Examples for the paper:
Interconnection and Damping Assignment Passivity-based Control Without Partial Differential Equations

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1 Introduction

Each of the following examples illustrates one of the cases proposed in the paper. Therefore, below, we provide four examples where the results of one of the paper's propositions are employed to achieve regulation of a physical system.

Caveat: This is not a standalone document but a complementary material.

2 Biochemical fermenter

This example is based on the material reported in [4, Section 4]. We refer the reader to the mentioned paper and the references therein for further details. The model of the biochemical fermenter depicted in Fig. 1 is given by

$$\begin{aligned} \dot{x}_1 &= \mu(x_2)x_1 - x_1\frac{q}{L} \\ \dot{x}_2 &= -\frac{\mu(x_2)}{Y}x_1 + (S_{\mathtt{f}} - x_2)\frac{q}{L}, \end{aligned} \tag{1}$$

where $x_1, x_2 > 0$ denote the cell and substrate concentrations, q is the volumetric inflow rate; $L > 0$ represents the reactor volume; $S_{\mathtt{f}} > 0$ denotes the feed of substrate entering the reactor; $Y > 0$ is the Yield coefficient. Moreover, the cell growth rate $\mu(x_2)$ is given by

$$\mu(x_2) := \frac{\mu_{\mathtt{max}}x_2}{c_1 + x_2 + c_2x_2^2} \tag{2}$$

with $\mu_{\mathtt{max}}, c_1, c_2 > 0$. Hence $\mu(x_2) \in (0, \mu_{\mathtt{max}})$. Considering

$$u = \frac{q}{\mu(x_2)x_1L}, \tag{3}$$

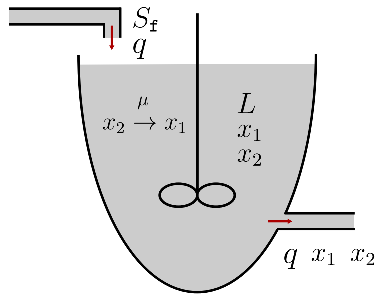

Figure 1: Second-order biomechanical fermenter. Schematic based on [4].

this system admits a pH representation, with

$$
\begin{aligned}
F(x) &= \mu(x_2)x_1 \begin{bmatrix} -1 & 0 \\ 0 & -\dfrac{1}{Yx_2} \end{bmatrix}; \quad g(x) = \mu(x_2)x_1 \begin{bmatrix} -x_1 \\ S_\mathtt{f} - x_2 \end{bmatrix}; \\
H(x) &= -x_1 + \frac{1}{2}x_2^2.
\end{aligned} \tag{4}
$$

Note that the set of assignable equilibria is given by

$$
\mathcal{E} = \left\{ x \in \mathbb{R}_+ \times \mathbb{R}_+ \mid S_\mathtt{f} = \frac{x_1}{Y} + x_2 \right\}. \tag{5}
$$

Furthermore, $u_\star x_{1_\star} = 1$. This system satisfies Assumptions 1 and 2. Moreover,

$$
\gamma(x) = -\frac{1}{2}x_1^2 + \frac{S_\mathtt{f} Y}{2}x_2^2 - \frac{Y}{3}x_2^3. \tag{6}
$$

Some computations show that Assumption 3 holds for

$$
\Phi(\gamma(x)) = u_\star \gamma(x) + \frac{1}{2}K\left(\gamma(x) - \gamma(x_\star)\right)^2, \tag{7}
$$

with $K > 0$. Hence, this system falls in Case 1. Consequently, it can be controlled following the result of Proposition 1.

3 Planar manipulator

This example is inspired by the 2 DOF Serial Flexible Joint system developed by Quanser. Some experimental results of passivity-based controllers applied to that system can be found in [2, 3]. Consider the planar manipulator shown in Fig. 2. This system has two degrees of freedom, and its workspace is a subset of the horizontal plane, i.e., there is no potential energy due to gravity. We denote the angular positions and momenta as $q \in \mathbb{R}^2$ and $p \in \mathbb{R}^2$, respectively. The

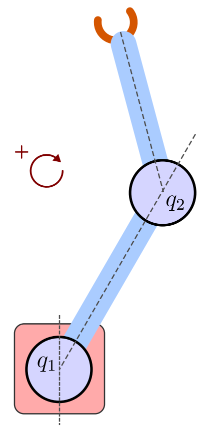

Figure 2: Planar manipulator with two degrees of freedom.

behavior of this system admits a pH representation with

$$F = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -c_1 & 0 \\ 0 & -1 & 0 & -c_2 \end{bmatrix}; \quad g = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}; \quad H(q,p) = \frac{1}{2} p^\top M^{-1}(q) p \tag{8}$$

where

$$M(q) = \begin{bmatrix} a_1 + a_2 + 2b\cos(q_2) & a_2 + b\cos(q_2) \\ a_2 + b\cos(q_2) & a_2 \end{bmatrix} \tag{9}$$

$u \in \mathbb{R}^2$ is the input vector, which corresponds to the torques of the motors; a_1, a_2, b are positive such that $M(q) \succ 0$; c_1, c_2 are nonnegative charaterizing the natural damping of the system.

The set of assignable equilibria for this system is characterized by[1]

$$\mathcal{E} = \left\{ (q,p) \in \mathbb{R}^2 \times \mathbb{R}^2 \mid q = q_\star,\ p = \mathbf{0} \right\}, \tag{10}$$

where $q_\star \in \mathbb{R}^2$.

Note this system satisfies Assumption 4, with

$$x_{\mathtt{u}} = q, \quad x_{\mathtt{a}} = p. \tag{11}$$

[1]We consider $\mathbb{R}^2$ for simplicity. In practice, it could be more convenient to consider $\mathcal{S}^1$ for each configuration variable.

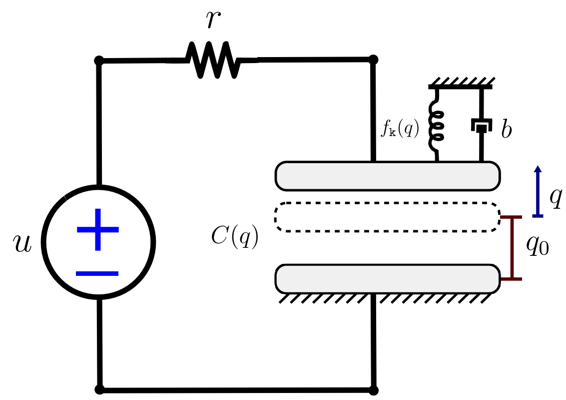

Figure 3: Micromechanical switch with a nonlinear spring

Accordingly,

$$\begin{aligned}\left(\frac{\partial H(q,p)}{\partial p}\right)\Big|_{(q,p)=(q_\star,\mathbf{0})} &= \left(M^{-1}(q)p\right)\big|_{(q,p)=(q_\star,\mathbf{0})} = \mathbf{0};\\ \left(\frac{\partial^2 H(q,p)}{\partial p^2}\right)\Big|_{(q,p)=(q_\star,\mathbf{0})} &= \left(M^{-1}(q)\right)\big|_{(q,p)=(q_\star,\mathbf{0})} = M^{-1}(q_\star) \succ 0.\end{aligned} \tag{12}$$

Hence, Assumption 5 holds. Moreover, because $F_{\mathtt{uu}} = \mathbf{0}$, this system can be stabilized with the result of Proposition 2. In particular, $\Phi_{\mathtt{u}}(q)$ represents the desired potential energy of the closed-loop system. A straightforward choice for this function is

$$\Phi_{\mathtt{u}}(q) = \frac{1}{2}\left(q - q_\star\right)^\top K\left(q - q_\star\right); \quad K \succ 0. \tag{13}$$

4 Microelectromechanical switch

This example is inspired by the electromechanical examples studied in [5, 6]. Consider the micromechanical switch depicted in Fig. 3. Let q and p the position and momentum of the movable plate, respectively. The distance between the plates is given by $q + q_0$, where $q_0 > 0$ where denotes the gap between the plates when the spring is at rest. Note that the constraint $q + q_0 > 0$ must be satisfied. Otherwise, the plates are overlapping. The mass of the movable plate is represented by m, and the symbol b represents the coefficient of the linear damper. The energy stored and force exerted by the spring are given by

$$E_{\mathtt{k}}(q) = k \ln\left(\cosh(q)\right), \quad f_{\mathtt{k}}(q) = k\tanh(q), \quad k > 0, \tag{14}$$

respectively. The capacitor's charge is denoted as Q. Moreover, the capacitance is given by the expression

$$C(q) = \frac{A\epsilon}{q + q_0}, \tag{15}$$

where A denotes the plate's area and ϵ the permittivity. Consequently, the energy stored by the capacitor is given by

$$E_{\mathtt{c}}(q, Q) = \frac{1}{2C(q)} Q^2 = \frac{(q + q_0)}{2A\epsilon} Q^2 \tag{16}$$

The symbol r represents the electrical resistance. The input to the system is the voltage provided by the voltage source, represented by u. This system admits a pH representation with

$$\begin{aligned} F &= \begin{bmatrix} 0 & 1 & 0 \\ -1 & -b & 0 \\ 0 & 0 & -\frac{1}{r} \end{bmatrix}; \quad g = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{r} \end{bmatrix}; \\ H(q, p, Q) &= \frac{p^2}{2m} + k \ln\left(\cosh(q)\right) + \frac{(q + q_0)}{2A\epsilon} Q^2. \end{aligned} \tag{17}$$

Moreover, the set of assignable equilibria for this system is given by

$$\mathcal{E} = \left\{ (q, p, Q) \in \mathbb{R}^3 \mid q \in (-q_0, 0], \quad p = 0, \quad Q = \sqrt{-2A\epsilon k \tanh(q)} \right\}. \tag{18}$$

This system satisfies Assumption 4—considering a new input $\bar{u} = \frac{u}{r}$—with

$$x_{\mathtt{u}} = \begin{bmatrix} q \\ p \end{bmatrix}; \qquad x_{\mathtt{a}} = Q. \tag{19}$$

Hence,

$$\frac{\partial H(x)}{\partial x_{\mathtt{u}}} = \underbrace{\begin{bmatrix} k \tanh(q) \\ \frac{1}{m} p \end{bmatrix}}_{\beta_{\mathtt{u}}} + \underbrace{\begin{bmatrix} \frac{Q^2}{2A\epsilon} \\ 0 \end{bmatrix}}_{\Gamma_{\mathtt{a}}} \tag{20}$$

and

$$B_{\mathtt{u}} = \begin{bmatrix} k \operatorname{sech}^2(q_\star) & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \succ 0. \tag{21}$$

Accordingly, Assumption 6 is satisfied. Notably, in this example, is given by the mechanical energy of the system, i.e.,

$$\alpha_{\mathtt{u}}(x_{\mathtt{u}}) = \frac{p^2}{2m} + k \ln\left(\cosh(q)\right). \tag{22}$$

Because $F_{\mathtt{ua}} = \mathbf{0}$ and Assumption 6 holds, the system can be stabilized using the result of Proposition 6. A suitable choice for $\Phi_{\mathtt{a}}(x_{\mathtt{a}})$ is given by

$$\Phi_{\mathtt{a}}(x_{\mathtt{a}}) = \frac{1}{2} K_{\mathtt{a}} \left(Q - Q_\star\right)^2 - \frac{(q_\star + q_0)}{A\epsilon} Q Q_\star, \quad K_{\mathtt{a}} > 0; \tag{23}$$

where

$$Q_\star = \sqrt{-2A\epsilon k \tanh(q_\star)}. \tag{24}$$

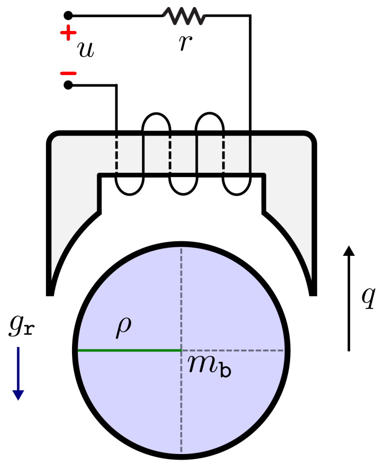

Figure 4: Magnetic levitation system.

5 Magnetic levitation system

Consider the magnetic levitation depicted in Fig. 4. In [1], the authors provide a pH model corresponding to this system, with

$$\begin{aligned} F &= \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -r \end{bmatrix}; \quad g = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; \\ H(x) &= m_{\mathtt{b}} g_{\mathtt{r}} q + \frac{1}{2m_{\mathtt{b}}} p^2 + \frac{1}{2k} \varphi^2 (\rho - q), \end{aligned} \tag{25}$$

where q, p denote the ball's position (with respect to its center of mass) and momentum, respectively, φ represents the inductor's magnetic flux. Similarly, $m_{\mathtt{b}}, g_{\mathtt{r}}, r, \rho$ represent the ball's mass, gravitational acceleration, electrical resistance, and ball's radius, respectively. The parameter k is associated with the number of turns in the coil. All these parameters are positive. Additionally, the position must satisfy the physical constraint $q < \rho$. The input to the system is the voltage u provided by the source. The set of assignable equilibria is given by

$$\mathcal{E} = \left\{ x \in \mathbb{R}^3 \mid p = 0, \varphi = \sqrt{2k m_{\mathtt{b}} g_{\mathtt{r}}} \right\}. \tag{26}$$

Note that this system satisfies Assumption 4. In particular,

$$\begin{aligned} x_{\mathtt{u}} &= \begin{bmatrix} q \\ p \end{bmatrix}; \; \frac{\partial H(x)}{\partial x_{\mathtt{u}}} = \begin{bmatrix} m_{\mathtt{b}} g_{\mathtt{r}} - \frac{1}{2k} \varphi^2 \\ \frac{p}{m_{\mathtt{b}}} \end{bmatrix}; \\ x_{\mathtt{a}} &= \varphi; \quad \frac{\partial H(x)}{\partial x_{\mathtt{a}}} = \frac{1}{k} \varphi (\rho - q). \end{aligned} \tag{27}$$

Hence,

$$\beta_{\mathtt{u}}(x_{\mathtt{u}}) = \begin{bmatrix} m_{\mathtt{b}} g_{\mathtt{r}} \\ \dfrac{p}{m_{\mathtt{b}}} \end{bmatrix}, \quad \beta_{\mathtt{a}}(x_{\mathtt{a}}) = \frac{1}{2k}\varphi^2 \tag{28}$$

Note that

$$B_{\mathtt{a}} = \frac{\varphi_\star}{k} = \frac{\sqrt{2km_{\mathtt{b}}g_{\mathtt{r}}}}{k} > 0. \tag{29}$$

Consequently, Assumption 7 holds, for $\Upsilon = -[1\ 0]^\top$. In particular,

$$\alpha_{\mathtt{u}}(x_{\mathtt{u}}) = m_{\mathtt{b}} g_{\mathtt{r}} q + \frac{1}{2m_{\mathtt{b}}} p^2; \quad \alpha_{\mathtt{a}}(x_{\mathtt{a}}) = \frac{1}{6k}\varphi^3. \tag{30}$$

Note that $\alpha_{\mathtt{u}}(x_{\mathtt{u}})$ corresponds to the energy of the mechanical system

Because $F_{\mathtt{ua}} = \mathbf{0}$, this system is characterized by Case 4. Accordingly, it can be stabilized using the result provided in Proposition 4. Moreover, $z = q + \varphi$, and a suitable choice for $\Psi(z)$ is

$$\Psi(z) = \frac{1}{2} K_{\mathtt{z}} \left(q + \varphi - q_\star - \sqrt{2km_{\mathtt{b}}g_{\mathtt{r}}} \right)^2 - m_{\mathtt{b}} g_{\mathtt{r}} (q + \varphi). \tag{31}$$

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