CRADLE++ Tests

November 3, 2025

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Simplest non-trivial case: Gamov-Teller decay. Full test: decay of a known nuclei. Simplifying assumption: focus on one decay path of that nuclei. Example:

$$^{60}\mathrm{Co}(5^+) \rightarrow {}^{60}\mathrm{Ni}(4^+)$$

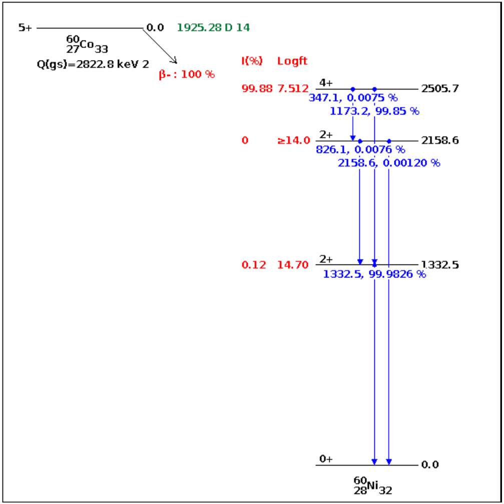


Figure: Decay Scheme of $^{60}\mathrm{Co}$ into $^{60}\mathrm{Ni}$.

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Properties of $^{60}\mathrm{Co}(5^+) \to {}^{60}\mathrm{Ni}(4^+)$

- Q $= 317.06$ keV (not fully ideal)
- $J_f = J_i - 1 \to \lambda_{J_i,J_f} = \Lambda_{J_i,J_f} = 1$
- 2 γ almost always ($5^+ \to 2^+$ only 1 γ)

Many cases to consider, though for realism: keep $C_A = C'_A = \mathrm{cte}(= 1)$.

- $C_T = C'_T = 0$ (Standard Model)
- $C_T = C'_T$ pure real (and large)
- $C_T = C'_T$ pure imaginary
- $C_T = -C'_T$, either real or imaginary

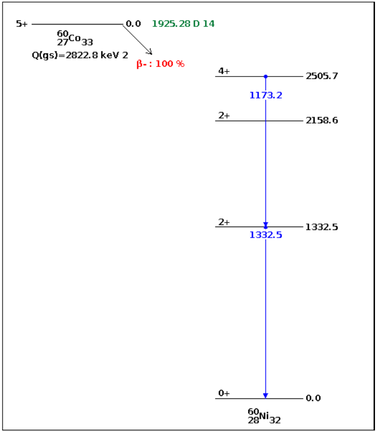
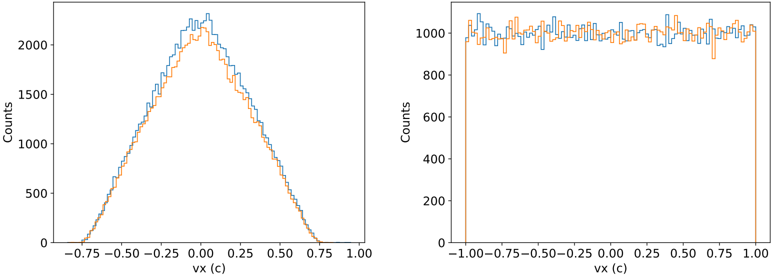


Figure: Decay Scheme of $^{60}\mathrm{Co}$ into $^{60}\mathrm{Ni}$ featuring the only decay of interest

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values

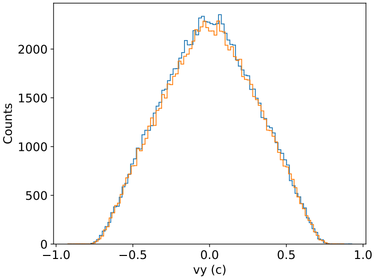


(a) e (b) ν

Figure: Distribution of the x component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values

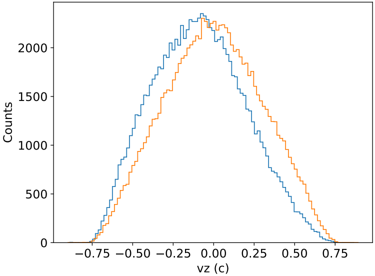


(a) e (b) ν

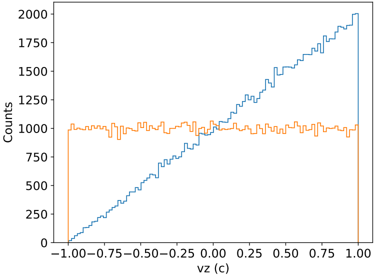
Figure: Distribution of the y component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model values



(a) e



(b) ν

Figure: Distribution of the z component of the velocity of the emmited leptons for a decay of (blue) fully polarized nuclei in the z direction and (orange) unpolarised nuclei

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Numerical evaluation

Use that distributions in z_e, z_ν, $\cos\theta_{e,\nu} \equiv z_{e,\nu}$ and ϕ are known.

$$f(z_e) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \left\langle A\beta_e \right\rangle z_e}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f(z_\nu) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \left\langle B\beta_e \right\rangle z_\nu}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f(z_{e,\nu}) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \left\langle a\beta_e \right\rangle z_{e,\nu}}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f(\phi) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \left\langle \left(a + \frac{c}{3}\right)\beta_e \right\rangle \frac{\pi^2}{16}\cos\phi + \left\langle D\beta_e \right\rangle \frac{\pi^2}{16}\sin\phi}{2\pi(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

Averages computed numerically using f(E) from the simulation data itself (avoid computing the Fermi function myself)

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Standard Model values

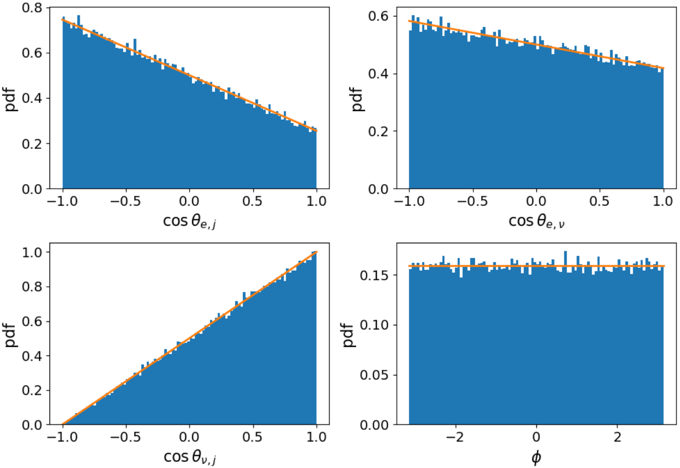

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Real Positive

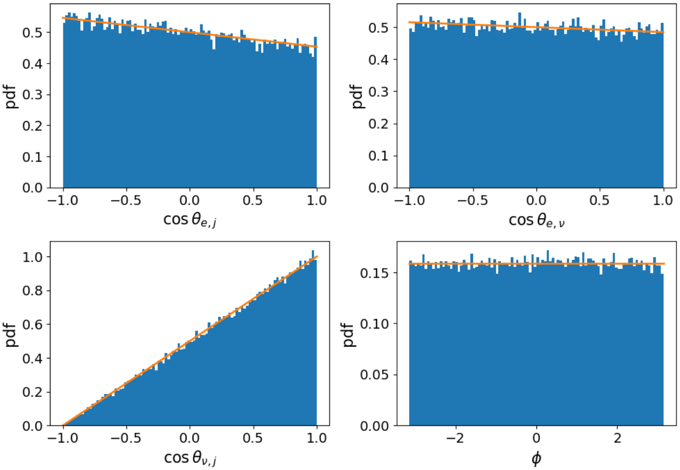

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Real Negative

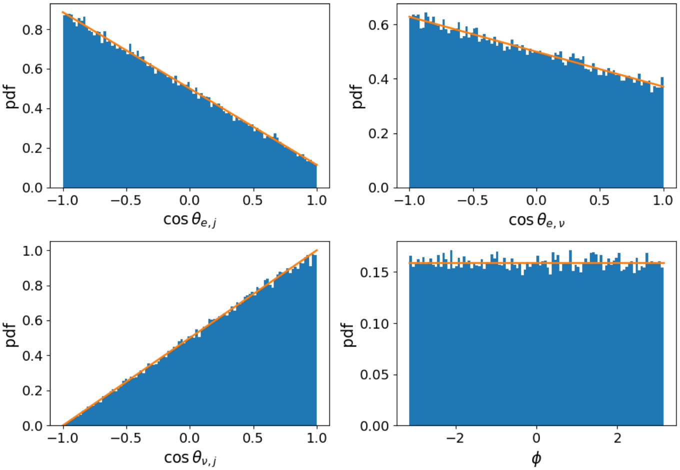

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = -1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Imaginary Positive

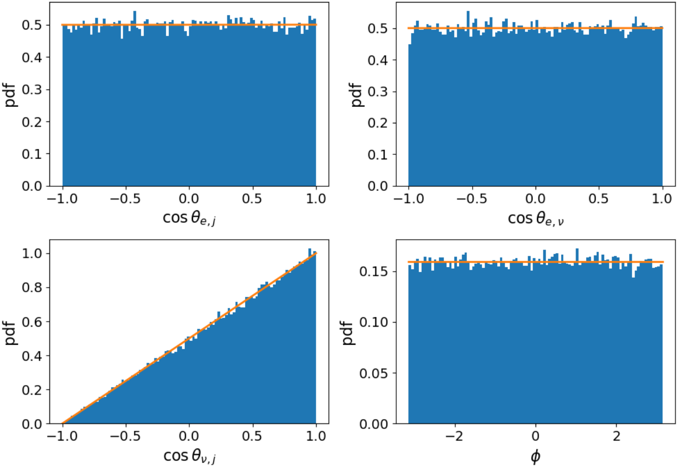

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Imaginary Negative

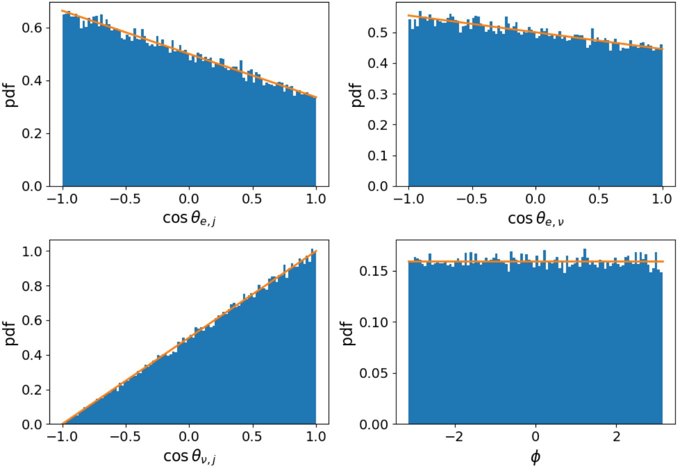

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = -i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = -C_T'$

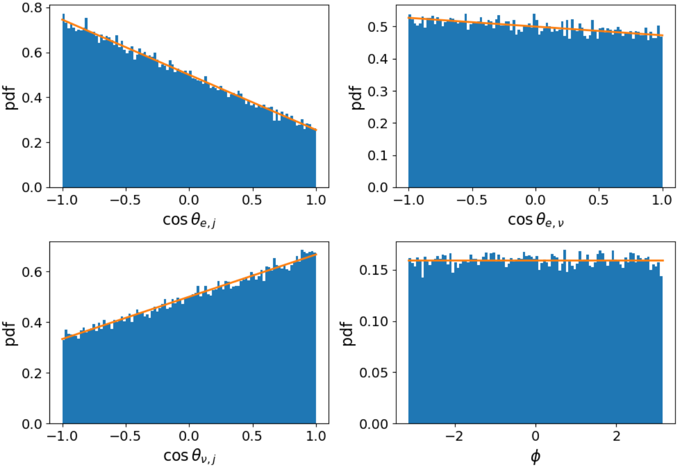

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = -C_T' = 1/\sqrt{2}$