CRADLE++ Tests

November 13, 2025

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Simplest non-trivial case: Gamow-Teller decay.
Full test: decay of a known nuclei.
Simplifying assumption: focus on one decay path of that nuclei. Example:

$$^{60}\mathrm{Co}(5^+) \to {}^{60}\mathrm{Ni}(4^+)$$

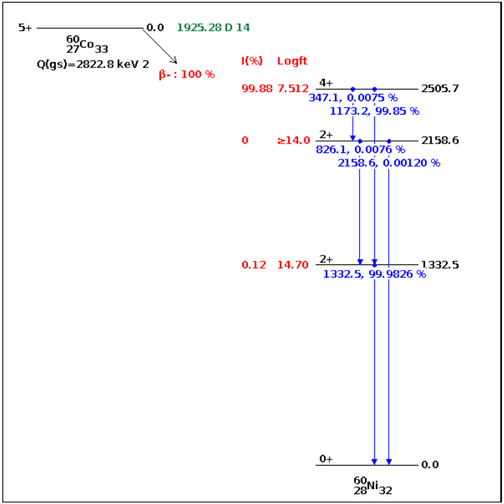


Figure: Decay Scheme of $^{60}\mathrm{Co}$ into $^{60}\mathrm{Ni}$.

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Properties of $^{60}\mathrm{Co}(5^+) \to {}^{60}\mathrm{Ni}(4^+)$

- Q = 317.06 keV (good for testing, $\langle \beta_e \rangle = 0.68, \langle \gamma_e^{-1} \rangle = 0.72, \langle \alpha Z \gamma_e^{-1} \rangle = 0.15$)
- $J_f = J_i - 1 \to \lambda_{J_i,J_f} = \Lambda_{J_i,J_f} = 1$
- 2 γ almost always ($5^+ \to 2^+$ only 1 γ)

Many cases to consider, though for realism: keep $C_A = C'_A = \mathrm{cte}(= 1)$.

- $C_T = C'_T = 0$ (Standard Model)
- $C_T = C'_T$ pure real (and large)
- $C_T = C'_T$ pure imaginary
- $C_T = -C'_T$, either real or imaginary

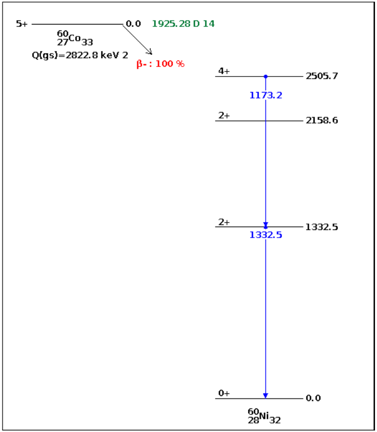
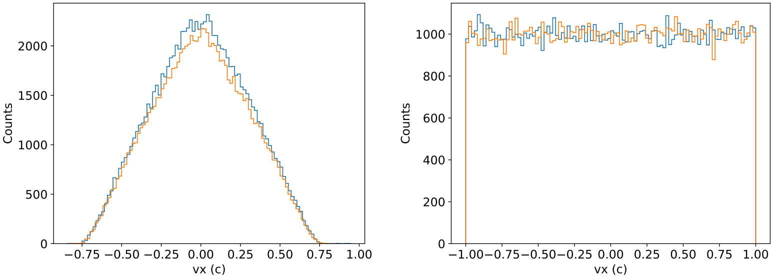


Figure: Decay Scheme of $^{60}\mathrm{Co}$ into $^{60}\mathrm{Ni}$ featuring the only decay of interest

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values

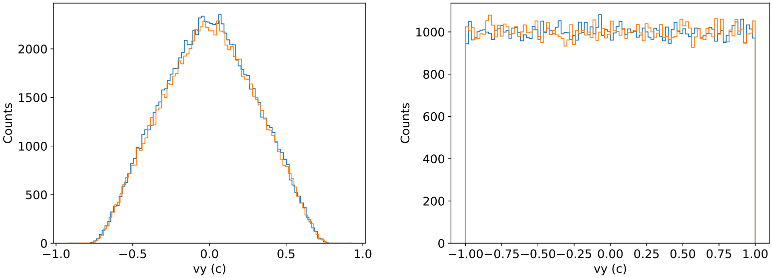


(a) e (b) ν

Figure: Distribution of the x component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values

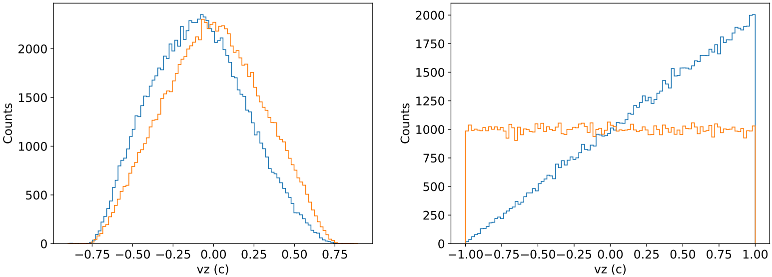


(a) e (b) ν

Figure: Distribution of the y component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values



(a) e (b) ν

Figure: Distribution of the z component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Numerical evaluation

Use that distributions in z_e, z_ν, $\cos\theta_{e,\nu} \equiv z_{e,\nu}$ and ϕ are known if $F \geq 0$ for all orientations of $p\mathbf{p_e}, \mathbf{p_\nu}$ ($\mathbf{J}$ fixed).

$$f(z_e) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \langle A\beta_e \rangle z_e}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f(z_\nu) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \langle B \rangle z_\nu}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f(z_{e,\nu}) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \langle a\beta_e \rangle z_{e,\nu}}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f(\phi) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \left\langle \left(a + \frac{c}{3}\right) \beta_e \right\rangle \frac{\pi^2}{16} \cos\phi + \langle D\beta_e \rangle \frac{\pi^2}{16} \sin\phi}{2\pi(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

Averages computed numerically using f(E) from the simulation data itself (avoid computing the Fermi function myself)

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values

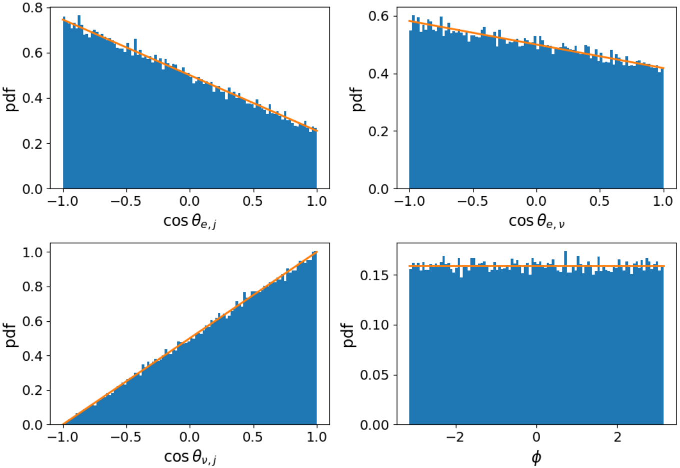


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Real Positive

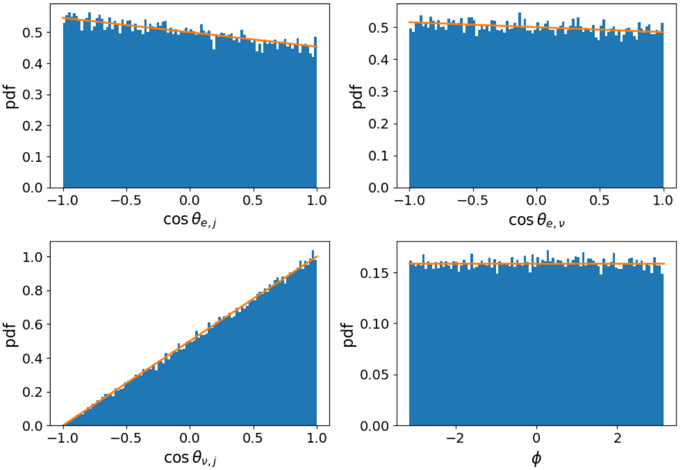


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Real Negative

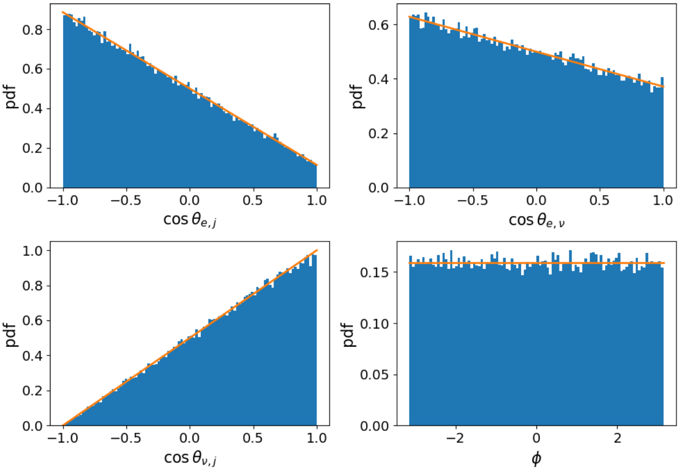


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = -1/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Imaginary Positive

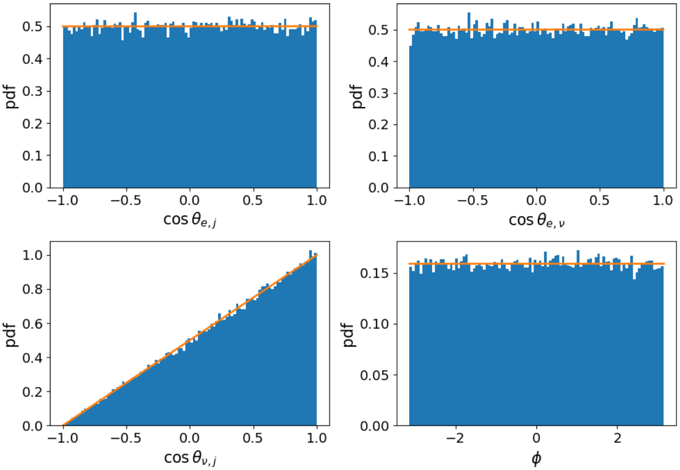


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Imaginary Negative

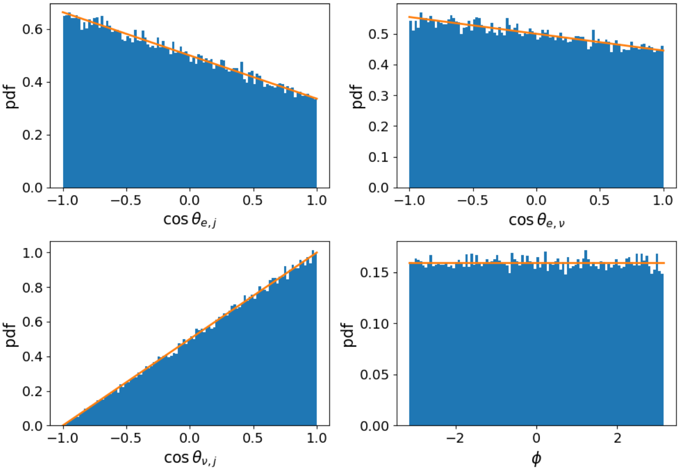


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = -i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = -C_T'$

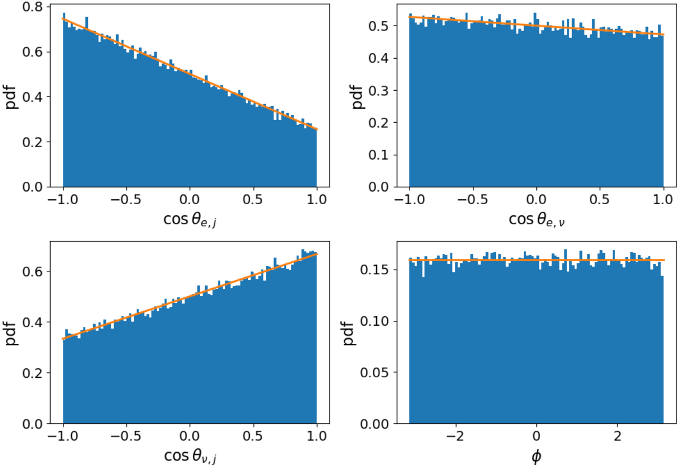
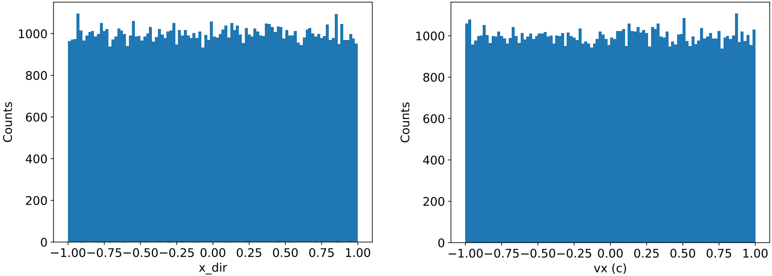


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = -C_T' = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in -Z



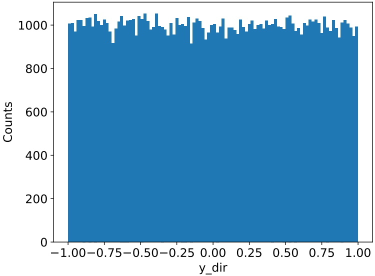
(a) e

(b) ν

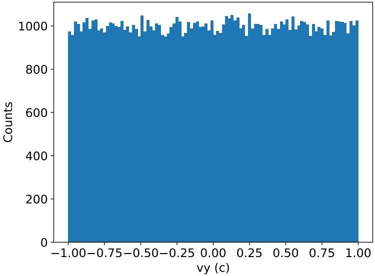
Figure: Distribution of the x component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the negative z direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in -Z



(a) e

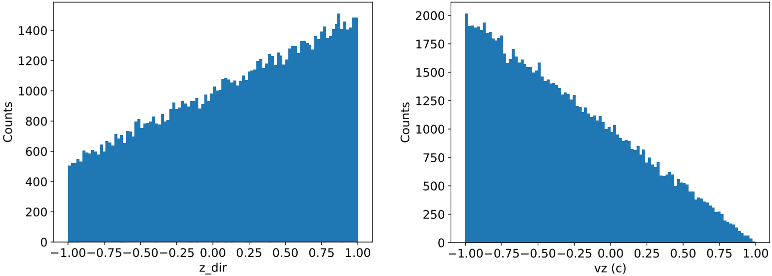


(b) ν

Figure: Distribution of the y component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the negative z direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in -Z

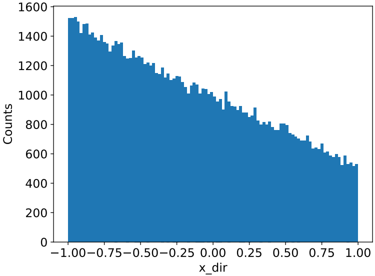


(a) e (b) ν

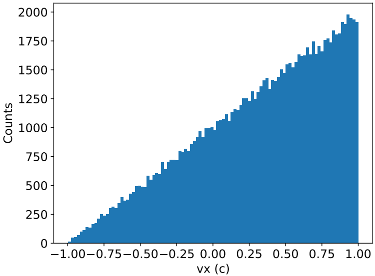
Figure: Distribution of the z component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the negative z direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+X$



(a) e

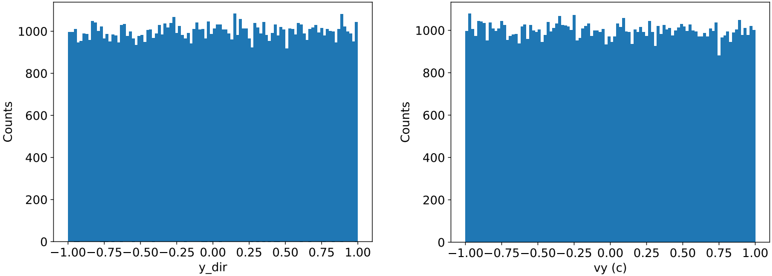


(b) ν

Figure: Distribution of the x component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the positive x direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+X$

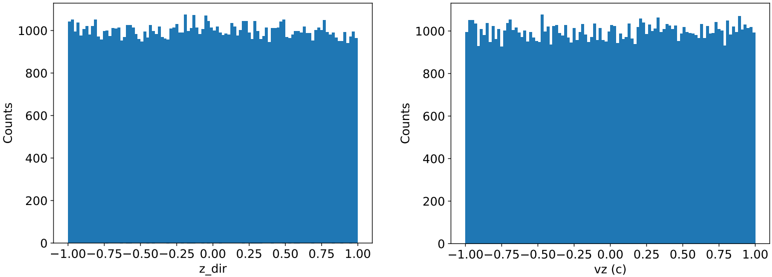


(a) e (b) ν

Figure: Distribution of the y component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the positive x direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+X$

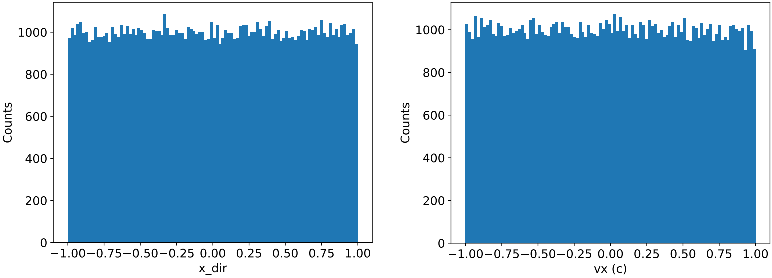


(a) e (b) ν

Figure: Distribution of the z component of the direction of the velocity of the emited leptons for a decay of fully polarized nuclei in the positive x direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+\mathrm{Y}$

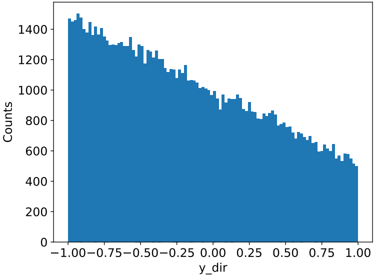


(a) e (b) ν

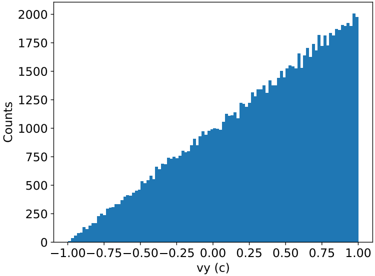
Figure: Distribution of the y component of the direction of the velocity of the emited for a decay of fully polarized nuclei in the positive y direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+\mathrm{Y}$



(a) e

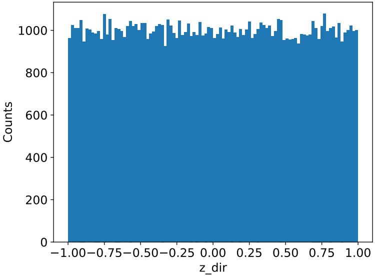


(b) ν

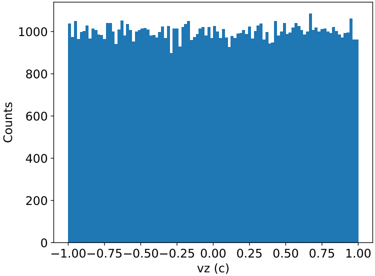
Figure: Distribution of the y component of the direction of the velocity of the emited leptons for a decay of fully polarized nuclei in the positive y direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+\mathrm{Y}$



(a) e



(b) ν

Figure: Distribution of the z component of the direction of the velocity of the emited leptons for a decay of fully polarized nuclei in the positive y direction

Gamow-Teller Decay: $^{12}\mathrm{B}$

Other examples:
$^{12}\mathrm{B}(1^+) \rightarrow {}^{12}\mathrm{C}(0^+)$
Properties:

- Q = 13369 keV (very high)
- No γ produced
- $J_f = J_i - 1$
- A = -1, B = 1, a = -1/3, c = 1

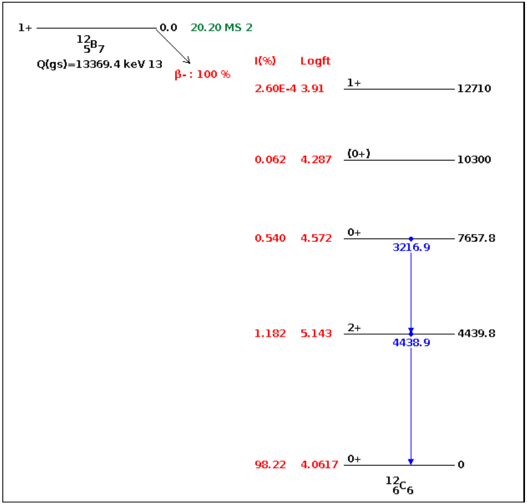


Figure: Decay Scheme of $^{12}\mathrm{B}$ into $^{12}\mathrm{C}$.

Gamow-Teller Decay: $^{12}\mathrm{B}$

Standard Model values

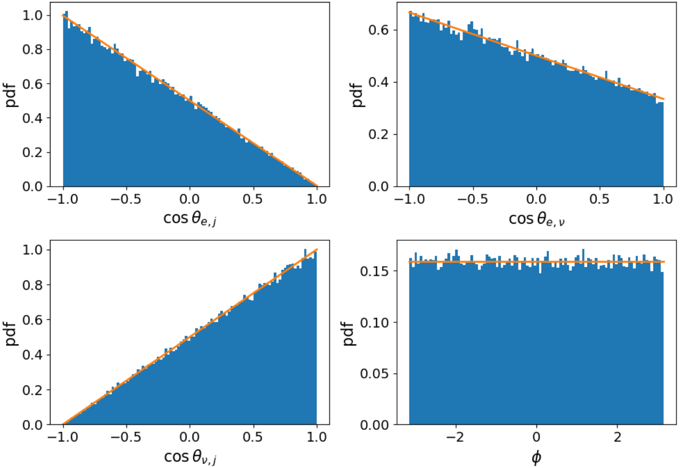


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: $^{20}\mathrm{F}$

Other examples:
$^{20}\mathrm{F}(2^+) \rightarrow {}^{20}\mathrm{Ne}(2^+)$
Properties:

- Q = 5390 keV (very high)
- 1 γ produced
- $J_f = J_i$ (could be in reality a mixed decay, 2 states may be in same T = 1 isospin multiplet)
- A = -1/3, B = 1/3, a = -1/3, c = -1

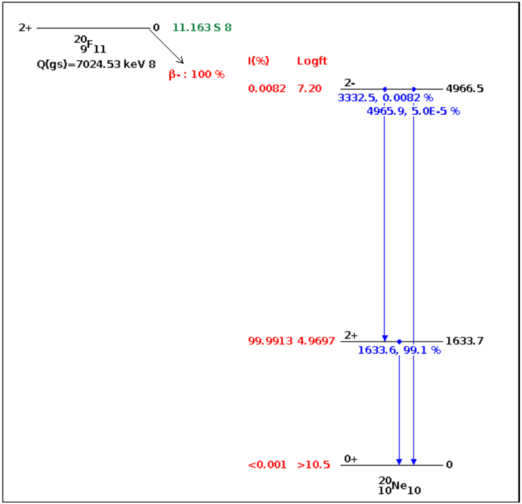


Figure: Decay Scheme of $^{20}\mathrm{F}$ into $^{20}\mathrm{Ne}$.

Gamow-Teller Decay: $^{20}\mathrm{F}$

Standard Model values

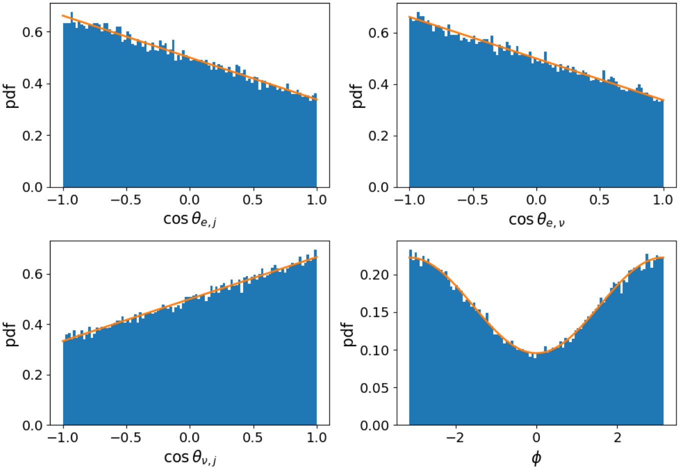


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: $^{18}\mathrm{F}$

A β^+ example: $^{18}\mathrm{F}(1^+) \to {}^{18}\mathrm{O}(0^+)$
Properties:

- Q $= 533$ keV
- No γ produced
- $J_f = J_i - 1 \to \lambda_{J_i,J_f} = \Lambda_{J_i,J_f} = 1$

Some tests for a β^+ decay. Using $C_A = C'_A = \mathrm{cte}(= 1)$.

- $C_T = C'_T = 0$ (Standard Model)
- $C_T = C'_T$ pure real (and large)
- $C_T = C'_T$ pure imaginary
- $C_T = -C'_T$, either real or imaginary

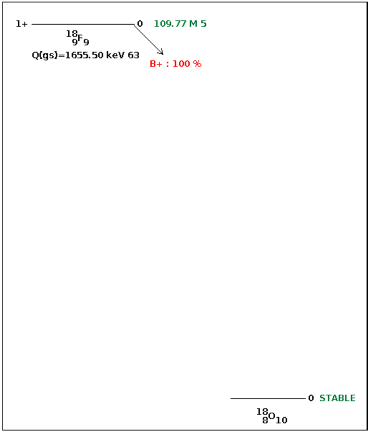


Figure: Decay Scheme of $^{18}\mathrm{F}$ into $^{18}\mathrm{O}$.

Gamow-Teller Decay: $^{18}\mathrm{F}$

Standard Model values

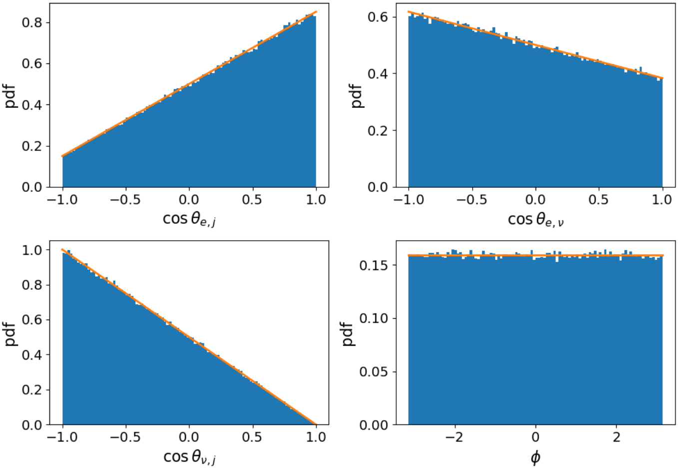


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: $^{18}\mathrm{F}$

$C_T = C_T'$ Real

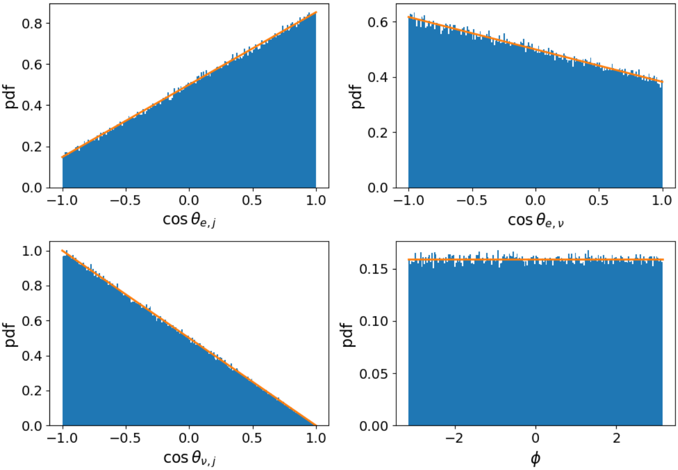


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = -1/\sqrt{2}$

Gamow-Teller Decay: $^{18}\mathrm{F}$

$C_T = C'_T$ Imaginary

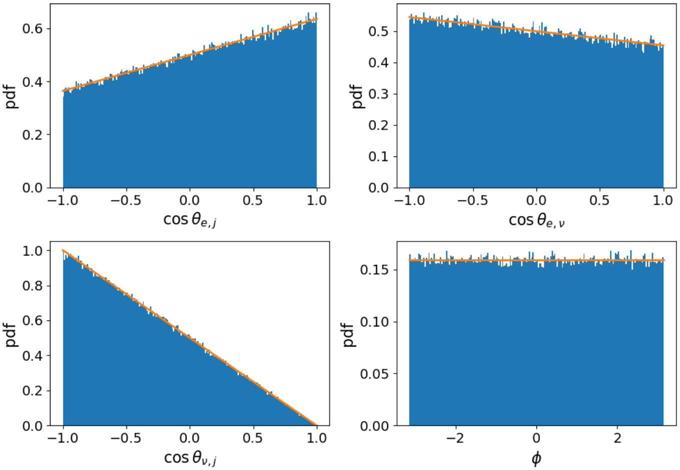


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = i/\sqrt{2}$

Gamow-Teller Decay: $^{18}\mathrm{F}$

$C_T = -C'_T$

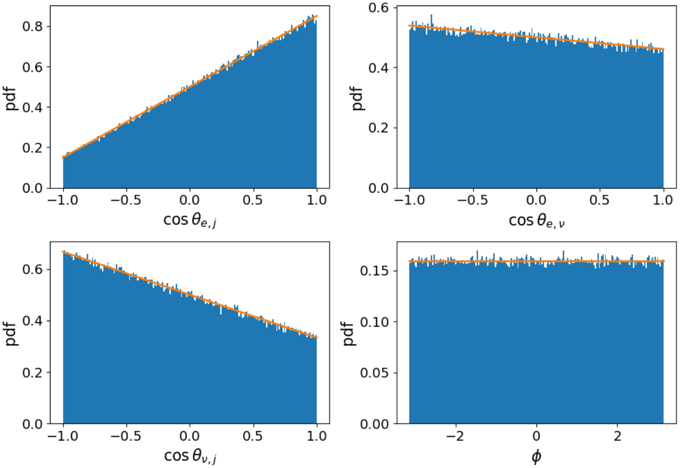


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = -C'_T = 1/\sqrt{2}$

Mixed Decays: $^{39}\mathrm{Ca}$

Now we consider a mixed β^+ decay: $^{39}\mathrm{Ca}\left(\frac{3}{2}\right)^+ \to {}^{39}\mathrm{K}\left(\frac{3}{2}\right)^+$
Properties:

- Q = 5502.5 keV (actually inconvenient, $\langle \gamma_e^{-1} \rangle = 0.2$ and $\alpha Z = 0.14$)
- No γ produced

First, do tests with 2 non-zero coupling for pairs with non-zero correlations A, B and D, with either real or imaginary values of the pair, that are not present in a Gamow-Teller decay. We use $|M_{GT} C_i| = |M_F C_i| = 1$ for convenience

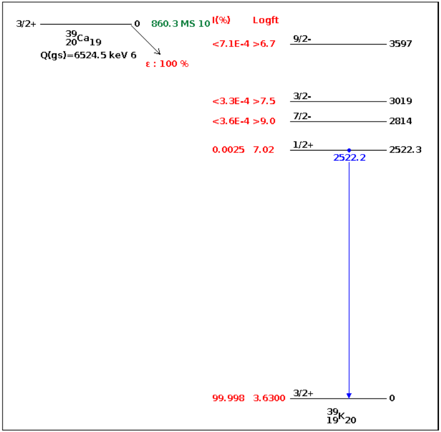


Figure: Decay Scheme of $^{39}\mathrm{Ca}$ into $^{39}\mathrm{K}$.

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_T$

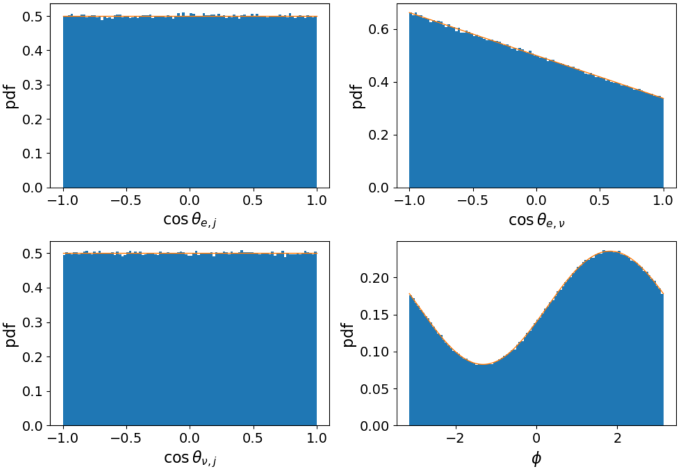


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_T = 0.6 - 0.8i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_T'$

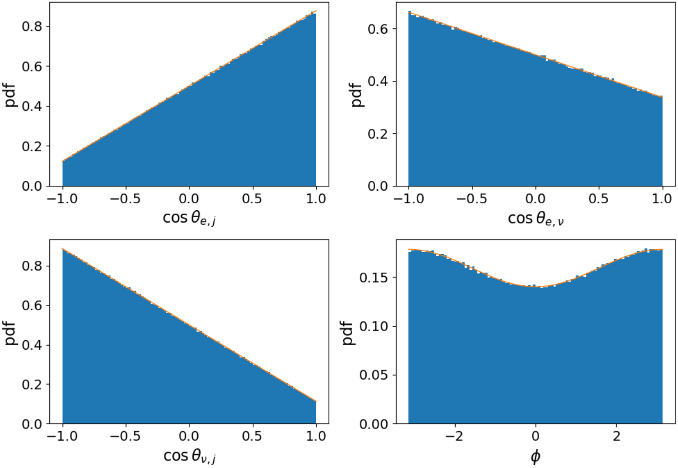


Figure: Distribution of various relevant angles,z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 1, M_{GT} C_T' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A$

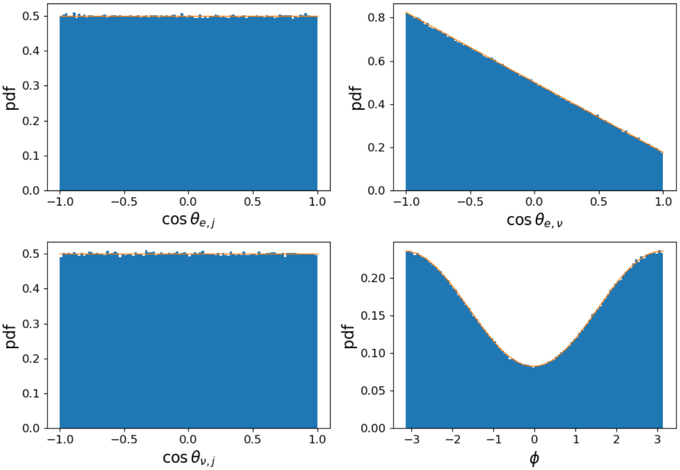


Figure: Distribution of various relevant angles,z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 1, M_{GT} C_A = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A'$

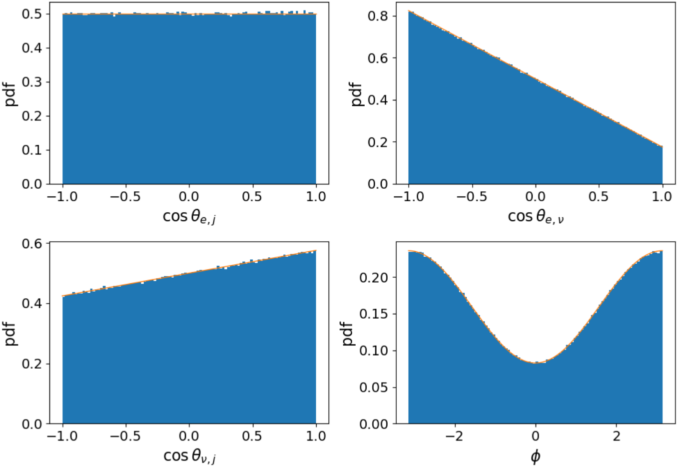


Figure: Distribution of various relevant angles,z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 1, M_{GT} C_A' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_A'$

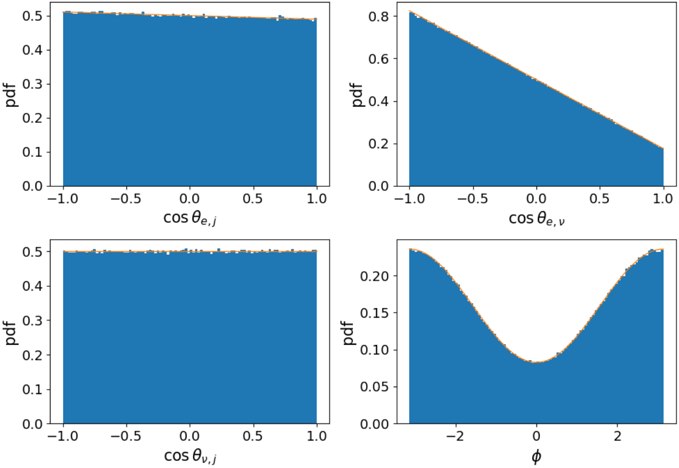


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_A' = 0.6 - 0.8i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_V C_A$

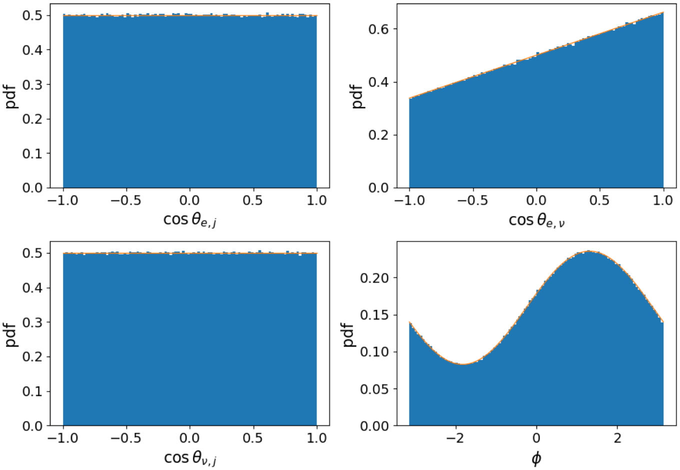


Figure: Distribution of various relevant angles,z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 0.8 + 0.6i$, $M_{GT} C_A = 0.6 - 0.8i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_A'$

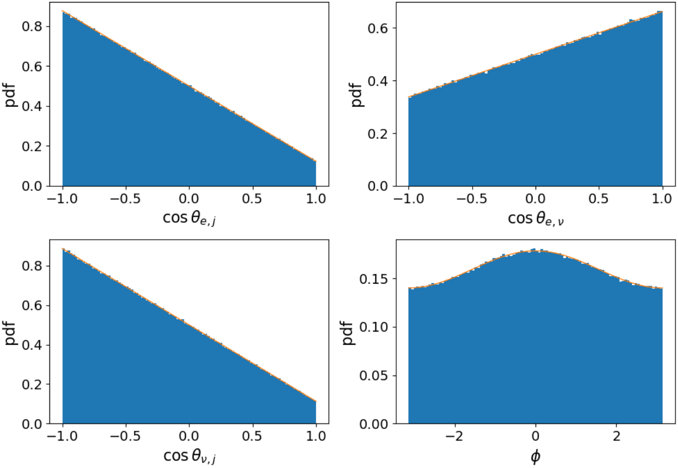


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 1, M_{GT} C_A' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_T$

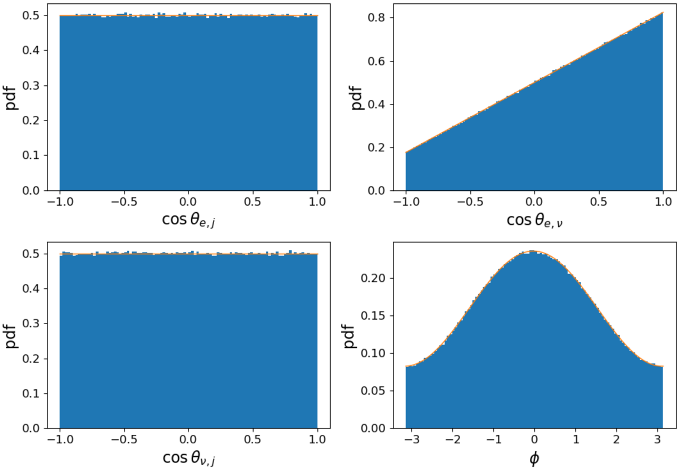


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 1, M_{GT} C_T = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_T'$

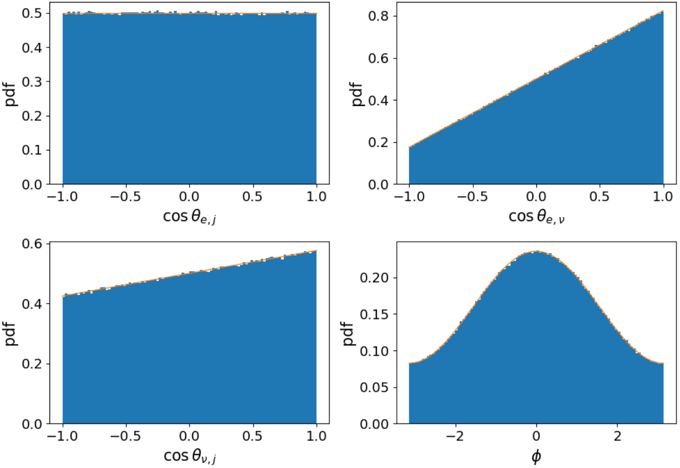


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 1, M_{GT} C_T' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_V C_T'$

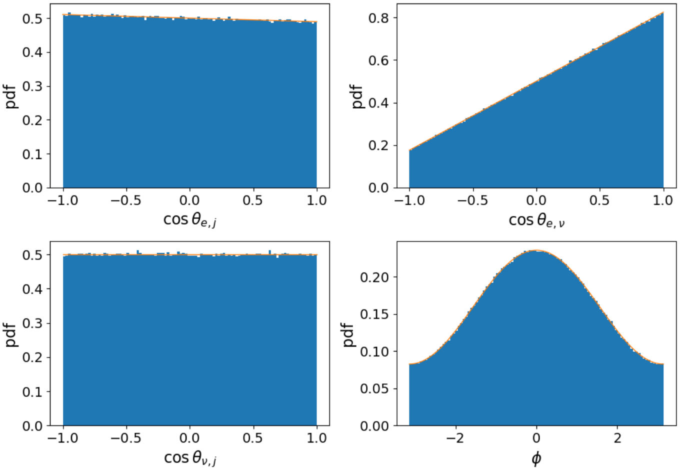


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 0.6 - 0.8i$, $M_{GT} C_T' = 0.8 + 0.6i$ and rest of couplings 0

Mixed Decays: $^{39}\mathrm{Ca}$

Now we consider a mixed β^+ decay: $^{39}\mathrm{Ca}\left(\frac{3}{2}\right)^+ \rightarrow {}^{39}\mathrm{K}\left(\frac{3}{2}\right)^+$
Properties:

- Q = 5502.5 keV (actually inconvenient, $\langle\gamma_e^{-1}\rangle = 0.2$ and $\alpha Z = 0.14$)
- No γ produced

Now, we peform tests using a close to real configuration: $M_{GT}C_A = M_{GT}C'_A = \rho = 0.661$, $C_V = C'_V = 1$, $M_F = 1$ Consider either:

- No scalar or tensor
- C_T pure real or imaginary
- C_S pure real or imaginary
- $C_S C_T$ pure imaginary

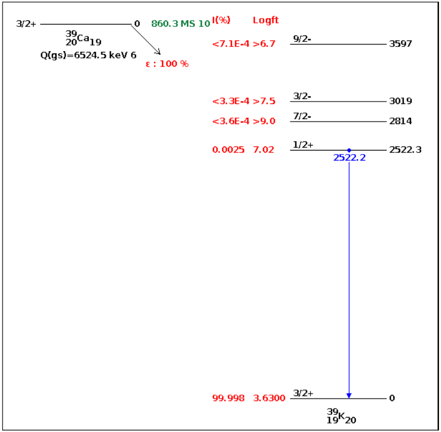


Figure: Decay Scheme of $^{39}\mathrm{Ca}$ into $^{39}\mathrm{K}$.

Mixed Decay: $^{39}\mathrm{Ca}$

Standard Model

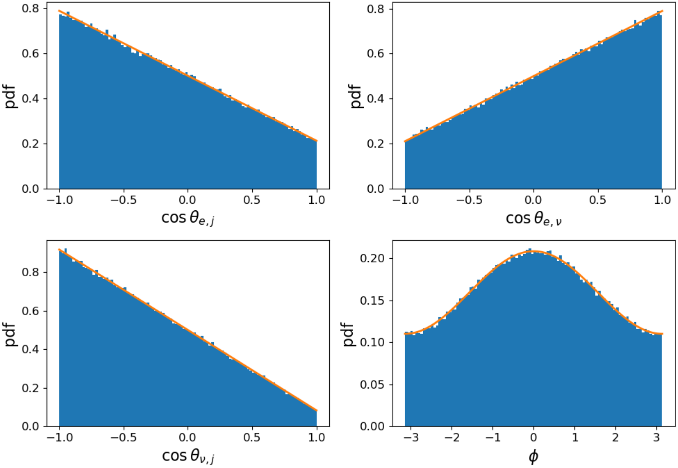


Figure: Distribution of various relevant angles,z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for the Standard Model prediction (including experimental value of the mixing ratio)

Mixed Decay: $^{39}\mathrm{Ca}$

C_T real or C'_T real

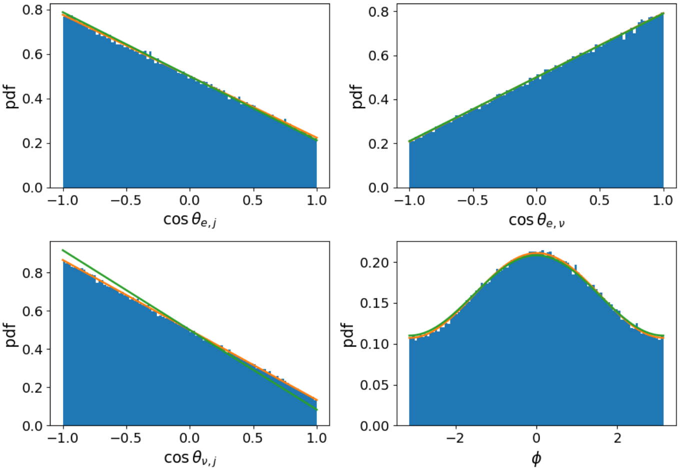


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, theoretical value for the $M_{GT}\,C_T = 0.5$ in orange and Standard Model prediction in green

Mixed Decay: $^{39}\mathrm{Ca}$

C_T imaginary or C'_T imaginary

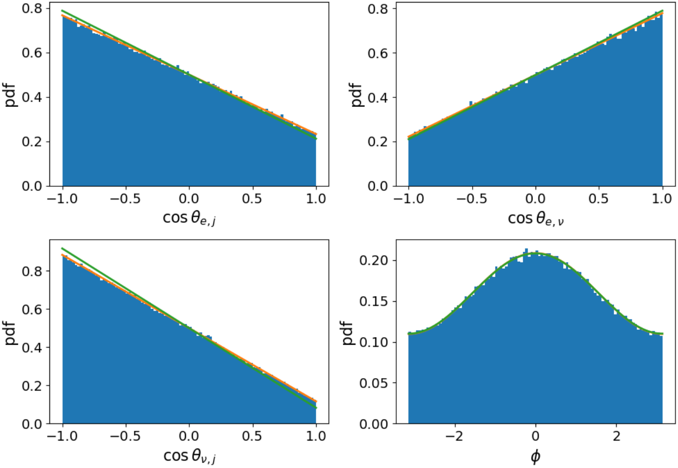


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, theoretical value for the $M_{GT}C_T = 0.5i$ in orange and Standard Model prediction in green

Mixed Decay: $^{39}\mathrm{Ca}$

C_S real or C'_S real

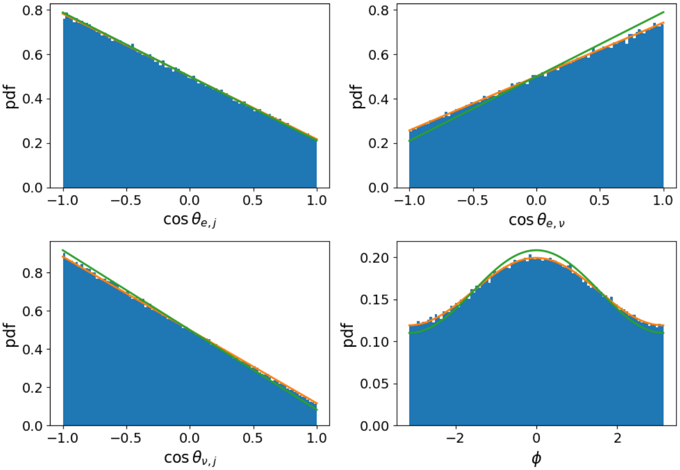


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, theoretical value for the $C_S = 0.5$ in orange and Standard Model prediction in green

Mixed Decay: $^{39}\mathrm{Ca}$

C_S imaginary or C'_S imaginary

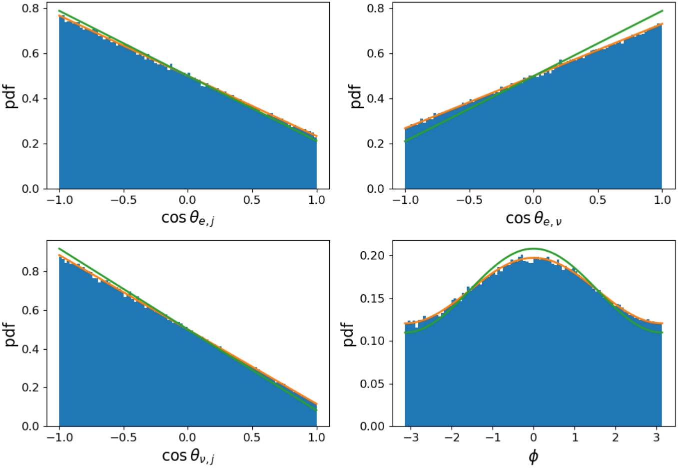


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, theoretical value for the $C_S = 0.5i$ in orange and Standard Model prediction in green

Mixed Decay: $^{39}\mathrm{Ca}$

C_S real, C_T imaginary

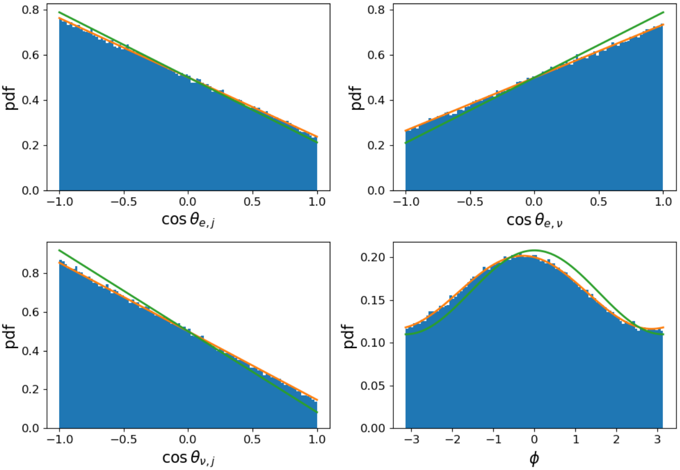


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, theoretical value for the $C_S = 0.5$ and $M_{GT} C_T = 0.5i$ in orange and Standard Model prediction in green