

CRADLE++ Tests 2

November 19, 2025

Simulation Wu Experiment

Consider atoms of ^{60}Co in a thermal bath and in the presence of a magnetic field in the $-z$ direction.

Model each nuclei as independent 11 state system (each of the values of m_j)

$$Z = \sum_{m_j=-5}^5 e^{\frac{m_j \mu_N(^{60}\text{Co})B}{5k_b T}} \rightarrow P(m_j) = \frac{1}{Z} e^{\frac{m_j \mu_N(^{60}\text{Co})B}{5k_b T}}$$

From here polarisation and alignment in Z direction (P_z and \mathcal{A}_z) can be computed

$$P_z = \frac{1}{J} \langle m_j \rangle = \sum_{m_j=-5}^5 \frac{m_j P(m_j)}{5} \quad \mathcal{A}_z = \frac{3 \langle m_j^2 \rangle - J(J+1)}{J(2J-1)}$$

which leads to non-zero A polarisation.

Simulation Wu Experiment

Implementation:

- ▶ $N = 200000$ atoms
- ▶ $|z_e| > \cos 15^\circ$
- ▶ Realistic value of $\mu_N(^{60}\text{Co})$
- ▶ 1 sim for each T , with its P_z and \mathcal{A}_z

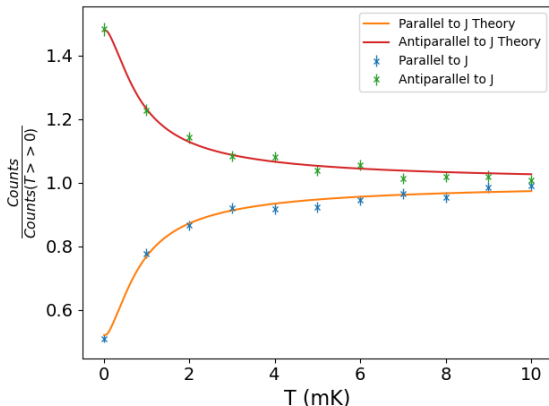


Figure: Simulation of the 1957 Wu experiment using $N = 200000$ ^{60}Co nuclei for each T

Gamow-Teller Decay: ^{60}Co

Properties of $^{60}\text{Co}(5^+) \rightarrow ^{60}\text{Ni}(4^+)$

- ▶ $Q = 317.06 \text{ keV}$ (good for testing, $\langle \beta_e \rangle = 0.68$, $\langle \gamma_e^{-1} \rangle = 0.72$, $\langle \alpha Z \gamma_e^{-1} \rangle = 0.15$)
- ▶ $J_f = J_i - 1 \rightarrow \lambda_{J_i, J_f} = \Lambda_{J_i, J_f} = 1$
- ▶ 2 γ almost always ($5^+ \rightarrow 2^+$ only 1 γ)

Many cases to consider, though for realism: keep $C_A = C'_A = \text{cte}(=1)$.

- ▶ $C_T = C'_T = 0$ (Standard Model)
- ▶ $C_T = C'_T$ pure real (and large)
- ▶ $C_T = C'_T$ pure imaginary
- ▶ $C_T = -C'_T$, either real or imaginary

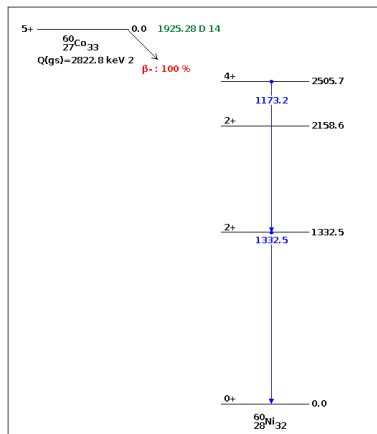


Figure: Decay Scheme of ^{60}Co into ^{60}Ni featuring the only decay of interest

Gamow-Teller Decay: ^{60}Co

Numerical evaluation

Use that distributions in z_e , z_ν , $\cos\theta_{e,\nu} \equiv z_{e,\nu}$ and ϕ are known if $F \geq 0$ for all orientations of $\mathbf{p}_e, \mathbf{p}_\nu$ (\mathbf{J} fixed).

$$f_1(z_e) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle A\beta_e \rangle z_e}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_2(z_\nu) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle B \rangle z_\nu}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_3(z_{e,\nu}) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle a\beta_e \rangle z_{e,\nu}}{2(1 + \langle b\gamma_e^{-1} \rangle)}$$

$$f_4(\phi) = \frac{1 + \langle b\gamma_e^{-1} \rangle + \langle (a + \frac{c}{3}) \beta_e \rangle \frac{\pi^2}{16} \cos \phi + \langle D\beta_e \rangle \frac{\pi^2}{16} \sin \phi}{2\pi(1 + \langle b\gamma_e^{-1} \rangle)}$$

Averages computed numerically using $f(E)$ from the simulation data itself (avoid computing the Fermi function myself)

Gamow-Teller Decay: ^{60}Co

Numerical verification

From distributions:

- ▶ Compute difference $\Delta_{i,j} = f_j(x_i) - f_{j,th}(x_i)$ for each point x_i in each distribution f_j
- ▶ Use $\sigma_{i,j} = \sqrt{f_j(x_i)}$ as uncertainty
- ▶ Compute residuals as:

$$Res_{i,j} = \frac{\Delta_{i,j}}{\sigma_{i,j}} \rightarrow \chi_j^2 = \sum_i Res_{i,j}^2$$

- ▶ Verify $\chi_j^2 \approx \#\{x_i\}$ and residuals mostly between -2 and 2.

Gamow-Teller Decay: ^{60}Co

Standard Model

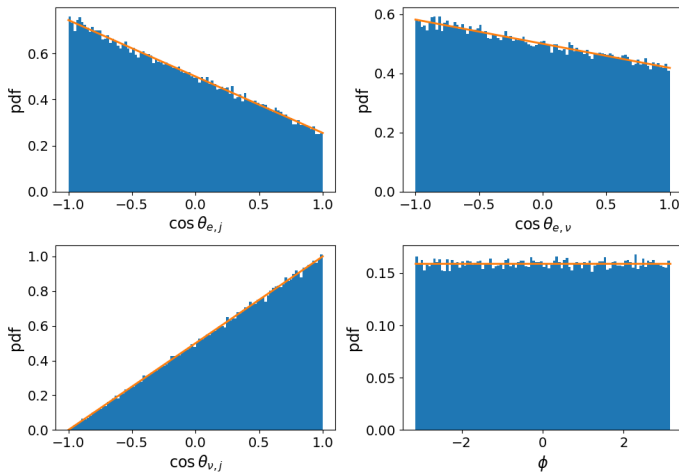


Figure: Distribution of various relevant angles, z_e , z_ν , $z_{e,\nu}$ and ϕ , each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: ^{60}Co

Standard Model

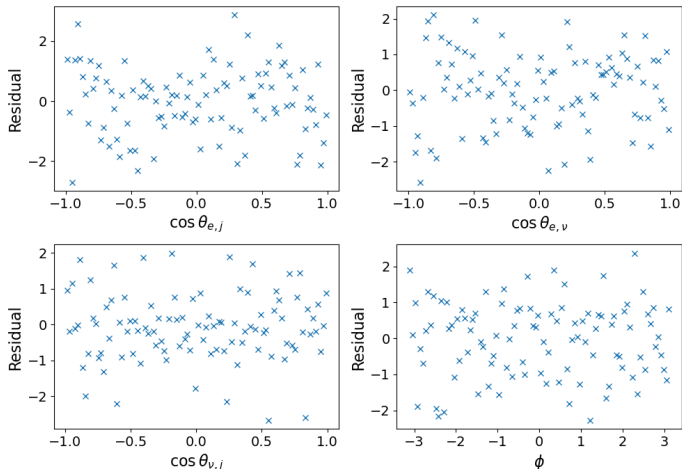


Figure: Residuals from the comparison between CRADLE simulation and theory of various relevant angles, z_e , z_ν , $z_{e,\nu}$ and ϕ . Values of $\chi^2/100$: 1.25, 0.86, 1.05, 1.03

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Real Positive

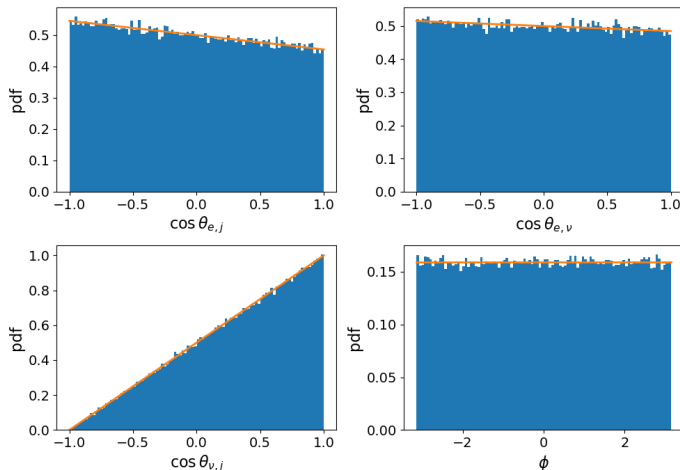


Figure: Distribution of various relevant angles, z_e , z_ν , $z_{e,\nu}$ and ϕ , each with a well-known distribution, and the theoretical value with $C_T = C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Real Positive

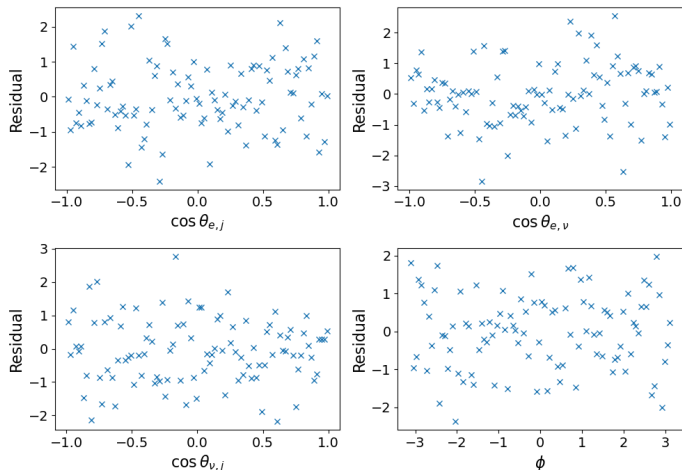


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e , z_ν , $z_{e,\nu}$ and ϕ distributions with $C_T = C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Real Negative

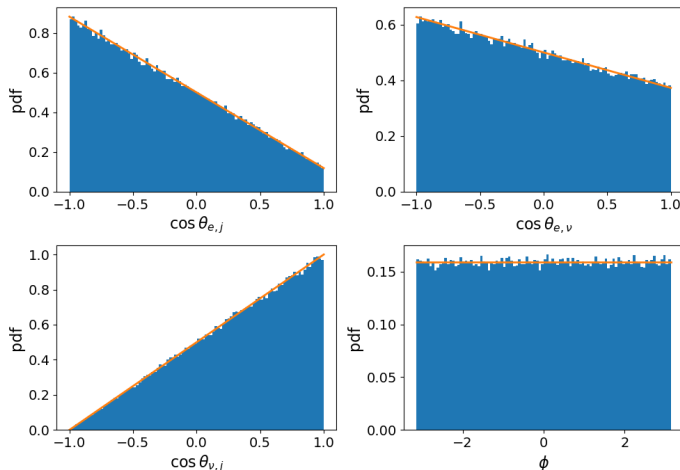


Figure: Distribution of various relevant angles, z_e , z_n , $z_{e,n}$ and ϕ , each with a well-known distribution, and the theoretical value with $C_T = C'_T = -1/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Real Negative

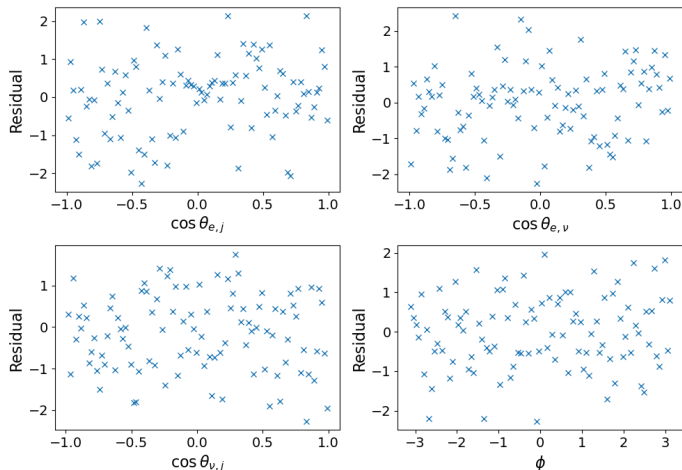


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e , z_n , $z_{e,n}$ and ϕ distributions with $C_T = C'_T = -1/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Imaginary Positive

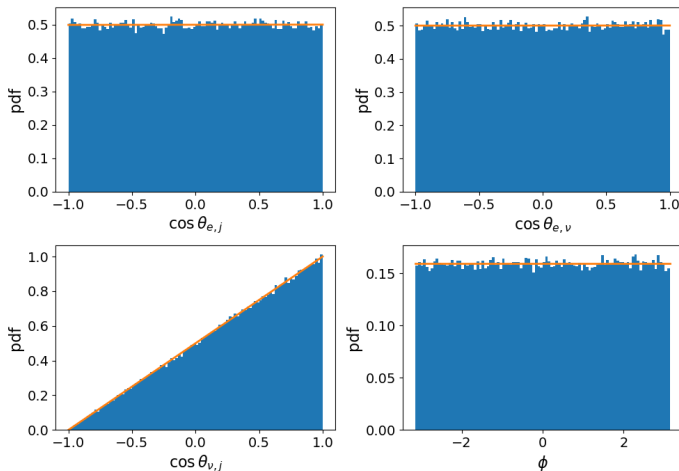


Figure: Distribution of various relevant angles, z_e , z_ν , $z_{e,\nu}$ and ϕ , each with a well-known distribution, and the theoretical value with $C_T = C'_T = i/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Imaginary Positive

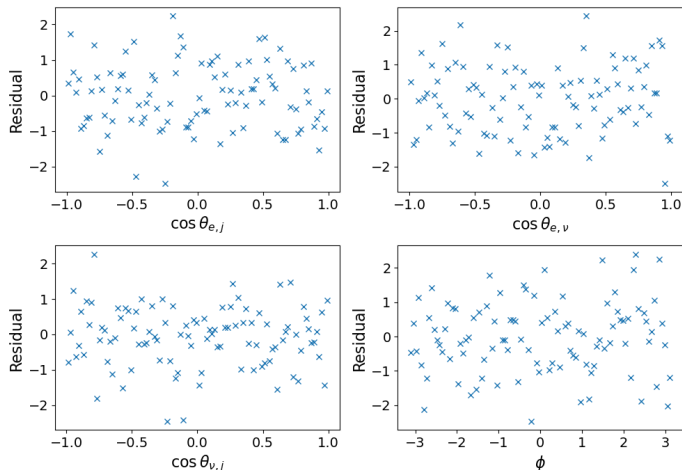


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e , z_ν , $z_{e,\nu}$ and ϕ distributions with $C_T = C'_T = i/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Imaginary Negative

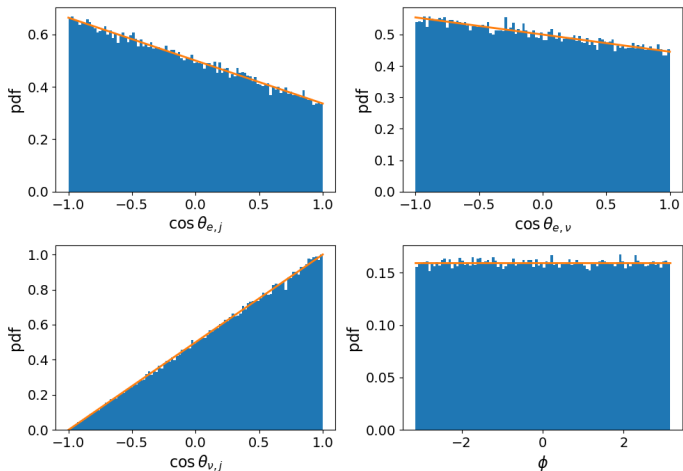


Figure: Distribution of various relevant angles, z_e , z_ν , $z_{e,\nu}$ and ϕ , each with a well-known distribution, and the theoretical value with $C_T = C'_T = -i/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C'_T$ Imaginary Negative

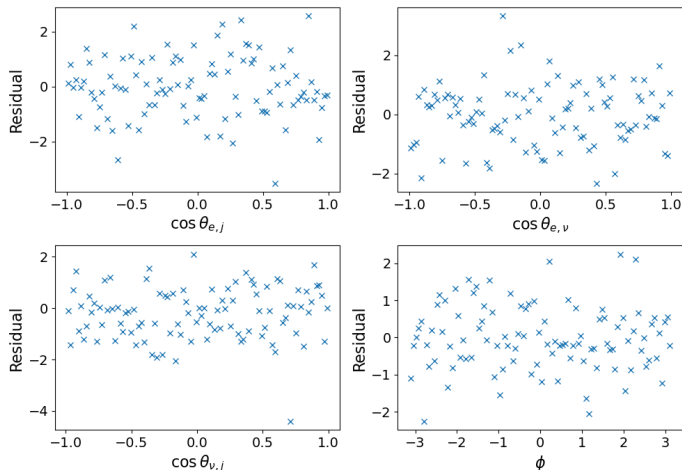


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e , z_ν , $z_{e,\nu}$ and ϕ distributions with $C_T = C'_T = -i/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$$C_T = -C'_T$$

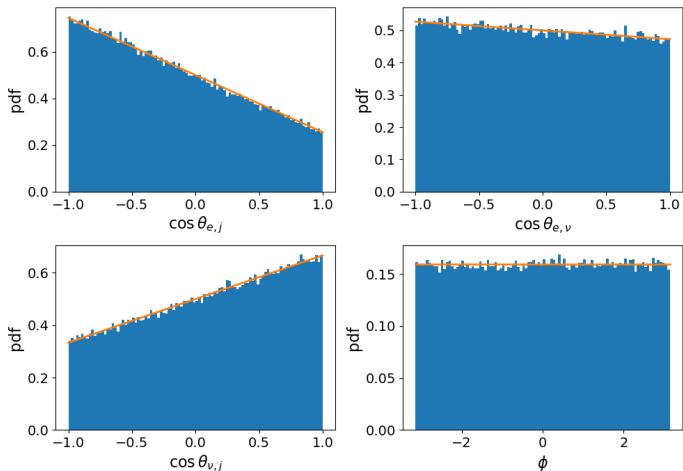


Figure: Distribution of various relevant angles, z_e , z_ν , $z_{e,\nu}$ and ϕ , each with a well-known distribution, and the theoretical value with $C_T = -C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$$C_T = -C'_T$$

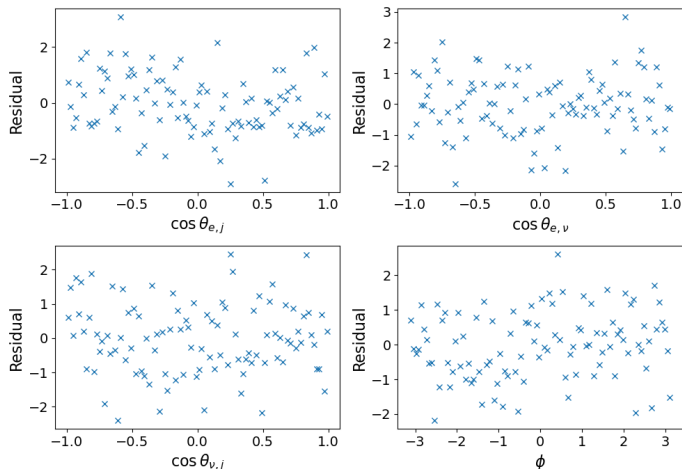


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e , z_ν , $z_{e,\nu}$ and ϕ distributions with $C_T = -C'_T = 1/\sqrt{2}$