CRADLE++ Tests

November 4, 2025

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Simplest non-trivial case: Gamow-Teller decay. Full test: decay of a known nuclei. Simplifying assumption: focus on one decay path of that nuclei. Example:

$$^{60}\mathrm{Co}(5^+) \to {}^{60}\mathrm{Ni}(4^+)$$

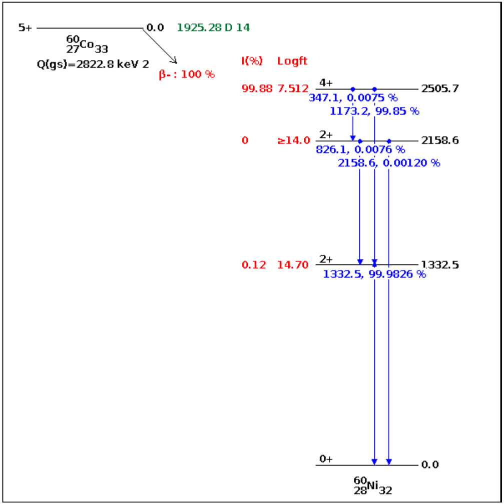


Figure: Decay Scheme of $^{60}\mathrm{Co}$ into $^{60}\mathrm{Ni}$.

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Properties of $^{60}\mathrm{Co}(5^+) \to {}^{60}\mathrm{Ni}(4^+)$

- Q = 317.06 keV (not fully ideal)
- $J_f = J_i - 1 \to \lambda_{J_i,J_f} = \Lambda_{J_i,J_f} = 1$
- 2 γ almost always ($5^+ \to 2^+$ only 1 γ)

Many cases to consider, though for realism: keep $C_A = C'_A = \mathrm{cte}(= 1)$.

- $C_T = C'_T = 0$ (Standard Model)
- $C_T = C'_T$ pure real (and large)
- $C_T = C'_T$ pure imaginary
- $C_T = -C'_T$, either real or imaginary

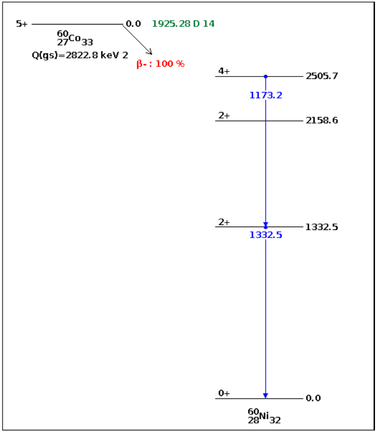
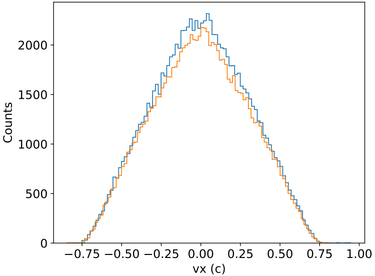


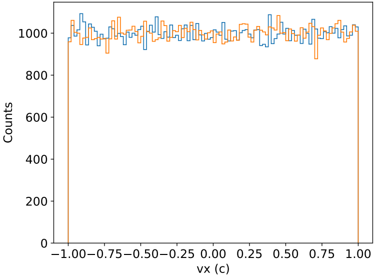
Figure: Decay Scheme of $^{60}\mathrm{Co}$ into $^{60}\mathrm{Ni}$ featuring the only decay of interest

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values



(a) e

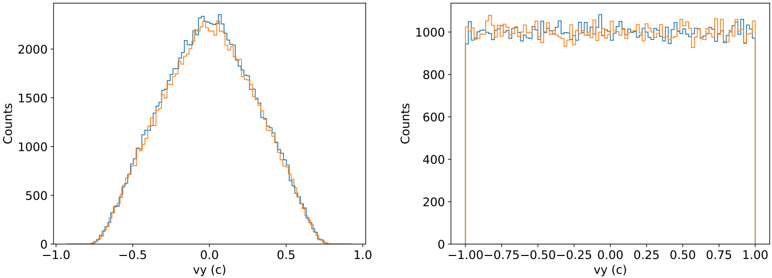


(b) ν

Figure: Distribution of the x component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values

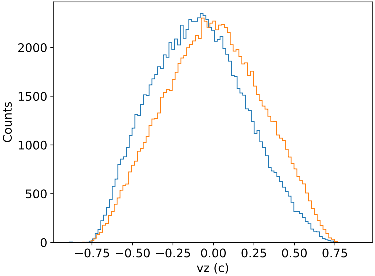


(a) e (b) ν

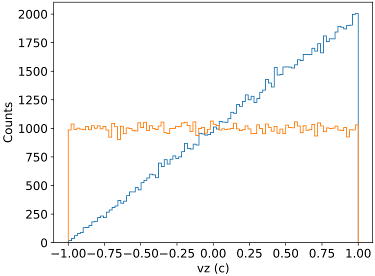
Figure: Distribution of the y component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values



(a) e



(b) ν

Figure: Distribution of the z component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Numerical evaluation

Use that distributions in z_e, z_ν, $\cos\theta_{e,\nu} \equiv z_{e,\nu}$ and ϕ are known.

$$f(z_e) = \frac{1 + \left\langle b\gamma_e^{-1}\right\rangle + \left\langle A\beta_e\right\rangle z_e}{2(1 + \left\langle b\gamma_e^{-1}\right\rangle)}$$

$$f(z_\nu) = \frac{1 + \left\langle b\gamma_e^{-1}\right\rangle + \left\langle B\beta_e\right\rangle z_\nu}{2(1 + \left\langle b\gamma_e^{-1}\right\rangle)}$$

$$f(z_{e,\nu}) = \frac{1 + \left\langle b\gamma_e^{-1}\right\rangle + \left\langle a\beta_e\right\rangle z_{e,\nu}}{2(1 + \left\langle b\gamma_e^{-1}\right\rangle)}$$

$$f(\phi) = \frac{1 + \left\langle b\gamma_e^{-1}\right\rangle + \left\langle \left(a + \frac{c}{3}\right)\beta_e\right\rangle \frac{\pi^2}{16}\cos\phi + \left\langle D\beta_e\right\rangle \frac{\pi^2}{16}\sin\phi}{2\pi(1 + \left\langle b\gamma_e^{-1}\right\rangle)}$$

Averages computed numerically using f(E) from the simulation data itself (avoid computing the Fermi function myself)

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values

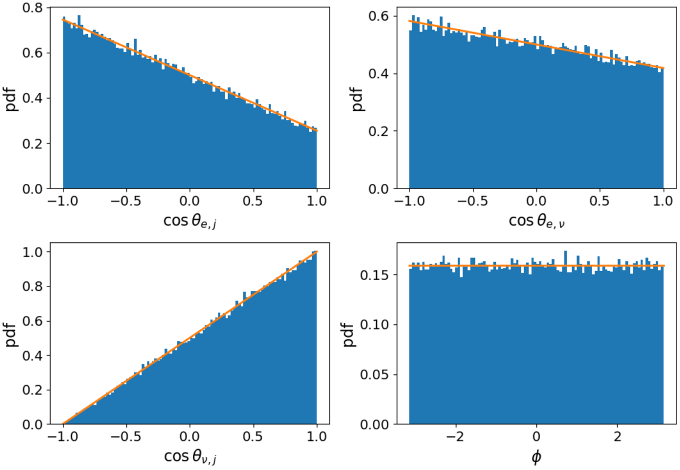


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Real Positive

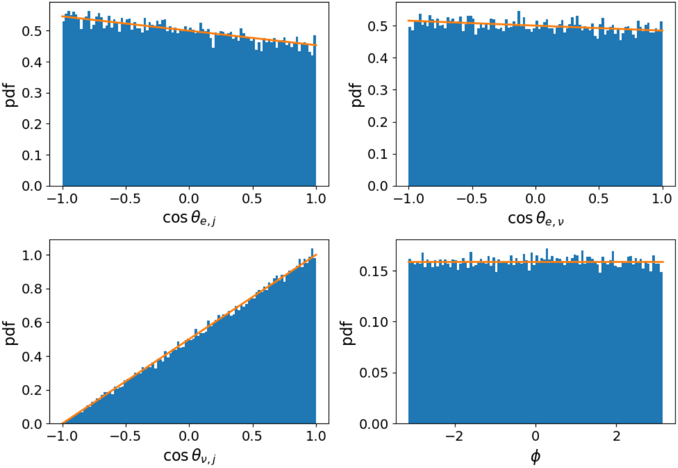


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Real Negative

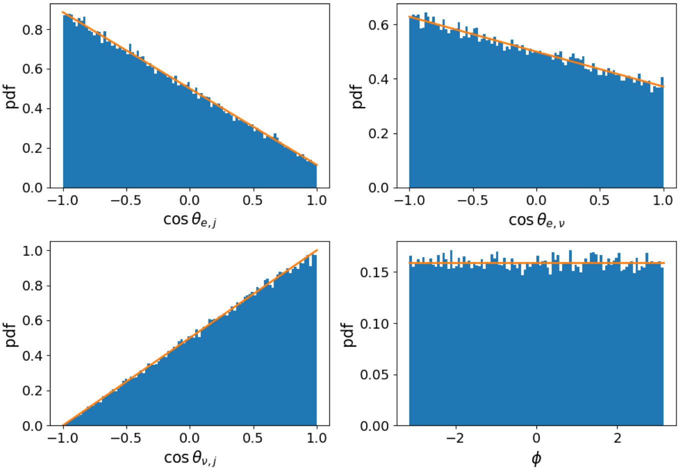


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = -1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Imaginary Positive

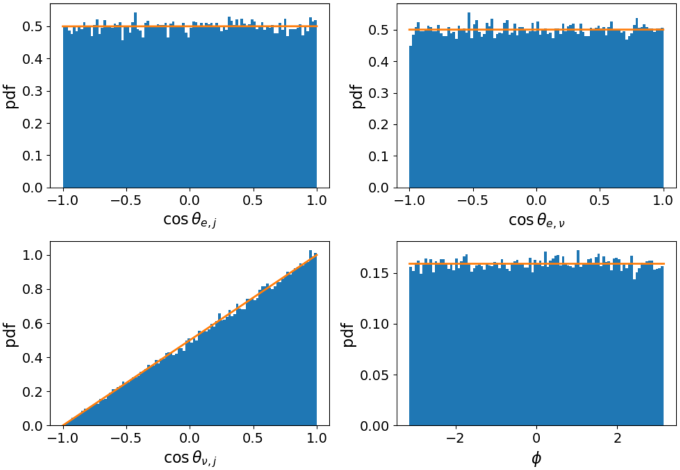


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Imaginary Negative

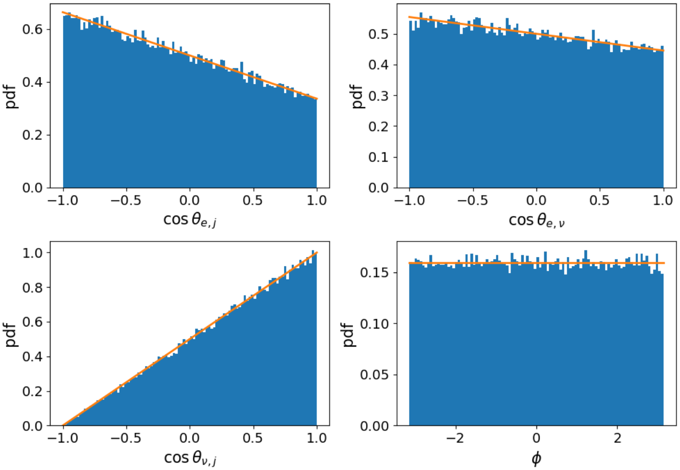


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = -i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = -C'_T$

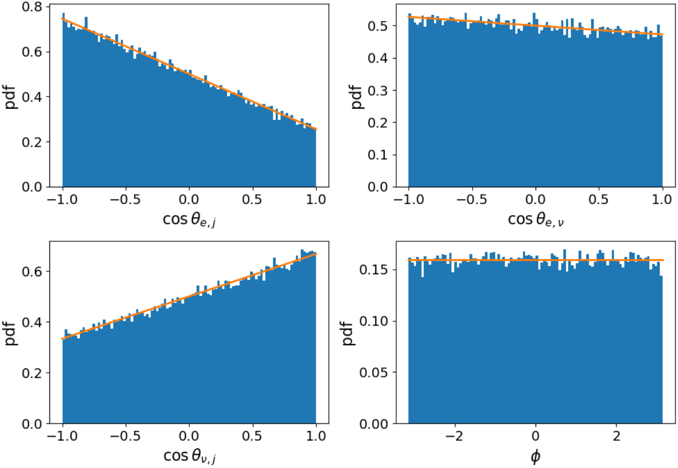
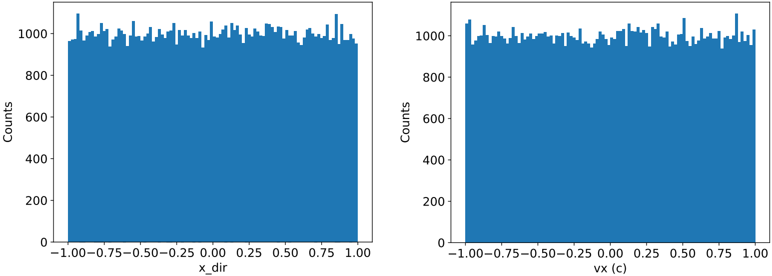


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = -C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in -Z

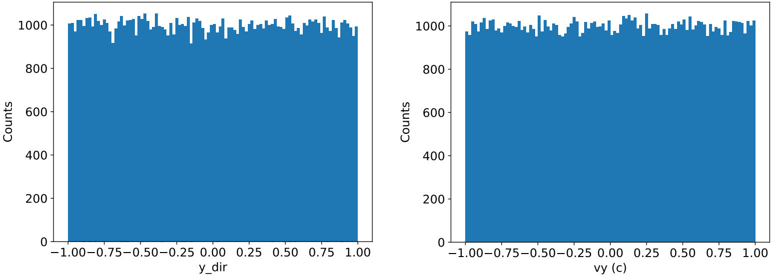


(a) e (b) ν

Figure: Distribution of the x component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the negative z direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in -Z



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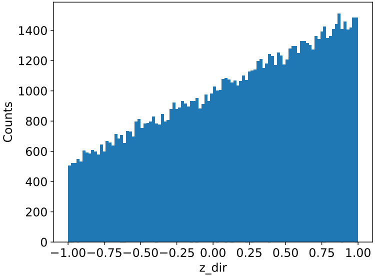
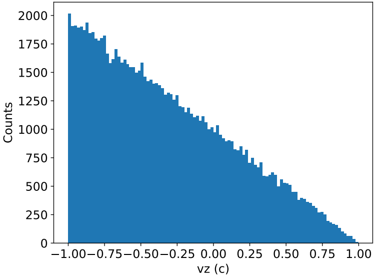
Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in -Z



(a) e

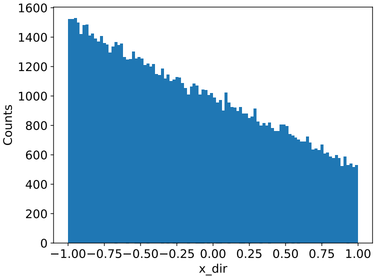


(b) ν

Figure: Distribution of the z component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the negative z direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+\mathrm{X}$

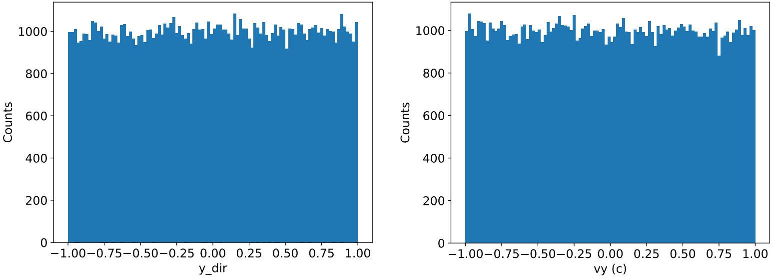


(a) e (b) ν

Figure: Distribution of the x component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the positive x direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+X$

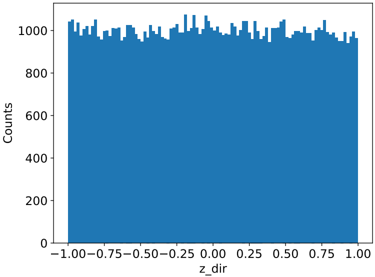


(a) e (b) ν

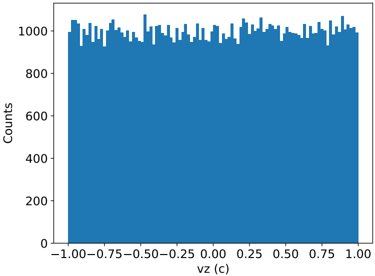
Figure: Distribution of the y component of the direction of the velocity of the emmited leptons for a decay of fully polarized nuclei in the positive x direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+X$



(a) e

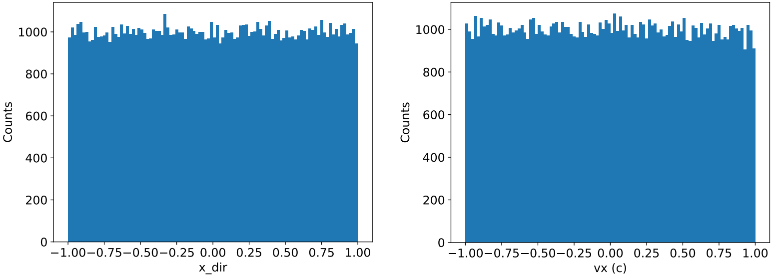


(b) ν

Figure: Distribution of the z component of the direction of the velocity of the emited leptons for a decay of fully polarized nuclei in the positive x direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+\mathrm{Y}$

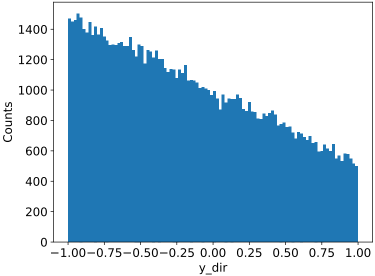


(a) e (b) ν

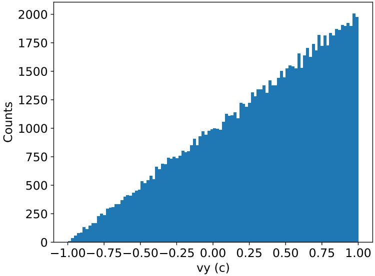
Figure: Distribution of the y component of the direction of the velocity of the emited for a decay of fully polarized nuclei in the positive y direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+\mathrm{Y}$



(a) e

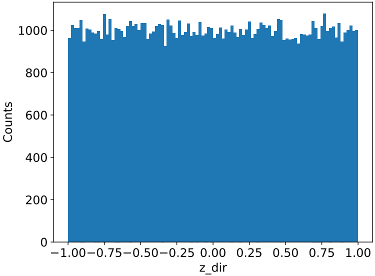


(b) ν

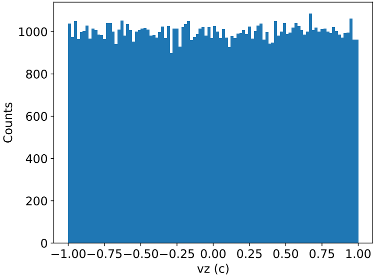
Figure: Distribution of the y component of the direction of the velocity of the emited leptons for a decay of fully polarized nuclei in the positive y direction

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Polarisation in $+$Y



(a) e



(b) ν

Figure: Distribution of the z component of the direction of the velocity of the emited leptons for a decay of fully polarized nuclei in the positive y direction

Gamow-Teller Decay: $^{12}\mathrm{B}$

Other examples:
$^{12}\mathrm{B}(1^+) \rightarrow {}^{12}\mathrm{C}(0^+)$
Properties:

- Q = 13369 keV (very high)
- No γ produced
- $J_f = J_i - 1$
- A = -1, B = 1, a = -1/3, c = 1

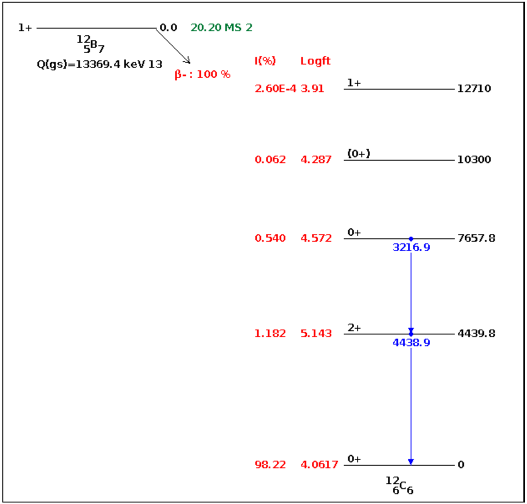


Figure: Decay Scheme of $^{12}\mathrm{B}$ into $^{12}\mathrm{C}$.

Gamow-Teller Decay: $^{12}\mathrm{B}$

Standard Model values

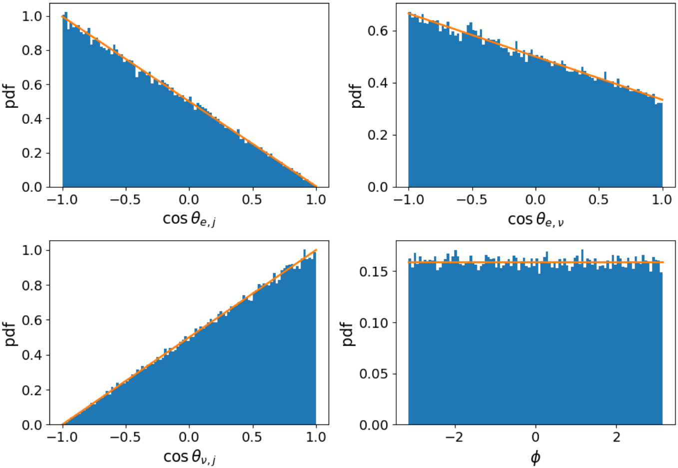


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: $^{20}\mathrm{F}$

Other examples:
$^{20}\mathrm{F}(2^+) \rightarrow {}^{20}\mathrm{Ne}(2^+)$
Properties:

- Q = 5390 keV (very high)
- 1 γ produced
- $J_f = J_i$ (could be in reality a mixed decay, 2 states may be in same T = 1 isospin multiplet)
- A = -1/3, B = 1/3, a = -1/3, c = -1

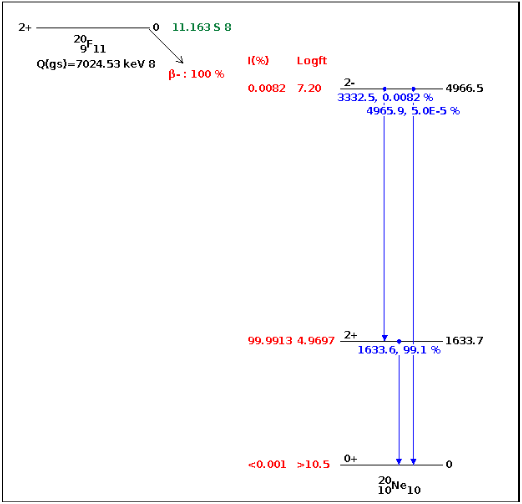


Figure: Decay Scheme of $^{20}\mathrm{F}$ into $^{20}\mathrm{Ne}$.

Gamow-Teller Decay: $^{20}\mathrm{F}$

Standard Model values

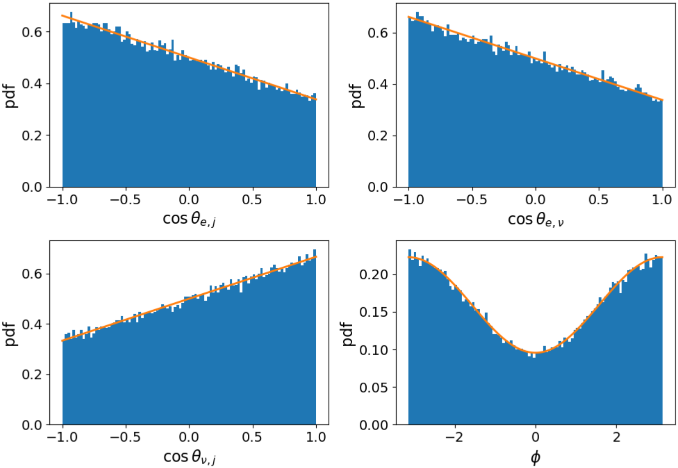


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: ^{18}F

A β^+ example: $^{18}\mathrm{F}(1^+) \rightarrow {}^{18}\mathrm{O}(0^+)$
Properties:

- Q = 1044 keV
- No γ produced
- $J_f = J_i - 1 \rightarrow \lambda_{J_i,J_f} = \Lambda_{J_i,J_f} = 1$

Some tests for a β^+ decay. Using $C_A = C'_A = \mathrm{cte}(=1)$.

- $C_T = C'_T = 0$ (Standard Model)
- $C_T = C'_T$ pure real (and large)
- $C_T = C'_T$ pure imaginary
- $C_T = -C'_T$, either real or imaginary

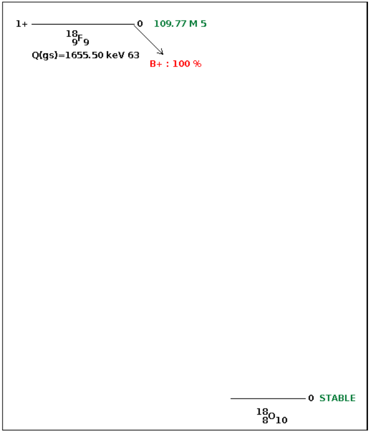


Figure: Decay Scheme of ^{18}F into ^{18}O.