# CRADLE++ Tests 2

November 26, 2025

# Simulation Wu Experiment

Consider atoms of $^{60}\text{Co}$ in a thermal bath and in the presence of a magnetic field in the -z direction.

Model each nuclei as independent 11 state system (each of the values of $m_j$)

$$Z = \sum_{m_j=-5}^{5} e^{\frac{m_j \mu_N(^{60}\text{Co})B}{5k_b T}} \rightarrow P(m_j) = \frac{1}{Z} e^{\frac{m_j \mu_N(^{60}\text{Co})B}{5k_b T}}$$

From here polarisation and alignment in Z direction ($P_z$ and $\mathcal{A}_z$) can be computed

$$P_z = \frac{1}{J} \langle m_j \rangle = \sum_{m_j=-5}^{5} \frac{m_j P(m_j)}{5} \qquad \mathcal{A}_z = \frac{3\left\langle m_j^2 \right\rangle - J(J+1)}{J(2J-1)}$$

which leads to non-zero A polarisation.

# Simulation Wu Experiment

Implementation:

- N = 200000 atoms
- $|z_e| > \cos 15°$
- Realistic value of $\mu_N(^{60}\text{Co})$
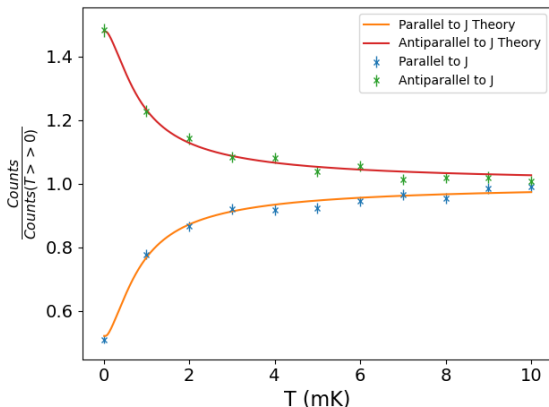- 1 sim for each T, with its $P_z$ and $\mathcal{A}_z$



Figure: Simulation of the 1957 Wu experiment using N = 200000 $^{60}\text{Co}$ nuclei for each T

# Gamow-Teller Decay: $^{60}\text{Co}$

Properties of $^{60}\text{Co}(5^+) \to {}^{60}\text{Ni}(4^+)$

- Q = 317.06 keV (good for testing, $\langle \beta_e \rangle = 0.68, \langle \gamma_e^{-1} \rangle = 0.72, \langle \alpha Z \gamma_e^{-1} \rangle = 0.15$)
- $J_f = J_i - 1 \to \lambda_{J_i, J_f} = \Lambda_{J_i, J_f} = 1$
- 2 $\gamma$ almost always ($5^+ \to 2^+$ only 1 $\gamma$)

Many cases to consider, though for realism: keep $C_A = C_A' = \text{cte}(= 1)$.

- $C_T = C_T' = 0$ (Standard Model)
- $C_T = C_T'$ pure real (and large)
- $C_T = C_T'$ pure imaginary
- $C_T = -C_T'$, either real or imaginary



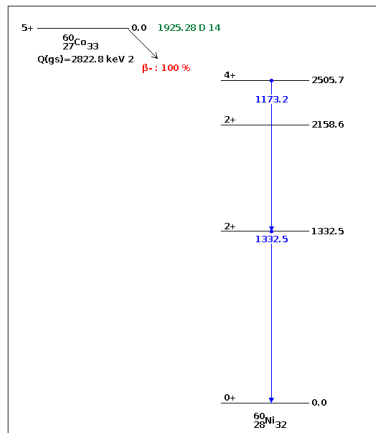Figure: Decay Scheme of $^{60}\text{Co}$ into $^{60}\text{Ni}$ featuring the only decay of interest

# Gamow-Teller Decay: $^{60}$Co

Use that distributions in $z_e$, $z_\nu$, $\cos\theta_{e,\nu} \equiv z_{e,\nu}$ and $\phi$ are known if $F \geq 0$ for all orientations of $\mathbf{p_e}, \mathbf{p_\nu}$ ($\mathbf{J}$ fixed).

$$f_1(z_e) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle A\beta_e \rangle z_e}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_2(z_\nu) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle B \rangle z_\nu}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_3(z_{e,\nu}) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle a\beta_e \rangle z_{e,\nu}}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_4(\phi) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle \left(a + \frac{c}{3}\right)\beta_e \rangle \frac{\pi^2}{16}\cos\phi + \langle D\beta_e \rangle \frac{\pi^2}{16}\sin\phi}{2\pi(1 + \langle b\gamma_e^{-1} \rangle)}$$

Averages computed numerically using f(E) from the simulation data itself (avoid computing the Fermi function myself)

# Gamow-Teller Decay: $^{60}\mathrm{Co}$

Numerical verification

From distributions:

- Compute difference $\Delta_{i,j} = f_j(x_i) - f_{j,th}(x_i)$ for each point $x_i$ in each distribution $f_j$
- Use $\sigma_{i,j} = \sqrt{f_j(x_i)}$ as uncertainity
- Compute residuals as:

$$Res_{i,j} = \frac{\Delta_{i,j}}{\sigma_{i,j}} \rightarrow \chi_j^2 = \sum_i Res_{i,j}^2$$

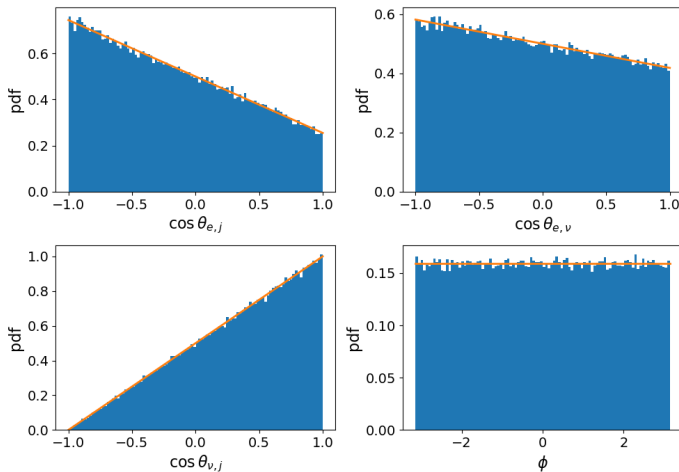- Verify $\chi_j^2 \approx \#\{x_i\}$ and residuals mostly between -2 and 2.

# Gamow-Teller Decay: $^{60}\text{Co}$

Standard Model



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value

# Gamow-Teller Decay: $^{60}\text{Co}$
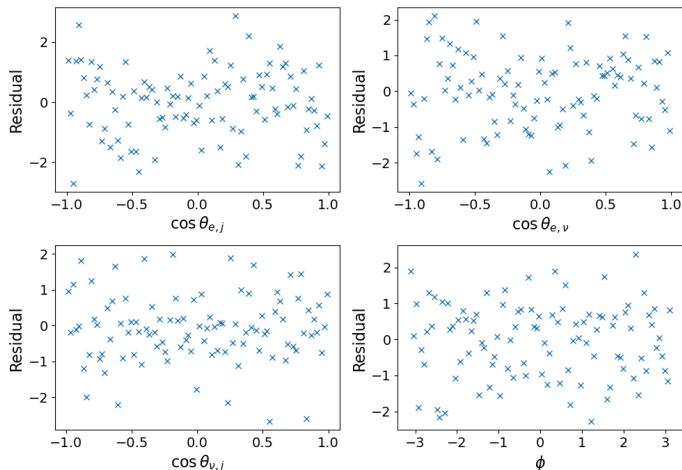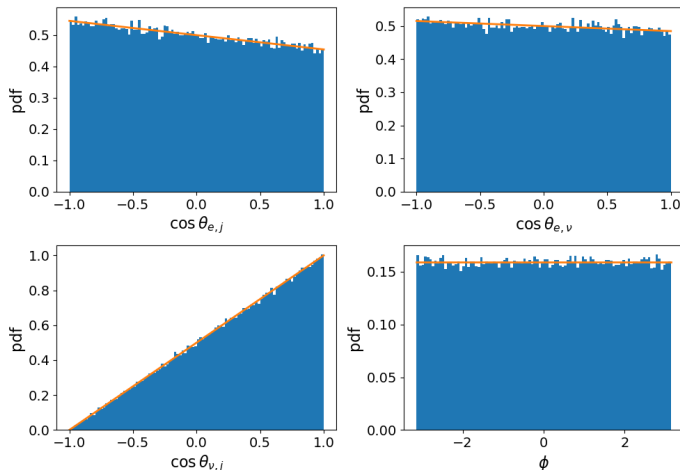
Standard Model



Figure: Residuals from the comparison between CRADLE simulation and theory of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$.

# Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Real Positive



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value with $C_T = C_T' = 1/\sqrt{2}$

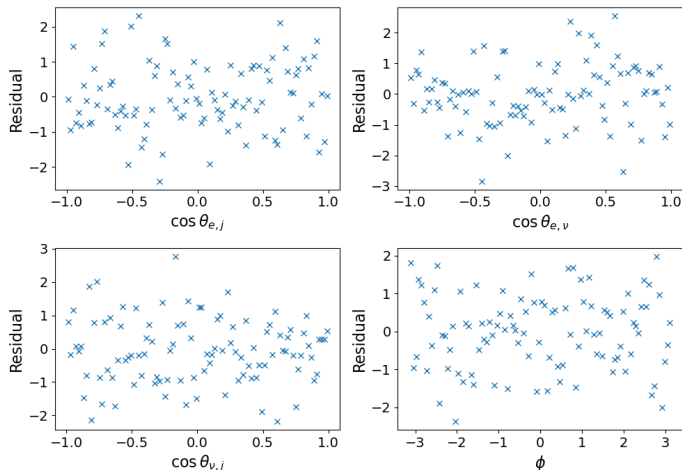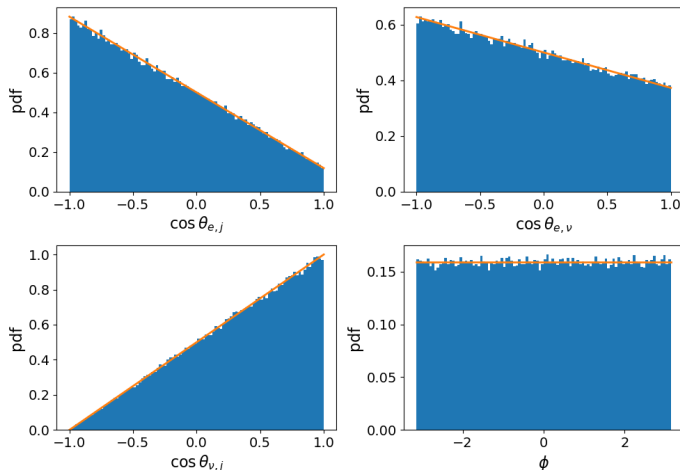# Gamow-Teller Decay: $^{60}\text{Co}$

$C_T = C_T'$ Real Positive



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $C_T = C_T' = 1/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\text{Co}$

$C_T = C_T'$ Real Negative



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value with $C_T = C_T' = -1/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\mathrm{Co}$
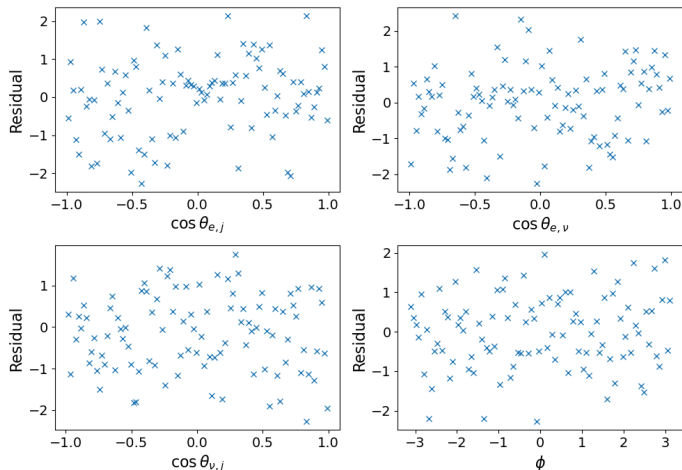
$C_T = C_T'$ Real Negative



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $C_T = C_T' = -1/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\mathrm{Co}$
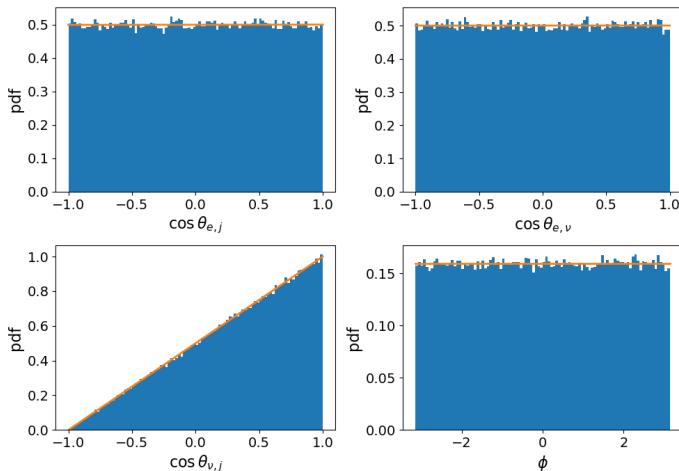
$C_T = C_T'$ Imaginary Positive



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value with $C_T = C_T' = i/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\mathrm{Co}$
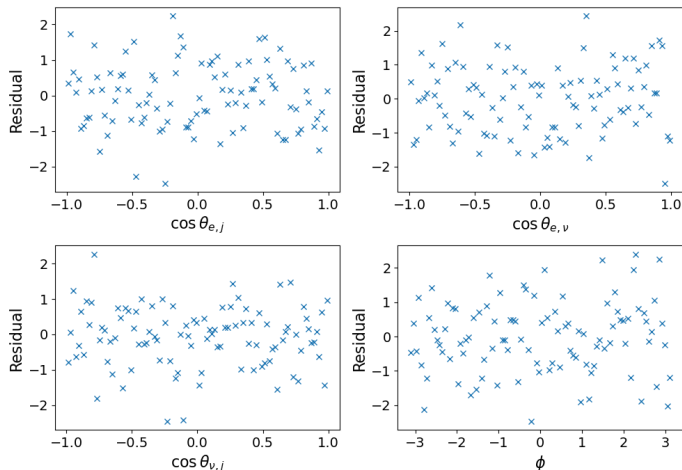
$C_T = C_T'$ Imaginary Positive



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $C_T = C_T' = i/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\text{Co}$
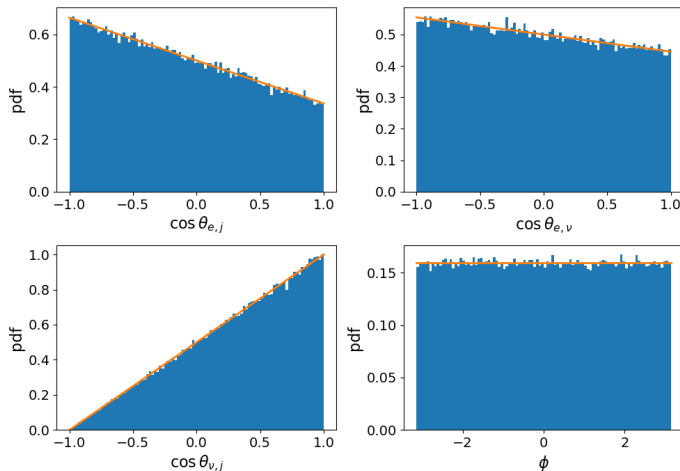
$C_T = C'_T$ Imaginary Negative



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value with $C_T = C'_T = -i/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\text{Co}$
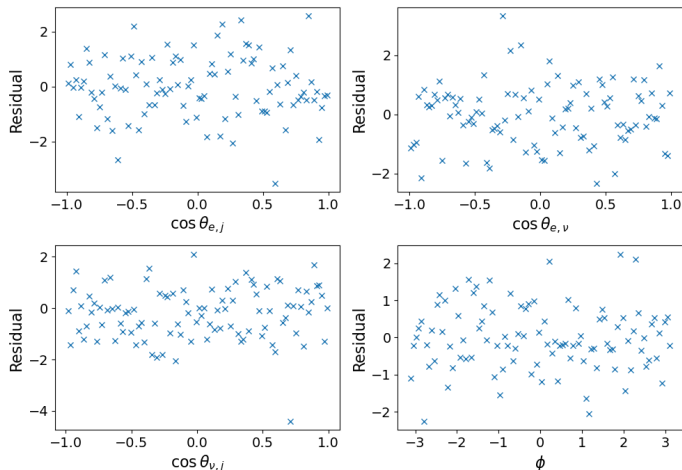
$C_T = C_T'$ Imaginary Negative



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $C_T = C_T' = -i/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\mathrm{Co}$
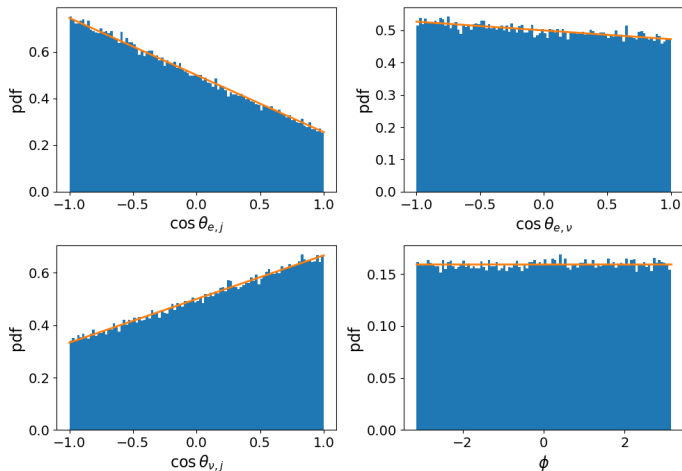
$C_T = -C_T'$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value with $C_T = -C_T' = 1/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\mathrm{Co}$
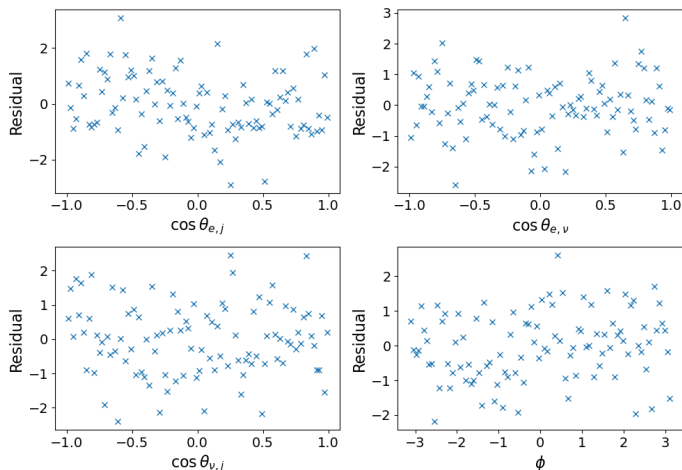
$C_T = -C_T'$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $C_T = -C_T' = 1/\sqrt{2}$

# Gamow-Teller Decay: $^{60}\mathrm{Co}$

Summary

We present here the $\chi^2$ values.

|  | $\chi^2(z_e)$ | $\chi^2(z_\nu)$ | $\chi^2(z_{e,\nu})$ | $\chi^2(\phi)$ |
|---|---|---|---|---|
| SM | 1.253 | 0.864 | 1.047 | 1.026 |
| $Re(C_T) > 0$ | 0.915 | 0.915 | 0.944 | 0.936 |
| $Re(C_T) < 0$ | 1.000 | 0.856 | 0.982 | 0.859 |
| $Im(C_T) > 0$ | 0.826 | 0.696 | 0.959 | 1.066 |
| $Im(C_T) < 0$ | 1.208 | 1.051 | 1.030 | 0.739 |
| $C_T + C'_T = 0$ | 1.149 | 1.051 | 0.905 | 0.915 |

Table: Values of $\chi^2/100$ for each distribution for each of the tests performed. The first one is the Standard Model values, rows 2 to 5 correspond to tests where $C_T = C'_T$ and the last one features $C_T = -C'_T$. For all tests $M_{GT} = 1$, $C_A = C'_A = 1.2754$ and $|C_T| = |C'_T| = 1/\sqrt{2}$

All close to 1

# Mixed Decays: $^{39}$Ca

Now we consider a mixed $\beta^+$
decay: $^{39}\text{Ca}\left(\frac{3}{2}\right)^+ \rightarrow {}^{39}\text{K}\left(\frac{3}{2}\right)^+$
Properties:

- Q = 5502.5 keV (a bit
  inconvenient, $\langle \gamma_e^{-1} \rangle = 0.2$
  and $\alpha Z = 0.14$)
- No $\gamma$ produced

First, do tests with 2 non-zero
coupling for pairs with $N = 10^6$
decays. Shown those with
non-zero correlations A, B and D
that are not present in a
Gamow-Teller decay. We use
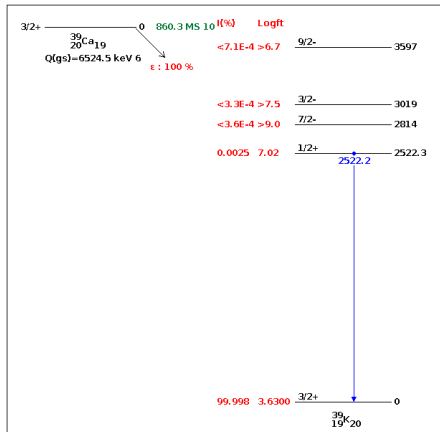$|M_{GT}C_{A,T}^{(\prime)}| = |M_F C_{V,S}^{(\prime)}| = 1$ for
convenience



Figure: Decay Scheme of $^{39}$Ca into
$^{39}$K.

# Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_T$

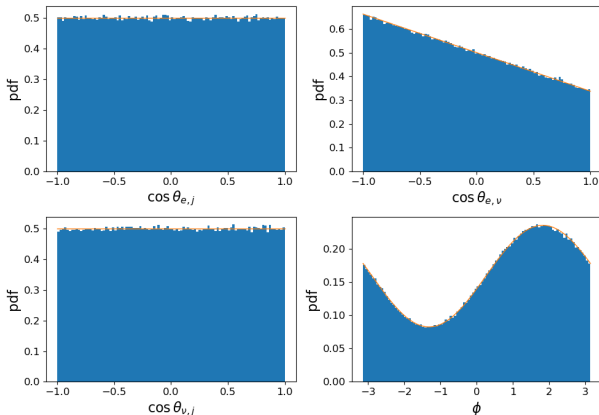Non zero D proportional to $\langle \beta_e \rangle$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_T = 0.6 - 0.8i$ and rest of couplings 0

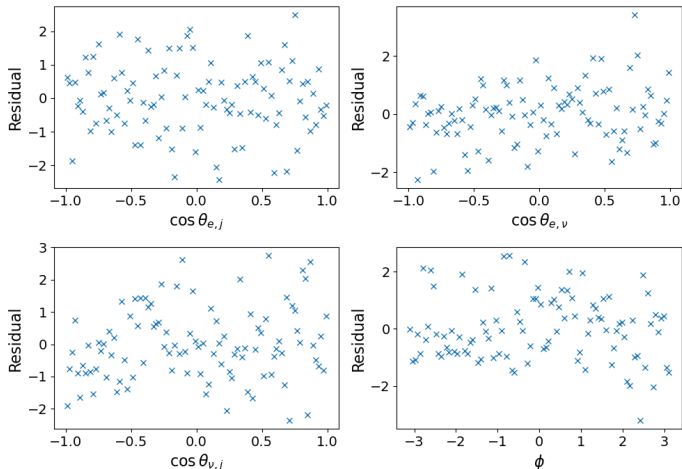# Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_T$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_T = 0.6 - 0.8i$ and rest of couplings 0

# Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_T'$

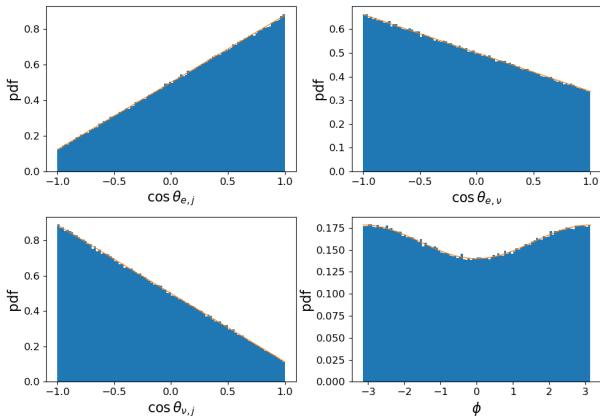Non zero $A > 0, B < 0$ proportional to $\langle \beta_e \rangle$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_S = 1, M_{GT} C_T' = 1$ and rest of couplings 0
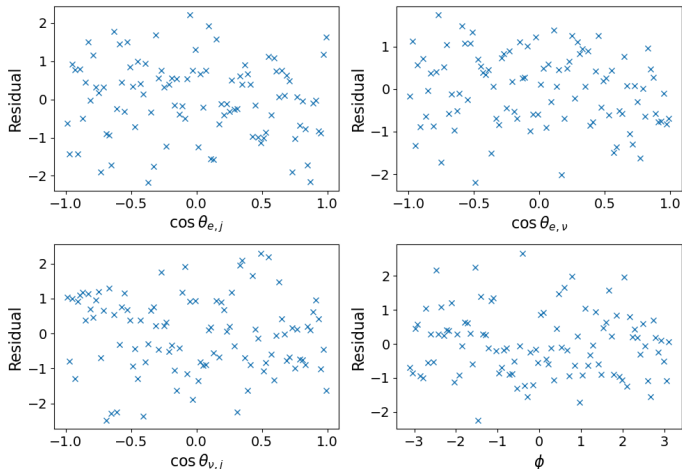
# Mixed Decay: $^{39}\text{Ca}$

Real $C_S C'_T$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_S = 1, M_{GT} C'_T = 1$ and rest of couplings 0

# Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A$

Non zero $D > 0$ proportional to $\langle \alpha Z \gamma_e^{-1} \rangle$: very hard to see
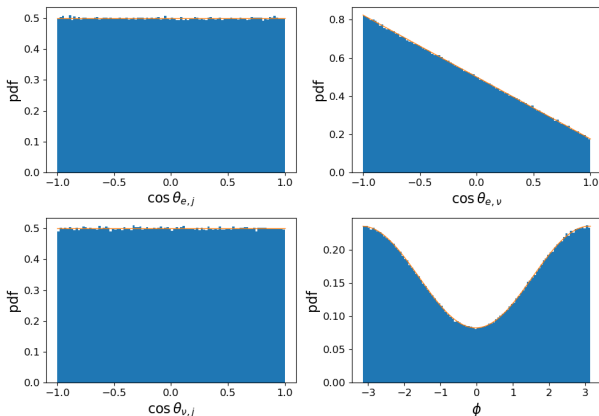


Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_S = 1, M_{GT} C_A = 1$ and rest of couplings 0
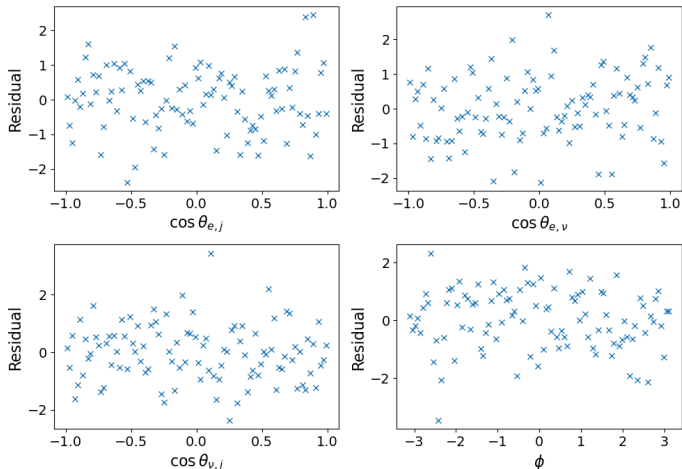
# Mixed Decay: $^{39}\text{Ca}$

Real $C_S C_A$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_S = 1, M_{GT} C_A = 1$ and rest of couplings 0

# Mixed Decay: $^{39}\text{Ca}$

Real $C_S C_A$: D Component

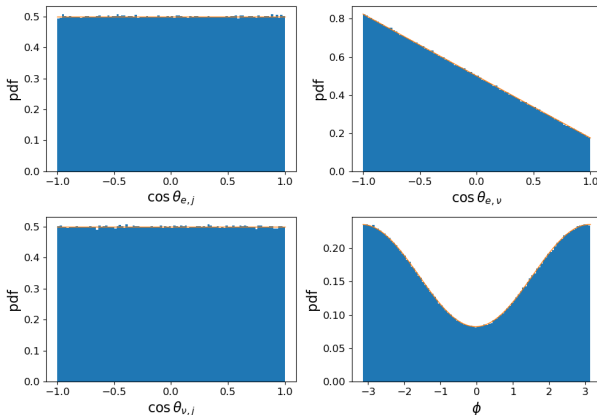To see the D component: increase N (number of decays) from $10^6$ to $3 \cdot 10^6$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ and the theoretical value for $M_F C_S = 1, M_{GT} C_A = 1$

# Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A$:  D Component

To see the D component:

▶ Increase N

▶ Keep only the sine
  component ($b = 0$)

$$f_4(\phi) \propto \left(a + \frac{c}{3}\right) \langle \beta_e \rangle \frac{\pi^2}{16} \cos \phi$$

$$+ \langle D\beta_e \rangle \frac{\pi^2}{16} \sin \phi + 1$$

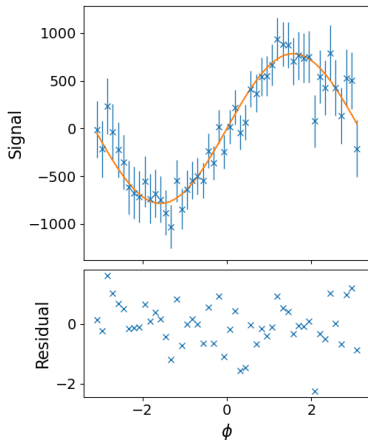▶ Reduce number of bins
  ($100 \rightarrow 50$)



Figure: In top, cosine component of the $\phi$
distribution (blue) and its theoretical vaule
(orange). At the bottom, residuals.

# Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A'$

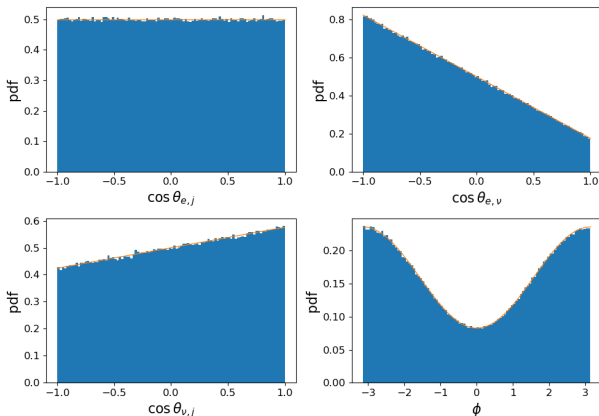Non zero $B > 0$ proportional to $\left\langle \gamma_e^{-1} \right\rangle$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_S = 1, M_{GT} C_A' = 1$ and rest of couplings 0
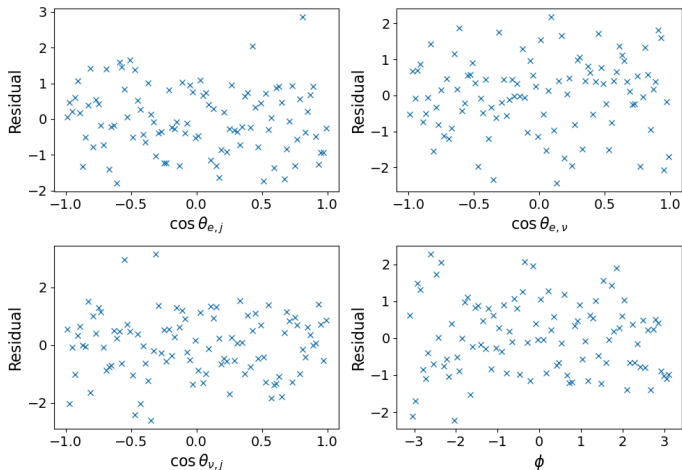
# Mixed Decay: $^{39}\text{Ca}$

Real $C_S C_A'$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_S = 1$, $M_{GT} C_A' = 1$ and rest of couplings 0

# Mixed Decay: $^{39}\mathrm{Ca}$

### Imaginary $C_S C_A'$

Non zero $A < 0$ proportional to $\langle \alpha Z \gamma_e^{-1} \rangle$: hard to see
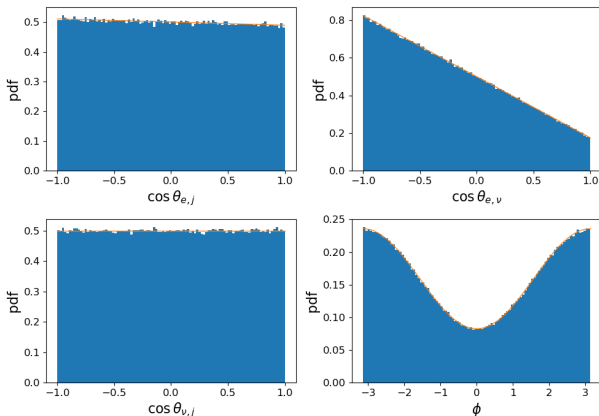


Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_A' = 0.6 - 0.8i$ and rest of couplings 0
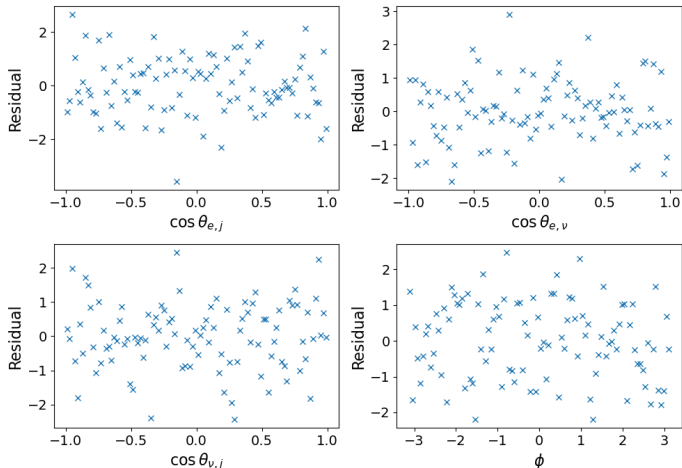
# Mixed Decay: $^{39}\text{Ca}$

Imaginary $C_S C'_A$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_S = 0.8 + 0.6i$, $M_{GT} C'_A = 0.6 - 0.8i$ and rest of couplings 0

# Mixed Decay: $^{39}\text{Ca}$

## Imaginary $C_S C'_A$: A Component

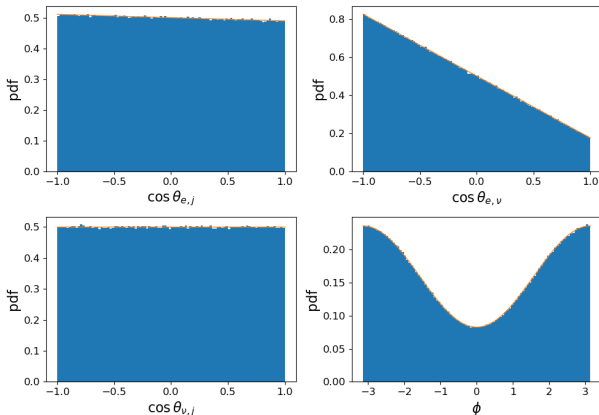To see the A component: increase N (number of decays) from $10^6$ to $3 \cdot 10^6$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ and the theoretical value for $M_F C_S = 0.8 + 0.6i$, $M_{GT} C'_A = 0.6 - 0.8i$

# Mixed Decay: $^{39}\text{Ca}$

Imaginary $C_S C_A'$: A Component

To see the A component:

- ▶ Increase N
- ▶ Keep only the linear component ($b = 0$)

$$f_1(z_e) \propto 1 + \langle A\beta \rangle\, z_e$$
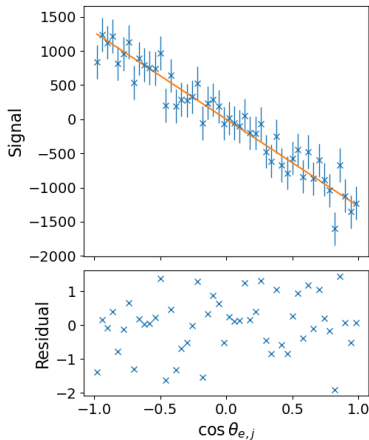
- ▶ Reduce number of bins ($100 \rightarrow 50$)



Figure: In top, linear component of the $z_e$ distribution (blue) and its theoretical vaule (orange). At the bottom, residuals.

# Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_V C_A$

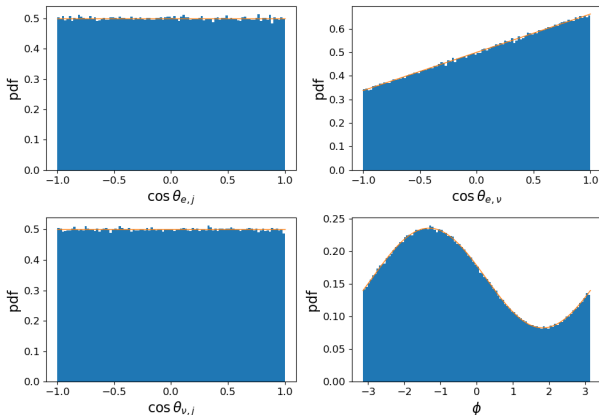Non-zero $D < 0$ proportional to $\langle \beta_e \rangle$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_V = 0.8 + 0.6i$, $M_{GT} C_A = 0.6 - 0.8i$ and rest of couplings 0

# Mixed Decay: $^{39}\text{Ca}$
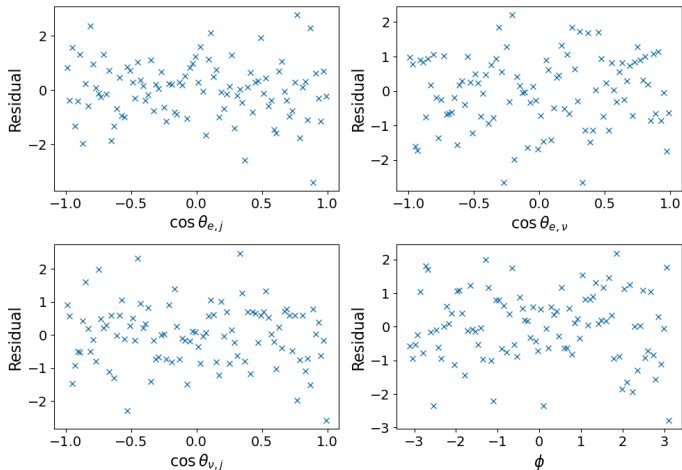
Imaginary $C_V C_A$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_V = 0.8 + 0.6i$, $M_{GT} C_A = 0.6 - 0.8i$ and rest of couplings 0

# Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C'_A$

Non-zero $A, B < 0$ proportional to $\langle \beta_e \rangle$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_V = 1$, $M_{GT} C'_A = 1$ and rest of couplings 0

# Mixed Decay: $^{39}\text{Ca}$
## Real $C_V C_A'$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_V = 1$, $M_{GT} C_A' = 1$ and rest of couplings 0

# Mixed Decay: $^{39}\text{Ca}$

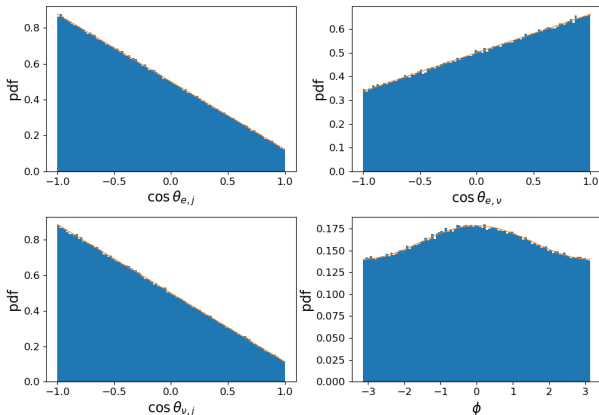Non-zero $D < 0$ proportional to $\langle \alpha Z \gamma_e^{-1} \rangle$: very hard to see



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_V = 1$, $M_{GT} C_T = 1$ and rest of couplings 0

# Mixed Decay: $^{39}\mathrm{Ca}$

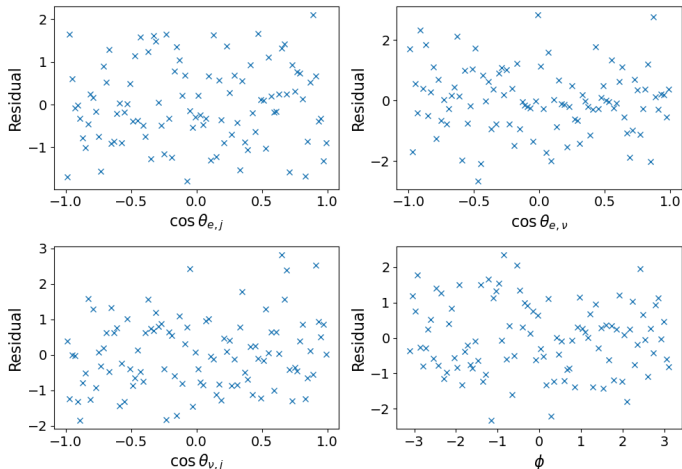## Real $C_V C_T$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_V = 1$, $M_{GT} C_T = 1$ and rest of couplings 0

# Mixed Decay: $^{39}\text{Ca}$

Real $C_V C_T'$

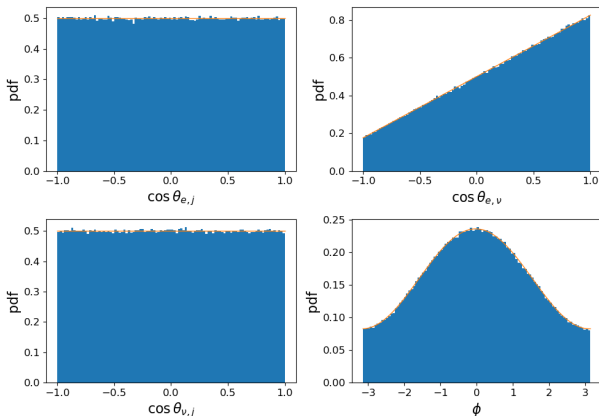Non-zero $B > 0$ proportional to $\left\langle \gamma_e^{-1} \right\rangle$



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_V = 1$, $M_{GT} C_T' = 1$ and rest of couplings 0
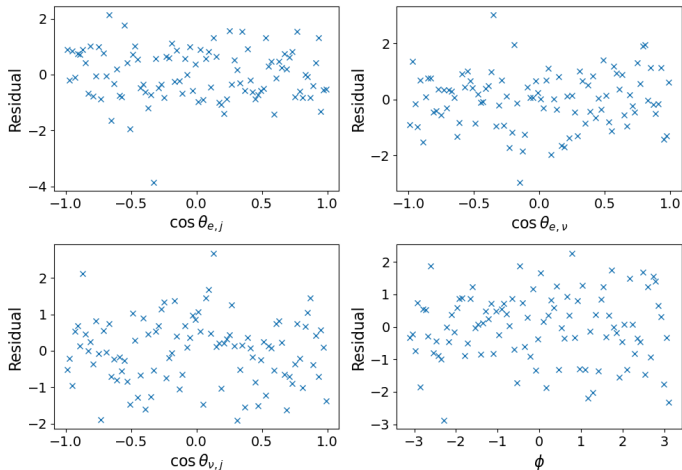
# Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V\, C_T'$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F\, C_V = 1, M_{GT}\, C_T' = 1$ and rest of couplings 0

# Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_V C_T'$

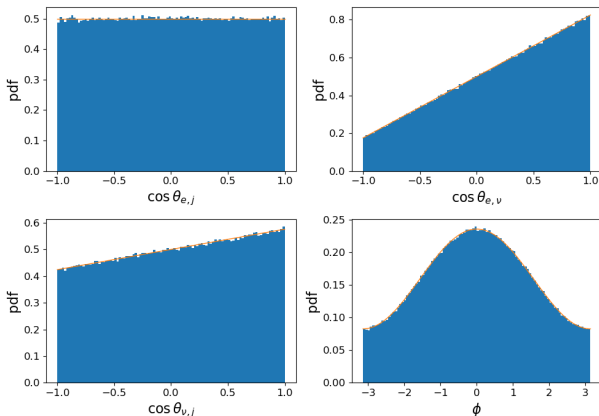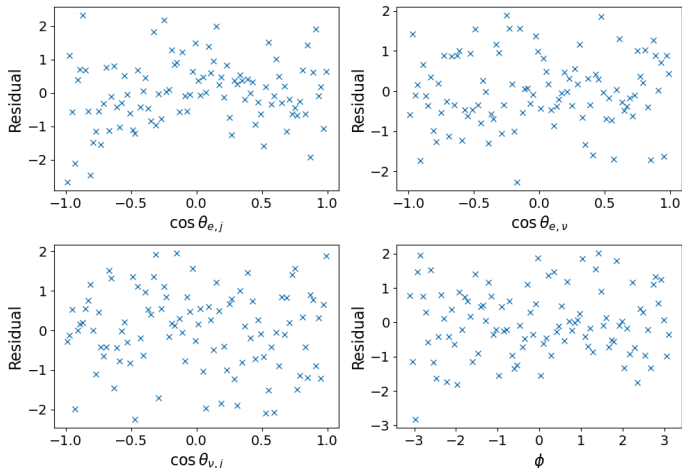Non-zero $A < 0$ proportional to $\langle \alpha Z \gamma_e^{-1} \rangle$: hard to see



Figure: Distribution of various relevant angles, $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$, each with a well-known distribution, and the theoretical value for $M_F C_V = 0.6 - 0.8i$, $M_{GT} C_T' = 0.8 + 0.6i$ and rest of couplings 0

# Mixed Decay: $^{39}\text{Ca}$

Imaginary $C_V C'_T$



Figure: Residuals from the comparison between CRADLE simulation and theory for $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ distributions with $M_F C_V = 0.6 - 0.8i$, $M_{GT} C'_T = 0.8 + 0.6i$ and rest of couplings 0

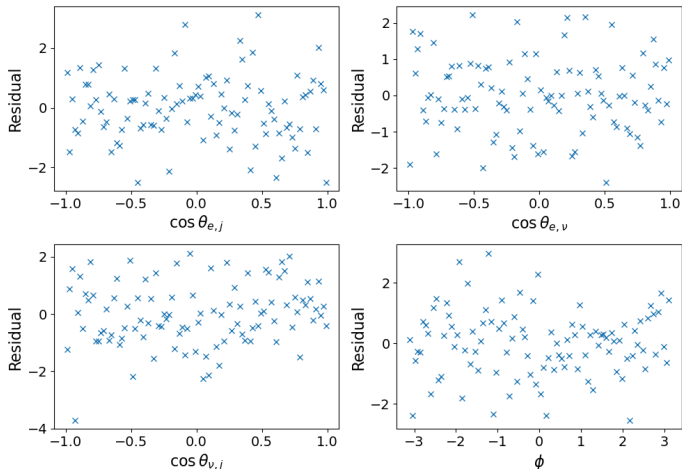# Mixed Decay: $^{39}\mathrm{Ca}$

Summary

Table: Values of $\chi^2/100$ for each distribution for each of the tests

|  | $\chi^2(z_e)$ | $\chi^2(z_\nu)$ | $\chi^2(z_{e,\nu})$ | $\chi^2(\phi)$ |
|---|---|---|---|---|
| Re $C_S C_S'$ | 1.409 | 1.207 | 0.840 | 1.111 |
| Im $C_S C_S'$ | 0.997 | 1.109 | 0.981 | 1.229 |
| Im $C_S C_T$ | 1.117 | 1.219 | 0.924 | 1.256 |
| Re $C_S C_T'$ | 0.975 | 1.185 | 0.727 | 0.908 |
| Re $C_S C_A$ | 0.820 | 0.951 | 0.877 | 1.057 |
| Re $C_S C_A'$ | 0.822 | 1.127 | 1.046 | 0.963 |
| Im $C_S C_A'$ | 1.162 | 0.905 | 0.869 | 1.133 |
| Re $C_S' C_T$ | 0.891 | 1.086 | 0.974 | 1.003 |
| Im $C_S' C_T'$ | 0.930 | 1.179 | 1.360 | 1.268 |
| Re $C_S' C_A'$ | 0.895 | 0.873 | 0.893 | 1.225 |
| Im $C_S' C_A$ | 1.024 | 0.898 | 0.844 | 0.908 |
| Im $C_S' C_A$ | 1.121 | 0.912 | 0.972 | 0.733 |
| Re $C_T C_T'$ | 1.301 | 1.069 | 0.966 | 1.130 |
| Im $C_T C_T'$ | 1.015 | 1.087 | 0.981 | 1.051 |

# Mixed Decay: $^{39}\text{Ca}$

Summary

| | $\chi^2(z_e)$ | $\chi^2(z_\nu)$ | $\chi^2(z_{e,\nu})$ | $\chi^2(\phi)$ |
|---|---|---|---|---|
| Re $C_T C_V$ | 0.843 | 0.801 | 0.952 | 1.096 |
| Re $C_T C_V'$ | 0.942 | 0.911 | 0.947 | 1.262 |
| Im $C_T C_V'$ | 0.832 | 0.809 | 1.093 | 0.877 |
| Re $C_T' C_V'$ | 0.833 | 0.882 | 1.107 | 1.118 |
| Re $C_T' C_V$ | 0.933 | 0.997 | 0.757 | 0.962 |
| Im $C_T' C_V$ | 1.163 | 1.169 | 1.017 | 1.147 |
| Re $C_T C_A'$ | 0.724 | 0.832 | 1.062 | 1.026 |
| Im $C_T C_A'$ | 1.181 | 1.243 | 1.001 | 0.711 |
| Re $C_V C_V'$ | 0.947 | 0.962 | 0.957 | 1.092 |
| Im $C_V C_V'$ | 0.980 | 1.304 | 0.930 | 1.124 |
| Re $C_A C_A'$ | 1.089 | 0.834 | 0.963 | 1.239 |
| Im $C_A C_A'$ | 0.965 | 1.019 | 0.841 | 0.954 |
| Re $C_V C_A'$ | 0.848 | 0.992 | 1.147 | 0.988 |
| Im $C_V C_A$ | 1.097 | 0.817 | 1.043 | 1.058 |

Table: Values of $\chi^2/100$ for each distribution for each of the tests

# Plausible scenarios with non-zero D

Looking at D in terms of $C_X, C'_X$

$$D = 2\delta_{J_i, J_f} M_F M_{GT} \sqrt{\frac{J_i}{J_i + 1}} \left[ Im \left( C_S C_T^* + C'_S C'_T{}^* - C_V C_A^* - C'_V C'_A{}^* \right) \right.$$

$$\left. \mp \frac{\alpha Z m_e}{p_e} Re \left( C_S C_A^* + C_S C'_A{}^* + C_T C_V^* + C'_T C'_V{}^* \right) \right]$$

Multiple simple options:

- ▶ $C_A = C'_A$ with an imaginary component, $C_V = C'_V = 1$
- ▶ Keep $C_A^{(\prime)}$ and $C_V^{(\prime)}$, and add either a $C_S = C'_S$ or a $C_T = C'_T$ coupling with a real component (simpler $C_S$)
- ▶ Keep $C_A^{(\prime)}$ and $C_V^{(\prime)}$, add add $C_S = -C'_S$ and $C_T = -C'_T$ couplings, one real and other imaginary
- ▶ Keep $C_A^{(\prime)}$ and $C_V^{(\prime)}$, and add $C_S = C'_S$ and $C_T = C'_T$ couplings, one real and other imaginary (not so simple)

# Imaginary $C_V C_A$

Idea: add just a phase shift between $C_A$ and $C_V$, value $\theta$.

- ▶ Global symmetry to a phase shift of all coupling constants $\rightarrow$ $C_V$ chosen 1
- ▶ $|C_A|$ untouched $\rightarrow \xi, a, b = 0, c$ constant
- ▶ $\delta D \propto \sin(\theta) \approx \theta$ and $\delta D \propto \langle \beta_e \rangle$ (good)
- ▶ $\delta A, \delta B \propto 1 - \cos(\theta) = O(\theta^2)$

$$A\xi = 2|M_{GT}|^2 \lambda_{JJ'} \left[ \pm Re\left( C_T C_T'^* - C_A C_A'^* \right) + \frac{\alpha Z m_e}{p_e} Im\left( C_T C_A'^* + C_T' C_A^* \right) \right]$$

$$+ 2\delta_{J'J} M_F M_{GT} \sqrt{\frac{J}{J+1}} \left[ Re(C_S C_T'^* + C_S' C_T^* - C_V C_A'^* - C_V' C_A^*) \right.$$

$$\left. \pm \frac{\alpha Z m}{p_e} Im\left( C_S C_A'^* + C_S' C_A^* - C_V C_T'^* - C_V' C_T^* \right) \right]$$
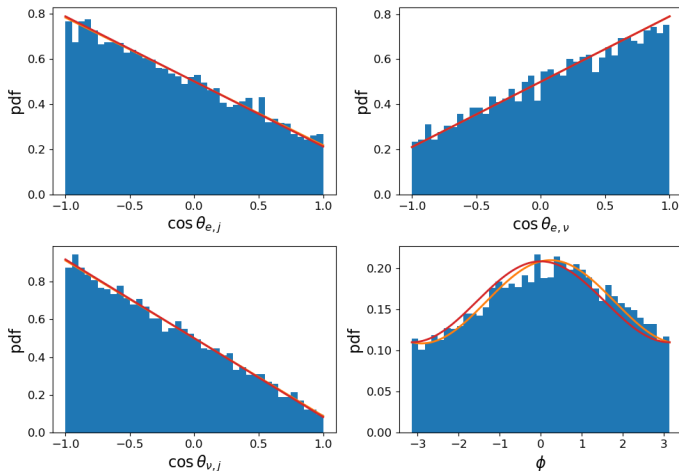
# Imaginary $C_V C_A$



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with $N = 10000$ decays of $^{39}\text{Ca}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (red) imaginary component of $C_A$
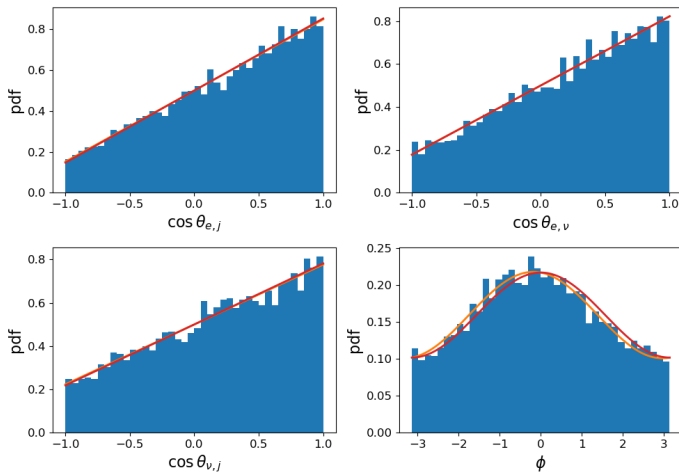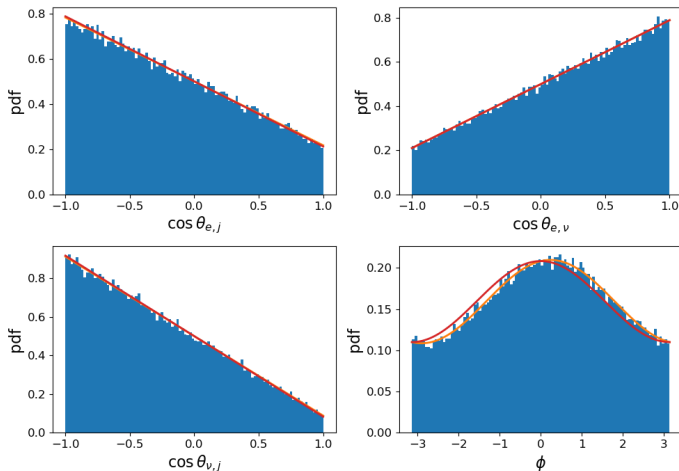
# Imaginary $C_V C_A$



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with $N = 10000$ decays of $^{23}\mathrm{Mg}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (red) imaginary component of $C_A$
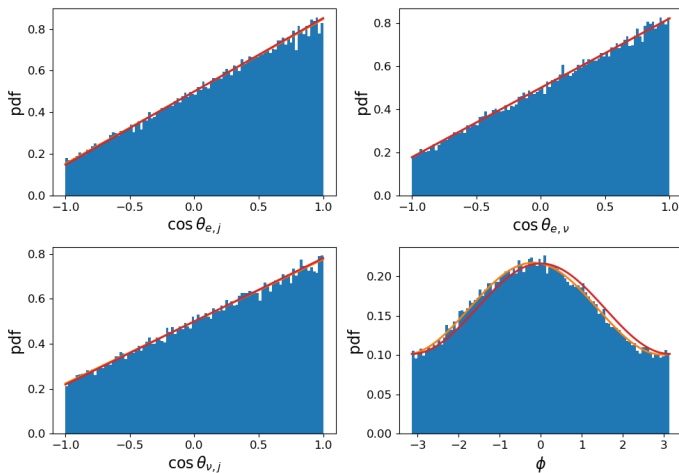
# Imaginary $C_V C_A$



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with $N = 100000$ decays of $^{39}\text{Ca}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (red) imaginary component of $C_A$
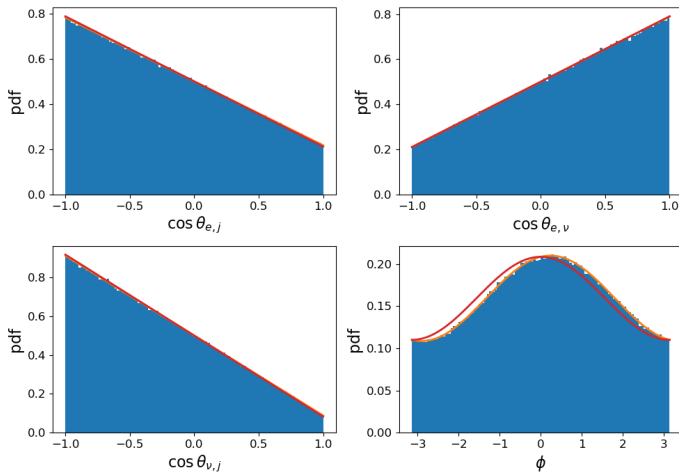
# Imaginary $C_V C_A$



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with N = 100000 decays of $^{23}\mathrm{Mg}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (red) imaginary component of $C_A$
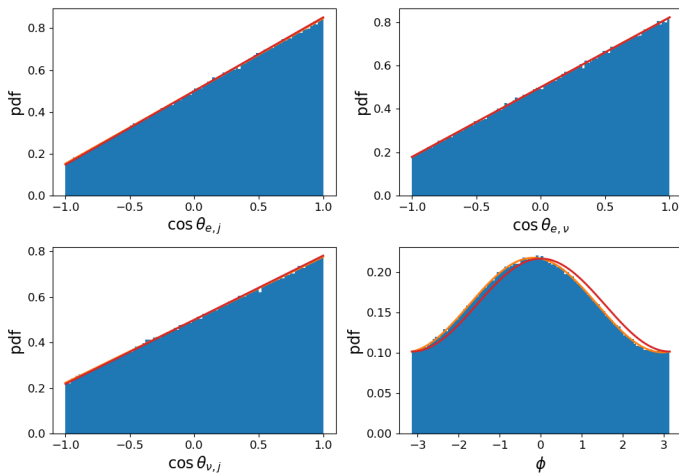
# Imaginary $C_V C_A$



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with $N = 1000000$ decays of $^{39}\text{Ca}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (red) imaginary component of $C_A$

# Imaginary $C_V C_A$



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with N = 1000000 decays of $^{23}\mathrm{Mg}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (red) imaginary component of $C_A$

# Imaginary $C_V C_A$

For theory: using $|C_V| = |C_A| = 1$ and $\theta$ the angle between $C_V$ and $C_A$

$$D \langle \beta_e \rangle = \frac{4 M_{GT} \sin \theta \sqrt{\frac{2}{3}}}{2 + 2|M_{GT}|^2} \langle \beta_e \rangle$$

For experiment: noting $b = 0$, we can relate $N(\phi > 0)$ and $N(\phi < 0)$ to the distribution

$$\frac{N(\phi > 0)}{N} = \int_0^\pi f_4(\phi) = \int_0^\pi \frac{1 + \left\langle \left(a + \frac{c}{3}\right) \beta_e \right\rangle \frac{\pi^2}{16} \cos \phi + \langle D\beta_e \rangle \frac{\pi^2}{16} \sin \phi}{2\pi}$$

$$\frac{N(\phi > 0)}{N} = \frac{1}{2} + D \langle \beta_e \rangle \frac{\pi}{16} \to D \langle \beta_e \rangle = \frac{N(\phi > 0) - N(\phi < 0)}{N(\phi > 0) + N(\phi < 0)} \frac{8}{\pi}$$
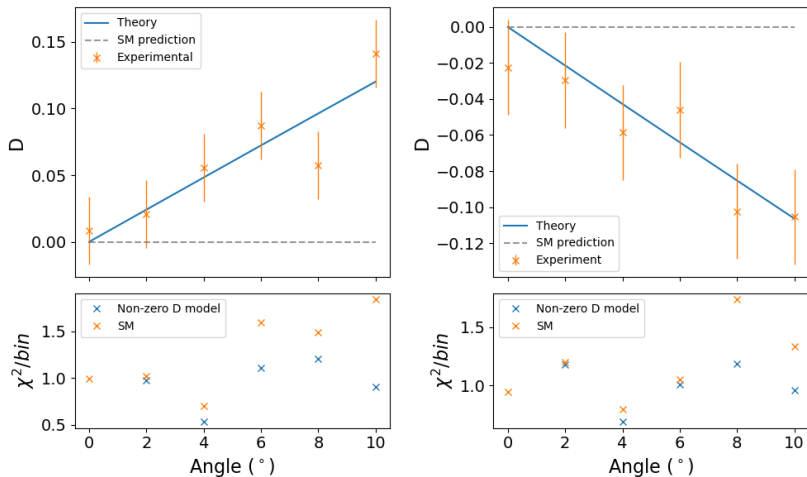
# Imaginary $C_V C_A$



Figure: Experimental values of D and $\chi^2$ of the distribution of $\phi$ for ${}^{39}\mathrm{Ca}$ and ${}^{23}\mathrm{Mg}$ for N = 10000 decays

# Imaginary $C_V C_A$



Figure: Experimental values of D and $\chi^2$ of the distribution of $\phi$ for $^{39}\mathrm{Ca}$ and $^{23}\mathrm{Mg}$ for N = 100000 decays

# Imaginary $C_V C_A$



Figure: Experimental values of D and $\chi^2$ of the distribution of $\phi$ for $^{39}\text{Ca}$ and $^{23}\text{Mg}$ for N = 1000000 decays

# Imaginary $C_V C_A$

We can observe $\chi^2$ for the other distributions:



Figure: Experimental values of $\chi^2$ for a model with imaginary component of $C_A$ for all distributions as function of the angle and number of decays for $^{39}\text{Ca}$ decays

# Imaginary $C_V C_A$
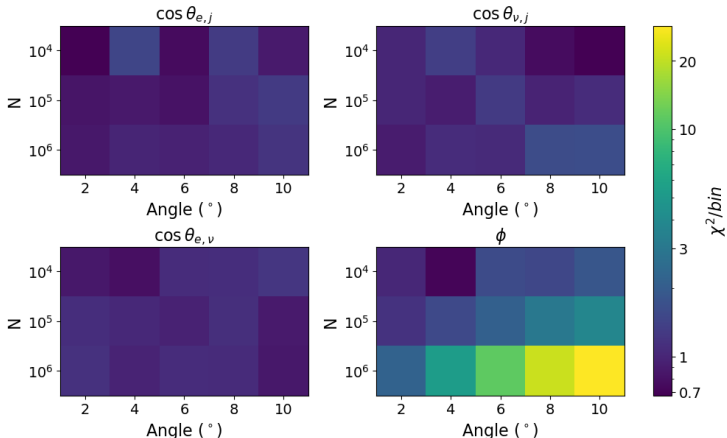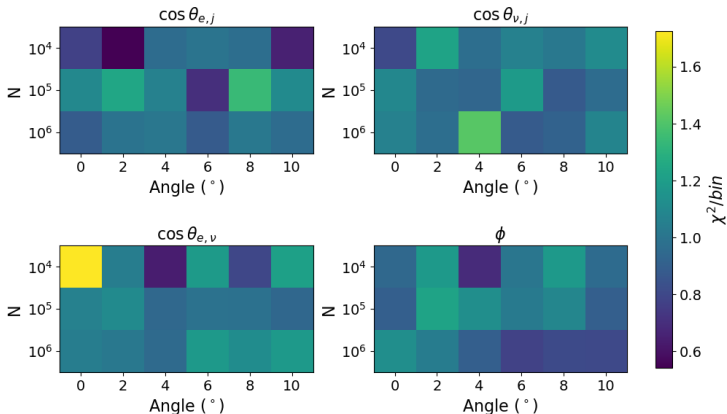
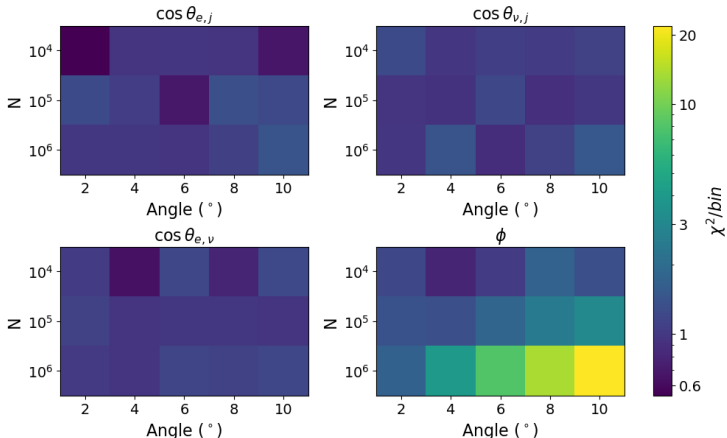We can observe $\chi^2$ for the other distributions:



Figure: Experimental values of $\chi^2$ for a model with standard couplings of for all distributions as function of the angle and number of decays for $^{39}\text{Ca}$ decays

# Imaginary $C_V C_A$

We can observe $\chi^2$ for the other distributions:



Figure: Experimental values of $\chi^2$ for a model with imaginary component of $C_A$ for all distributions as function of the angle and number of decays for $^{39}\mathrm{Ca}$ decays

# Imaginary $C_V C_A$

We can observe $\chi^2$ for the other distributions:



Figure: Experimental values of $\chi^2$ for a model with standard couplings of for all distributions as function of the angle and number of decays for $^{39}\mathrm{Ca}$ decays

# Imaginary $C_S C_T$: right-handed W' component

Consider $C_S$ real, $C_T$ pure imaginary, $C_S' = -C_S$, and $C_T' = -C_T$,

- ▶ Symmetry under simultaneous rotation of $C_S$, $C_S'$, $C_T$ and $C_T'$ while $C_V^{(\prime)}, C_A^{(\prime)}$ fixed.
- ▶ $C_X C_Y^* + C_X' C_Y'^* = C_X C_Y'^* + C_X' C_Y^* = 0$ if $X \in \{S, T\}$ and $Y \in \{V, A\}$
- ▶ Only new terms:
  - ○ $2 C_S C_T^*$ in D (main effect)
  - ○ $|C_X^{(\prime)}|^2$ in $\xi$, $a\xi$ and $c\xi \to \xi$ increases, $c\xi$ decreases, $a\xi$ increases if $\rho^2 |C_T|^2 > 3|C_S|^2$. $a$ and $c$ decrease in absolute value
  - ○ $2|C_T|^2$ in $A\xi$ and $B\xi \to$ both $A\xi$ and $B\xi$ increase, but $\xi$ increases faster. $A$ and $B$ decrease
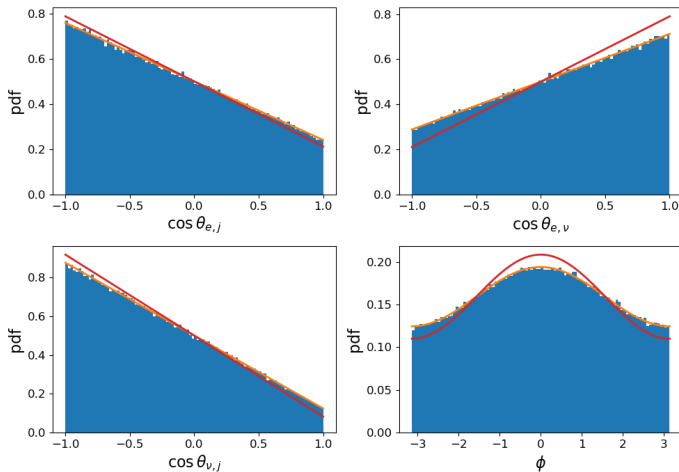
# Imaginary $C_S C_T$: right-handed W' component



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with N = 400000 decays of $^{39}\mathrm{Ca}$ with $C_V = C_V' = 1$, $C_A = C_A' = 1$, $C_S = -C_S' = 0.4$ and $C_T = -C_T' = 0$, and theoretical values with these coupling constants (orange) and SM couplings (red)

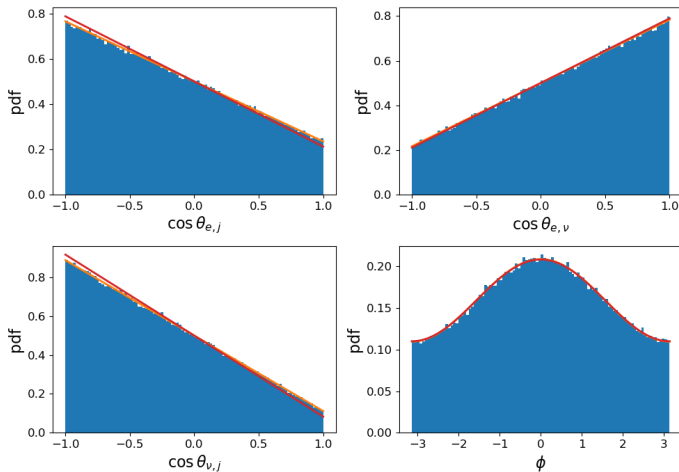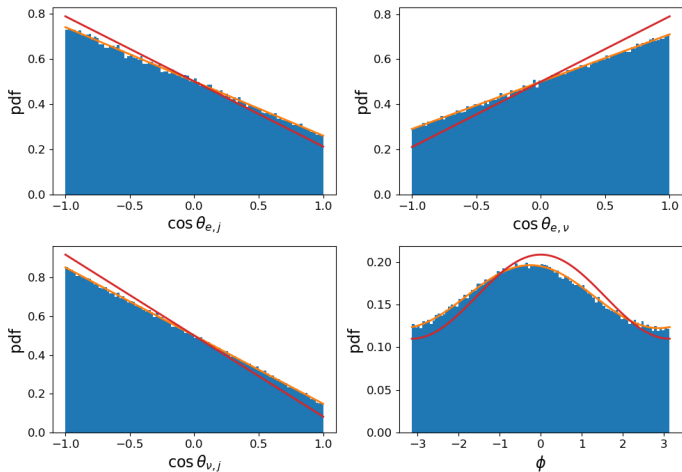# Imaginary $C_S C_T$: right-handed W' component



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with $N = 400000$ decays of $^{39}\mathrm{Ca}$ with $C_V = C_V' = 1$, $C_A = C_A' = 1$, $C_S = -C_S' = 0$ and $C_T = -C_T' = 0.4i$, and theoretical values with these coupling constants (orange) and SM couplings (red)

# Imaginary $C_S C_T$: right-handed W' component



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with N = 400000 decays of $^{39}\mathrm{Ca}$ with $C_V = C_V' = 1$, $C_A = C_A' = 1$, $C_S = -C_S' = 0.4$ and $C_T = -C_T' = 0.4i$, and theoretical values with these coupling constants (orange) and SM couplings (red)

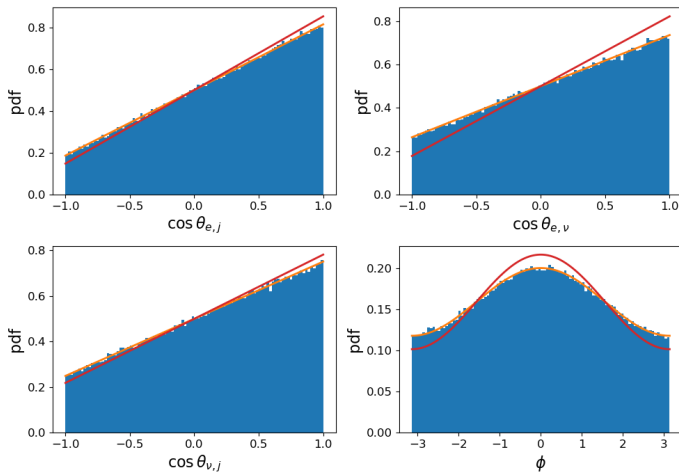# Imaginary $C_S C_T$: right-handed W' component



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with N = 400000 decays of $^{23}\mathrm{Mg}$ with $C_V = C_V' = 1$, $C_A = C_A' = 1$, $C_S = -C_S' = 0.4$ and $C_T = -C_T' = 0$, and theoretical values with these coupling constants (orange) and SM couplings (red)

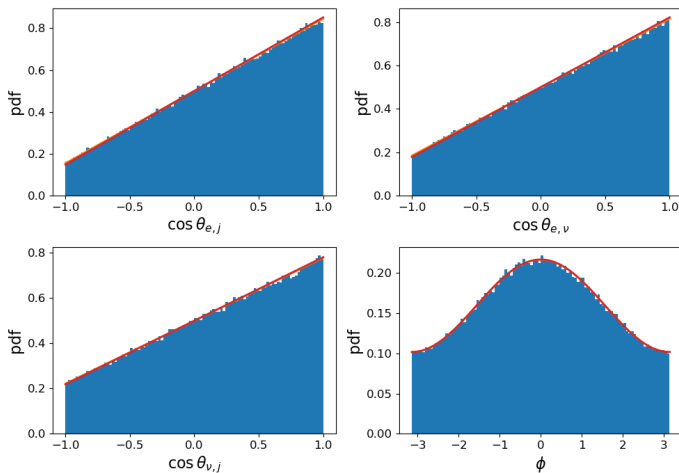# Imaginary $C_S C_T$: right-handed W' component



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with N = 400000 decays of $^{23}\mathrm{Mg}$ with $C_V = C_V' = 1$, $C_A = C_A' = 1$, $C_S = -C_S' = 0$ and $C_T = -C_T' = 0.4i$, and theoretical values with these coupling constants (orange) and SM couplings (red)

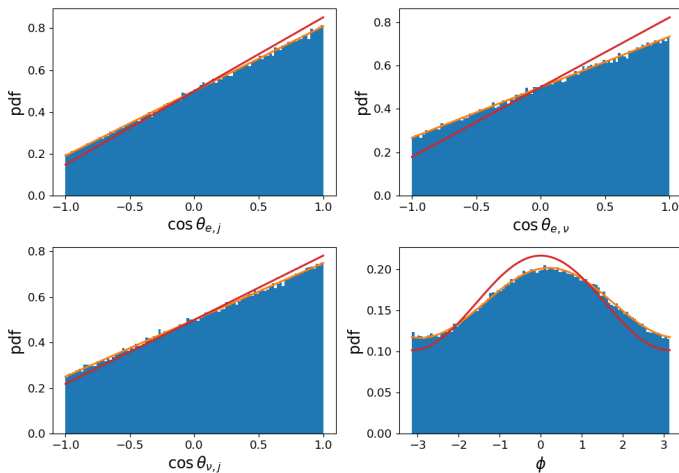# Imaginary $C_S C_T$: right-handed W' component



Figure: Distribution of $z_e$, $z_\nu$, $z_{e,\nu}$ and $\phi$ for a simulation with $N = 400000$ decays of $^{23}\mathrm{Mg}$ with $C_V = C_V' = 1$, $C_A = C_A' = 1$, $C_S = -C_S' = 0.4$ and $C_T = -C_T' = 0.4i$, and theoretical values with these coupling constants (orange) and SM couplings (red)

# Imaginary $C_S C_T$: right-handed W' component

For theory: consider the only new contribution

$$D \langle \beta_e \rangle = -\frac{4 M_{GT} |C_S||C_T|\sqrt{\frac{2}{3}}}{2(1+|C_S|^2) + 2|M_{GT}|^2(1+|C_T|^2)} \langle \beta_e \rangle$$

For experiment: $b = 0 \rightarrow$ same procedure as previous simulations

$$\frac{N(\phi > 0)}{N} = \int_0^\pi f_4(\phi) = \int_0^\pi \frac{1 + \left\langle \left(a + \frac{c}{3}\right) \beta_e \right\rangle \frac{\pi^2}{16} \cos\phi + \langle D\beta_e \rangle \frac{\pi^2}{16} \sin\phi}{2\pi}$$

$$\frac{N(\phi > 0)}{N} = \frac{1}{2} + D \langle \beta_e \rangle \frac{\pi}{16} \rightarrow D \langle \beta_e \rangle = \frac{N(\phi > 0) - N(\phi < 0)}{N(\phi > 0) + N(\phi < 0)} \frac{8}{\pi}$$

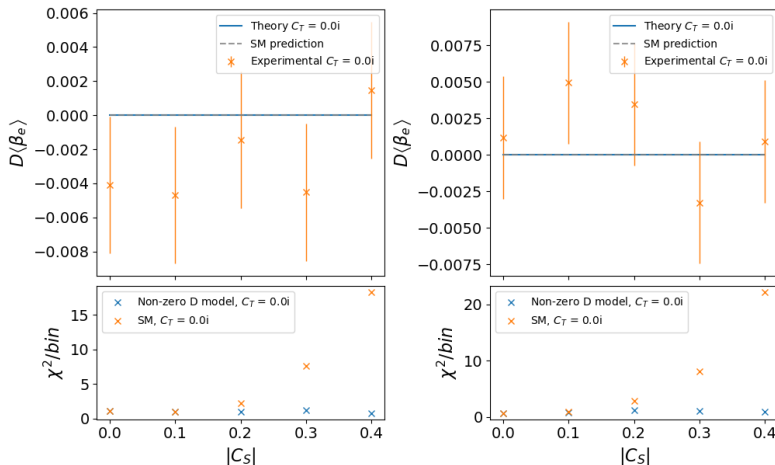# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of D and $\chi^2$ of the distribution of $\phi$ for $^{39}\text{Ca}$ and $^{23}\text{Mg}$ decays with $|C_T| = 0$ and varying $|C_S|$

# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of D and $\chi^2$ of the distribution of $\phi$ for $^{39}\text{Ca}$ and $^{23}\text{Mg}$ decays with $|C_T| = 0.1$ and varying $|C_S|$

# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of D and $\chi^2$ of the distribution of $\phi$ for $^{39}\text{Ca}$ and $^{23}\text{Mg}$ decays with $|C_T| = 0.2$ and varying $|C_S|$

# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of D and $\chi^2$ of the distribution of $\phi$ for $^{39}\text{Ca}$ and $^{23}\text{Mg}$ decays with $|C_T| = 0.3$ and varying $|C_S|$

# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of D and $\chi^2$ of the distribution of $\phi$ for $^{39}\text{Ca}$ and $^{23}\text{Mg}$ decays with $|C_T| = 0.4$ and varying $|C_S|$
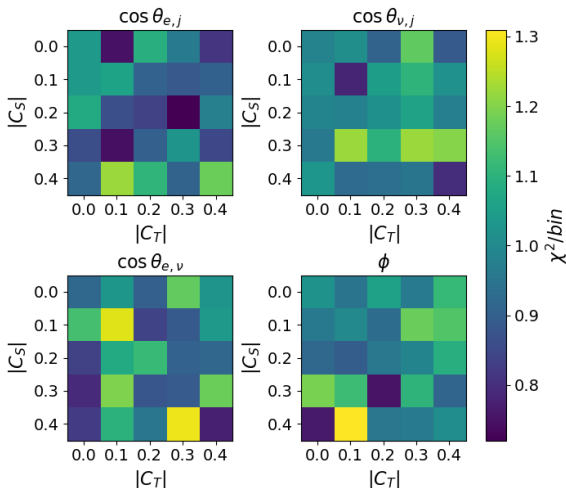
# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of $\chi^2$ for a model with non-zero $Im(C_S C_T)$ coming from right-handed component of interaction for all distributions as function of $|C_S|$ and $|C_T|$ for 400000 $^{39}\text{Ca}$ decays

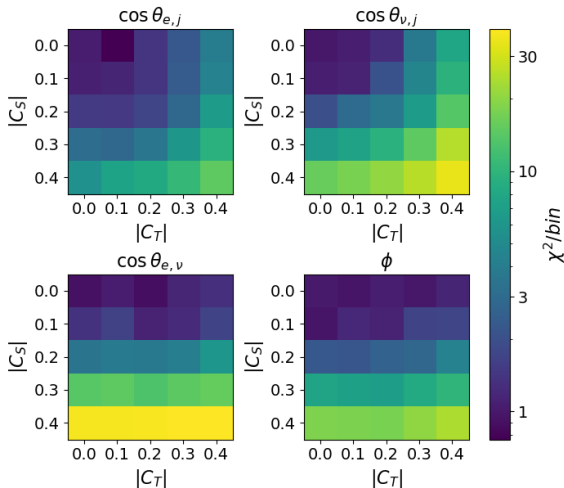# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of $\chi^2$ for a model with standard couplings for all distributions as function of $|C_S|$ and $|C_T|$ for 400000 $^{39}\mathrm{Ca}$ decays
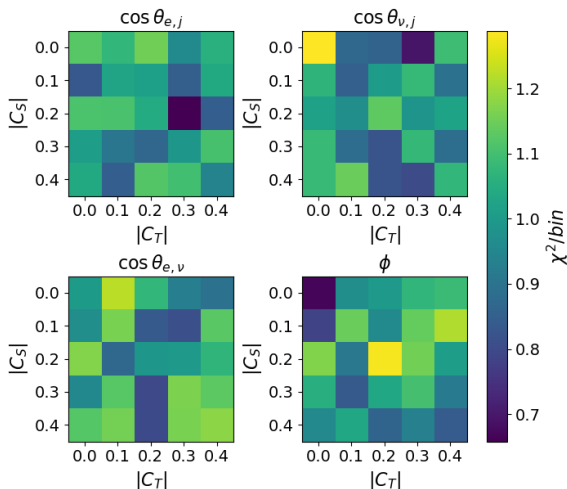
# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of $\chi^2$ for a model with non-zero $Im(C_S C_T)$ coming from right-handed component of interaction for all distributions as function of $|C_S|$ and $|C_T|$ for 400000 $^{23}\mathrm{Mg}$ decays

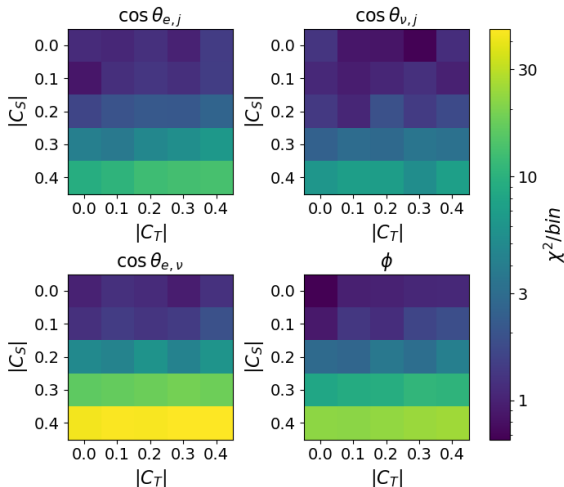# Imaginary $C_S C_T$: right-handed W' component



Figure: Experimental values of $\chi^2$ for a model with standard couplings for all distributions as function of $|C_S|$ and $|C_T|$ for 400000 $^{23}\mathrm{Mg}$ decays