INVERSE PROBLEMS

M2 AI — SIGNAL PROCESSING

DIRECT PROBLEM

Let

- be the signal of interest (clean image, music, etc.)
- be a known linear operator (sensing matrix, mixing matrix, diffusion matrix, etc.)
- be the (noisy) observed/measured signal
- be some noise (assumed to be white Gaussian noise)

The direct problem is :

INVERSE PROBLEM

The goal of the inverse problem is to estimate the original signal [illegible] from the measurement

[illegible]

If [illegible] the problem is said (over)-determined

If [illegible] the problem is said under-determined

INVERSE PROBLEM: EXAMPLES

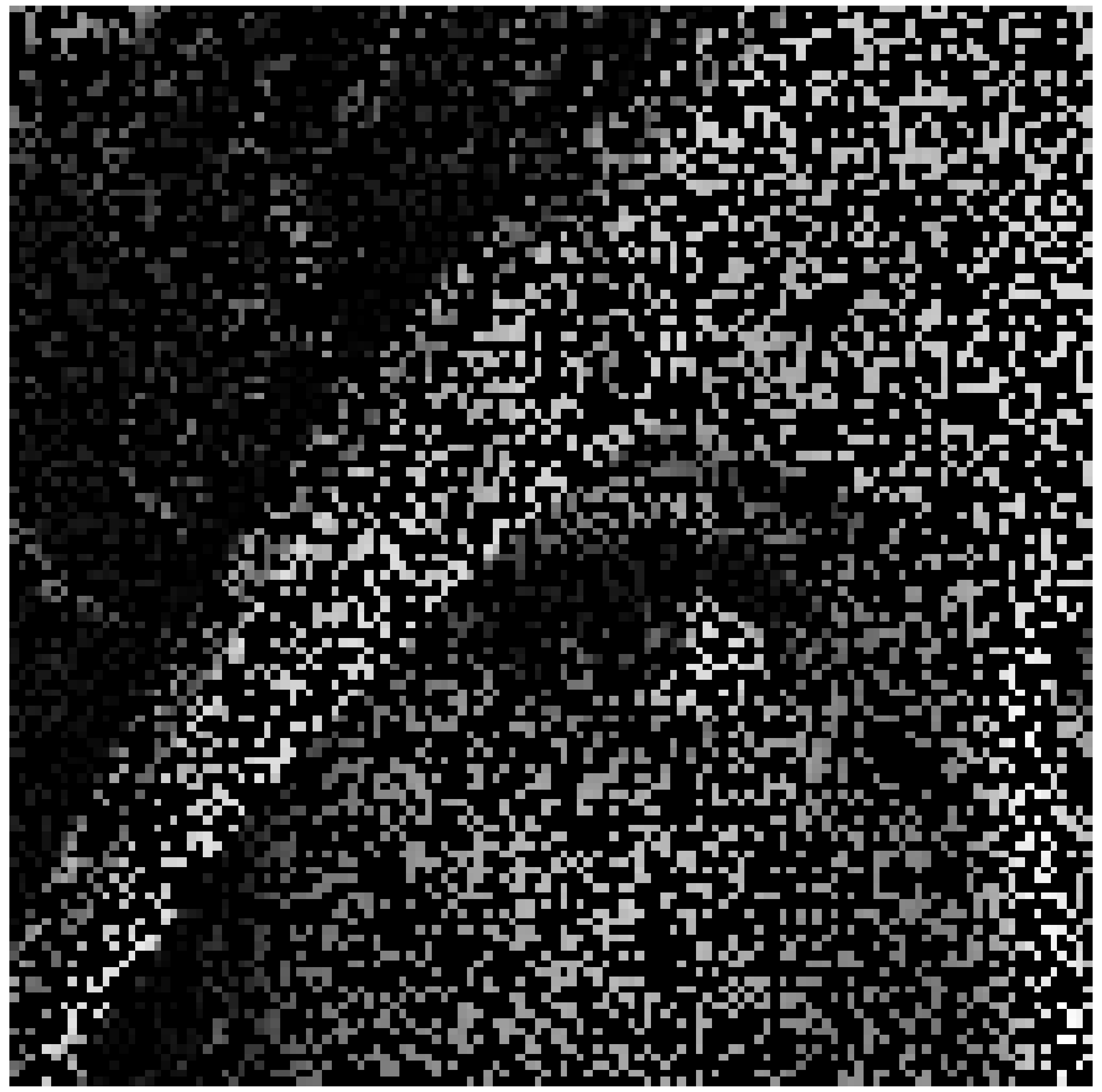
Denoising:

Deconvolution: or

INVERSE PROBLEM: EXAMPLES

Compressive sensing: A is a random matrix with

Inpaiting: is a binary mask



OPTIMIZATION FRAMEWORK

We seek an estimation of by

where

- is the loss: models the link between the signal and the observation through the operator

- is the loss: models the prior on the signal

- is some hyper-parameter

OPTIMIZATION FRAMEWORK

We seek an estimation of by

Loss:

- : energy of the residual, adapted to white Gaussian noise
- : robust regression

Regularization:

- : energy of the signal
- : energy of the derivative
- : sparsity of the signal
- sparsity of the derivative (total variation)

USE OF A DICTIONARY

can be difficult to chose

Idea: use a dictionary (such as Wavelets or time-frequency), where the signal is known to be sparse (well represented by few coefficients)

Let be such a dictionary, with . are called the synthesis coefficients

The direct problem writes

The minimization becomes

and

INVERSE PROBLEM: ALGORITHM

How to minimize

- It is a non-smooth convex problem
- Known as the Lasso or Basis-Pursuit Denoising

Consider the "simple" denoising problem (e.g. is orthogonal)

We can show that the solution is given by the so-called Soft-Thresholding operator :

FISTA WITH WARM RESTART

In practice, the algorithm must be run with various values of lambda.

When ￼ we have￼.

When ￼ the solution is ￼

One can choose these values distributed on a log scale￼, with a fixed number of ￼, such as we have ￼

The idea is to initialize the algorithm for ￼ with the results get from ￼

FISTA WITH THRESHOLDING RULES

The Soft-thresholding can be replaced by any thresholding rules

Some examples:

- Hard Thresholding:
- Empirical Wiener:

TO DO: INPAINTING

- Data
  - Image or signal you want
- Todo
  - Simulate various inpainting problems as follows
    - Generate a random binary matrix A of the same size as the signal, with a parameter [illegible] controlling the Bernoulli law
    - Add some white Gaussian noise (at various levels)
    - Generate the direct problem [illegible] (where [illegible]is the original signal)
    - Estimate [illegible] using the sparse approach (reminder: an audio signal (resp. image) is sparse in the time-frequency (resp. Wavelet) domain)
  - Discuss the results obtained by changing:
    - the sparse representation (various wavelets, various STFT parameters…)
    - the thresholding rules (soft, hard, empirical Wiener)
    - the choice of the [illegible] parameter
  - Discussion should be made concerning the value of [illegible] and the level of noise