Constraint & Data Mining

Cours2-3

Master 2 - DS

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Frequent Itemset Mining

Knowledge Discovery in Databases (KDD)

Definition and steps

- Knowledge Discovery in Databases (KDD) revolves around the investigation and creation of knowledge, processes, algorithms, and the mechanisms for retrieving potential knowledge from data collections

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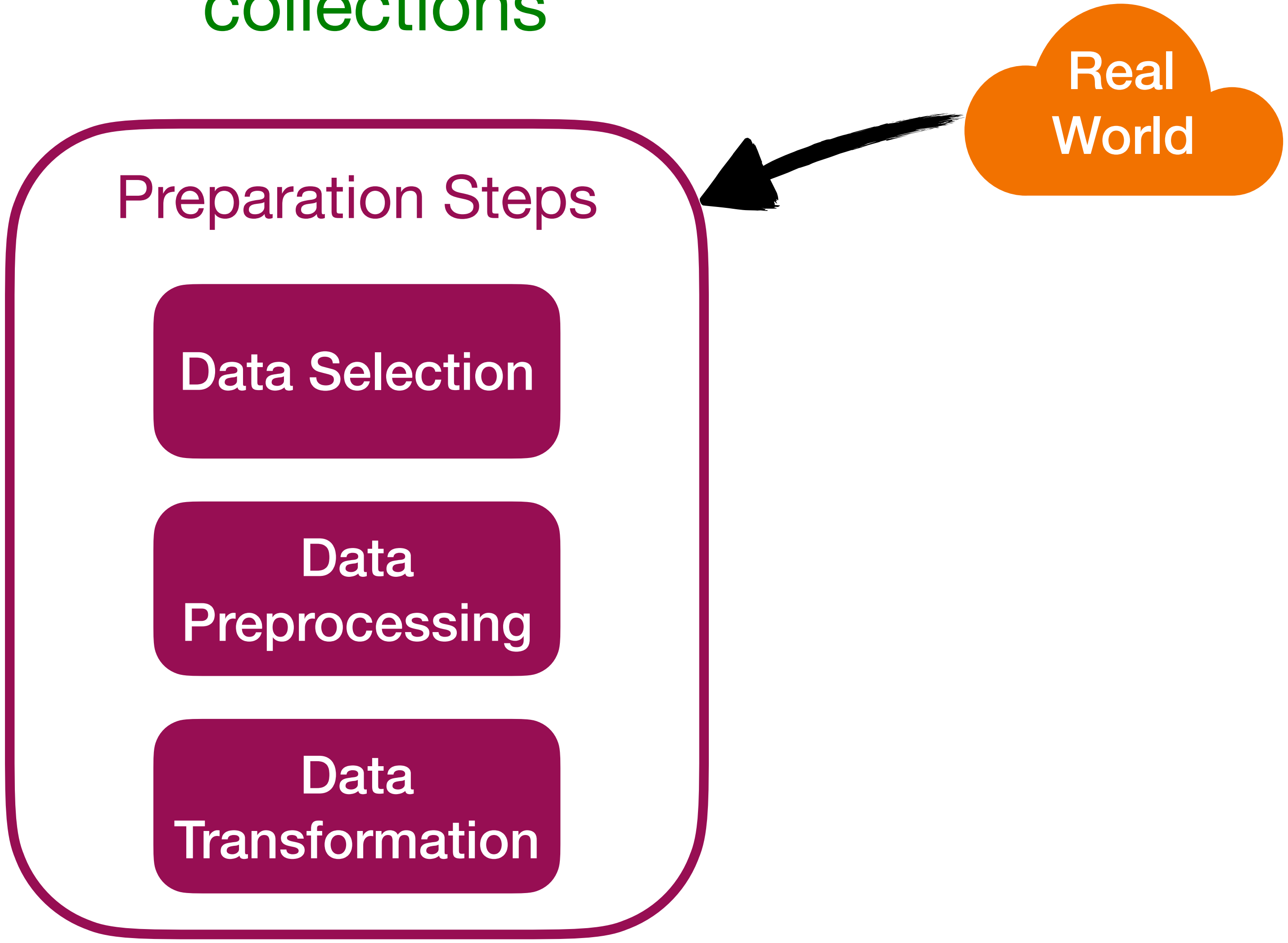

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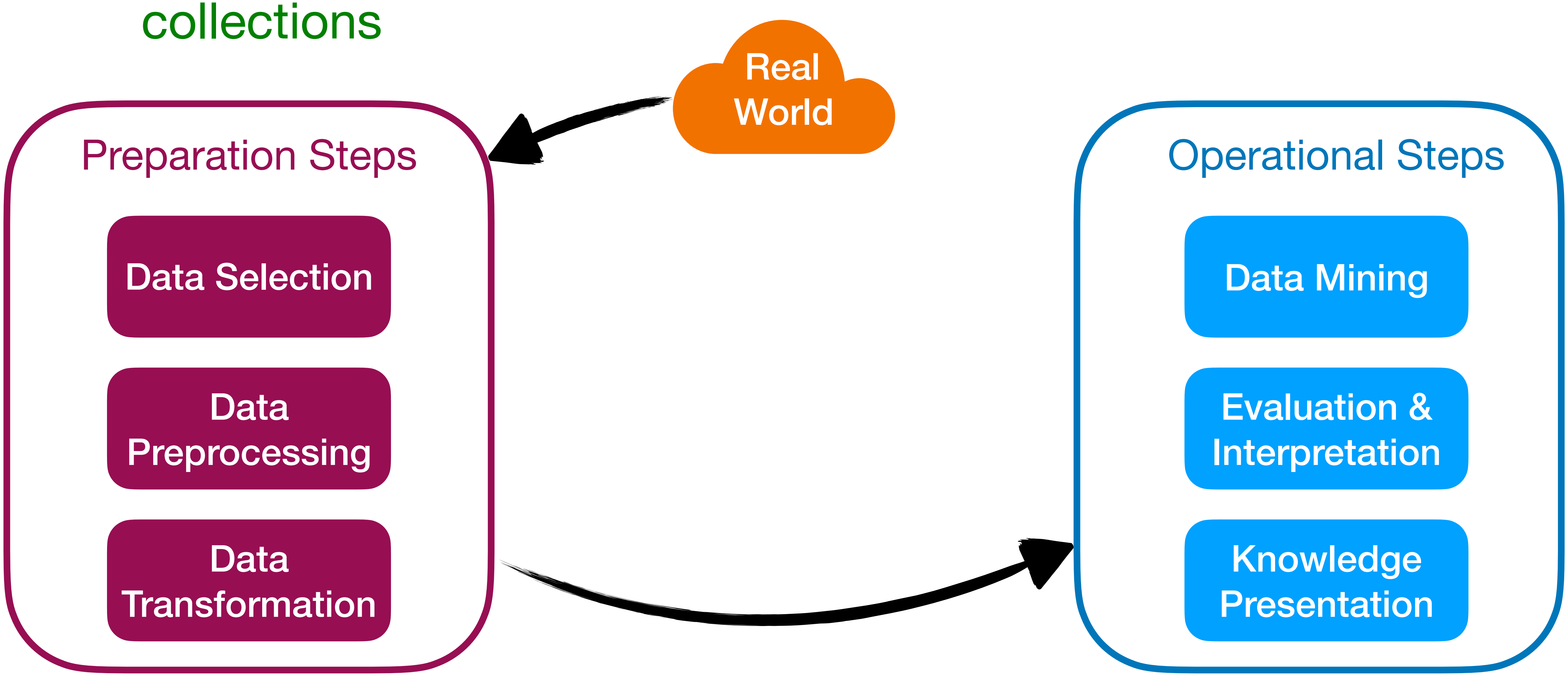

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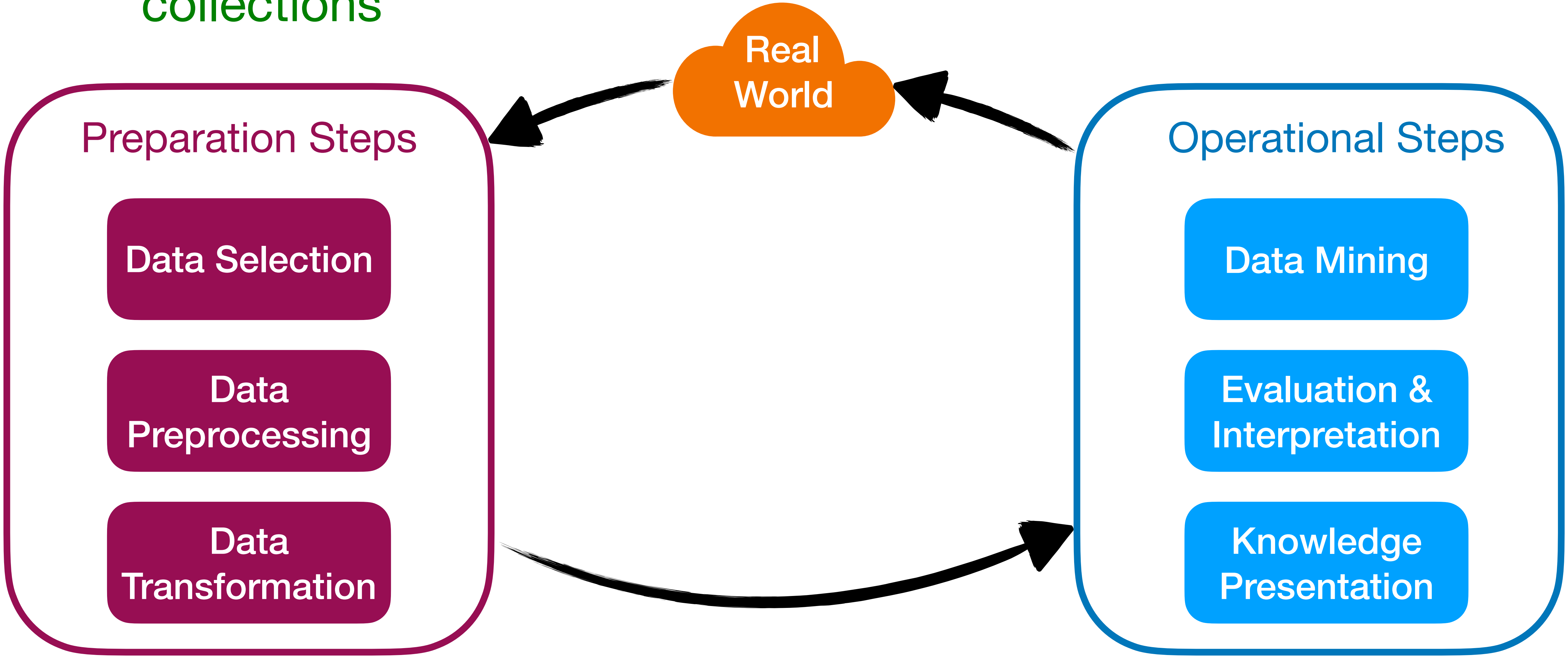

Frequent Itemset Mining

Motivations

- **Purpose:** A key method in market basket analysis, aimed at identifying patterns in customer purchasing behavior
- **Goal:** To discover frequently co-occurring products, which are items that customers often buy together
- **Applications of Frequent Itemsets:**
  - **Product Arrangement:** Optimize product placement on store shelves or within catalogs
  - **Cross-Selling:** Suggest related products to customers (e.g., recommending additional products during online shopping)
  - **Product Bundling:** Offer product bundles based on co-purchase patterns
  - **Other Applications:** Fraud detection, dependency analysis, fault localization, and more

Frequent Itemset Mining

Basic Notions

- Items: $I = \{p_1, \ldots, p_n\}$
- Itemset, transaction: $P, T \subseteq I$
- Transactional dataset: $D = \{T_1, \ldots, T_m\}$
- Language of itemsets: $\mathcal{L}_I = 2^I$
- Cover of an itemset: $cover(P) = \{T_i \in D : \; P \subseteq T_i\}$
- Absolute Frequency: $freq(P) = |cover(P)|$
- Relative Frequency: $freq(P) = \dfrac{|cover(P)|}{|D|}$

Frequent Itemset Mining

Problem Definition

- **Given:**
  - A set of items $I = \{p_1, \ldots, p_n\}$
  - A transactional dataset $D = \{T_1, \ldots, T_m\}$
  - A minimum support α
- **The need:**
  - The set of itemset P s.t.: $freq(P) \geq \alpha$

Frequent Itemset Mining

Example(1)

- $I = \{a, b, c, d, e\}$
- $D = \{T_1, \ldots, T_{10}\}$

Frequent Itemset Mining

Example(1)

- $I = \{a, b, c, d, e\}$
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H_D

1:	a			d	e
2:		b	c	d	
3:	a		c		e
4:	a		c	d	e
5:	a				e
6:	a		c	d	
7:		b	c		
8:	a		c	d	e
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Frequent Itemset Mining

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8:	a		c	d	e
9:		b	c		e
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V_D

a	b	c	d	e
1	2	2	1	1
3	7	3	2	3
4	9	4	4	4
5		6	6	5
6		7	8	8
8		8	10	9
10		9		10

Frequent Itemset Mining

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6		7	8	8
8		8	10	9
10		9		10

M_D

	a	b	c	d	e
1:	1	0	0	1	1
2:	0	1	1	1	0
3:	1	0	1	0	1
4:	1	0	1	1	1
5:	1	0	0	0	1
6:	1	0	1	1	0
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Frequent Itemset Mining

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$cover(bc) = ?$

$freq(bc) = ?$

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Frequent Itemset Mining

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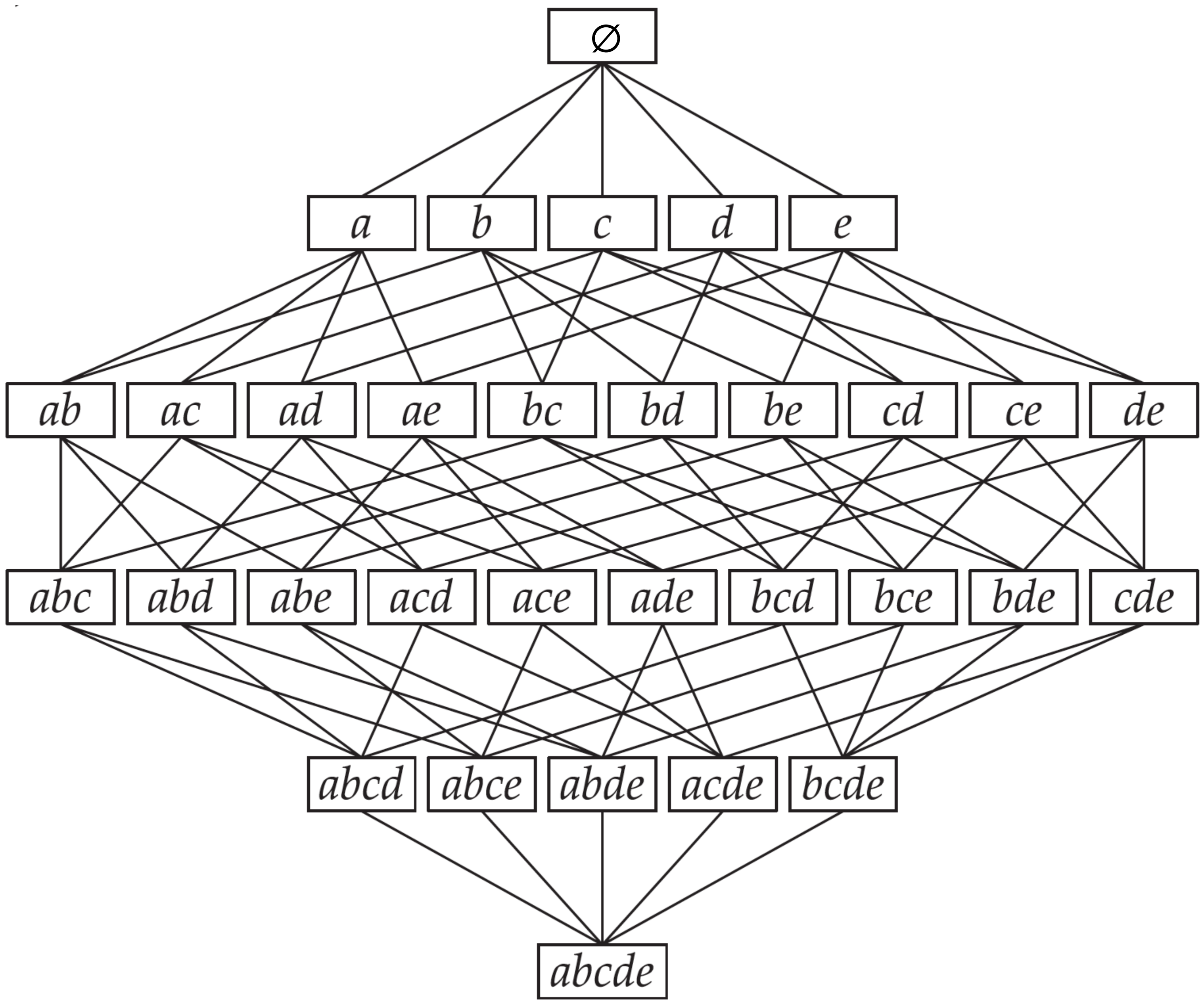

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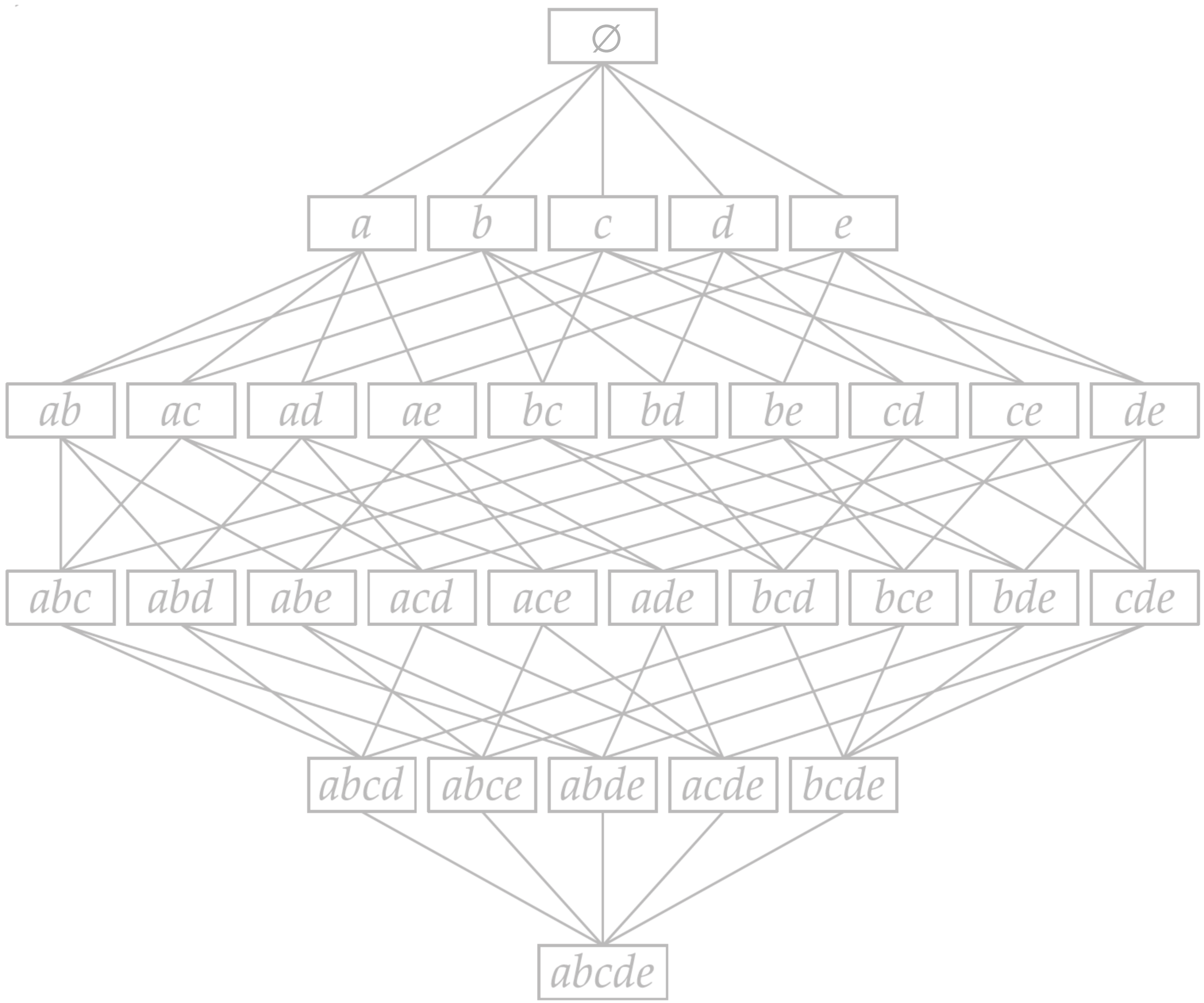

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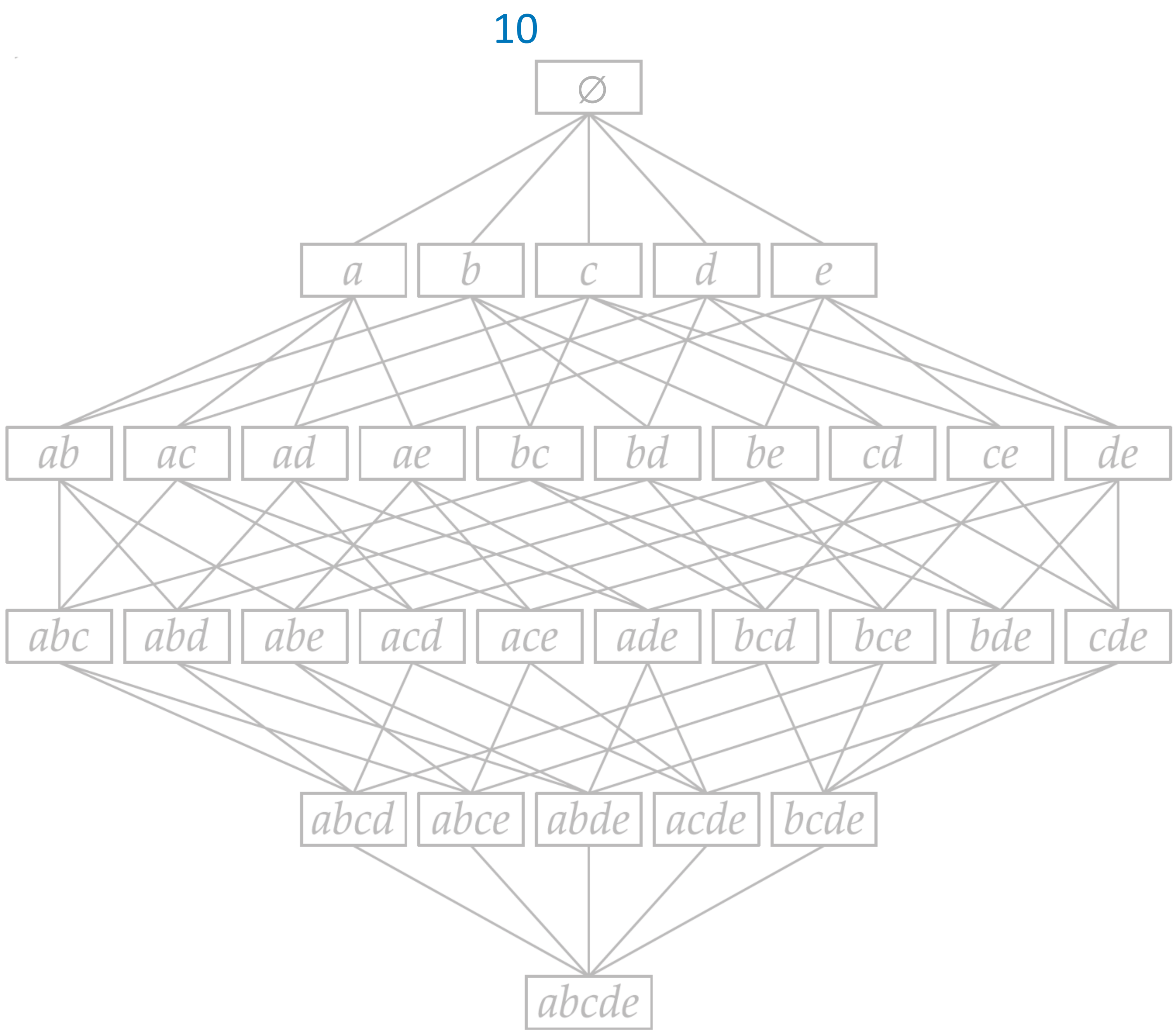

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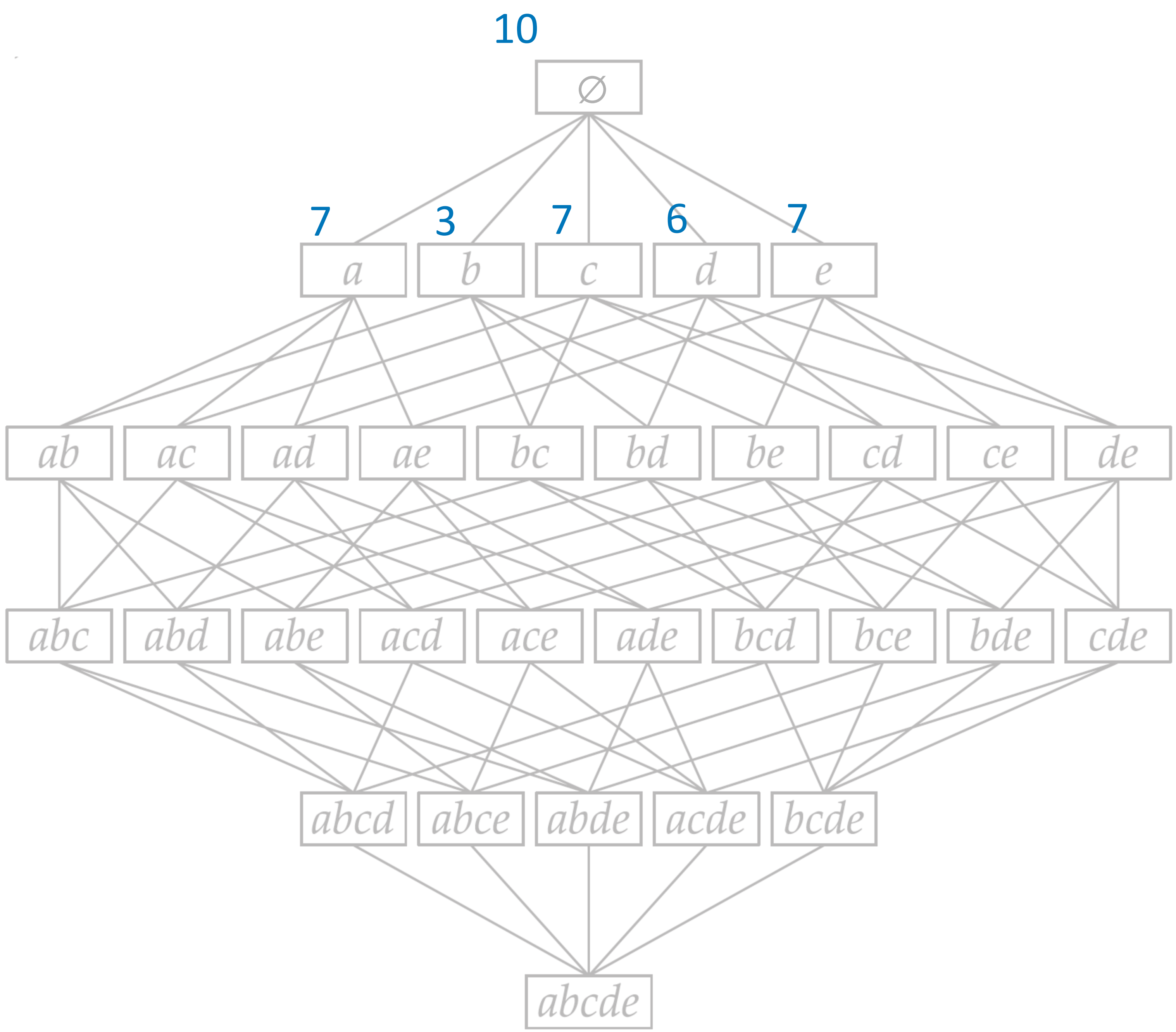

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Frequent Itemset Mining

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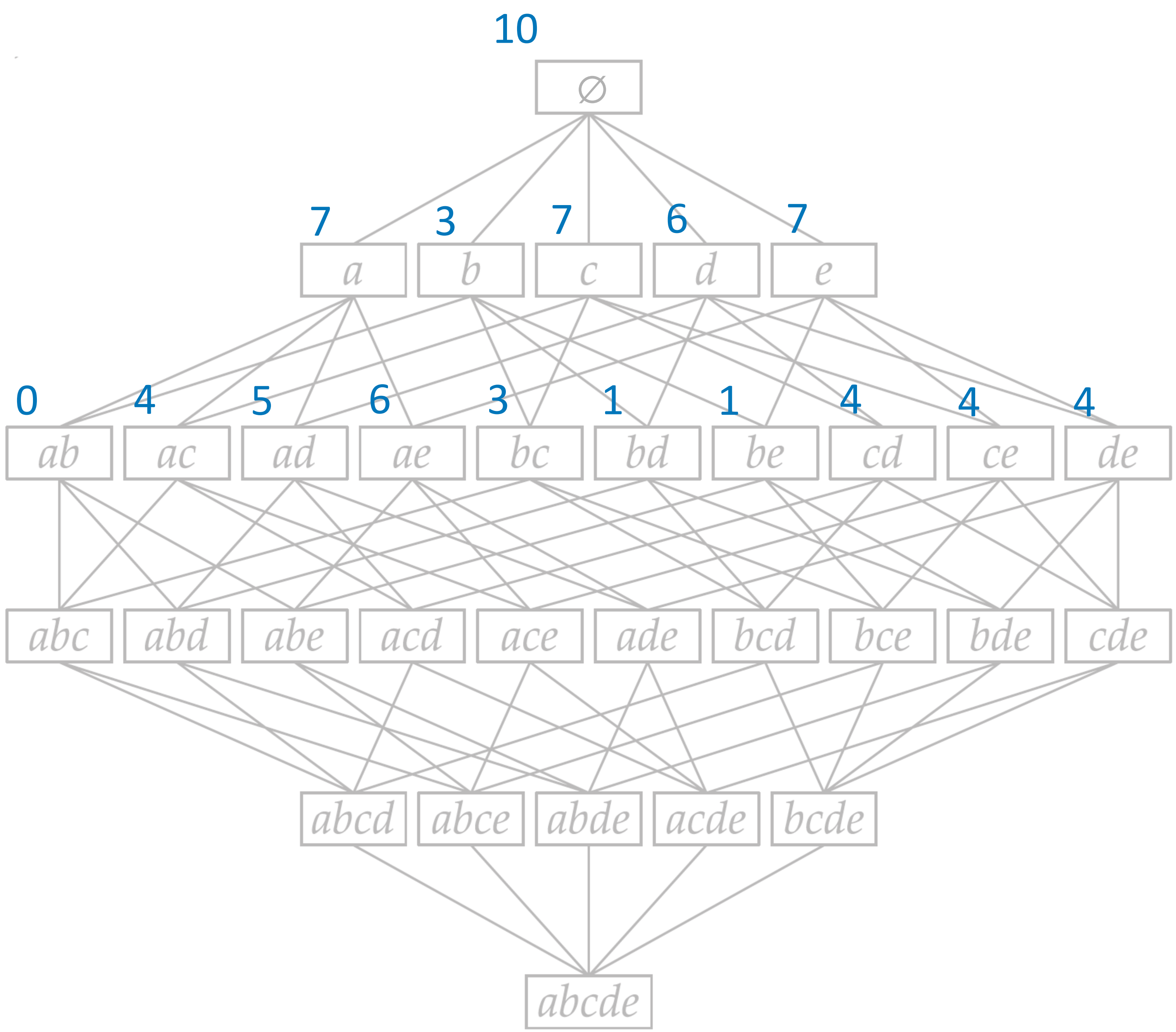

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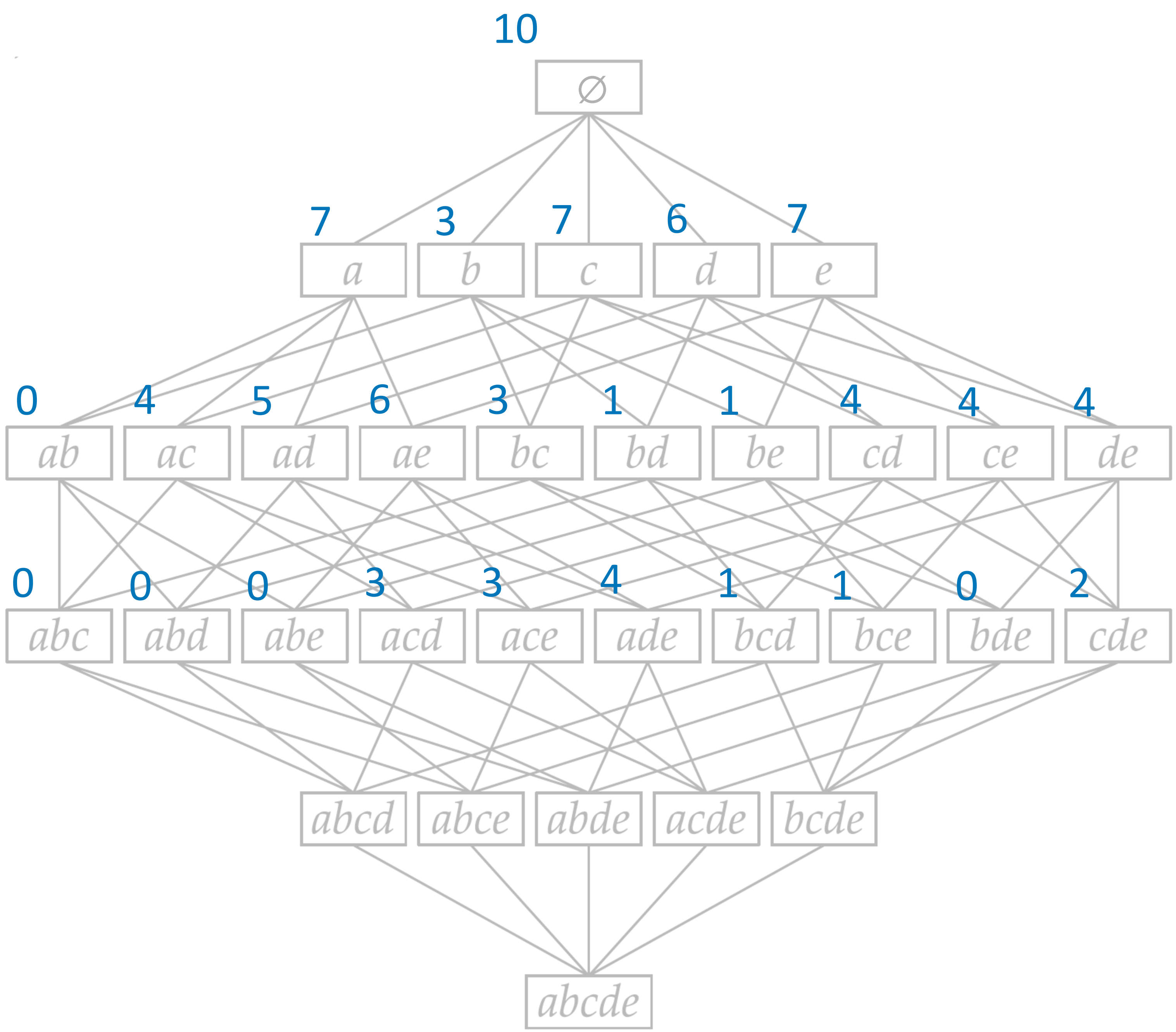

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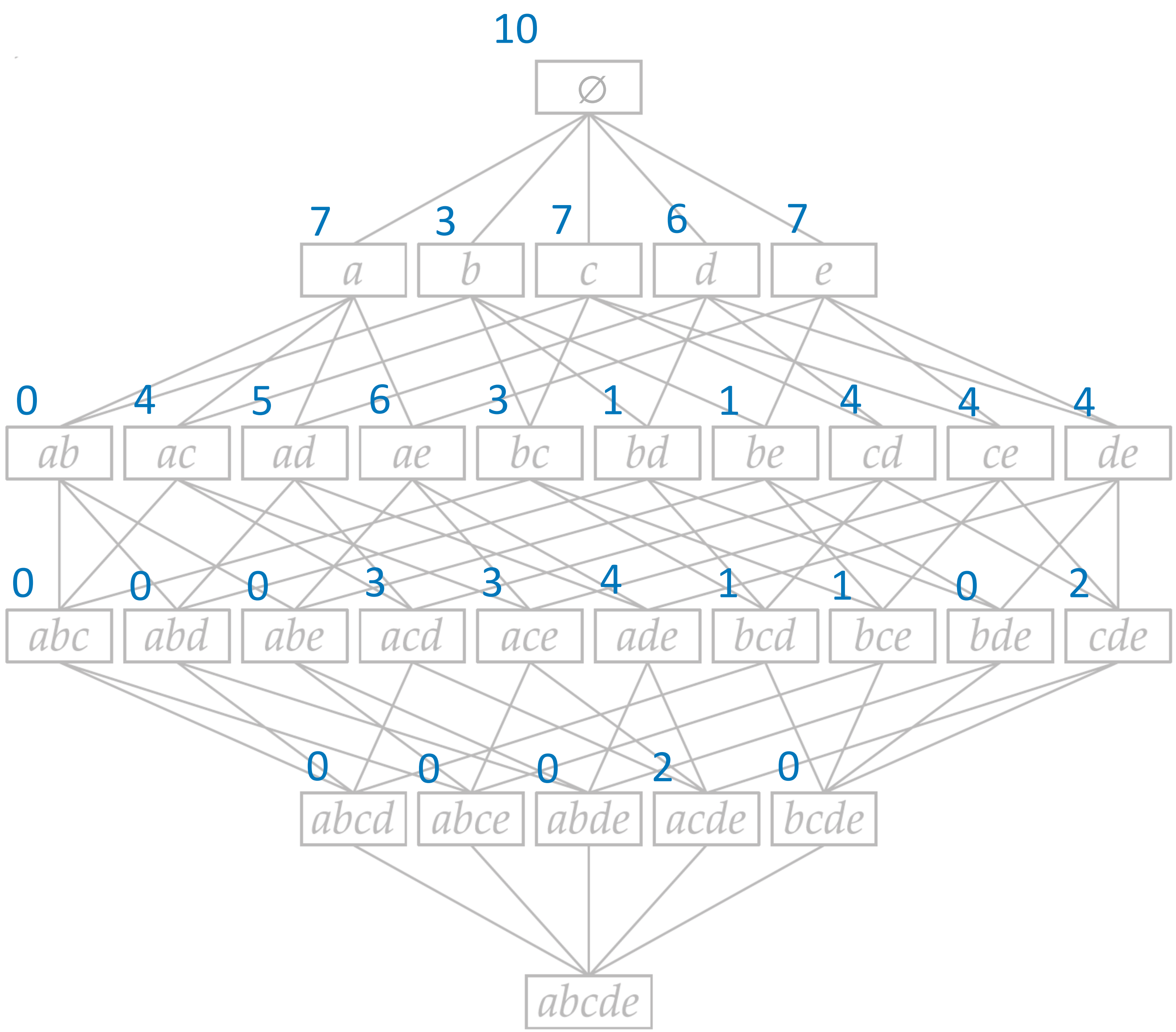

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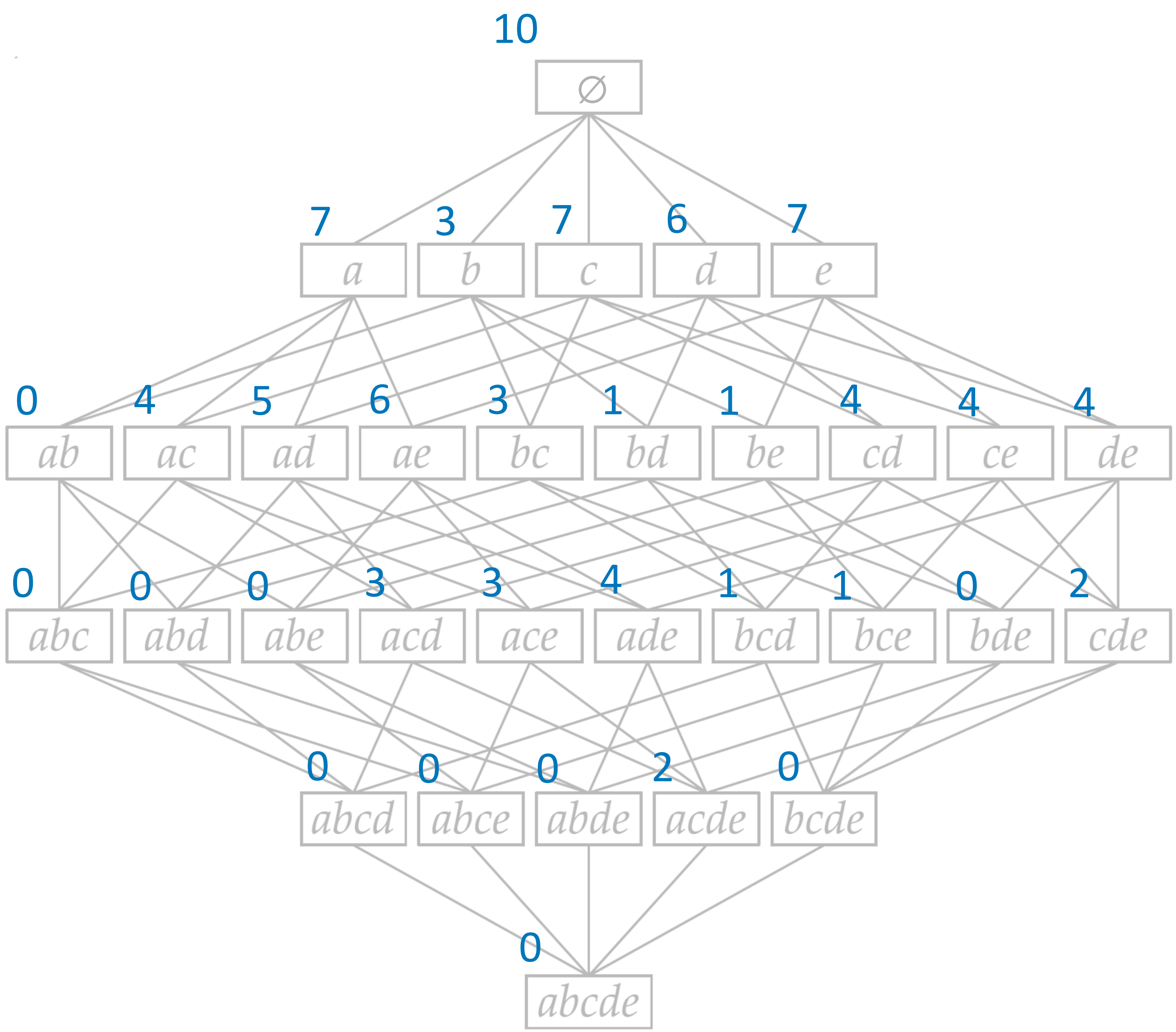

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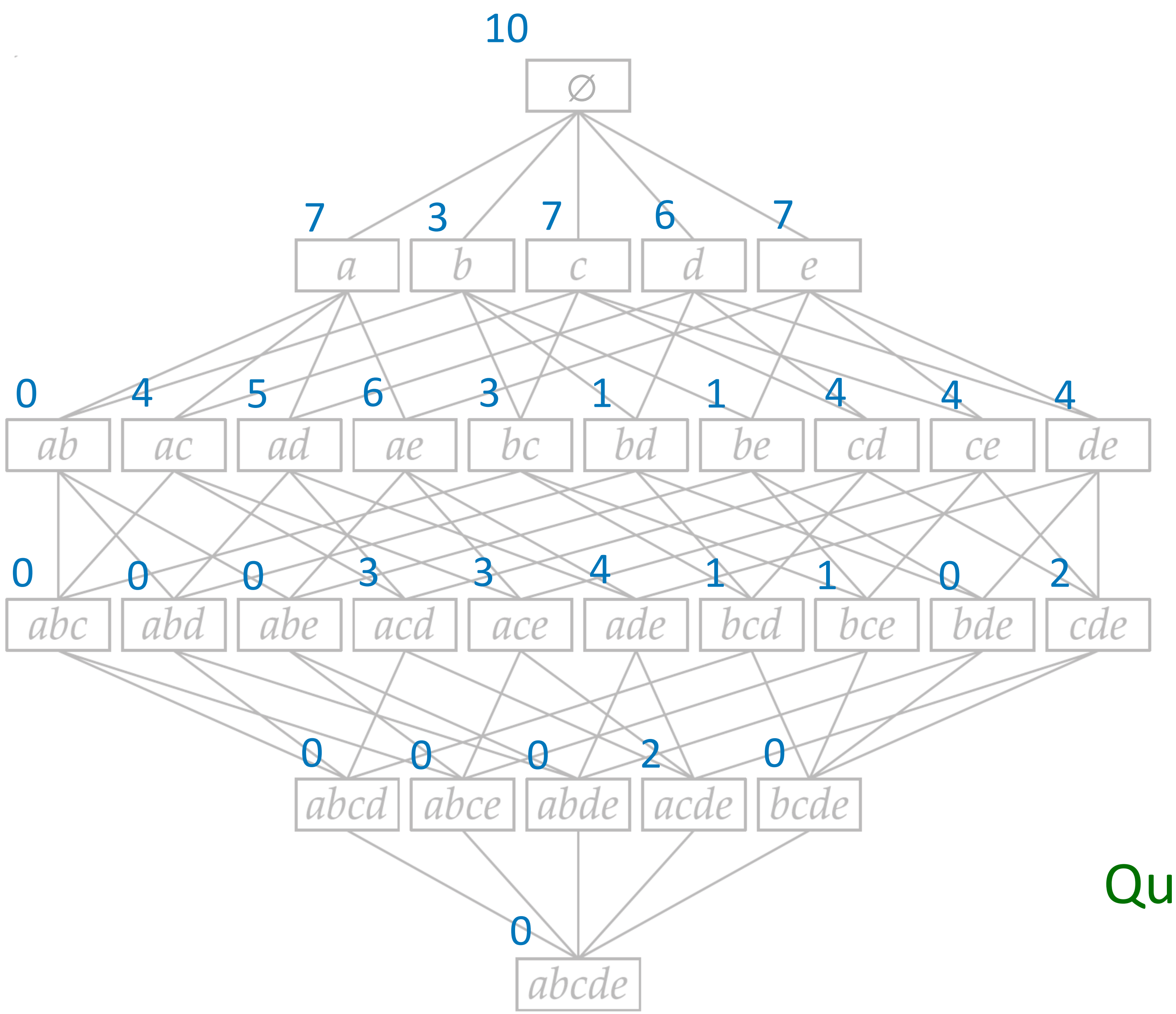

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Query-1: Frequent itemset?

Frequent Itemset Mining

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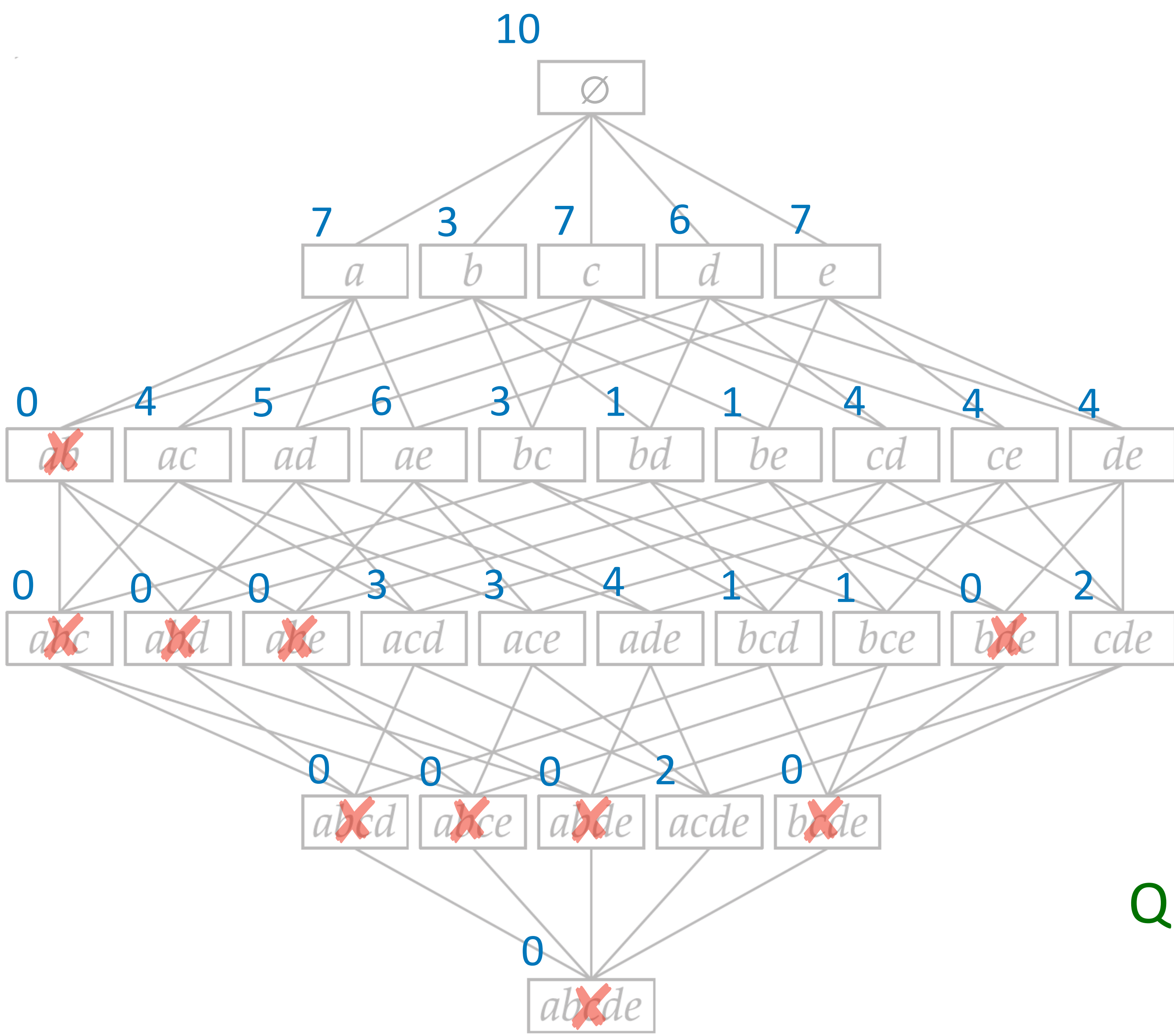

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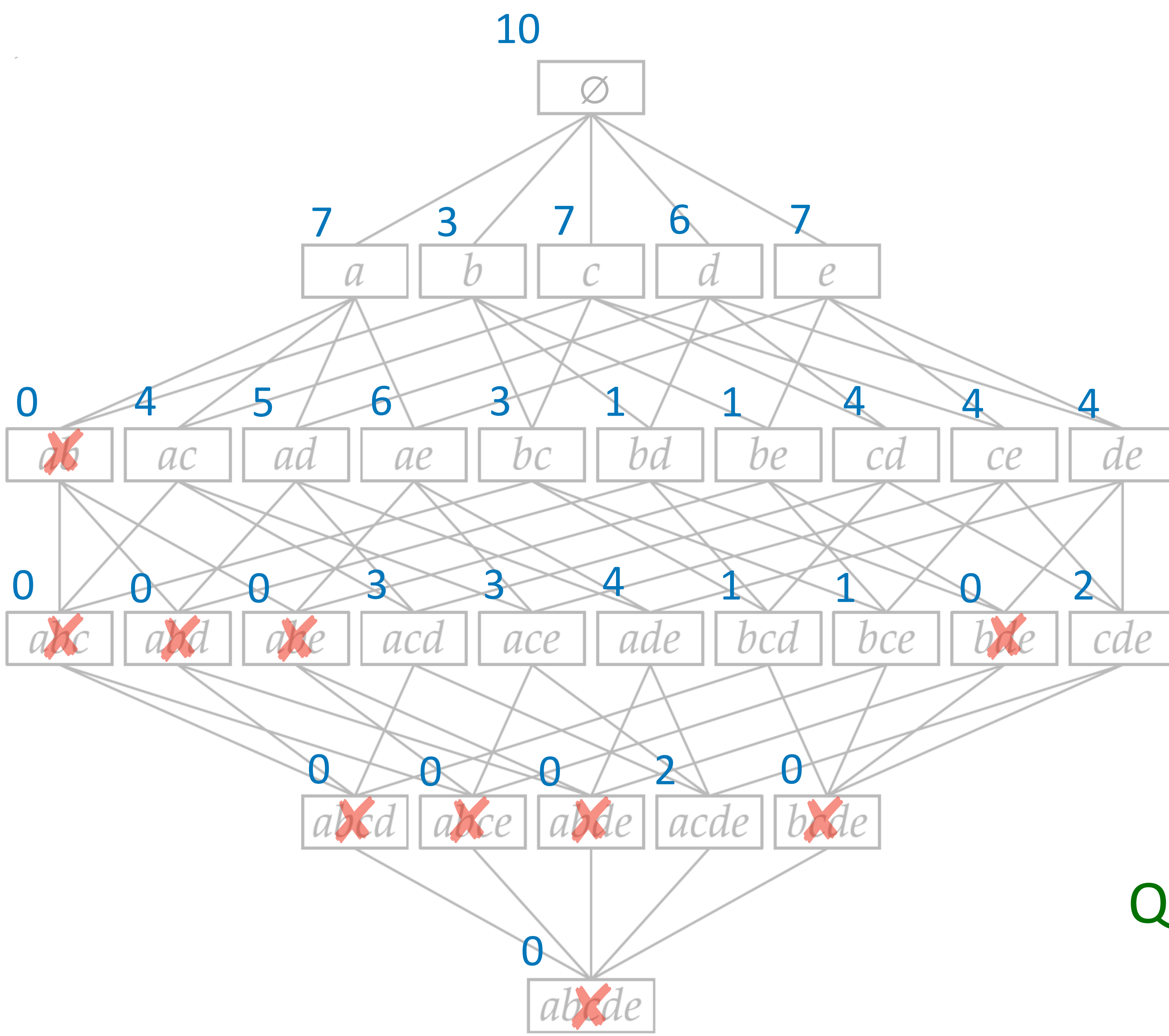
Query-1: Frequent itemset?

Frequent Itemset Mining

Example(1)

Query-2: Frequent itemset with minimum support $\alpha = 3$?



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Frequent Itemset Mining

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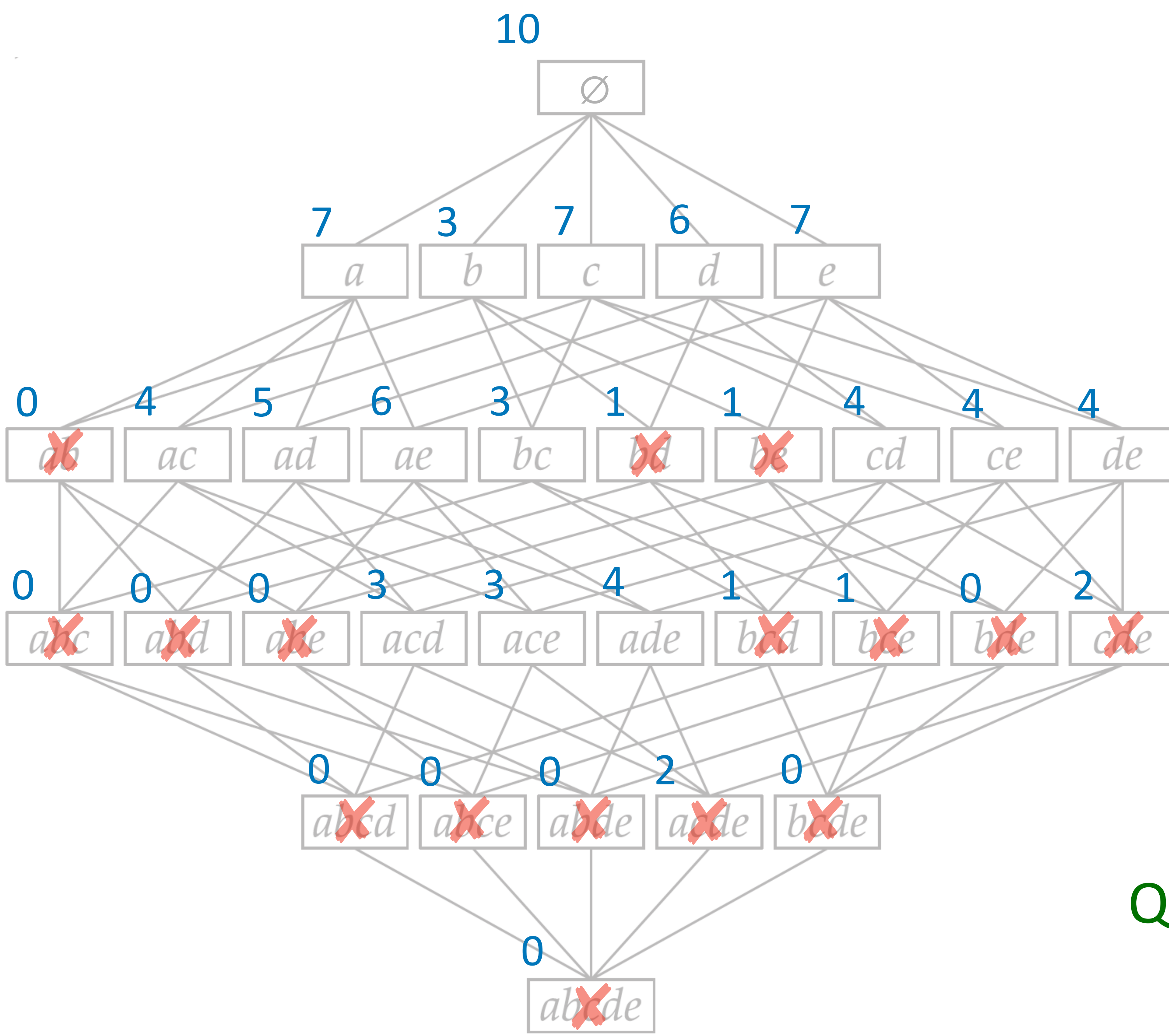

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Searching for Frequent Itemsets

Naïve Search

- A naïve search that consists of enumerating and testing the frequency of itemset candidates in a given dataset is usually infeasible

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Number of items (n)	Search space (2^n)
10	$\approx 10^3$
20	$\approx 10^6$
30	$\approx 10^9$
100	$\approx 10^{30}$
128	$\approx 10^{68}$ (atoms in the universe)
1000	$\approx 10^{301}$

Searching for Frequent Itemsets

Anti-Monotonicity Property

- Given a transaction database D over items I and two itemsets P and Q:

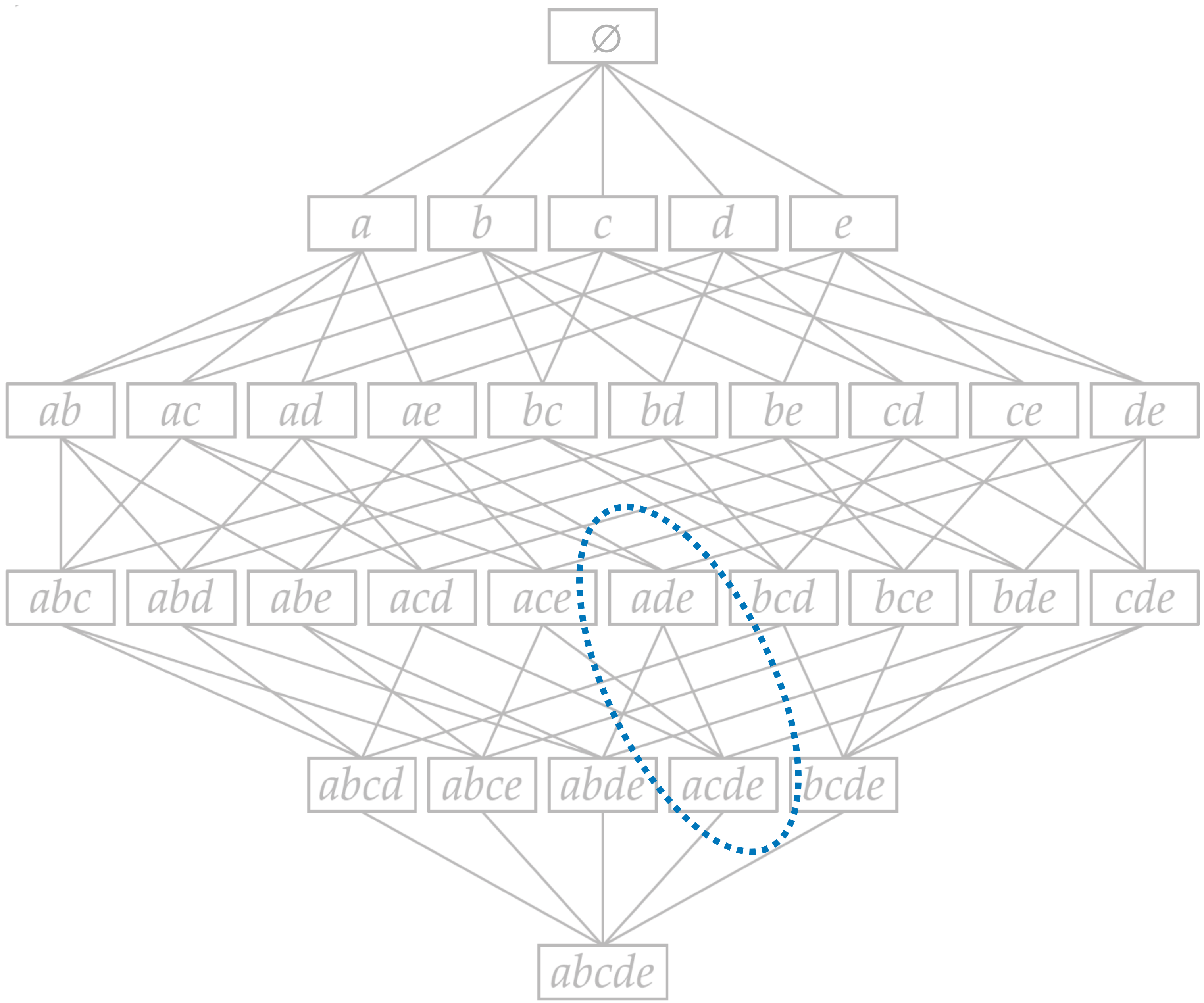
$$Q \subseteq P \Rightarrow cover(P) \subseteq cover(Q)$$

- That is,

$$Q \subseteq P \Rightarrow freq(P) \leq freq(Q)$$

Anti-Monotonicity Property

Example(2)



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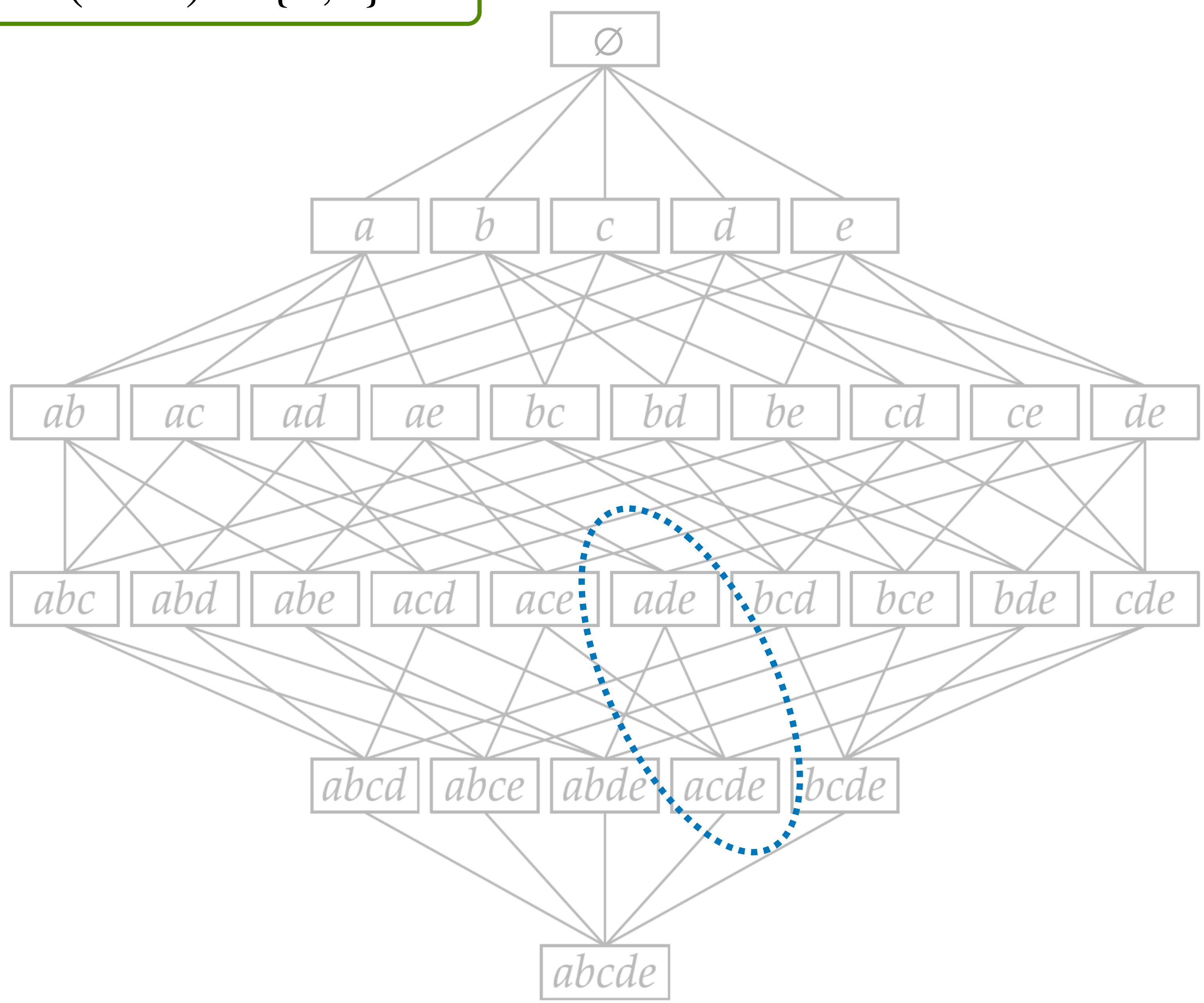
Anti-Monotonicity Property

Example(2)

$$cover(ade) = \{1,4,8,10\}$$
$$cover(acde) = \{4,8\}$$



H_D

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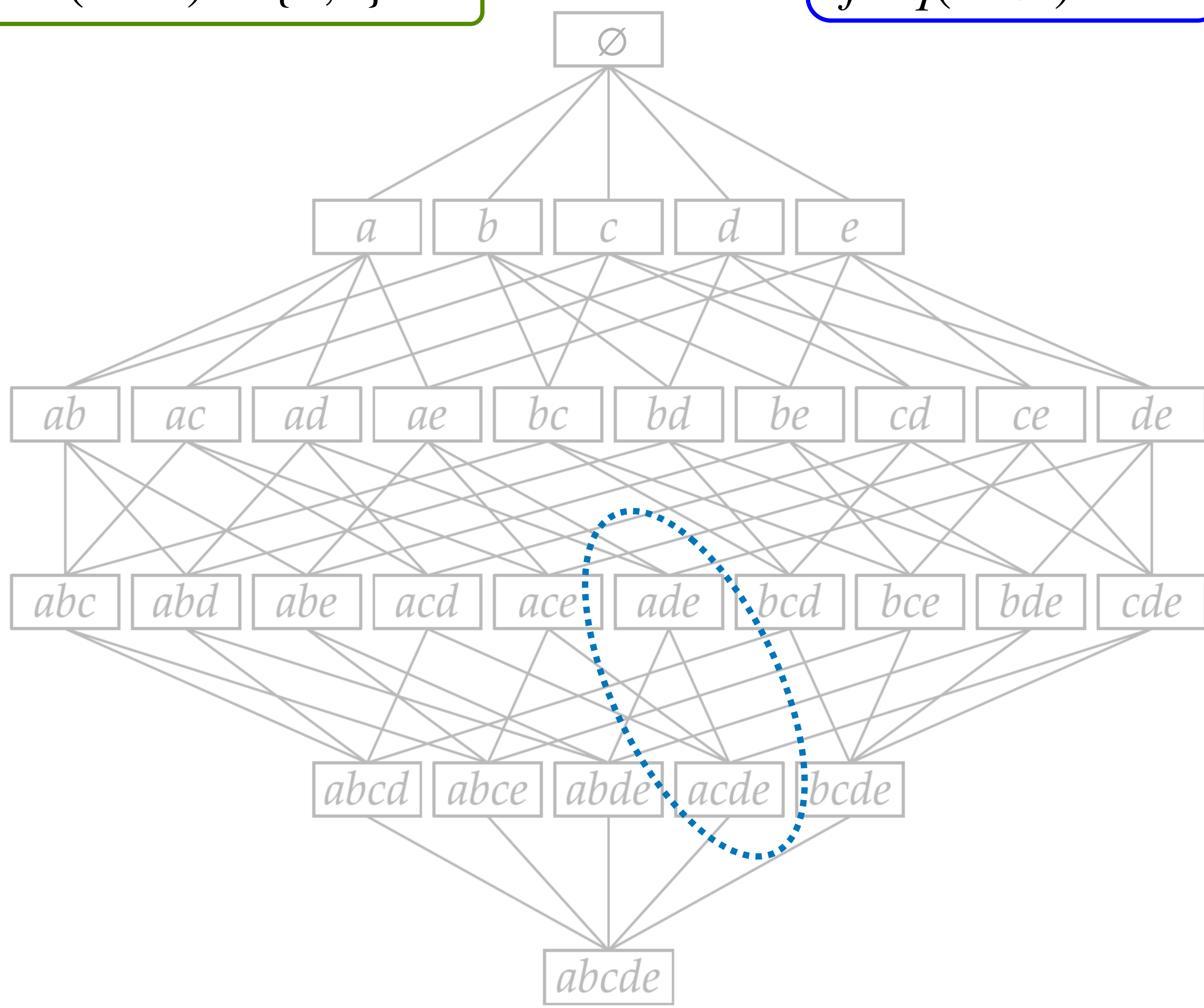
Anti-Monotonicity Property

Example(2)

$cover(ade) = \{1,4,8,10\}$
$cover(acde) = \{4,8\}$

$freq(ade) = 4$
$freq(acde) = 2$



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Searching for Frequent Itemsets

Apriori Property

- Given a transaction database D over items I, a minsup α and two itemsets P and Q:

$$Q \subseteq P \Rightarrow freq(P) \leq freq(Q)$$

- It follows: $Q \subseteq P \wedge freq(P) \geq \alpha \Rightarrow freq(Q) \geq \alpha$

- Contraposition: $Q \subseteq P \wedge freq(Q) < \alpha \Rightarrow freq(P) < \alpha$

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All subsets of a frequent itemset are frequent!

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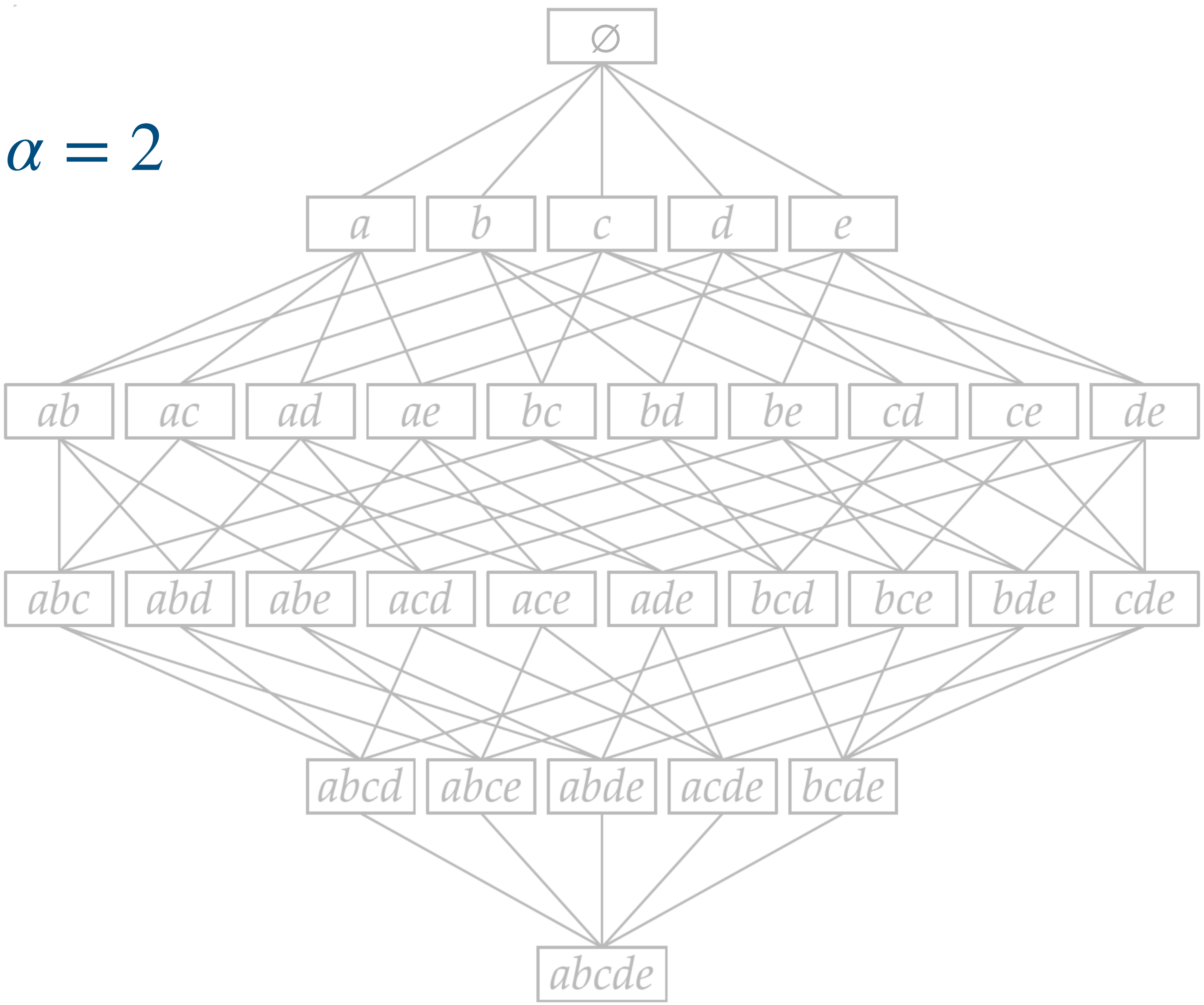
All supersets of an infrequent itemset are infrequent!

Apriori Property

Example(3)

All subsets of a frequent itemset are frequent!

$\alpha = 2$



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Apriori Property

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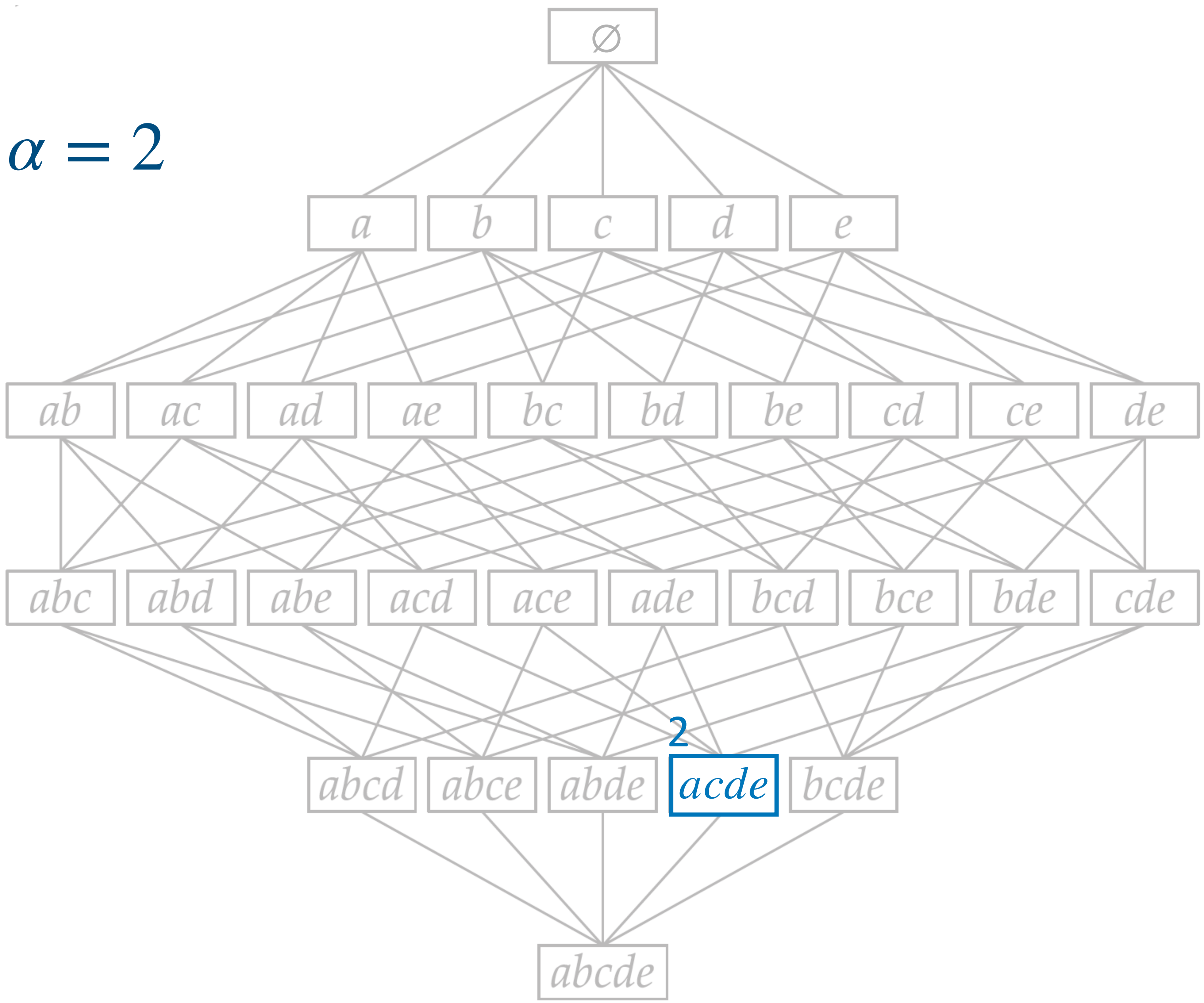

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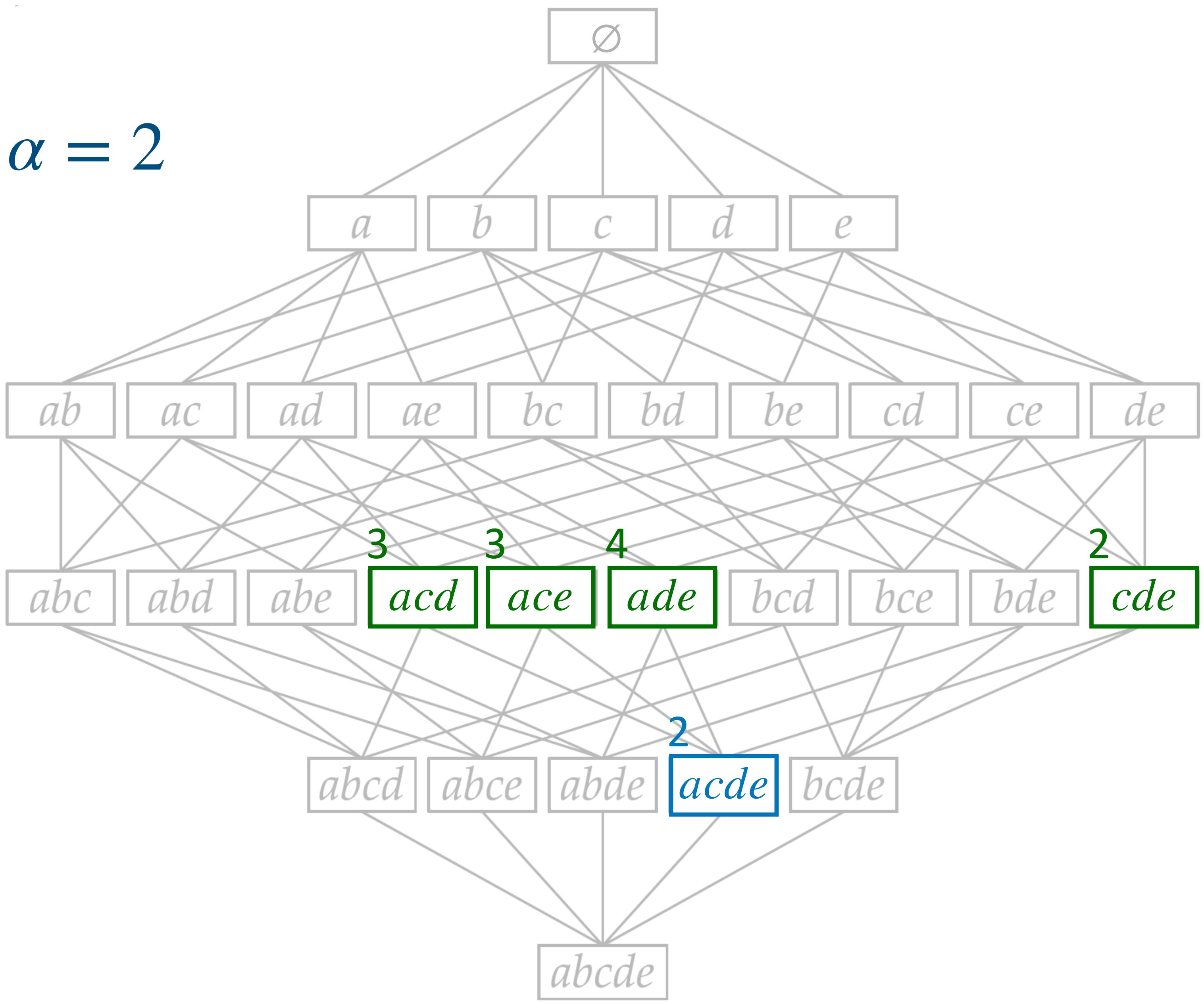

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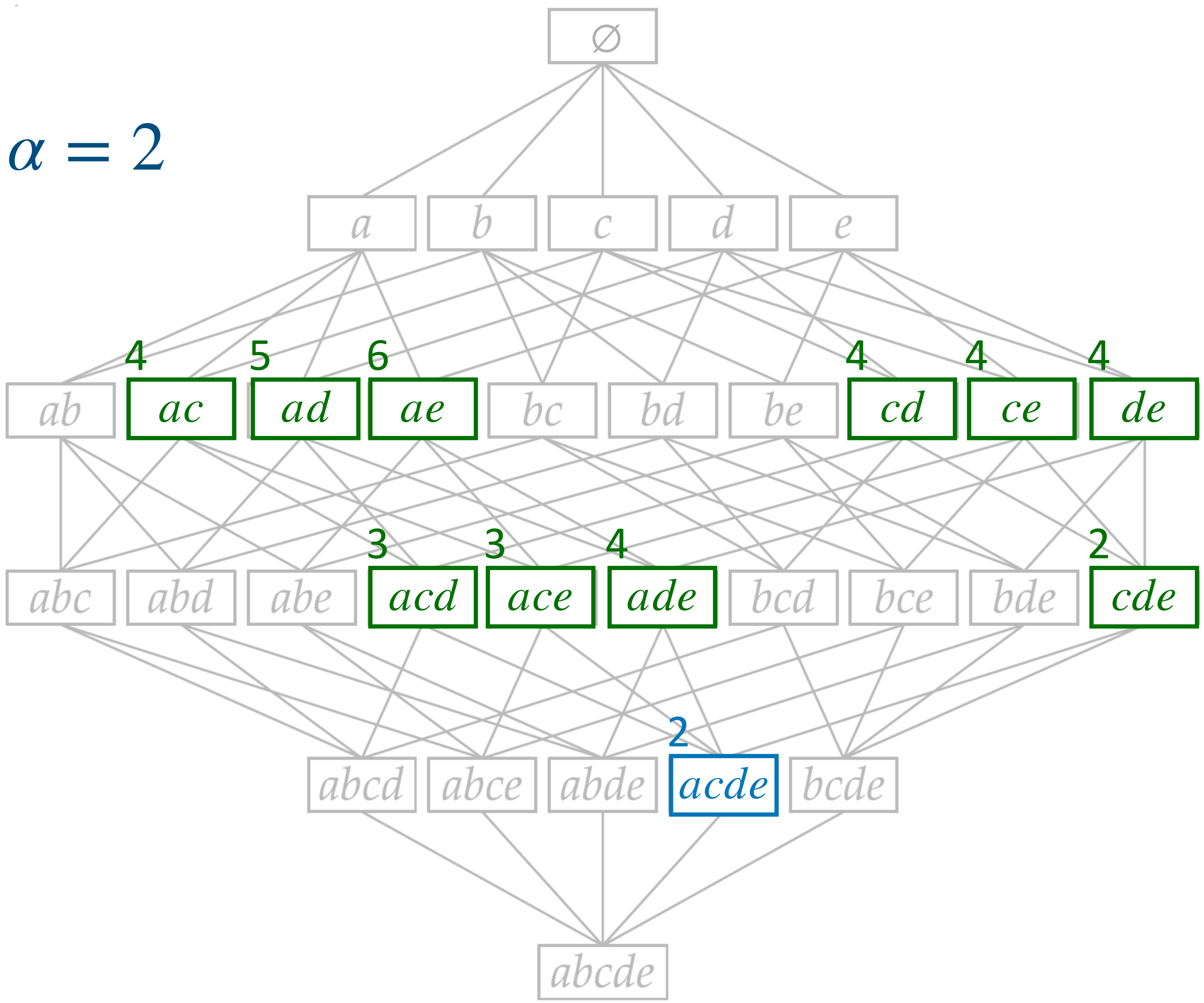

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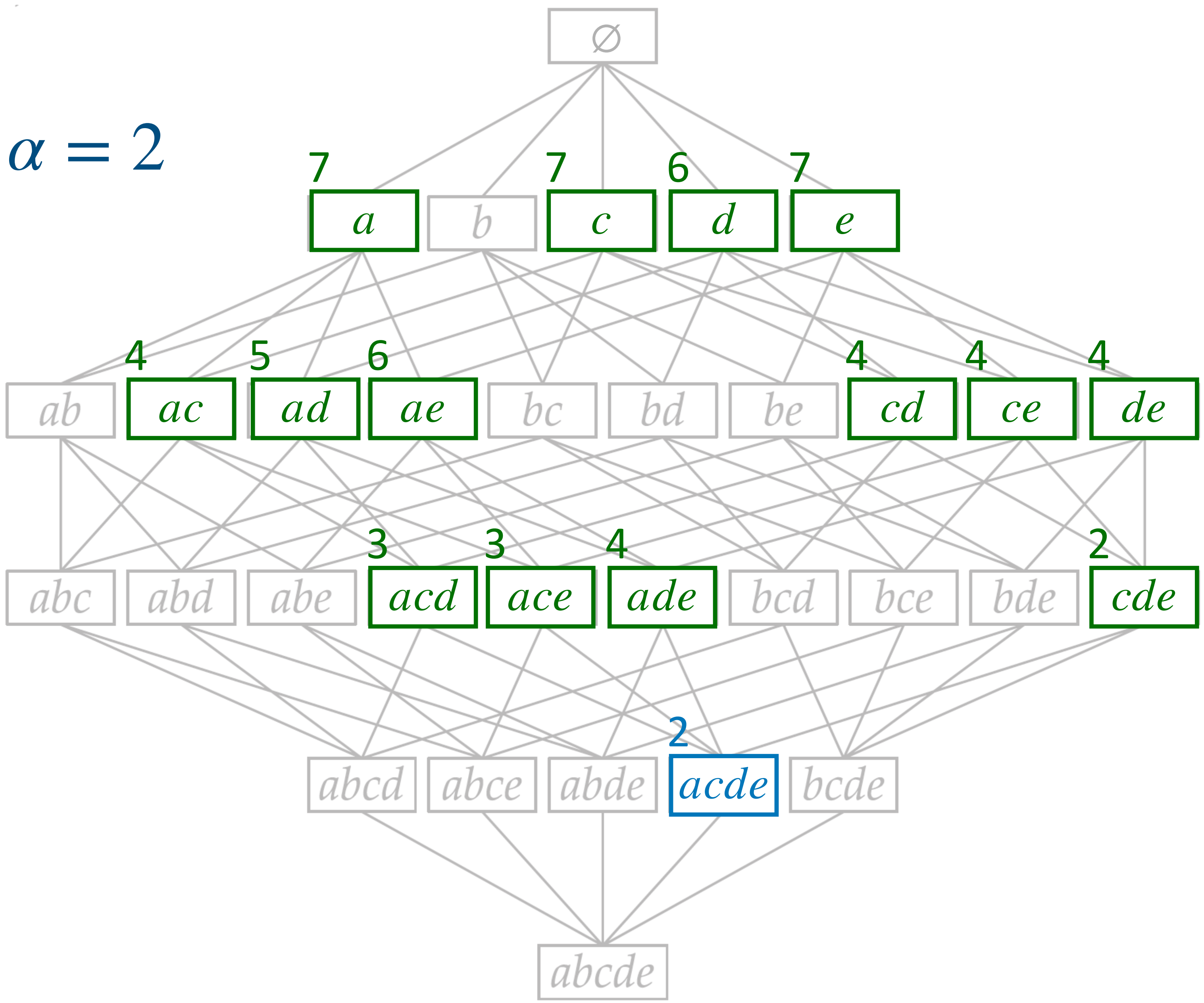

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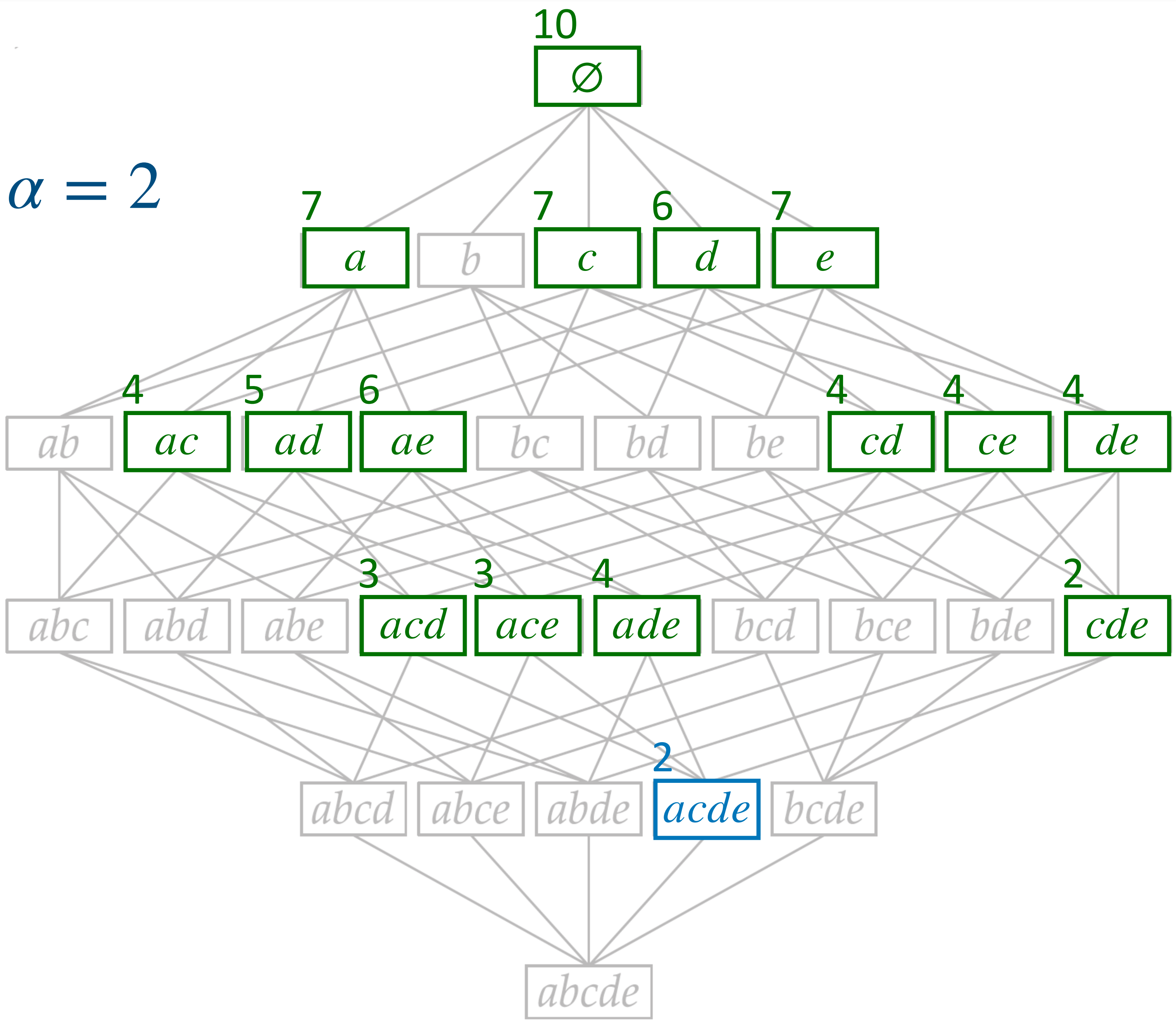
$\alpha = 2$



H_D

1:	a			d	e
2:		b	c	d	
3:	a		c		e
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5:	a				e
6:	a		c	d	
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Apriori Property

Example(3)

All supersets of an infrequent itemset are infrequent!

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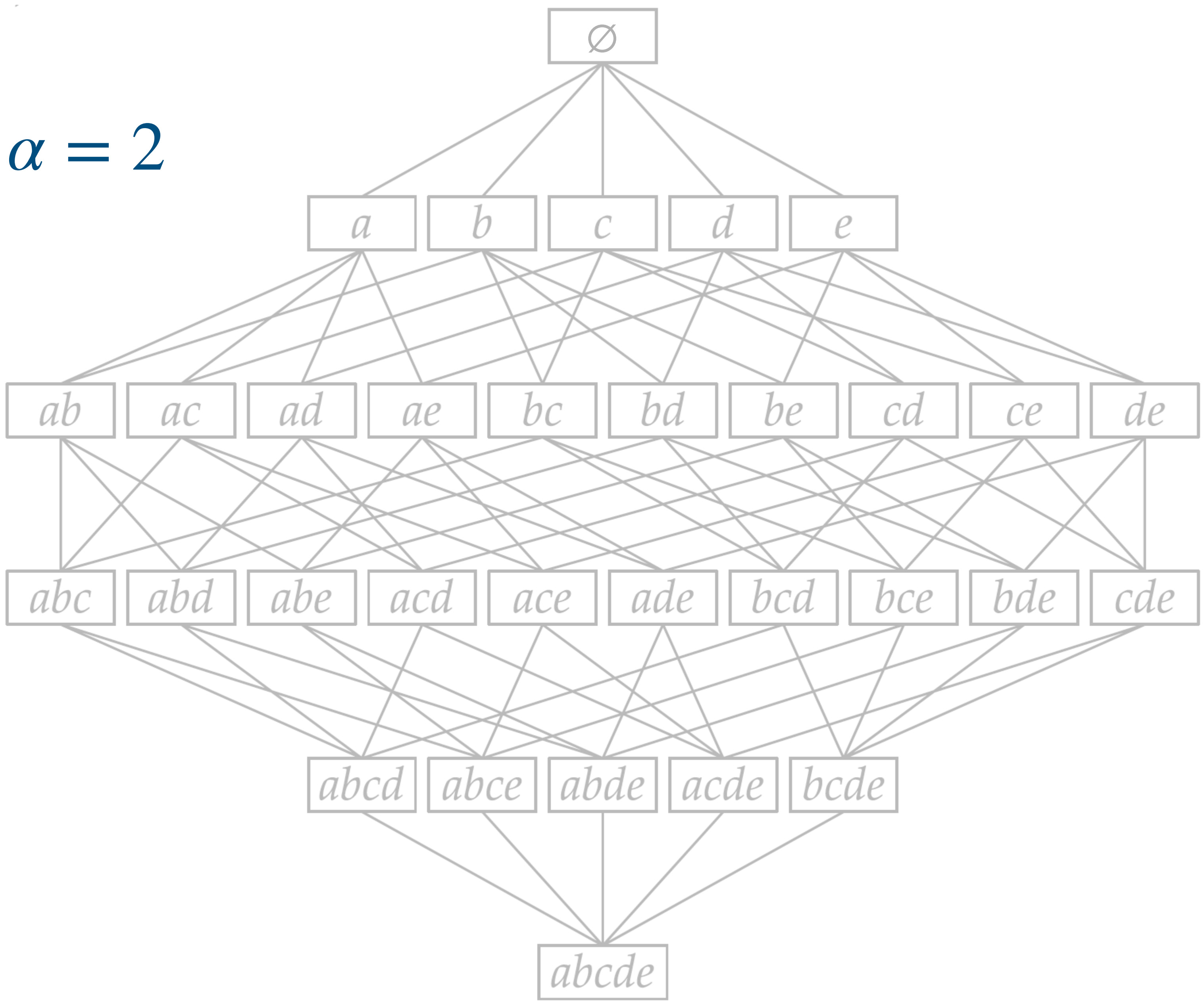

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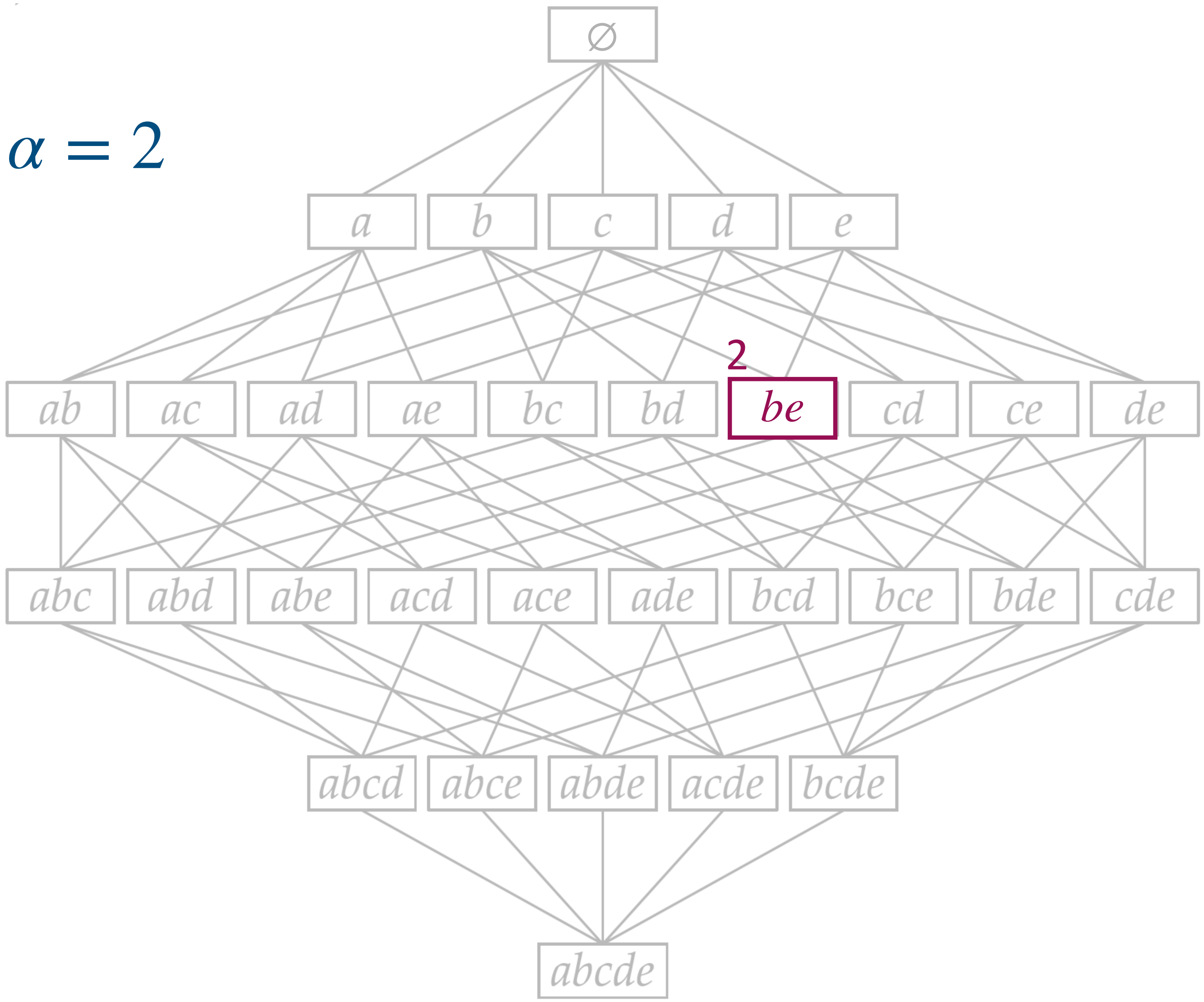

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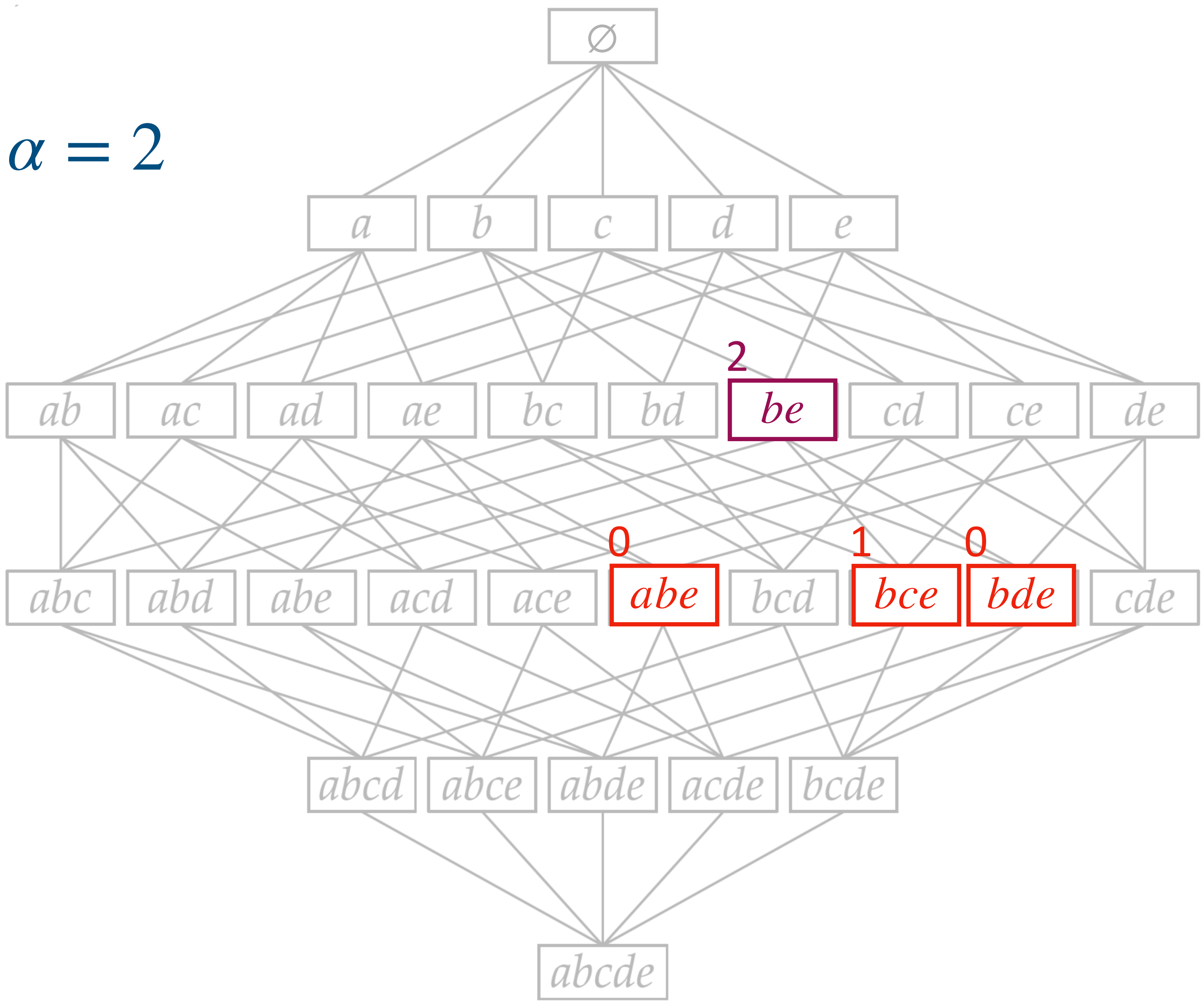

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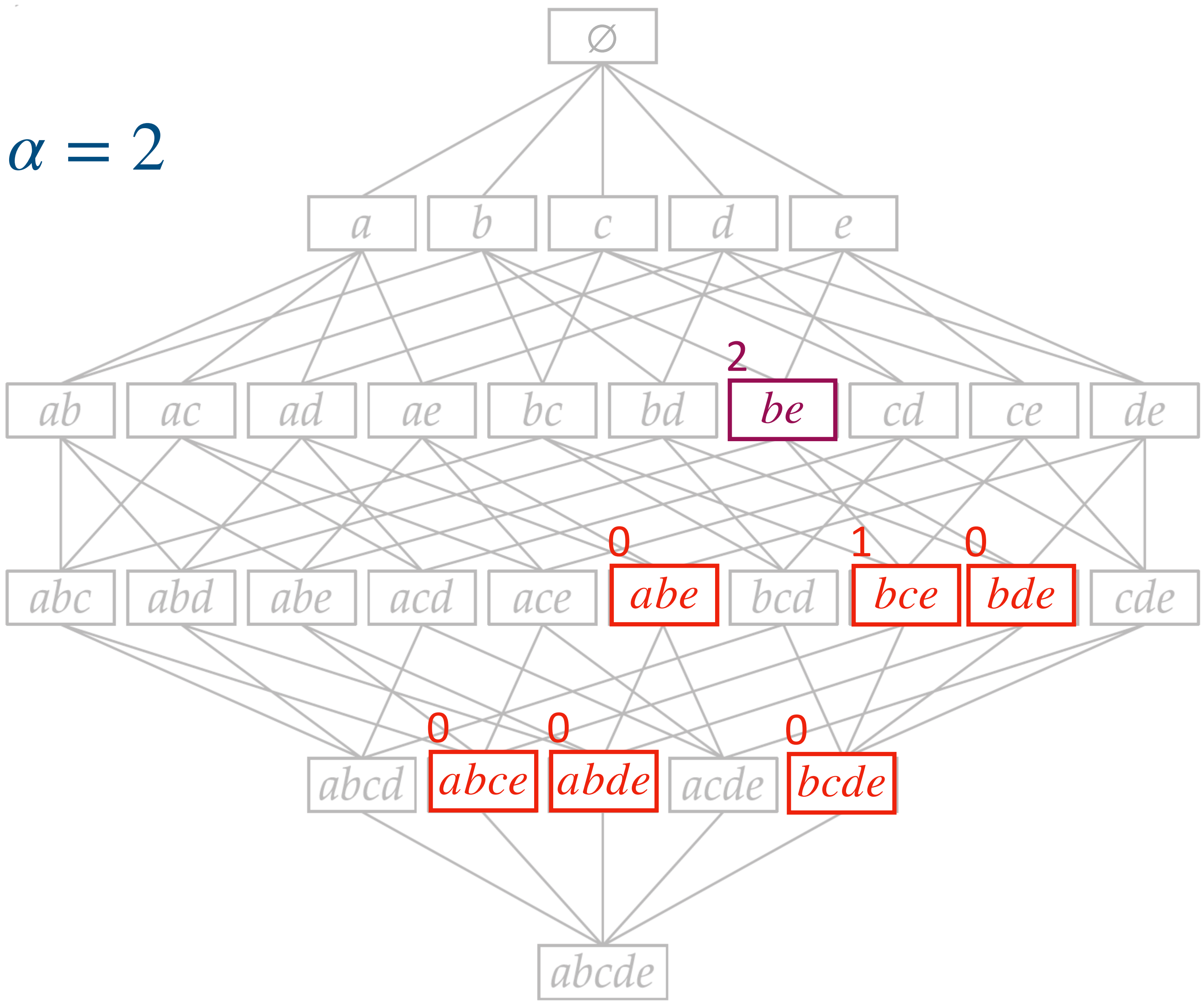

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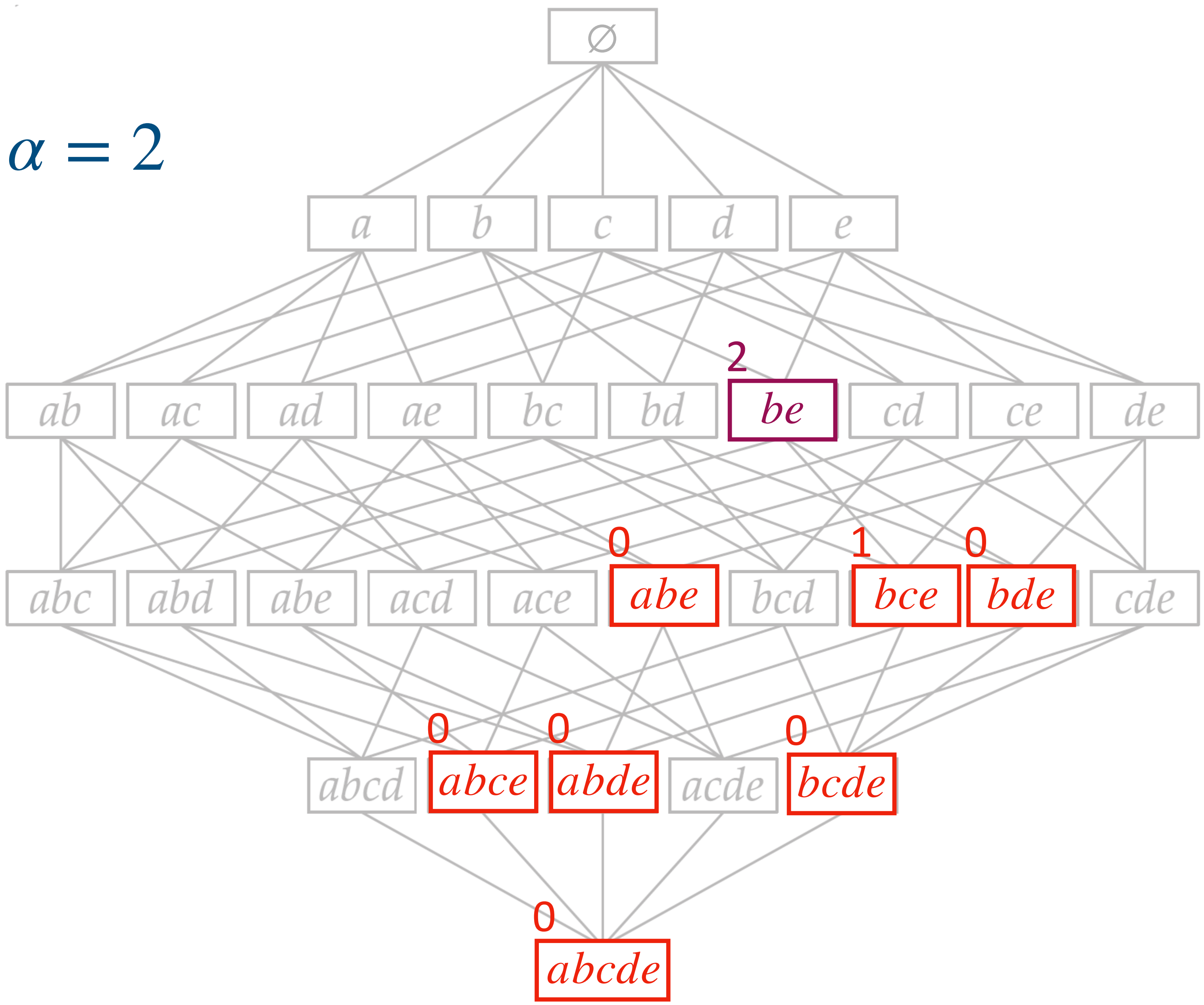

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Searching for Frequent Itemsets

Poset $(2^I, \subseteq)$

- Test A partial order is a binary relation R over a set S :
- $\forall x, y, z \in S$
- $x \ R \ x$ (reflexivity)
- $x \ R \ y \wedge y \ R \ x \Rightarrow x = y$ (anti-symmetry)
- $x \ R \ y \wedge y \ R \ z \Rightarrow x \ R \ z$ (transitivity)
- What are S and R in Itemset Mining?

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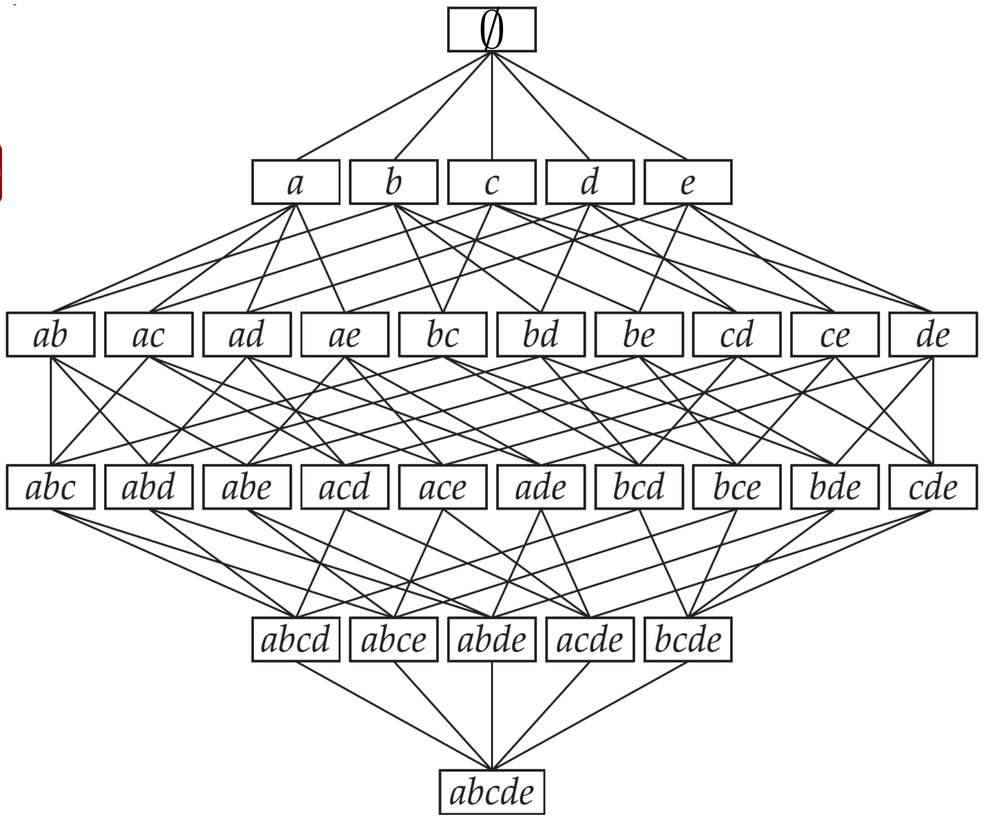
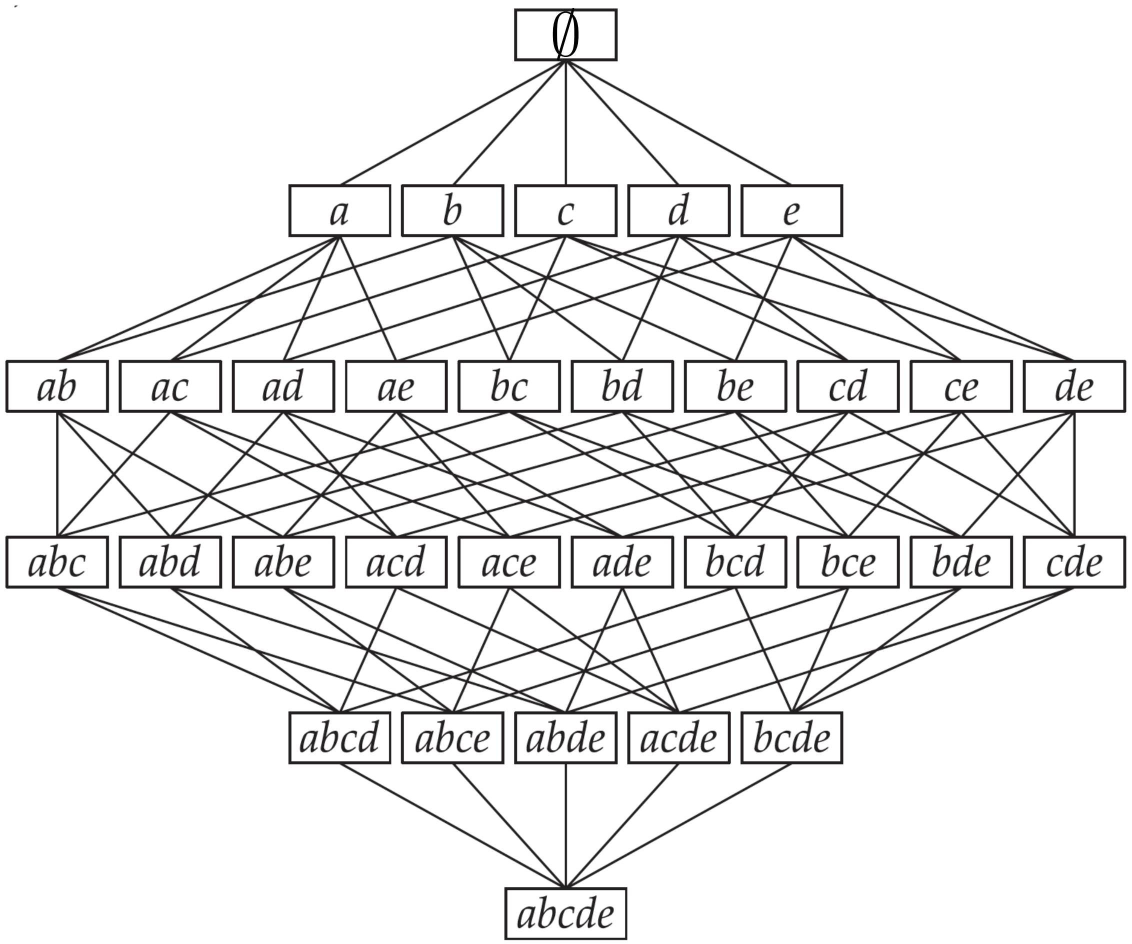


Searching for Frequent Itemsets

Partially ordered sets (review)

- Comparable itemsets: $x \subseteq y \vee y \subseteq x$
- Incomparable itemsets: $x \nsubseteq y \wedge y \nsubseteq x$



Searching for Frequent Itemsets

Apriori Algorithm [Agrawal and Srikant 1994]

- Determine the support of the one-element item sets (i.e. singletons) and discard the infrequent items
- Form candidate itemsets with two items (both items must be frequent), determine their support, and discard the infrequent itemsets
- Form candidate item sets with three items (all contained pairs must be frequent), determine their support, and discard the infrequent itemsets
- And so on!

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- And so on!

Based on candidate generation and pruning

Searching for Frequent Itemsets

Apriori Algorithm [Agrawal and Srikant 1994]

Algorithm 1: Apriori Algorithm

Input: Transaction database $\mathcal{D}$, minimum support threshold α

Output: Frequent itemsets

$k \leftarrow 1$;

$L_k \leftarrow \{p_i \mid p_i \in \mathcal{I} \wedge \mathtt{freq}(p_i) \geq \alpha\}$;

while $L_k \neq \emptyset$ **do**

  $C \leftarrow \mathtt{aprioriGen}(L_k)$;

  $k \leftarrow k + 1$;

  $L_k \leftarrow \{c \mid c \in C \wedge \mathtt{freq}(c) \geq \alpha\}$;

return $\bigcup_i L_i$;

Searching for Frequent Itemsets

Apriori Algorithm [Agrawal and Srikant 1994]

Function `aprioriGen(`L_k`):`

- $E \leftarrow \emptyset$;
- **for** *each pair of itemsets* $P', P'' \in L_k$ *such that* $P' = \{p_{i_1}, \ldots, p_{i_{k-1}}, p_{i_k}\}$ *and* $P'' = \{p_{i_1}, \ldots, p_{i_{k-1}}, p_{i'_k}\}$ **do**
  - **if** $p_{i_k} \neq p_{i'_k}$ **then**
    - $P \leftarrow P' \cup P''$;
    - **if** $\forall p_i \in P,\ P \setminus \{p_i\} \in L_k$ **then**
      - $E \leftarrow E \cup \{P\}$;
- **return** E ;

Apriori Algorithm [Agrawal and Srikant 1994]

Improving candidates generation

- Using `aprioriGen` function, an item of $k+1$ size can be generated in a δ possible ways:

$$\delta = \frac{k(k+1)}{2}$$

Apriori Algorithm [Agrawal and Srikant 1994]

Improving candidates generation

- Using `aprioriGen` function, an item of $k+1$ size can be generated in a δ possible ways:

$$\delta = \frac{k(k+1)}{2}$$

6 possibilities to generate (abcd)

	abc	abd	acd	bcd
abc	—	abcd	abcd	abcd
abd	abcd	—	abcd	abcd
acd	abcd	abcd	—	abcd
bcd	abcd	abcd	abcd	—

Apriori Algorithm [Agrawal and Srikant 1994]

Improving candidates generation

- Using `aprioriGen` function, an item of $k+1$ size can be generated in a δ possible ways:

$$\delta = \frac{k(k+1)}{2}$$

Need: Generate itemset candidate only once

How: Assign a unique parent itemset to each candidate, ensuring that it is generated only from its parent itemset

Apriori Algorithm [Agrawal and Srikant 1994]

Improving candidates generation

- Assigning unique parents turns the poset lattice into a tree:

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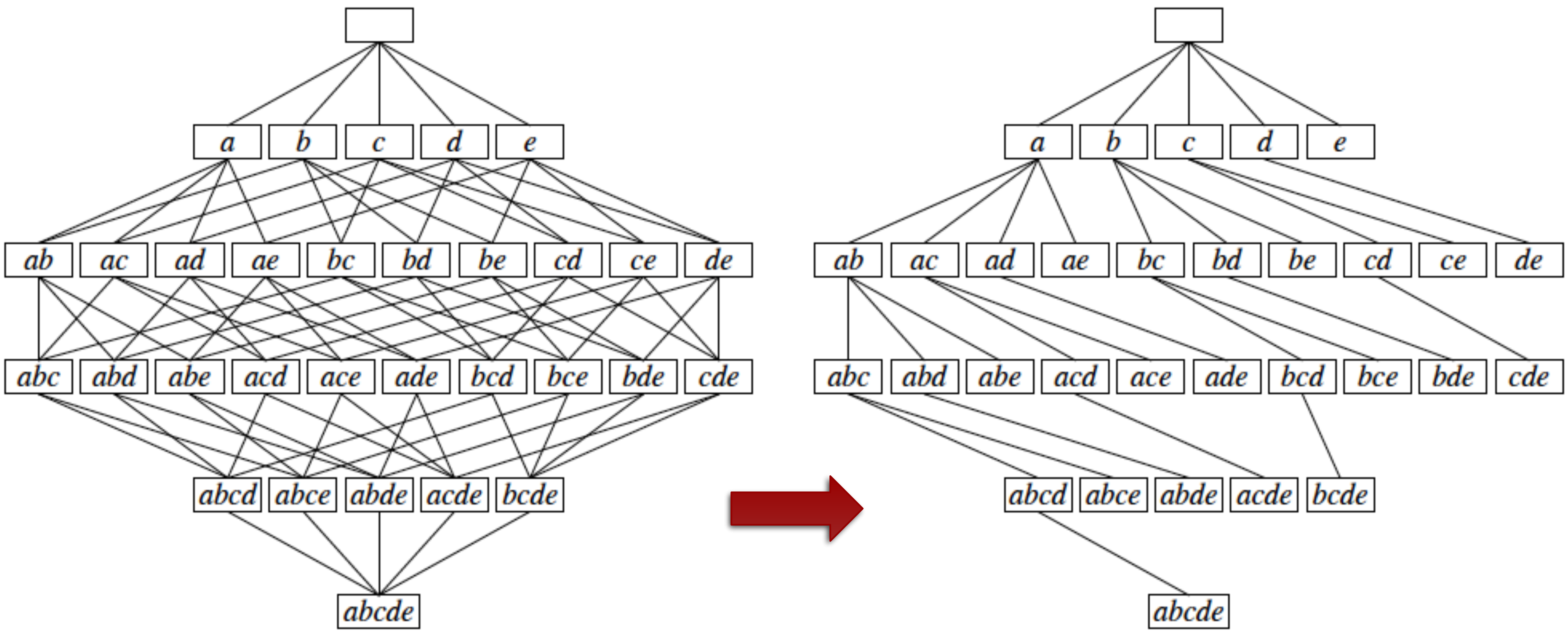


Apriori Algorithm [Agrawal and Srikant 1994]

Canonical form for itemsets

- An itemset can be represented as a word over an alphabet I
  - Question: how many words of k items can we have?
  - Answer: k-permutations of k items: $k!$
- By imposing an arbitrary order (e.g., lexicography order) on the items, we can define a canonical form—a unique representation of itemsets that eliminates symmetries
  - Lex on items : $abc < acb < bac < bca\ldots$
  - $\kappa(abc) = \kappa(acb) = \kappa(bac) = \kappa(bca) = abc$
  - $\kappa(abc,1) = a;\ \kappa(abc,2) = b;\ \kappa(abc,3) = c$

Apriori Algorithm [Agrawal and Srikant 1994]

Recursive processing with Canonical forms

- For each itemset of a given size, generate all possible extensions of the itemset by adding one item, subject to the condition that:

Apriori Algorithm [Agrawal and Srikant 1994]

Recursive processing with Canonical forms

- For each itemset of a given size, generate all possible extensions of the itemset by adding one item, subject to the condition that:

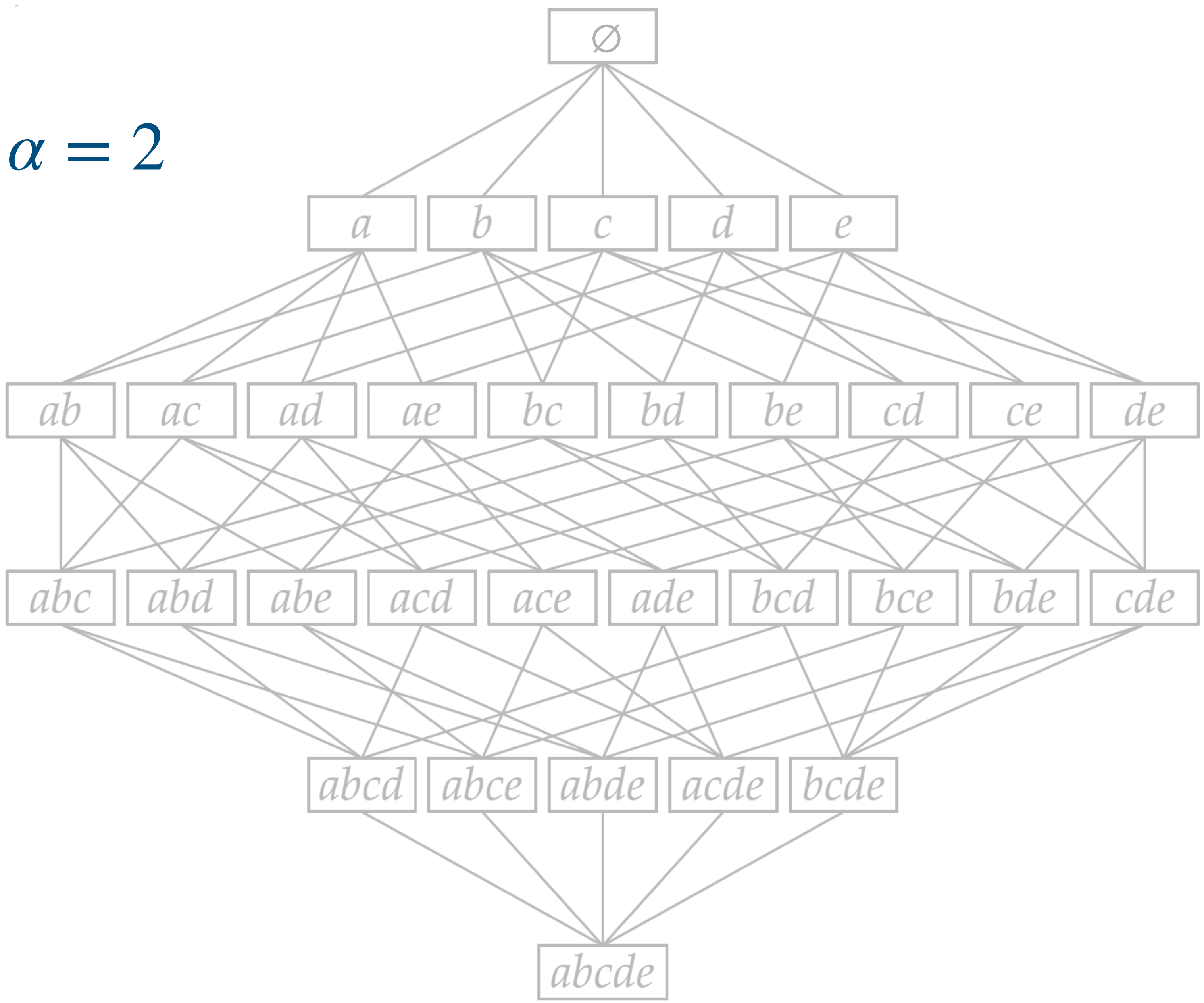
$$child(P, \alpha) = \{P' : (P' = P \cup \{p_i\}) \wedge (\kappa(P, |P|) < p_i) \wedge (freq(P') \geq \alpha)\}$$

Recursive processing with Canonical forms

Example(4)

$$child(P, \alpha) = \{P' : (P' = P \cup \{p_i\}) \wedge (\kappa(P, |P|) < p_i) \wedge (freq(P') \geq \alpha)\}$$

$\alpha = 2$



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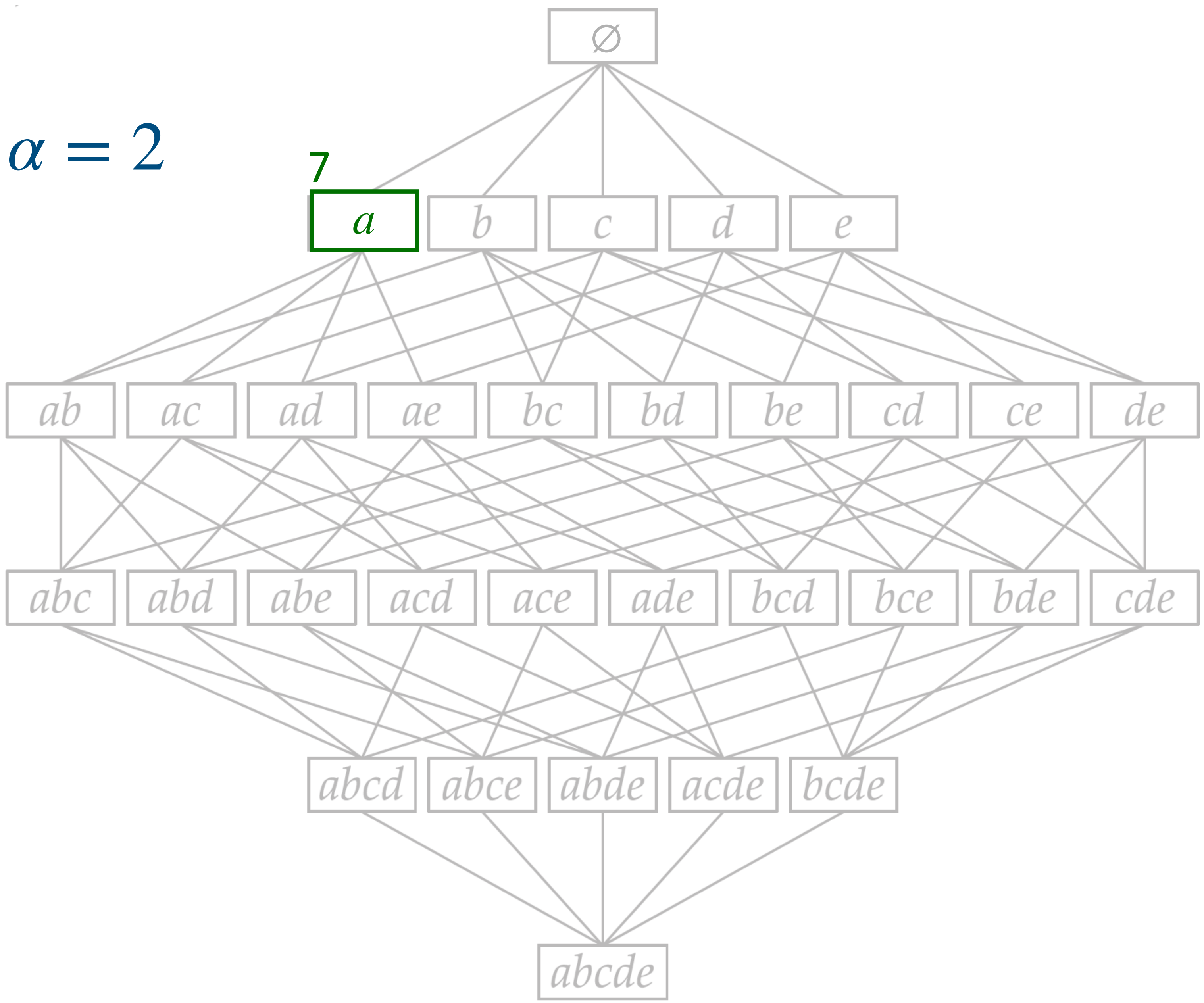

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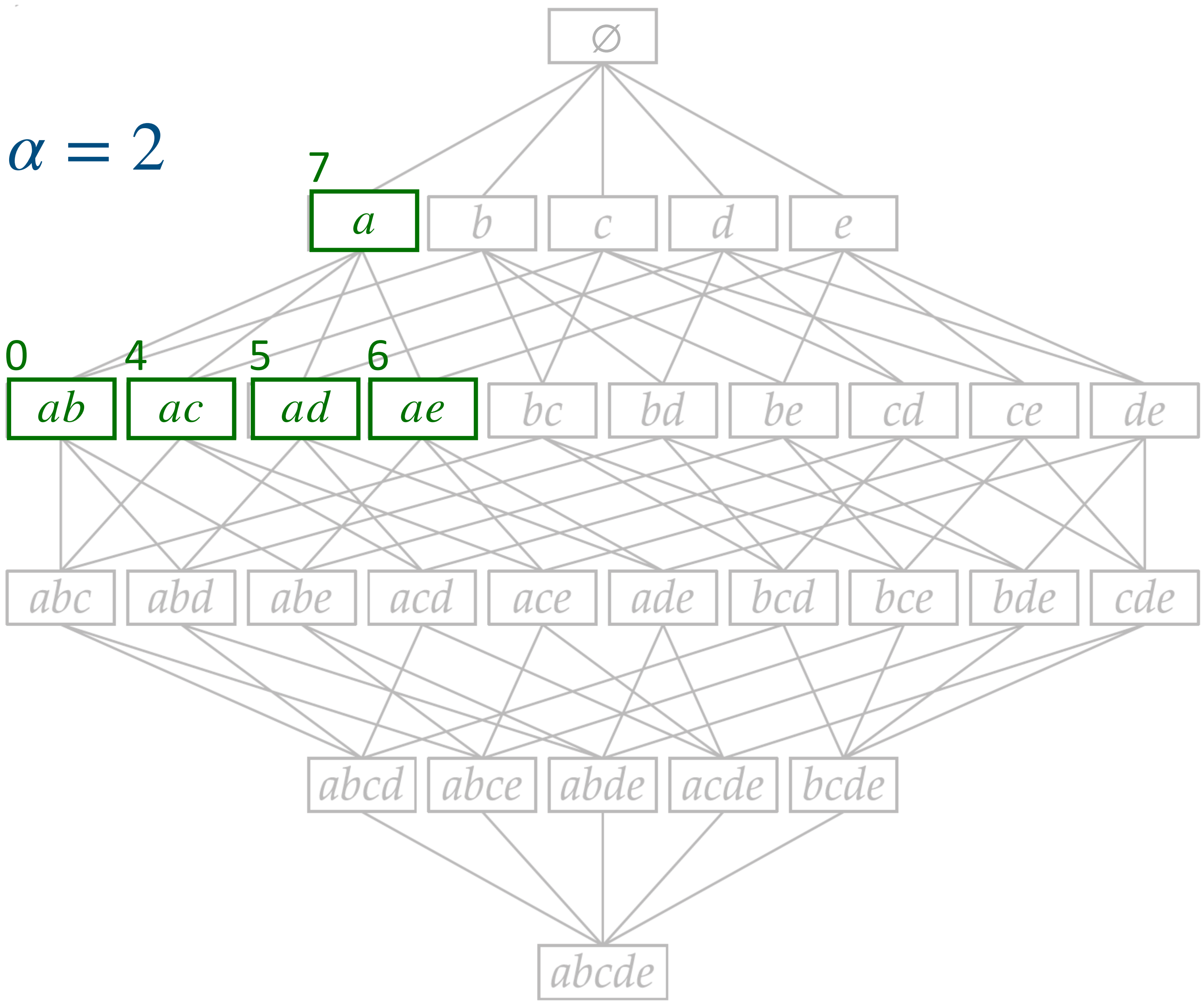

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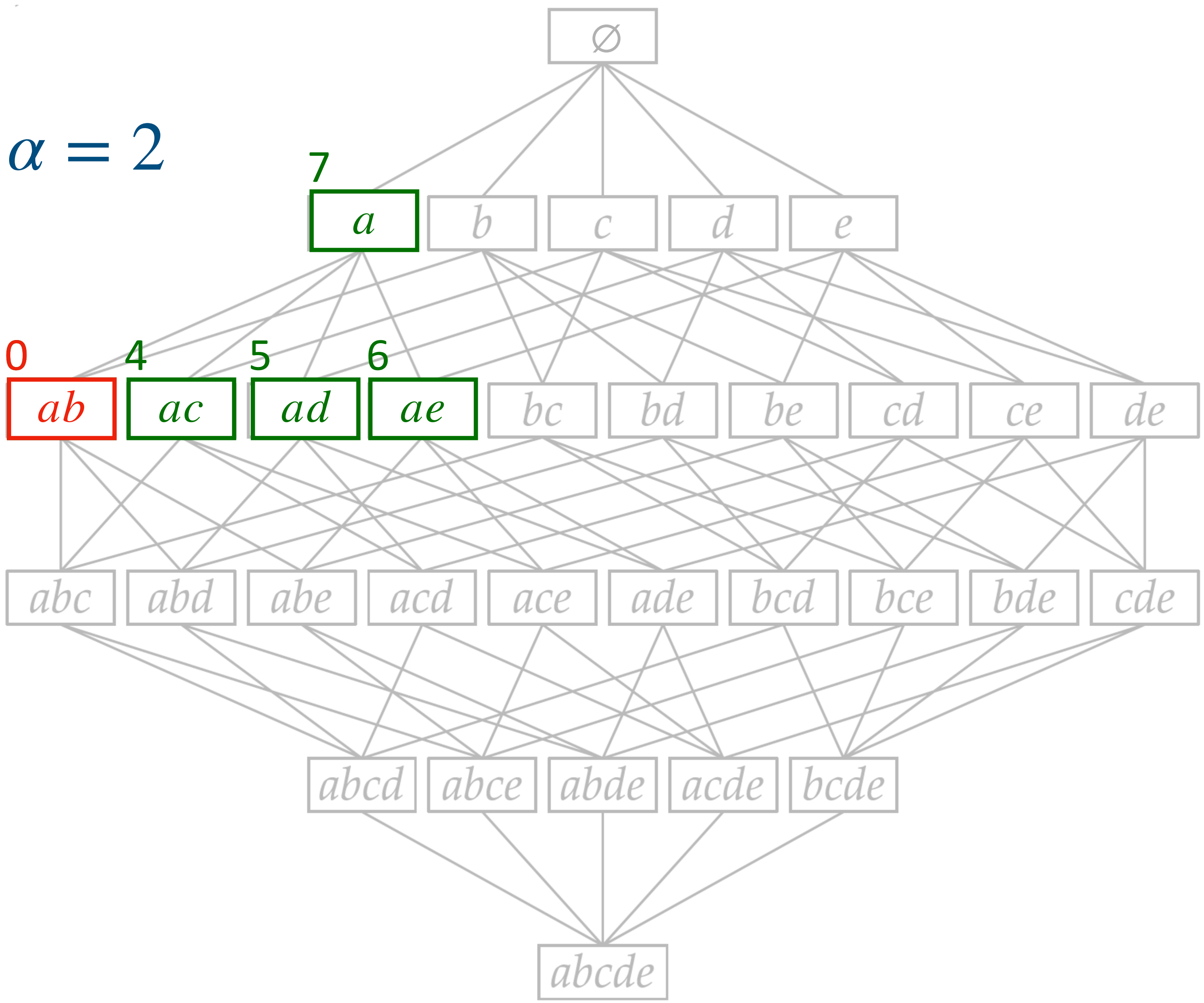

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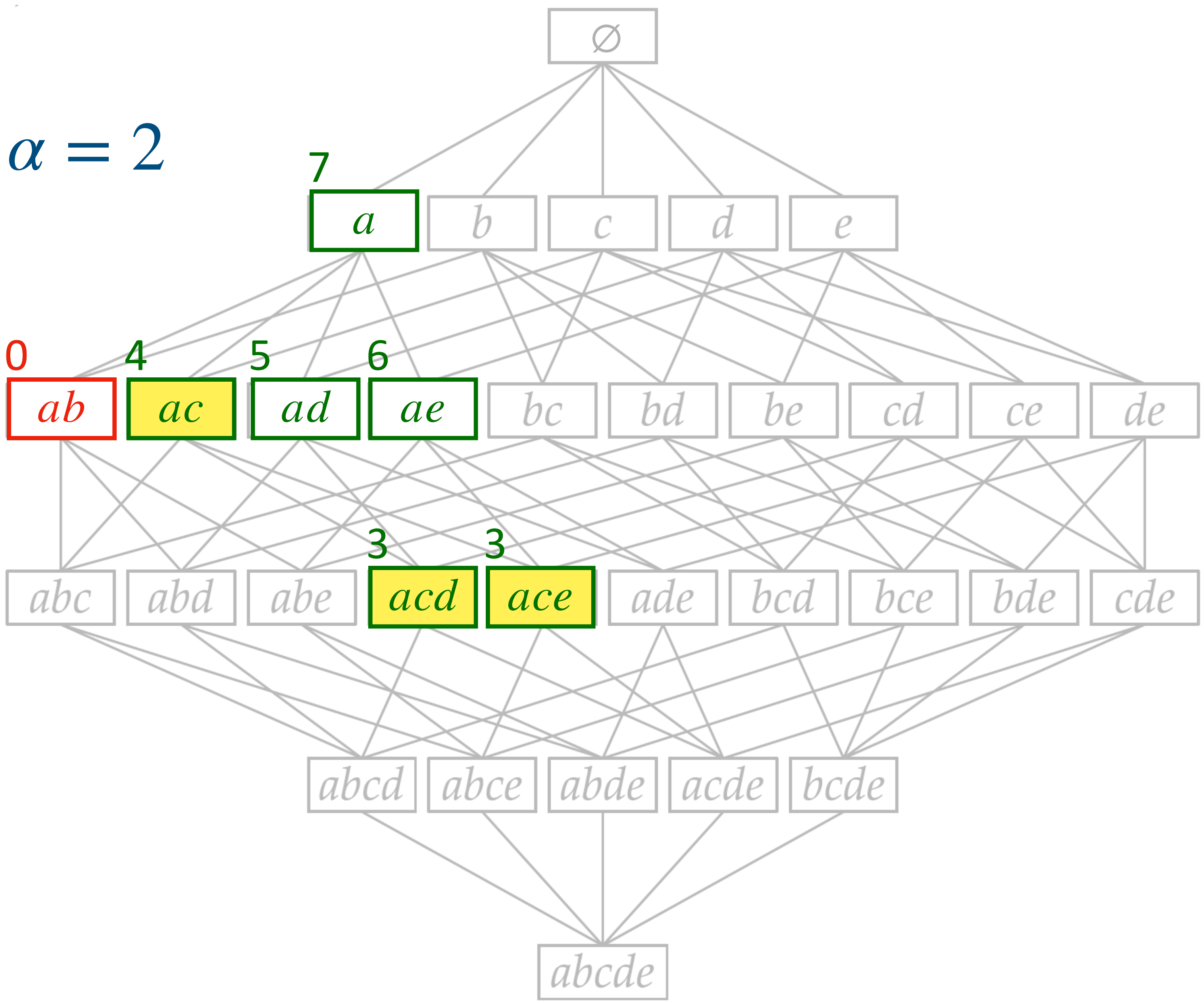

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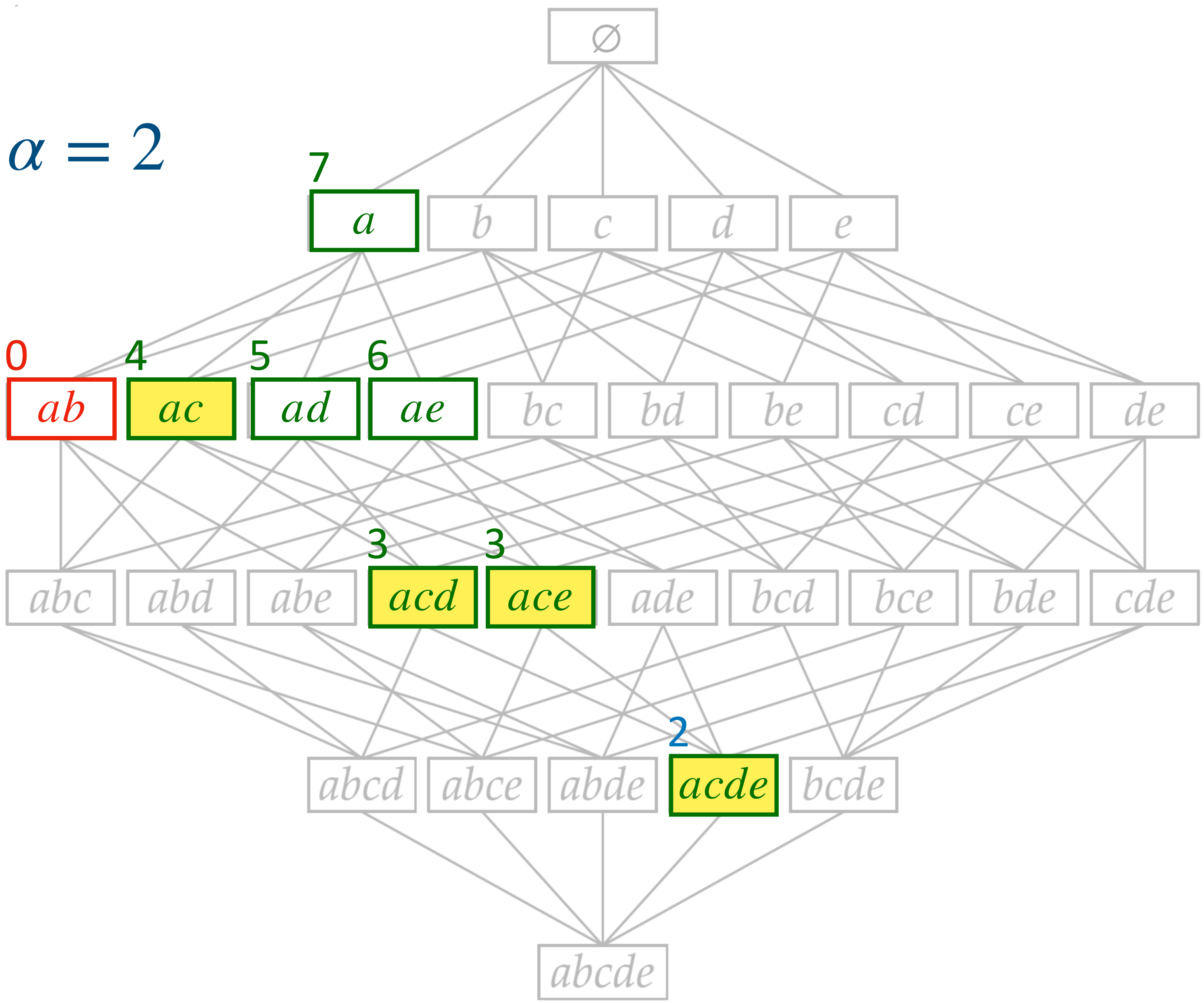

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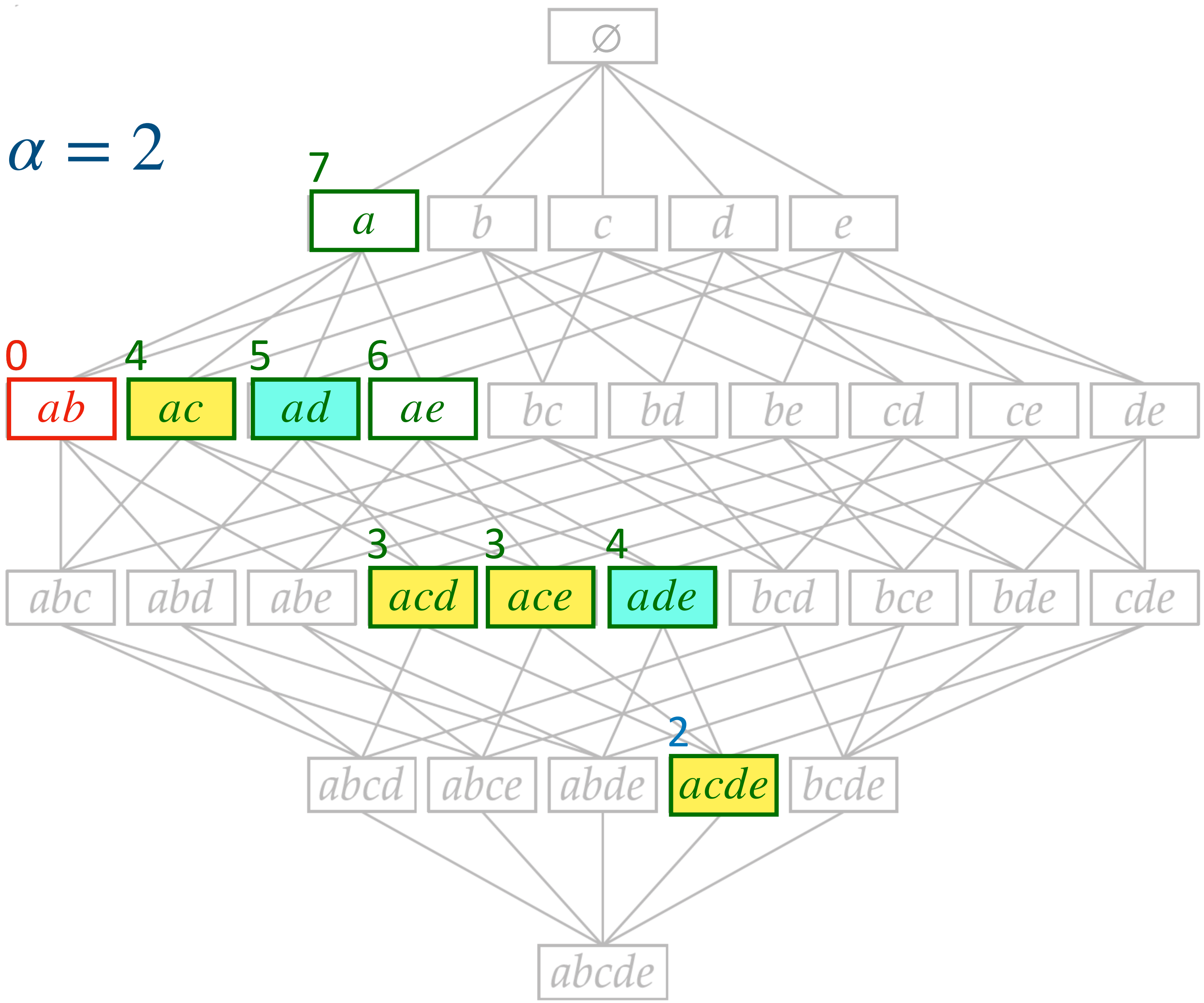

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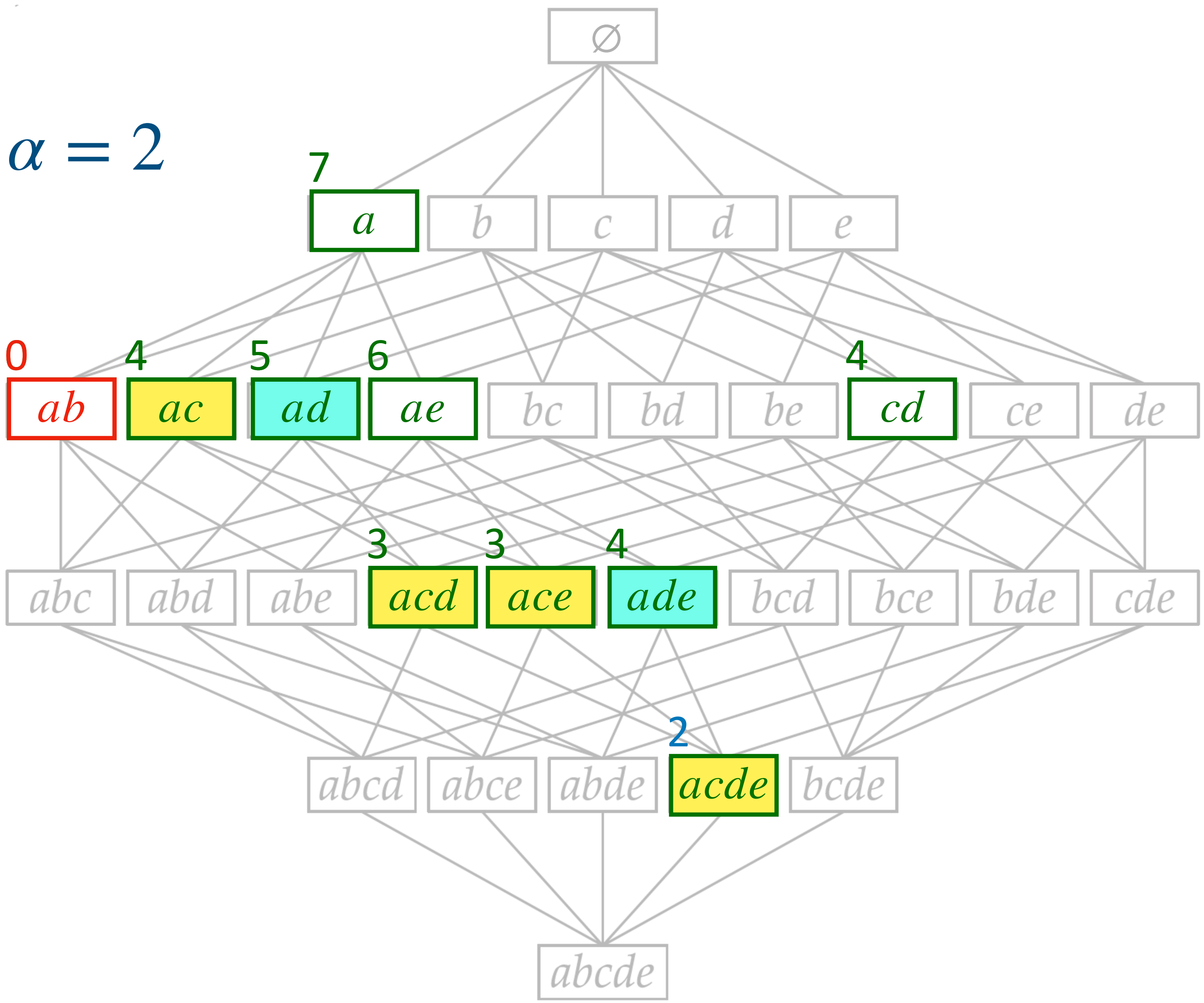

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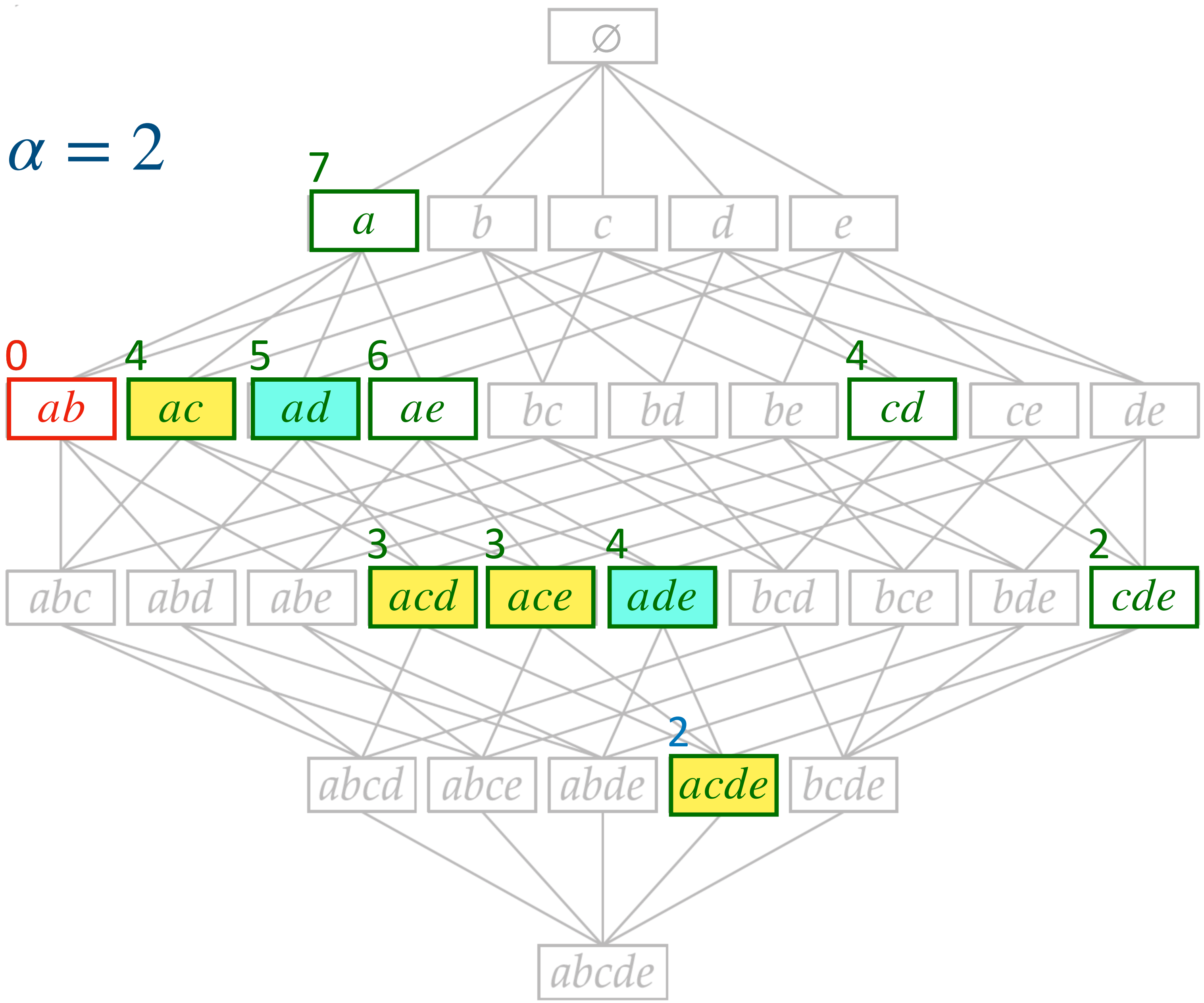

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Searching for Frequent Itemsets

Items Ordering

- **Any order can be used, but:**
  - The structure of the search space depends heavely on the chosen order
  - Algorithm efficiency varies significantly based on the item order
- **Advanced methods dynamically adjust the order during teh search:**
  - Use different but « compatible » orders in different branches

Searching for Frequent Itemsets

Items Ordering (heuristics)

- Sort the items w.r.t. their frequency (decreasing/increasing) : Frequent itemsets are composed of frequent items
- Sort items based on the size of the area they cover : considering both their frequency and the sizes of the transactions that include them

Condensed representation of Frequent Itemsets:

Closed and Maximal Itemsets

Maximal Itemsets

Definition

- The set of Maximal (frequent) Itemsets:

$$M_\alpha = \{P \subset I \,|\, freq(P) \geq \alpha \wedge \forall P' \supset P : freq(P') < \alpha\}$$

- That is:

$$\forall \alpha, \forall P \in F_\alpha : (P \in M_\alpha) \vee (\exists P' \supset P : freq(P') \geq \alpha)$$

(With F_α the set of all frequent itemsets)

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An itemset is considered maximal if it is frequent, and none of its proper supersets is frequent

- That is:

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Maximal Itemsets

Definition

- Every frequent itemset has a maximal superset:

$$\forall \alpha, \forall P \in F_\alpha : (\exists P' \in M_\alpha : P \subseteq P')$$

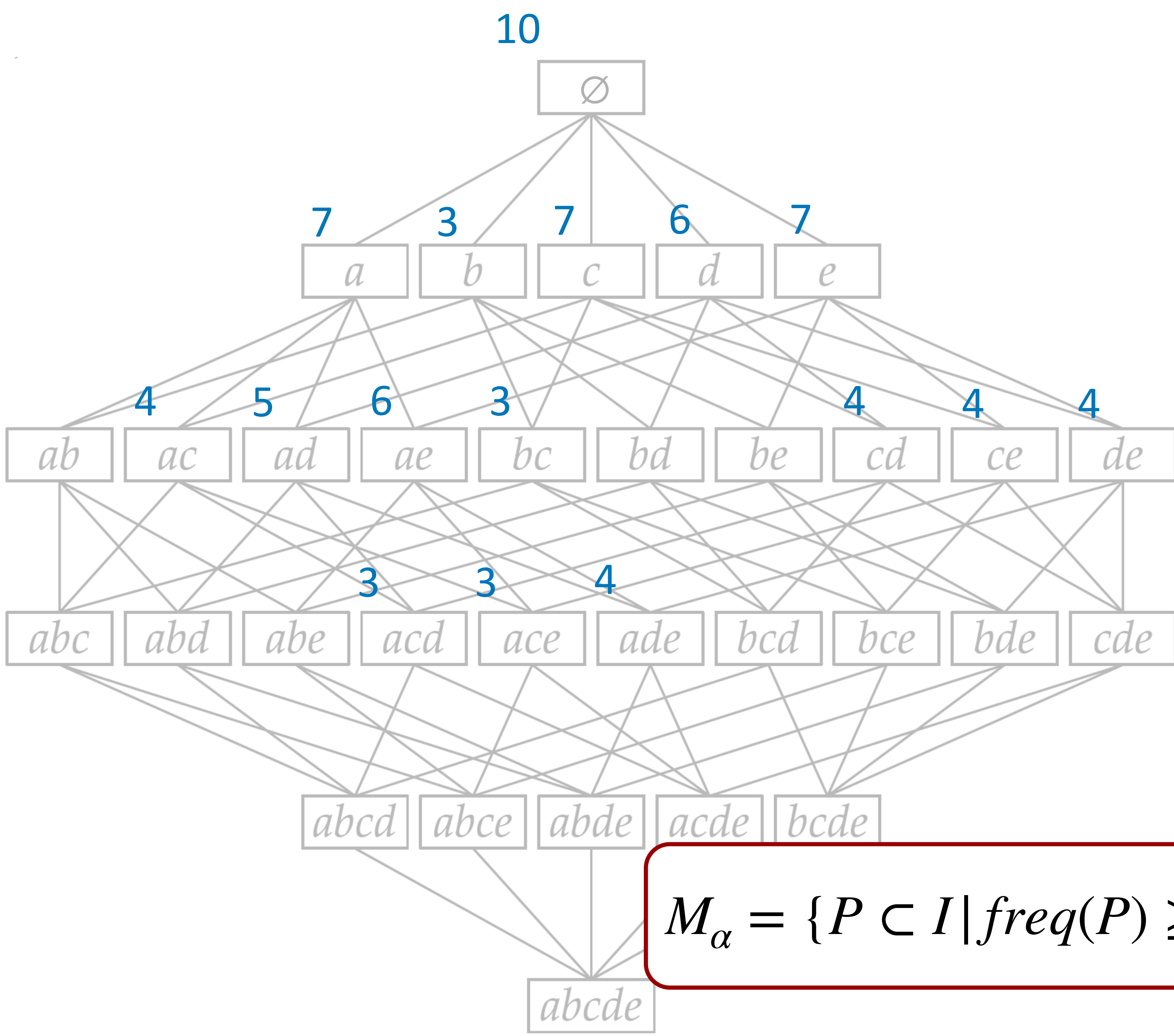
- The maximal itemsets provide a condensed representation of the frequent itemsets, where:

$$\forall \alpha : F_\alpha = \bigcup_{P \in M_\alpha} 2^P$$

Maximal Itemset Mining

Example(5)

Query-3: Maximal itemsets with $\alpha = 3$?



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Maximal Itemset Mining

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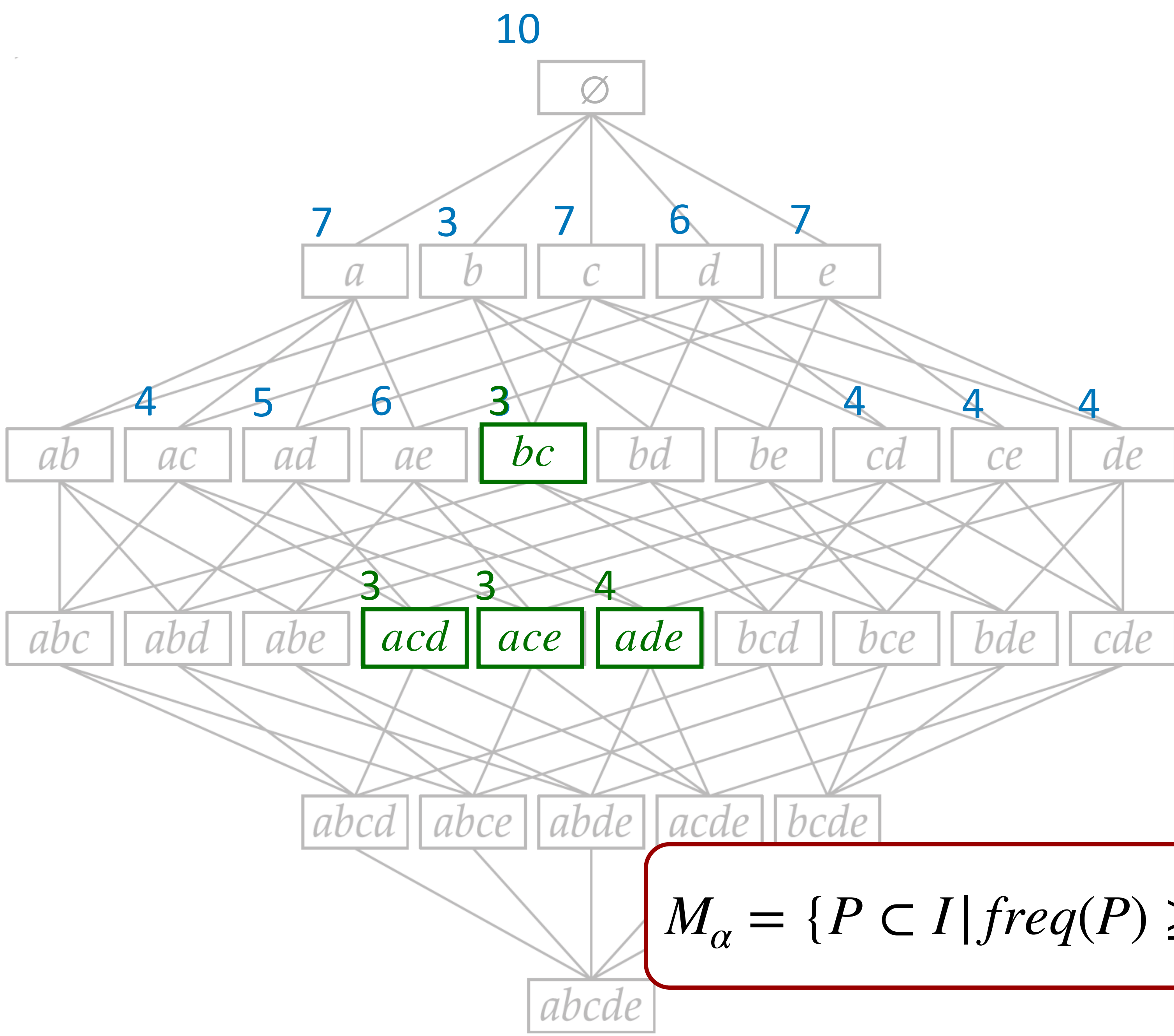

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Maximal Itemsets

Limitation

- The set of maximal itemsets captures all frequent itemsets, but it does not retain the support values for all of them
- **The Need:** Is it possible to have a condensed representation of the frequent itemsets that still preserves the support values for all?”

Maximal Itemsets

Limitation

- The set of Closed (frequent) Itemsets:

$$C_\alpha = \{P \subset I \,|\, freq(P) \geq \alpha \wedge \forall P' \supset P : freq(P') < freq(P)\}$$

An itemset is closed if it is frequent, but none of its proper supersets has the same support value

- That is:

$$\forall \alpha, \forall P \in F_\alpha : (P \in C_\alpha) \vee (\exists P' \supset P : freq(P') = freq(P))$$

Closed Itemsets

Definition

- Every frequent itemset has a closed superset:

$$\forall \alpha, \forall P \in F_\alpha : (\exists P' \in C_\alpha : P \subseteq P')$$

- Closed itemsets provide a condensed representation of frequent itemsets, where:

$$\forall \alpha : F_\alpha = \bigcup_{P \in C_\alpha} 2^P$$

Closed Itemsets

Definition

- Every frequent itemset has a closed superset with the same support
- The set of all closed itemsets preserves the knowledge of support values:

$$\forall \alpha, \forall P \in F_\alpha : cover(P) = \max_{P' \in C_\alpha, P' \supseteq P} cover(P')$$

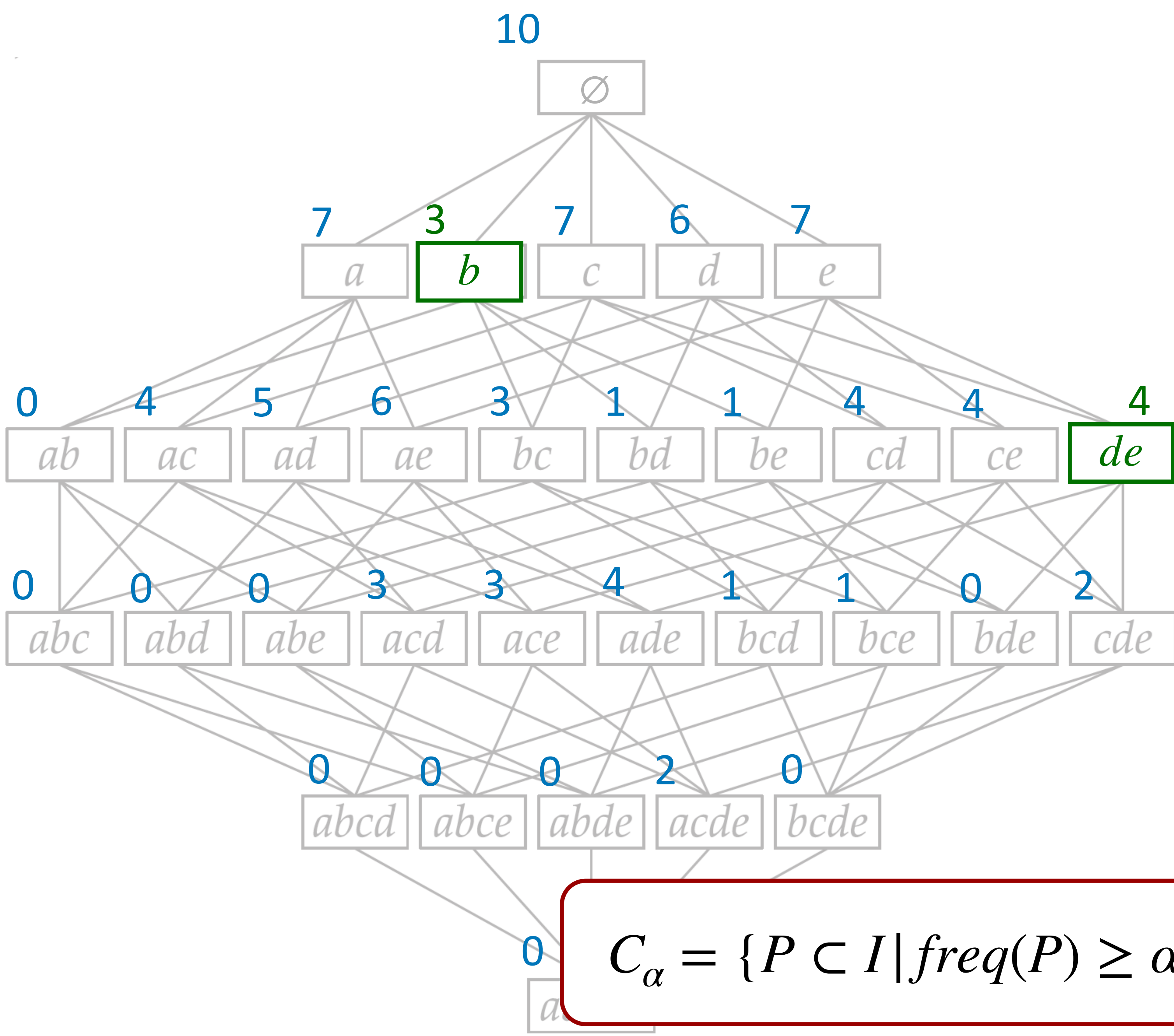
- Unlike maximal itemsets:

$$\forall \alpha, \forall P \in F_\alpha : cover(P) \supseteq \max_{P' \in M_\alpha, P' \supseteq P} cover(P')$$

Closed Itemset Mining

Example(6)

Q: Are 'b' and 'de' closed itemsets?"



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4:	a		c	d	e
5:	a				e
6:	a		c	d	
7:		b	c		
8:	a		c	d	e
9:		b	c		e
10:	a			d	E

$$C_\alpha = \{P \subset I \,|\, freq(P) \geq \alpha \wedge \forall P' \supset P : freq(P') < freq(P)\}$$

Closed Itemsets

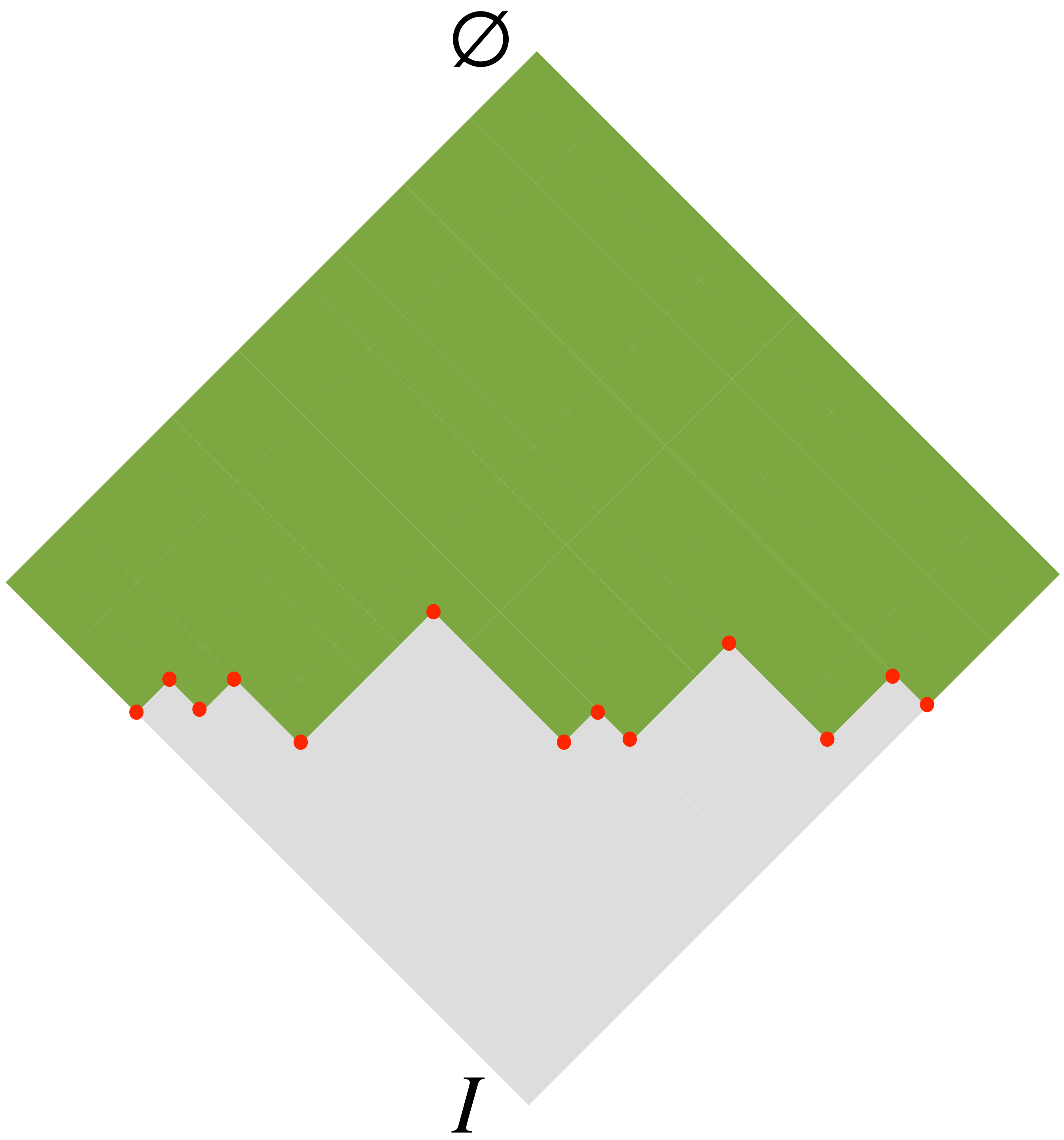
Definition

- Every frequent itemset has a closed superset with the same support
- The set of all closed itemsets preserves the knowledge of support values:

$$\forall \alpha, \forall P \in F_\alpha : cover(P) = \max_{P' \in C_\alpha, P' \supseteq P} cover(P')$$

- Unlike maximal itemsets:

$$\forall \alpha, \forall P \in F_\alpha : cover(P) \supseteq \max_{P' \in M_\alpha, P' \supseteq P} cover(P')$$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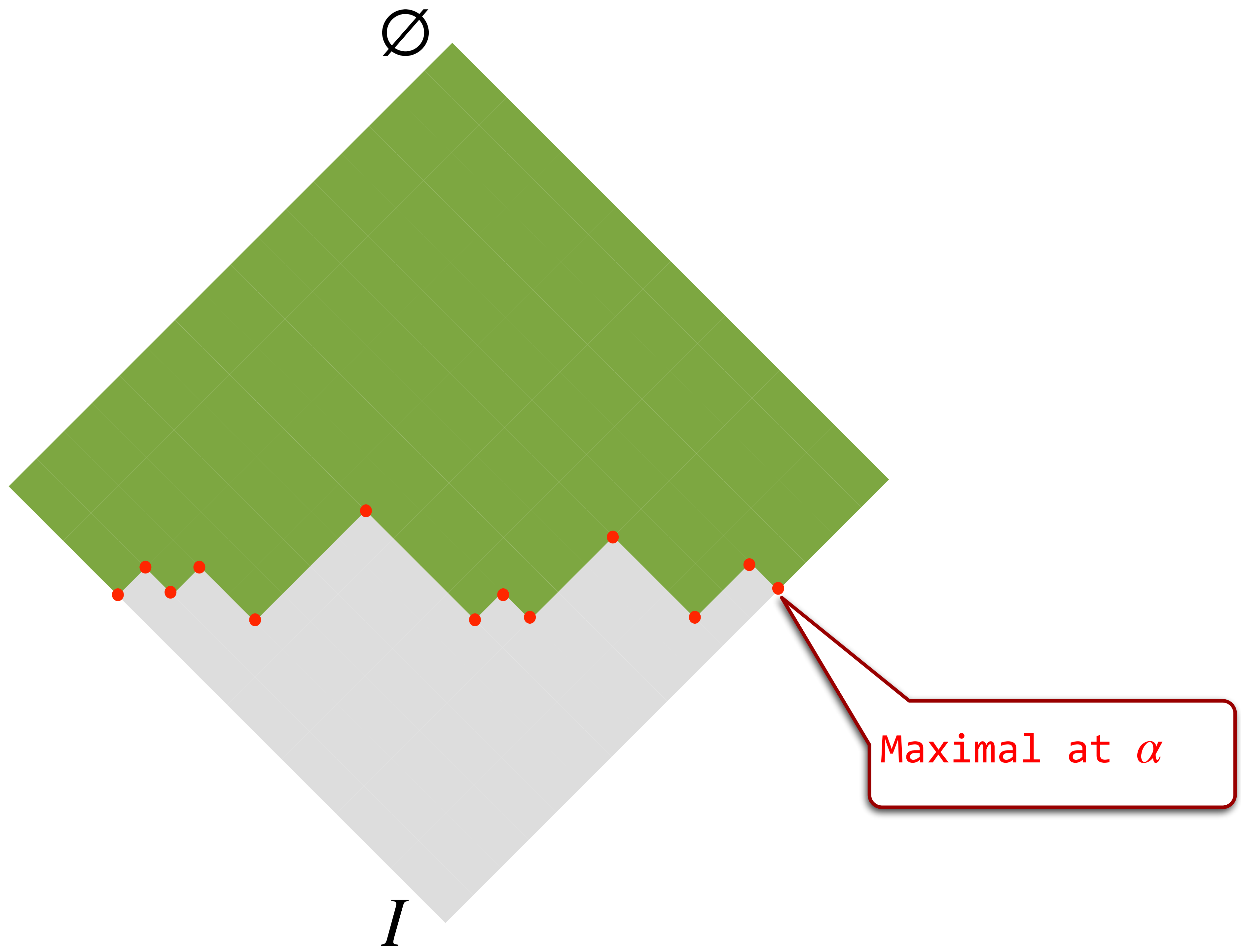
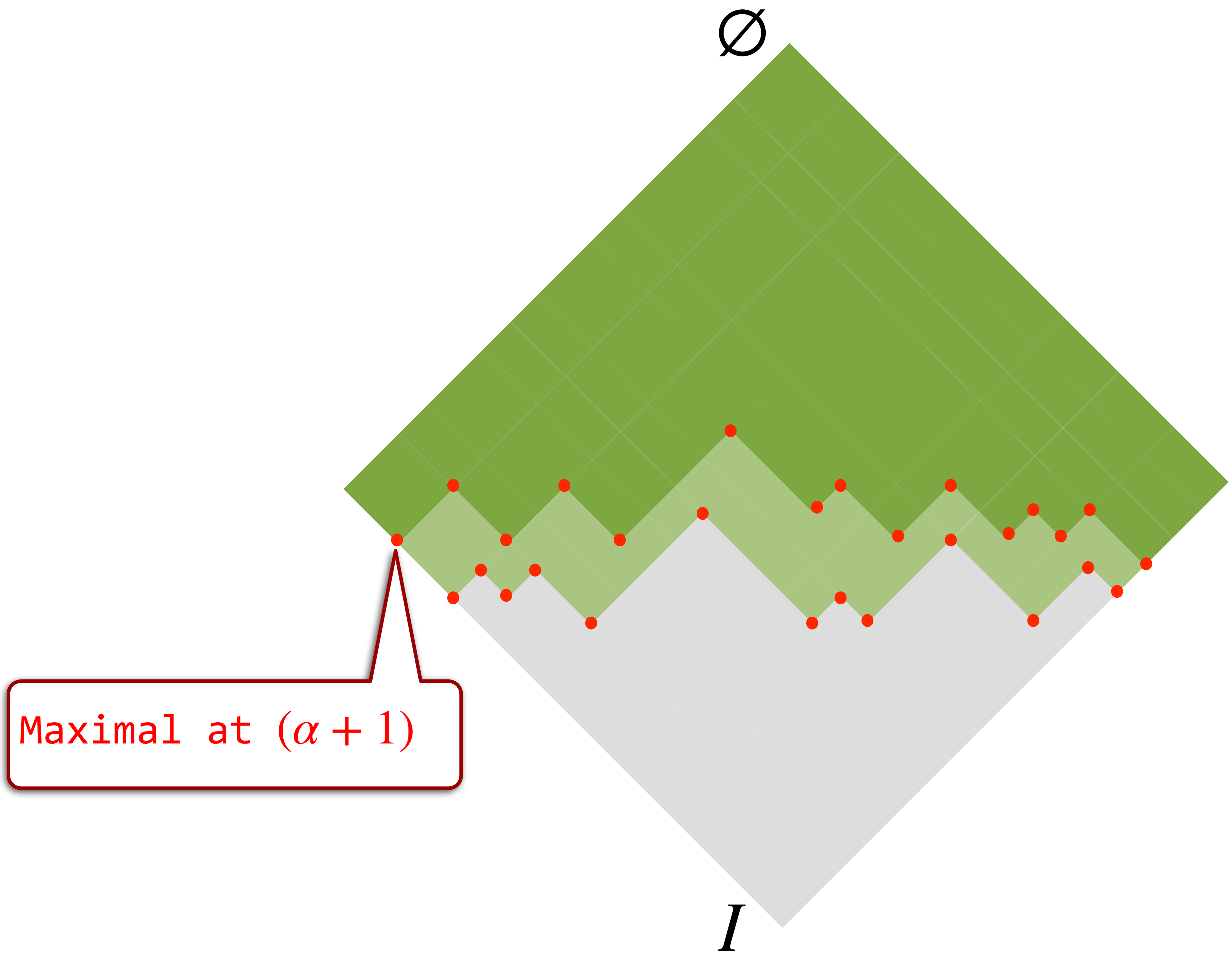
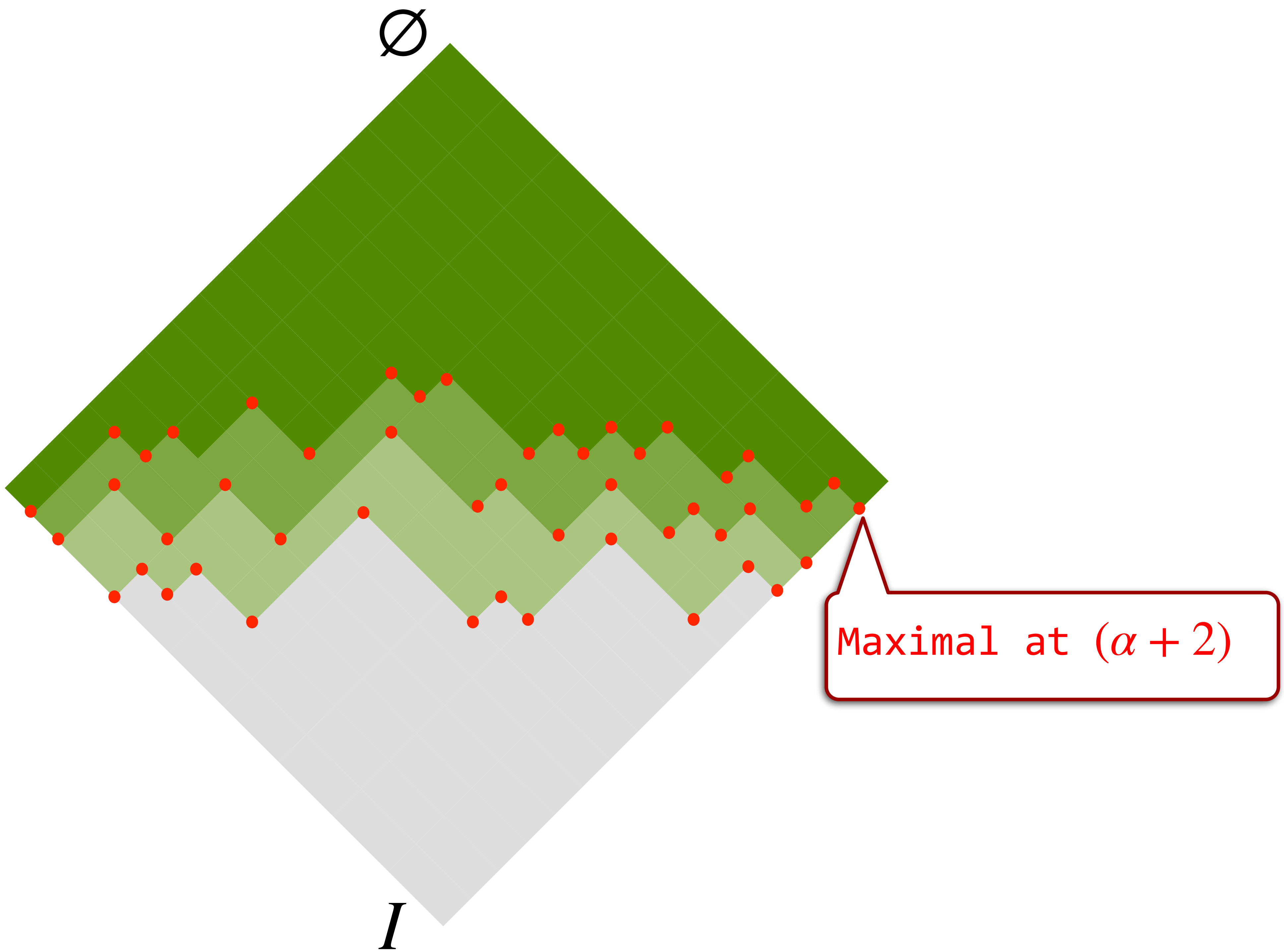
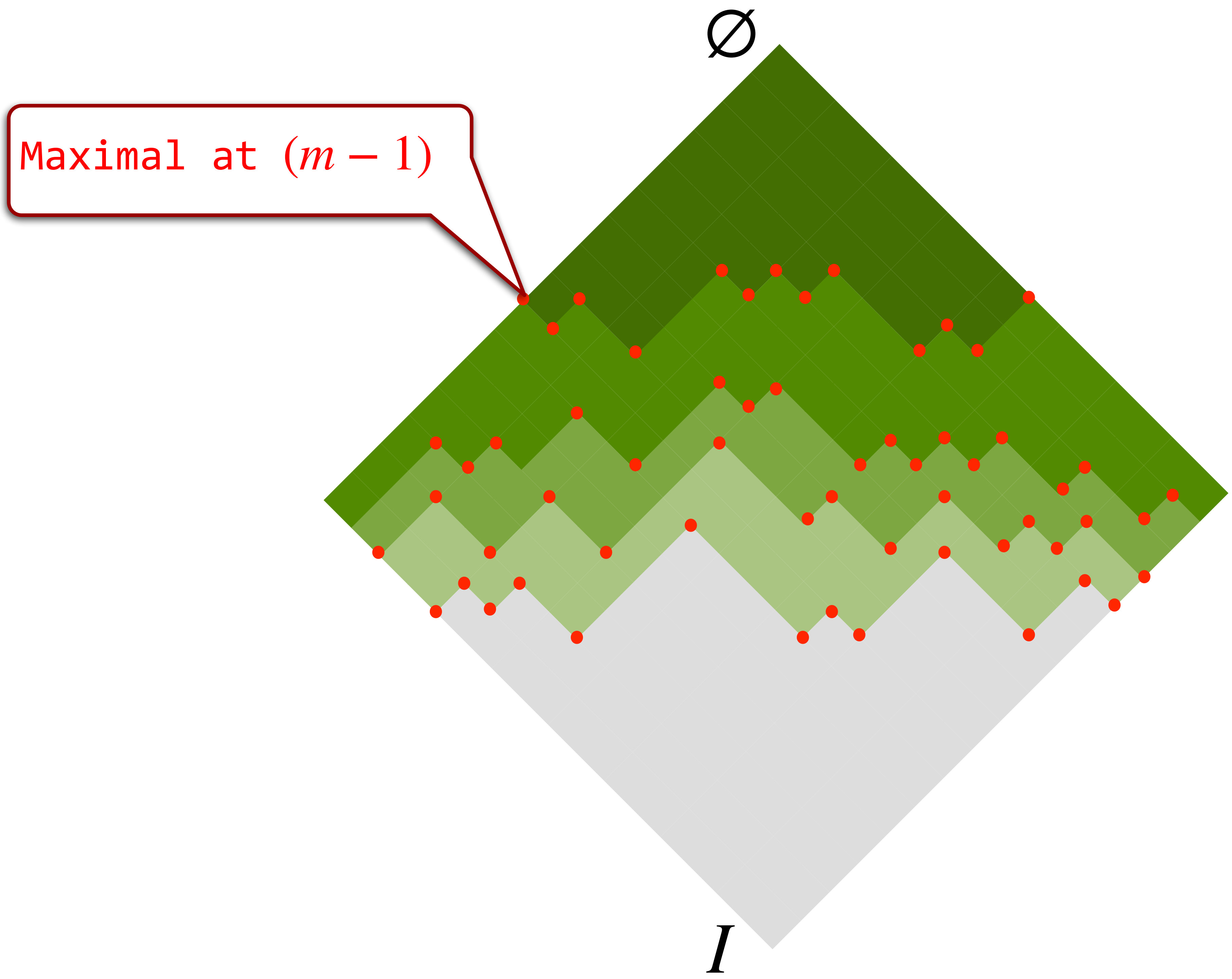
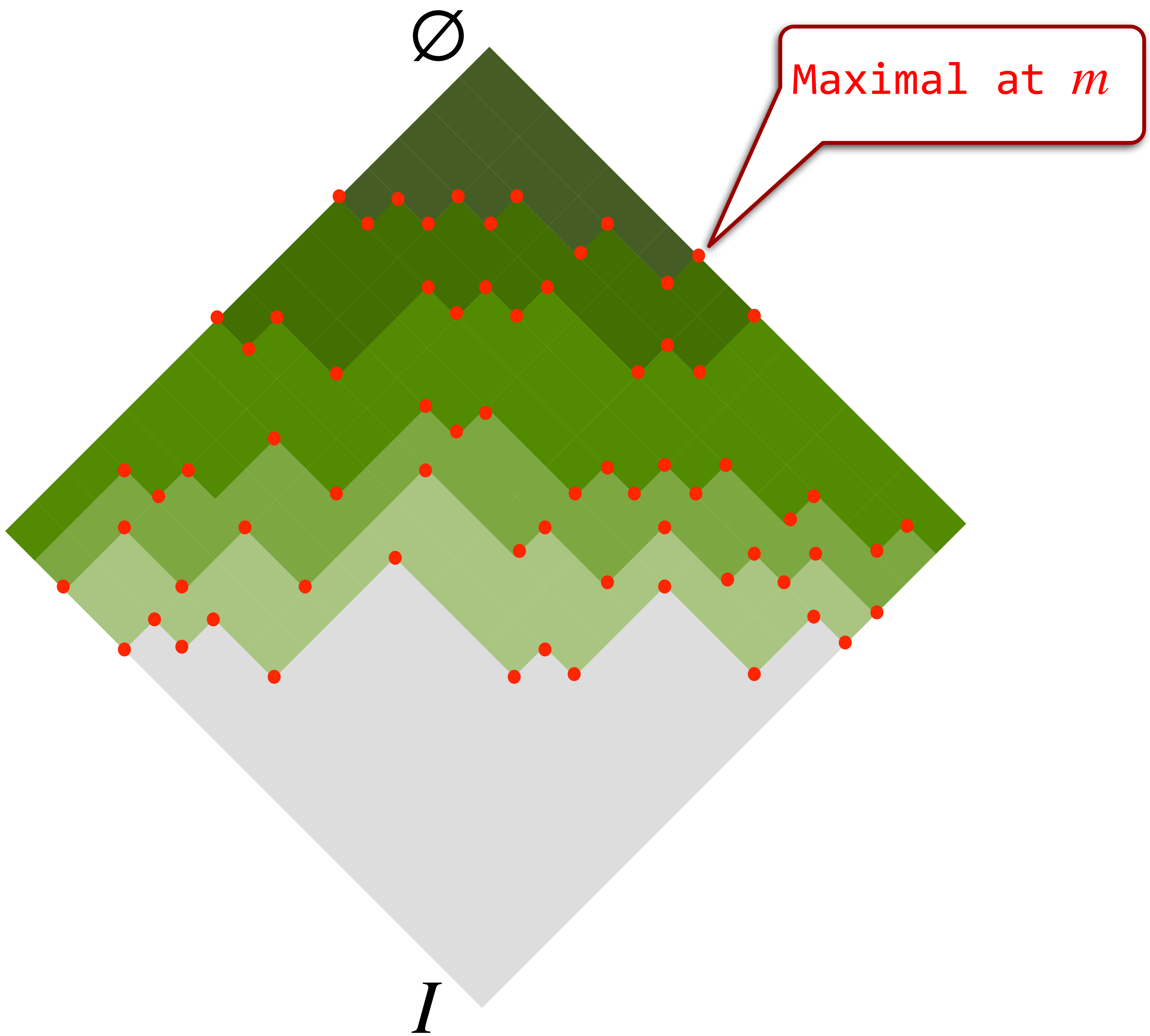
I


$\varnothing$
I
Maximal at α

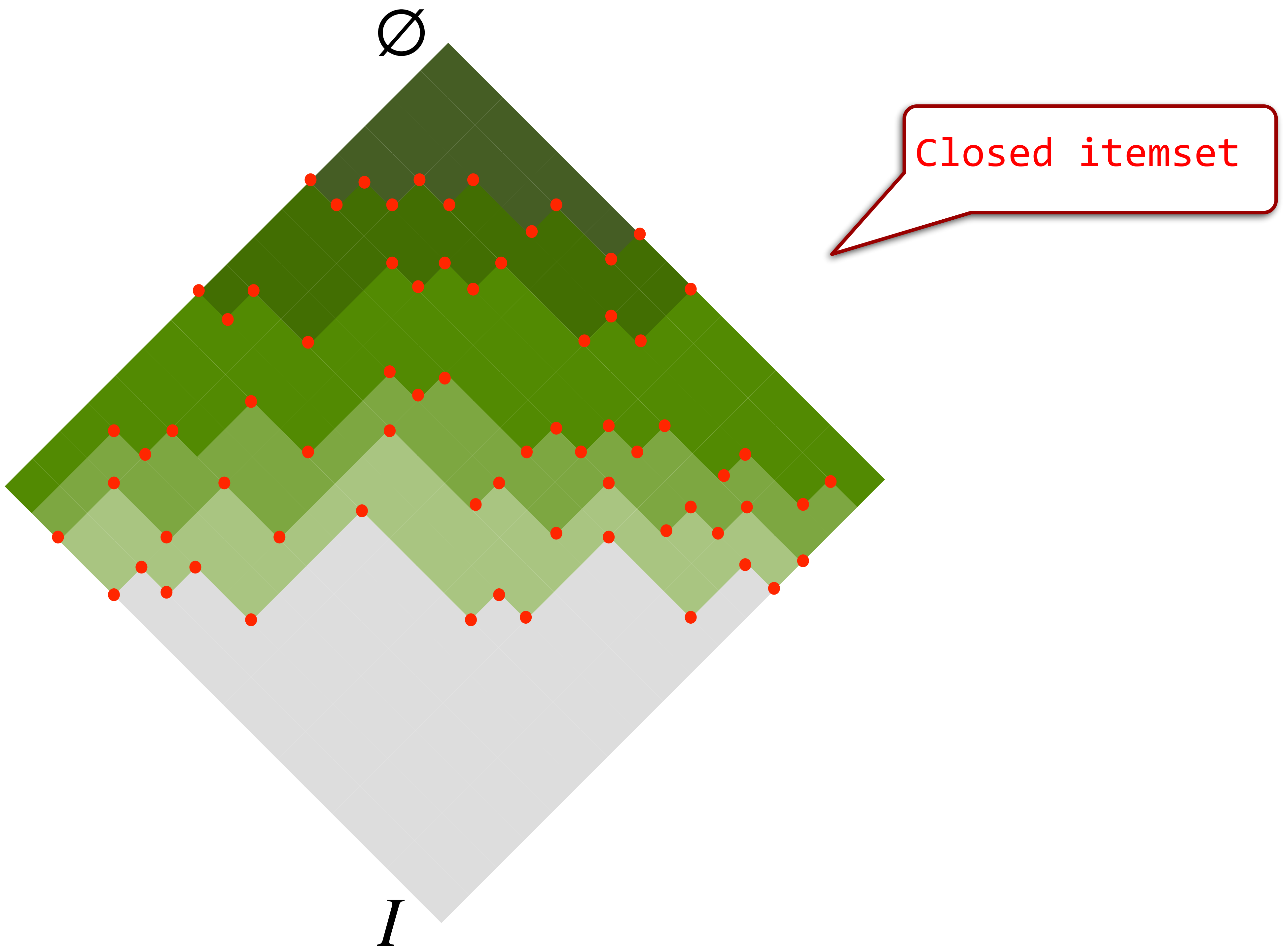

$\emptyset$
Maximal at $(\alpha + 1)$
I


$\varnothing$
Maximal at $(\alpha + 2)$
I


Maximal at $(m-1)$
$\varnothing$
I



∅
Maximal at m
I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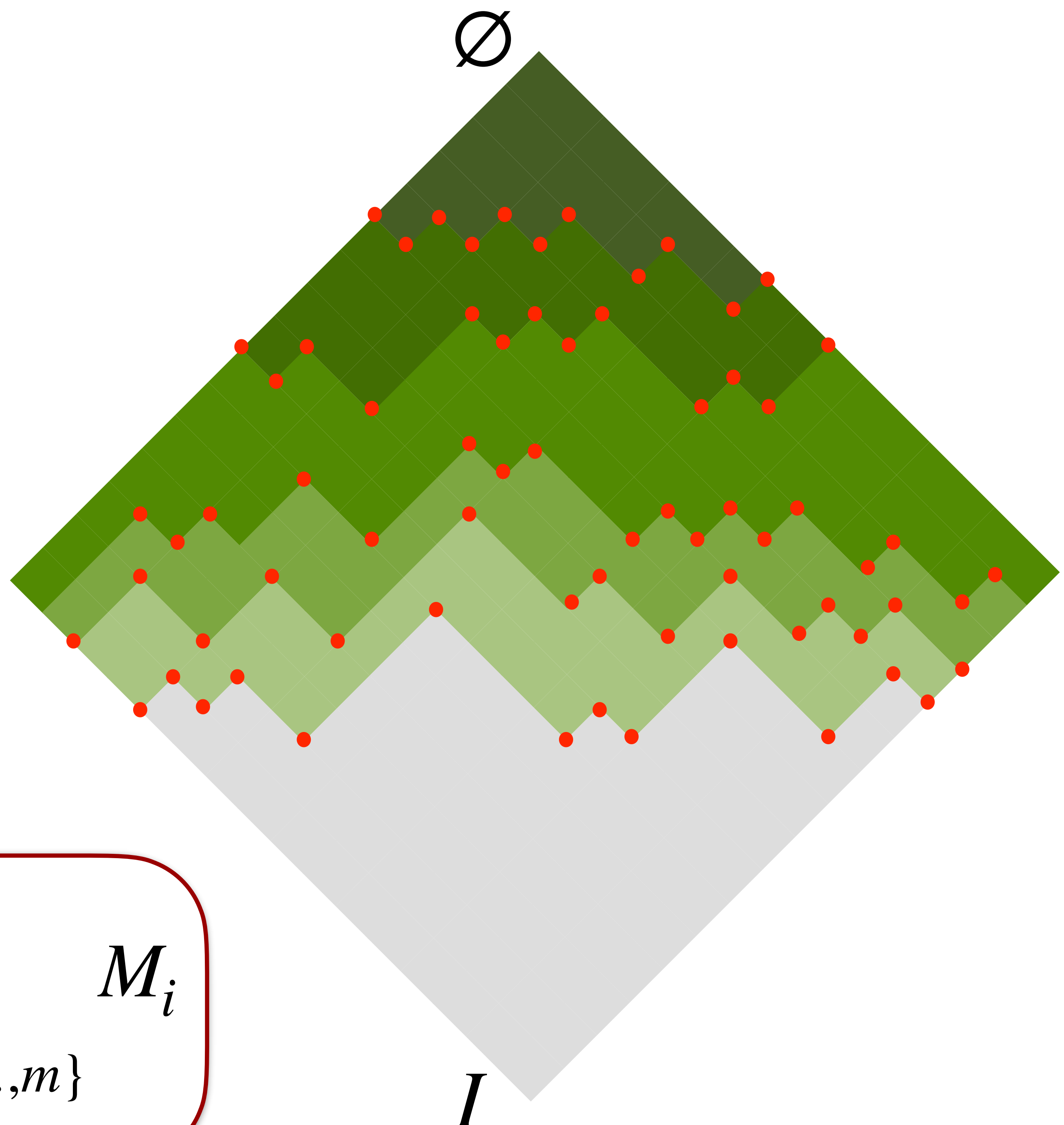
Closed itemset



$$C_\alpha = \bigcup_{i \in \{\alpha,\ldots,m\}} M_i$$

Frequent/Closed/Maximal

Itemsets

Dataset	#Frequent	#Closed	#Maximal
Zoo-1	151 807	3 292	230
Mushroom	155 734	3 287	453
Lymph	9 967 402	46 802	5 191
Hepatitis	$27 . 10^7$+	1 827 264	189 205

http://fimi.ua.ac.be/data/

LCM Algorithm Linear Closed Item Set Miner

[Uno et al., 03] (version 1) [Uno et al., 04, 05] (versions 2 & 3)

LCM

Basic Ideas

- Itemset candidates are evaluated in a given order (depth-first traversal of the prefix tree—lex order for instance—)
- Items are eliminated step-by-step from the transaction database through recursive processing of conditional transaction databases
- Maintains both horizontal and vertical representations of the transaction database in parallel
- **Vertical representation:** Used to filter transactions based on the chosen split item
- **Horizontal representation:** Used to populate the vertical representation for the next recursion step (avoiding intersection operations as in the Eclat algorithm).

LCM

Notations

- **Tail:** The tail of an itemset P is the last item in P:

$$P = \{a_1, \ldots a_k\} : tail(P) = a_k$$

- **Sub-itemsets:** The sub-itemset of P up to the j^{th} item is defined as:

$$P = \{a_1, \ldots a_k\} : P(a_j) = \{a_1, \ldots, a_j\}$$

- **Prefix:** The prefix of P in the prefix tree is defined as:

$$P = \{a_1, \ldots a_k\} : \text{prefix}(P) = P \backslash tail(P) = P(tail(P)) = \{a_1, \ldots, a_{k-1}\}$$

LCM

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LCM

Closure

- A set S has a closure under an operation f if:

$$\forall x \in S, f(x) \in S$$

- In this case, we say that the set S is closed under f
- A closure operation is:
  - **Increasing (or extensive):** The closure of an object contains the object itself, i.e., $S \subseteq \text{closure}(S)$
  - **Idempotent:** The closure of the closure of an object is the closure itself, i.e., $\text{closure}(\text{closure}(S)) = \text{closure}(S)$
  - **Monotone:** If $X \subseteq Y$, then $\text{closure}(X) \subseteq \text{closure}(Y)$

LCM

Itemset Closure

$$closure(P) = \bigcap_{t \in cover(P)} t$$

LCM

Itemset Closure

$$closure(P) = \bigcap_{t \in cover(P)} t$$

t1:		B	C		E	F	G	H
t2:	A			D			G	
t3:	A		C	D				H
t4:	A				E	F		
t5:		B			E	F		
t6:		B			E	F	G	

LCM

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$$closure(EG) = \,?$$

LCM

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LCM

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LCM

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LCM

Closure Extension [Pasquier et al., 99]

- **Closure Extension:** A rule for constructing a closed itemset from an existing one:
  - Simply add an item and compute its closure!

LCM

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Closed itemset

LCM

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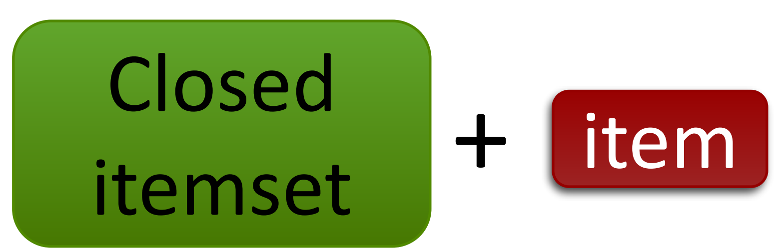

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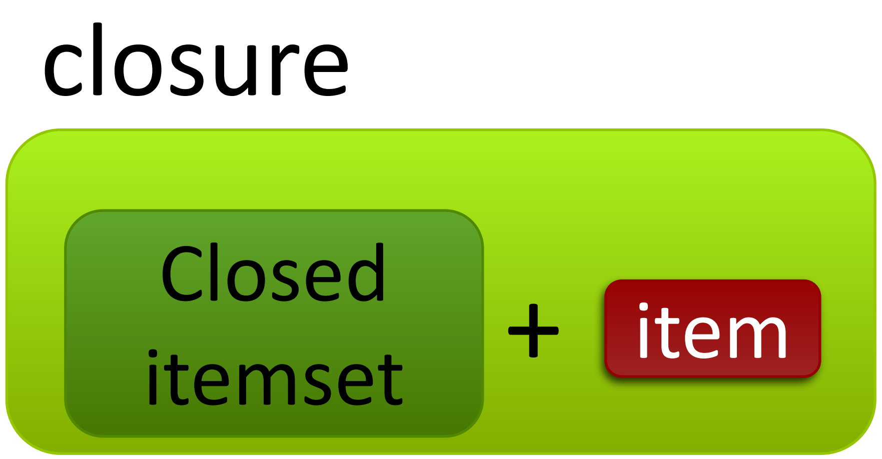

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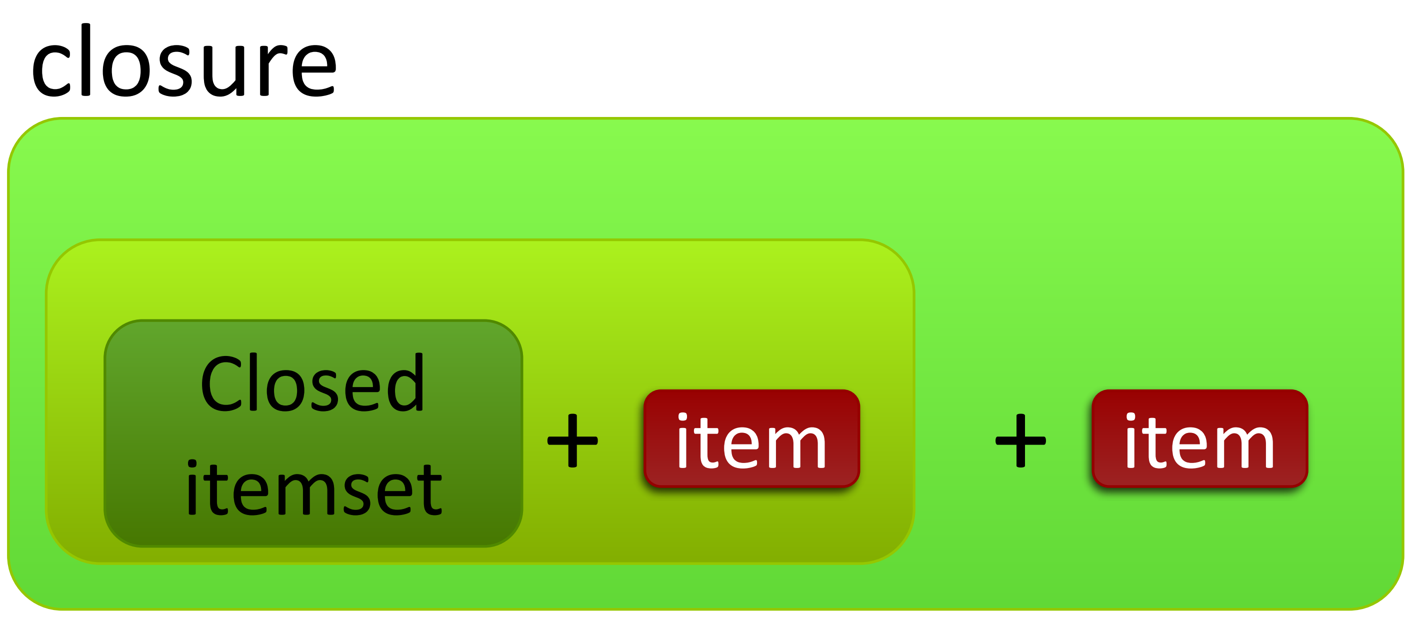

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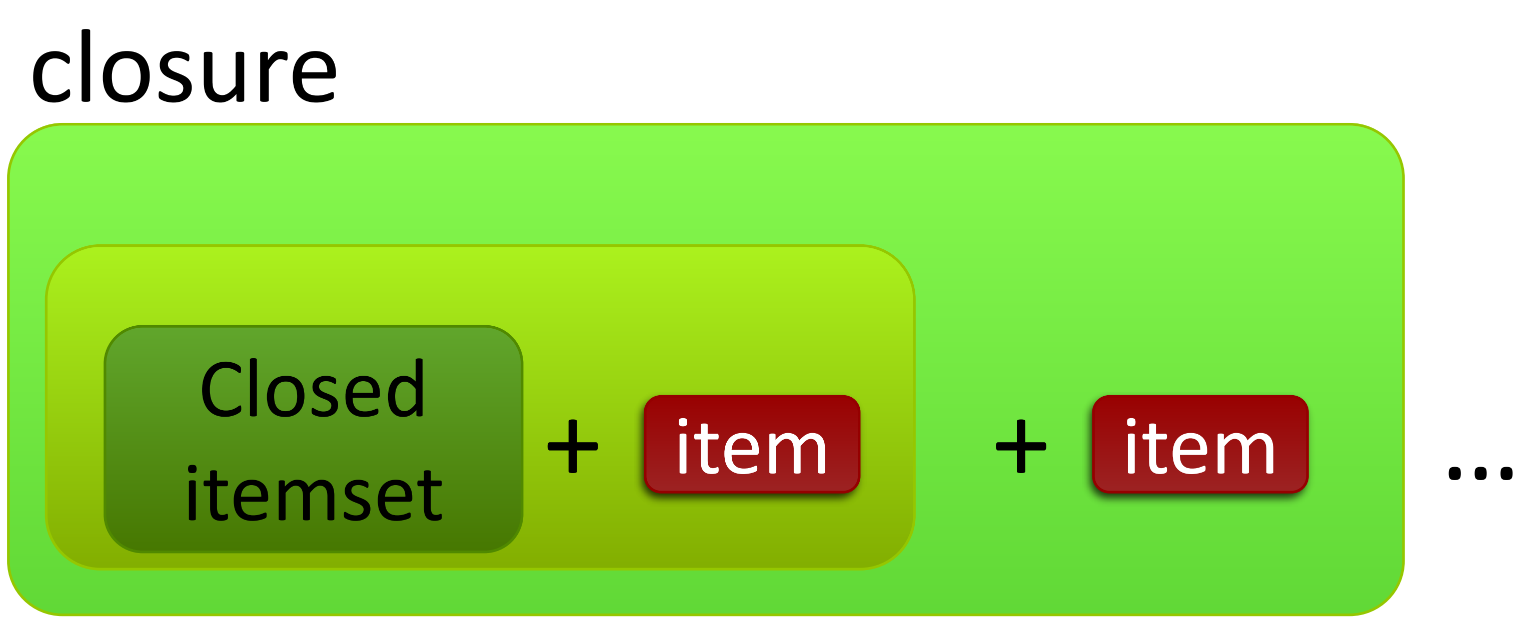

LCM

Algorithm

Algorithm 1: LCM

InOut: P: Closed Frequent Itemset;
In: α: minsup

$\texttt{print}(P)$
foreach $p_i > \texttt{tail}(P)$ **do**
  if $freq(P \cup \{p_i\}) \geq \alpha$ **then** $\text{LCM}(\texttt{closure}(P \cup \{p_i\}, \alpha)$

[Uno et al., 03]

Constraint & Data Mining

Cours2

Master 2 - DS

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20/01/2025