

Angular correlation Function

$$F = 1 + a \frac{\mathbf{p}_e \cdot \mathbf{p}_\nu}{E_e E_\nu} + \frac{\mathbf{J}}{J} \cdot \left(A \frac{\mathbf{p}_e}{E_e} + B \frac{\mathbf{p}_\nu}{E_\nu} + D \frac{\mathbf{p}_e \times \mathbf{p}_\nu}{E_e E_\nu} \right)$$

Spherical Coordinates (\mathbf{J} parallel to positive Z axis)

$$\beta_e = (r = \beta_e; \theta = \theta_e; \phi = 0), \cos(\theta_e) \equiv z_e, \beta_e = \frac{|\mathbf{p}_e|}{E} = \sqrt{1 - \frac{m_e^2}{E^2}}$$

$$\beta_\nu = (r = 1; \theta = \theta_\nu; \phi = \phi), \cos(\theta_\nu) \equiv z_\nu$$

$$\beta_e \cdot \beta_\nu = \beta_e (\cos \theta_e \cos \theta_\nu + \sin \theta_e \sin \theta_\nu \cos \phi) =$$

$$\beta_e (z_e z_\nu + \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \cos \phi)$$

$$\beta_e \cdot \mathbf{j} = \beta_e \cos \theta_e = \beta_e z_e$$

$$\beta_\nu \cdot \mathbf{j} = \cos \theta_\nu = z_\nu$$

$$\mathbf{j} \cdot (\beta_e \times \beta_\nu) = \beta_e \sin \theta_e \sin \theta_\nu \sin \phi = \beta_e \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \sin \phi$$

Single Variable: A

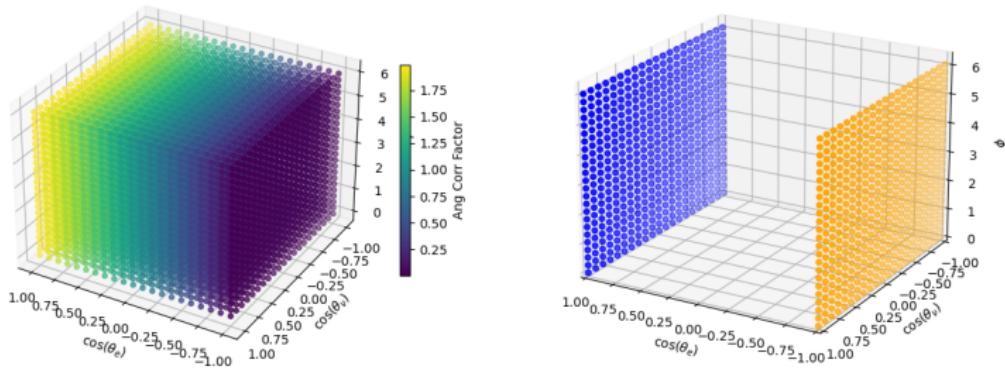


Figure: (Right) Values of the angular correlation Factor with $A = 1$, $E = 5000$ keV and rest of variables 0. (Left) Location of maximum (blue, value = 1.995) and minimum (orange, value = 0.005)

Single Variable: A

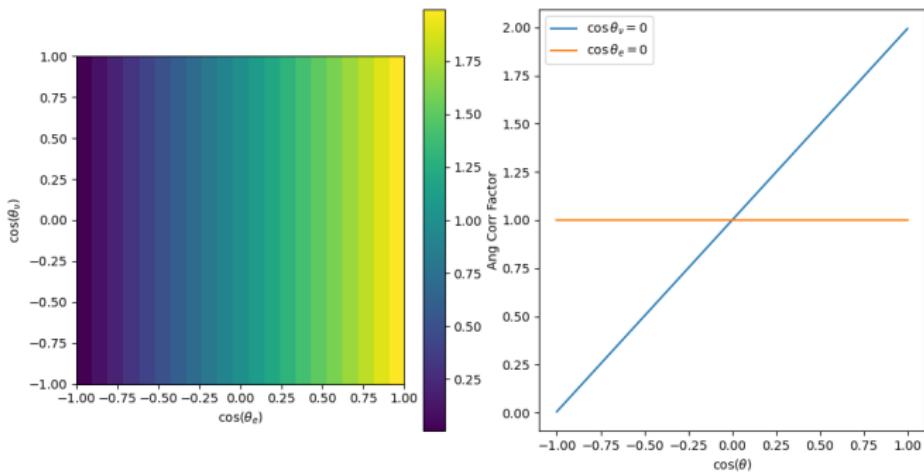


Figure: (Right) 2D projection of previous 3D image at any ϕ (Left) 1D projections at any ϕ , and either $z_e = 0$ or $z_\nu = 0$

Single Variable: B

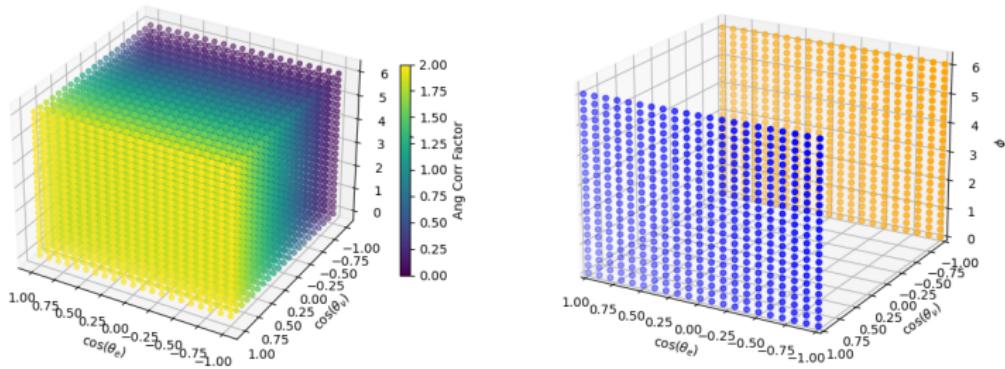


Figure: (Right) Values of the angular correlation Factor with $B = 1$, $E = 5000$ keV and rest of variables 0. (Left) Location of maximum (blue, value = 2) and minimum (orange, value = 0)

Single Variable: B

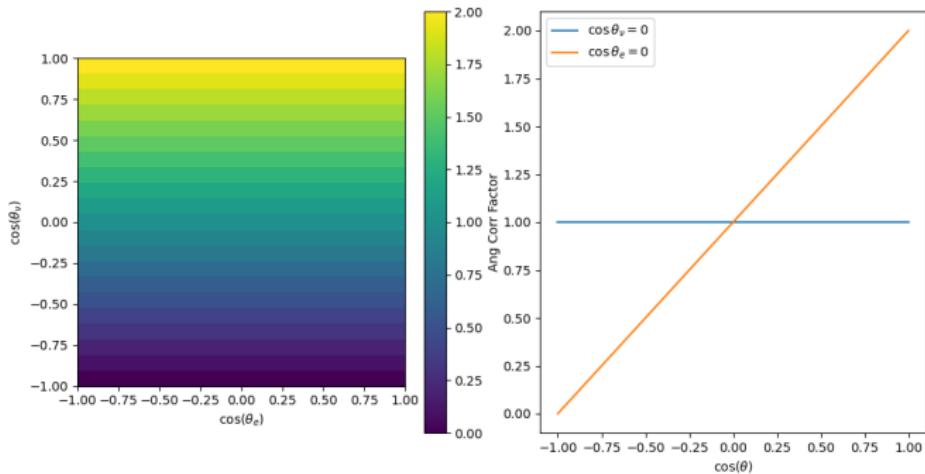


Figure: (Right) 2D projection of previous 3D image at any ϕ (Left) 1D projections at any ϕ , and either $z_e = 0$ or $z_\nu = 0$

Single Variable: a

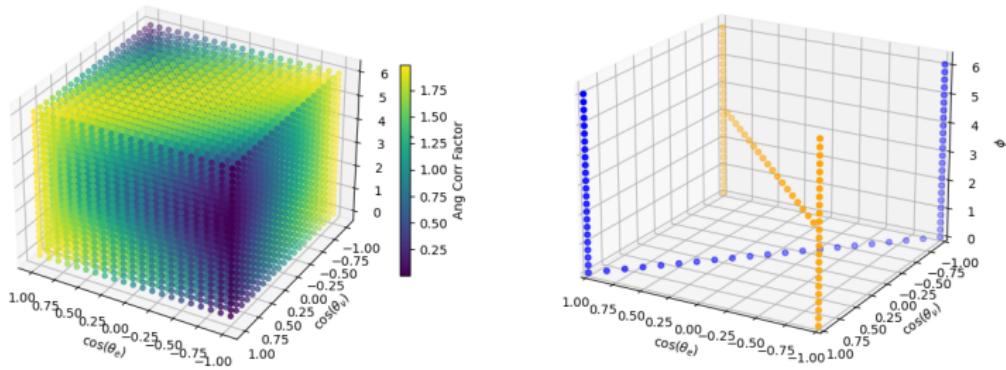


Figure: (Right) Values of the angular correlation Factor with $a = 1$, $E = 5000$ keV and rest of variables 0. (Left) Location of maximum (blue, value = 1.995) and minimum (orange, value = 0.005)

Single Variable: a

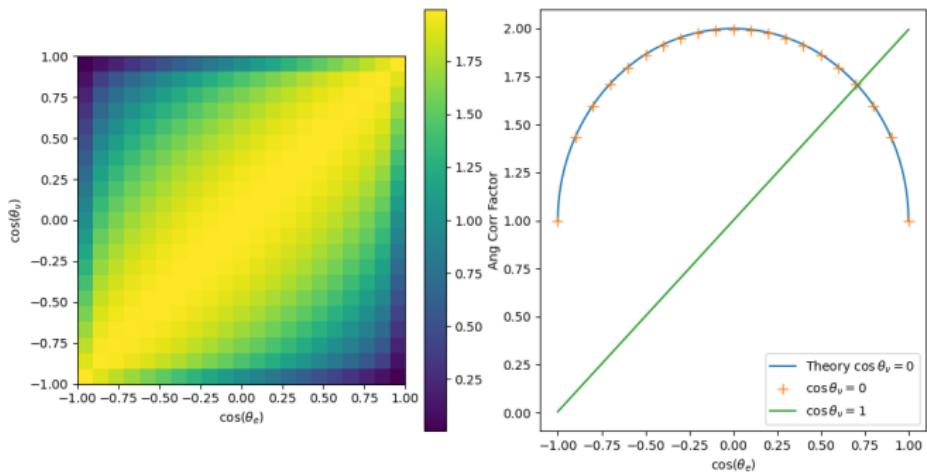


Figure: (Right) 2D projection of previous 3D image at $\phi = 0$ (Left) 1D projections at $\phi = 0$, and either $z_\nu = 0$ or $z_\nu = 1$

Single Variable: D

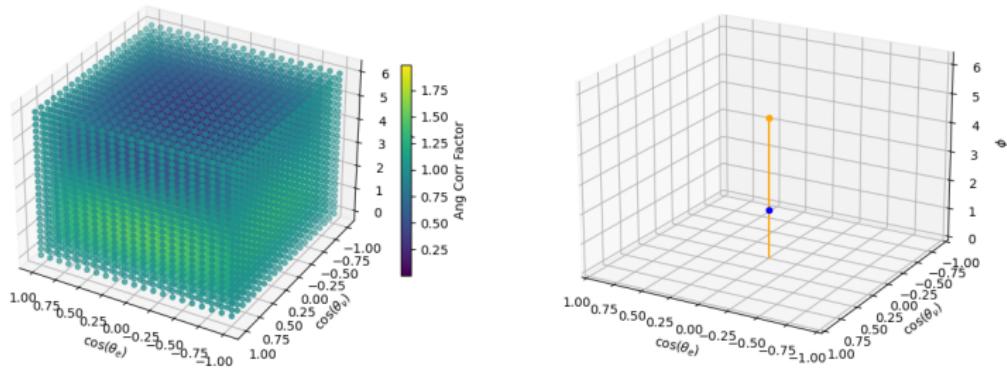


Figure: (Right) Values of the angular correlation Factor with $D = 1$, $E = 5000$ keV and rest of variables 0. (Left) Location of maximum (blue, value = 1.995) and minimum (orange, value = 0.005)

Single Variable: D

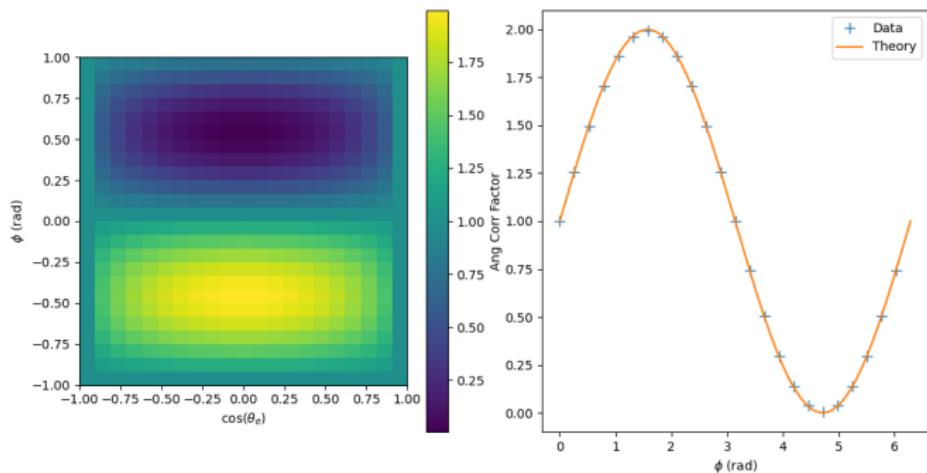


Figure: (Right) 2D projection of previous 3D image at $z_\nu = 0$ (Left) 1D projection, ie plot of F with respect to ϕ , at $z_e = 0$ and $z_\nu = 0$

Two variables: A and B

Consider different ratios by either:

- ▶ Fixing $A = B = 1$ and modifying the energy (first 3 plots)
- ▶ Same as before, but now $B = -1$
- ▶ Fixing $B = 1$ and $E \gg m_e \rightarrow \beta_e \approx 1$ and modifying $A > B$

$$F = 1 + A\beta_e z_e + Bz_\nu$$

No ϕ dependence: we can work in a 2D crosssection and capture all of the details.

Two variables: A and B

Low Energy, Positive B

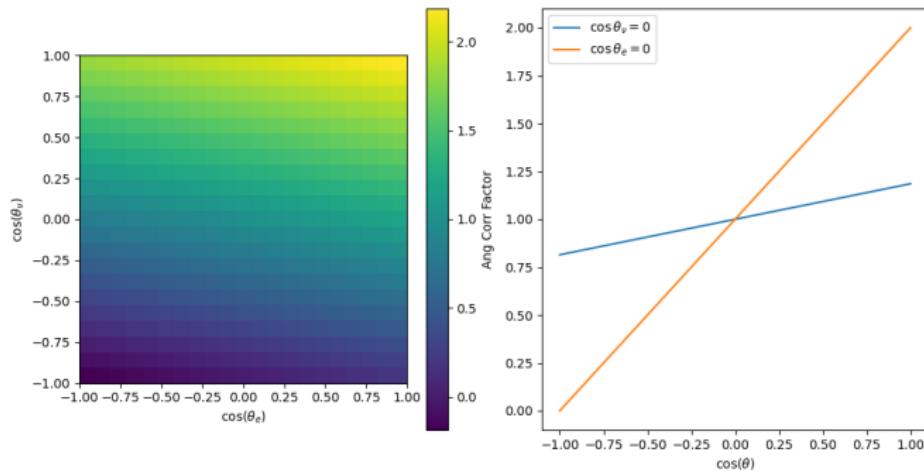


Figure: (Right) Values of the angular correlation Factor with $A = B = 1$, $E = 520$ keV and rest of variables 0. Maximum = 2.18526, Minimum = -0.18526 (Left) 1D projections at any ϕ , and either $z_e = 0$ (orange, slope $\equiv m = 1$) or $z_\nu = 0$ (blue, $m = 0.1852$)

Two variables: A and B

Medium Energy, Positive B

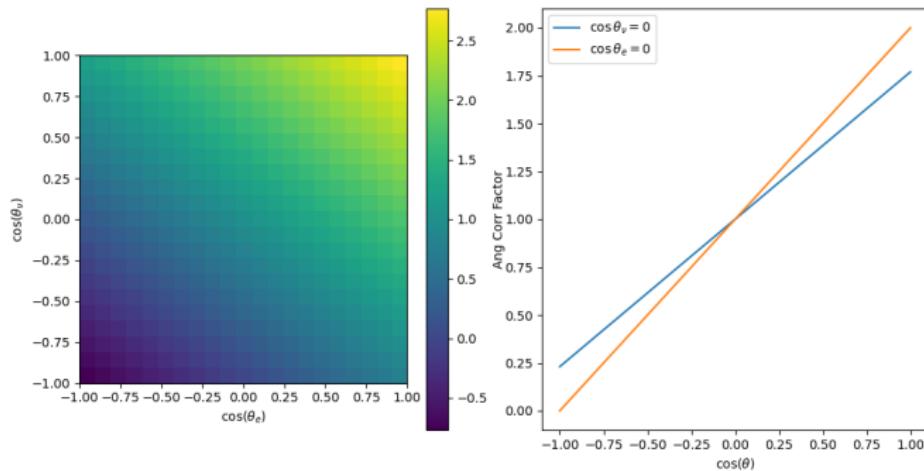


Figure: (Right) Values of the angular correlation Factor with $A = B = 1$, $E = 800$ keV and rest of variables 0. Maximum = 2.76942, Minimum = -0.76942 (Left) 1D projections at any ϕ , and either $z_e = 0$ (orange, m = 1) or $z_{\nu} = 0$ (blue, m = 0.7694)

Two variables: A and B

High Energy, Positive B

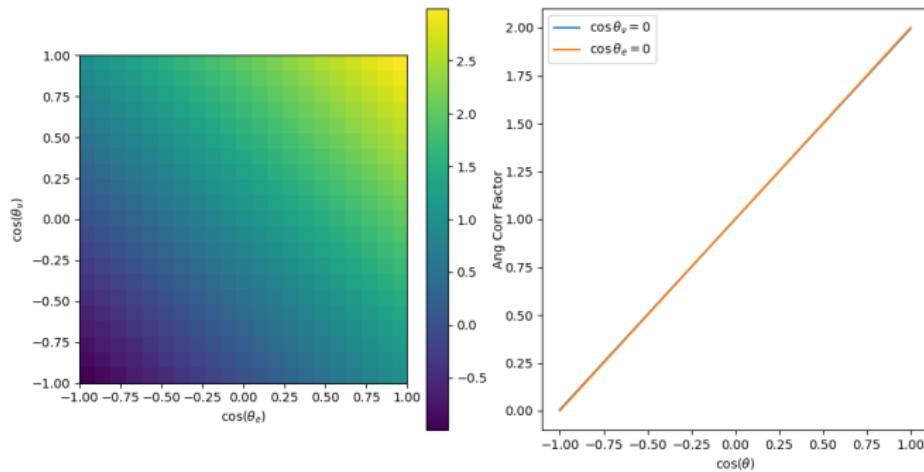


Figure: (Right) Values of the angular correlation Factor with $A = B = 1$, $E = 5000$ keV and rest of variables 0. Maximum = 2.99476, Minimum = -0.99476 (Left) 1D projections at any ϕ , and either $z_e = 0$ (orange, m = 1) or $z_\nu = 0$ (blue, m = 0.9948)

Two variables: A and B

High A, Positive B

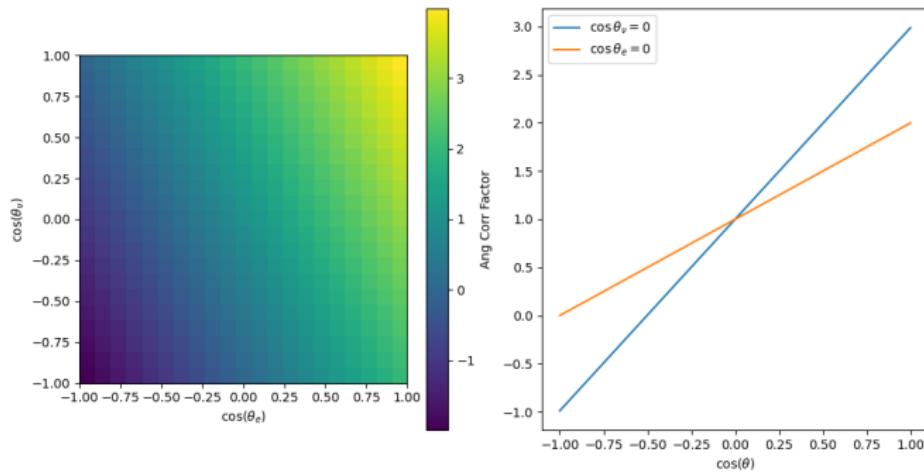


Figure: (Right) Values of the angular correlation Factor with $A = 2$, $B = 1$, $E = 5000$ keV and rest of variables 0. Maximum = 3.98953, Minimum = -1.98953 (Left) 1D projections at any ϕ , and either $z_e = 0$ (orange, $m = 1$) or $z_\nu = 0$ (blue, $m = 1.9895$)

Two variables: A and B

Very High A, Positive B

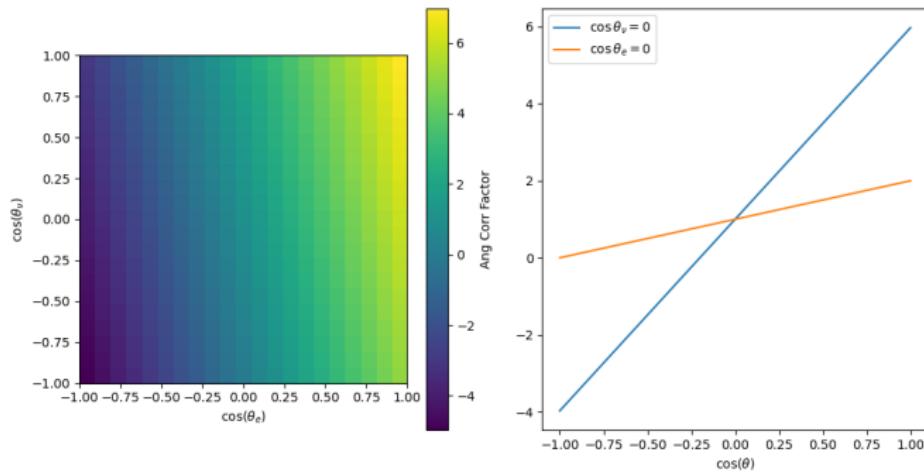


Figure: (Right) Values of the angular correlation Factor with $A = 5$, $B = 1$, $E = 5000$ keV and rest of variables 0. Maximum = 3.98953, Minimum = -1.98953 (Left) 1D projections at any ϕ , and either $z_e = 0$ (orange, $m = 1$) or $z_\nu = 0$ (blue, $m = 1.9895$)

Two variables: A and B

Low Energy, Negative B

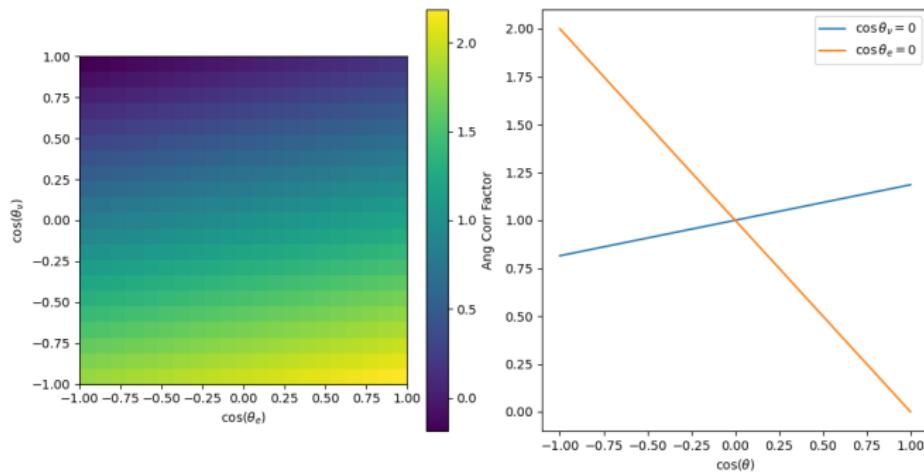


Figure: (Right) Values of the angular correlation Factor with $A = 1$, $B = -1$, $E = 520$ keV and rest of variables 0. Maximum = 2.18526, Minimum = -0.18526 (Left) 1D projections at any ϕ , and either $z_e = 0$ (orange, slope $\equiv m = 1$) or $z_\nu = 0$ (blue, $m = -0.1852$)

Two variables: A and B

High Energy, Negative B

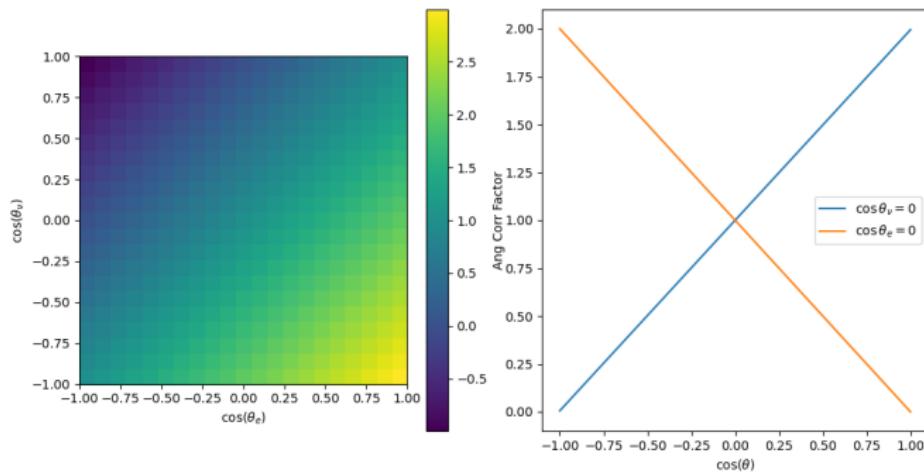


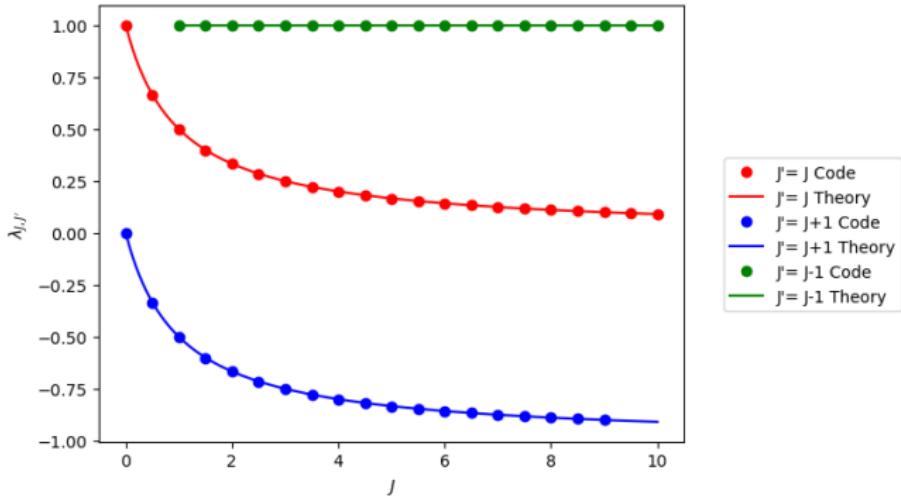
Figure: (Right) Values of the angular correlation Factor with $A = 1$, $B = -1$, $E = 5000$ keV and rest of variables 0. Maximum = 2.99476, Minimum = -0.99476 (Left) 1D projections at any ϕ , and either $z_e = 0$ (orange, $m = 1$) or $z_\nu = 0$ (blue, $m = -0.9948$)

Note of Concern

- ▶ Negative values for the angular correlation factor found in the tests. Concern for Montecarlo procedure.
- ▶ Possible in real simulations? Example from values in previous test: Gamov-Teller, $C_A = C'_A = C$, (C_V, C'_V irrelevant),
 $\xi = 2C|M_{GT}|^2$, $|A| = |B| = |\lambda_{J_i, J_f}|$

$$|A| = |B| = \frac{2}{\xi} |M_{GT}|^2 |\lambda_{J_i, J_f}| |Re(C_A \overline{C'_A})|$$

- ▶ $|\lambda_{J_i, J_f}| = 1$, negative values can be found with certainty,
 $|\lambda_{J_i, J_f}| > 0.5$ negative values are possible depending on energy



Hope: Still need to consider a , which equals

$$a = -\frac{|M_{GT}|^2(|C_A|^2 + |C'_A|^2)}{3\xi} = -\frac{1}{3}$$

. Spoiler: situation worse ($A = -B$)

Two variables: a and B

Since term proportional to a depends on E , we can consider different ratios by either:

- ▶ Fixing $a = B = 1$ and modifying the energy
- ▶ Same as before, but now $B = -1$
- ▶ Fixing $B = 1$ and $E \gg m_e \rightarrow \beta_e \approx 1$ and modifying $a > B$

We recall

$$F = 1 + a\beta_e(z_e z_\nu + \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \cos \phi) + Bz_\nu$$

Maxima and minima with $z_e = \pm 1, z_\nu = \pm 1 \rightarrow \mathbf{p}_e \parallel \mathbf{p}_\nu \parallel \mathbf{J}$

Two variables: a and B

Low Energy

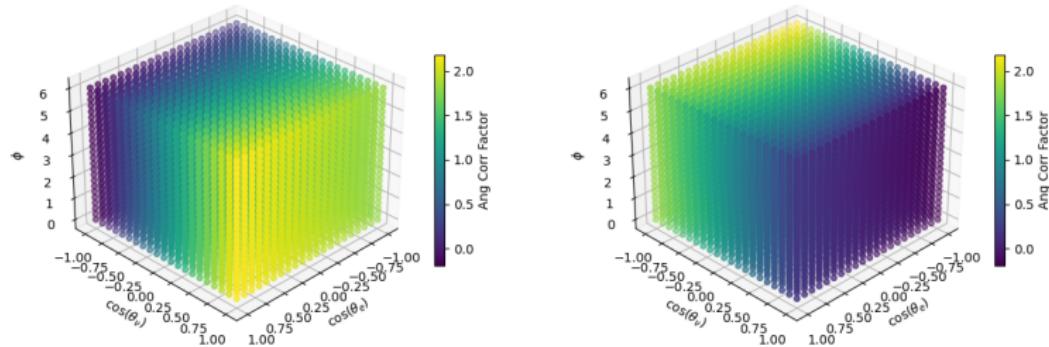


Figure: Values of the angular correlation Factor with (Right) $a = 1$, $B = 1$ and (Left) $a = 1$, $B = -1$; with $E = 520$ keV and rest of variables 0 for both.

Two variables: a and B

High Energy

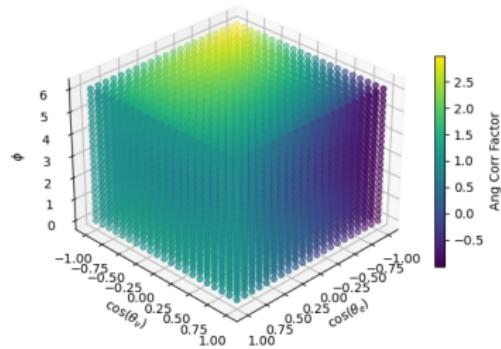
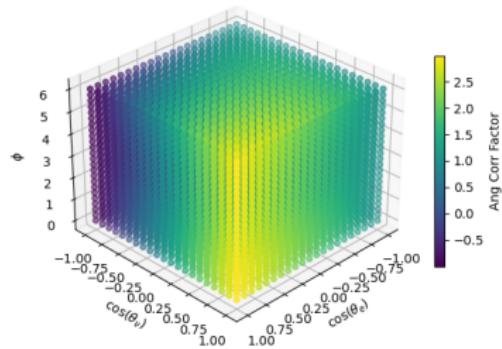


Figure: Values of the angular correlation Factor with (Right) $a = 1$, $B = 1$ and (Left) $a = 1$, $B = -1$; with $E = 5000$ keV and rest of variables 0 for both.

Two variables: a and B

More Ratios

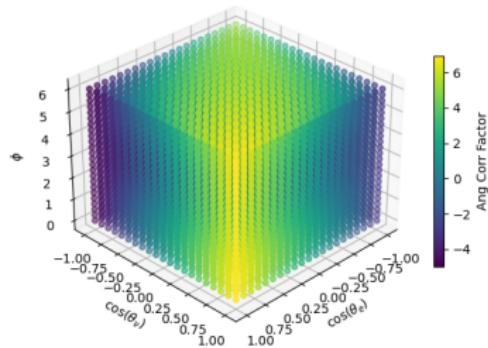
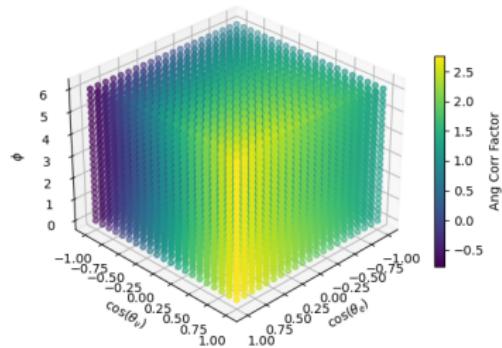


Figure: Values of the angular correlation Factor with (Right) $a = 1$, $B = 1$, $E = 800$ keV and (Left) $a = 5$, $B = 1$, $E = 5000$ keV; with the rest of variables 0 for both.

Two variables: a and B

Maximum and Minimum

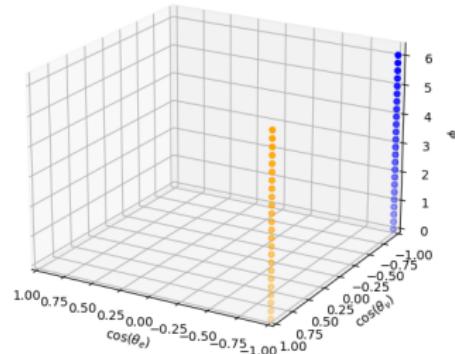
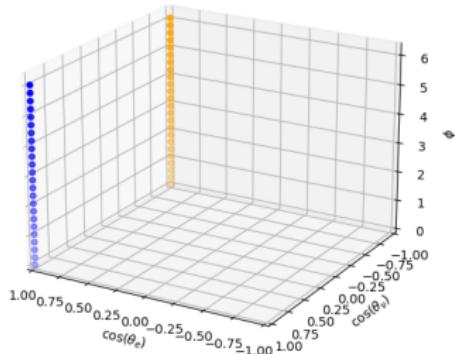


Figure: Location of maximum (blue, value = 2.99476) and minimum (orange, value = -0.99476) for (Right) $a = B = 1$, $E = 5000$ keV and (Left) $a = 1$, $B = -1$, $E = 5000$ keV

Two variables: a and A

Since both terms proportional to a depends on E , we can consider only consider different ratios by changing one (A), while leaving the other (a) fixed. For convenience $E \gg m_e$.

$$F = 1 + \beta_e(a(z_e z_\nu + \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \cos \phi) + A z_e)$$

Maximum and minimum with $z_e = \pm 1, z_\nu = \pm 1 \rightarrow \mathbf{p}_e \parallel \mathbf{p}_\nu \parallel \mathbf{J}$

Two variables: a and A

$$|A| \ll a$$

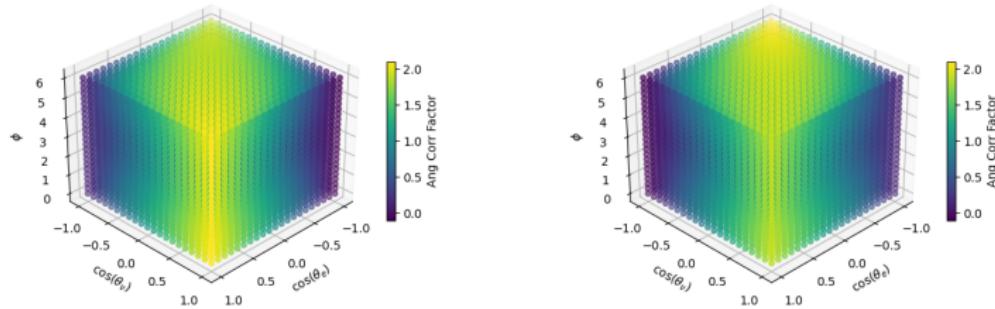


Figure: Values of the angular correlation Factor with (Right) $a = 1$, $A = 0.1$ and (Left) $a = 1$, $A = -0.1$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: a and A

$$|A| = a$$

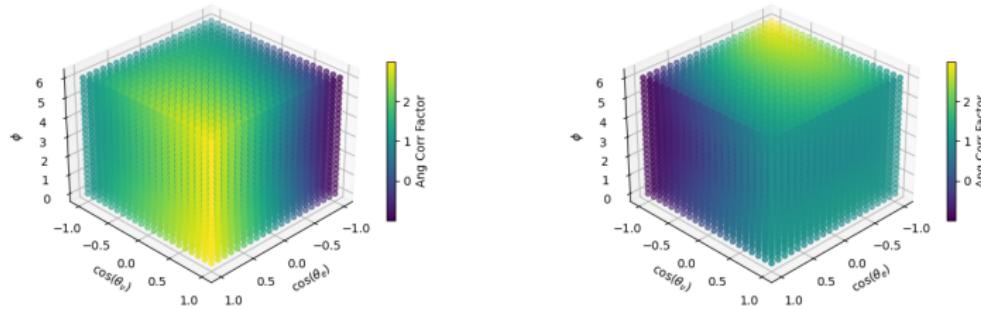


Figure: Values of the angular correlation Factor with (Right) $a = 1$, $A = 1$ and (Left) $a = 1$, $A = -1$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: a and A

$$|A| \gg a$$

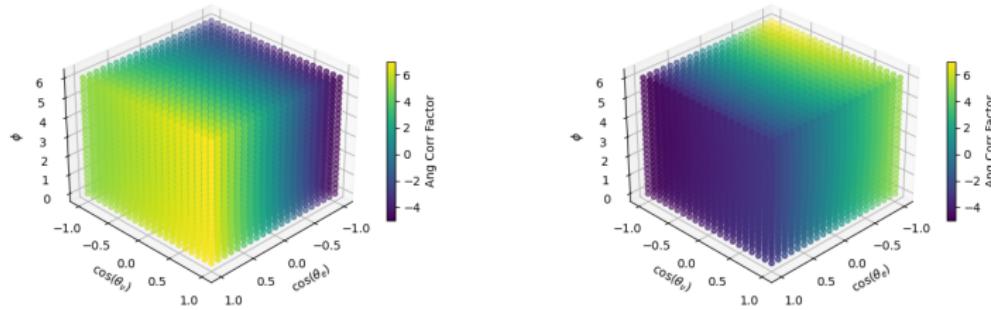


Figure: Values of the angular correlation Factor with (Right) $a = 1$, $A = 5$ and (Left) $a = 1$, $A = -5$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: a and A

Maximum and Minimum

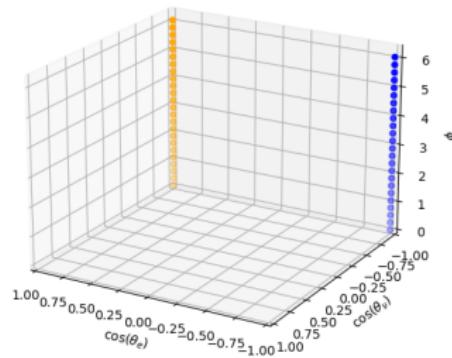
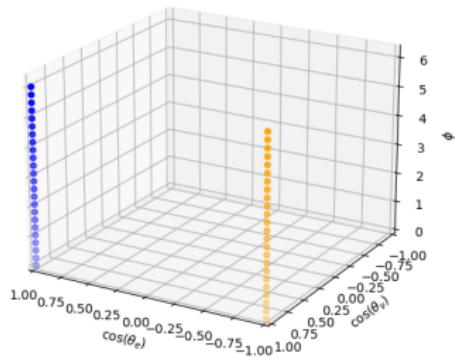


Figure: Location of maximum (blue, value = 2.99997) and minimum (orange, value = -0.99997) for (Right) $a = B = 1$, $E = 5000$ keV and (Left) $a = 1$, $B = -1$, $E = 5000$ keV

Two variables: B and D

Since term proportional to a depends on E , we can consider different ratios by either:

- ▶ Fixing $B = D = 1$ and modifying the energy
- ▶ Same as before, but now $B = -1$
- ▶ Fixing $B = 1$ and $E \gg m_e \rightarrow \beta_e \approx 1$ and modifying $D > B$

We recall

$$F = 1 + D\beta_e \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \sin \phi + Bz_\nu$$

Maxima and minima no longer at $z_e = \pm 1, z_\nu = \pm 1$. In fact, expect $z_e = 0, \phi = \pm \pi/2$. Additionally, at the maximum ($D\beta_e \equiv D'$):

$$z_\nu = \frac{B}{\sqrt{D'^2 + B^2}}$$

$$F = 1 + \sqrt{D'^2 + B^2}$$

Two variables: B and D

3D examples

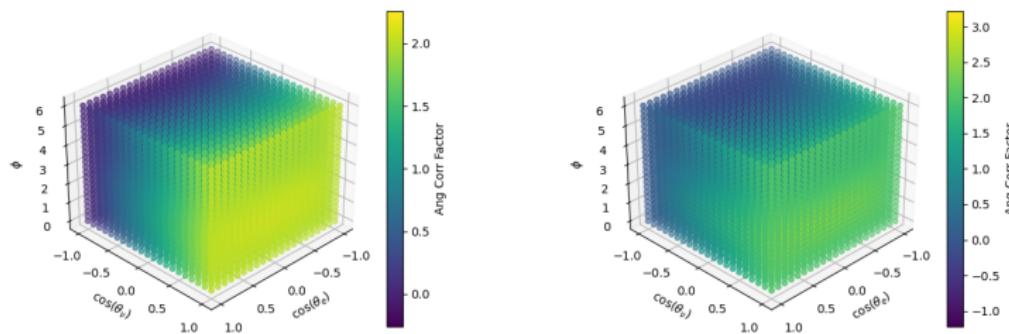


Figure: Values of the angular correlation Factor with (Right) $D = 1$, $B = 1$, $E = 800$ keV and (Left) $D = 2$, $B = 1$, $E = 5000$ keV, with the rest of variables 0 for both.

Image difficult to treat: consider only properties of the extrema.

Two variables: B and D

Low Energy

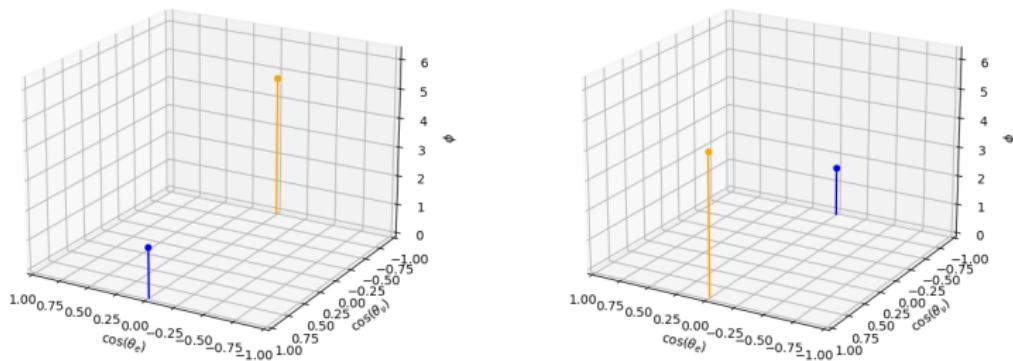


Figure: Location of maximum (blue) and minimum (orange) for (Right)
 $D = B = 1$ (Left) $D = 1, B = -1$, both at $E = 520$ keV

Two variables: B and D

High Energy

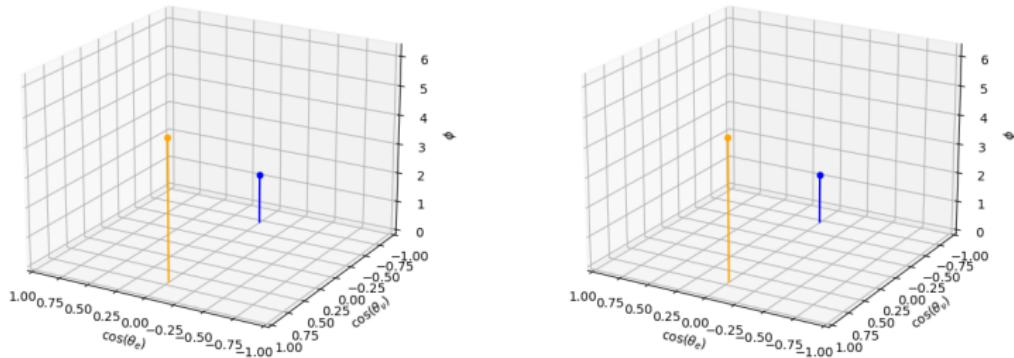


Figure: Location of maximum (blue) and minimum (orange) for (Right)
D = B = 1 and (Left) D = 1, B = -1, both at E = 5000 keV

Two variables: B and D

More Ratios

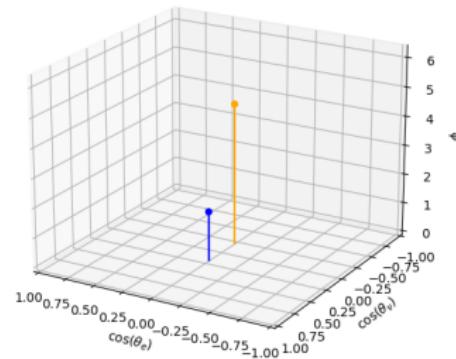
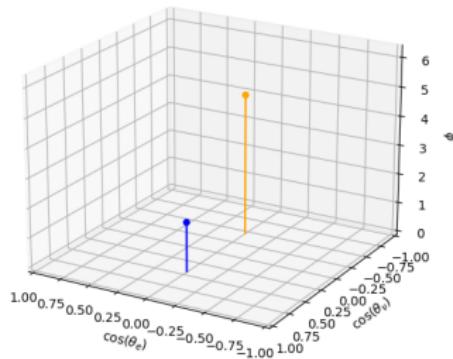


Figure: Location of maximum (blue) and minimum (orange) for (Right) $D = 2$, $B = 1$, $E = 5000$ keV and (Left) $D = 5$, $B = 1$, $E = 5000$ keV

Two variables: B and D

Behaviour of maximum

Considering now only cases with $B > 0$:

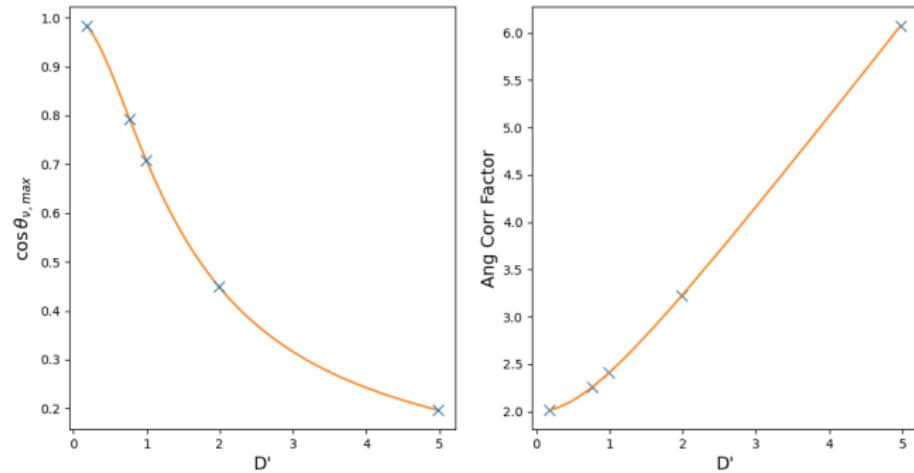


Figure: Behaviour of the z_ν coordinate for the maximum and the maximum value of the angular correlation factor for different values of D'

Two variables: A and D

Since both terms proportional to a depends on E , we can consider only consider different ratios by changing one (A), while leaving the other (D) fixed. For convenience $E \gg m_e$.

$$F = 1 + D\beta_e \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \sin \phi + A\beta_e z_e$$

Maxima and minima no longer at $z_e = \pm 1, z_\nu = \pm 1$. In fact, expect $z_\nu = 0, \phi = \pm \pi/2$. Additionally, at the maximum:

$$z_e = \frac{A}{\sqrt{D^2 + A^2}}$$

$$F = 1 + \beta_e \sqrt{D^2 + A^2}$$

We look directly at the properties of the extrema.

Two variables: A and D

$$|A| \ll D$$

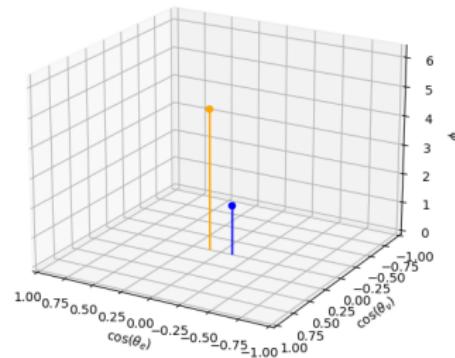
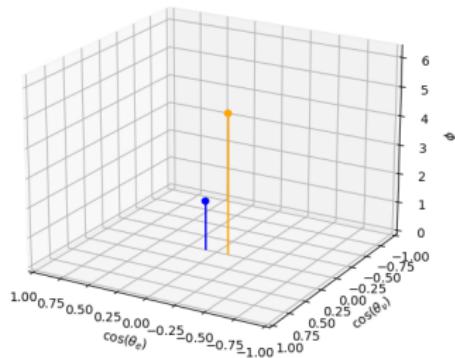


Figure: Positions of the maximum (blue) and minimum (orange) for (Right) $D = 1$, $A = 0.1$ and (Left) $D = 1$, $A = -0.1$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: A and D

$$|A| = D$$

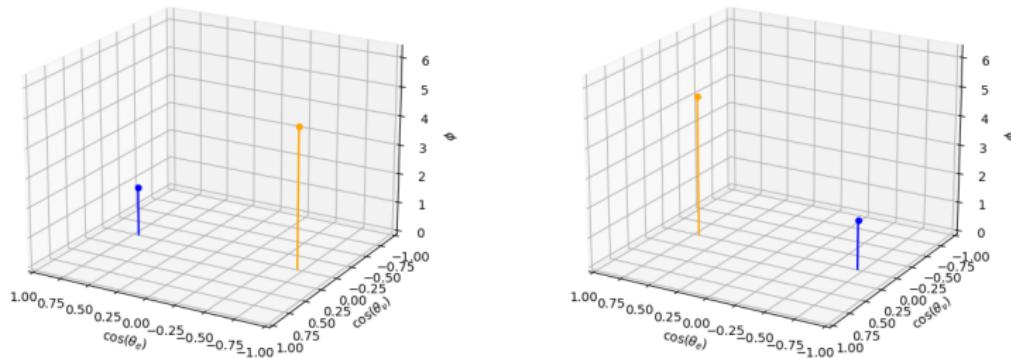


Figure: Positions of the maximum (blue) and minimum (orange) (Right) $D = 1$, $A = 1$ and (Left) $D = 1$, $A = -1$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: A and D

$$|A| \gg D$$

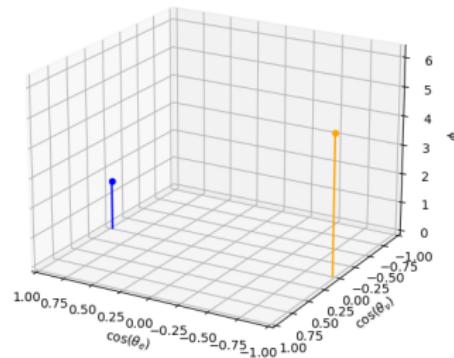
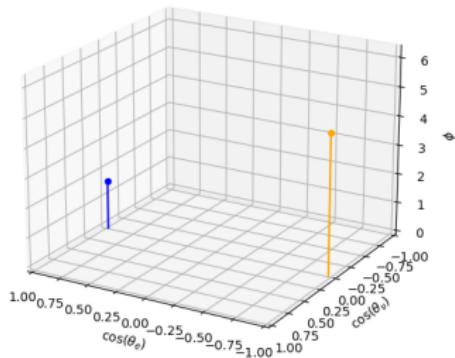


Figure: Positions of the maximum (blue) and minimum (orange) (Right) $D = 1$, $A = 5$ and (Left) $D = 1$, $A = -5$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: A and D

Behaviour of maximum

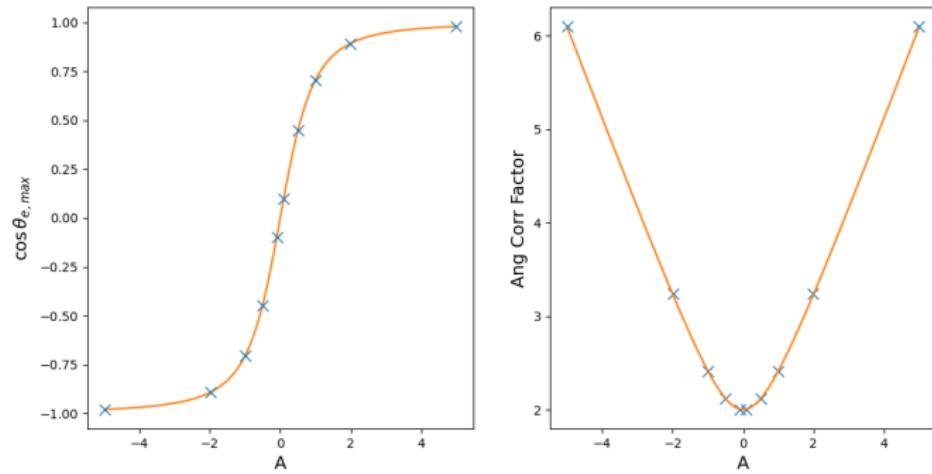


Figure: Behaviour of the z_ν coordinate for the maximum and the maximum value of the angular correlation factor for different values of A

Two variables: a and D

Since both terms proportional to a depends on E , we can consider only consider different ratios by changing one (D), while leaving the other (a) fixed. For convenience $E \gg m_e$.

$$F = 1 + \beta_e (a(z_e z_\nu + \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \cos \phi) + D \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \sin \phi)$$

Maxima and minima no longer at $z_e = \pm 1, z_\nu = \pm 1$. In fact, expect $z_\nu = 0, z_e = 0$. Additionally, at the maximum:

$$\tan \phi = \frac{D}{a}$$

$$F = 1 + \beta_e \sqrt{D^2 + a^2}$$

We look directly at the properties of the extrema.

Two variables: a and D

$$|D| \ll a$$

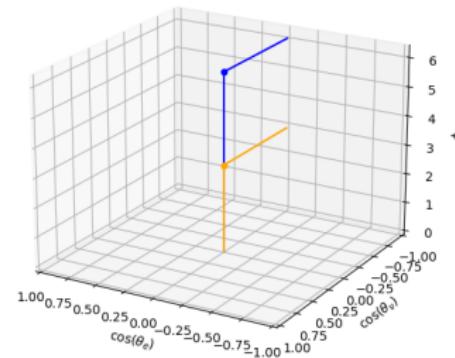
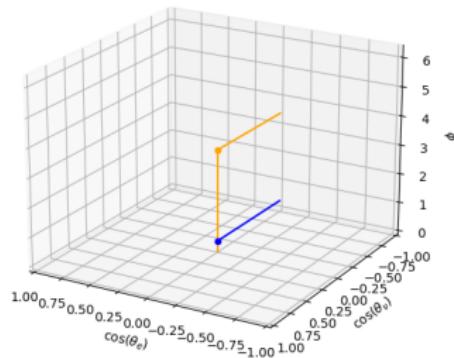


Figure: Positions of the maximum and minimum with Factor with (Right) $a = 1$, $D = 0.25$ and (Left) $a = 1$, $D = -0.25$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: a and D

$$|D| = a$$

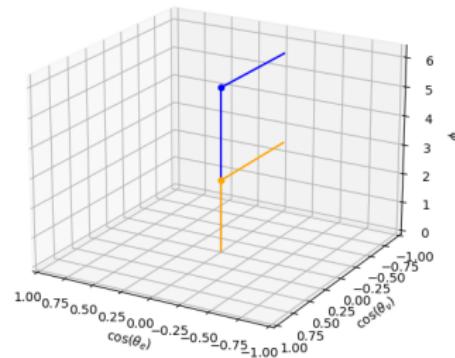
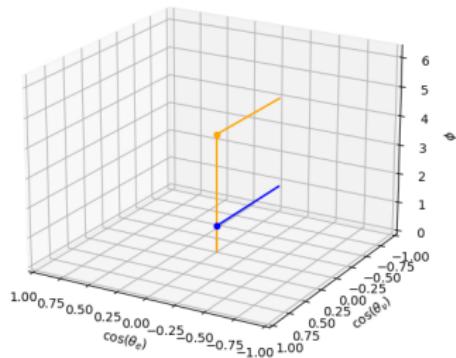


Figure: Positions of the maximum and minimum with with (Right) $a = 1$, $D = 1$ and (Left) $a = 1$, $D = -1$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: a and D

$$|D| \gg a$$

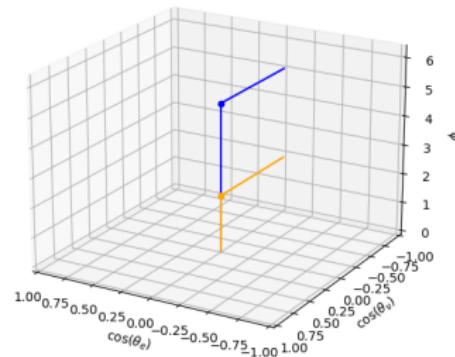
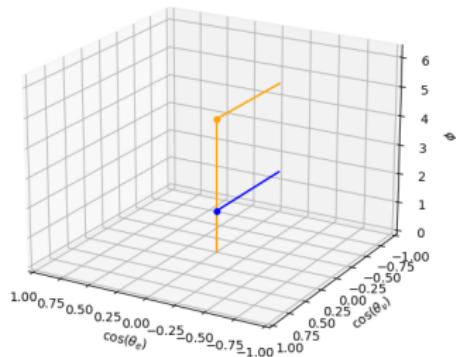


Figure: Positions of the maximum and minimum with (Right) $a = 1$, $D = 4$ and (Left) $a = 1$, $D = -4$, with $E = 100000$ keV and rest of variables 0 for both.

Two variables: a and D

Behaviour of maximum

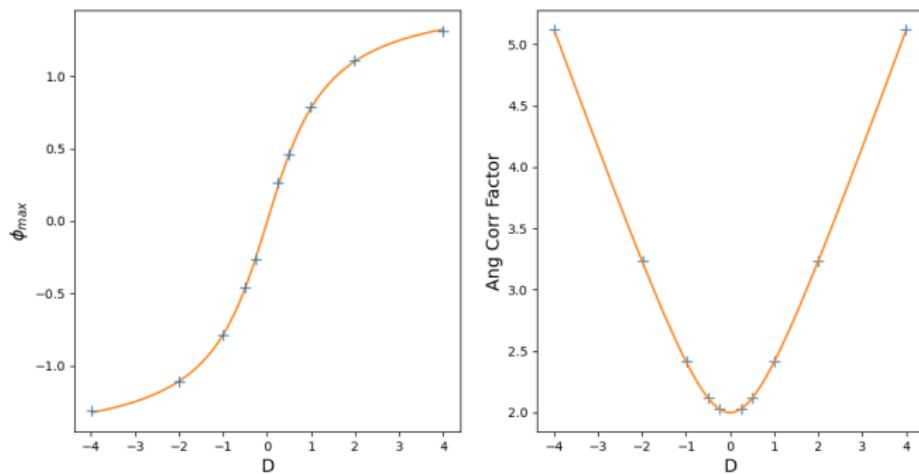


Figure: Behaviour of the ϕ coordinate for the maximum and the maximum value of the angular correlation factor for different values of D. Note discrepancies at the endpoint are a result of a sampling too coarse.

Three variables: a, A, B

We come back to the case we hinted at before, that is compatible with all Gamov-Teller decays within the Standard Model framework.

Gamov-Teller, $C_A = C'_A = C$, (C_V, C'_V irrelevant), $\xi = 2C|M_{GT}|^2$, $|A| = |B| = |\lambda_{J_i, J_f}|$, $A < 0$, $B > 0$

$$|A| = |B| = \frac{2}{\xi} |M_{GT}|^2 |\lambda_{J_i, J_f}| |\operatorname{Re}(C_A \overline{C'_A})| = |\lambda_{J_i, J_f}|$$

$$a = -\frac{|M_{GT}|^2 (|C_A|^2 + |C'_A|^2)}{3\xi} = -\frac{1}{3}$$

We consider different values of $|A|$, particularly with $|A| > 0.5$, and a range of electron energies from very low to very high.

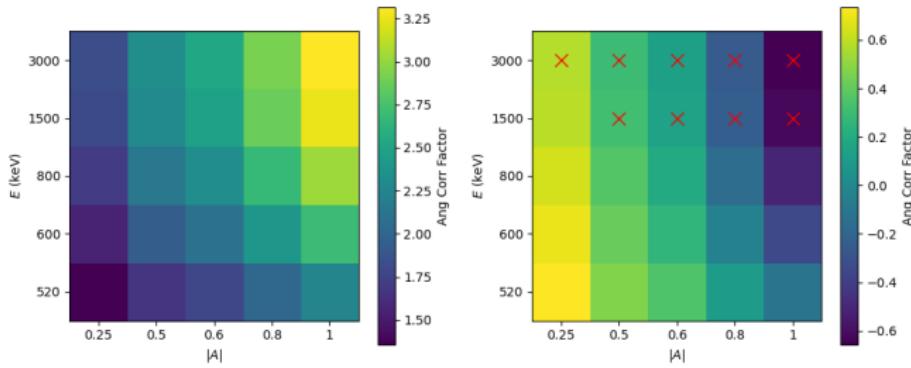


Figure: Values of the maximum and minimum for different combinations of $|A|$ and E . , Values that are negative are marked with a cross