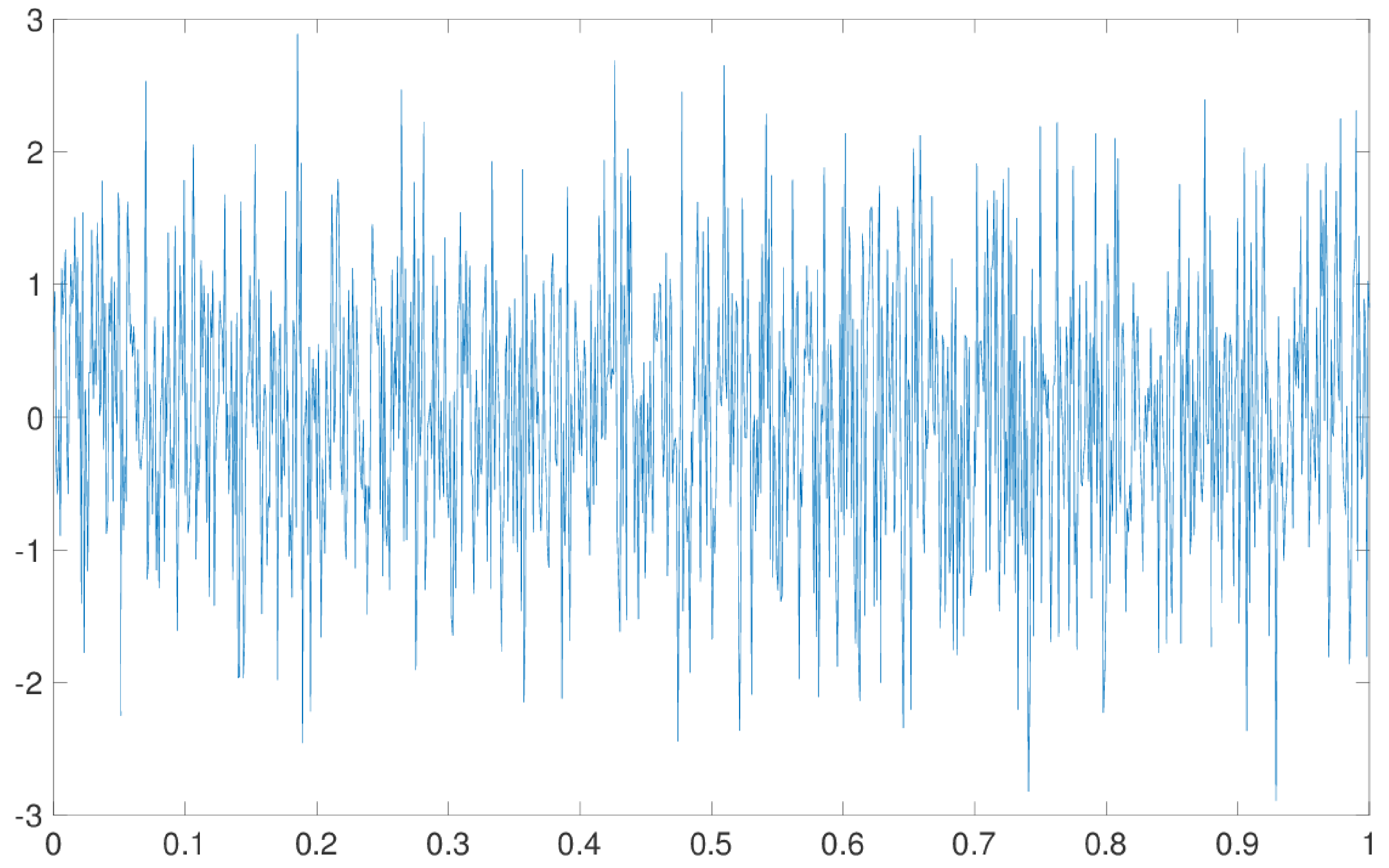
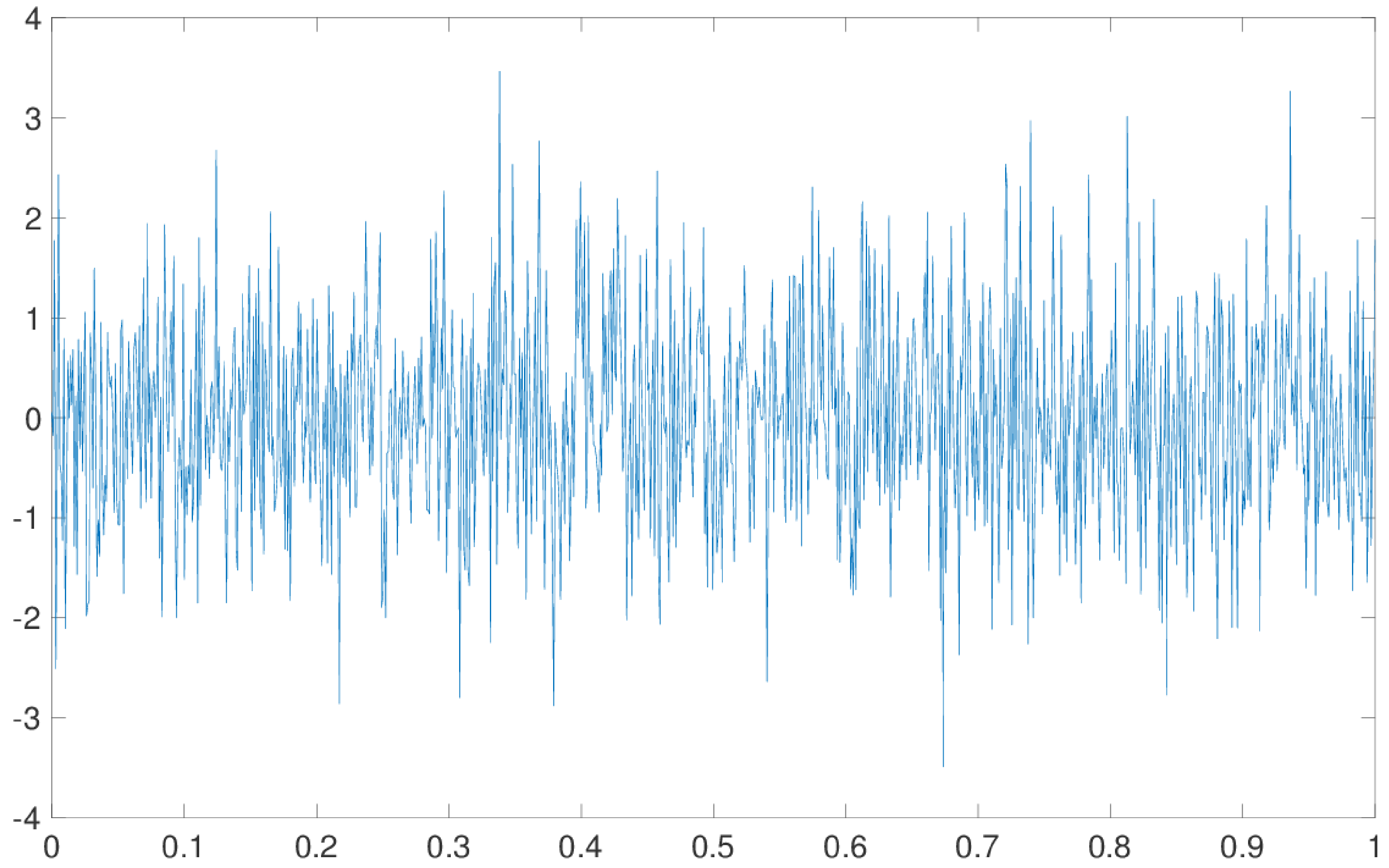
RANDOM SIGNALS

M2 AI — SIGNAL PROCESSING

INTRODUCTION

- Usefulness:
  - Noise modeling
  - Statistical estimation
  - Bayesian approach
- Different, but similar





DIGITAL RANDOM SIGNAL

- A digital random signal is a random process:
- For each , is a random variable
- In practice we observe one (partial) trajectory, that is one realization
- The random signal is, usually, assumed to be ergodic and stationary:
  - We can work with only one realization
  - The autocorrelation function is deterministic
  - We can work with temporal statistics

SPECTRUM OF RANDOM SIGNAL

- Let be a random signal
  - The Fourier transform of is not meaningful
  - The spectrum of a random signal is the Fourier transform of its autocorrelation function:

- Can be difficult to estimate in practice
- Two estimators:
  - Periodogram estimator (power spectrum of the observed trajectory)
  - Welch estimator (average of short-time power spectrum)

NOISE

- Gaussian white noise
  - All the random variables ￼ are i.i.d. from ￼
  - Autocorrelation function: ￼
  - Spectrum: ￼
- A Gaussian colored noise is a filtered white noise

LINEAR DENOISING

Let be a noisy measure of a "clean" signal corrupted by some additive noise :

Signal to Noise Ratio (SNR):

Goal: find the best (oracle) filter such that is the best estimation of

Solution: Wiener filter, given in the frequency domain by

More on the numerical tour !! (See the linear image denoising tour)

TO DO: NOISE SPECTRUM DENSITY ESTIMATION AND WIENER FILTERING

- Data
  - 3 noises
  - Audio file or image of your choice
- Todo
  - For each noise
    - Estimate the spectrum density by periodogram and Welsh method
    - Identify the color of the noise (white, pink, red, mixture of noises…)
  - With the image or audio file
    - Simulate a noisy version of the signal with various SNR (0dB, 5 dB, 10 dB, 15 dB, 20 dB), using a Gaussian white noise
    - Denoise the signal using the Wiener filter