

INTRODUCTION

M2 AI — SIGNAL PROCESSING

CONTENT

▶ Organisation

- ▶ Short introduction for each sequence (30 min)
- ▶ Working with Matlab or Python (your choice. Even Julia)
- ▶ Groups of two students
- ▶ Free organization: refer to theory by yourself if needed, ask questions (during the class or by mail)

▶ What is expected

- ▶ Theory must be understood
- ▶ Practical work must be done
- ▶ A jupyter notebook (Python) or Matlab Publish (Matlab) to present your work

▶ Ressources

- ▶ My website: <http://hebergement.universite-paris-saclay.fr/mkowalski>
- ▶ Mail: matthieu.kowalski@universite-paris-saclay.fr ; matthieu.kowalski@inria.fr
- ▶ Numerical tours: <https://www.numerical-tours.com>

SIGNAL: INTUITIVE DEFINITION

- ▶ A signal is a physical representation that carries “information” from a source to a recipient.
- ▶ It is a quantity physically measurable by a sensor, which can vary over time and/or space



EXAMPLE: MUSICAL (SOUND) SIGNAL

- ▶ Measured quantity: variation of sound pressure



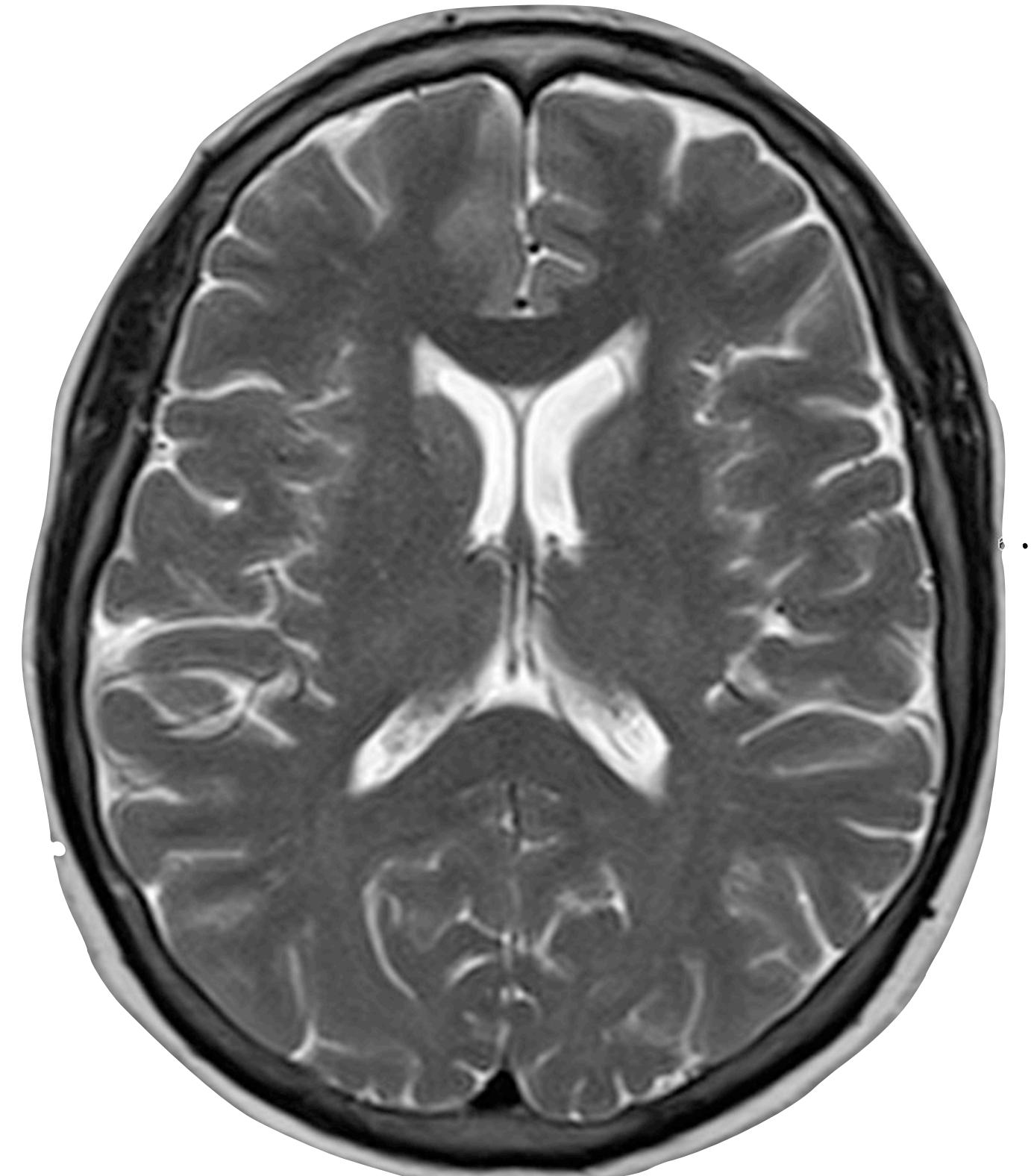
EXAMPLE: PHOTOGRAPHY

- ▶ Measured quantity: photoelectric effect



EXAMPLE: IRM

- ▶ Measured quantity: magnetic field



EXAMPLE: ECHOGRAPH

- ▶ Physical quantity measured: Doppler effect by ultrasound



GOALS OF SIGNAL PROCESSING

- ▶ Model
- ▶ Analyze
- ▶ Restore
- ▶ Transmit
- ▶ Compress
- ▶ Solve inverse problems
- ▶ ...

DETERMINISTIC VS RANDOM

- ▶ **Deterministic model:** signals that can be predicted “certainly” using simple descriptors (its function, its frequencies, etc.)
Example: recording of a song, photography, etc.
Tools: Fourier, Wavelets, Time-Frequency analysis, Cepral analysis (Hilbert/Harmonic analysis)
- ▶ **Random model:** signals from “stochastic processes”: each “realization” shares common quantities, but each signal is different.
Example: background noise
Tools: Probability, statistics, Estimation, Bayesian inference
- ▶ The two models are complementary
Example: noisy signal

DETERMINISTIC SIGNALS: MATHEMATICAL DEFINITION

- ▶ An **analog** signal is a function of a **continuous variable**, usually "continuous" time. Let $s(t)$ be an analog temporal signal

$$\begin{aligned}s &: \mathbb{R} \rightarrow \mathbb{R} \\ t &\mapsto s(t)\end{aligned}$$

- ▶ A **numerical** signal is a function of a **discrete variable**, generally "discrete" time, that is to say, a mathematical sequence. Let $s[t]$ be a numerical time signal

$$\begin{aligned}s &: \mathbb{Z} \rightarrow \mathbb{R} \\ t &\mapsto s[t] = s_t\end{aligned}$$

FROM ANALOG TO DIGITAL

- ▶ Two steps

1. Sampling

2. Quantization

- ▶ Sampling Theorem:

Let $x(t)$ an **analog, band-limited signal**, i.e. with cutoff frequency ν_0 . Then, $x(t)$ can be **reconstructed perfectly** from the samples $x(t_n)$ collected at the moment

$$t_n = \frac{n}{2\nu_0} = \frac{n}{\nu_e}$$

- ▶ $\nu_e = 2\nu_0$ is called the **sampling frequency**.
- ▶ The reconstruction formula is given by

$$x(t) = \sum_{n=-\infty}^{+\infty} x(t_n) \text{sinc}(\nu_e(t - t_n))$$

FROM ANALOG TO DIGITAL

- ▶ Two steps

1. Sampling

2. Quantization

- ▶ Quantization

Each values $x(t_n)$ must be mapped from a real value (infinite precision) to a decimal value (with finite precision).

DEFINITIONS

A signal $s(t)$ is

- ▶ Causal iff $s(t) = 0 \quad \forall t < 0$
- ▶ Stable iff $\|s\|_1 < +\infty$ $\left(\|s\|_1 = \sum_{k=-\infty}^{+\infty} |s[k]| \right)$
- ▶ Of finite energy iff $\|s\|_2^2 < +\infty$ $\left(\|s\|_2^2 = \sum_{k=-\infty}^{+\infty} |s[k]|^2 \right)$
- ▶ Realisable iff $s(t)$ is stable and causal

SEQUENCES

1. Spectral analysis and ideal filtering Audio: guitar tuner Image: zoom in imaging
2. Real time Filtering Audio: Delay effect (IIR + FIR) Image: Segmentation
3. Random Signal Audio: noise spectrum estimation and vocoder
4. Time-frequency Audio: audio denoising by spectral subtraction
5. Wavelets for images Image: image compression and denoising by thresholding
6. Introduction to inverse problems Debluring or Superresolution using TV or wavelets