Social Data Management
Communities

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M2 Data Science

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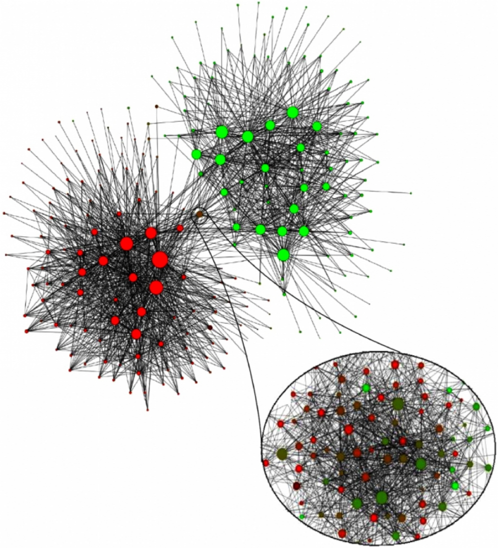
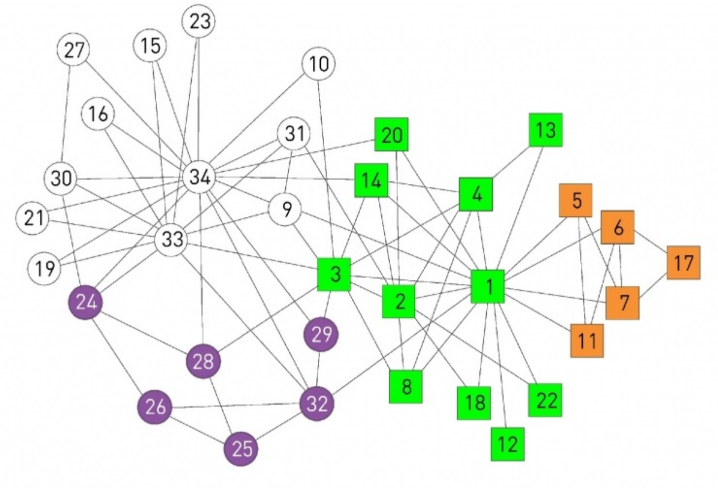


Communities in the Belgium call graph

Graph Communities



Communities in Zachary's Karate club

Detecting Communities – Hypothesis 1

Hypothesis
A graph's community structure is uniquely encoded in its topology.

Detecting Communities – Hypothesis 2

Hypothesis
A community is a locally dense connected subgraph in a network.

Detecting Communities

A few approaches:

1. **Maximum Cliques**: a community is a subgraph whose nodes are all connected to each other.
2. **Strong Communities**: relaxation of cliques, depending on the *internal degree* (number of neighbors in the community) vs. *external degree* (neighbours outside of the community) – a strong community has a greater internal degree than external degree.

Detecting Communities

A naïve algorithm for detecting 2 communities:

1. divide the graph in two (find a **cut**) and decide if they are strong communities, and
2. choose the best cut over all possible cuts.

This can generalize to more communities, but it needs to generate an **exponential** number of cuts.

Polynomial Algorithms

We need polynomial algorithms to be able to detect efficiently the communities in a graph.

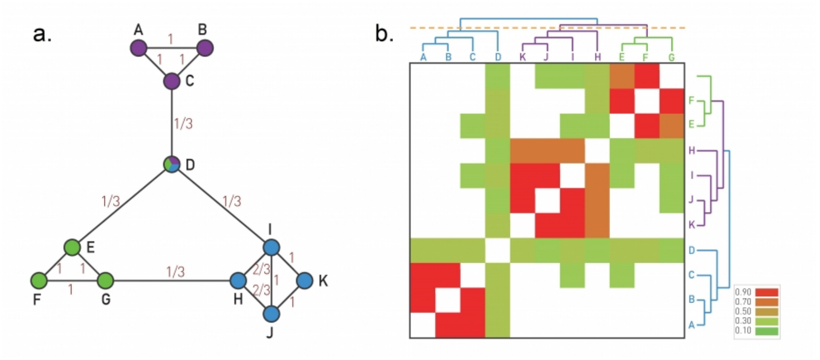
One approach is **hierarchical clustering**:

- uses a similarity matrix X, where x_{ij} encodes the similarity between nodes i and j, and
- based on this, **agglomerative algorithms** merge nodes into the same community, while
- **divisive algorithms** isolate communities by removing low similarity links.

Agglomerative Algorithm – Ravasz

1. **Define the similarity matrix**: various ways, but the algorithm uses the *topological overlap matrix*, encoding the number of common neighbors over the maximum possible.
2. **Define group similarity**: computed as the *average cluster similarity* – the average of x_{ij} over all node pairs
3. **Apply the Hierarchical Clustering**:
    3.1 assign each node to a community of their own,
    3.2 find the community pairs with highest similarity and merge them,
    3.3 compute the similarity between all communities
    3.4 repeat until only one community exists.
4. The community structure will be encoded in the *Dendogram*, showing the order in which communities were merged (see next slide).

Agglomerative Algorithm – Ravasz

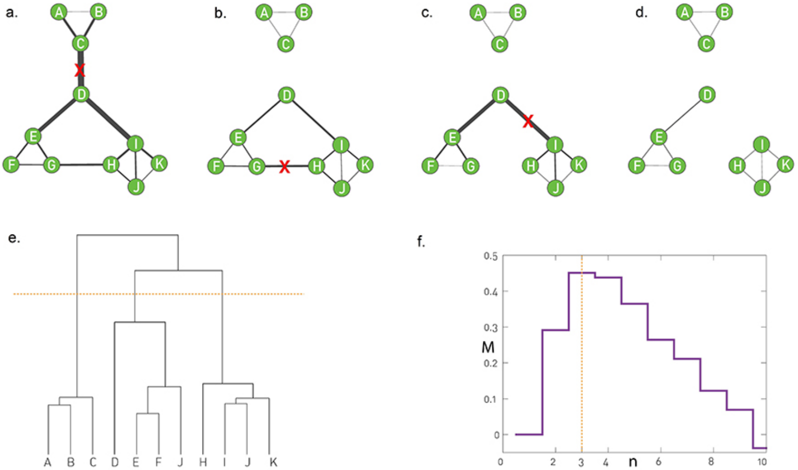


Divisive Algorithm – Girvan-Newman

Divisive algorithms remove edges:

1. **Define centrality**: x_{ij} needs to select nodes in different communities, e.g., betweenness.
2. **Apply the Hierarchical Clustering**:
   2.1 remove the link with the largest centrality
   2.2 recompute the centrality of all other links
   2.3 repeat until no links exist
3. The community structure will be encoded in the **Dendogram**, showing the order in which edges were removed (see next slide).

Divisive Algorithm – Girvan-Newman



Detecting Communities – Hypothesis 3

Hypothesis
Random networks lack a community structure.

Modularity

Consider a graph having some partition into communities C having L_C links. If L_C is greater than the number of links expected by a random wiring *having the same degree distribution*, then it is **a potential community**.

This is measured by the **modularity**:

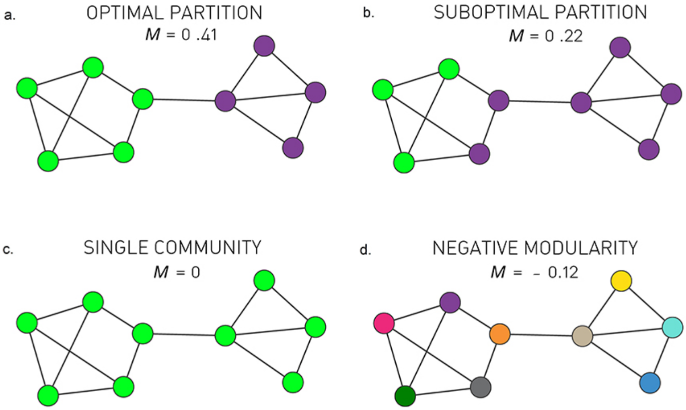
$$M_C = \frac{1}{2L} \sum_{(i,j)\in C} (A_{ij} - p_{ij}),$$

where p_{ij} can be computed by randomizing the original network, e.g.,:

$$p_{ij} = \frac{k_i k_j}{2L}.$$

For the entire graph: we sum the modularities over all communities.

Modularity – Examples



Modularity – Algorithm

Hypothesis
The partition with maximum modularity corresponds to the optimal community structure.

Modularity – Algorithm

For computation efficiency concerns, all algorithms use a greedy approach:

1. Assign each node to its own community.
2. Inspect each community pair connected by at least one link and merge the ones having the highest increase in modularity ΔM *for the whole network*.
3. Repeat until all nodes are in a single community.
4. Choose the partition with the highest modularity.

Louvain algorithm: optimizes the modularity for each node – moves one node in the nearest community w.r.t. modularity

Community Detection Algorithms – Complexity

algorithm	**type**	**complexity**
Ravazs	agglomerative	$\mathcal{O}(N^2)$
Girvan–Newman	divisive	$\mathcal{O}(N^3)$
greedy optimized	modularity	$\mathcal{O}(N \log^2 N)$
Louvain	modularity	$\mathcal{O}(L)$
Infomap	flow	$\mathcal{O}(N \log N)$

Open Issues in Community Detection

- **Do communities really exist?**: given a network, do we know it is always organized in communities?
- **Are the hypotheses valid?**: is a community only identified by its wiring diagram?
- **Does everybody belong to a community?**
- **How do we know which measure is the valid one?**: centrality, similarity, modularity, flow, etc.

Acknowledgments

Figures in slides 3, 4, 11, 13, and 16 taken from the book "Network Science" by A.-L. Barabási. The contents is partly inspired by the flow of Chapter 9 of the same book. `http://barabasi.com/networksciencebook/`

References i

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