Angular Correlation Function

$$F = 1 + a\frac{\mathbf{p_e}\cdot\mathbf{p_\nu}}{E_eE_\nu} + b\frac{m_e}{E} + c\left(\frac{\mathbf{p_e}\cdot\mathbf{p_\nu}}{3E_eE_\nu} - \frac{(\mathbf{p_e}\cdot\mathbf{j})(\mathbf{p_\nu}\cdot\mathbf{j})}{E_eE_\nu}\right)$$

$$+\frac{\mathbf{J}}{J}\cdot\left(A\frac{\mathbf{p_e}}{E_e} + B\frac{\mathbf{p_\nu}}{E_\nu} + D\frac{\mathbf{p_e}\times\mathbf{p_\nu}}{E_eE_\nu}\right)$$

Spherical Coordinates ($\mathbf{J}$ parallel to positive Z axis)

$$\boldsymbol{\beta}_\mathbf{e} = (r = \beta_e; \theta = \theta_e; \phi = 0),\ \cos(\theta_e) \equiv z_e,\ \beta_e = \frac{|\mathbf{p_e}|}{E} = \sqrt{1 - \frac{m_e^2}{E^2}}$$

$$\boldsymbol{\beta}_{\boldsymbol{\nu}} = (r = 1; \theta = \theta_\nu; \phi = \phi), \quad \cos(\theta_\nu) \equiv z_\nu$$

$$\boldsymbol{\beta}_\mathbf{e}\cdot\boldsymbol{\beta}_{\boldsymbol{\nu}} = \beta_e(\cos\theta_e\cos\theta_\nu + \sin\theta_e\sin\theta_\nu\cos\phi) =$$

$$\beta_e(z_ez_\nu + \sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\cos\phi)$$

$$\boldsymbol{\beta}_\mathbf{e}\cdot\mathbf{j} = \beta_e\cos\theta_e = \beta_ez_e$$

$$\boldsymbol{\beta}_{\boldsymbol{\nu}}\cdot\mathbf{j} = \cos\theta_\nu = z_\nu$$

$$\mathbf{j}\cdot(\boldsymbol{\beta}_\mathbf{e}\times\boldsymbol{\beta}_{\boldsymbol{\nu}}) = \beta_e\sin\theta_e\sin\theta_\nu\sin\phi = \beta_e\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\sin\phi$$

Angular Correlation Factor

$$(\boldsymbol{\beta}_{\mathbf{e}} \cdot \mathbf{j})(\boldsymbol{\beta}_{\boldsymbol{\nu}} \cdot \mathbf{j}) = z_e z_\nu$$

Putting all together:

$$F = 1 + a\beta(z_e z_\nu + \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\cos\phi) + b\frac{m_e}{E} +$$

$$+c\beta\left(-\frac{2}{3}z_e z_\nu + \frac{1}{3}\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\right) + A\beta z_e + B z_\nu +$$

$$+D\beta\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\sin\phi =$$

$$= 1 + b\frac{m_e}{E} + \left(a - \frac{2}{3}c\right)\beta z_e z_\nu + A\beta z_e + B z_\nu +$$

$$+\beta\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\left(\left(a + \frac{c}{3}\right)\cos\phi + D\sin\phi\right)$$

Single Variable: c

Angular Correlation Factor

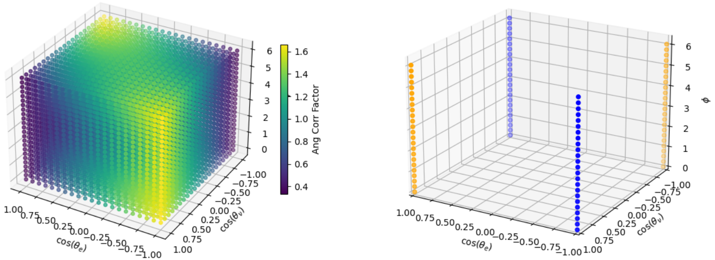


Figure: (Right) Values of the angular correlation Factor with a = 1, E = 5000 keV and rest of variables 0. (Left) Location of maximum (blue, value = 1.995) and minimum (orange, value = 0.005)

Single Variable: c

Angular Correlation Factor

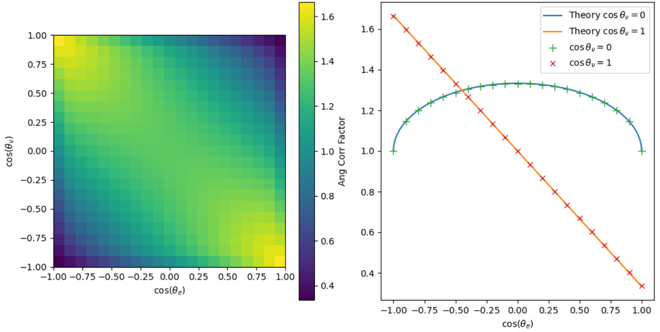


Figure: (Right) 2D projection of previous 3D image at $\phi = 0$ (Left) 1D projections at $\phi = 0$, and either $z_\nu = 0$ or $z_\nu = 1$

Single Variable c

Sampling

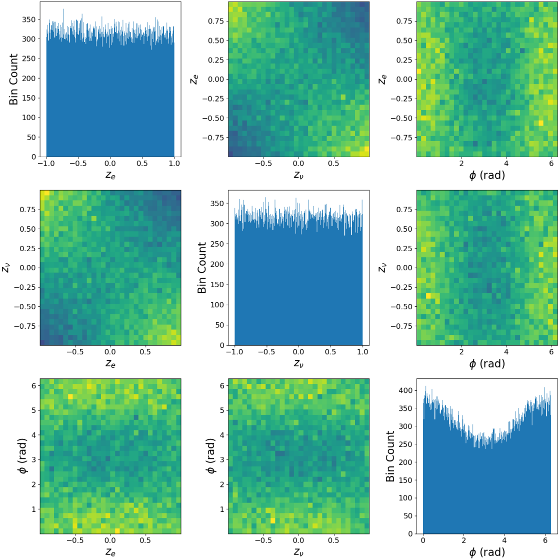


Figure: Pair plots for N = 100000 decays with c = 1, E = 1000 keV

Single Variable c

Marginal distributions

For z_e (and z_ν by symmetry of the expresions), we can observe reason why the marginal distribution becomes constant:

$$f(z_e) = N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi F =$$

$$= N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi \left(1 + c\beta \left(-\frac{2}{3} z_e z_\nu + \frac{1}{3}\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\cos\phi\right)\right)$$

$$= N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi = 4\pi N = N$$

Single Variable a

Marginal distributions

For ϕ, we can derive the expected shape:

$$f(\phi) = N \int_{-1}^{1} dz_\nu \int_{-1}^{1} dz_e F$$

$$= N \int_{-1}^{1} dz_\nu \int_{-1}^{1} dz_e \left(1 + c\beta\left(-\frac{2}{3} z_e z_\nu + \frac{1}{3}\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\cos\phi\right)\right)$$

$$= N\left(4 + a\beta\left(\frac{\pi}{2}\right)^2 \cos\phi/3\right) = N\left(1 + a\beta\frac{\pi^2}{48}\cos\phi\right)$$

Single Variable a

Marginal distributions

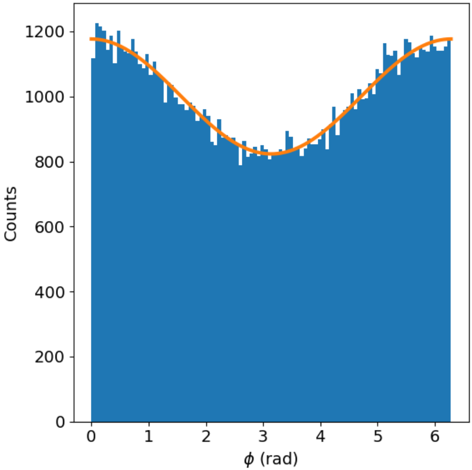


Figure: Histogram showing the values of ϕ with a $=1$, E $=1000$ keV for N $=100000$ decays, and curve showing the theoretical distribution

Two variables: c and B

Since term proportional to a depends on E, we can consider different ratios by either:

- Fixing $c = B = 1$ and modifying the energy
- Same as before, but now $B = -1$
- Fixing $B = 1$ and $E \gg m_e \to \beta_e \approx 1$ and modifying $c > B$

We recall

$$F = 1 + c\beta_e\left(-\frac{2}{3}z_e z_\nu + \frac{1}{3}\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\cos\phi\right) + Bz_\nu$$

Maxima and minima with $z_e = \pm 1, z_\nu \pm 1 \to \mathbf{p_e} \parallel \mathbf{p_\nu} \parallel \mathbf{J}$

Two variables: c and B

Angular Correlation Factor

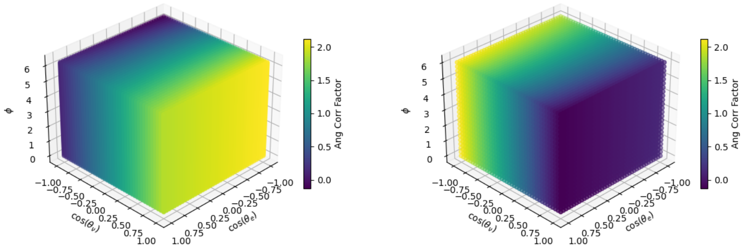


Figure: Values of the angular correlation Factor with (Right) c = 1, B = 1 and (Left) c = 1, B = -1; with E = 520 keV and rest of variables 0 for both.

Two variables: c and B

Angular Correlation Factor

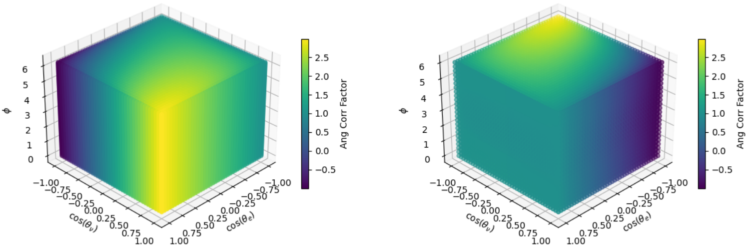


Figure: Values of the angular correlation Factor with (Right) c = 1, B = 1 and (Left) c = 1, B = -1; with E = 5000 keV and rest of variables 0 for both.

Two variables: c and B

Angular Correlation Factor

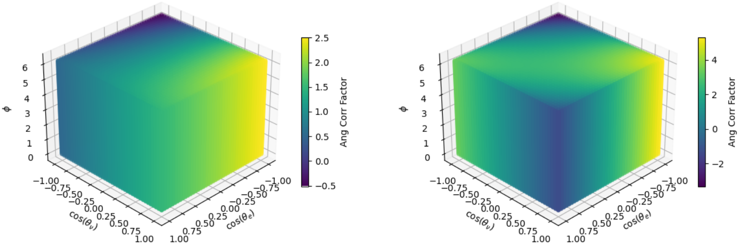


Figure: Values of the angular correlation Factor with (Right) a = c, B = 1, E = 800 keV and (Left) c = 5, B = 1, E = 5000 keV; with the rest of variables 0 for both.

Two variables: c and B

Maximum and Minimum

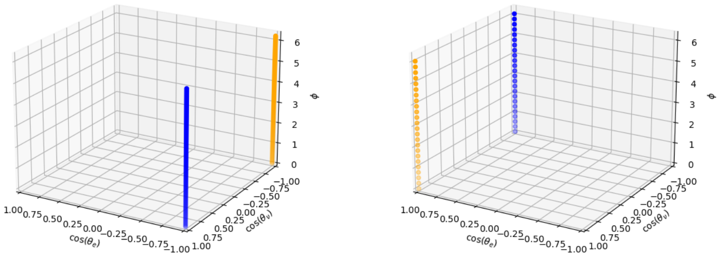


Figure: Location of maximum (blue, value = 2.66318) and minimum (orange, value = -0.66318) for (Right) c = B = 1, E = 5000 keV and (Left) c = 1, B = -1, E = 5000 keV

Two variable: c and B

Sampling

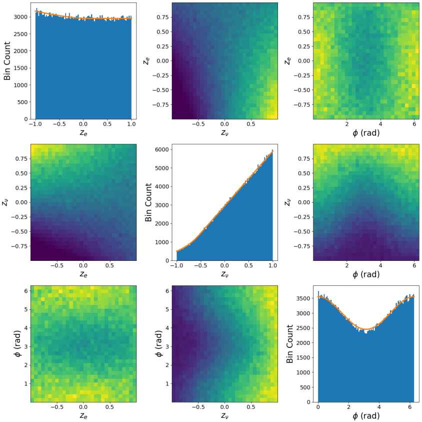


Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with c = B = 1, E = 5000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$

Two variables: c and A

Since both terms proportional to a depends on E, we can consider only consider different ratios by changing one (c), while leaving the other (A) fixed. For convenience $E \gg m_e$.

$$F = 1 + \beta_e \left(c \left(-\frac{2}{3} z_e z_\nu + \frac{1}{3} \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \cos\phi \right) + A z_e \right)$$

Maximum and minimum with $z_e = \pm 1, z_\nu \pm 1 \rightarrow \mathbf{p_e} \parallel \mathbf{p_\nu} \parallel \mathbf{J}$

Two variables: c and A

Angular Correlation Factor

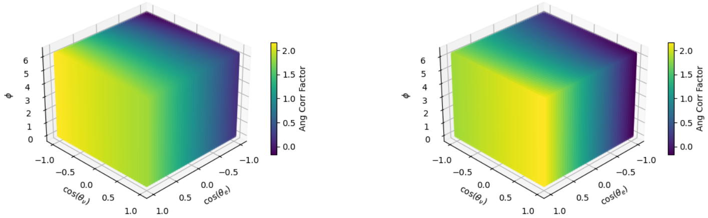
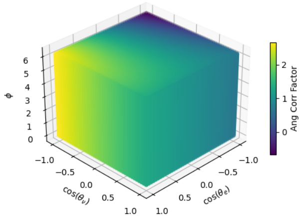


Figure: Values of the angular correlation Factor with (Right) A = 1, c = 0.25 and (Left) A = 1, c = -0.25, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and A

Angular Correlation Factor



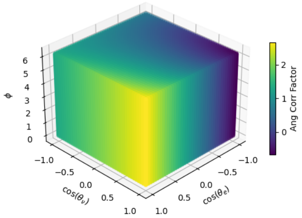


Figure: Values of the angular correlation Factor with (Right) A = 1, c = 1 and (Left) A = 1, c = -1, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and A

Angular Correlation Factor

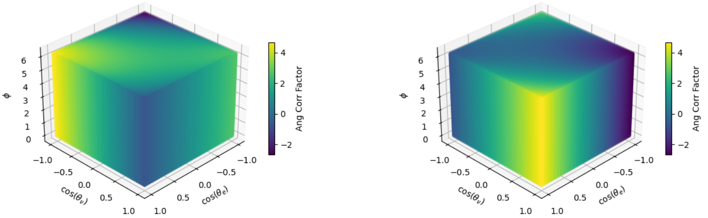


Figure: Values of the angular correlation Factor with (Right) A = 1, c = 4 and (Left) A = 1, c = -4, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and A

Maximum and Minimum

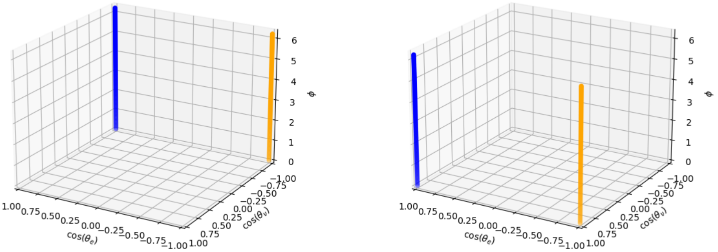


Figure: Location of maximum (blue, value = 2.66664) and minimum (orange, value = -0.66664) for (Right) A = c = 1, E = 100000 keV and (Left) A = 1, c = -1, E = 100000 keV

Two variable: c and A

Sampling

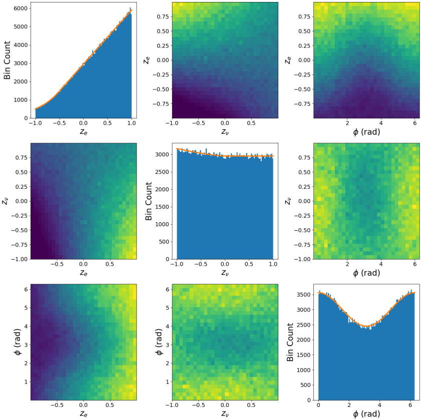


Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with c = A = 1, E = 100000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$

Two variables: c and a

Since both terms proportional to a depends on E, we can consider only consider different ratios by changing one (c), while leaving the other (A) fixed. For convenience $E \gg m_e$.

$$F = 1 + \left(a - \frac{2}{3}c\right)\beta z_e z_\nu + \beta\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\left(a + \frac{c}{3}\right)\cos\phi$$

Maximum and minimum depends on the relative signs of a and c

- c and a opposite sign: $z_e = \pm 1, z_\nu = \pm 1 \rightarrow \mathbf{p_e} \parallel \mathbf{p_\nu} \parallel \mathbf{J}$
- c and a same sign: $z_e = 0, z_\nu = 0 \rightarrow \mathbf{p_e} \parallel \mathbf{p_\nu} \perp \mathbf{J}$

Two variables: c and a

Angular Correlation Factor

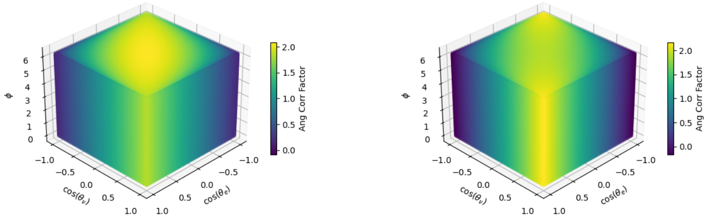


Figure: Values of the angular correlation Factor with (Right) a = 1, c = 0.25 and (Left) a = 1, c = -0.25, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and A

Angular Correlation Factor

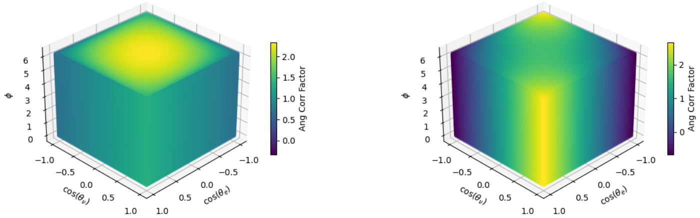


Figure: Values of the angular correlation Factor with (Right) a = 1, c = 1 and (Left) a = 1, c = -1, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and A

Angular Correlation Factor

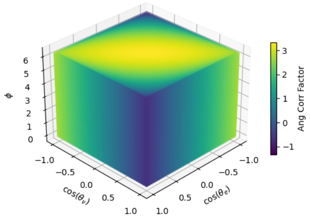


Figure: Values of the angular correlation Factor with (Right) a = 1, c = 4 and (Left) a = 1, c = -4, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and a

Maximum and Minimum

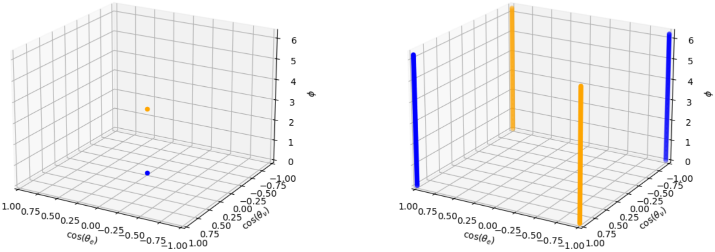


Figure: Location of maximum (blue) and minimum (orange) for (Right) a = c = 1, E = 100000 keV (values 2.33332, -0.33332) and (Left) a = 1, c = -1, E = 100000 keV (values 2.66664, -0.66664)

Two variable: c and a

Sampling

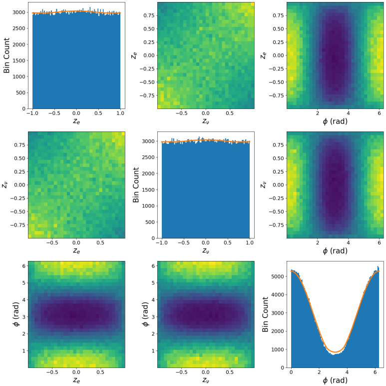


Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with c = a = 1, E = 100000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$

Two variables: c and D

Angular Correlation Factor

Since both terms proportional to a depends on E, we can consider only consider different ratios by changing one (c), while leaving the other (D) fixed. For convenience $E \gg m_e$.

$$F = 1 + \beta_e \left(-\frac{2}{3} c z_e z_\nu + \left(\frac{1}{3} c \cos\phi + D \sin\phi \right) \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \right)$$

Maxima and minima depend on the ratio between D and c:

- If $c^2 < 3D^2$: maximum at $z_e = z_\nu = 0$ and

$$\tan\phi = \frac{3D}{c}$$

$$F = 1 + \beta_e \sqrt{D^2 + \left(\frac{c}{3} \right)^2}$$

Two variables: c and D

Angular Correlation Factor

Since both terms proportional to a depends on E, we can consider only consider different ratios by changing one (c), while leaving the other (D) fixed. For convenience $E \gg m_e$.

$$F = 1 + \beta_e \left(-\frac{2}{3} c z_e z_\nu + \left(\frac{1}{3} c \cos\phi + D \sin\phi \right) \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \right)$$

Maxima and minima depend on the ratio between D and c:

- If $c^2 > 3D^2$: maximum at $z_e = z_\nu = \pm 1$ and

$$F = 1 + \frac{2}{3} \beta |c|$$

We look only at properties of the extrema

Two variables: c and D

Maximum and Minimum

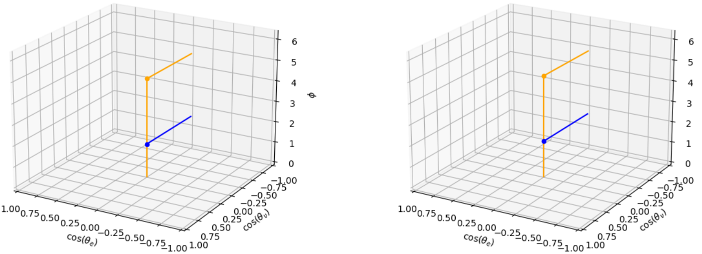


Figure: Positions of the maximum and minimum with Factor with (Right) D = 1, c = 0.25 and (Left) D = 1, c = -0.25, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and D

Maximum and Minimum

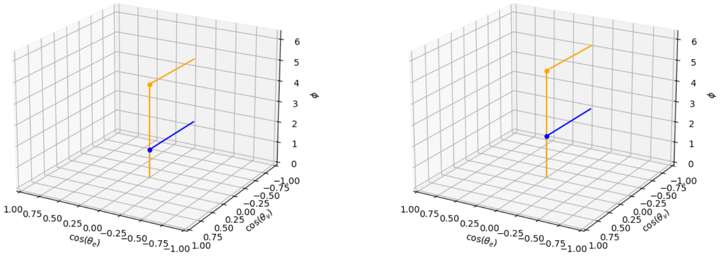


Figure: Positions of the maximum and minimum with with (Right) D = 1, c = 1 and (Left) D = 1, c = -1, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and D

Maximum and Minimum

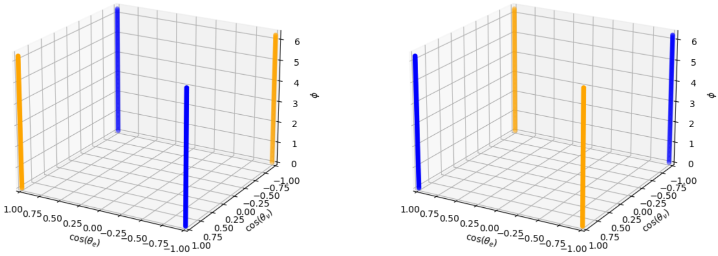


Figure: Positions of the maximum and minimum with (Right) D = 1, c = 2 and (Left) D = 1, c = -2, with E = 100000 keV and rest of variables 0 for both.

Two variables: c and D

Behaviour of maximum

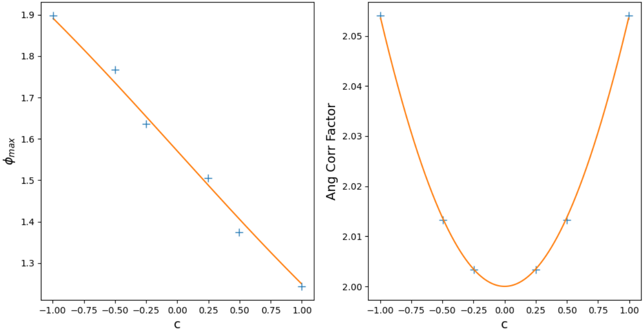


Figure: Behaviour of the ϕ coordinate for the maximum and the maximum value of the angular correlation factor for diferent values of c. Note discrepancies are a result of a sampling too coarse.

Two variable: c and D

Sampling

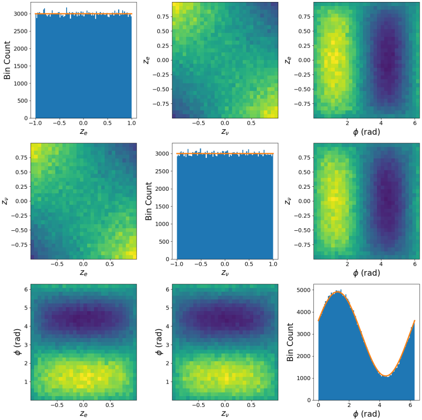


Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with c = D = 1, E = 100000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$