

# CRADLE++ Tests 2

November 19, 2025

## Simulation Wu Experiment

Consider atoms of  $^{60}\text{Co}$  in a thermal bath and in the presence of a magnetic field in the -z direction.

Model each nuclei as independent 11 state system (each of the values of  $m_j$ )

$$Z = \sum_{m_j=-5}^5 e^{\frac{m_j \mu_N ({}^{60}\text{Co}) B}{5k_b T}} \rightarrow P(m_j) = \frac{1}{Z} e^{\frac{m_j \mu_N ({}^{60}\text{Co}) B}{5k_b T}}$$

From here polarisation and alignment in Z direction ( $P_z$  and  $\mathcal{A}_z$ ) can be computed

$$P_z = \frac{1}{J} \langle m_j \rangle = \sum_{m_j=-5}^5 \frac{m_j P(m_j)}{5} \quad \mathcal{A}_{\ddagger} = \frac{3 \left\langle m_j^2 \right\rangle - J(J+1)}{J(2J-1)}$$

which leads to non-zero A polarisation.

# Simulation Wu Experiment

Implementation:

- ▶  $N = 200000$  atoms
- ▶  $|z_e| > \cos 15^\circ$
- ▶ Realistic value of  $\mu_N(^{60}\text{Co})$
- ▶ 1 sim for each  $T$ , with its  $P_z$  and  $\mathcal{A}_z$

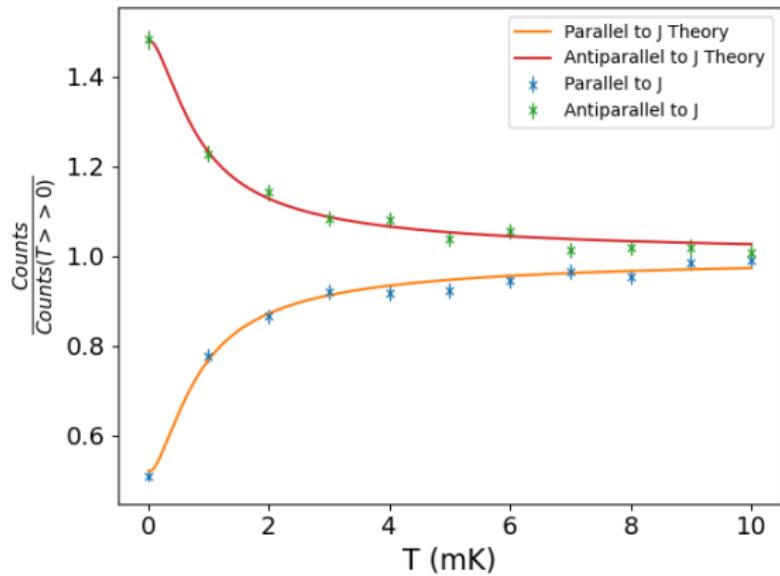


Figure: Simulation of the 1957 Wu experiment using  $N = 200000$   $^{60}\text{Co}$  nuclei for each  $T$

# Gamow-Teller Decay: $^{60}\text{Co}$

Properties of  $^{60}\text{Co}(5^+) \rightarrow ^{60}\text{Ni}(4^+)$

- $Q = 317.06$  keV (good for testing,  $\langle \beta_e \rangle = 0.68$ ,  $\langle \gamma_e^{-1} \rangle = 0.72$ ,  $\langle \alpha Z \gamma_e^{-1} \rangle = 0.15$ )
- $J_f = J_i - 1 \rightarrow \lambda_{J_i, J_f} = \Lambda_{J_i, J_f} = 1$
- 2  $\gamma$  almost always ( $5^+ \rightarrow 2^+$  only 1  $\gamma$ )

Many cases to consider, though for realism: keep  $C_A = C'_A = \text{cte}(= 1)$ .

- $C_T = C'_T = 0$  (Standard Model)
- $C_T = C'_T$  pure real (and large)
- $C_T = C'_T$  pure imaginary
- $C_T = -C'_T$ , either real or imaginary

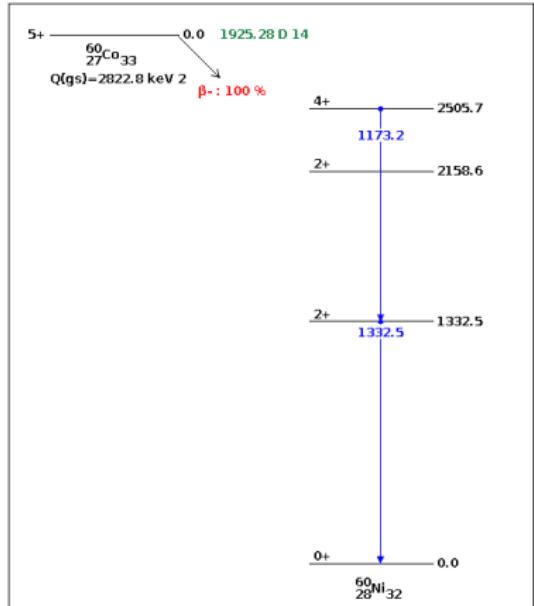


Figure: Decay Scheme of  $^{60}\text{Co}$  into  $^{60}\text{Ni}$  featuring the only decay of interest

# Gamow-Teller Decay: $^{60}\text{Co}$

## Numerical evaluation

Use that distributions in  $z_e$ ,  $z_\nu$ ,  $\cos \theta_{e,\nu} \equiv z_{e,\nu}$  and  $\phi$  are known if  $F \geq 0$  for all orientations of  $p\mathbf{p}_e, \mathbf{p}_\nu$  ( $\mathbf{J}$  fixed).

$$f_1(z_e) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle A\beta_e \rangle z_e}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_2(z_\nu) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle B \rangle z_\nu}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_3(z_{e,\nu}) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle a\beta_e \rangle z_{e,\nu}}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_4(\phi) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle (a + \frac{c}{3})\beta_e \rangle \frac{\pi^2}{16} \cos \phi + \langle D\beta_e \rangle \frac{\pi^2}{16} \sin \phi}{2\pi(1 + \langle b\gamma_e^{-1} \rangle)}$$

Averages computed numerically using  $f(E)$  from the simulation data itself (avoid computing the Fermi function myself)

# Gamow-Teller Decay: $^{60}\text{Co}$

## Numerical verification

From distributions:

- ▶ Compute difference  $\Delta_{i,j} = f_j(x_i) - f_{j,\text{th}}(x_i)$  for each point  $x_i$  in each distribution  $f_j$
- ▶ Use  $\sigma_{i,j} = \sqrt{f_j(x_i)}$  as uncertainty
- ▶ Compute residuals as:

$$\text{Res}_{i,j} = \frac{\Delta_{i,j}}{\sigma_{i,j}} \rightarrow \chi_j^2 = \sum_i \text{Res}_{i,j}^2$$

- ▶ Verify  $\chi_j^2 \approx \#\{x_i\}$  and residuals mostly between -2 and 2.

# Gamow-Teller Decay: $^{60}\text{Co}$

Standard Model

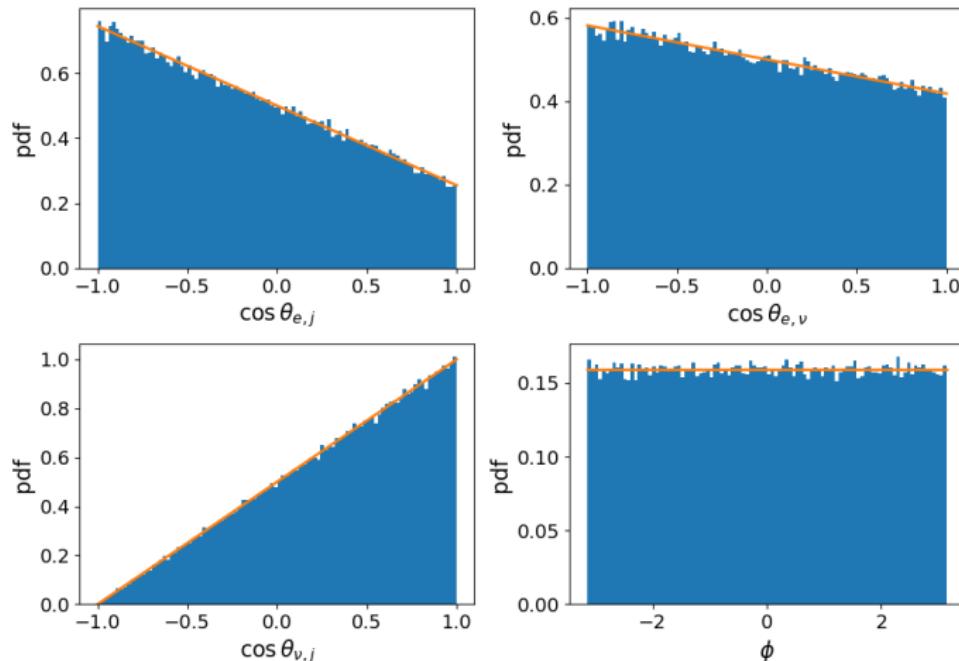


Figure: Distribution of various relevant angles,  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$ , each with a well-known distribution, and the theoretical value

# Gamow-Teller Decay: $^{60}\text{Co}$

Standard Model

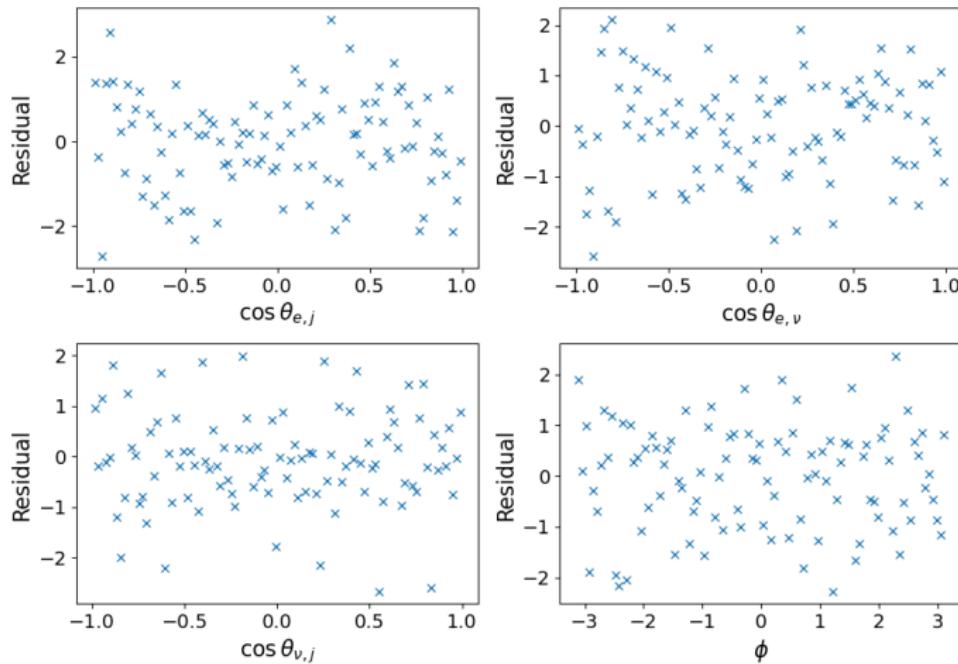


Figure: Residuals from the comparison between CRADLE simulation and theory of various relevant angles,  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$ . Values of  $\chi^2/100$ : 1.25, 0.86, 1.05, 1.03

# Gamow-Teller Decay: $^{60}\text{Co}$

$C_T = C'_T$  Real Positive

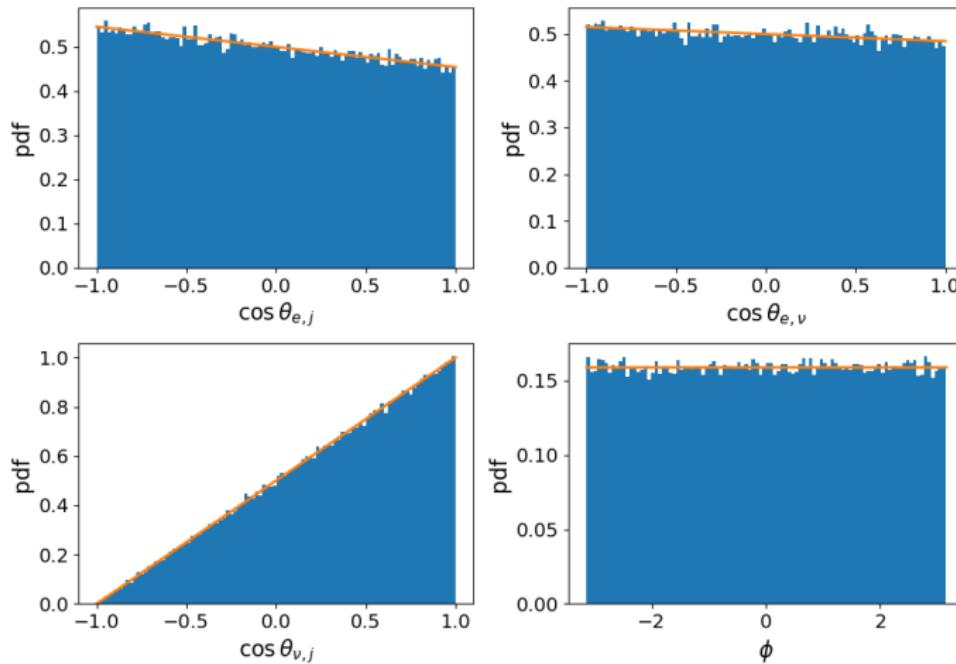


Figure: Distribution of various relevant angles,  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$ , each with a well-known distribution, and the theoretical value with  
 $C_T = C'_T = 1/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\text{Co}$

$C_T = C'_T$  Real Positive

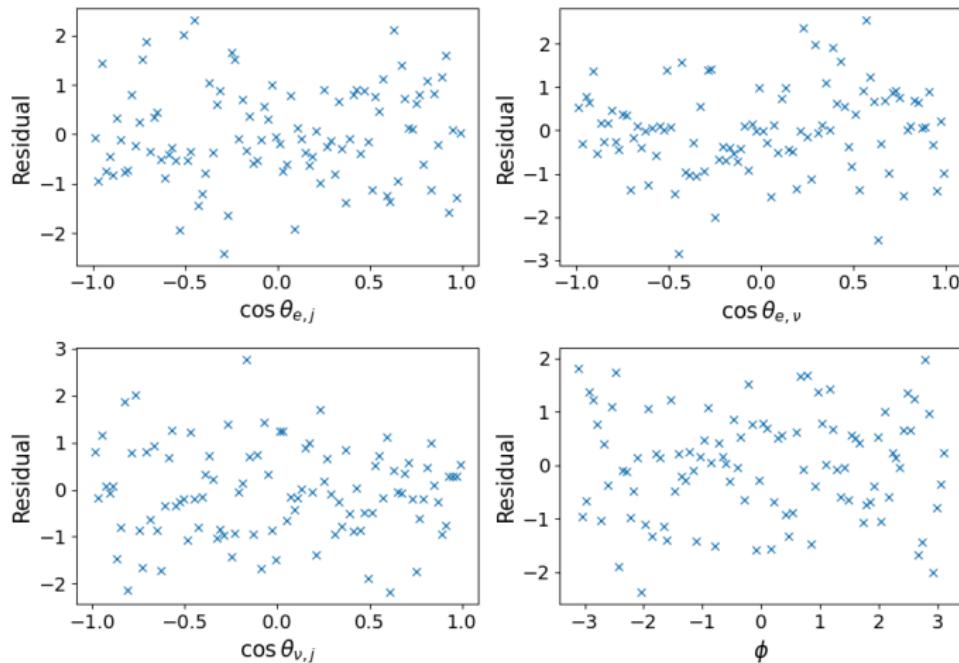


Figure: Residuals from the comparison between CRADLE simulation and theory for  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$  distributions with  $C_T = C'_T = 1/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\text{Co}$

$C_T = C'_T$  Real Negative

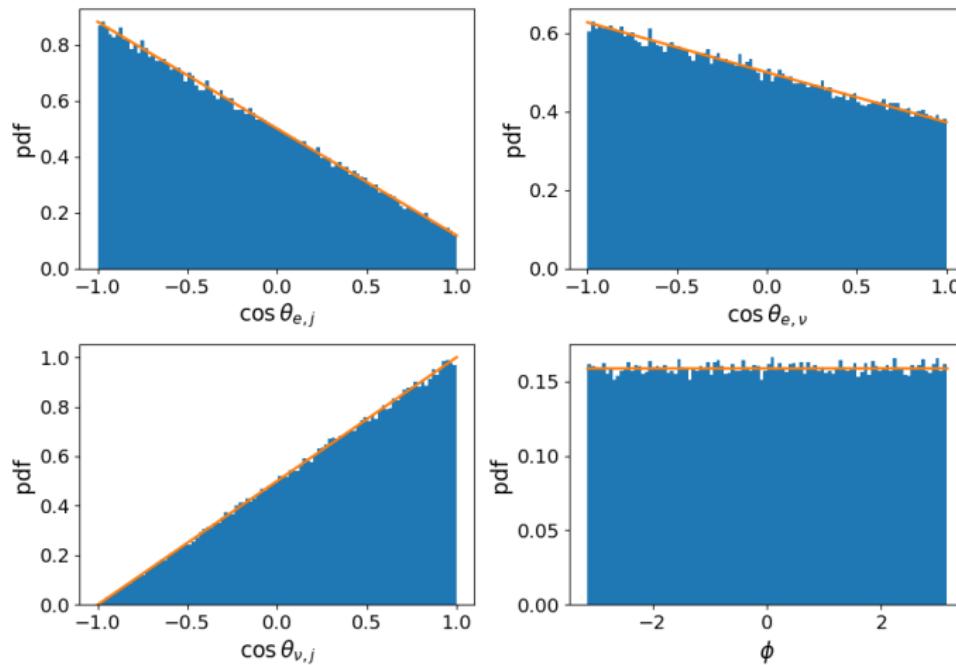


Figure: Distribution of various relevant angles,  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$ , each with a well-known distribution, and the theoretical value with  
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# Gamow-Teller Decay: $^{60}\text{Co}$

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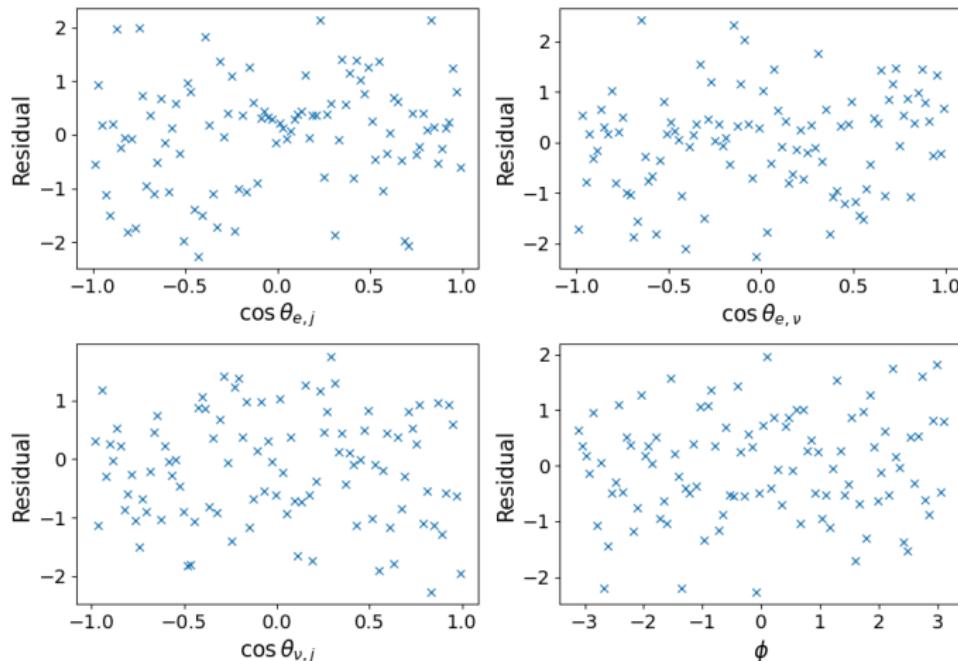


Figure: Residuals from the comparison between CRADLE simulation and theory for  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$  distributions with  $C_T = C'_T = -1/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\text{Co}$

$C_T = C'_T$  Imaginary Positive

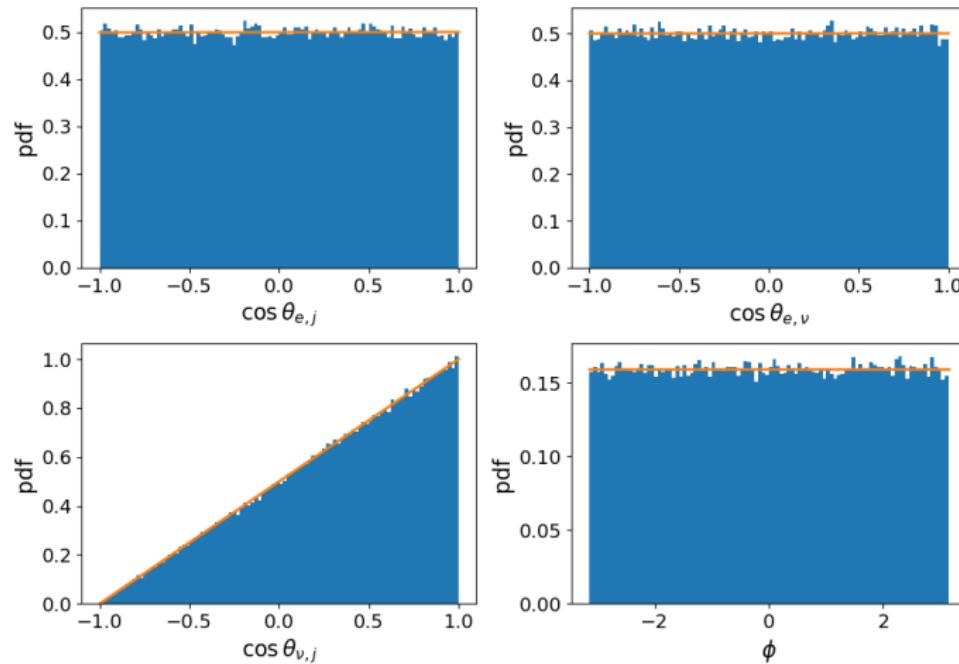


Figure: Distribution of various relevant angles,  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$ , each with a well-known distribution, and the theoretical value with  
 $C_T = C'_T = i/\sqrt{2}$

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$C_T = C'_T$  Imaginary Positive

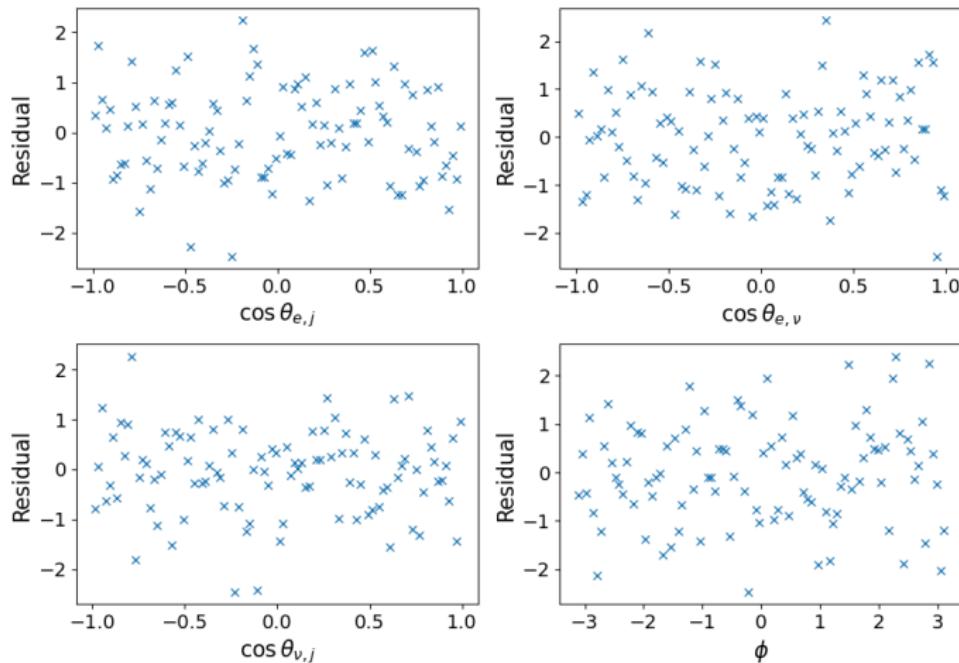
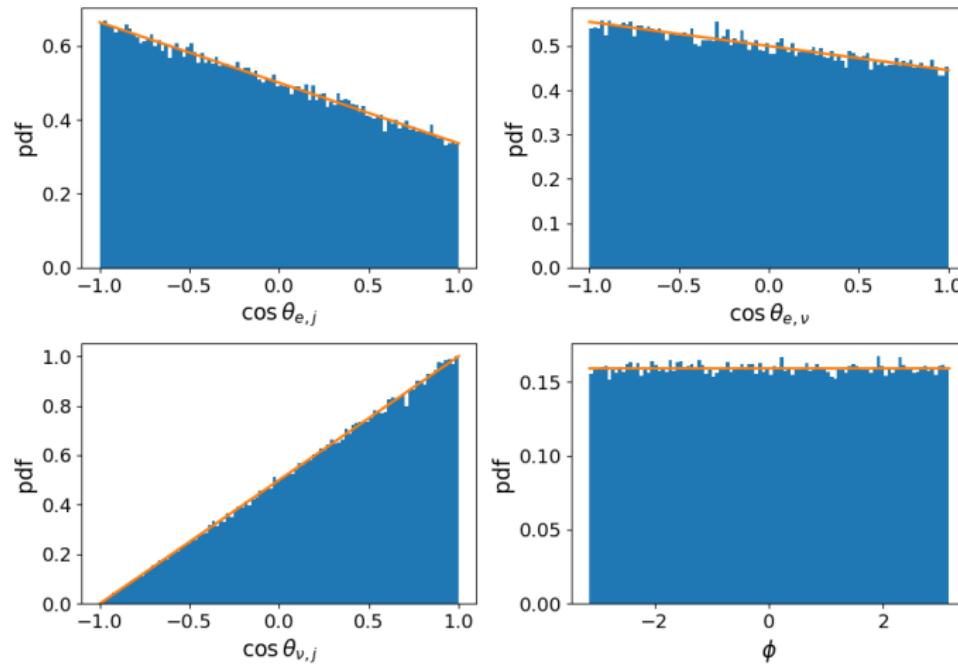


Figure: Residuals from the comparison between CRADLE simulation and theory for  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$  distributions with  $C_T = C'_T = i/\sqrt{2}$

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**Figure:** Distribution of various relevant angles,  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$ , each with a well-known distribution, and the theoretical value with  
 $C_T = C'_T = -i/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\text{Co}$

$C_T = C'_T$  Imaginary Negative

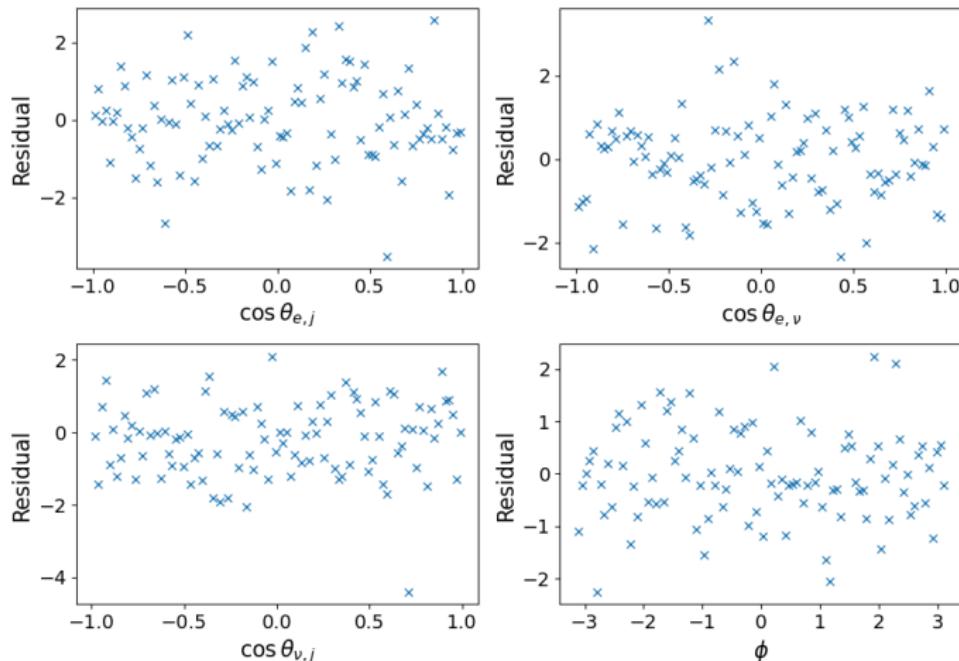


Figure: Residuals from the comparison between CRADLE simulation and theory for  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$  distributions with  $C_T = C'_T = -i/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\text{Co}$

$$C_T = -C'_T$$

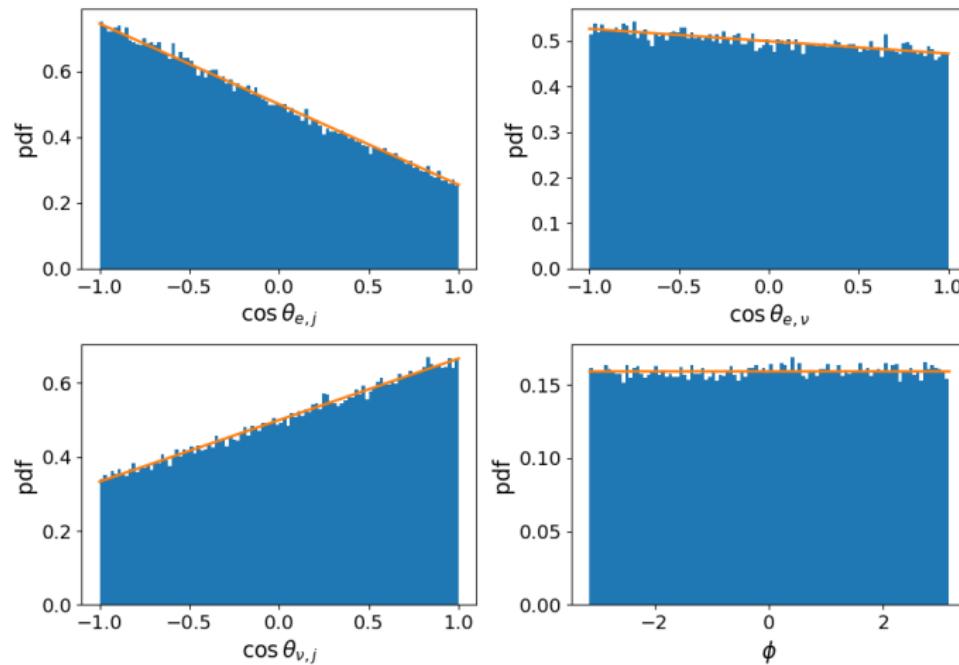


Figure: Distribution of various relevant angles,  $z_e$ ,  $z_\nu$ ,  $z_{e,\nu}$  and  $\phi$ , each with a well-known distribution, and the theoretical value with  
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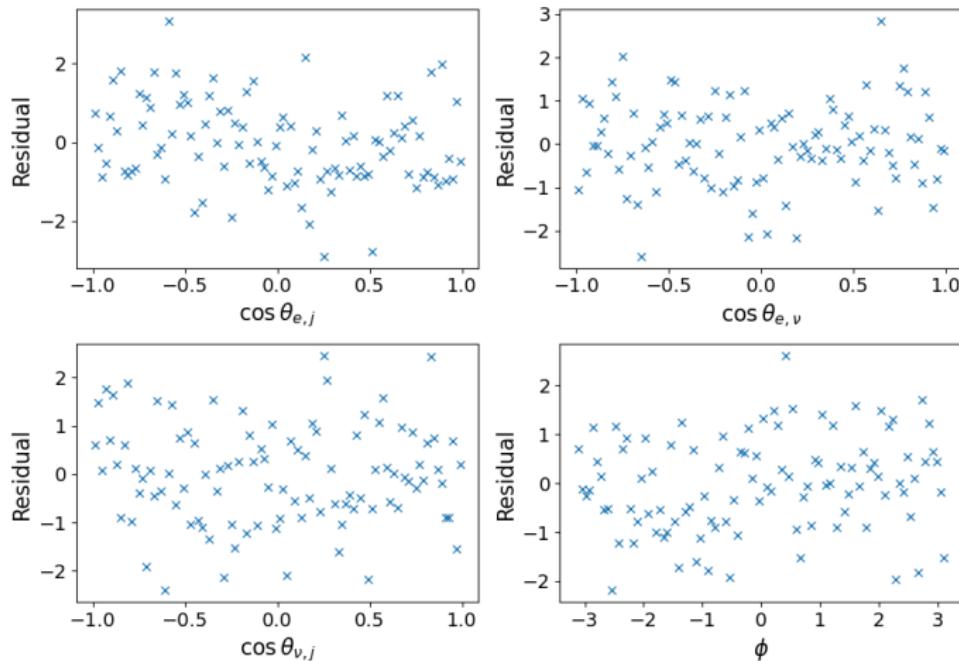


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