Geometría de la traza del movimiento de un electron expuesto a un campo electromagnético externo

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En unidades del Sistema Internacional, la ecuación del movimiento para una partícula de masa m, carga q, expuesta a un campo electromagnético externo es

$$m\vec{a} = q\vec{E} + q\vec{v} \times \vec{B} \tag{1}$$

Esta ecuación diferencial es lineal. Esto nos permite resolverla por partes. Sabemos que, cuando el campo elétrico es constante, la solución $\vec{f}$ a la ecuación

$$m\frac{d^2\vec{f}}{dt^2} = q\vec{E} \qquad \text{está dada por} \qquad \vec{f} = \frac{q}{m}\vec{E}\frac{t^2}{2} + \left.\frac{d\vec{f}}{dt}\right|_0 t + \vec{f}_0 \tag{2}$$

Si conseguimos hallar la solución $\vec{g}$ a la ecuación

$$m\frac{d^2\vec{g}}{dt^2} = q\frac{d\vec{g}}{dt} \times \vec{B} \tag{3}$$

Entonces $\vec{r} = \vec{f} + \vec{g}$ satisface (1).

$$m\frac{d^2\vec{g}}{dt^2} = q\frac{d\vec{g}}{dt} \times \vec{B} \tag{4}$$

$$\frac{d^2\vec{g}}{dt^2} = \frac{q}{m}\begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{dg_x}{dt} & \frac{dg_y}{dt} & \frac{dg_z}{dt} \\ B_x & B_y & B_z \end{vmatrix} \tag{5}$$

$$\begin{cases} \dfrac{d^2 g_x}{dt^2} = \dfrac{q}{m}\begin{vmatrix} \frac{dg_y}{dt} & \frac{dg_z}{dt} \\ B_y & B_z \end{vmatrix} \\ \dfrac{d^2 g_y}{dt^2} = \dfrac{q}{m}\begin{vmatrix} \frac{dg_z}{dt} & \frac{dg_x}{dt} \\ B_z & B_x \end{vmatrix} \\ \dfrac{d^2 g_z}{dt^2} = \dfrac{q}{m}\begin{vmatrix} \frac{dg_x}{dt} & \frac{dg_y}{dt} \\ B_x & B_y \end{vmatrix} \end{cases} \tag{6}$$

$$\begin{cases} \frac{d^2g_x}{dt^2} = \frac{q}{m}B_z\frac{dg_y}{dt} - \frac{q}{m}B_y\frac{dg_z}{dt} \\ \frac{d^2g_y}{dt^2} = \frac{q}{m}B_x\frac{dg_z}{dt} - \frac{q}{m}B_z\frac{dg_x}{dt} \\ \frac{d^2g_z}{dt^2} = \frac{q}{m}B_y\frac{dg_x}{dt} - \frac{q}{m}B_x\frac{dg_y}{dt} \end{cases} \tag{7}$$

$$\begin{cases} \frac{d^2g_x}{dt^2} = 0\frac{dg_x}{dt} + \frac{q}{m}B_z\frac{dg_y}{dt} - \frac{q}{m}B_y\frac{dg_z}{dt} \\ \frac{d^2g_y}{dt^2} = -\frac{q}{m}B_z\frac{dg_x}{dt} + 0\frac{dg_y}{dt} + \frac{q}{m}B_x\frac{dg_z}{dt} \\ \frac{d^2g_z}{dt^2} = \frac{q}{m}B_y\frac{dg_x}{dt} - \frac{q}{m}B_x\frac{dg_y}{dt} + 0\frac{dg_z}{dt} \end{cases} \tag{8}$$

$$\frac{d^2\vec{g}}{dt^2} = \begin{pmatrix} 0 & \frac{q}{m}B_z & -\frac{q}{m}B_y \\ -\frac{q}{m}B_z & 0 & \frac{q}{m}B_x \\ \frac{q}{m}B_y & -\frac{q}{m}B_x & 0 \end{pmatrix} \frac{d\vec{g}}{dt} \tag{9}$$

$$\frac{d^2\vec{g}}{dt^2} = A\frac{d\vec{g}}{dt} \tag{10}$$

$$\frac{d\vec{g}}{dt} = e^{At}\left.\frac{d\vec{g}}{dt}\right|_0 \tag{11}$$

$$\vec{g} = A^{-1}e^{At}\left.\frac{d\vec{g}}{dt}\right|_0 + \vec{g}_0 \tag{12}$$

$$A = \begin{pmatrix} 0 & \frac{q}{m}B_z & -\frac{q}{m}B_y \\ -\frac{q}{m}B_z & 0 & \frac{q}{m}B_x \\ \frac{q}{m}B_y & -\frac{q}{m}B_x & 0 \end{pmatrix} \tag{13}$$

autovalores de A

$$\det(\lambda - A) = \begin{vmatrix} \lambda & -\frac{q}{m}B_z & \frac{q}{m}B_y \\ \frac{q}{m}B_z & \lambda & -\frac{q}{m}B_x \\ -\frac{q}{m}B_y & \frac{q}{m}B_x & \lambda \end{vmatrix} \tag{14}$$

$$= \lambda\begin{vmatrix} \lambda & -\frac{q}{m}B_x \\ \frac{q}{m}B_x & \lambda \end{vmatrix} \tag{15}$$

$$+ \frac{q}{m}B_z\begin{vmatrix} \frac{q}{m}B_z & -\frac{q}{m}B_x \\ -\frac{q}{m}B_y & \lambda \end{vmatrix} \tag{16}$$

$$+ \frac{q}{m}B_y\begin{vmatrix} \frac{q}{m}B_z & \lambda \\ -\frac{q}{m}B_y & \frac{q}{m}B_x \end{vmatrix} \tag{17}$$

$$= \lambda(\lambda^2 + \frac{q^2}{m^2}B_x^2) \tag{18}$$

$$+ \frac{q}{m}B_z(\frac{q}{m}B_z\lambda - \frac{q^2}{m^2}B_xB_y) + \frac{q}{m}B_y(\frac{q^2}{m^2}B_xB_z + \frac{q}{m}B_y\lambda) \tag{19}$$

$$= \lambda^3 + \frac{q^2}{m^2}B_x^2\lambda \tag{20}$$

$$
\begin{aligned}
&+\frac{q^2}{m^2}B_z^2\lambda-\frac{q^3}{m^3}B_xB_yB_z+\frac{q^3}{m^3}B_xB_yB_z+\frac{q^2}{m^2}B_y^2\lambda && (21)\\
&=\lambda(\lambda^2+\left\|\frac{q}{m}\vec{B}\right\|_2^2) && (22)\\
&=\lambda(\lambda-i\left\|\frac{q}{m}\vec{B}\right\|_2)(\lambda+i\left\|\frac{q}{m}\vec{B}\right\|_2) && (23)
\end{aligned}
$$

autoespacios de A