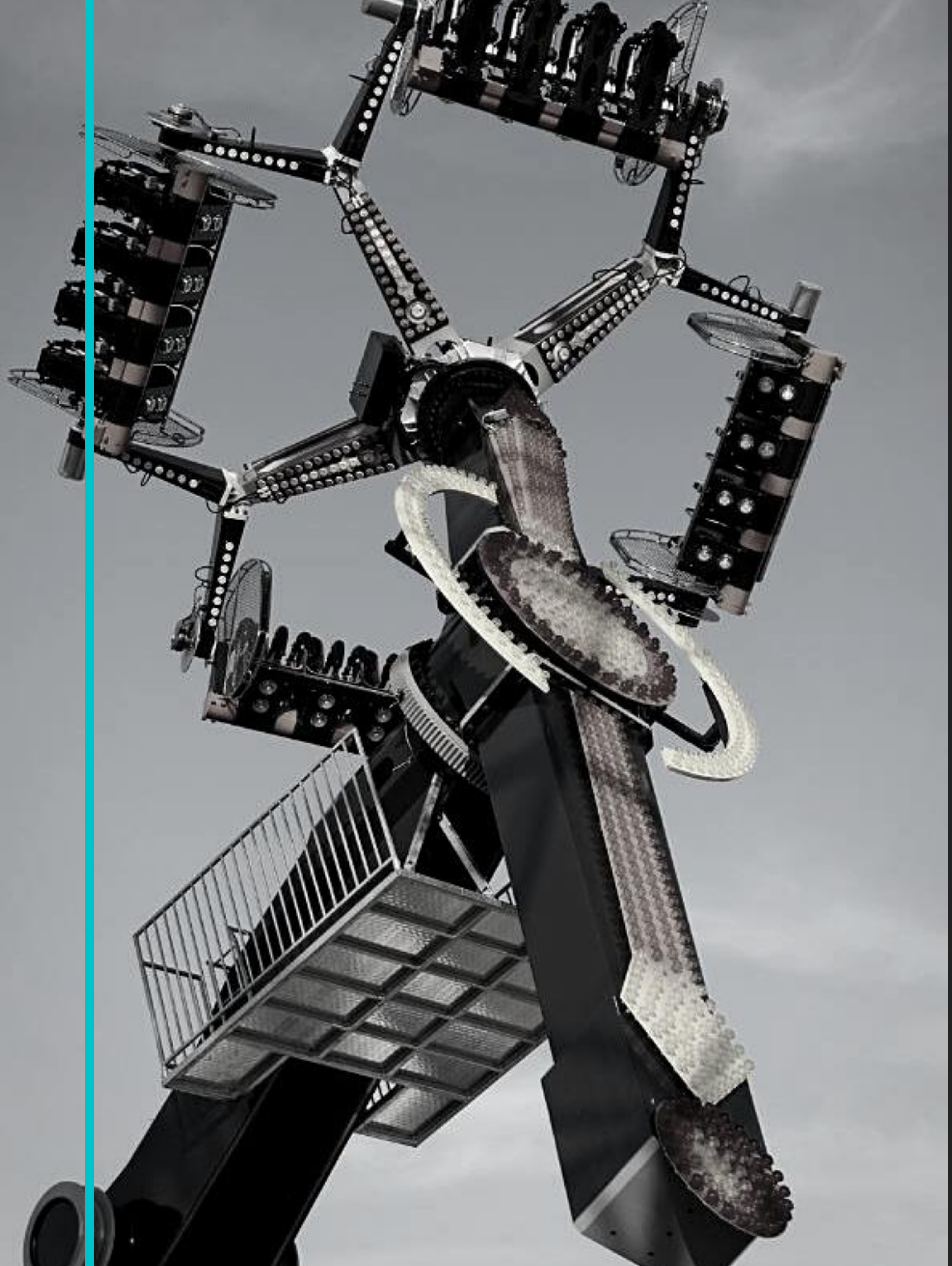

JUNE 26, 2020

EXTREME SWING

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HIGHLIGHTS

SYSTEM ANALYSIS

System and goal of analysis • Target customers and system requirements • Engineering Specifications • Performance Indices • Evaluation matrices • Scoring system

KINEMATIC ANALYSIS

System model • Constraint equations • Workspace • Velocity and acceleration analysis • Monte Carlo sensitivity analysis and optimization

DYNAMIC ANALYSIS

Dynamic model • Inverse dynamics • Numerical solution comparison • Maplesim model • Performance analysis

RESULTS

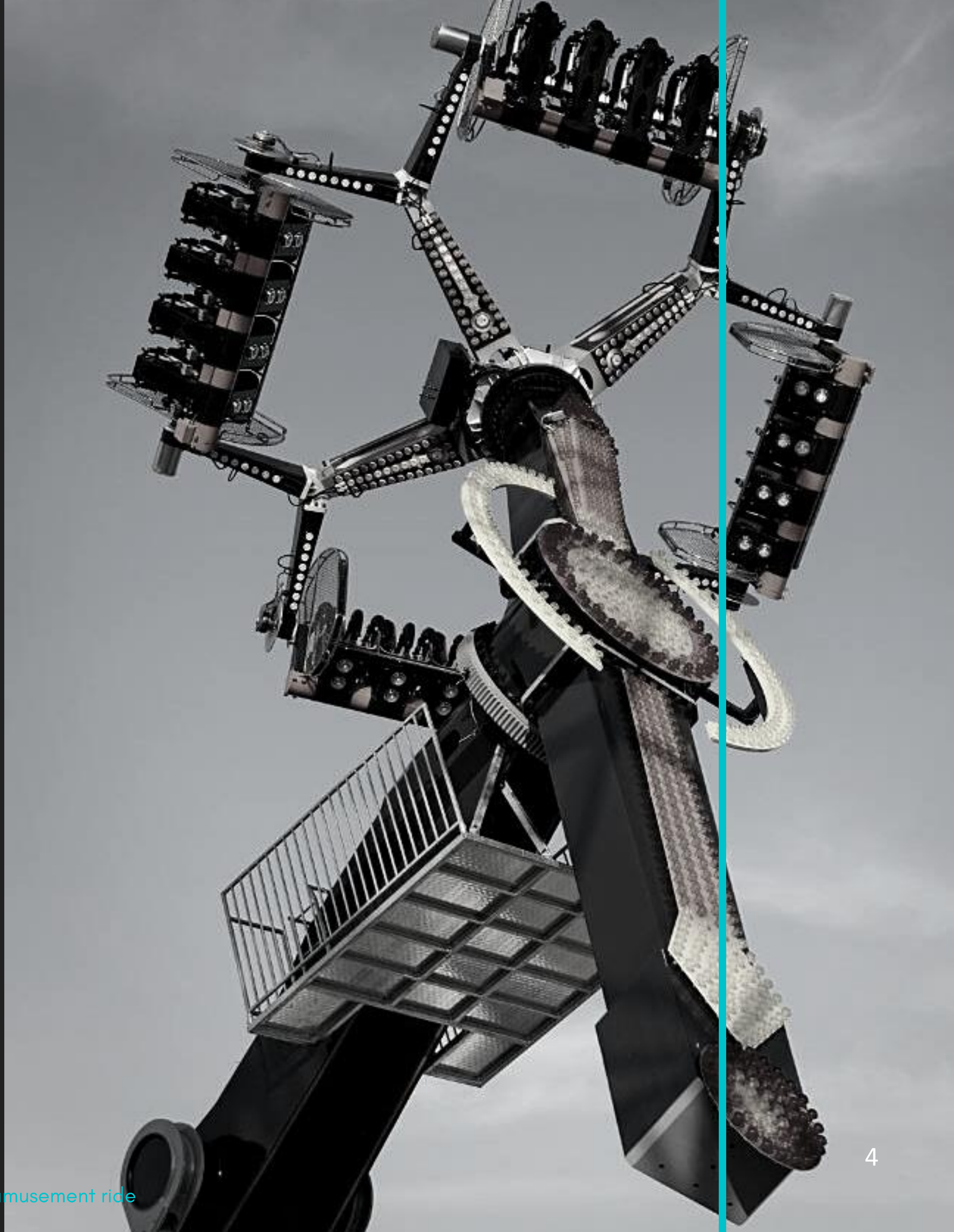
Concluding remarks

SYSTEM ANALYSIS

- System and goal of analysis
- Target customers and system requirements
- Engineering Specifications
- Performance Indices
- Evaluation matrices
- Scoring system

System and goal of analysis

- Pegasus 16 Pendulum like amusement ride
- 3D open chain mechanism
- Four bodies
  - Lifting arm
  - Pendulum
  - Carousel
  - Seats
- Evaluate and optimize the system performances



Target Customers and System Requirements

OWNER

- Keep costs low
- Low power consumption
- Low transportation and mounting costs
- Ensure safety

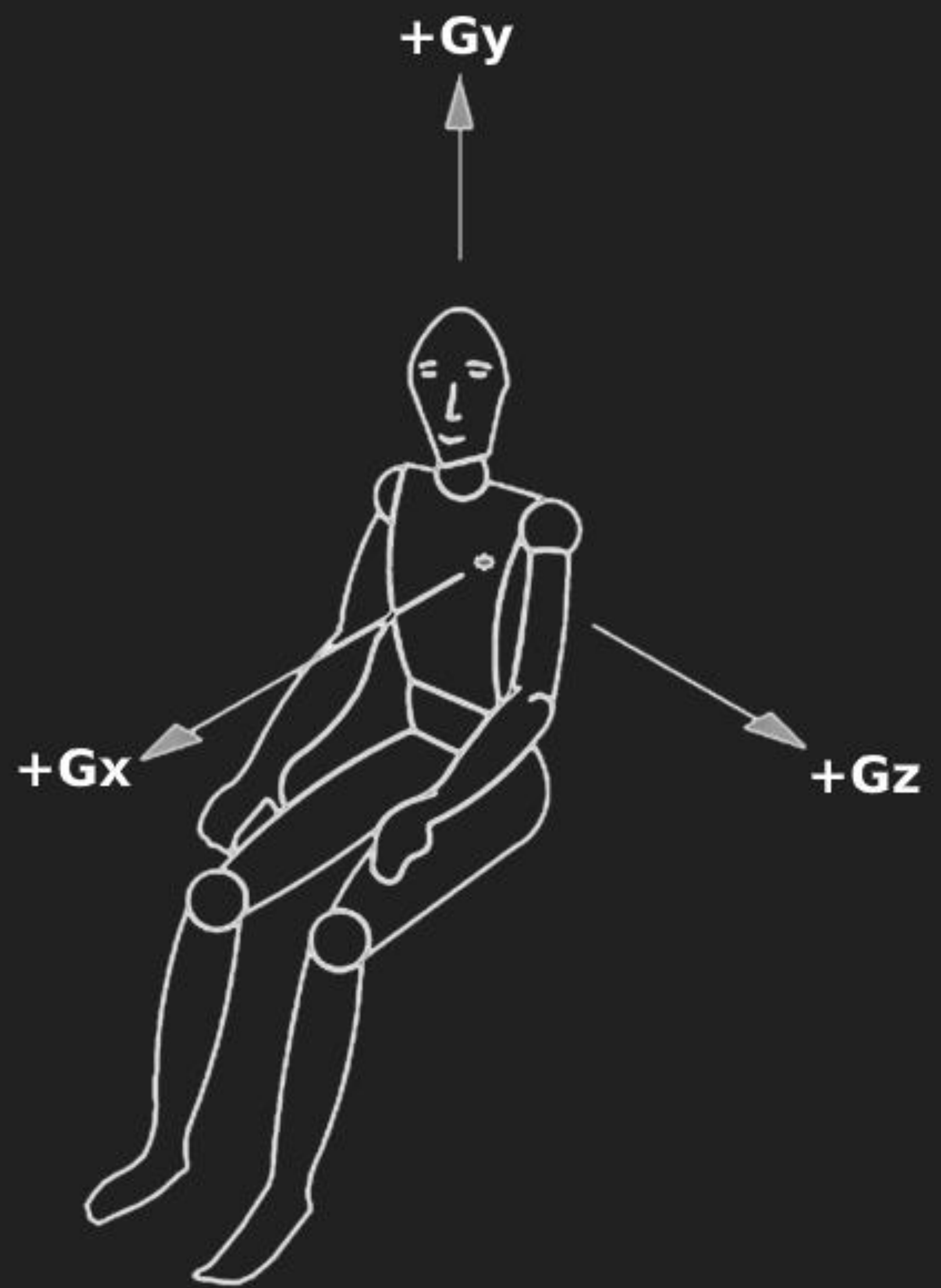
PERSON RIDING

- Fun and exciting ride
- Minimal motion sickness

Target Customers and System Requirements

What makes a roller coaster ride "fun"?

- increased G forces
- different and varying G forces
- accelerations felt in three directions Gx Gy Gz

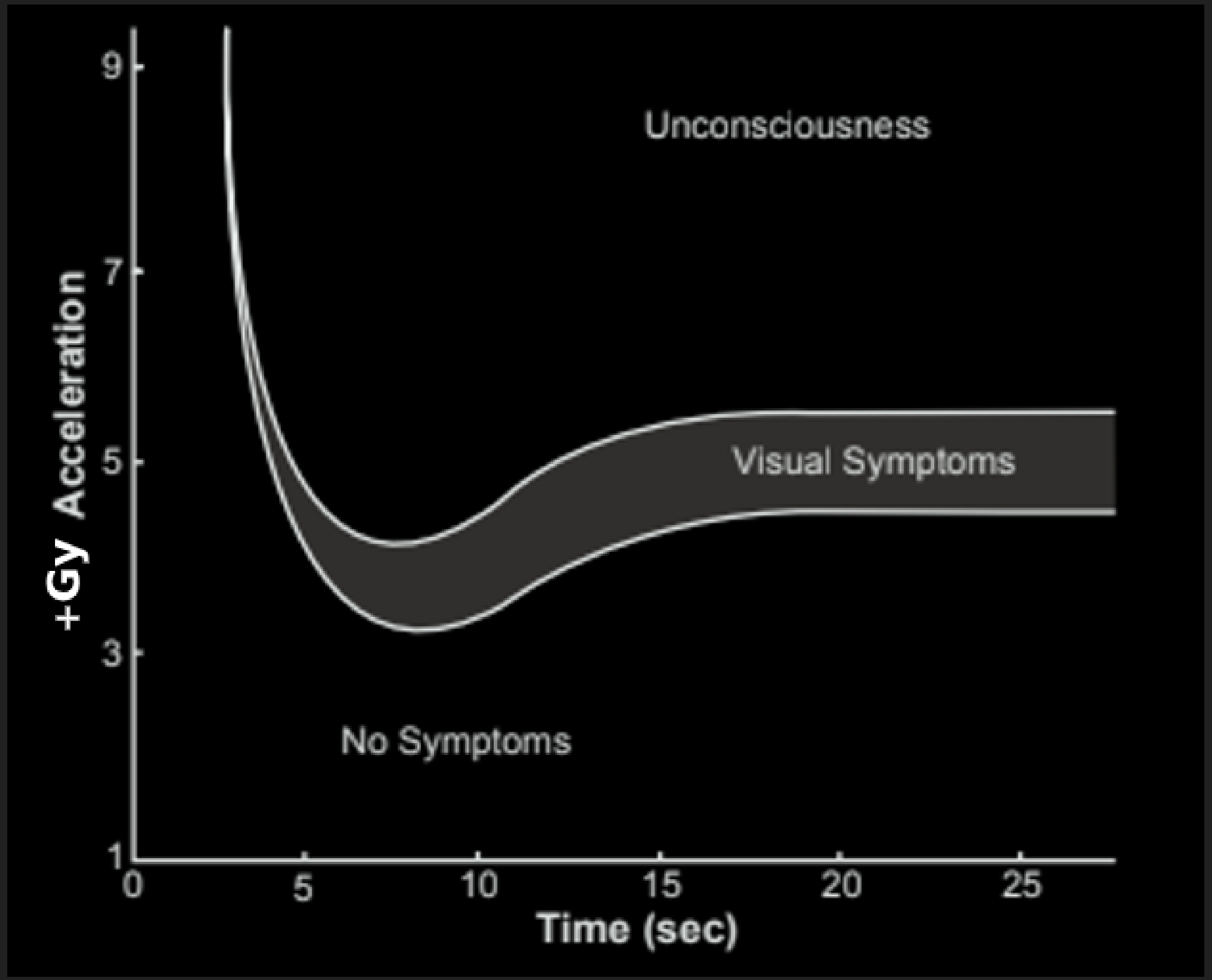


We can control the magnitude, duration, and direction of these accelerations to optimize the user's experience.

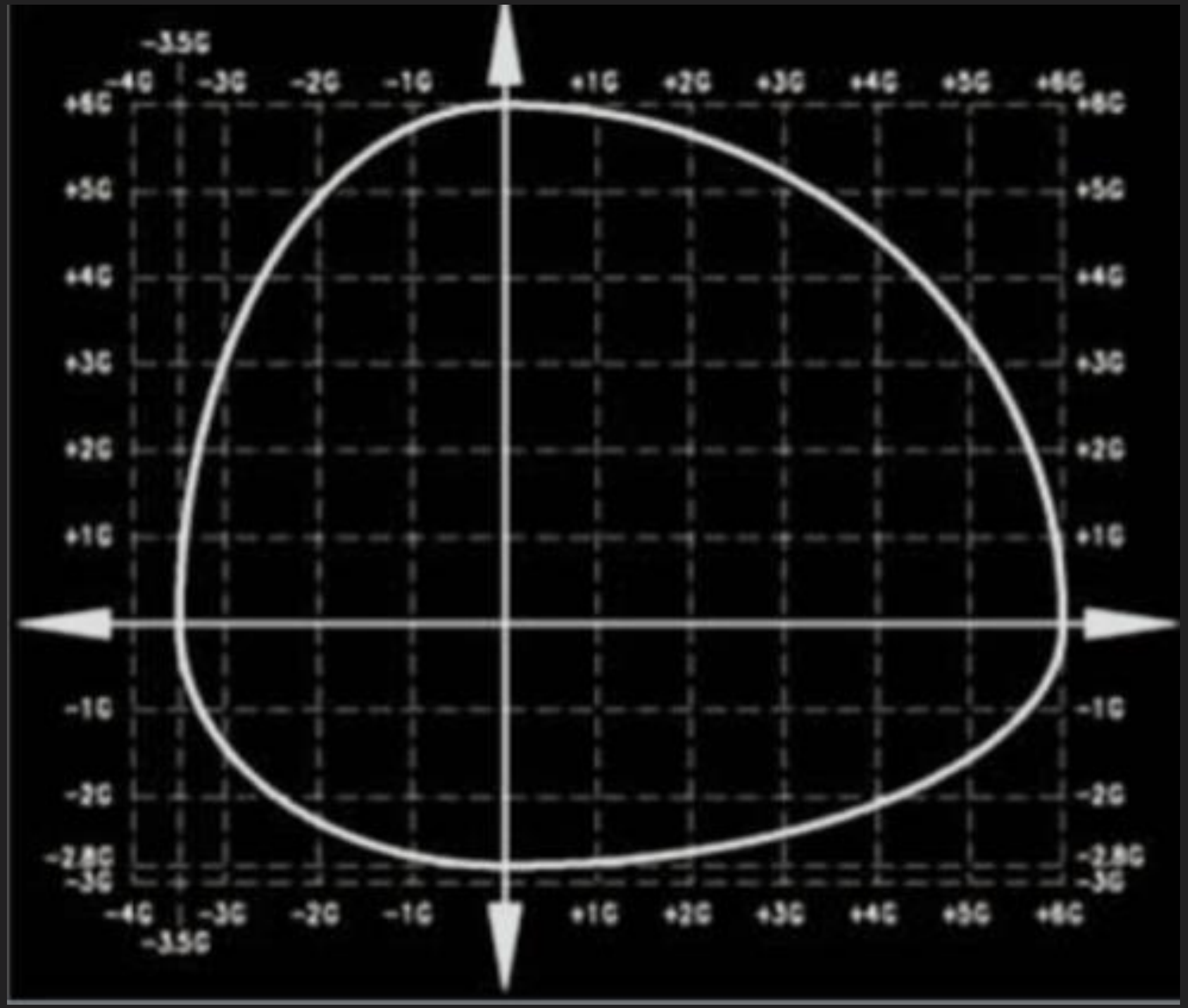
Target Customers and System Requirements

What makes a ride safe?

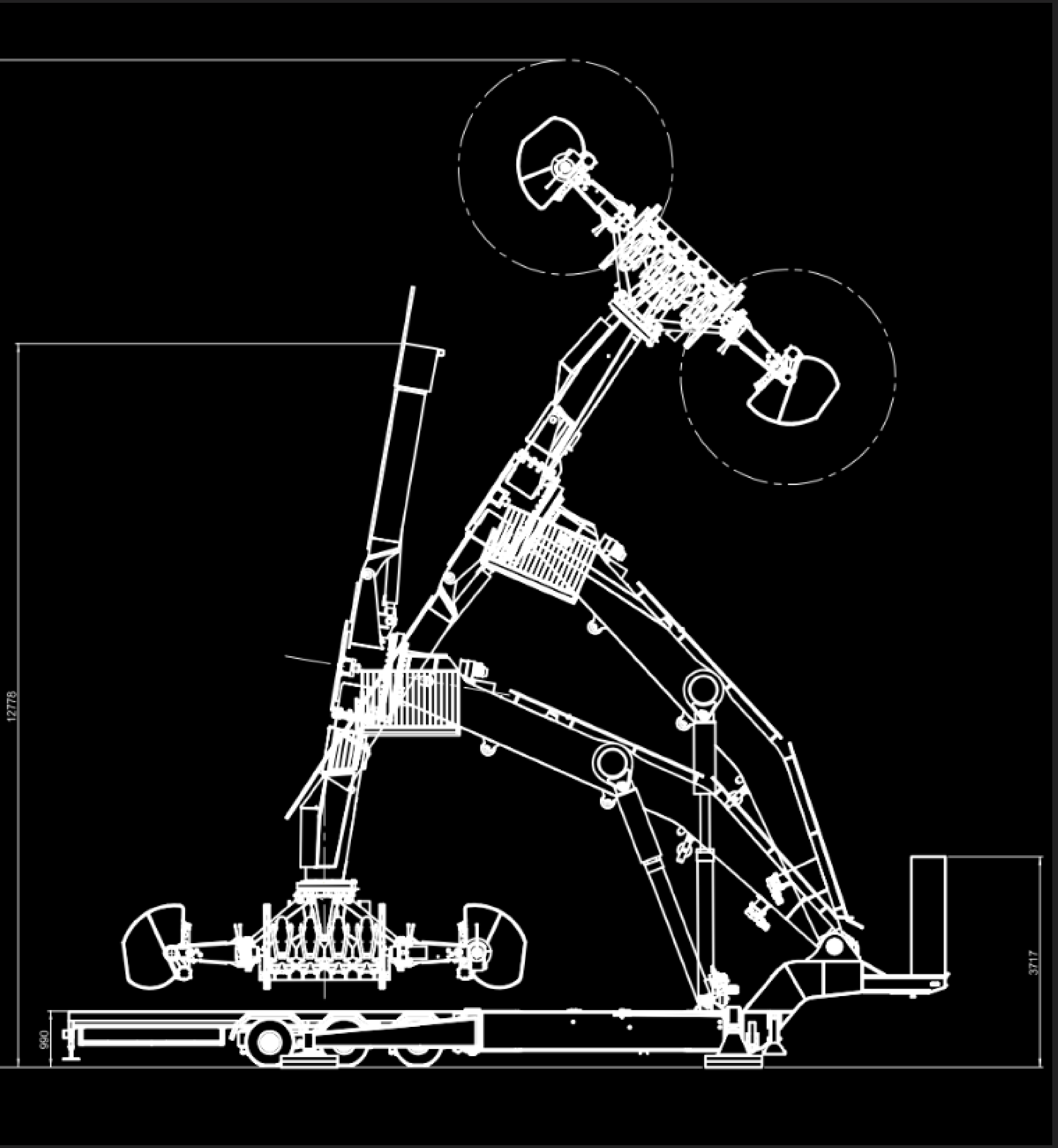
- Limits in the vertical acceleration



- Limits in combination of G-forces



Engineering Specifications: Kinematic



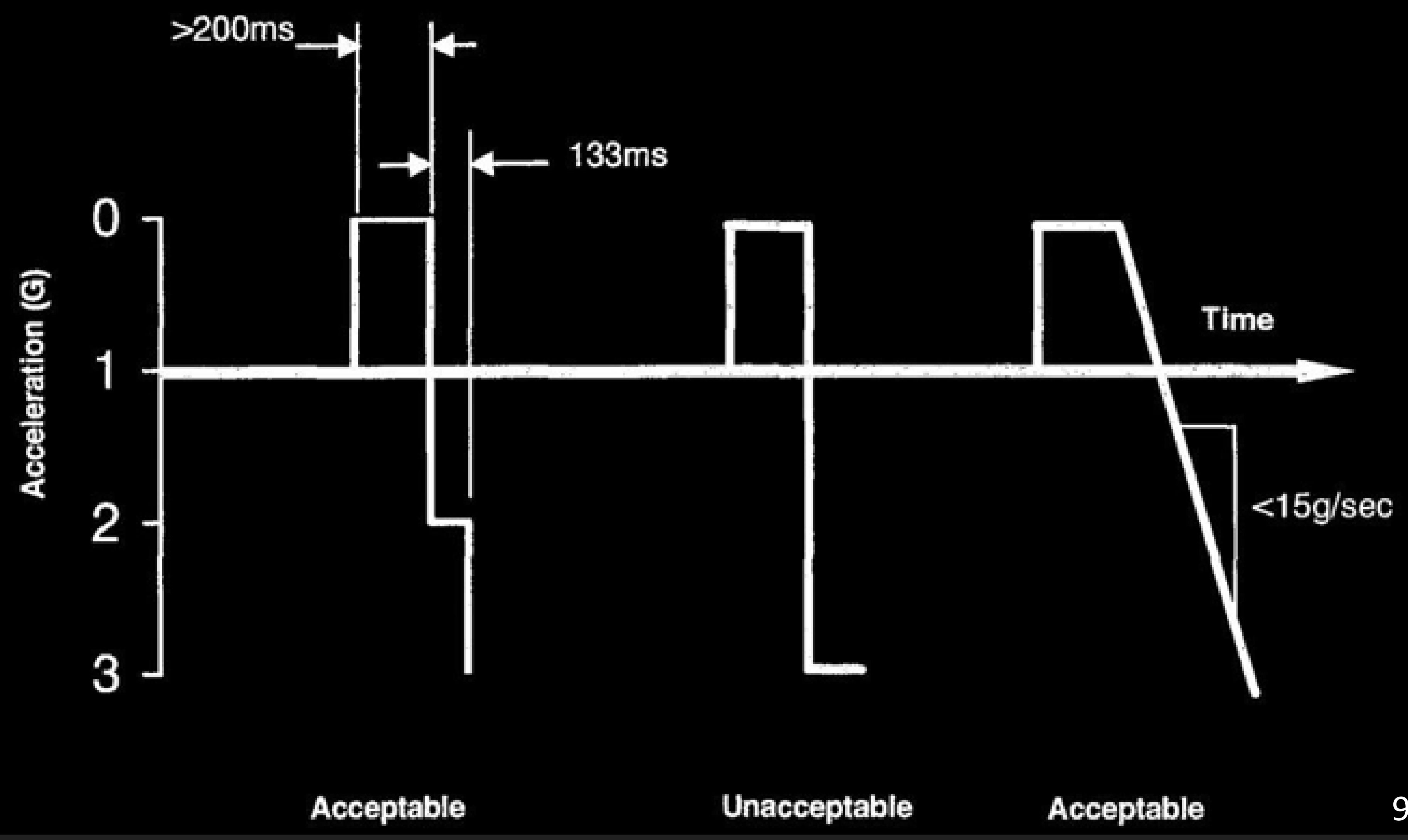
- Dimensions
  - L1 = 10.03 m
  - L2 = 4.247 m
  - L3 = 2.53 m
- Angular velocities
  - $\omega_1 = 11\,rpm$
  - $\omega_2 = 10\,rpm$
- Number of seats 16 (4 x 4)

- Ride Duration 90s

Engineering Specifications: Dynamic

- Power consumption
  - Carousel motor – 28kW
  - Pendulum motor – 28kW
  - Lifting arm – 55kW
- Weight
  - 53,870 kg without riders
  - 55,950 kg with riders
- Linear accelerations at seats
  - Maximum for X and Y is 5 G
  - Maximum for Z is 2.5 G
  - Duration ≥ 200 ms
- Jerk at the seats
  - Maximum for X and Y is 15 G/s
  - Maximum for Z – not relevant

Admissible acceleration	0.2 s		1.5 s		>12 s	
	min	max	min	max	min	max
a_x	$-2.0g$	$+6.0g$	$-1.5g$	$+5.0g$	$-1.5g$	$+2.5g$
a_y	$-2.0g$	$+6.0g$	$-1.5g$	$+5.0g$	$-1.1g$	$+2.0g$
a_z	$-3.0g$	$+3.0g$	$-2.5g$	$+2.5g$	$-2.0g$	$+2.0g$



Performance Indices

Performance indices are used to compare the different solutions and optimize the ride.

THRILLING EXPERIENCE

- Fun

TRADE-OFFS

- Overall power consumption
- Dimensions
- Motor velocities
- Safety

How to numerically calculate «fun»?

Performance Indices: The fun equation

$$\text{Fun} = \gamma_1 \text{Fun}_x + \gamma_2 \text{Fun}_y + \gamma_3 \text{Fun}_z$$

$$\text{Fun}_x = \sum_{i=1}^{N} \alpha_1 acceleration_{x_i} + \alpha_2 duration_{x_i} - \alpha_3\, jerk_{x_i}$$

$$\alpha_1 = 0.3\,, \alpha_2 = 0.5, \alpha_3 = 0.2$$
$$\gamma_1 = 0.3\,, \gamma_2 = 0.6, \gamma_3 = 0.1$$

Evaluation Matrix

Specification	Weight
Fun	5
Safety	∞
Power Consumption	4
Dimension – L1	3
Dimension – L2	1
Dimension – L3	3
Angular velocity 2	2
Angular velocity 3	3

Scoring system

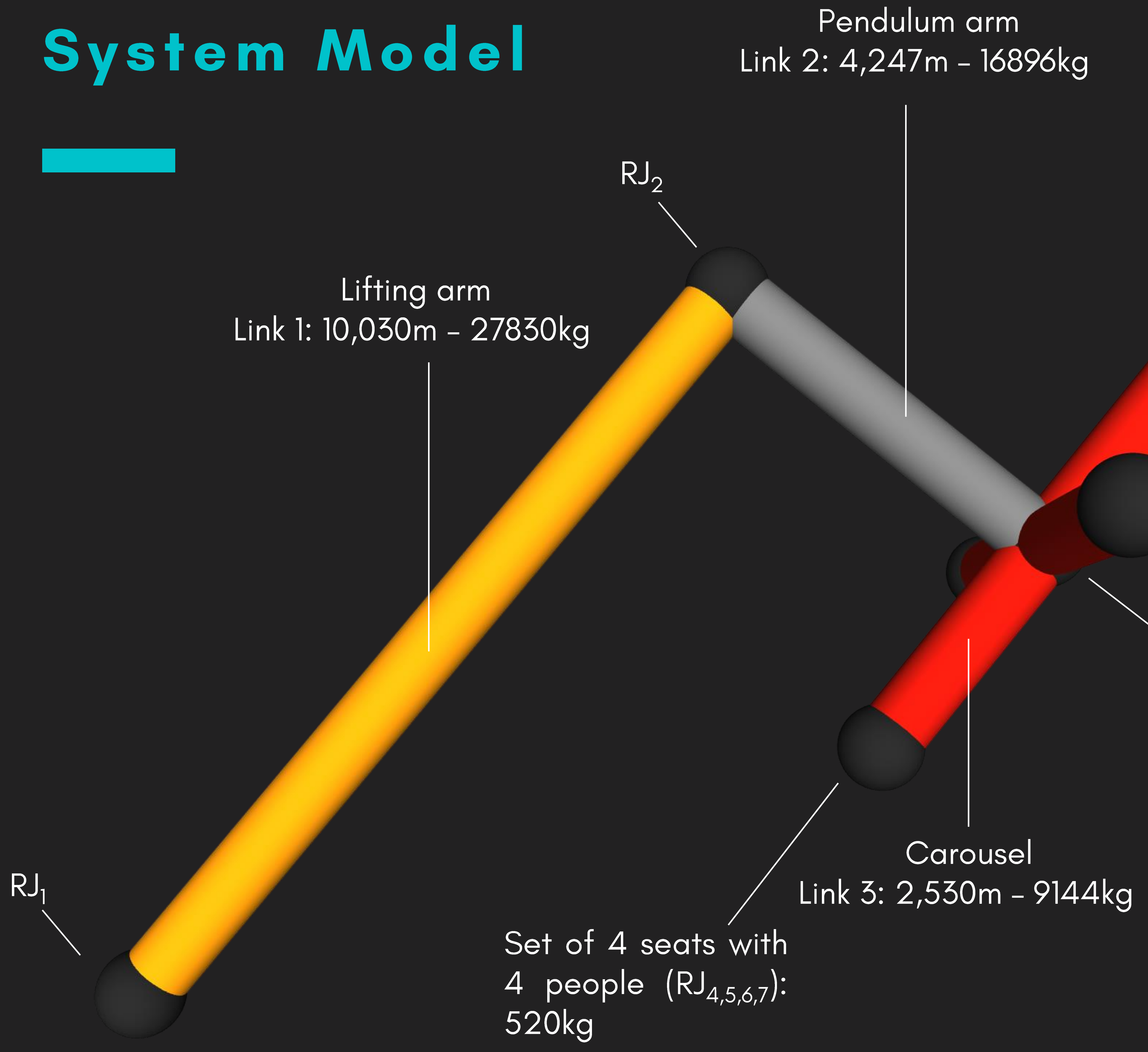
Parameter	Score 1	Score 2	Score 3
Fun	Less than baseline	Equal to baseline	Higher than baseline
Power Consumption	449-499	399-449	<399
Length of L1	Longer than baseline	Equal to baseline	Shorter than baseline
Length of L2	Longer than baseline	Equal to baseline	Shorter than baseline
Length of L3	Longer than baseline	Equal to baseline	Shorter than baseline
ω_2	Higher than baseline	Equal to baseline	Slower than baseline
ω_3	Higher than baseline	Equal to baseline	Slower than baseline

Parameter	Score -1	Score 0
Safety	Fail	Pass

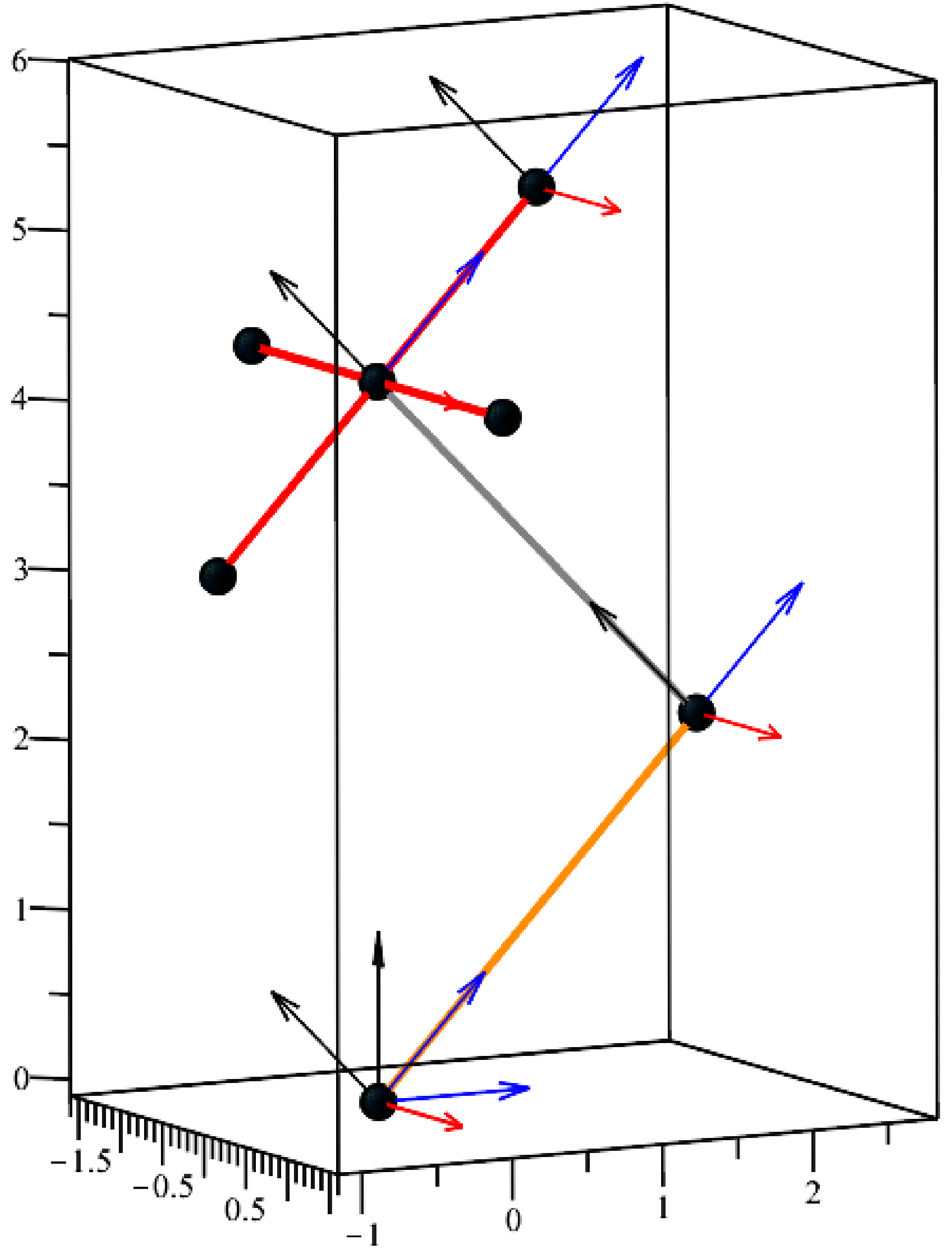
KINEMATIC ANALYSIS

- System model
- Constraint equations
  - Global approach
  - Recursive approach
  - Natural approach
- Workspace
- Velocity and acceleration analysis
- Monte Carlo sensitivity analysis and optimization

System Model



The lengths have been calculated from a scaled drawing provided by "Technical Park", the Pegasus 16 producers.



Representation of the robot in Maple:

- X axis: blue
- Y axis: black
- Z axis: red

The vertical axis in the "person reference frame" corresponds to the y axis in the modelled system.

Constraint equations: Global approach

Definition of the body reference frames using Euler angles:

$$RF_i = Translate(x_i, y_i\ z_i).Rotate(z,\psi).Rotate(x,\varphi).Rotate(\ y,\theta)$$

Definition of the auxiliary reference frames:

- Lifting arm and Pendulum arm:

$$RFa1_i = RF_i$$
$$RFa2_i = RF_i.Translate\left(L_{i_x}, L_{i_y}, L_{i_z}\right)$$

- Carousel:

$$RFa1_3 = RF3$$
$$RFa2_3 = RF_3.Rotate(y,\beta).Translate\left(L_{i_x},0,0\right), where\ \beta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$$

Constraint equations:

- First revolute joint:

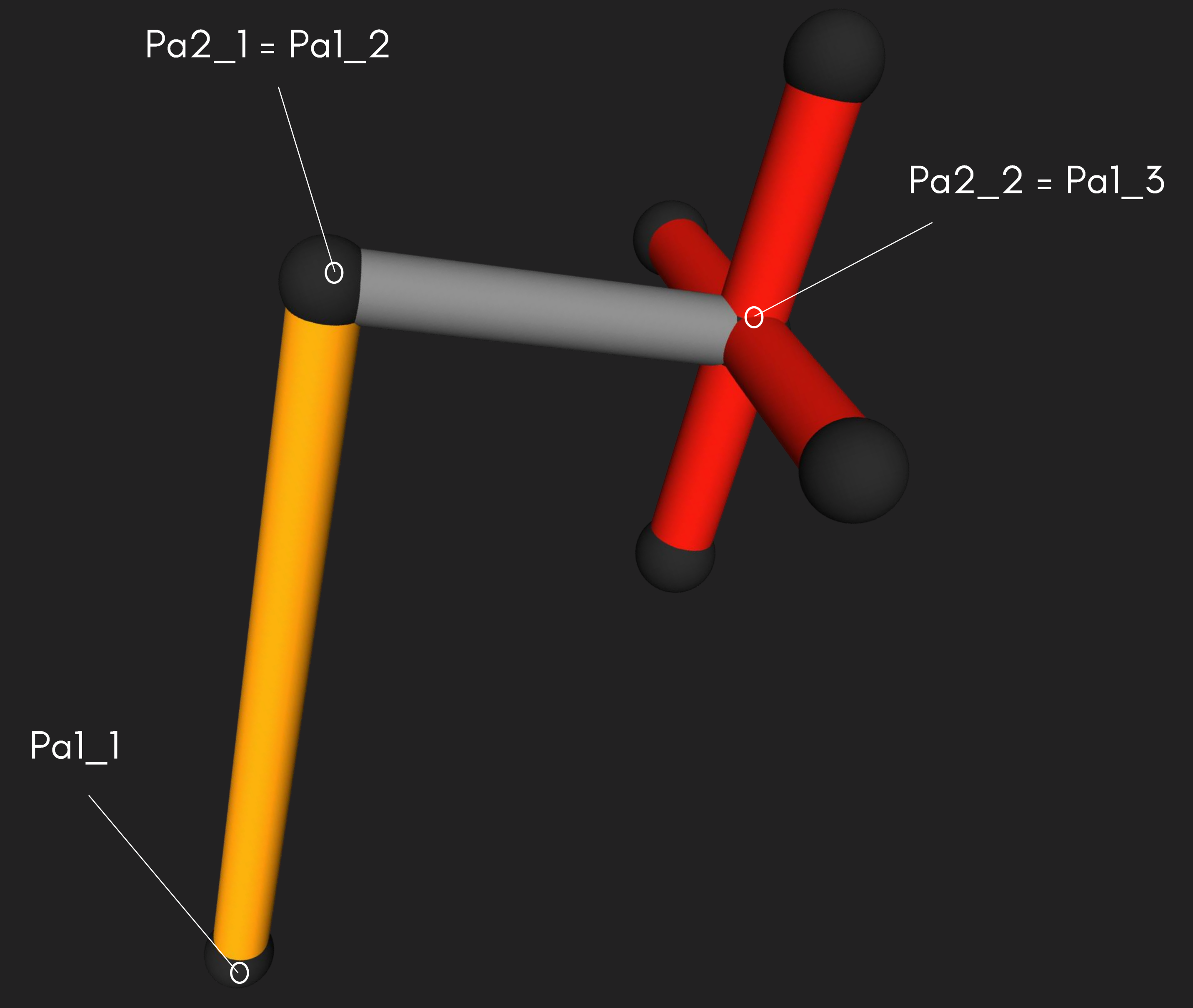
$$\overline{O(RF_1)O_{GND}} \cdot \overline{O(RF_1)O_{GND}} = 0$$
$$\overline{k_1} \cdot \overline{i_{GND}} = 0$$
$$\overline{k_1} \cdot \overline{j_{GND}} = 0$$

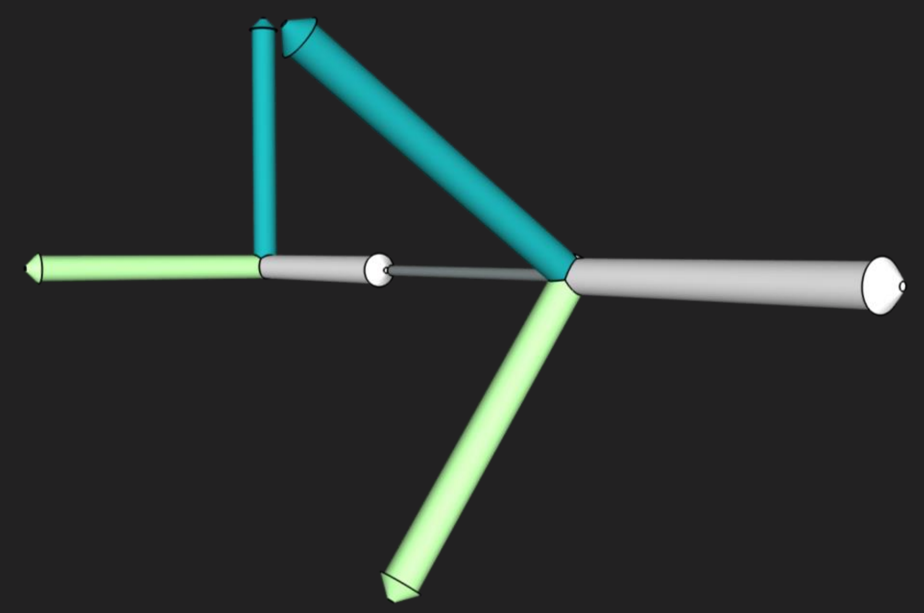
- Second and third revolute joints:

$$\overline{O(RF_i)O(RFa2_{i-1})} \cdot \overline{O(RF_i)O(RFa2_{i-1})} = 0$$
$$\overline{rot_i} \perp \overline{rot_{i-1,1}} = 0$$
$$\overline{rot_i} \perp \overline{rot_{i-1,2}} = 0$$

$$42\ variables = 6\,\frac{variables}{joint}\ x\ 7\ joints \quad \& \quad 35\ constraints = 5\ \frac{constraints}{joint}\ x\ 7\ joints$$

$$Dof\ = 42\ variables\ - 35\ constraints = 7$$





Constraint equations: Recursive approach

The recursive approach was derived by following the kinematic chain, starting from the ground and, since the system is an open chain mechanism, ending on the four different single nodes.

- First revolute joint: it introduces the first degree of freedom

$$RF_{RJ1} = Rotate(z, \theta_1)$$
$$RFa2_1 = RF_{RJ1}.Translate(L1,0,0)$$

- Second revolute joint: it introduces the second degree of freedom

$$RF_{RJ2} = Rotate(x, \theta_2)$$
$$RFa1_2 = RFa2_1.RF_{RJ2}$$
$$RFa2_2 = RFa1_2.Translate(0, L2,0)$$

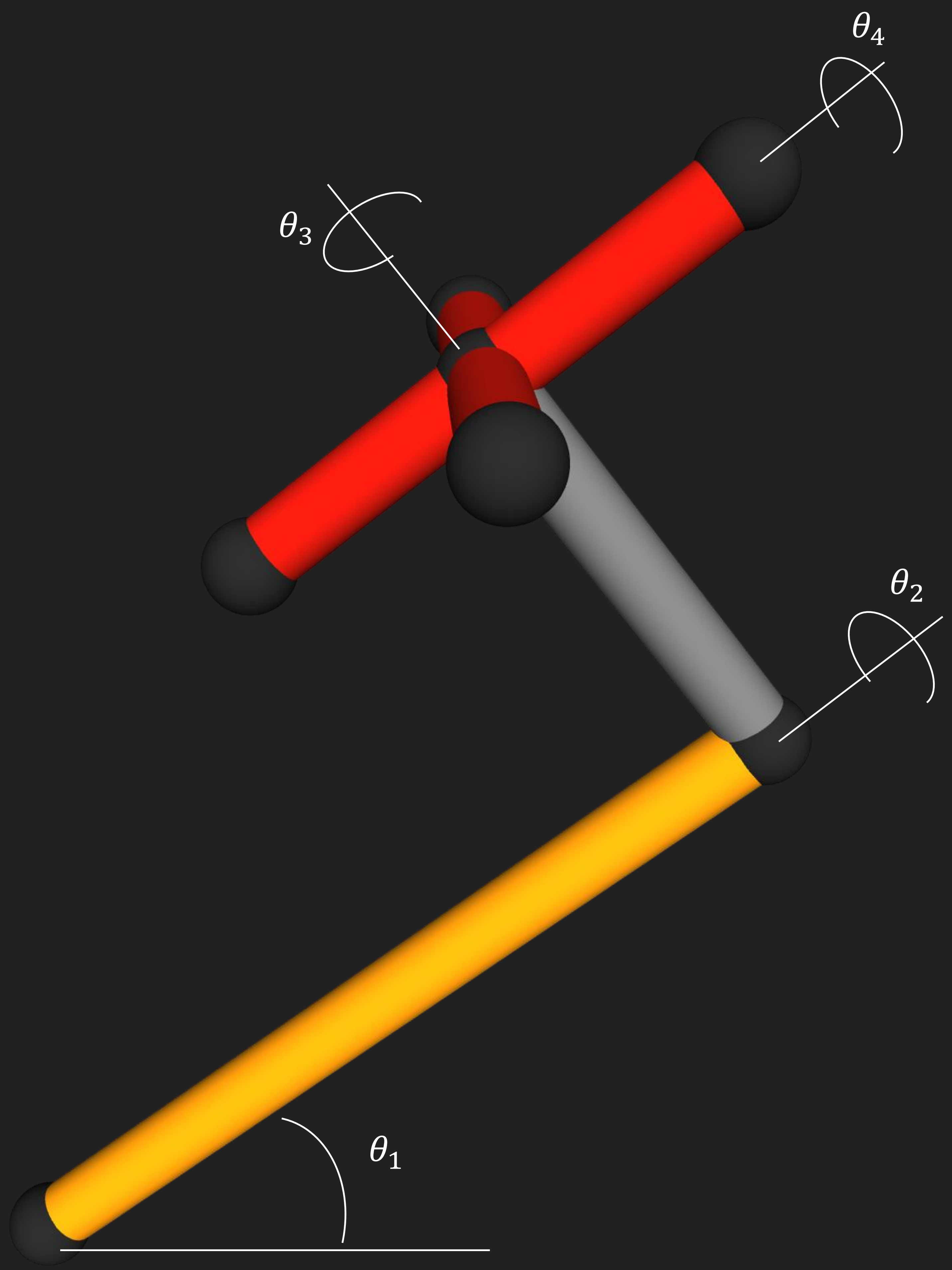
- Third revolute joint: it introduces the third degree of freedom

$$RF_{RJ3} = Rotate(y, \theta_3)$$
$$RFa1_3 = RFa2_2.RF_{RJ3}$$

$$RFa2_{3_i} = RFa1_3.Rotate(y, \beta_i).Translate(L3,0,0), where\ \beta_i = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\ and\ i \in [1,4]$$

- Finally, each set of seats introduces the last degree of freedom, as they rotate around their z-axis

$$RF_{RJ4} = Rotate(z, \theta_4)$$
$$RFa1_{4_i} = RFa2_{3_i}.RF_{RJ4}, where\ i\ \in [1,4]$$



Constraint equations: Natural approach

Five points were assigned to the ride: one at the origin of each revolute joint (Pa_i) and one more at the end effector (Pa_5). As constraint equations, the following were considered:

- Constitutive equations:

$$\|\overline{V_{i-1,i}}\| = \|\overline{Pa_{i-1}Pa_i}\| = L_{i-1}, \qquad where\ i \in [2,4]$$

- Orthogonality between the first link and the z-axis of the ground:

$$\overline{V_{12}}.\overline{k_{GND}} = 0$$

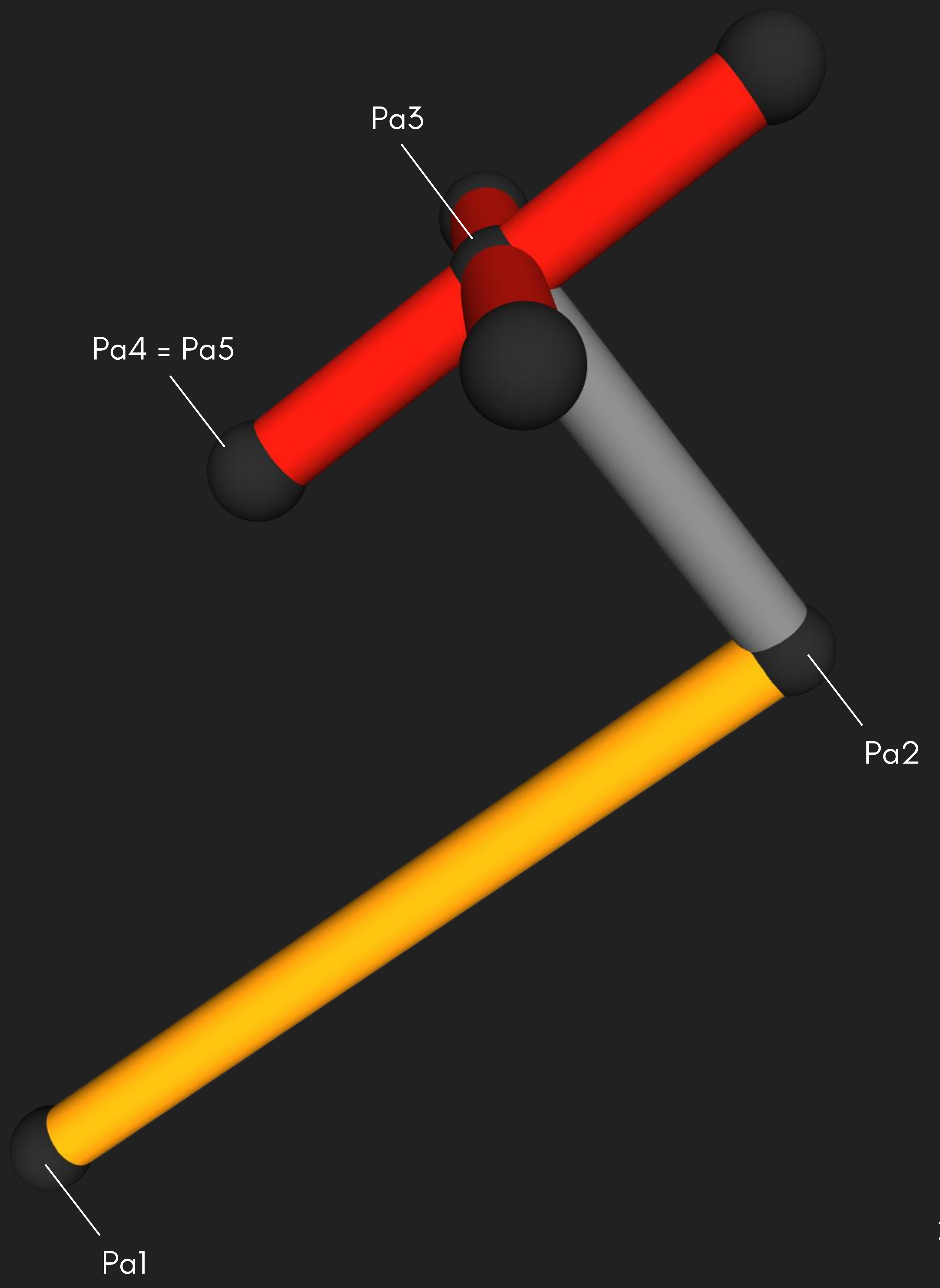
- Orthogonality between consecutive links:

$$\overline{V_{i-1\,i}}.\overline{V_{i\,i+1}} = 0, \qquad where\ i \in [2,4]$$

- Pa_5 corresponds with the previous point:

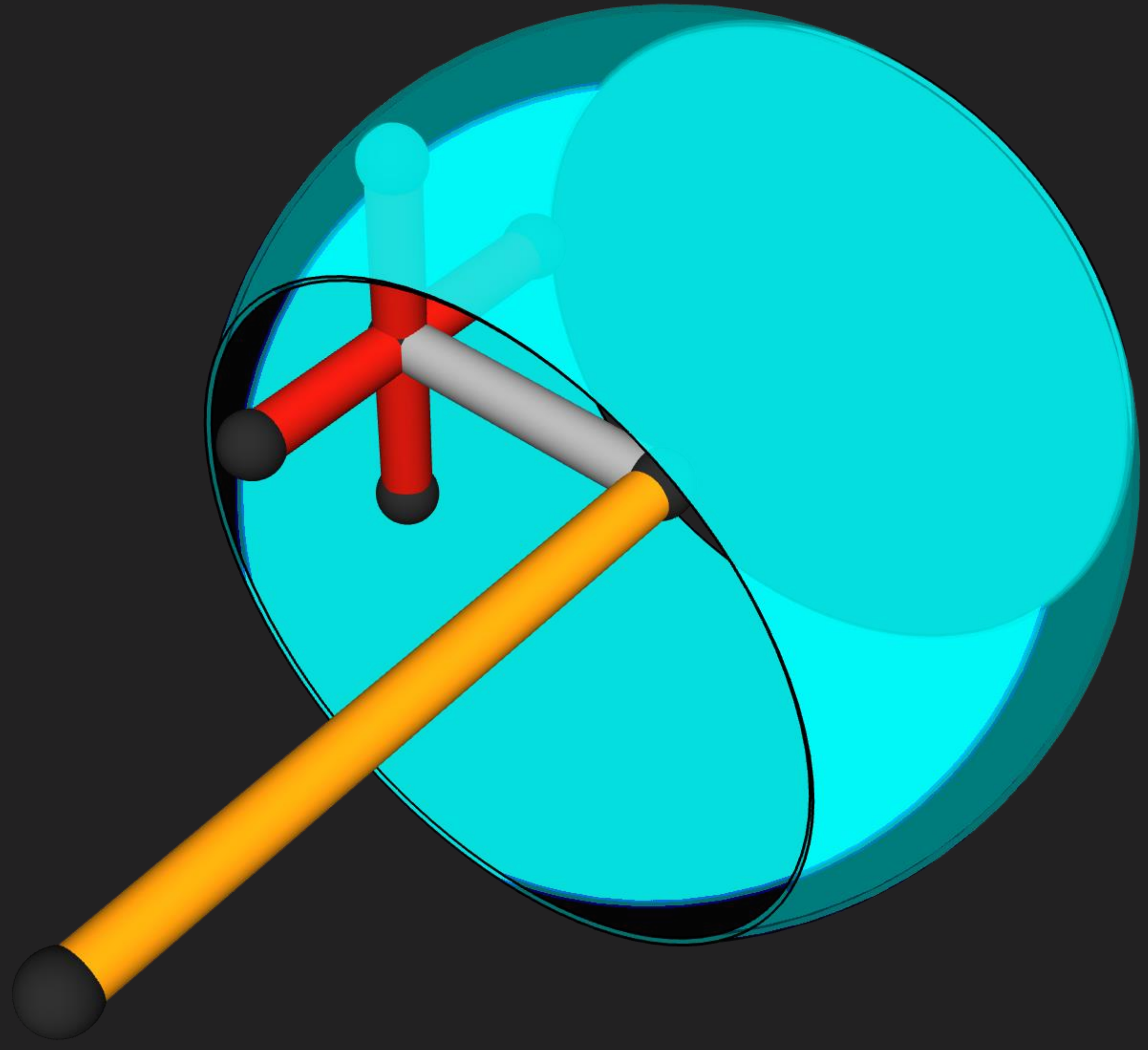
$$\|\overline{V_{45}}\| = \|\overline{Pa_4 Pa_5}\| = 0$$

According to the previous formulation, eight constraint equations were obtained and twelve variables were defined, giving as a result four degrees of freedom.

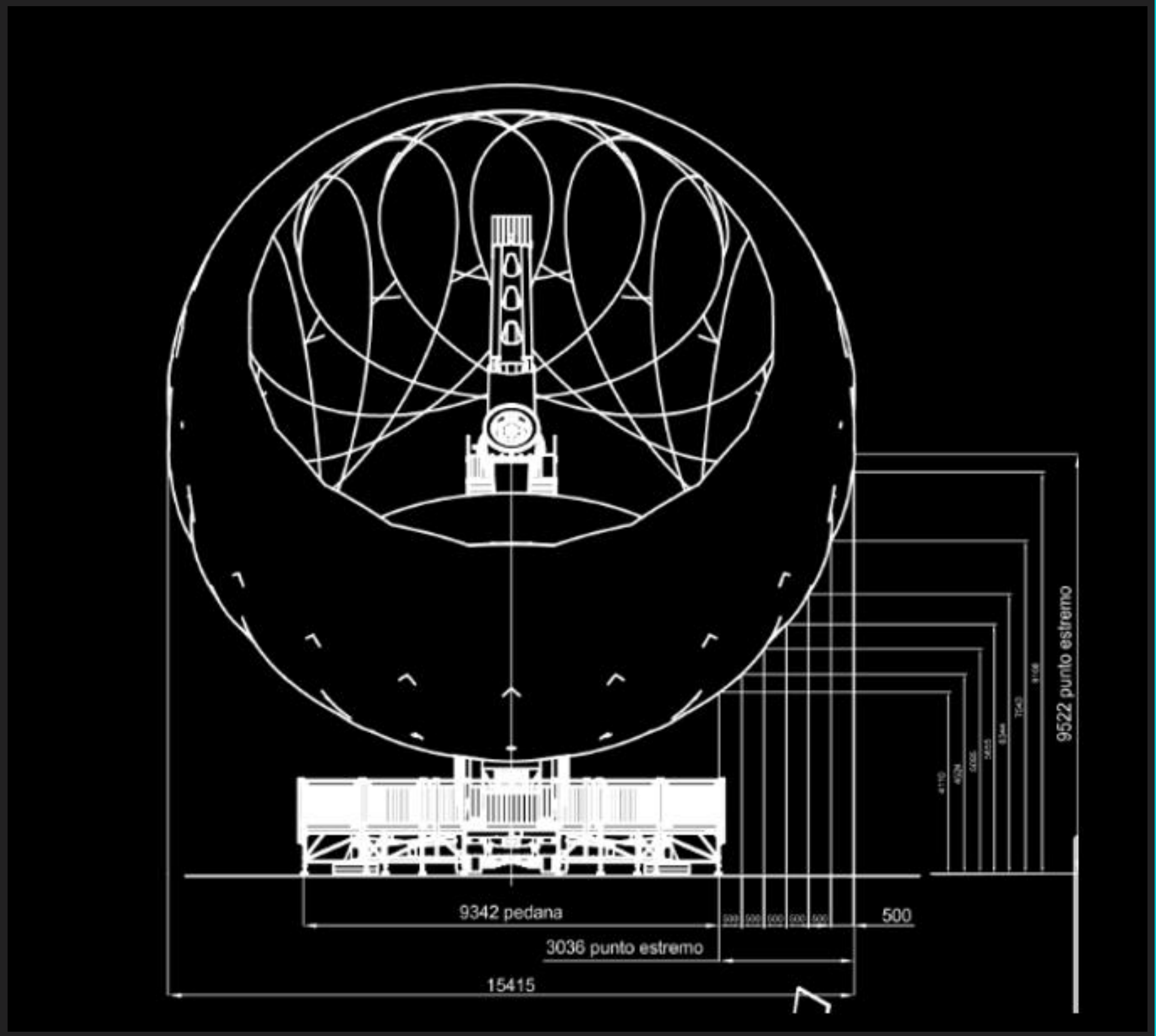


Position analysis & Workspace

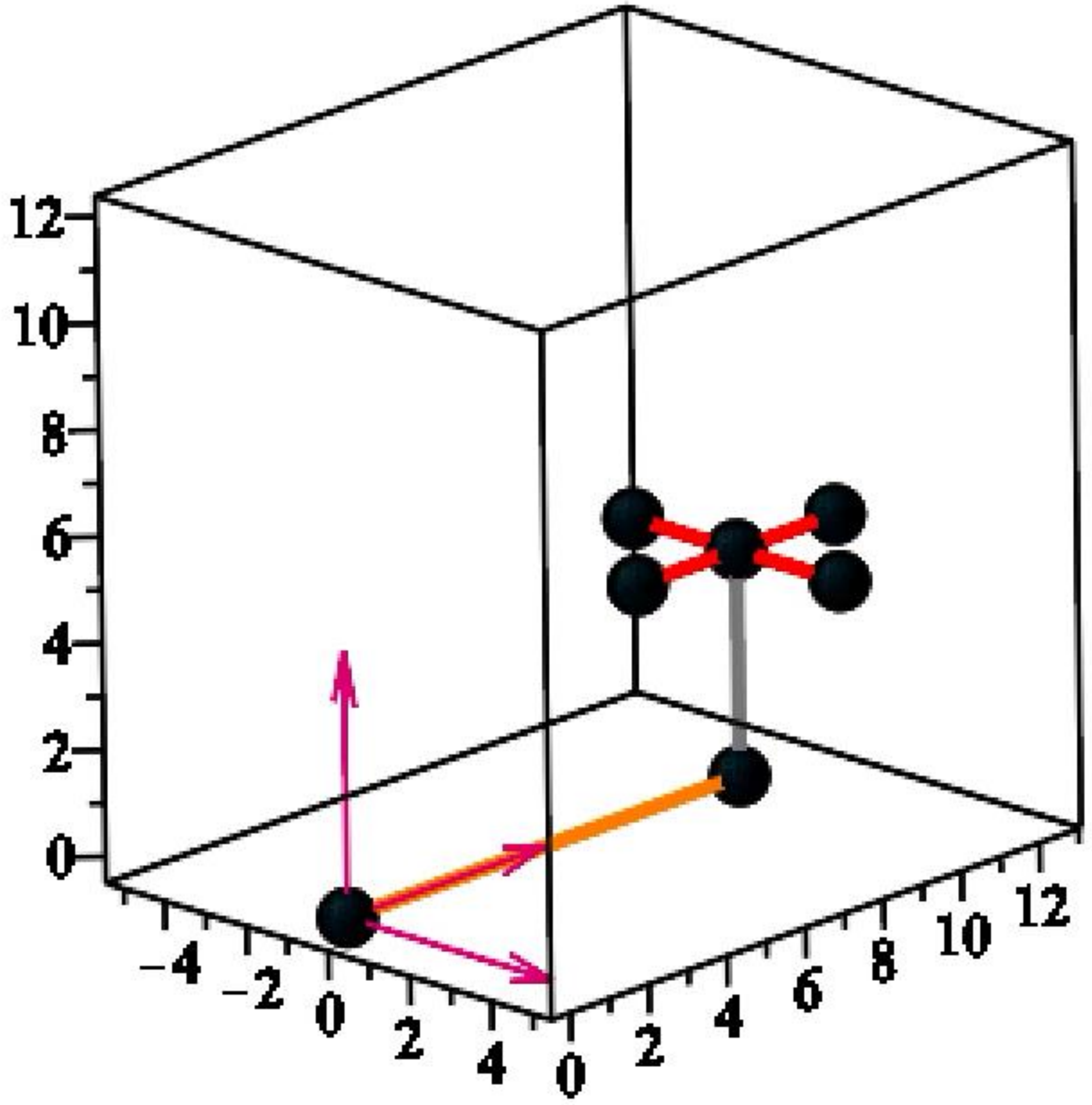
Workspace of System obtained using a CAD software



Target Workspace of Pegasus 16 provided by Technical Park



System Model Animation considering 4 dof



The workspace of the system was obtained analytically by plotting the position of the four sets of seats for all values of θ_2 and θ_3 (rotation about the second and third revolute joints) in Matlab.

Velocity and acceleration analysis: global approach

Global Approach:

For the global approach, in order to compute the velocities of the dependent coordinates (q_D) given the independent coordinates (q_I), the velocity ratio matrix (τ) was first computed using the constraint matrix (ϕ) as follows:

$$\tau = -\frac{d\phi}{dq_D}^{-1}\frac{d\phi}{dq_I}$$

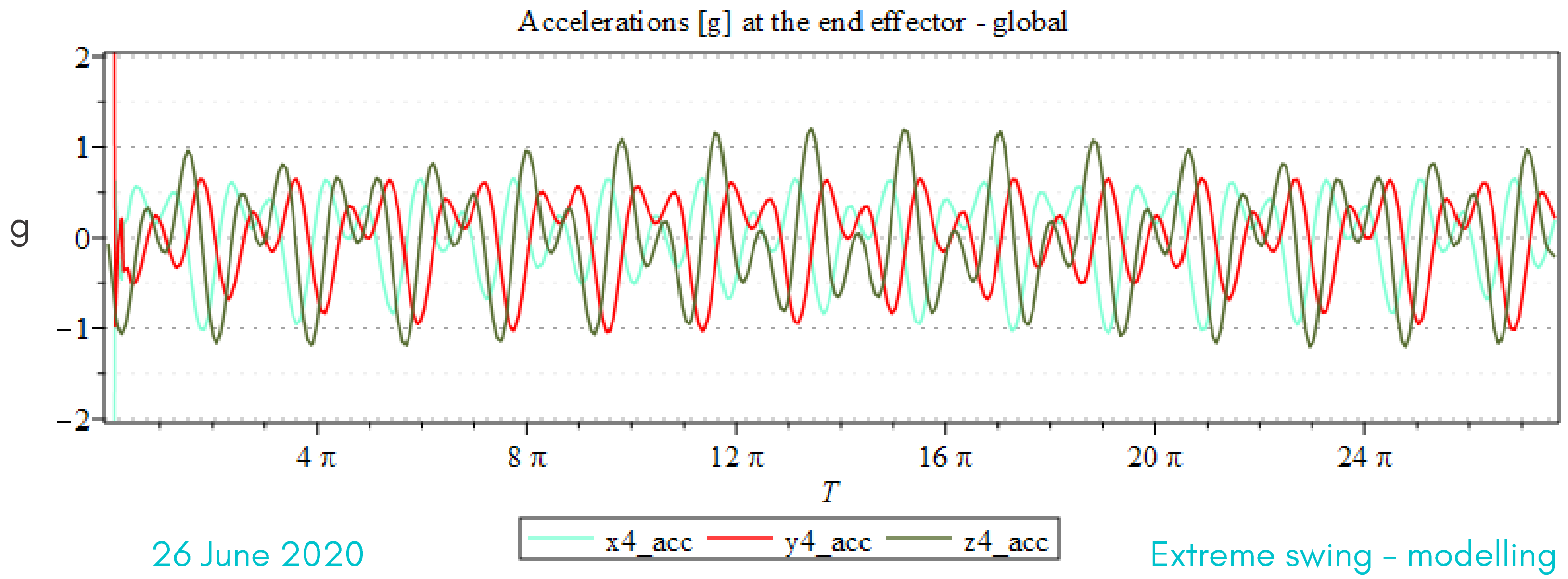
The velocities were then computed as:

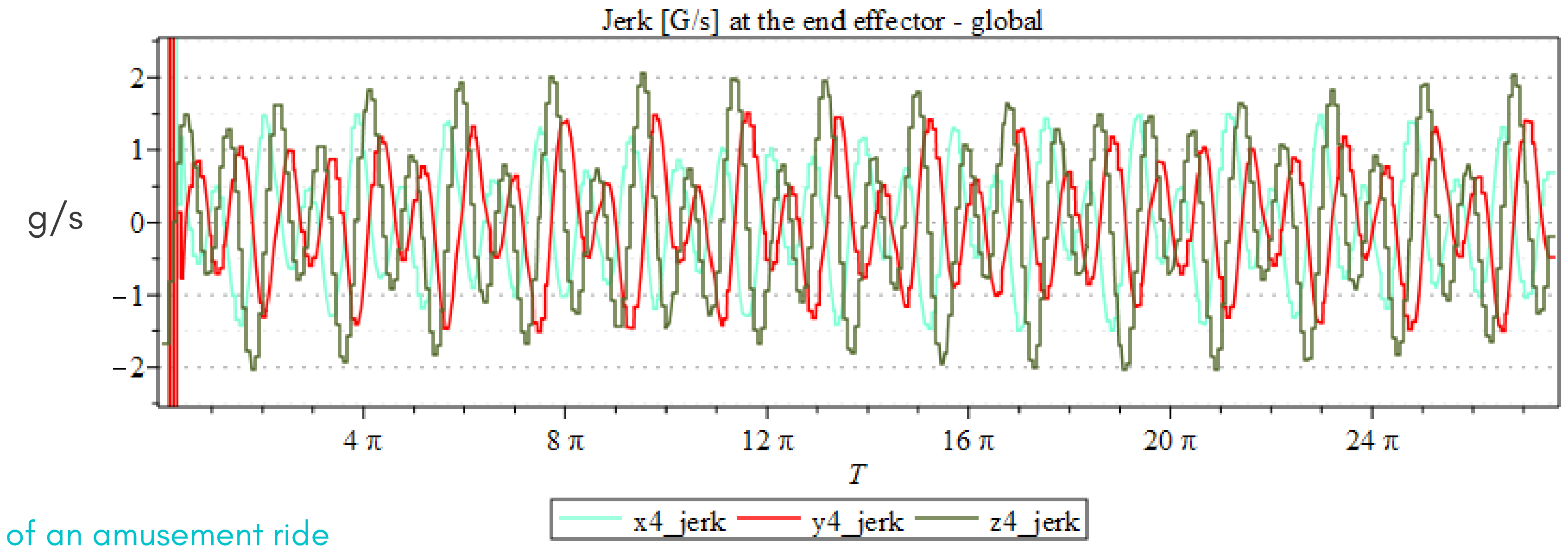
$$\dot{q_D} = \tau * \frac{d}{dt} q_I$$

The accelerations of the dependent coordinates were then be computed as:

$$\ddot{q_D} = \frac{d}{dt} \dot{q_D}$$

In order to obtain plots of the accelerations and jerks, both ϕ_2 and θ_3 (independent coordinates for the global approach) were modelled as rehonomous constraints, where the rotation angles $\omega_2$and ω_3 were considered constant ($\phi_2(t) = \omega_2 t,\ \theta_3(t) = \omega_3 t$). Time was taken to be independent, and, using the augmented constraint matrix, a finite displacement analysis was conducted on maple. The profiles were interpolated using splines.

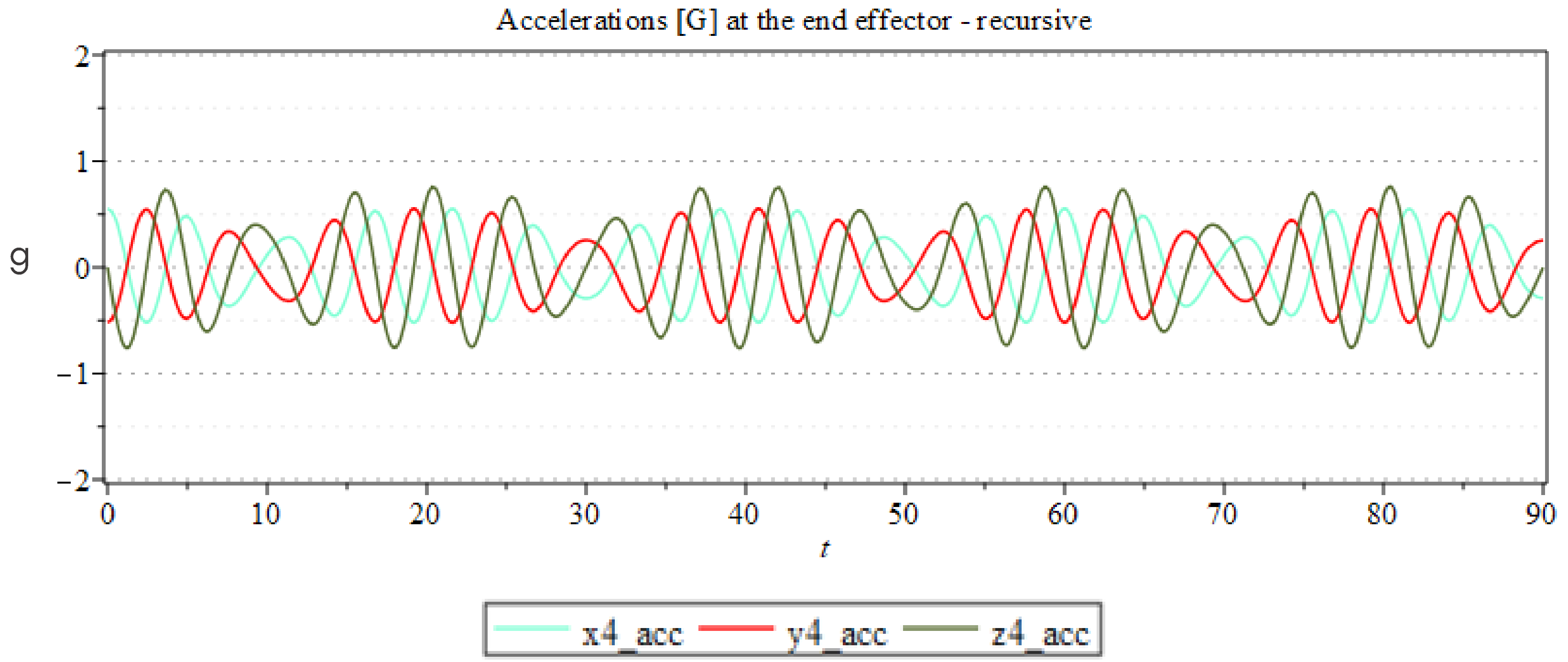


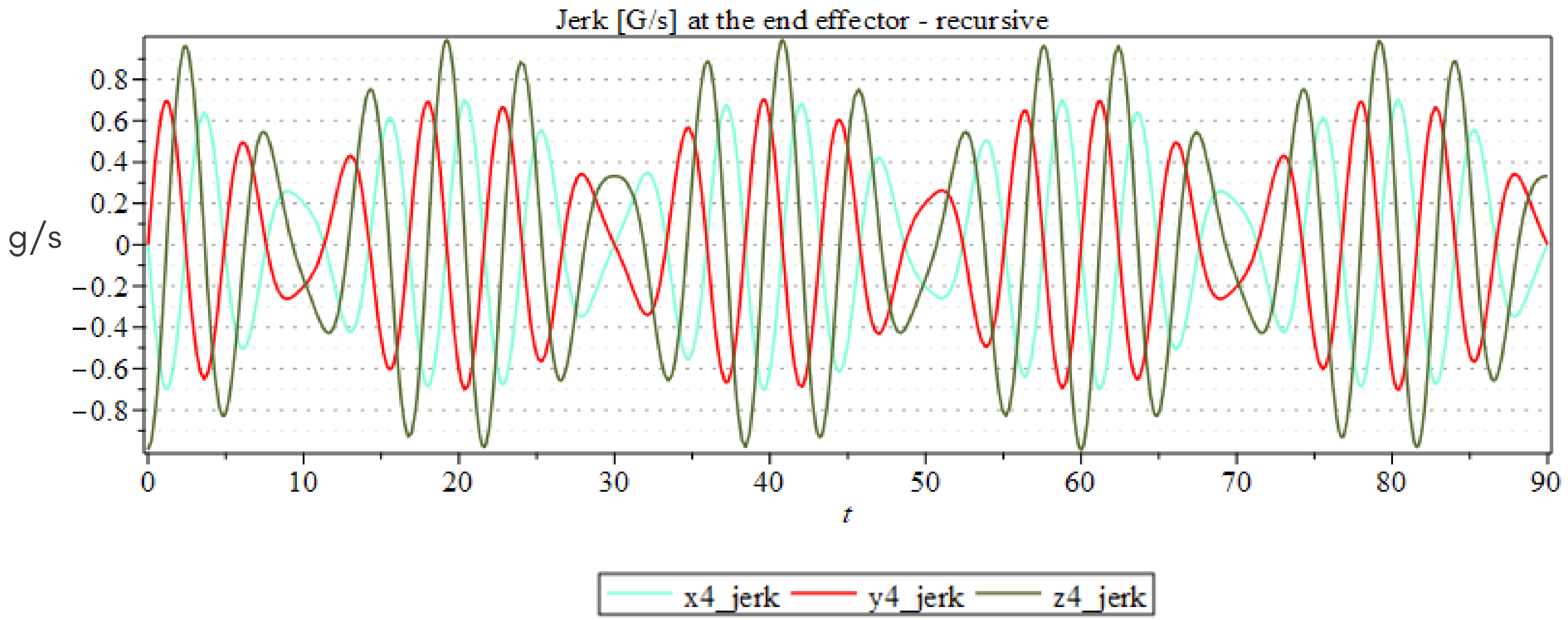


Velocity and acceleration analysis: recursive approach

Recursive Approach:

The recursive formulation of the system had no constraint equations. So, the position of one of the sets of seats (the origin of $RFa2_{3_1}$ projected in ground) was differentiated directly to obtain the velocity of the person, and the same method was used to compute the acceleration and the jerk.





Evaluation of performance indices

The fun equation was calculated for 6 different possible sets of parameter values **using the recursive formulation.**

- Solution 3: higher fun equation but larger ω_2. The lenghts are not modified and it requires a lower ω_3.
- Solution 6: shorter lengths of L1 and L2 and same rotation speeds as the target.

		Target	Solution 1		Solution 2		Solution 3		Solution 4		Solution 5		Solution 6	
Specification	Weight	Value	Value	Score	Value	Score	Value	Score	Value	Score	Value	Score	Value	Score
Fun	5	30.42	29.57	1	25.02	1	32.74	3	35.87	3	28.82	1	30.44	2
Safety	∞	Pass	Pass	0	Pass	0	Pass	0	Not pass	-1	Pass	0	Pass	0
Power** Consumption	4	N/A**		N/A		N/A		N/A		N/A		N/A		N/A
L_1	3	10.03m*	10.03m	2	10.03m	2	10.03m	2	20.03 m	1	10.03 m	2	7 m	3
L_2	1	4.25m*	4.25m	2	4.25m	2	4.25 m	2	4.25 m	2	3.247 m	3	4.25 m	2
L_3	3	2.53m*	2.53m	2	2.53m	2	2.53 m	2	10 m	1	2.00	3	2.3 m	3
ω_2	2	11 rpm*	9 rpm	3	15 rpm	1	12 rpm	1	11 rpm	2	11 rpm	2	11 rpm	2
ω_3	3	10 rpm*	5 rpm	3	13 rpm	1	6 rpm	3	10 rpm	2	10 rpm	2	10 rpm	2
TOTAL SCORE			34		24		40		33		33		40	

*The values have been provided by "Technical Park", the Pegasus 16 producers.
** Power consumption will be evaluated with the dynamic analysis.

Monte Carlo sensitivity analysis and optimization

- A Monte Carlo sensitivity analysis was conducted to investigate the relative effects different parameters had on our fun equation. We evaluated the effects of the angular rotation speed of the second and third motors (ω_2 and ω_3) as well as the lengths of links 1, 2 and 3. We investigated 10 values sampled from a uniform distribution around each parameter, and measured the fun equation at each of these values. We computed and compared the standard deviation of each of these sets of values. The parameter which gave the highest standard deviation thus had the largest effect on our fun equation.
- The parameters to which the fun equation was the most sensitive depend on the chosen solution and are highlighted in the table below.

	Baseline Solution		Solution 6		Solution 3	
Parameter	Uniform Distribution Interval	Standard Deviation of Results	Uniform Distribution Interval	Standard Deviation of Results	Uniform Distribution Interval	Standard Deviation of Results
ω_2 [rpm]	[7,16]	0.45658	[6,16]	0.69145	[7,17]	1.44545
ω_3 [rpm]	[5,15]	**1.24896**	[5,15]	0.95808	[1,11]	0.44066
L_1 [m]	[8.53,11.53]	0	[5.5,8,5]	0	[8.53,11.53]	0
L_2 [m]	[3,5.5]	0.54849	[2.75,5.75]	**0.98228**	[3,5.5]	**1.72545**
L_3 [m]	[1.530,3.530]	0.11592	[0.8,3.8]	0.28594	[1.530,3.530]	0.77580

Monte Carlo sensitivity analysis and optimization

The optimization problem was solved using Maple's NLP Solver. The function being maximized is a simplified version of the fun equation, $\alpha_1 acceleration$ – $\alpha_3 jerk$ (computed in all three directions for the duration of the ride, 90 seconds), where $\alpha_1 = 0.3$ and $\alpha_3 = 0.2$. The idea was to optimize a part of the fun equation which was easier to compute. The function was optimized with respect to the two parameters (found through the Monte Carlo sensitivity analysis) that influence the fun equation the most.

	Optimization Parameter		Optimal Value	Parameter value	Fun Equation
Solution 3	L_2 [m]	x	0.94424	5.75	35.67016584
		y	-0.00351	2.75	30.39157529
		z	3.24731	5.75	35.67016584
	ω_2 [rpm]	x	4.56340	13.91216	32.187025
		y	7.01940	17.00	32.776183
		z	4.53744	12.04722	32.645091
Solution 6	L_2 [m]	x	0.95775	2.75	29.187355
		y	0.88142	2.75	29.187355
		z	5.84325	5.75	32.226819
	ω_3 [rpm]	x	4.67731	11.02237	29.622515
		y	3.85659	9.06875	30.332861
		z	6.14721	12.53866	28.922781

The highest fun equation is reached using solution 3 and fixing L_2 to be 5.75m. Solution 6 was also the highest when using L_2 as 5.75m.

Dynamic Analysis

DYNAMIC MODEL

INVERSE DYNAMICS

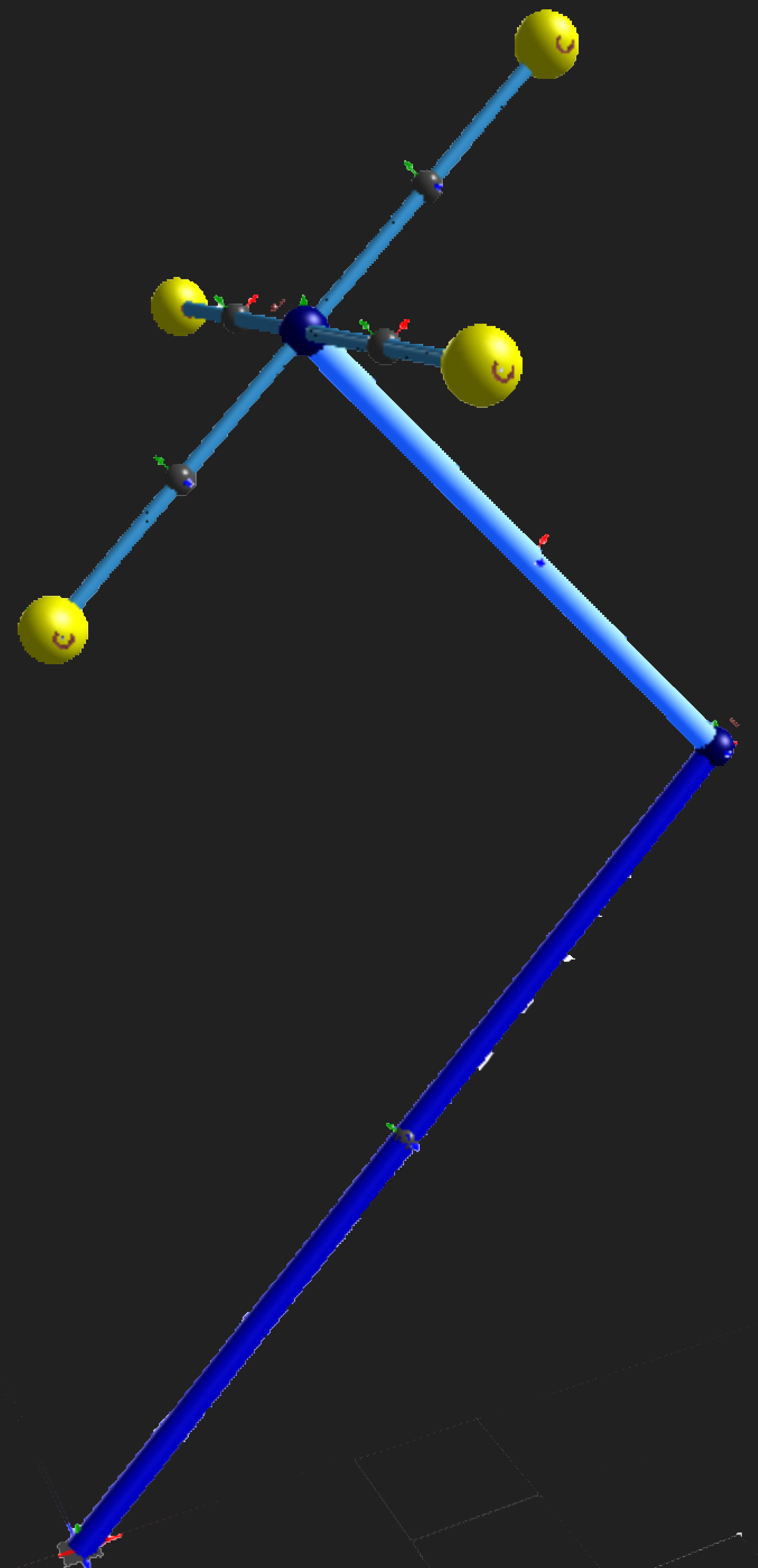
NUMERICAL SOLUTION COMPARISON

MAPLESIM MODEL

PERFORMANCE EVALUATION

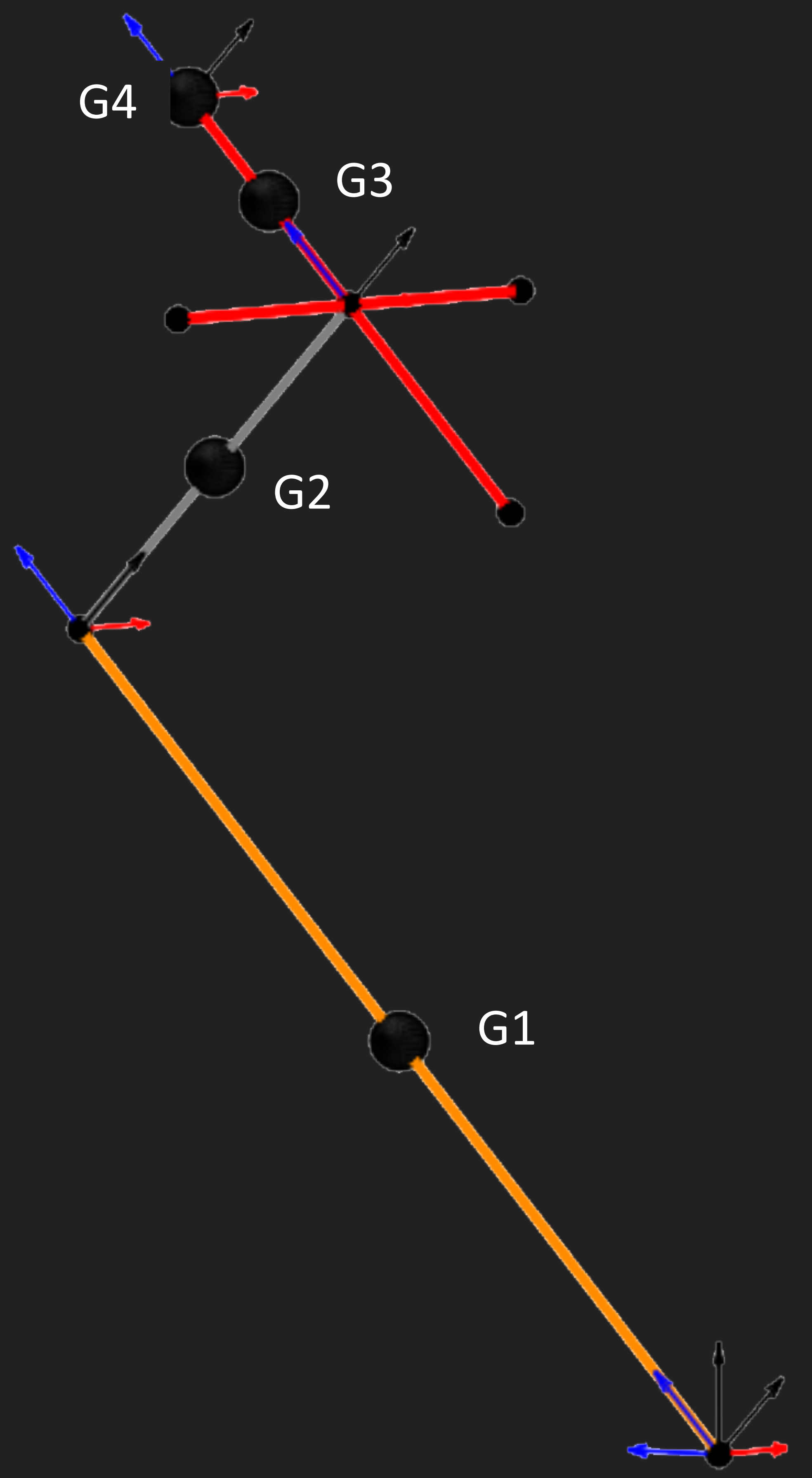
System Model and Overview of Analysis

- Recursive formulation conducted during kinematic analysis uses a minimal set of coordinates.
  - System is Open Chain
  - Obtained an ODE
- Rehonomous constraints were added to obtain a DAE.
- Torque profiles of the system were evaluated using inverse dynamics
- DAE equations were solved using
  - Maple (numeric solution, index reduction)
  - MATLAB (Projection method, Baumgarte)
- Dynamic performance indices were evaluated in order to select the solution that provides the most fun (power consumption).



Dynamic Model: Considerations

- Center of mass in the middle of each link
- Four bodies are considered to be bars
- Lifting arm is not actuated and orientation is fixed at at $\theta_1(t) = \frac{\pi}{4}$
- The second and the third link motors are actuated through $\boldsymbol{T_1}$ and $\boldsymbol{T_2}$ torques and rotate respectively with velocities ω_2 and ω_3
- Motion profiles of joints 2 and 3 constrained by $\Phi = \begin{cases} \theta_2(t) - \omega_2 t \\ \theta_3(t) - \omega_3 t \end{cases}$
- The seats are free to rotate and subject only to inertial forces.



Dynamic Model: Derivation

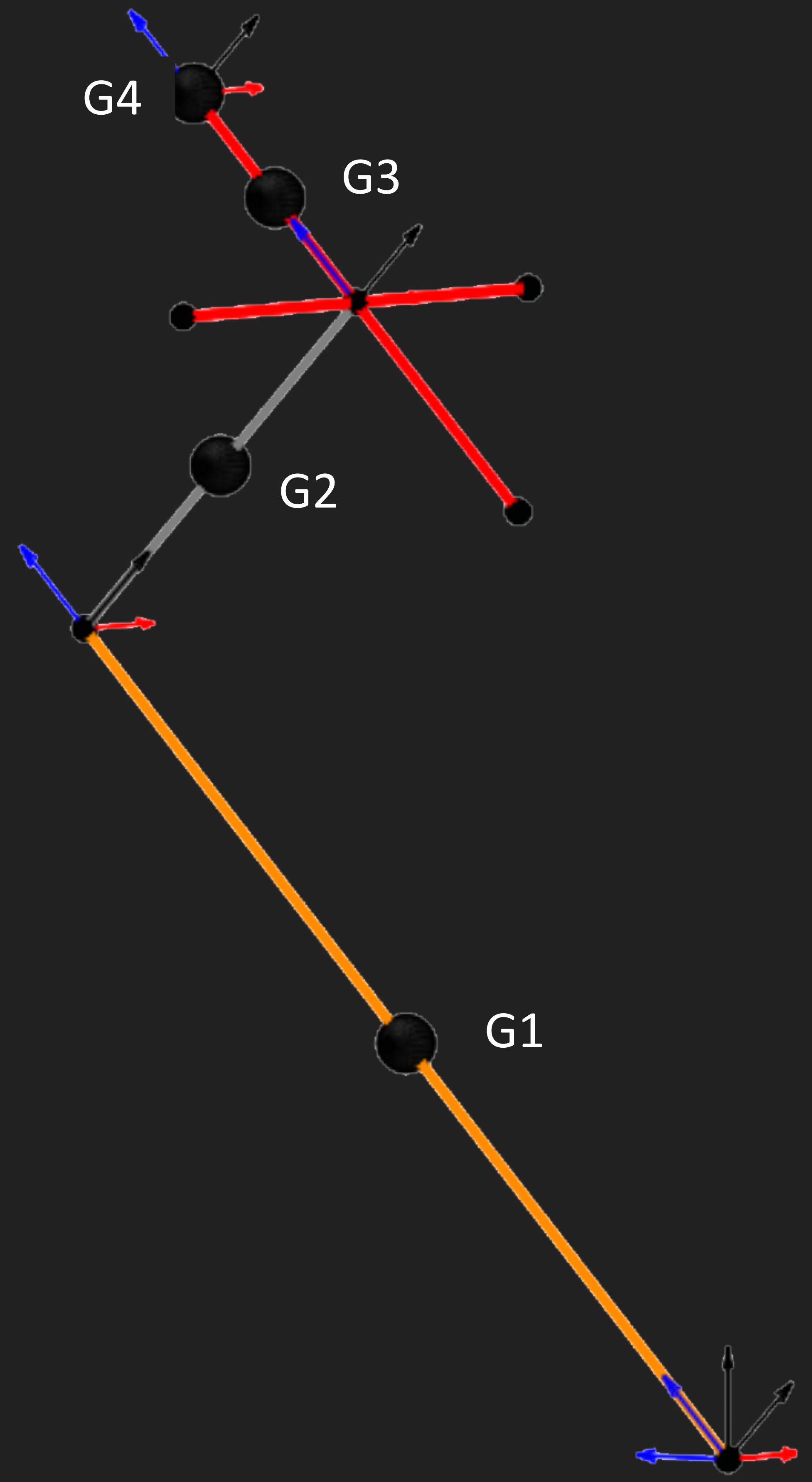
- **Lagrange approach** considering the four bodies, the two actuation torques, and a damper at the seats:

$$\frac{\partial L}{\partial \bar{q}_i} - \frac{d}{dt}\frac{\partial L}{\partial \dot{\bar{q}}_i} = 0$$

$$L = K - H \text{ and } \bar{q} = [\theta_2(t), \theta_3(t), \theta_4(t)]$$

- The index 1 DAE which describes the sytem is given by:

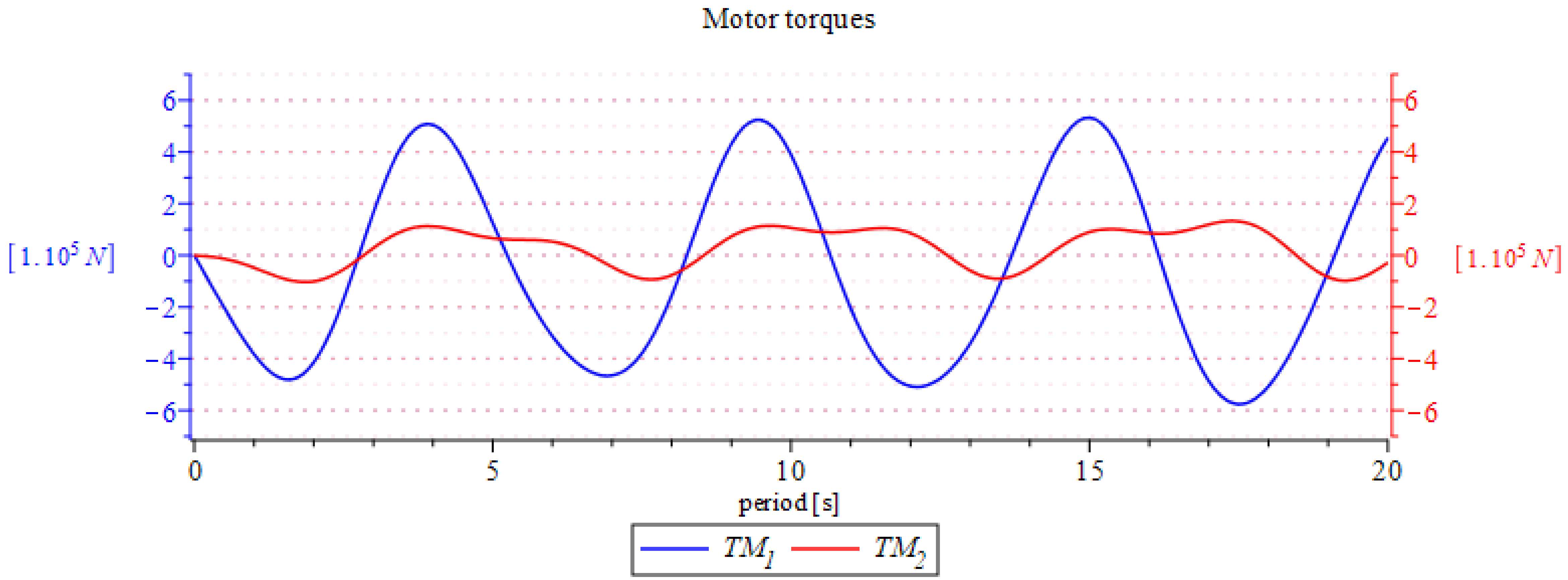
$$Pegasus_{DAE} = \begin{cases} \ddot{\theta}_2 = f_1(\bar{q}_i, \dot{\bar{q}}_i\, t) \\ \ddot{\theta}_3 = f_2(\bar{q}_i, \dot{\bar{q}}_i\, t) \\ \ddot{\theta}_4 = f_3(\bar{q}_i, \dot{\bar{q}}_i\, t) \\ \theta_2(t) - \omega_2 t \\ \theta_3(t) - \omega_3 t \end{cases}$$



Inverse Dynamics

- The torque profiles that achieve the desired motion defined in the kinematic analysis were solved for using this system, and the variables $\bar{q}_{inv} = \left[\ddot{\theta}_2(t), \ddot{\theta}_3(t), \ddot{\theta}_4(t), T_1(t), T_2(t)\right]$:

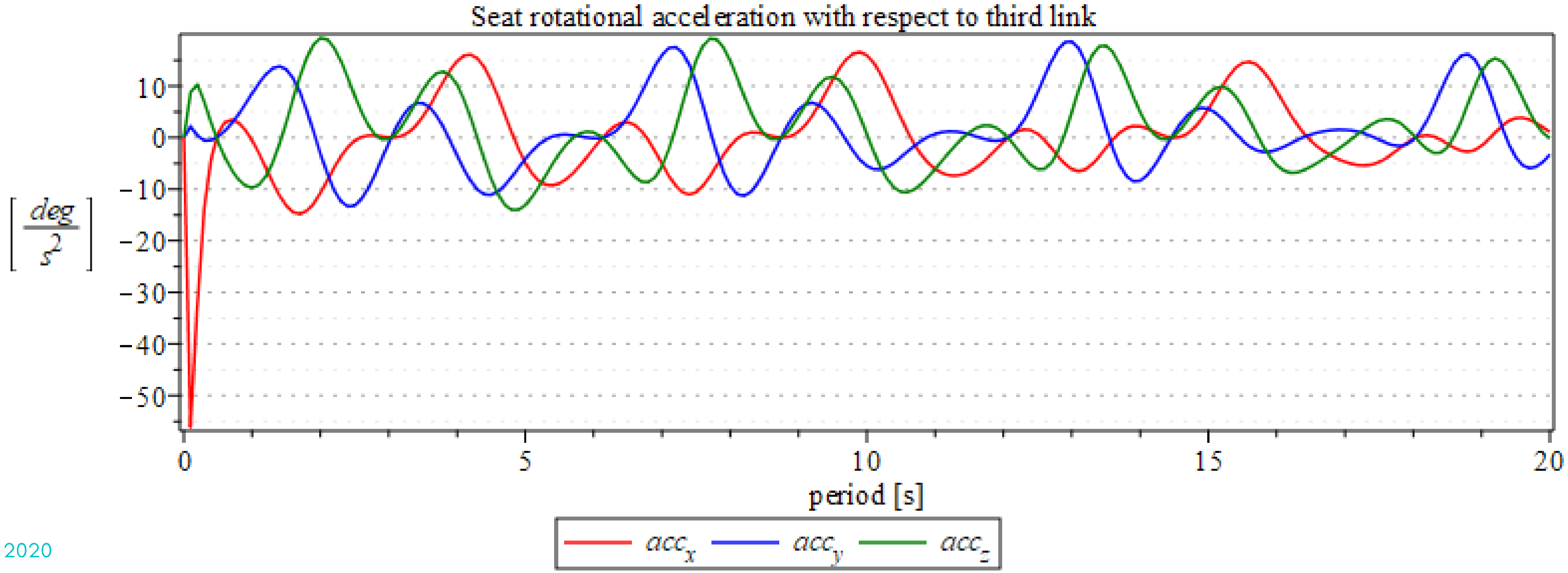
The torque profiles obtained are shown below:



Dynamic Numerical Solution (Maple)

With initial conditions shown below, the numerical solution was obtained in maple:

$$ic = \begin{cases} \theta_2(0) = 0 \\ \theta_3(0) = 0 \\ \theta_4(0) = 0 \\ \dot{\theta}_2(0) = \omega_2 \\ \dot{\theta}_3(0) = \omega_3 \\ \dot{\theta}_4(0) = 0 \end{cases}$$



Dynamic Solution Comparison

- To use the MATLAB DAE index 3 classes, the approach with Lagrange multipliers λ_i was considered.
  - The multipliers act like the torques of our system.

- With this approach we obtained an index-3 DAE:

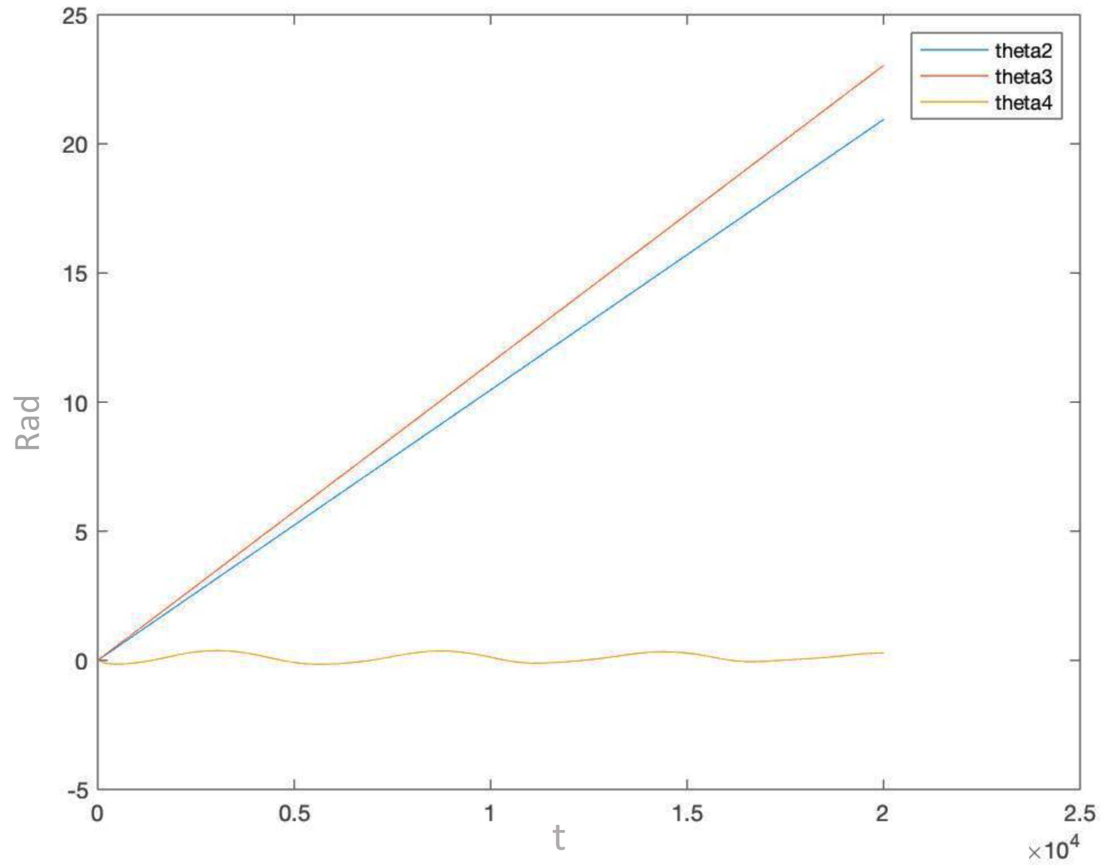
$$\boldsymbol{M} \cdot \ddot{\bar{q}} + \boldsymbol{\Phi_q}^T \bar{\lambda} = f(t, \bar{q}, \dot{\bar{q}})$$
$$\boldsymbol{\Phi}(t, \bar{q}) = 0$$

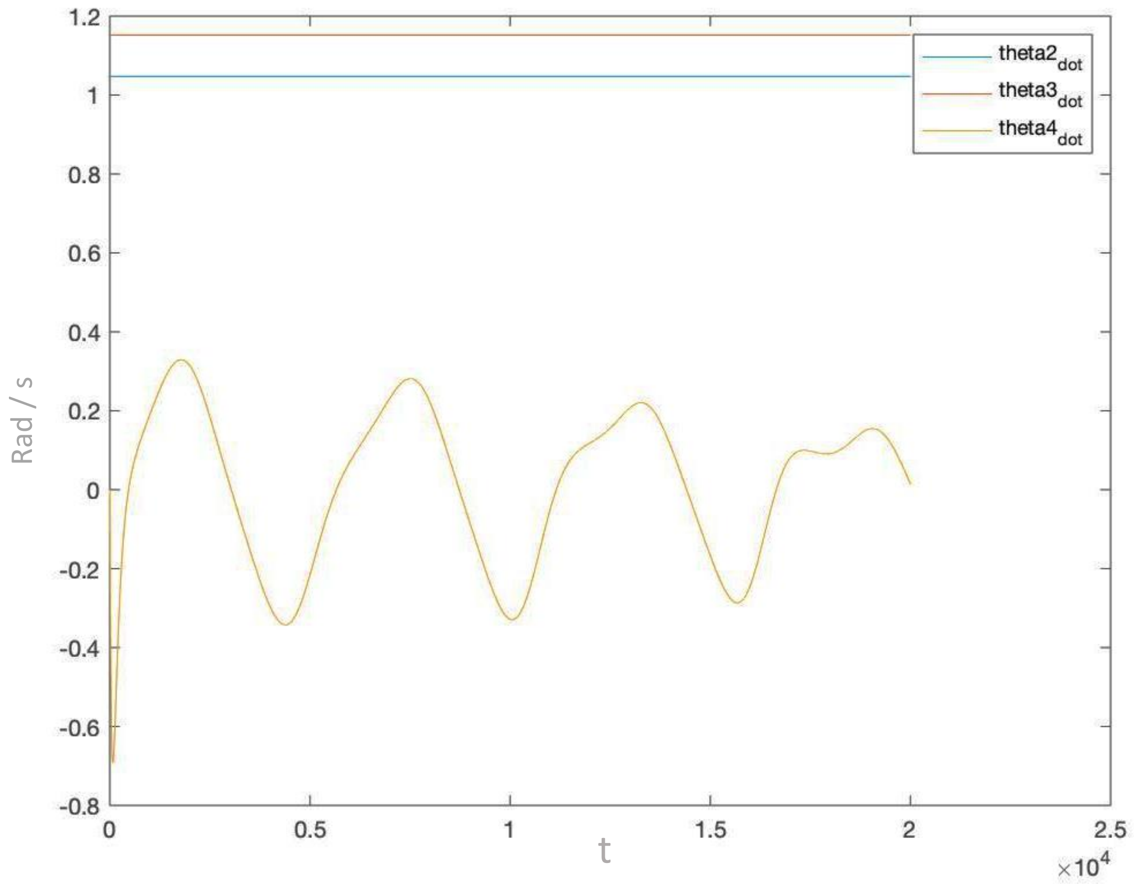
- The necessary inputs for the MATLAB class were then taken from maple.

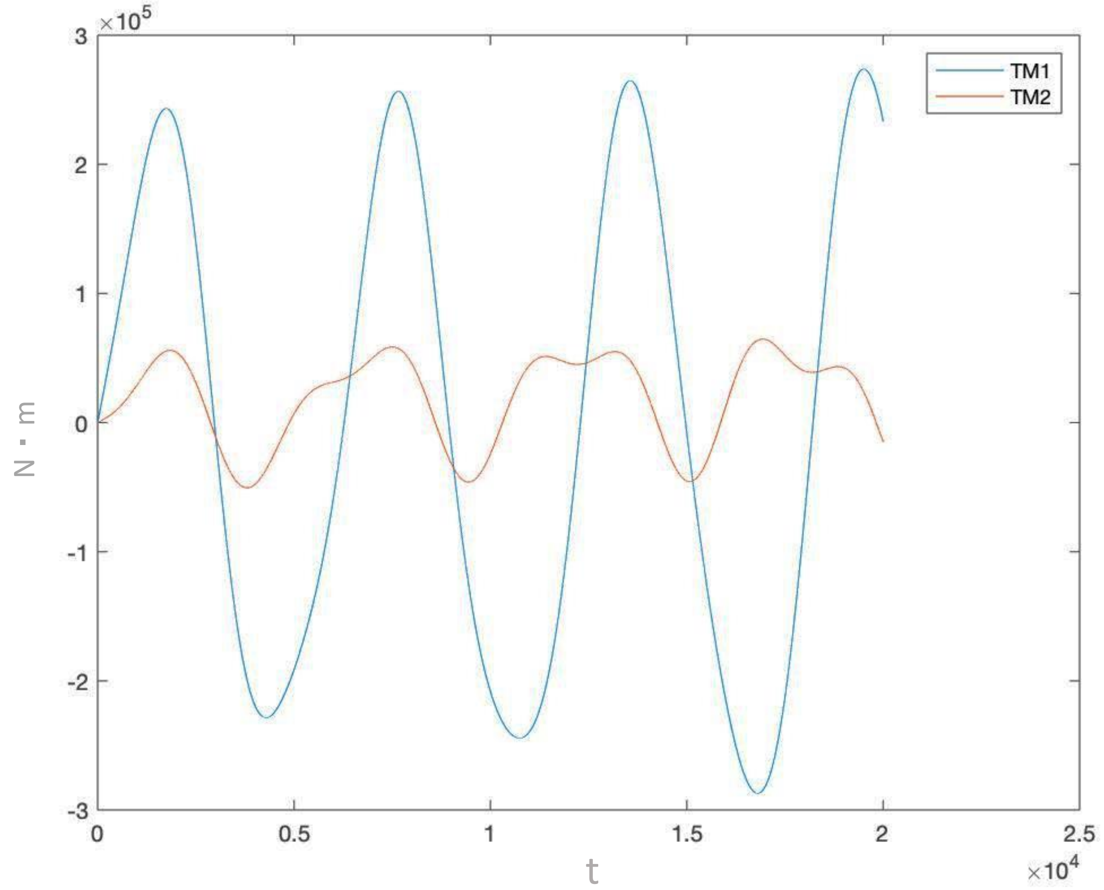
- The MATLAB **baumgarte** and **projection** solvers were used to solve the DAE.

Baumgarte Method

- Using the baumgarte method, **the index was reduced and a stabilized ODE was created** using the coefficients $\eta = 0.5$, and $\omega = 1$.

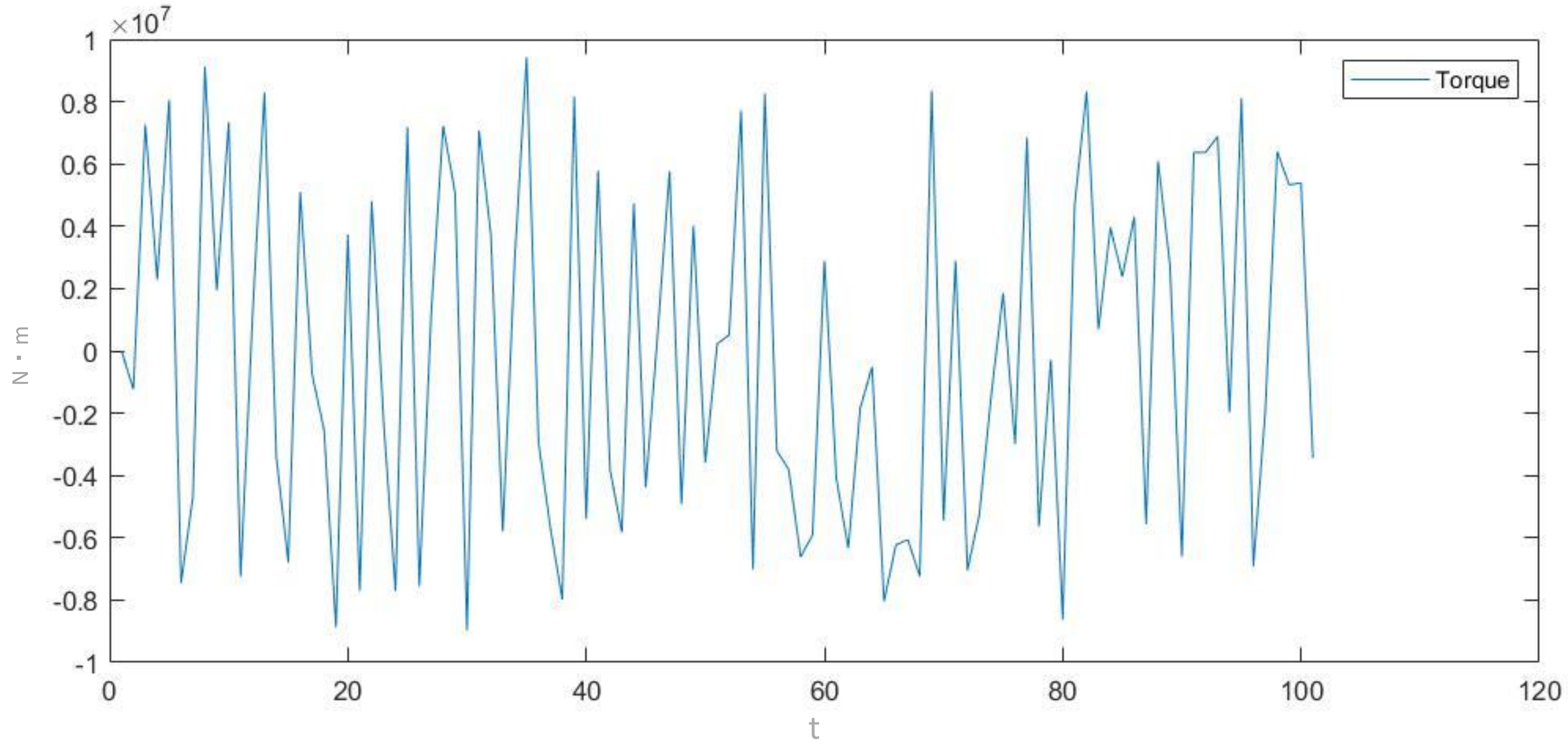




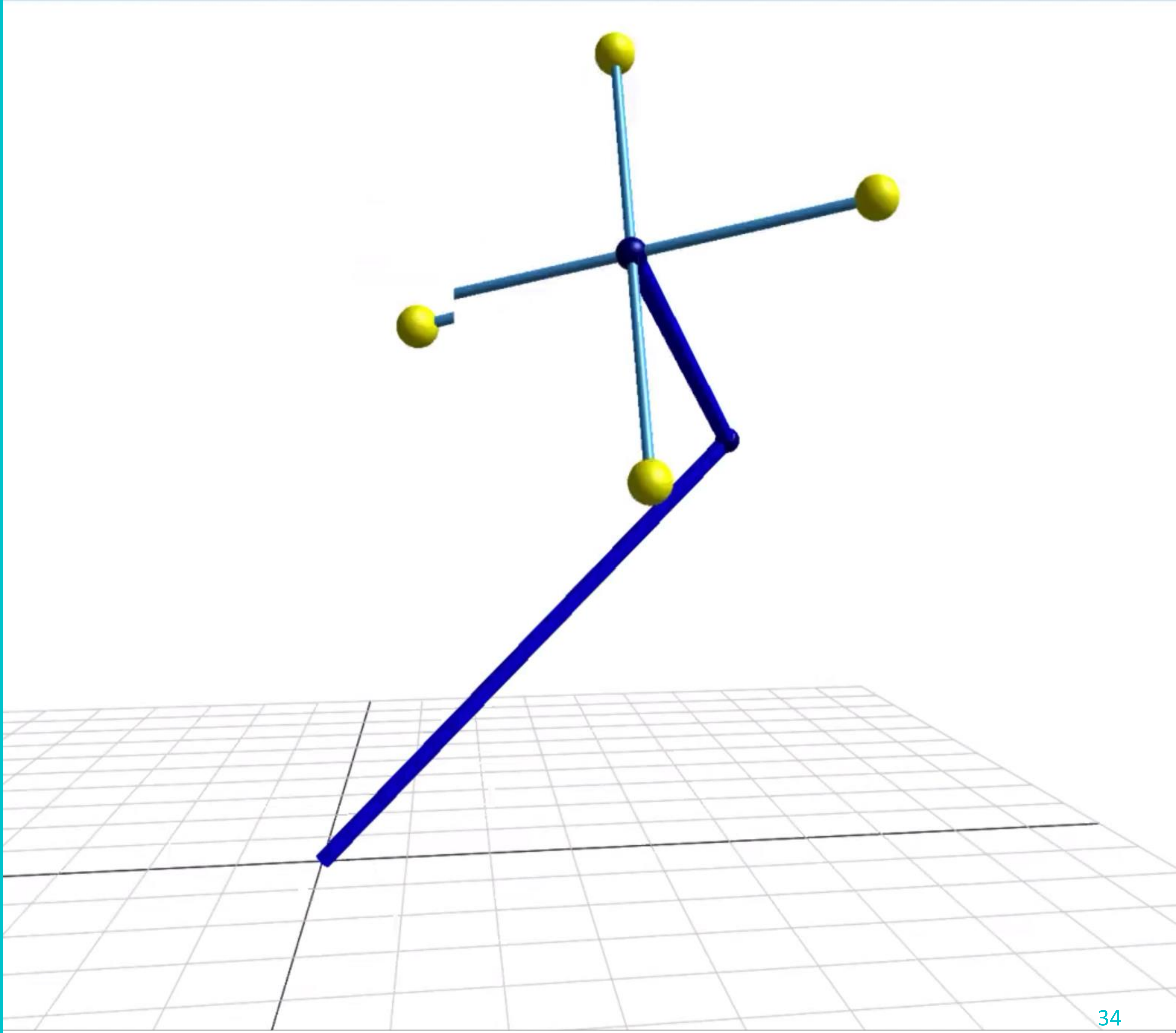


Projection Method

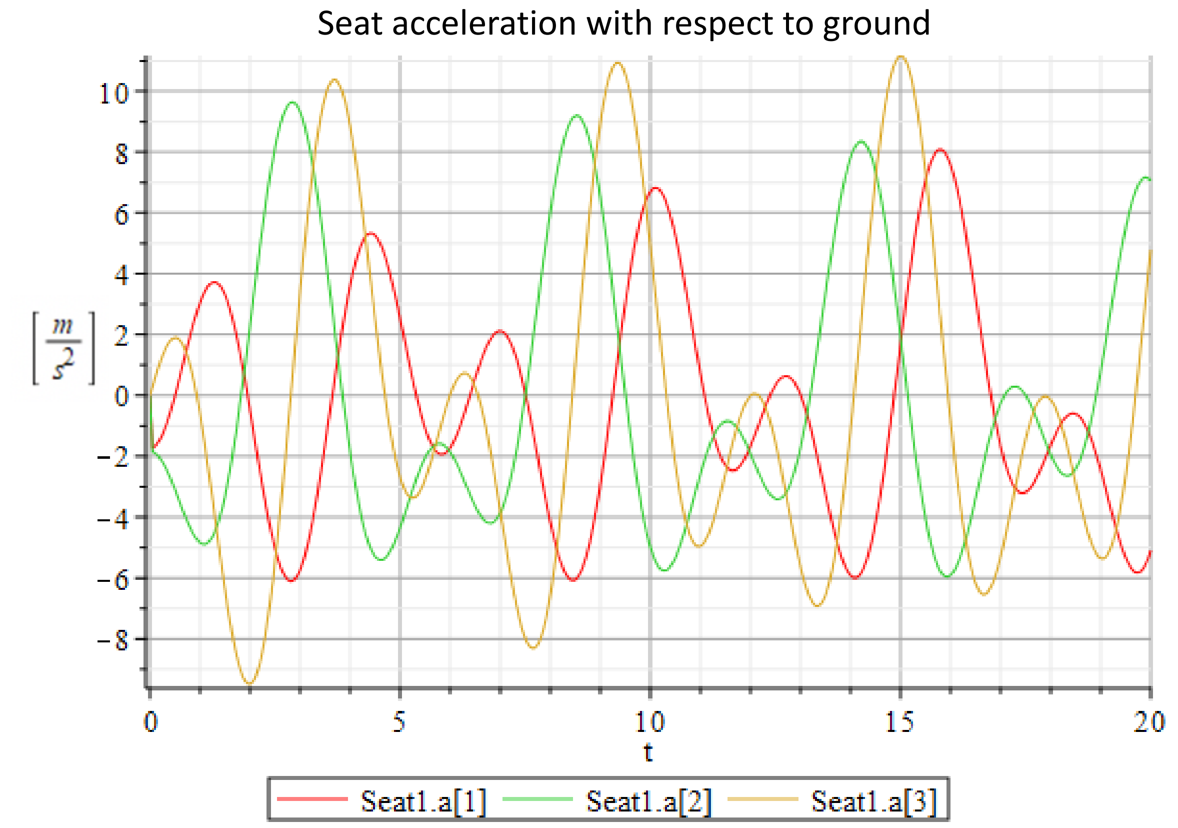
- Matlab library does not allow for lambda values to be passed when computing the **gforce** matrix.
- The projection solver did not work.
- The complete projection solver solution is left as a future step for this system.

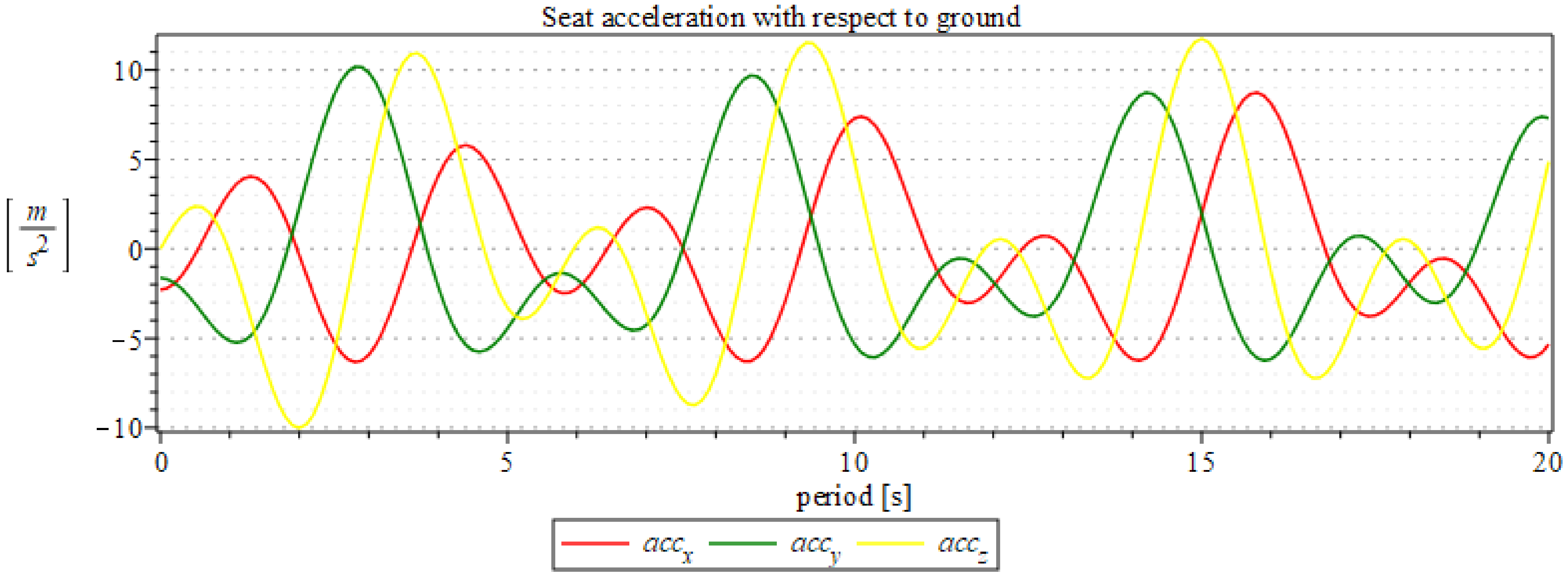


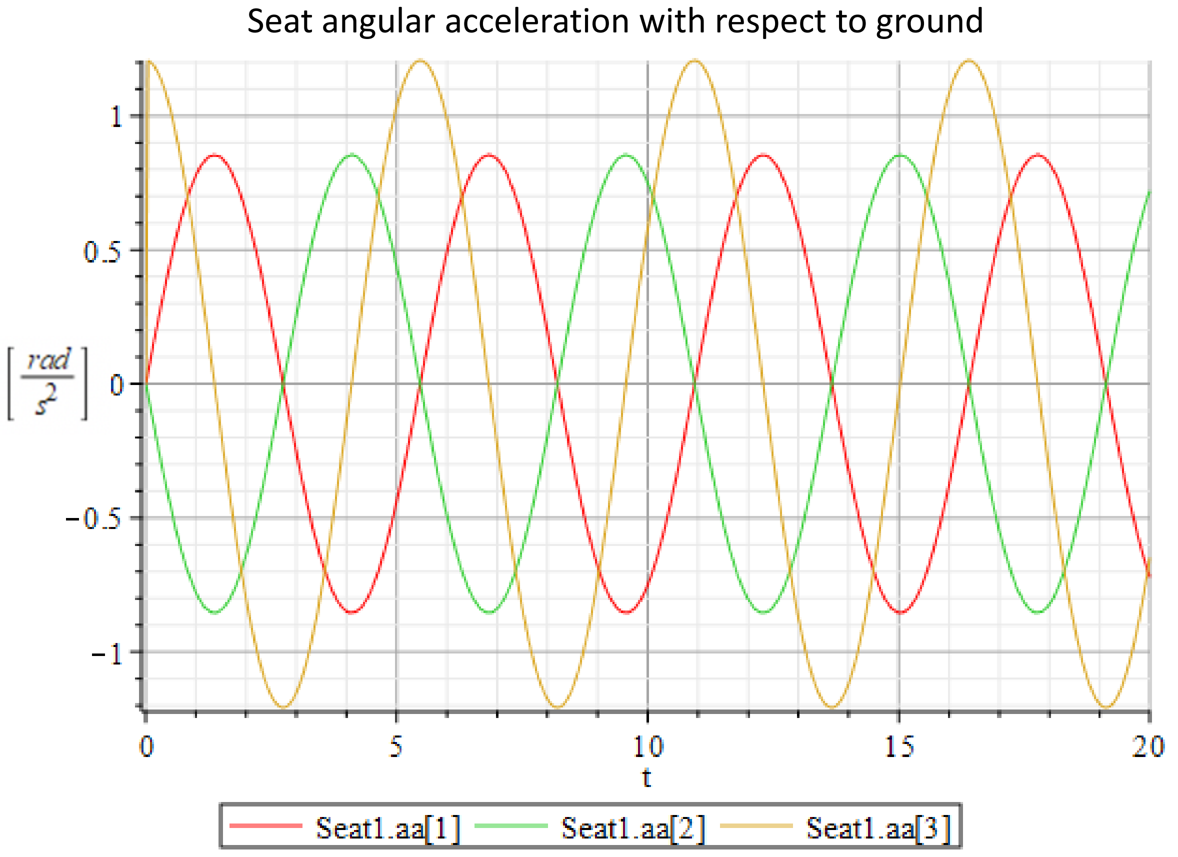
MapleSim Model

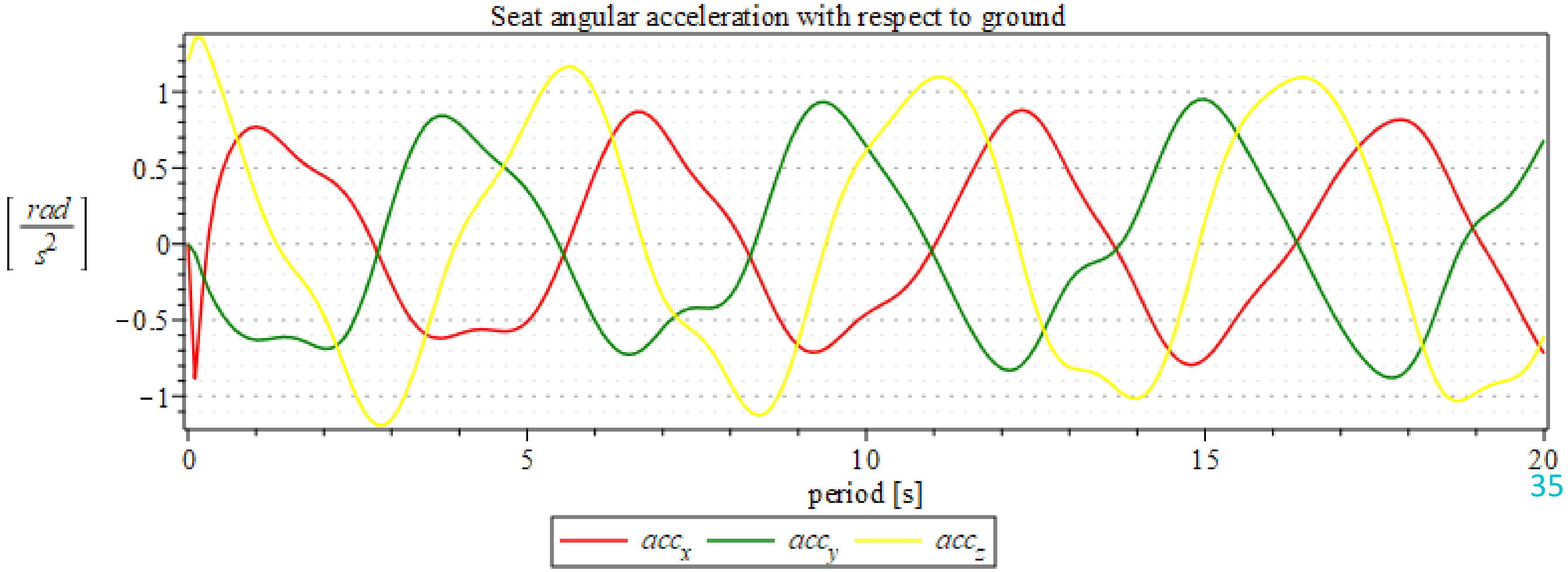


MapleSim Model



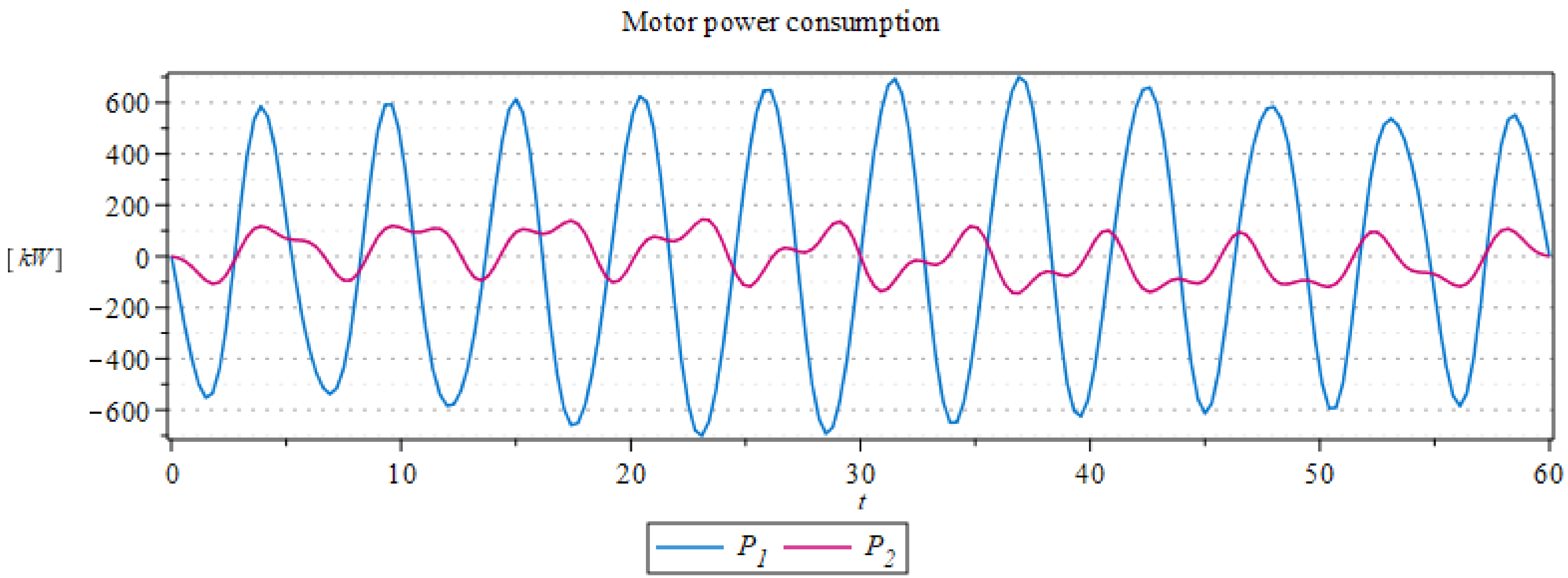






Power Consumption

- The numerical solution was used to calculate the power consumption of the two actuated motors for the duration of the whole ride, shown in the following graph.
  - MBSymba power() command

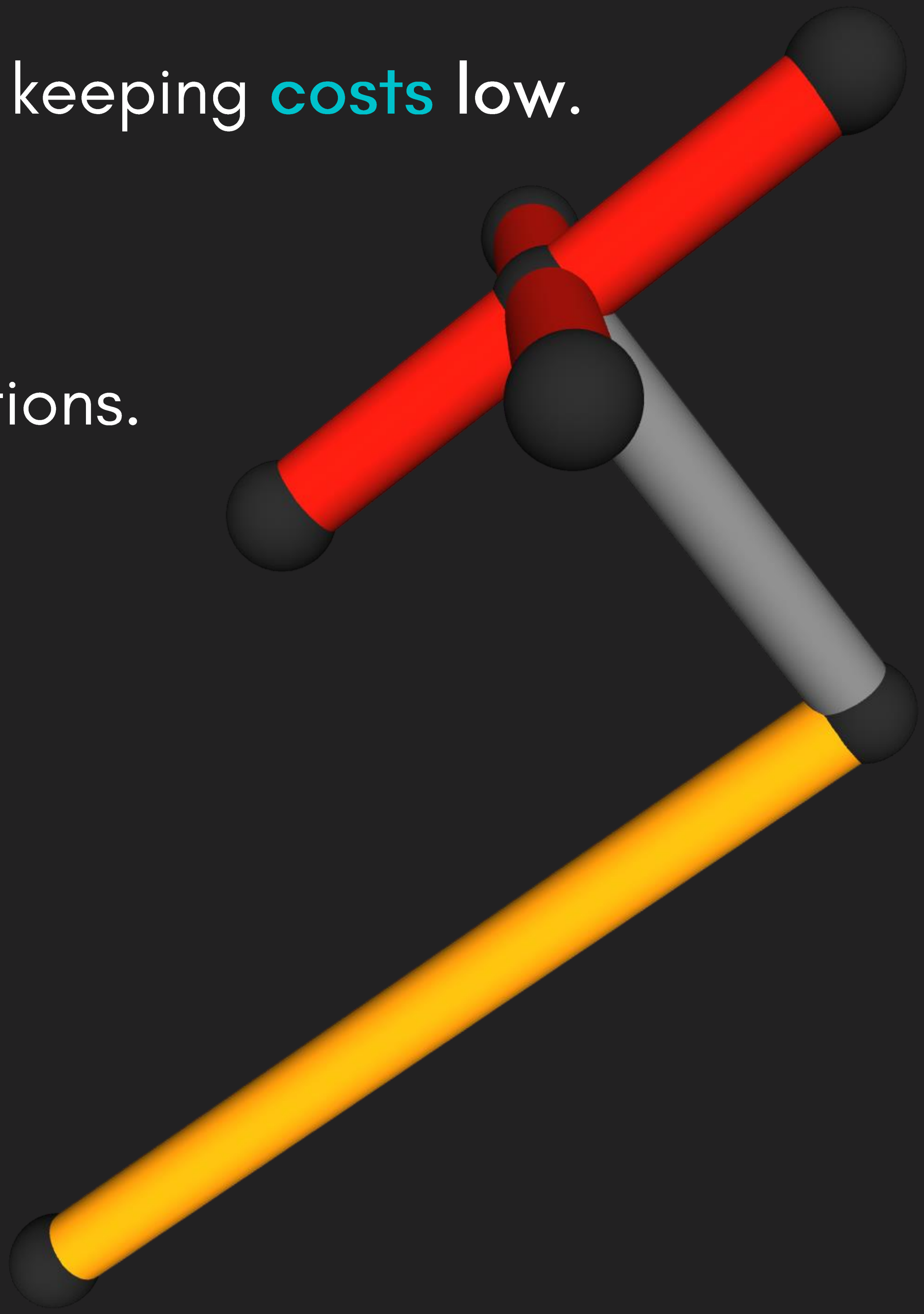


Performance Analysis

		Target	Solution 1		Solution 2		Solution 3		Solution 4		Solution 5		Solution 6	
Specification	Weight	Value	Value	Score	Value	Score	Value	Score	Value	Score	Value	Score	Value	Score
Fun	5	30.42	29.57	1	25.02	1	32.74	3	35.87	3	28.82	1	30.44	2
Safety	∞	Pass	Pass	0	Pass	0	Pass	0	Not pass	-1	Pass	0	Pass	0
Power** Consumption [MW]	4	499	322.6	3	574.8	0	394.3	3	449	1	375.6	3	420.8	2
L_1	3	10.03m*	10.03m	2	10.03m	2	10.03m	2	20.03 m	1	10.03 m	2	7 m	3
L_2	1	4.25m*	4.25m	2	4.25m	2	4.25 m	2	4.25 m	2	3.247 m	3	4.25 m	2
L_3	3	2.53m*	2.53m	2	2.53m	2	2.53 m	2	10 m	1	2.00	3	2.3 m	3
ω_2	2	11 rpm*	9 rpm	3	15 rpm	1	12 rpm	1	11 rpm	2	11 rpm	2	11 rpm	2
ω_3	3	10 rpm*	5 rpm	3	13 rpm	1	6 rpm	3	10 rpm	2	10 rpm	2	10 rpm	2
TOTAL SCORE			46		24		52		37		45		48	

Final Conclusions and Key Takeaways

- Goal of the design process was to find the most exciting ride keeping costs low.
- A fun equation was derived to numerically evaluate fun.
- Performance indices were defined for compare different solutions.
- With the kinematic analysis solutions 3 and 6 were identified
- The solution with the highest score overall was solution 3
  - MORE FUN
  - LOWER POWER CONSUMPTION



Thank you for your attention

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