Angular correlation Function

$$F = 1 + a\frac{\mathbf{p_e}\cdot\mathbf{p_\nu}}{E_e E_\nu} + \frac{\mathbf{J}}{J}\cdot\left(A\frac{\mathbf{p_e}}{E_e} + B\frac{\mathbf{p_\nu}}{E_\nu} + D\frac{\mathbf{p_e}\times\mathbf{p_\nu}}{E_e E_\nu}\right)$$

Spherical Coordinates ($\mathbf{J}$ parallel to positive Z axis)

$$\boldsymbol{\beta}_{\mathbf{e}} = (r = \beta_e; \theta = \theta_e; \phi = 0),\ \cos(\theta_e) \equiv z_e,\ \beta_e = \frac{|\mathbf{p_e}|}{E} = \sqrt{1 - \frac{m_e^2}{E^2}}$$

$$\boldsymbol{\beta}_{\boldsymbol{\nu}} = (r = 1; \theta = \theta_\nu; \phi = \phi), \quad \cos(\theta_\nu) \equiv z_\nu$$

$$\boldsymbol{\beta}_{\mathbf{e}}\cdot\boldsymbol{\beta}_{\boldsymbol{\nu}} = \beta_e(\cos\theta_e\cos\theta_\nu + \sin\theta_e\sin\theta_\nu\cos\phi) =$$

$$\beta_e(z_e z_\nu + \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\cos\phi)$$

$$\boldsymbol{\beta}_{\mathbf{e}}\cdot\mathbf{j} = \beta_e\cos\theta_e = \beta_e z_e$$

$$\boldsymbol{\beta}_{\boldsymbol{\nu}}\cdot\mathbf{j} = \cos\theta_\nu = z_\nu$$

$$\mathbf{j}\cdot(\boldsymbol{\beta}_{\mathbf{e}}\times\boldsymbol{\beta}_{\boldsymbol{\nu}}) = \beta_e\sin\theta_e\sin\theta_\nu\sin\phi = \beta_e\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\sin\phi$$

Single Variable A

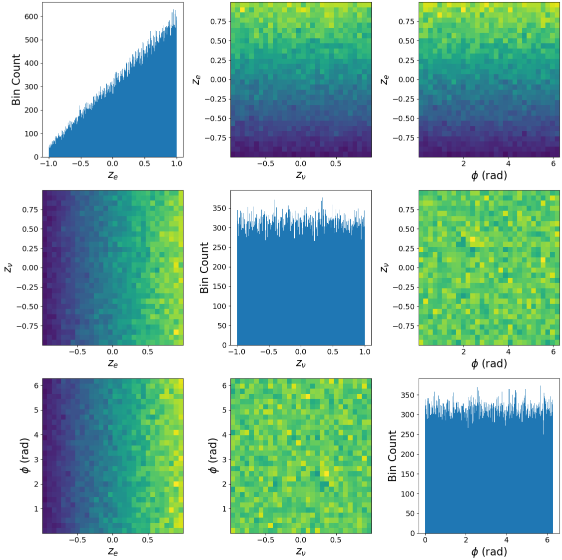

Figure: Pair plots for N = 100000 decays with A = 1, E = 1000 keV

Single Variable A

In detail: distribution of the z_e component

$$f(z_e) = N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi F =$$

$$= 4\pi N(1+A\beta z_e) = N(1+A\beta z_e)$$

N normalization constant. Here

$$N = \frac{\#\text{counts}}{\#\text{bins}}$$

as "average" of $f(z_e) = \frac{1}{2}\int_{-1}^{1} f(z_e)dz_e = N$

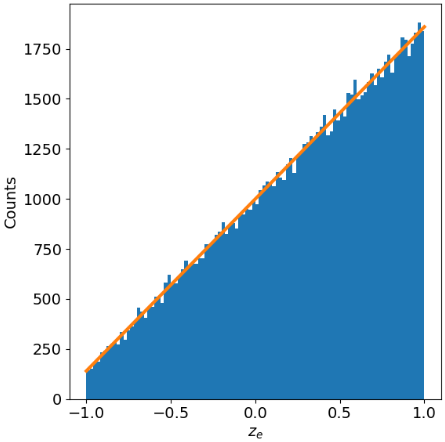

Figure: Histogram showing the values of z_e with A = 1, E = 1000 keV for N = 100000 decays, and curve showing the theoretical distribution

Single Variable B

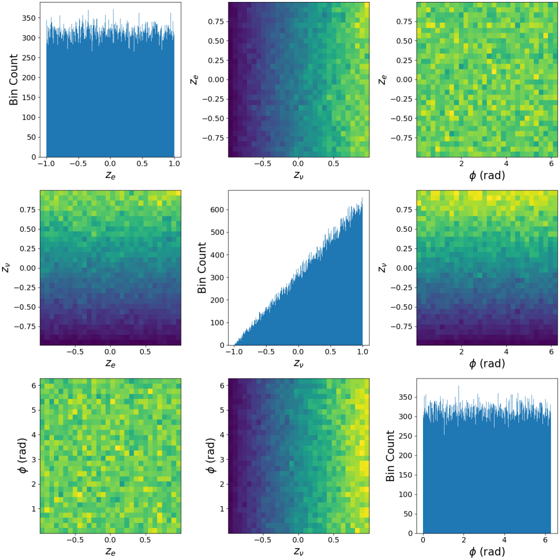

Figure: Pair plots for N $=$ 100000 decays with A $=$ B, E $=$ 1000 keV

Single Variable B

In detail: distribution of the z_ν component (only with non trivial dependence)
Theoretical distribution:

$$f(z_\nu) = N \int_{-1}^{1} dz_e \int_{0}^{2\pi} d\phi F =$$

$$= 4\pi N(1+Bz_\nu) = N(1+Bz_\nu)$$

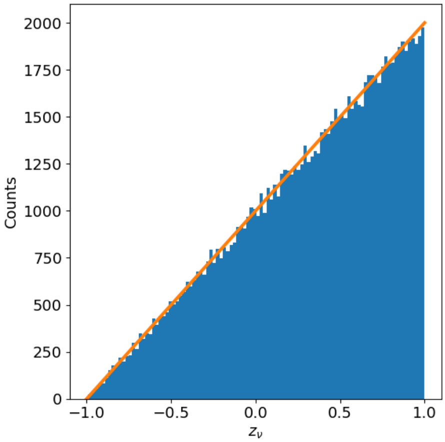

Figure: Histogram showing the values of z_e with B = 1, E = 1000 keV for N = 100000 decays, and curve showing the theoretical distribution

Single Variable a

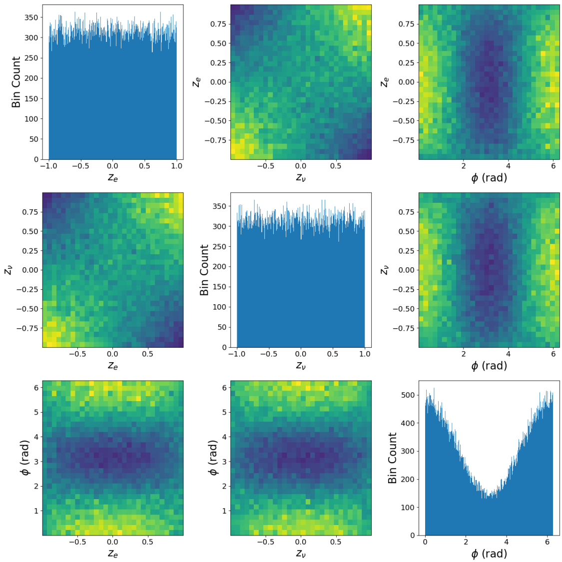

Figure: Pair plots for N = 100000 decays with A = 1, E = 1000 keV

Single Variable a

Marginal distributions

For z_e (and z_ν by symmetry of the expresions), we can observe reason why the marginal distribution becomes constant:

$$f(z_e) = N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi F =$$

$$= N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi (1 + a\beta(z_e z_\nu + \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2} \cos\phi)) =$$

$$= N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi = 4\pi N = N$$

Single Variable a

Marginal distributions

For ϕ, we can derive the expected shape:

$$f(\phi) = N \int_{-1}^{1} dz_\nu \int_{-1}^{1} dz_e F =$$

$$= N \int_{-1}^{1} dz_\nu \int_{-1}^{1} dz_e (1 + a\beta(z_e z_\nu + \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2} \cos\phi)) =$$

$$= N \left(4 + a\beta \left(\frac{\pi}{2}\right)^2 \cos\phi\right) = N \left(1 + a\beta \frac{\pi^2}{16} \cos\phi\right)$$

Single Variable a

Marginal distributions

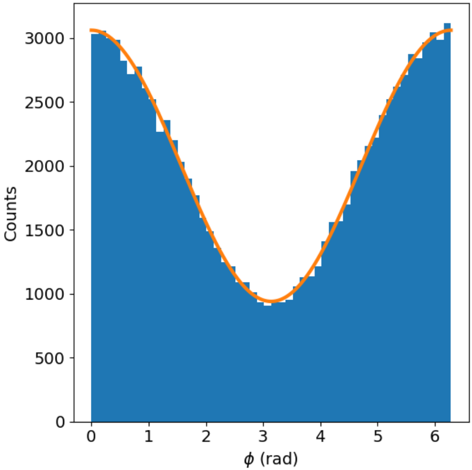

Figure: Histogram showing the values of ϕ with a = 1, E = 1000 keV for N = 100000 decays, and curve showing the theoretical distribution

Single Variable a

Marginal distributions

An extra variable we can plot is the cosine between the 2 vectors $\cos\theta_{e,\nu} = \boldsymbol{\beta_e} \cdot \boldsymbol{\beta_\nu}$. So F symplifies to:

$$F = 1 + a\beta\cos\theta_{e,\nu}$$

So the marginal distribution should be

$$f(\cos\theta_{e,\nu}) = N(1 + a\beta\cos\theta_{e,\nu})$$

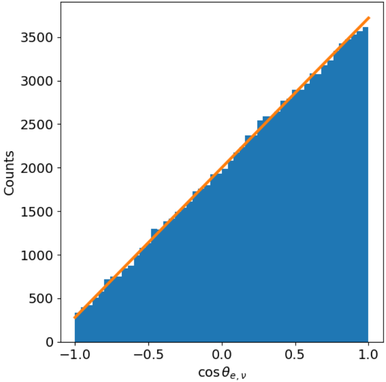

Figure: Histogram showing the values of $\cos\theta_{e,\nu}$ with a = 1, E = 1000 keV for N = 100000 decays, and curve showing the theoretical distribution

Single Variable D

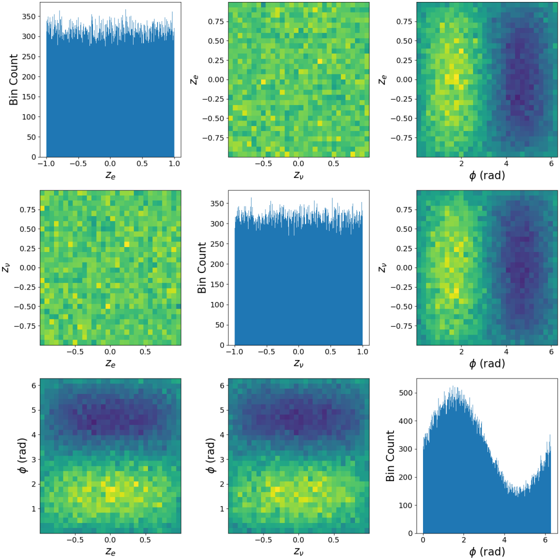

Figure: Pair plots for N = 100000 decays with D = 1, E = 1000 keV

For z_e (and z_ν by symmetry of the expresions), we can observe reason why the marginal distribution becomes constant:

$$f(z_e) = N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi F =$$

$$= N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi (1 + D\beta \sqrt{1 - z_e^2} \sqrt{1 - z_\nu^2} \sin\phi) =$$

$$= N \int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi = 4\pi N = N$$

Single Variable D

Marginal distributions

For ϕ, we can derive the expected shape:

$$f(\phi) = N \int_{-1}^{1} dz_\nu \int_{-1}^{1} dz_e F =$$

$$= N \int_{-1}^{1} dz_\nu \int_{-1}^{1} dz_e (1 + D\beta\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\sin\phi)) =$$

$$= N \left(4 + a\beta \left(\frac{\pi}{2}\right)^2 \sin\phi\right) = N \left(1 + a\beta \frac{\pi^2}{16} \sin\phi\right)$$

Single Variable D

Marginal distributions

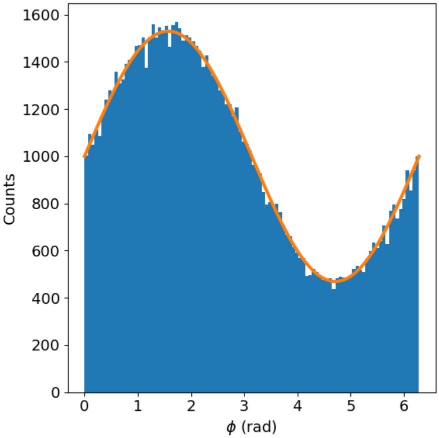

Figure: Histogram showing the values of ϕ with D = 1, E = 1000 keV for N = 100000 decays, and curve showing the theoretical distribution

Two variable: A and B

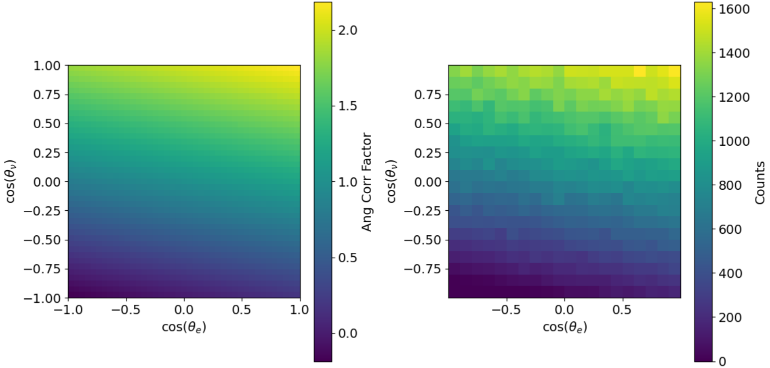

Figure: (Right) Output of the angular distribution function and (Left) histogram of N = 300000 decays, both plots with A = B= 1, E = 5000 keV

Two variable: A and B

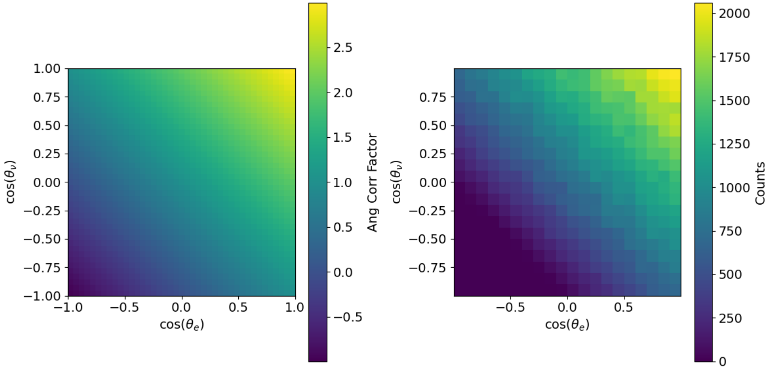

Figure: (Right) Output of the angular distribution function and (Left) histogram of N = 300000 decays, both plots with A = B= 1, E = 5000 keV

Two variable: A and B

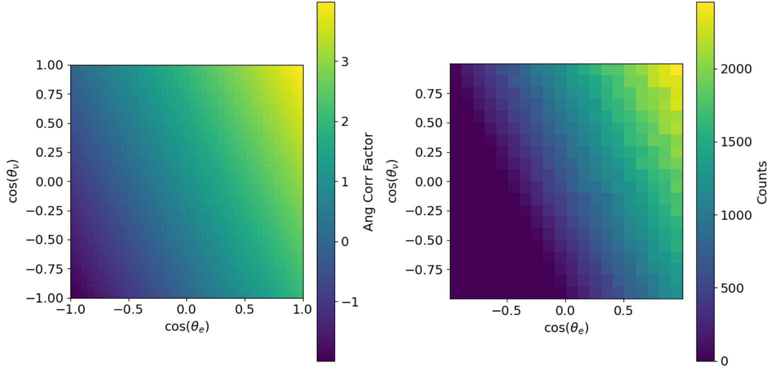

Figure: (Right) Output of the angular distribution function and (Left) histogram of N = 300000 decays, both plots with A = B= 1, E = 5000 keV

Two variable: A and B

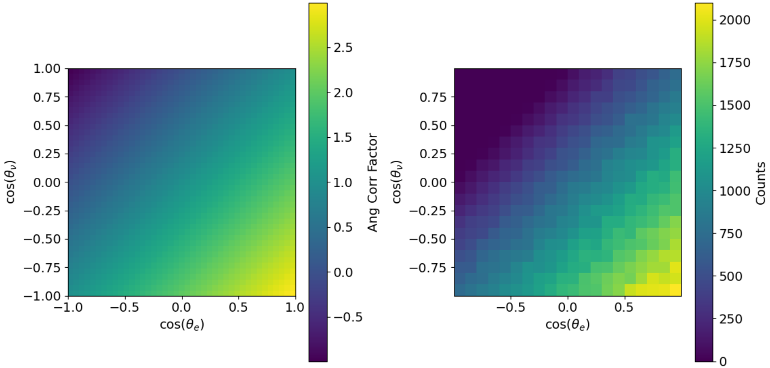

Figure: (Right) Output of the angular distribution function and (Left) histogram of N = 300000 decays, both plots with A = B= 1, E = 5000 keV

Two variable: A and B

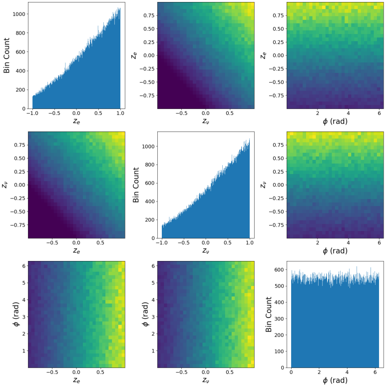

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with A = B = 1, E = 5000 keV

Two variables A and B

Marginal distributions (attempt)

To compute $f(z_e)$, avoid $F < 0$ areas, which occur if $Az_e\beta + Bz_\nu < -1$. If B = 1, $z_\nu > -1 - Az_e\beta$ is the lower bound

$$f(z_e) = N\int_{-1}^{1} dz_\nu \int_0^{2\pi} d\phi F = 2\pi N \int_{-1-Az_e\beta}^{1} dz_\nu (1 + A\beta z_e + z_\nu) =$$

$$= 2\pi N \left((1 + A\beta z_e)(2 + A\beta z_e) + \int_{-1-Az_e\beta}^{1} dz_\nu z_\nu \right)$$

$$= 2\pi N \left(2 + 2A\beta z_e + \frac{1}{2}(A\beta z_e)^2 \right) = N \left(2 + 2A\beta z_e + \frac{1}{2}(A\beta z_e)^2 \right)$$

Note:

$$\frac{1}{2}\int_{-1}^{1} f(z_e) = \left[2 + \frac{1}{6}(A\beta)^2 \right] N$$

so we need to divide $f(z_e)$ by the factor in [] to write N in the histogram as

$$N = \frac{\#\text{counts}}{\#\text{bins}}$$

Two variables A and B

Marginal distributions

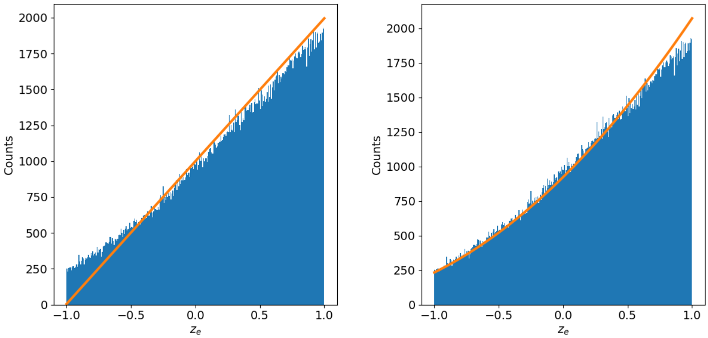

Figure: Histogram showing the values of z_e with A = B = 1, E = 5000 keV for N = 300000 decays. On the right, a naive computation of $f(z_e)$, which ignores the presence of F ¡ 0 areas, on the left a more refined one that fits the distribution better.

Two Variables A and B

Marginal distributions

Better method: use F numerically to compute the marginal distributions f(x).

- Ignore the $F < 0$ points (set them to 0)
- Integrate numerically over the remaining variables

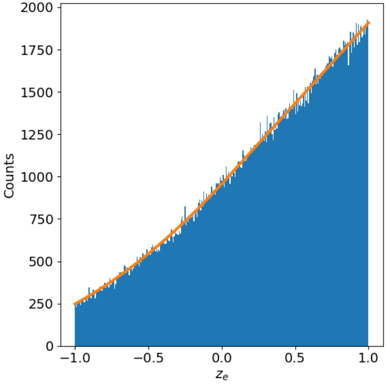

Figure: Histogram showing the values of $\cos\theta_{e,\nu}$ with A $=$ B $=$ 1, E $=$ 1000 keV for N $=$ 100000 decays, and updated marginal distribution

Two Variables A and B

2D marginal distributions

Works also with 2D histograms

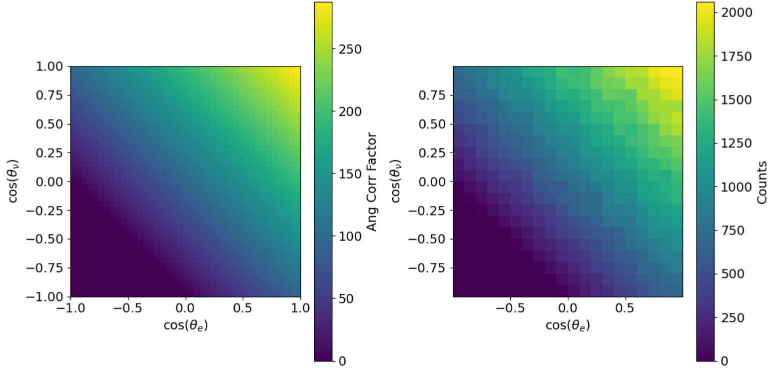

Figure: (Right) Theoretical distribution of particles and (Left) 2D histogram of N = 300000 decays, both plots with A = B = 1, E = 5000 keV

Two variable: rest of pairs

For the rest of variables, we show only the pairplot with the marginal distributions and the theoretical distribution in the 1D marginal plots. We expect 2 kinds of results

- (a,A), (a,B), (A,D) and (B,D):
  Here, one of the 1D histograms will be aproximately constant, and the other 2 will be close to the 1 variable case (though need to account for $F < 0$ areas)
- (a,D)
  In this case, integration along ϕ cancels most terms, and the remaining $z_e z_\nu$ cancels the only non constant term. So only non-constant marginal distribution is that of ϕ

$$F = 1+\beta_e(a(z_e z_\nu+\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\cos\phi)+D\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\sin\phi)$$

Two variable: a and D

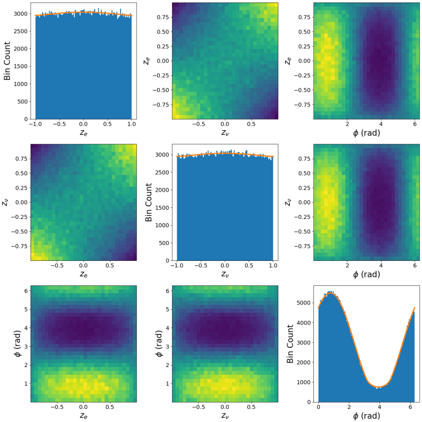

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with a = D = 1, E = 100000 keV. The 1 variable histograms show the theoretical distribution obtained from numerically integrating F with the constrain $F > 0$

Two variable: a and A

$$F = 1 + \beta_e(a(z_e z_\nu + \sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\cos\phi) + Az_e)$$

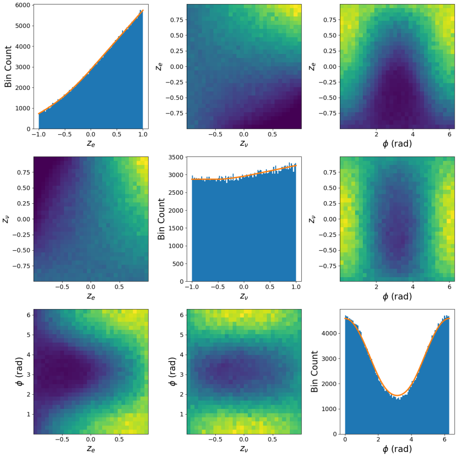

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with a = A = 1, E = 100000 keV

Two variable: a and B

$$F = 1 + a\beta_e\left(z_e z_\nu + \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\cos\phi\right) + B z_\nu$$

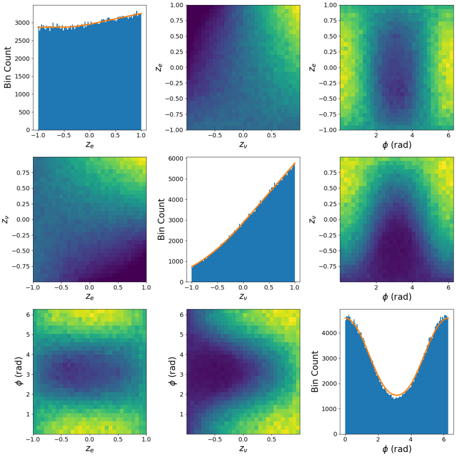

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with a = B = 1, E = 5000 keV.

Two variable: A and D

$$F = 1 + A\beta_e z_e$$
$$+ D\beta_e \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\sin\phi$$

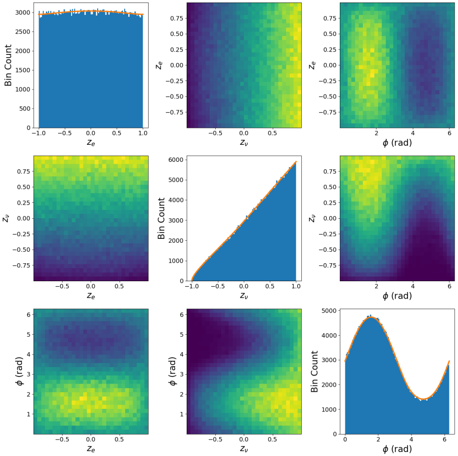

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with A = D = 1, E = 100000 keV

Two variable: B and D

$$F = 1 + Bz_\nu$$
$$+D\beta_e\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\sin\phi$$

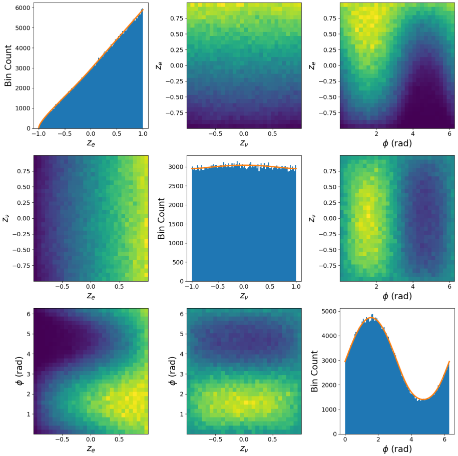

Figure: Pairplot with the marginal distributions for a simulation of N = 300000 decays with B = D = 1, E = 5000 keV