October 21, 2025

We need to test the a, b, c functions implemented. We test them by setting only one pair of values to a non-zero value.

1 Fermi ($J_i = 0$; $J_f = 0$)

We first analyse the simpler case: a Fermi transition. Here we only need to consider a and b, as c should be always undefined ($J = 0$ breaks the factor in c formula)

CS	ξ	a_t	b_t	c_t	a	b	c
CSP	2	-1	0	NaN	-1	0	NaN
CT	1	-1	0	NaN	-1	0	NaN
CTP	1	-1	0	NaN	-1	0	NaN
CV	2	0	cte	NaN	0	cte	NaN
CVP	2	0	0	NaN	0	0	NaN
CA	1	-1	0	NaN	-1	0	NaN
CAP	1	-1	0	NaN	-1	0	NaN

Table 1: Results of the test with CS as one of the variables. The first column is the second coupling constant, 2nd is ξ; 3rd to 5th are the expectation from inspecting the function, 6th to 8th the values from the test

1.1 First Test: vs NaN vs cte vs $\neq$ cte

Since we need do characterise each pair and check if the values given are correct for the whole energy, we'd need to compare the output with a graph. To reduce the amount of graphs, we can inspect first if the functions will return 0, be undefined (ξ in the denominator of the expressions can be 0), return a non-zero constant value or a variable one. It is to note if C_S, C'_S, C_V or C'_V is non-zero, we get an extra constant term in 1. The expected value from the single variable parts is computed and checked

CSP	ξ	a_t	b_t	c_t	a	b	c
CT	1	-1	0	NaN	-1	0	NaN
CTP	1	-1	0	NaN	-1	0	NaN
CV	2	0	0	NaN	0	0	NaN
CVP	2	0	cte	NaN	0	cte	NaN
CA	1	-1	0	NaN	-1	0	NaN
CAP	1	-1	0	NaN	-1	0	NaN

1.1.1 Real Terms

For a first test, we set to +1 only the two constants we select in the pair, rest are set to 0. Here are the results.

CT	ξ	a_t	b_t	c_t	a	b	c
CTP	0	NaN	NaN	NaN	NaN	NaN	NaN
CV	1	1	0	NaN	1	0	NaN
CVP	1	1	0	NaN	1	0	NaN
CA	0	NaN	NaN	NaN	NaN	NaN	NaN
CAP	0	NaN	NaN	NaN	NaN	NaN	NaN

CTP	ξ	a_t	b_t	c_t	a	b	c
CV	1	1	0	0	1	0	0
CVP	1	1	0	0	1	0	0
CA	0	NaN	NaN	NaN	NaN	NaN	NaN
CAP	0	NaN	NaN	NaN	NaN	NaN	NaN

CV	ξ	a_t	b_t	c_t	a	b	c
CVP	2	1	0	NaN	1	0	NaN
CA	1	1	0	NaN	1	0	NaN
CAP	1	1	0	NaN	1	0	NaN

CVP	ξ	a_t	b_t	c_t	a	b	c
CA	1	1	0	NaN	1	0	NaN
CAP	1	1	0	NaN	1	0	NaN

CA	ξ	a_t	b_t	c_t	a	b	c
CAP	0	NaN	NaN	NaN	NaN	NaN	NaN

We observe the results agree with the predictions

1.1.2 Imaginary Terms

For a second test, we give the pair the values $c_1 = 0.8 + 0.6i$, $c_2 = 0.6 - 0.8i$. These are chosen so that the modulus is 1, giving us integer values for ξ. The values are also chosen to prove only the imaginary terms, as $c_1\overline{c_2} = (0.8 + 0.6i)(0.6 + 0.8i) = i$, so any term on the real part will be zero. This also implies b = 0 in this test.

CS	ξ	a_t	b_t	c_t	a	b	c
CSP	2	-1	0	NaN	-1	0	NaN
CT	1	-1	0	NaN	-1	0	NaN
CTP	1	-1	0	NaN	-1	0	NaN
CV	2	$\neq$cte	0	NaN	$\neq$cte	0	NaN
CVP	2	0	0	NaN	0	0	NaN
CA	1	-1	0	NaN	-1	0	NaN
CAP	1	-1	0	NaN	-1	0	NaN

CSP	ξ	a_t	b_t	c_t	a	b	c
CT	1	-1	0	NaN	-1	0	NaN
CTP	1	-1	0	NaN	-1	0	NaN
CV	2	0	0	NaN	0	0	NaN
CVP	2	$\neq$cte	0	NaN	$\neq$cte	0	NaN
CA	1	-1	0	NaN	-1	0	NaN
CAP	1	-1	0	NaN	-1	0	NaN

CT	ξ	a_t	b_t	c_t	a	b	c
CTP	0	NaN	NaN	NaN	NaN	NaN	NaN
CV	1	1	0	NaN	1	0	NaN
CVP	1	1	0	NaN	1	0	NaN
CA	0	NaN	NaN	NaN	NaN	NaN	NaN
CAP	0	NaN	NaN	NaN	NaN	NaN	NaN

CTP	ξ	a_t	b_t	c_t	a	b	c
CV	1	1	0	NaN	1	0	NaN
CVP	1	1	0	NaN	1	0	NaN
CA	0	NaN	NaN	NaN	NaN	NaN	NaN
CAP	0	NaN	NaN	NaN	NaN	NaN	NaN

CV	ξ	a_t	b_t	c_t	a	b	c
CVP	2	1	0	NaN	1	0	NaN
CA	1	1	0	NaN	1	0	NaN
CAP	1	1	0	NaN	1	0	NaN

CVP	ξ	a_t	b_t	c_t	a	b	c
CA	1	1	0	NaN	1	0	NaN
CAP	1	1	0	NaN	1	0	NaN

CA	ξ	a_t	b_t	c_t	a	b	c
CAP	0	NaN	NaN	NaN	NaN	NaN	NaN

The results obtained match the predictions once more, confirming the function works

1.2 Second Test: correct numerical values

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Out of the combinations selected as non-zero constant and variable, we do the computations to check the validity. In our test, we have used $Z = 14$ and assumed the transition is β^-, that is $betaType = +1$, with $Q = 2000$keV. There are few values we need to validate, as most have been checked already.

1.2.1 cs=cv=1;csp=cvp=1

Only non-zero is b, relevant term is

$$b = \frac{2\gamma|M_F|^2}{\xi} Re(C_S\overline{C_V} + C'_S\overline{C'_V}) = \gamma$$

With the values chosen we obtain 0.994768 up to 6 decimals, exactly as the value printed in terminal 0.994768

1.2.2 cs=0.6+0.8i,cv=0.8-0.6i; csp=0.6+0.8i,cvp=0.8-0.6i;

Only non-zero term is a, relevant term is

$$a = -\frac{2|M_F|^2\alpha Z m_e}{\xi p_e} Im(C_S\overline{C_V} + C'_S\overline{C'_V})$$

we plot the result

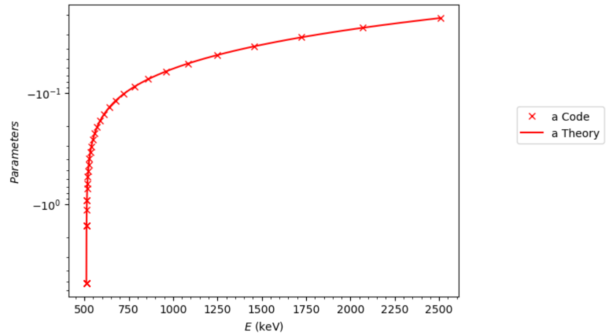

Figure 1: Coeficients a with respect to energy of the electron for a Fermi Z = 14 Q = 2000 keV decay with non-zero coupling constants cs=0.8+0.6i,cv=0.6-0.8i.

2 Gamov Teller Decay (J_i = 2,J_f = 1)

We now consider a Gamov-Teller transition, to select the terms in a, b and c that are proportional to M_{GT}.

2.1 First Test: 0 vs NaN vs cte vs ≠ cte

We perform the same checks initially as in the pure Fermi case. We once more check the results of the maximum function (=0, =NaN (shown in the results as -1) or ≠0) and the characterisation of the function (constant or variable) from the test.

2.1.1 Real Values

For a first test, we set to +1 only the two constants we select in the pair, rest are set to 0. As in the previous case, some NaN can be found. c is not automatically NaN. From the values $\Lambda_{J_i,J_f} = 1$ and the aligmennt is equal to $J^2 = 4$. This gives all the factors in front of c equal to -1. Equivalently, the factor in front of a is 1/3

CS	ξ	a_t	b_t	c_t	a	b	c
CSP	0	NaN	NaN	NaN	NaN	NaN	NaN
CT	1	1/3	0	-1	1/3	0	-1
CTP	1	1/3	0	-1	1/3	0	-1
CV	0	NaN	NaN	NaN	NaN	NaN	NaN
CVP	0	NaN	NaN	NaN	NaN	NaN	NaN
CA	1	-1/3	0	1	-1/3	0	1
CAP	1	-1/3	0	1	-1/3	0	1

Table 2: Results of the test with CS as one of the variables. The first column is the second coupling constant, 2nd is ξ; 3rd to 5th are the expectation from inspecting the function, 6th to 8th the values from the test

CSP	ξ	a_t	b_t	c_t	a	b	c
CT	1	1/3	0	-1	1/3	0	-1
CTP	1	1/3	0	-1	1/3	0	-1
CV	0	NaN	NaN	NaN	NaN	NaN	NaN
CVP	0	NaN	NaN	NaN	NaN	NaN	NaN
CA	1	-1/3	0	1	1/3	0	-1
CAP	1	-1/3	0	1	1/3	0	-1

CT	ξ	a_t	b_t	c_t	a	b	c
CTP	2	1/3	0	-1	1/3	0	-1
CV	1	1/3	0	-1	1/3	0	-1
CVP	1	1/3	0	-1	1/3	0	-1
CA	2	0	cte	0	0	cte	0
CAP	2	0	0	0	0	0	0

CTP	ξ	a_t	b_t	c_t	a	b	c
CV	1	1/3	0	-1			
CVP	1	1/3	0	-1			
CA	2	0	0	0			
CAP	2	0	cte	0			

CV	ξ	a_t	b_t	c_t	a	b	c
CVP	0	NaN	NaN	NaN	NaN	NaN	NaN
CA	1	-1/3	0	1	-1/3	0	1
CAP	1	-1/3	0	1	-1/3	0	1

CVP	ξ	a_t	b_t	c_t	a	b	c
CA	1	-1/3	0	1	-1/3	0	1
CAP	1	-1/3	0	1	-1/3	0	1

CA	ξ	a_t	b_t	c_t	a	b	c
CAP	2	-1/3	0	1	-1/3	0	1

All results agreed already from the first implementation

2.1.2 Complex Values

For a second test, we give the pair the values $c_1 = 0.8 + 0.6i$, $c_2 = 0.6 - 0.8i$. These are chosen so that the modulus is 1, giving us integer values for ξ. The values are also chosen to prove only the imaginary terms, as $c_1\overline{c_2} = (0.8 + 0.6i)(0.6 + 0.8i) = i$, so any term on the real part will be zero. This also implies $b = 0$ in this test.

CS	ξ	a_t	b_t	c_t	a	b	c
CSP	0	NaN	NaN	NaN	NaN	NaN	NaN
CT	1	1/3	0	-1	1/3	0	-1
CTP	1	1/3	0	-1	1/3	0	-1
CV	0	NaN	NaN	NaN	NaN	NaN	NaN
CVP	0	NaN	NaN	NaN	NaN	NaN	NaN
CA	1	-1/3	0	1	-1/3	0	1
CAP	1	-1/3	0	1	-1/3	0	1

Table 3: Results of the test with CS as one of the variables. The first column is the second coupling constant, 2nd is ξ; 3rd to 5th are the expectation from inspecting the function, 6th to 8th the values from the test

CSP	ξ	a_t	b_t	c_t	a	b	c
CT	1	1/3	0	-1	1/3	0	-1
CTP	1	1/3	0	-1	1/3	0	-1
CV	0	NaN	NaN	NaN	NaN	NaN	NaN
CVP	0	NaN	NaN	NaN	NaN	NaN	NaN
CA	1	-1/3	0	1	1/3	0	-1
CAP	1	-1/3	0	1	1/3	0	-1

CT	ξ	a_t	b_t	c_t	a	b	c
CTP	2	1/3	0	-1	1/3	0	-1
CV	1	1/3	0	-1	1/3	0	-1
CVP	1	1/3	0	-1	1/3		
CA	2	$\neq$cte	0	$\neq$cte	$\neq$cte	0	$\neq$cte
CAP	2	0	0	0			

CTP	ξ	a_t	b_t	c_t	a	b	c
CV	1	1/3	0	-1	1/3	0	-1
CVP	1	1/3	0	-1	1/3	0	-1
CA	2	0	0	0	1/3	0	-1
CAP	2	$\neq$cte	0	$\neq$cte	$\neq$cte	0	$\neq$cte

CV	ξ	a_t	b_t	c_t	a	b	c
CVP	0	NaN	NaN	NaN	NaN	NaN	NaN
CA	1	-1/3	0	1	-1/3	0	1
CAP	1	-1/3	0	1	-1/3	0	1

CVP	ξ	a_t	b_t	c_t	a	b	c
CA	1	-1/3	0	1	-1/3	0	1
CAP	1	-1/3	0	1	-1/3	0	1

CA	ξ	a_t	b_t	c_t	a	b	c
CAP	2	-1/3	0	1	-1/3	0	1

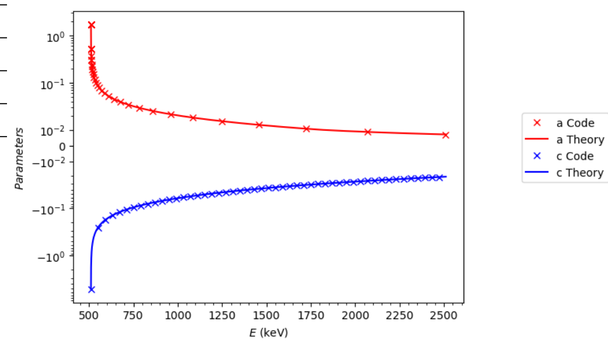

Figure 2: Coeficients a and c with respect to energy of the electron for a Fermi Z = 14 Q = 2000 keV decay with non-zero coupling constants ct=0.8+0.6i,ca=0.6-0.8i.

2.2 Second Test: correct numerical values

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2.2.1 ct=ca=1;ctp=cap=1

Only non-zero is b, relevant term is

$$b = \frac{2\gamma|M_{GT}|^2}{\xi} Re(C_T\overline{C_A} + C'_T\overline{C'_A}) = \gamma$$

With the values chosen we obtain 0.994768 up to 6 decimals, exactly as the value printed in terminal 0.994768

2.2.2 ct=0.6+0.8i,ca0.8-0.6i; ctp=0.6+0.8i,cap=0.8-0.6i;

Only both a and c are non-zero, relevant term is

$$a = \frac{2|M_{GT}|^2 \alpha Z m_e}{3\xi p_e} Im(C_T\overline{C_A} + C'_T\overline{C'_A})$$

$$c = \frac{2|M_{GT}|^2 \alpha Z m_e}{\xi p_e} Im(C_T\overline{C_A} + C'_T\overline{C'_A}) \Lambda_{J_i,J_f} \frac{J(J+1) - 3\left\langle(\mathbf{J}\cdot\mathbf{j})^2\right\rangle}{J)(2J-1)}$$