Sea $F(x) = e^{-2x}\ln(3x)$. Sus derivadas son

$$\begin{aligned}
\frac{\mathrm{d}F}{\mathrm{d}x} &= \frac{\mathrm{d}}{\mathrm{d}x}[e^{-2x}]\ln(3x) + e^{-2x}\frac{\mathrm{d}}{\mathrm{d}x}[\ln(3x)] && (1)\\
&= -2e^{-2x}\ln(3x) + e^{-2x}\frac{1}{x} && (2)\\
&= e^{-2x}\left(-2\ln(3x) + \frac{1}{x}\right) && (3)
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}^2F}{\mathrm{d}x^2} &= \frac{\mathrm{d}}{\mathrm{d}x}\left[e^{-2x}\left(-2\ln(3x) + \frac{1}{x}\right)\right] && (4)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}[e^{-2x}]\left(-2\ln(3x) + \frac{1}{x}\right) + e^{-2x}\frac{\mathrm{d}}{\mathrm{d}x}\left[-2\ln(3x) + \frac{1}{x}\right] && (5)\\
&= -2e^{-2x}\left(-2\ln(3x) + \frac{1}{x}\right) + e^{-2x}\left(-2\frac{1}{x} - \frac{1}{x^2}\right) && (6)\\
&= e^{-2x}\left(4\ln(3x) - 4\frac{1}{x} - \frac{1}{x^2}\right) && (7)
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}^3F}{\mathrm{d}x^3} &= \frac{\mathrm{d}}{\mathrm{d}x}\left[e^{-2x}\left(4\ln(3x) - 4\frac{1}{x} - \frac{1}{x^2}\right)\right] && (8)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}[e^{-2x}]\left(4\ln(3x) - 4\frac{1}{x} - \frac{1}{x^2}\right) + e^{-2x}\frac{\mathrm{d}}{\mathrm{d}x}\left[4\ln(3x) - 4\frac{1}{x} - \frac{1}{x^2}\right] && (9)\\
&= -2e^{-2x}\left(4\ln(3x) - 4\frac{1}{x} - \frac{1}{x^2}\right) + e^{-2x}\left[4\frac{1}{x} + 4\frac{1}{x^2} + 2\frac{1}{x^3}\right] && (10)\\
&= e^{-2x}\left(-8\ln(3x) + 12\frac{1}{x} + 6\frac{1}{x^2} + 2\frac{1}{x^3}\right) && (11)
\end{aligned}$$

$$\begin{aligned}
\frac{\mathrm{d}^4F}{\mathrm{d}x^4} &= \frac{\mathrm{d}}{\mathrm{d}x}\left[e^{-2x}\left(-8\ln(3x) + 12\frac{1}{x} + 6\frac{1}{x^2} + 2\frac{1}{x^3}\right)\right] && (12)\\
&= \frac{\mathrm{d}}{\mathrm{d}x}[e^{-2x}]\left(-8\ln(3x) + 12\frac{1}{x} + 6\frac{1}{x^2} + 2\frac{1}{x^3}\right) && (13)\\
&\quad + e^{-2x}\frac{\mathrm{d}}{\mathrm{d}x}\left[-8\ln(3x) + 12\frac{1}{x} + 6\frac{1}{x^2} + 2\frac{1}{x^3}\right] && (14)\\
&= -2e^{-2x}\left(-8\ln(3x) + 12\frac{1}{x} + 6\frac{1}{x^2} + 2\frac{1}{x^3}\right) && (15)\\
&\quad + e^{-2x}\left(-8\frac{1}{x} - 12\frac{1}{x^2} - 12\frac{1}{x^3} - 6\frac{1}{x^4}\right) && (16)\\
&= e^{-2x}\left(16\ln(3x) - 32\frac{1}{x} - 24\frac{1}{x^2} - 16\frac{1}{x^3} - 6\frac{1}{x^4}\right) && (17)
\end{aligned}$$

En consecuencia

$$F(2) = e^{-4}\ln(6) \tag{18}$$

$$\frac{\mathrm{d}F}{\mathrm{d}x}(2) = e^{-4}\left(-2\ln(6) + 1/2\right) = \frac{1 - 4\ln(6)}{2e^4} \tag{19}$$

$$\frac{\mathrm{d}^2F}{\mathrm{d}x^2}(2) = e^{-4}\left(4\ln(6) - 4\frac{1}{2} - \frac{1}{2^2}\right) = \frac{16\ln(6) - 9}{4e^4} \tag{20}$$

$$\frac{\mathrm{d}^3F}{\mathrm{d}x^3}(2) = e^{-4}\left(-8\ln(6) + 12\frac{1}{2} + 6\frac{1}{2^2} + 2\frac{1}{2^3}\right) \tag{21}$$

$$= e^{-4}\left(-8\ln(6) + 6 + 3\frac{1}{2} + \frac{1}{2^2}\right) \tag{22}$$

$$= \frac{-32\ln(6) + 31}{4e^4} \tag{23}$$

$$\frac{\mathrm{d}^4F}{\mathrm{d}x^4}(2) = e^{-4}\left(16\ln(6) - 32\frac{1}{2} - 24\frac{1}{2^2} - 16\frac{1}{2^3} - 6\frac{1}{2^4}\right) \tag{24}$$

$$= \frac{128\ln(6) - 195}{8e^4} \tag{25}$$

Así, el polinomio de Taylor, de orden 4 centrado en 2, para F es

$$P(x) = e^{-4}\ln(6) + \frac{1 - 4\ln(6)}{2e^4}(x-2) \tag{26}$$

$$+ \frac{16\ln(6) - 9}{4e^4}\frac{(x-2)^2}{2} + \frac{-32\ln(6) + 31}{4e^4}\frac{(x-2)^3}{6} \tag{27}$$

$$+ \frac{128\ln(6) - 195}{8e^4}\frac{(x-2)^4}{24} \tag{28}$$

$$= \frac{\ln(6)}{e^4} + \frac{1 - 4\ln(6)}{2e^4}(x-2) \tag{29}$$

$$+ \frac{16\ln(6) - 9}{8e^4}(x-2)^2 + \frac{-32\ln(6) + 31}{24e^4}(x-2)^3 \tag{30}$$

$$+ \frac{128\ln(6) - 195}{192e^4}(x-2)^4 \tag{31}$$