

SPECTRAL ANALYSIS

M2 AI — SIGNAL PROCESSING

USEFULNESS

Spectral analysis: frequency content of a function

Think about musical notes!

Measure the similarity (correlation, angle) between pure (complex) sine and a signal

Sines are eigen signals of time-invariant linear systems (filters)

The definition depends on the mathematical model

ONE FORMULA TO RULE THEM ALL

Let $x[n]$ be a signal (continuous, discrete, periodic...). Let $e^{j\omega n}$ be the (continuous, discrete, periodic...) **complex sine** at the frequency ω , then

$$X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

And

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

With the appropriate inner product !

ANALOG FOURIER TRANSFORM

Fourier transform

[OBJ]

Inverse Fourier transform

[OBJ]

DIGITAL FOURIER TRANSFORM

Let a discrete finite sequence $x[n]$ (i.e., a digital signal).

Fourier Transform:

$$X[k]$$

Inverse Fourier transform

$$x[n]$$

The signal $x[n]$ and its Fourier transform $X[k]$ are considered as **N-periodic sequences!**

MAIN PROPERTIES

Linearity

Energy and inner product preservation (Plancherel-Parseval theorem)

Translation in time becomes modulation in frequency

Modulation in time becomes translation in frequency

Time scale

Derivation

Hermitian symmetry for a real signal

Convolution for digital signals of same size N, (using zero-padding if necessary)

[OBJ]

the convolution is supposed to be **N-periodic!**

SPECTRUM

Let $x(t)$ be a signal and $X(f)$ its Fourier transform. The (power) spectrum of $x(t)$ is the (squared) modulus of $X(f)$:

$$S(f) = |X(f)|^2$$

If $x(t)$ is real-valued, then the spectrum is symmetrical with respect to the frequency f :

TO DO: GUITAR TUNER

Data:

single notes in wav files

Guitar notes and corresponding frequencies:

E1: 329.63 Hz ; **B2:** 246.94 Hz ; **G3:** 196.00 Hz ; **D4:** 146.83 Hz ; **A5:** 110.00 Hz ; **E6:** 82.41 Hz

Values of 1 cent for accuracy of each string: 0.15, 0.15, 0.15, 0.08, 0.08, 0.04 (see: http://zerocapcable.com/?page_id=225 and https://www.engineeringtoolbox.com/octave-bands-frequency-limits-d_1602.html)

Goal

For each wav files:

Perform a spectral analysis

Automatically determine the played note with the accuracy in cent

TO DO: ZOOM IN IMAGES

Data:

Any image you want

Goal:

For a given image:

Perform a spectral analysis

Resize the image to get a smaller image of half size

in the space domain by subsampling

by resizing in the Fourier domain

Resize the image to get a bigger image of double size

by sinc interpolation (zero padding in the Fourier domain)

by linear interpolation in the space domain