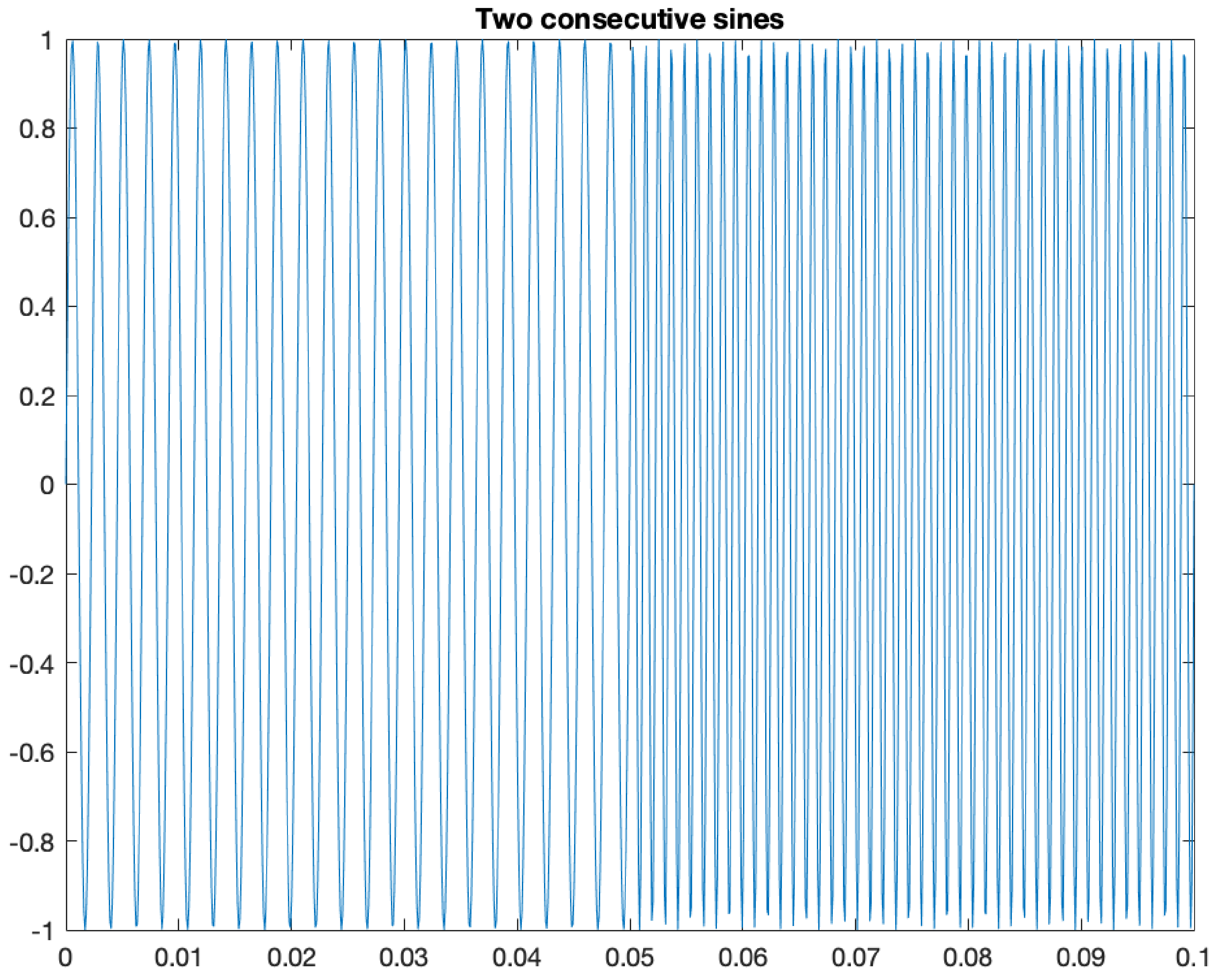
STFT EXAMPLE: SUM OF 2 SINES

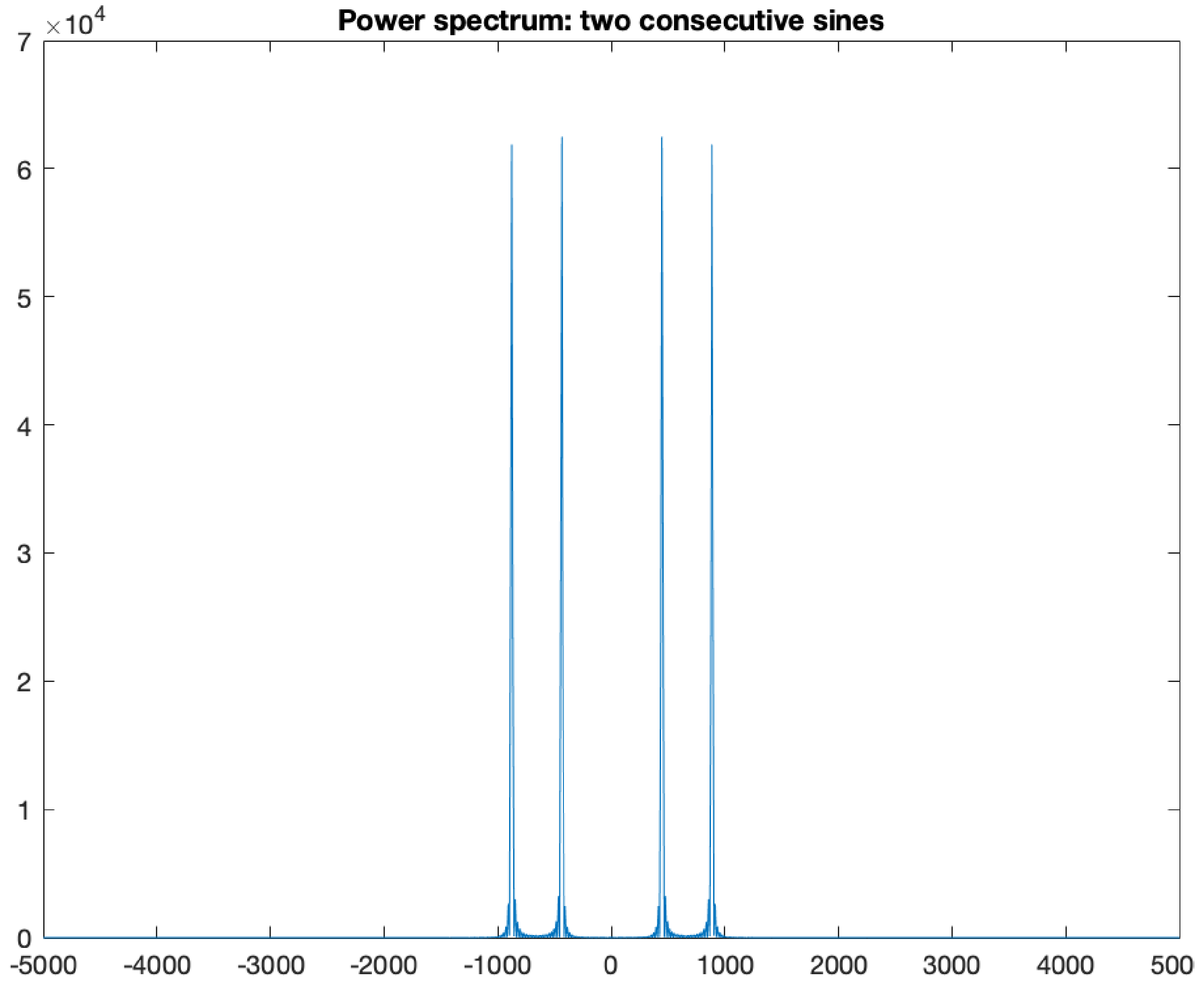


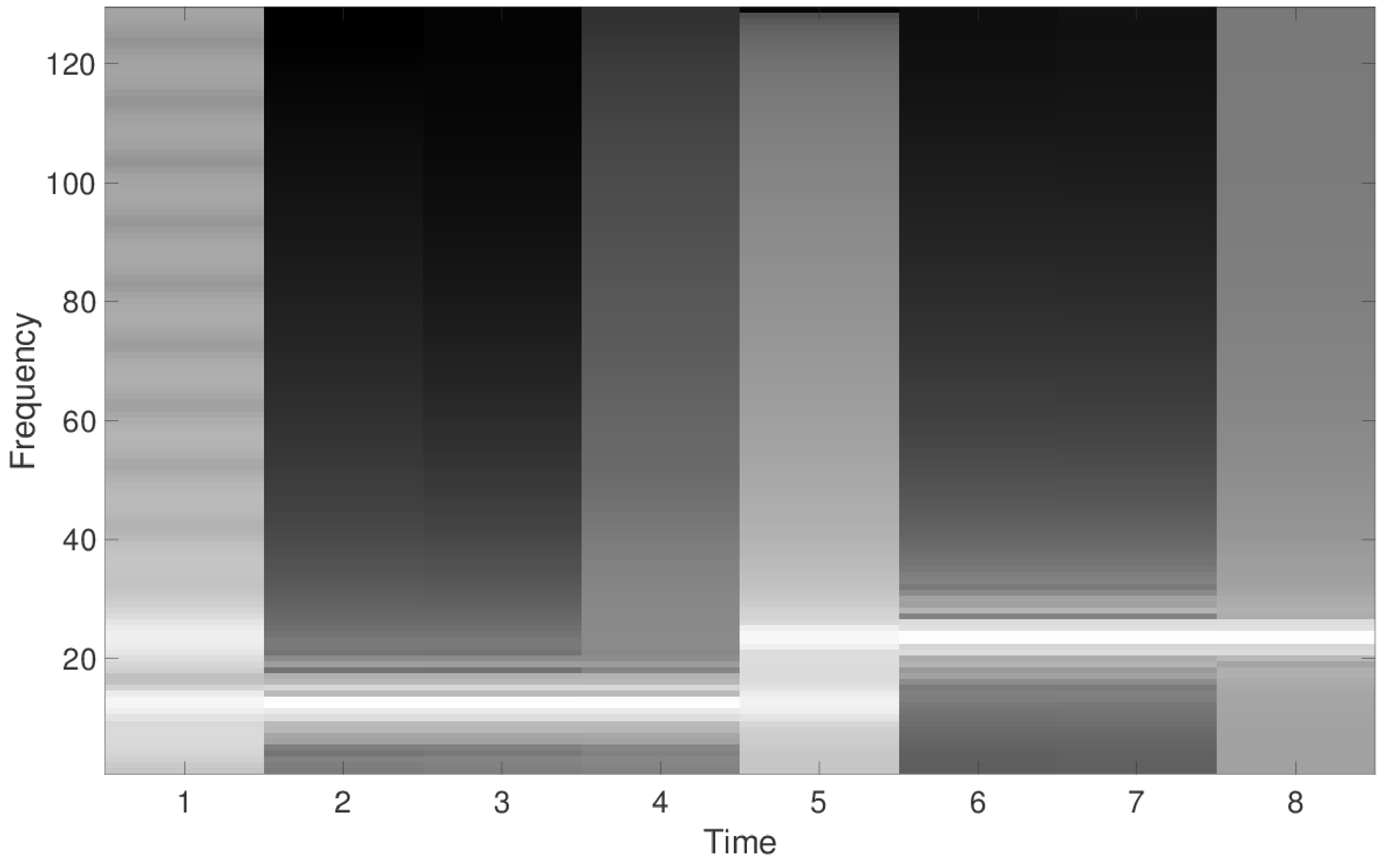




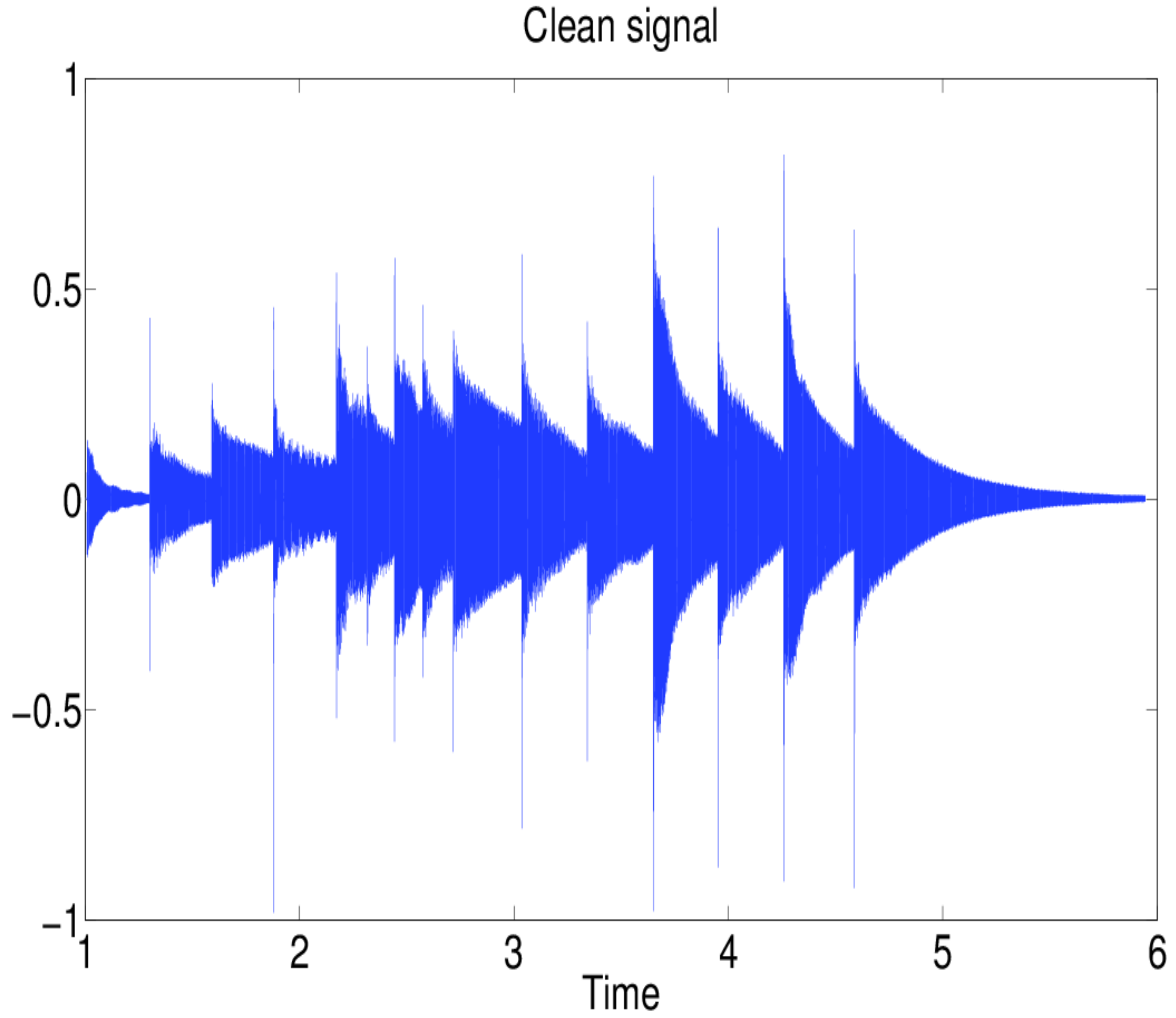
STFT EXAMPLE: SUCCESSION OF 2 SINES

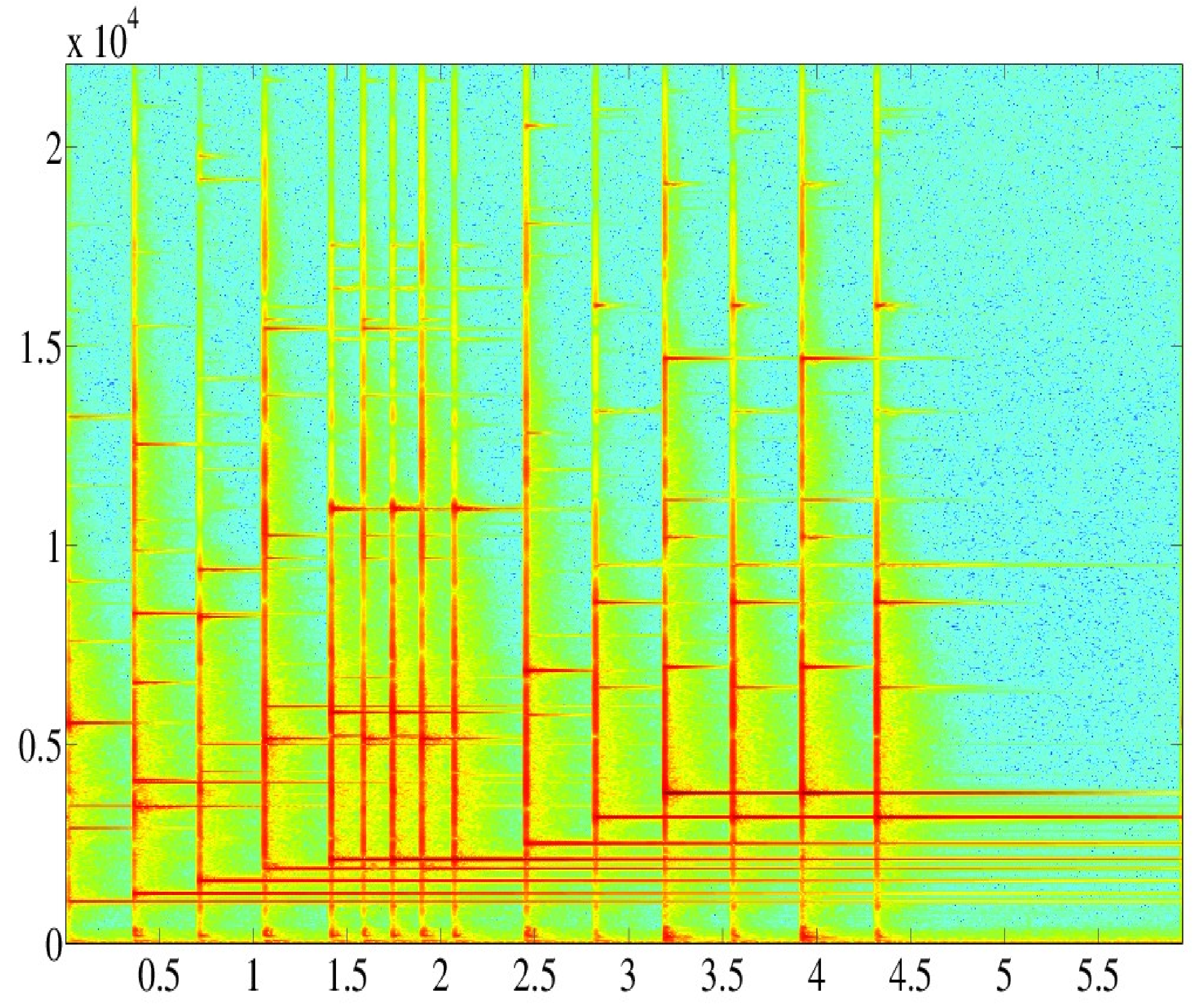


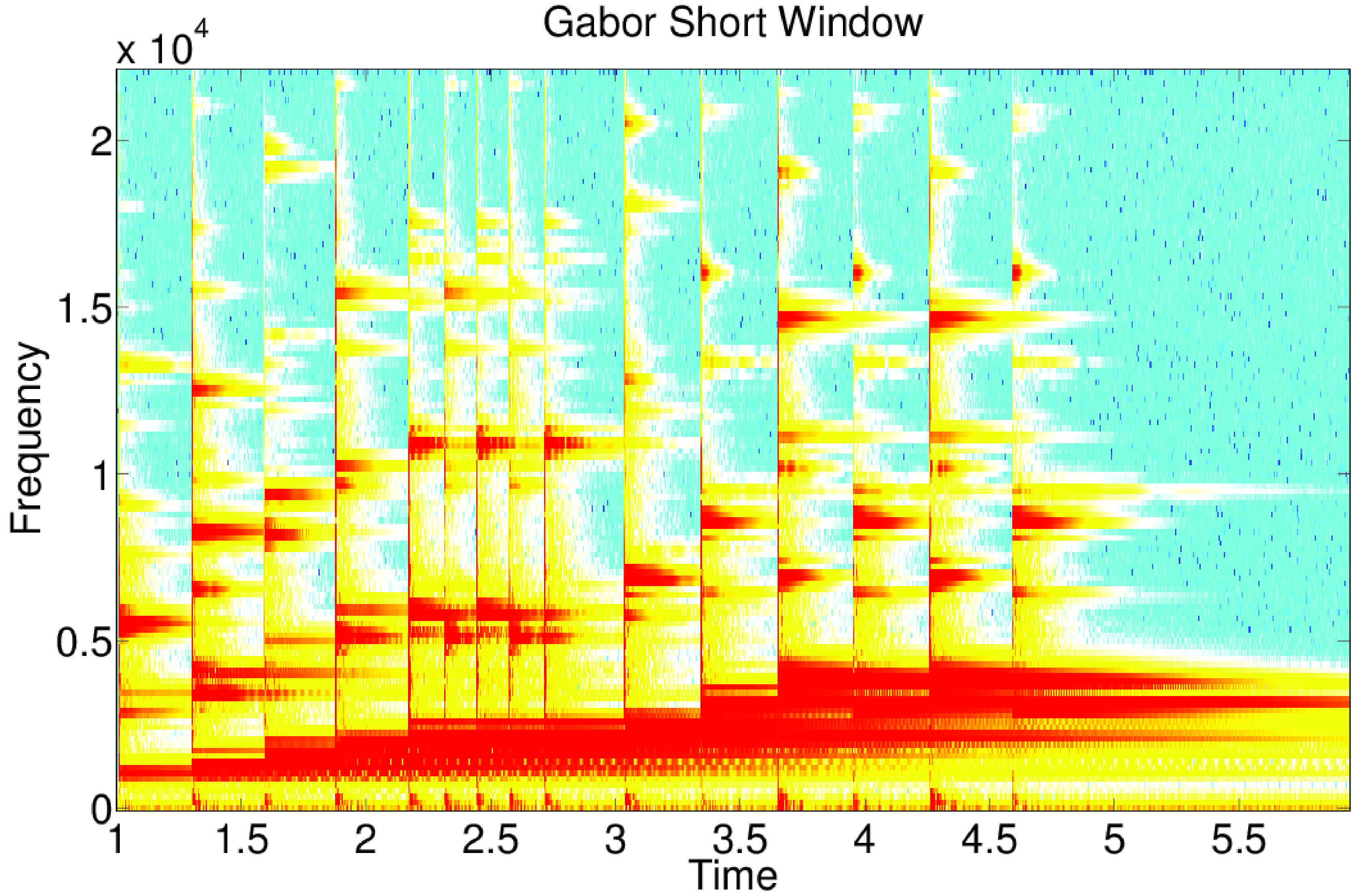




STFT EXAMPLE: GLOCKENSPIEL







DENOISING IN THE TIME-FREQUENCY DOMAIN

Let be a noisy measure of a "clean" signal corrupted by some additive noise :

In the STFT domain, we have

Proposed estimator

- Hard Thresholding

- Spectral subtraction

TO DO: DENOISING IN THE STFT DOMAIN

- Data
  - The 3 noises of the random chapter
  - “Clean” music signal
- Todo
  - Simulate a noisy version of the music using the noises at various SNR Level
  - Implement the denoising by hard thresholding and spectral subtraction
  - Denoise the different given noisy version of the clean signal
  - Discuss the parameters