Para $k = 1, 2, 3, \ldots$

$$
\begin{aligned}
b_k &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\mathrm{d}x && (1)\\
&= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\sin(kx)\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} f(x)\sin(kx)\mathrm{d}x && (2)\\
&= \frac{-1}{\pi}\int_{-\pi}^{0} \sin(kx)\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} \sin(kx)\mathrm{d}x && (3)\\
&= \frac{-1}{\pi}\int_{-\pi}^{0} \frac{-1}{k}\frac{\mathrm{d}}{\mathrm{d}x}\cos(kx)\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} \frac{-1}{k}\frac{\mathrm{d}}{\mathrm{d}x}\cos(kx)\mathrm{d}x && (4)\\
&= \frac{1}{k\pi}\int_{-\pi}^{0} \frac{\mathrm{d}\cos(kx)}{\mathrm{d}x}\mathrm{d}x + \frac{-1}{k\pi}\int_{0}^{\pi} \frac{\mathrm{d}\cos(kx)}{\mathrm{d}x}\mathrm{d}x && (5)\\
&= \frac{1}{k\pi}\cos(kx)|_{-\pi}^{0} + \frac{-1}{k\pi}\cos(kx)|_{0}^{\pi} && (6)\\
&= \frac{1}{k\pi}(\cos(0) - \cos(-k\pi)) + \frac{-1}{k\pi}(\cos(k\pi) - \cos(0)) && (7)\\
&= \frac{1}{k\pi}(1 - \cos(k\pi)) + \frac{-1}{k\pi}(\cos(k\pi) - 1) && (8)\\
&= \frac{2}{k\pi}(1 - \cos(k\pi)) && (9)\\
&= \frac{2}{k\pi}(1 - (-1)^k) && (10)\\
&= \begin{cases} 0 & \text{si } k \text{ es par} \\ \frac{4}{k\pi} & \text{si } k \text{ es impar} \end{cases} && (11)
\end{aligned}
$$