Angular correlation Function

$$F = 1 + a\frac{\mathbf{p_e}\cdot\mathbf{p_\nu}}{E_eE_\nu} + b\frac{m_e}{E} + c\left(\frac{\mathbf{p_e}\cdot\mathbf{p_\nu}}{3E_eE_\nu} - \frac{(\mathbf{p_e}\cdot\mathbf{j})(\mathbf{p_\nu}\cdot\mathbf{j})}{E_eE_\nu}\right)$$

$$+\frac{\mathbf{J}}{J}\cdot\left(A\frac{\mathbf{p_e}}{E_e} + B\frac{\mathbf{p_\nu}}{E_\nu} + D\frac{\mathbf{p_e}\times\mathbf{p_\nu}}{E_eE_\nu}\right)$$

Spherical Coordinates ($\mathbf{J}$ parallel to positive Z axis)

$$\boldsymbol{\beta}_\mathbf{e} = (r = \beta_e; \theta = \theta_e; \phi = 0),\ \cos(\theta_e) \equiv z_e,\ \beta_e = \frac{|\mathbf{p_e}|}{E} = \sqrt{1 - \frac{m_e^2}{E^2}}$$

$$\boldsymbol{\beta}_{\boldsymbol{\nu}} = (r = 1; \theta = \theta_\nu; \phi = \phi), \quad \cos(\theta_\nu) \equiv z_\nu$$

$$\boldsymbol{\beta}_\mathbf{e}\cdot\boldsymbol{\beta}_{\boldsymbol{\nu}} = \beta_e(\cos\theta_e\cos\theta_\nu + \sin\theta_e\sin\theta_\nu\cos\phi) =$$

$$\beta_e(z_ez_\nu + \sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\cos\phi)$$

$$\boldsymbol{\beta}_\mathbf{e}\cdot\mathbf{j} = \beta_e\cos\theta_e = \beta_e z_e$$

$$\boldsymbol{\beta}_{\boldsymbol{\nu}}\cdot\mathbf{j} = \cos\theta_\nu = z_\nu$$

$$\mathbf{j}\cdot(\boldsymbol{\beta}_\mathbf{e}\times\boldsymbol{\beta}_{\boldsymbol{\nu}}) = \beta_e\sin\theta_e\sin\theta_\nu\sin\phi = \beta_e\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\sin\phi$$

Angular Correlation Factor

$$(\boldsymbol{\beta}_{\mathbf{e}} \cdot \mathbf{j})(\boldsymbol{\beta}_{\boldsymbol{\nu}} \cdot \mathbf{j}) = z_e z_\nu$$

Single Variable c

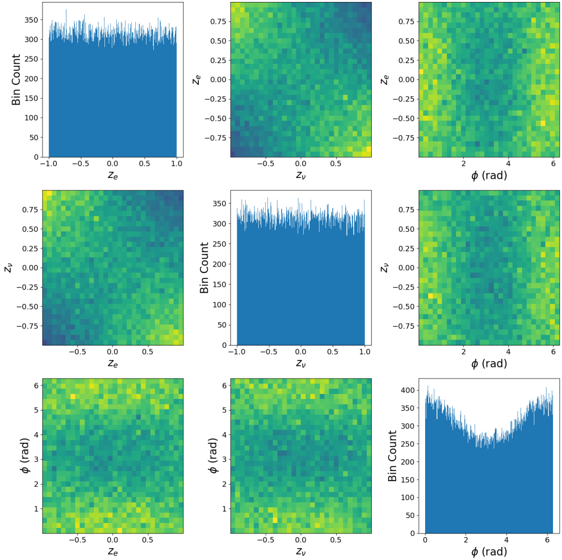

Figure: Pair plots for N = 100000 decays with c = 1, E = 1000 keV

For z_e (and z_ν by symmetry of the expresions), we can observe reason why the marginal distribution becomes constant:

$$f(z_e) = N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi F =$$

$$= N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi (1 + c\beta(-2 z_e z_\nu/3 + \sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\cos\phi)/3) =$$

$$= N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi = 4\pi N = N$$

For ϕ, we can derive the expected shape:

$$f(\phi) = N \int_{-1}^{1} dz_\nu \int_{-1}^{1} dz_e F =$$

$$= N \int_{-1}^{1} dz_\nu \int_{-1}^{1} dz_e (1 + a\beta(*2 z_e z_\nu/3 + \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2} \cos\phi)/3) =$$

$$= N \left(4 + a\beta \left(\frac{\pi}{2}\right)^2 \cos\phi/3\right) = N \left(1 + a\beta \frac{\pi^2}{48} \cos\phi\right)$$

Single Variable a

Marginal distributions

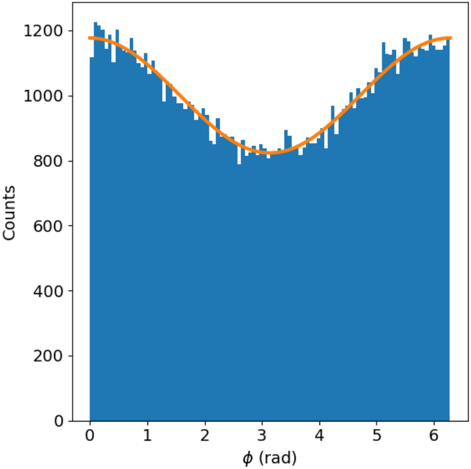

Figure: Histogram showing the values of ϕ with a $=1$, E $=1000$ keV for N $=100000$ decays, and curve showing the theoretical distribution

Two variable: rest of pairs

For the rest of variables, we show only the pairplot with the marginal distributions and the theoretical distribution in the 1D marginal plots. We expect 2 kinds of results

- (c,A), (c,B):
  Here, one of the 1D histograms will be aproximately constant, and the other 2 will be close to the 1 variable case (though need to account for $F < 0$ areas)
- (c,D),(c,a)
  In this case, integration along ϕ cancels most terms, and the remaining $z_e z_\nu$ cancels the only non constant term. So only non-constant marginal distribution is that of ϕ

Two variable: c and a

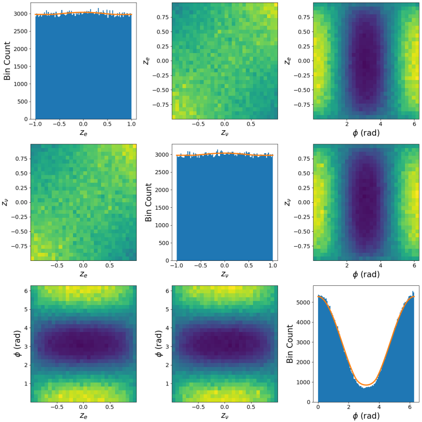

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with c = a = 1, E = 100000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$

Two variable: c and A

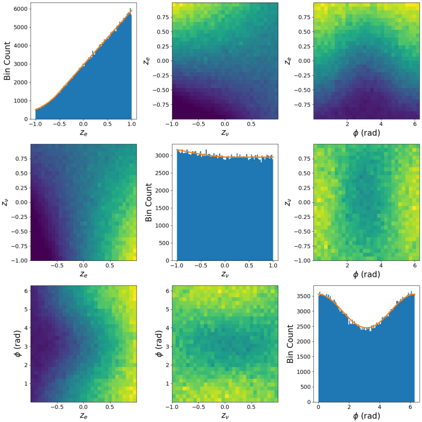

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with c = A = 1, E = 100000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$

Two variable: c and B

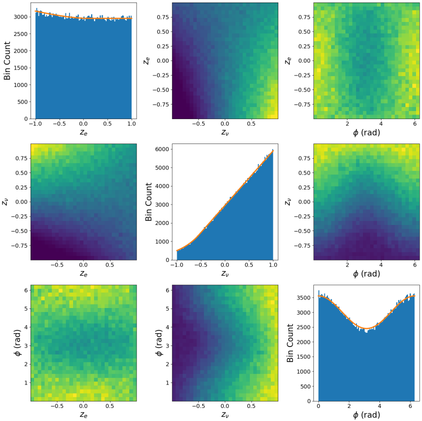

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with c = B = 1, E = 5000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$

Two variable: c and D

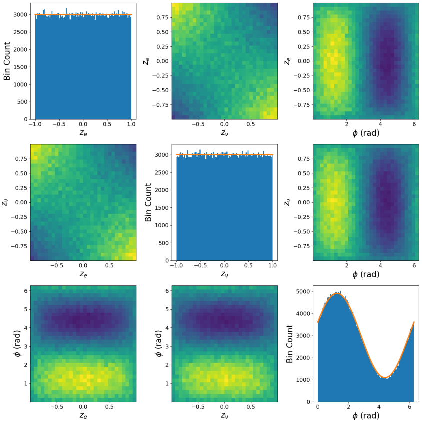

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with c = D = 1, E = 100000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$