# Angular correlation Function

$$F = 1 + a\frac{\mathbf{p_e}\cdot\mathbf{p_\nu}}{E_eE_\nu} + \frac{\mathbf{J}}{J}\cdot\left(A\frac{\mathbf{p_e}}{E_e} + B\frac{\mathbf{p_\nu}}{E_\nu} + D\frac{\mathbf{p_e}\times\mathbf{p_\nu}}{E_eE_\nu}\right)$$

Spherical Coordinates (**J** parallel to positive Z axis)

$$\boldsymbol{\beta}_\mathbf{e} = (r = \beta_e; \theta = \theta_e; \phi = 0),\ \cos(\theta_e) \equiv z_e,\ \beta_e = \frac{|\mathbf{p_e}|}{E} = \sqrt{1 - \frac{m_e^2}{E^2}}$$

$$\boldsymbol{\beta}_{\boldsymbol{\nu}} = (r = 1; \theta = \theta_\nu; \phi = \phi), \quad \cos(\theta_\nu) \equiv z_\nu$$

$$\boldsymbol{\beta}_\mathbf{e}\cdot\boldsymbol{\beta}_{\boldsymbol{\nu}} = \beta_e(\cos\theta_e\cos\theta_\nu + \sin\theta_e\sin\theta_\nu\cos\phi) =$$

$$\beta_e(z_ez_\nu + \sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\cos\phi)$$

$$\boldsymbol{\beta}_\mathbf{e}\cdot\mathbf{j} = \beta_e\cos\theta_e = \beta_ez_e$$

$$\boldsymbol{\beta}_{\boldsymbol{\nu}}\cdot\mathbf{j} = \cos\theta_\nu = z_\nu$$

$$\mathbf{j}\cdot(\boldsymbol{\beta}_\mathbf{e}\times\boldsymbol{\beta}_{\boldsymbol{\nu}}) = \beta_e\sin\theta_e\sin\theta_\nu\sin\phi = \beta_e\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\sin\phi$$

# Single Variable: A

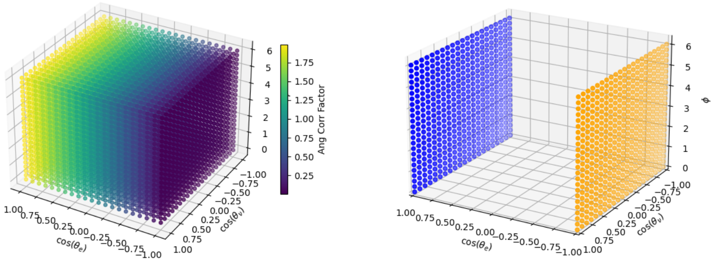



Figure: (Right) Values of the angular correlation Factor with A = 1, E = 5000 keV and rest of variables 0. (Left) Location of maximum (blue, value = 1.995) and minimum (orange, value = 0.005)

# Single Variable: A

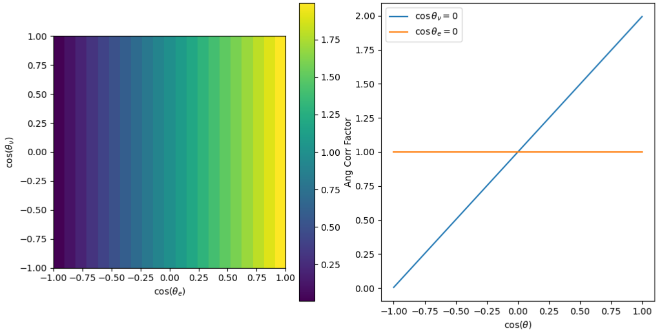



Figure: (Right) 2D projection of previous 3D image at any $\phi$ (Left) 1D projections at any $\phi$, and either $z_e = 0$ or $z_\nu = 0$

# Single Variable: B

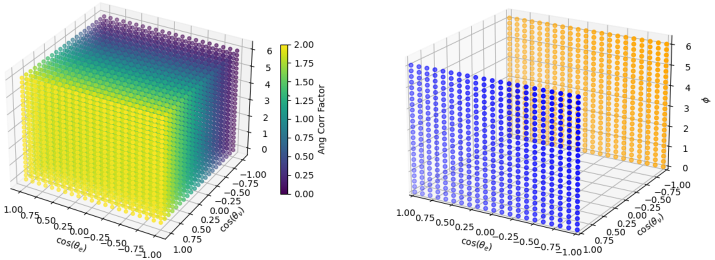



Figure: (Right) Values of the angular correlation Factor with B = 1, E = 5000 keV and rest of variables 0. (Left) Location of maximum (blue, value = 2) and minimum (orange, value = 0)

# Single Variable: B

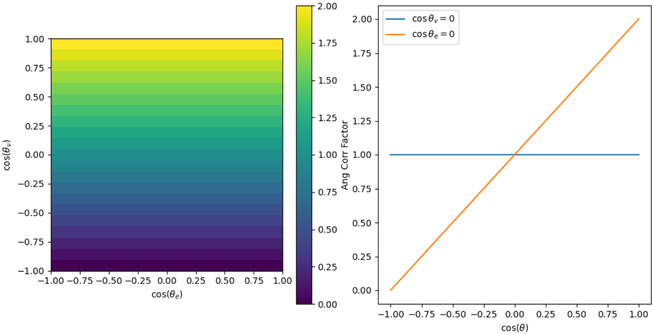



Figure: (Right) 2D projection of previous 3D image at any $\phi$ (Left) 1D projections at any $\phi$, and either $z_e = 0$ or $z_\nu = 0$

# Single Variable: a

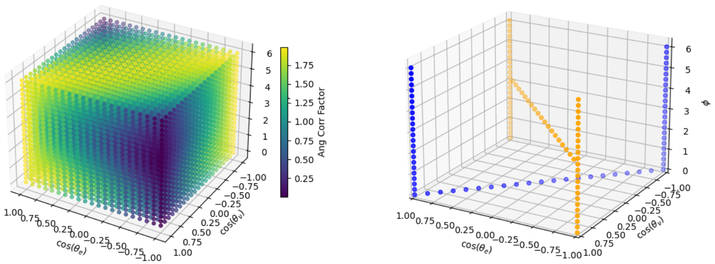



Figure: (Right) Values of the angular correlation Factor with a = 1, E = 5000 keV and rest of variables 0. (Left) Location of maximum (blue, value = 1.995) and minimum (orange, value = 0.005)

# Single Variable: a

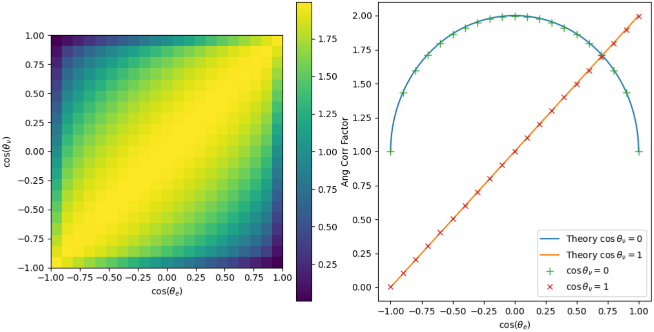



Figure: (Right) 2D projection of previous 3D image at $\phi = 0$ (Left) 1D projections at $\phi = 0$, and either $z_\nu = 0$ or $z_\nu = 1$

# Single Variable: D

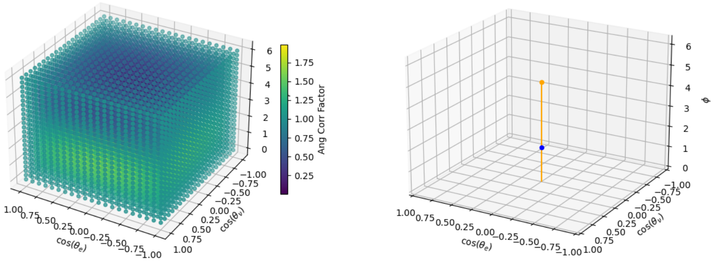



Figure: (Right) Values of the angular correlation Factor with D = 1, E = 5000 keV and rest of variables 0. (Left) Location of maximum (blue, value = 1.995) and minimum (orange, value = 0.005)

# Single Variable: D

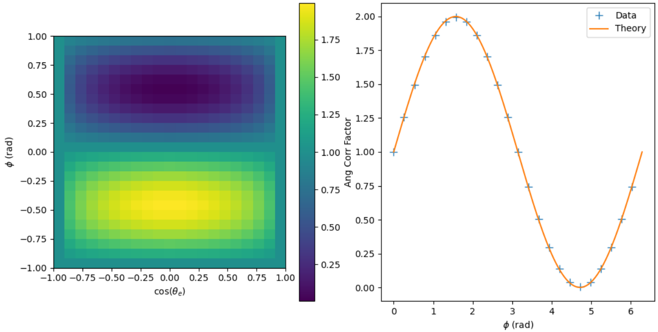



Figure: (Right) 2D projection of previous 3D image at $z_\nu = 0$ (Left) 1D projection, ie plot of F with respect to $\phi$, at $z_e = 0$ and $z_\nu = 0$

# Two variables: A and B

Consider different ratios by either:

- Fixing $A = B = 1$ and modifying the energy (first 3 plots)
- Same as before, but now $B = -1$
- Fixing $B = 1$ and $E \gg m_e \to \beta_e \approx 1$ and modifying $A > B$

$$F = 1 + A\beta_e z_e + B z_\nu$$

No $\phi$ dependence: we can work in a 2D crosssection and capture all of the details.

# Two variables: A and B

## Low Energy, Positive B

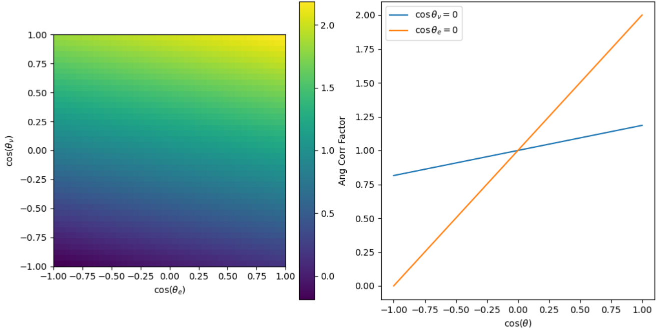



Figure: (Right) Values of the angular correlation Factor with $A = B = 1$, $E = 520$ keV and rest of variables 0. Maximum = 2.18526, Minimum = -0.18526 (Left) 1D projections at any $\phi$, and either $z_e = 0$ (orange, slope $\equiv$ m = 1) or $z_\nu = 0$ (blue, m = 0.1852)

# Two variables: A and B

## Medium Energy, Positive B

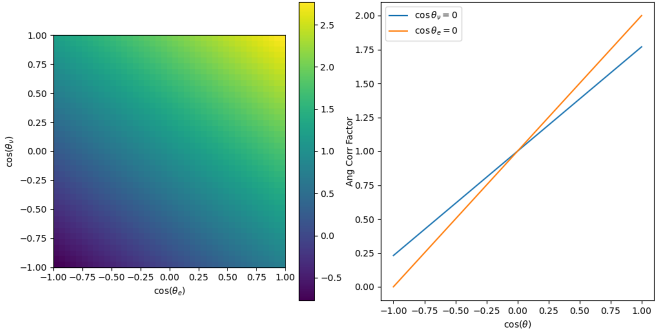



Figure: (Right) Values of the angular correlation Factor with $A = B = 1$, $E = 800$ keV and rest of variables 0. Maximum $= 2.76942$, Minimum $=$ -0.76942 (Left) 1D projections at any $\phi$, and either $z_e = 0$ (orange, m $=$ 1) or $z_\nu = 0$ (blue, m $= 0.7694$)

# Two variables: A and B

## High Energy, Positive B

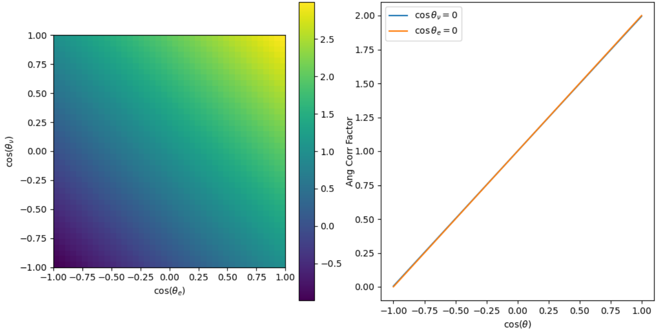



Figure: (Right) Values of the angular correlation Factor with $A = B = 1$, $E = 5000$ keV and rest of variables 0. Maximum = 2.99476, Minimum = -0.99476 (Left) 1D projections at any $\phi$, and either $z_e = 0$ (orange, m = 1) or $z_\nu = 0$ (blue, m = 0.9948)

# Two variables: A and B

## High A, Positive B

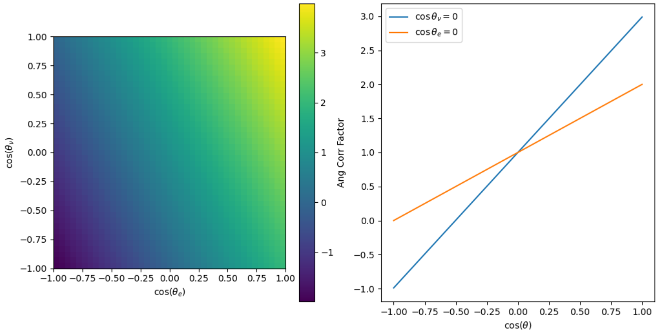



Figure: (Right) Values of the angular correlation Factor with $A = 2$, $B = 1$, $E = 5000$ keV and rest of variables 0. Maximum = 3.98953, Minimum = -1.98953 (Left) 1D projections at any $\phi$, and either $z_e = 0$ (orange, m = 1) or $z_\nu = 0$ (blue, m = 1.9895)

# Two variables: A and B

## Very High A, Positive B

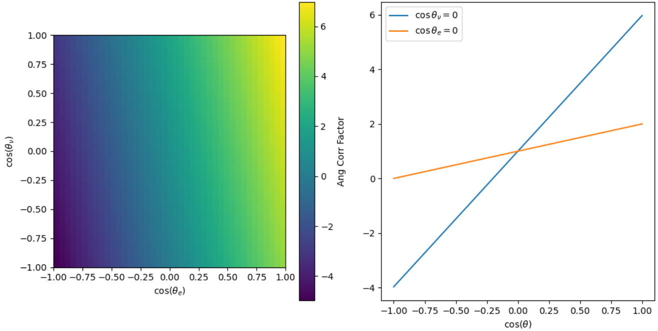



Figure: (Right) Values of the angular correlation Factor with $A = 5$, $B = 1$, $E = 5000$ keV and rest of variables 0. Maximum = 3.98953, Minimum = -1.98953 (Left) 1D projections at any $\phi$, and either $z_e = 0$ (orange, m = 1) or $z_\nu = 0$ (blue, m = 1.9895)

# Two variables: A and B

## Low Energy, Negative B

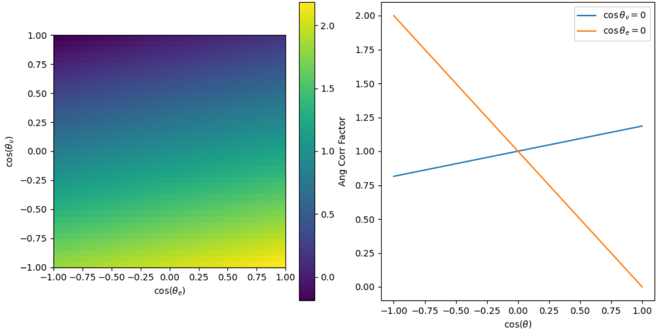



Figure: (Right) Values of the angular correlation Factor with $A = 1$, $B = -1$, $E = 520$ keV and rest of variables 0. Maximum = 2.18526, Minimum = -0.18526 (Left) 1D projections at any $\phi$, and either $z_e = 0$ (orange, slope $\equiv$ m = 1) or $z_\nu = 0$ (blue, m = -0.1852)

# Two variables: A and B

## High Energy, Negative B

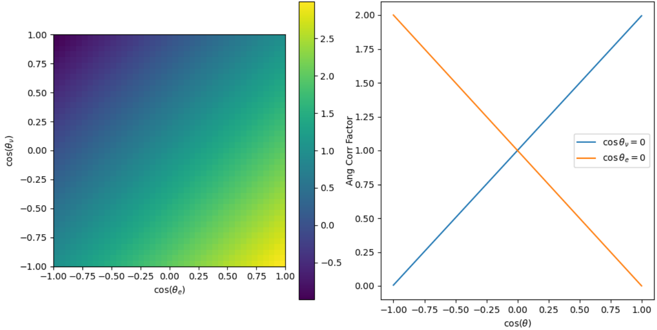


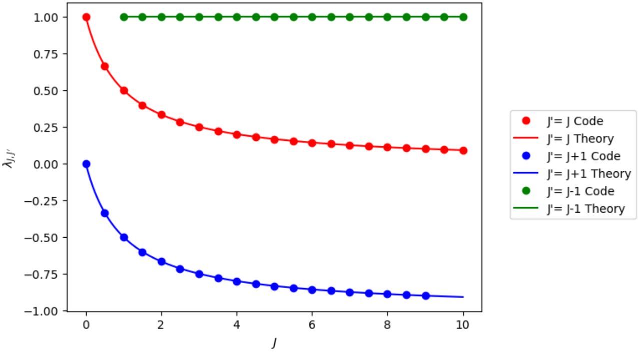

Figure: (Right) Values of the angular correlation Factor with $A = 1$, $B = -1$, $E = 5000$ keV and rest of variables 0. Maximum = 2.99476, Minimum = -0.99476 (Left) 1D projections at any $\phi$, and either $z_e = 0$ (orange, m = 1) or $z_\nu = 0$ (blue, m = -0.9948)

# Note of Concern

- Negative values for the angular correlation factor found in the tests. Concern for Montecarlo procedure.
- Possible in real simulations? Example from values in previous test: Gamov-Teller, $C_A = C'_A = C$, ($C_V, C'_V$ irrelevant), $\xi = 2C|M_{GT}|^2$, $|A| = |B| = |\lambda_{J_i,J_f}|$

$$|A| = |B| = \frac{2}{\xi}|M_{GT}|^2|\lambda_{J_i,J_f}||Re(C_A\overline{C'_A})|$$

- $|\lambda_{J_i,J_f}| = 1$, negative values can be found with certainity, $|\lambda_{J_i,J_f}| > 0.5$ negative values are posible depending on energy




Hope: Still need to consider $a$, which equals

$$a = -\frac{|M_{GT}|^2(|C_A|^2 + |C'_A|^2)}{3\xi} = -\frac{1}{3}$$

. Spoiler: situation worse $(A = -B)$

# Two variables: a and B

Since term proportional to a depends on E, we can consider different ratios by either:

- Fixing $a = B = 1$ and modifying the energy
- Same as before, but now $B = -1$
- Fixing $B = 1$ and $E \gg m_e \rightarrow \beta_e \approx 1$ and modifying $a > B$

We recall

$$F = 1 + a\beta_e(z_e z_\nu + \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\cos\phi) + Bz_\nu$$

Maxima and minima with $z_e = \pm 1, z_\nu \pm 1 \rightarrow \mathbf{p_e} \parallel \mathbf{p_\nu} \parallel \mathbf{J}$

# Two variables: a and B

## Low Energy

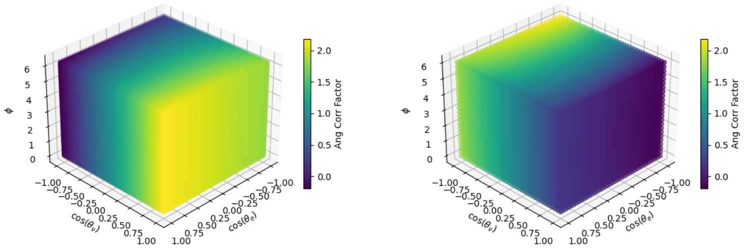



Figure: Values of the angular correlation Factor with (Right) a = 1, B = 1 and (Left) a = 1, B = -1; with E = 520 keV and rest of variables 0 for both.

# Two variables: a and B

## High Energy

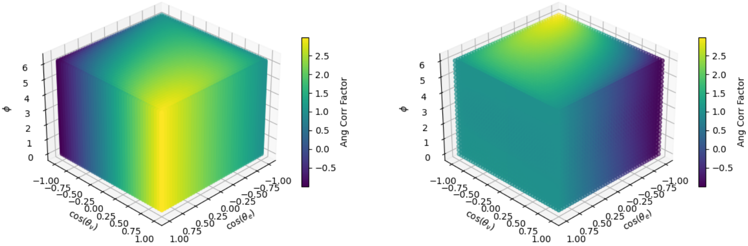



Figure: Values of the angular correlation Factor with (Right) a = 1, B = 1 and (Left) a = 1, B = -1; with E = 5000 keV and rest of variables 0 for both.

# Two variables: a and B

## More Ratios

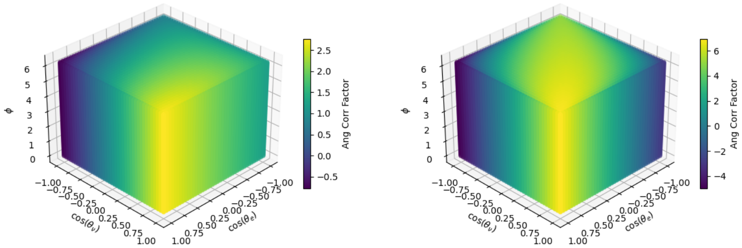



Figure: Values of the angular correlation Factor with (Right) a = 1, B = 1, E = 800 keV and (Left) a = 5, B = 1, E = 5000 keV; with the rest of variables 0 for both.

# Two variables: a and B

## Maximum and Minimum

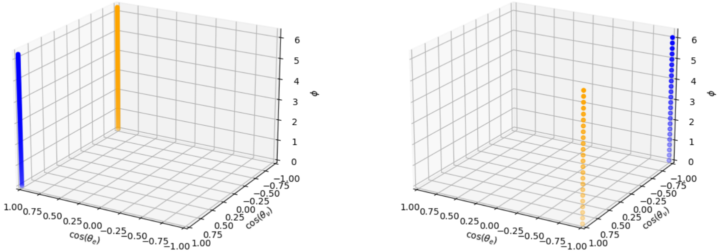



Figure: Location of maximum (blue, value = 2.99476) and minimum (orange, value = -0.99476) for (Right) a = B = 1, E = 5000 keV and (Left) a = 1, B = -1, E = 5000 keV

# Two variables: a and A

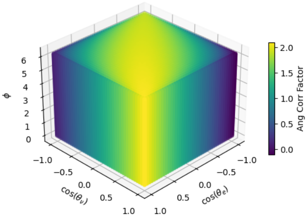
Since both terms proportional to a depends on E, we can consider only consider different ratios by changing one ($A$), while leaving the other ($a$) fixed. For convenience $E \gg m_e$.

$$F = 1 + \beta_e(a(z_e z_\nu + \sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\cos\phi) + Az_e)$$

Maximum and minimum with $z_e = \pm 1, z_\nu \pm 1 \rightarrow \mathbf{p_e} \parallel \mathbf{p_\nu} \parallel \mathbf{J}$

# Two variables: a and A

## $|A| \ll a$



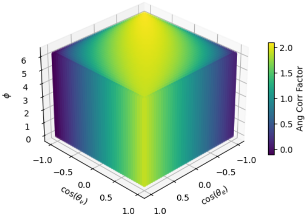




Figure: Values of the angular correlation Factor with (Right) a = 1, A = 0.1 and (Left) a = 1, A = -0.1, with E = 100000 keV and rest of variables 0 for both.

# Two variables: a and A

$|A| = a$

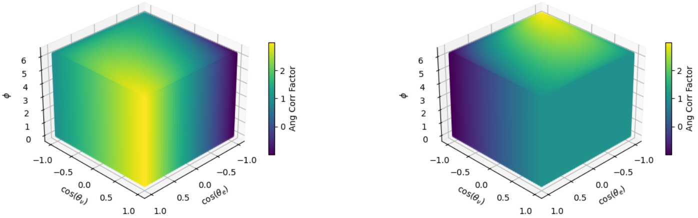
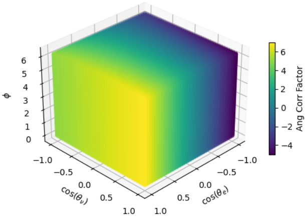



Figure: Values of the angular correlation Factor with (Right) a = 1, A = 1 and (Left) a = 1, A = -1, with E = 100000 keV and rest of variables 0 for both.

# Two variables: a and A

## $|A| \gg a$



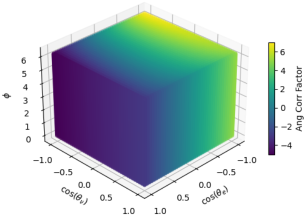




Figure: Values of the angular correlation Factor with (Right) a = 1, A = 5 and (Left) a = 1, A = -5, with E = 100000 keV and rest of variables 0 for both.

# Two variables: a and A

## Maximum and Minimum

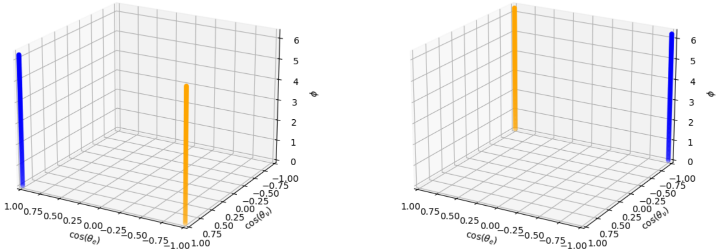



Figure: Location of maximum (blue, value = 2.99997) and minimum (orange, value = -0.99997) for (Right) a = B = 1, E = 5000 keV and (Left) a = 1, B = -1, E = 5000 keV

# Two variables: B and D

Since term proportional to a depends on E, we can consider different ratios by either:

- Fixing $B = D = 1$ and modifying the energy
- Same as before, but now $B = -1$
- Fixing $B = 1$ and $E \gg m_e \to \beta_e \approx 1$ and modifying $D > B$

We recall

$$F = 1 + D\beta_e\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\sin\phi + Bz_\nu$$

Maxima and minima no longer at $z_e = \pm 1, z_\nu \pm 1$. In fact, expect $z_e = 0$, $\phi = \pm\pi/2$. Additionally, at the maximum ($D\beta_e \equiv D'$):

$$z_\nu = \frac{B}{\sqrt{D'^2 + B^2}}$$

$$F = 1 + \sqrt{D'^2 + B^2}$$

# Two variables: B and D

## 3D examples

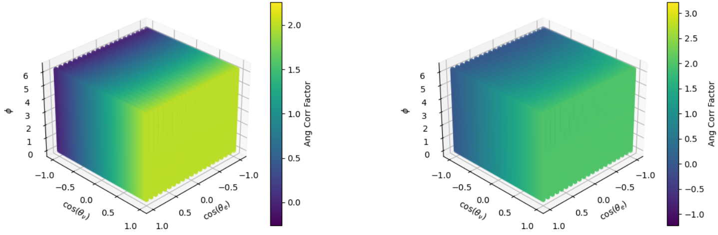



Figure: Values of the angular correlation Factor with (Right) D = 1, B = 1, E = 800 keV and (Left) D = 2, B = 1, E = 5000 keV, with the rest of variables 0 for both.

Image difficult to treat: consider only properties of the extrema.

# Two variables: B and D

## Low Energy

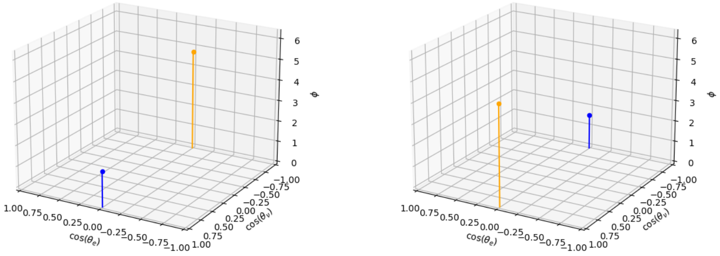
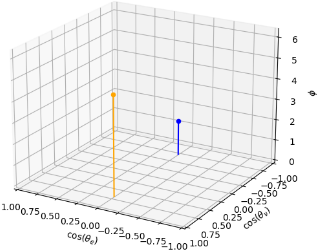



Figure: Location of maximum (blue) and minimum (orange) for (Right) D = B = 1 (Left) D = 1, B = -1, both at E = 520 keV

# Two variables: B and D

## High Energy



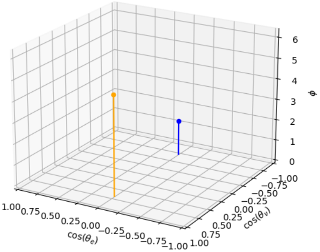




Figure: Location of maximum (blue) and minimum (orange) for (Right) D = B = 1 and (Left) D = 1, B = -1, both at E = 5000 keV

# Two variables: B and D

## More Ratios

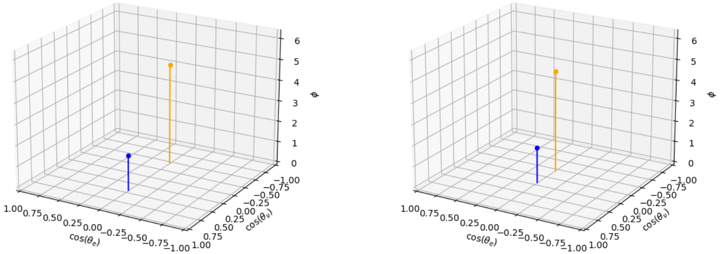



Figure: Location of maximum (blue) and minimum (orange) for (Right) D = 2, B = 1, E = 5000 keV and (Left) D = 5, B = 1, E = 5000 keV

# Two variables: B and D

## Behaviour of maximum

Considering now only cases with $B > 0$:

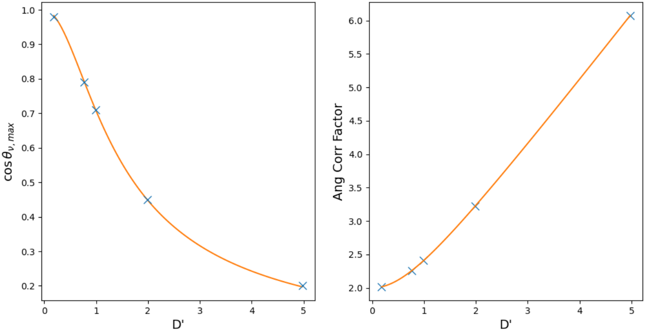



Figure: Behaviour of the $z_\nu$ coordinate for the maximum and the maximum value of the angular correlation factor for diferent values of D'

# Two variables: A and D

Since both terms proportional to a depends on E, we can consider only consider different ratios by changing one ($A$), while leaving the other ($D$) fixed. For convenience $E \gg m_e$.

$$F = 1 + D\beta_e\sqrt{1 - z_e^2}\sqrt{1 - z_\nu^2}\sin\phi + A\beta_e z_e$$

Maxima and minima no longer at $z_e = \pm 1, z_\nu \pm 1$. In fact, expect $z_\nu = 0$, $\phi = \pm\pi/2$. Additionally, at the maximum:

$$z_e = \frac{A}{\sqrt{D^2 + A^2}}$$

$$F = 1 + \beta_e\sqrt{D^2 + A^2}$$

We look directly at the properties of the extrema.

# Two variables: A and D

## $|A| \ll D$

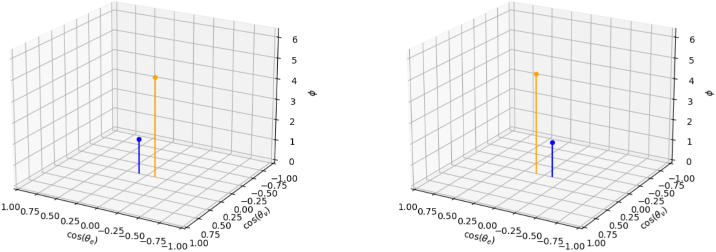



Figure: Positions of the maximum (blue) and minimum (orange) for (Right) D = 1, A = 0.1 and (Left) D = 1, A = -0.1, with E = 100000 keV and rest of variables 0 for both.

# Two variables: A and D

## $|A| = D$

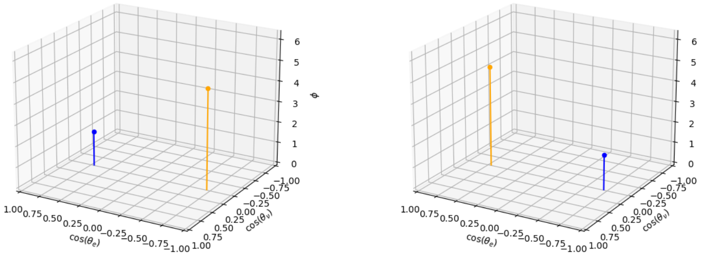



Figure: Positions of the maximum (blue) and minimum (orange) (Right) D = 1, A = 1 and (Left) D = 1, A = -1, with E = 100000 keV and rest of variables 0 for both.

# Two variables: A and D

## $|A| \gg D$

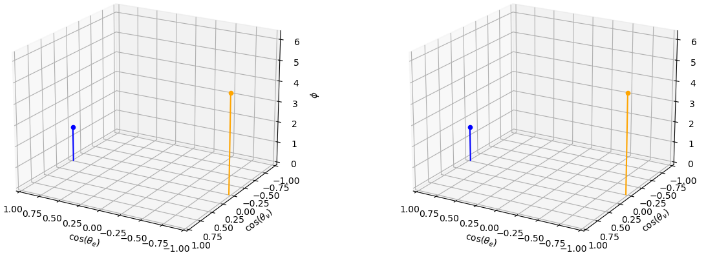



Figure: Positions of the maximum (blue) and minimum (orange) (Right) D = 1, A = 5 and (Left) D = 1, A = -5, with E = 100000 keV and rest of variables 0 for both.

# Two variables: A and D

## Behaviour of maximum

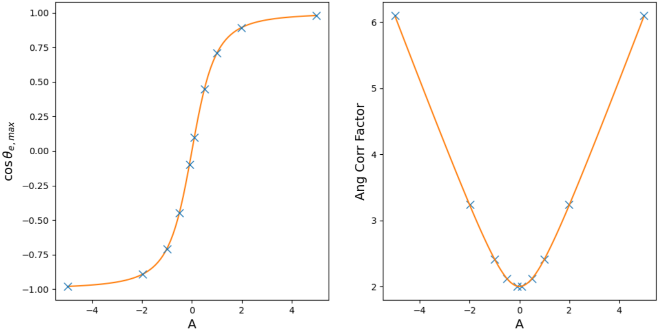



Figure: Behaviour of the $z_\nu$ coordinate for the maximum and the maximum value of the angular correlation factor for diferent values of A

# Two variables: a and D

Since both terms proportional to a depends on E, we can consider only consider different ratios by changing one ($D$), while leaving the other ($a$) fixed. For convenience $E \gg m_e$.

$$F = 1+\beta_e(a(z_e z_\nu+\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\cos\phi)+D\sqrt{1-z_e^2}\sqrt{1-z_\nu^2}\sin\phi)$$

Maxima and minima no longer at $z_e = \pm 1, z_\nu \pm 1$. In fact, expect $z_\nu = 0$, $z_e = 0$. Additionally, at the maximum:

$$\tan\phi = \frac{D}{a}$$

$$F = 1 + \beta_e\sqrt{D^2 + a^2}$$

We look directly at the properties of the extrema.

# Two variables: a and D

## $|D| \ll a$

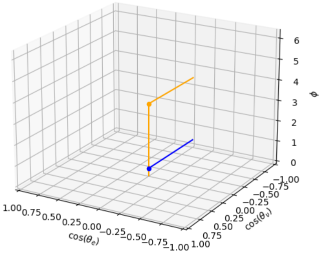



Figure: Positions of the maximum and minimum with Factor with (Right) a = 1, D = 0.25 and (Left) a = 1, D = -0.25, with E = 100000 keV and rest of variables 0 for both.

# Two variables: a and D

$|D| = a$

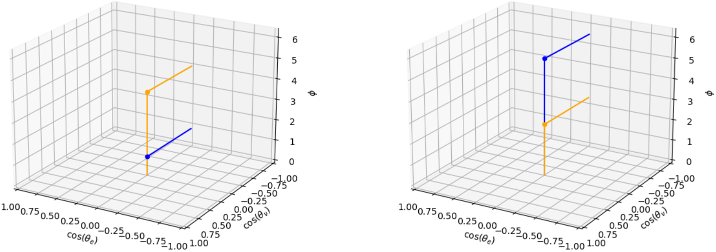



Figure: Positions of the maximum and minimum with with (Right) a = 1, D = 1 and (Left) a = 1, D = -1, with E = 100000 keV and rest of variables 0 for both.

# Two variables: a and D

## $|D| \gg a$

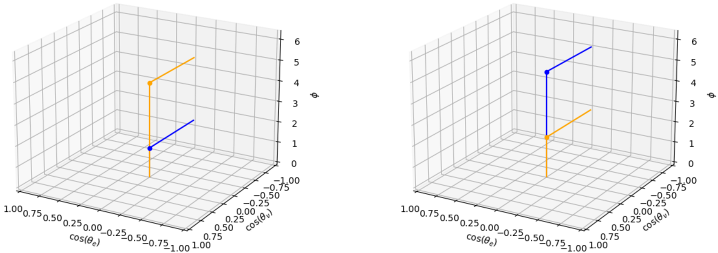



Figure: Positions of the maximum and minimum with (Right) a = 1, D = 4 and (Left) a = 1, D = -4, with E = 100000 keV and rest of variables 0 for both.

# Two variables: a and D

## Behaviour of maximum

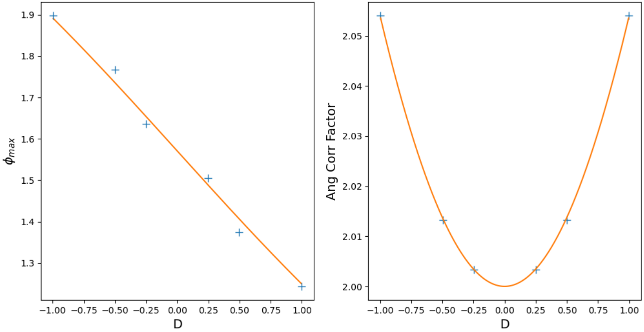



Figure: Behaviour of the $\phi$ coordinate for the maximum and the maximum value of the angular correlation factor for diferent values of D. Note discrepancies at the endpoint are a result of a sampling too coarse.

# Three variables: a, A, B

We come back to the case we hinted at before, that is compatible with all Gamov-Teller decays within the Standard Model framework.

Gamov-Teller, $C_A = C'_A = C$, ($C_V, C'_V$ irrelevant), $\xi = 2C|M_{GT}|^2$, $|A| = |B| = |\lambda_{J_i,J_f}|$, $A < 0$, $B > 0$

$$|A| = |B| = \frac{2}{\xi}|M_{GT}|^2|\lambda_{J_i,J_f}||Re(C_A\overline{C'_A})| = |\lambda_{J_i,J_f}|$$

$$a = -\frac{|M_{GT}|^2(|C_A|^2 + |C'_A|^2)}{3\xi} = -\frac{1}{3}$$

.

We consider different values of $|A|$, particularly with $|A| > 0.5$, and a range of electron energies from very low to very high.

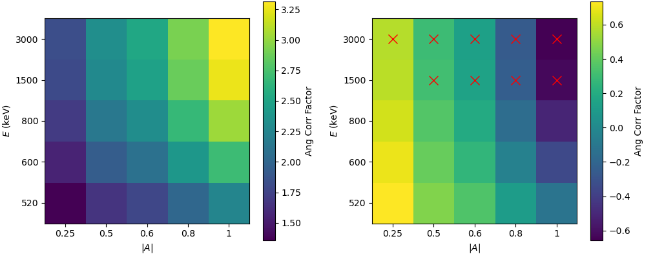



Figure: Values of the maximum and minimum for different combinations of $|A|$ and E. , Values that are negative are marked with a cross