CRADLE++ Tests 2

November 21, 2025

Simulation Wu Experiment

Consider atoms of $^{60}\mathrm{Co}$ in a thermal bath and in the presence of a magnetic field in the -z direction.
Model each nuclei as independent 11 state system (each of the values of m_j)

$$Z = \sum_{m_j=-5}^{5} e^{\frac{m_j \mu_N(^{60}\mathrm{Co})B}{5k_b T}} \rightarrow P(m_j) = \frac{1}{Z} e^{\frac{m_j \mu_N(^{60}\mathrm{Co})B}{5k_b T}}$$

From here polarisation and alignment in Z direction (P_z and $\mathcal{A}_z$) can be computed

$$P_z = \frac{1}{J}\langle m_j \rangle = \sum_{m_j=-5}^{5} \frac{m_j P(m_j)}{5} \qquad \mathcal{A}_z = \frac{3\left\langle m_j^2 \right\rangle - J(J+1)}{J(2J-1)}$$

which leads to non-zero A polarisation.

Simulation Wu Experiment

Implementation:

- N = 200000 atoms
- $|z_e| > \cos 15^\circ$
- Realistic value of $\mu_N(^{60}\mathrm{Co})$
- 1 sim for each T, with its P_z and $\mathcal{A}_z$

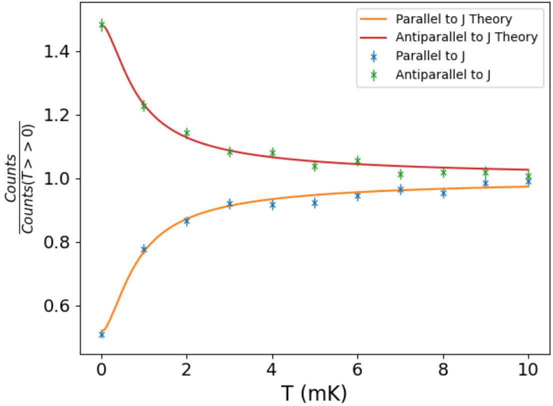


Figure: Simulation of the 1957 Wu experiment using N = 200000 $^{60}\mathrm{Co}$ nuclei for each T

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Properties of $^{60}\mathrm{Co}(5^+) \rightarrow {}^{60}\mathrm{Ni}(4^+)$

- Q = 317.06 keV (good for testing, $\langle \beta_e \rangle = 0.68, \langle \gamma_e^{-1} \rangle = 0.72, \langle \alpha Z \gamma_e^{-1} \rangle = 0.15$)
- $J_f = J_i - 1 \rightarrow \lambda_{J_i,J_f} = \Lambda_{J_i,J_f} = 1$
- 2 γ almost always ($5^+ \rightarrow 2^+$ only 1 γ)

Many cases to consider, though for realism: keep $C_A = C'_A = \mathrm{cte}(= 1)$.

- $C_T = C'_T = 0$ (Standard Model)
- $C_T = C'_T$ pure real (and large)
- $C_T = C'_T$ pure imaginary
- $C_T = -C'_T$, either real or imaginary

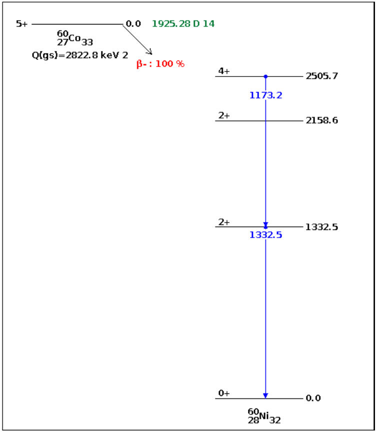


Figure: Decay Scheme of $^{60}\mathrm{Co}$ into $^{60}\mathrm{Ni}$ featuring the only decay of interest

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Numerical evaluation

Use that distributions in z_e, z_ν, $\cos\theta_{e,\nu} \equiv z_{e,\nu}$ and ϕ are known if $F \geq 0$ for all orientations of $\boldsymbol{p_e}, \boldsymbol{p_\nu}$ ($\boldsymbol{J}$ fixed).

$$f_1(z_e) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \langle A\beta_e \rangle z_e}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f_2(z_\nu) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \langle B \rangle z_\nu}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f_3(z_{e,\nu}) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \langle a\beta_e \rangle z_{e,\nu}}{2(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

$$f_4(\phi) = \frac{1 + \left\langle b\gamma_e^{-1} \right\rangle + \left\langle \left(a + \frac{c}{3}\right) \beta_e \right\rangle \frac{\pi^2}{16} \cos\phi + \langle D\beta_e \rangle \frac{\pi^2}{16} \sin\phi}{2\pi(1 + \left\langle b\gamma_e^{-1} \right\rangle)}$$

Averages computed numerically using f(E) from the simulation data itself (avoid computing the Fermi function myself)

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Numerical verification

From distributions:

- Compute difference $\Delta_{i,j} = f_j(x_i) - f_{j,th}(x_i)$ for each point x_i in each distribution f_j
- Use $\sigma_{i,j} = \sqrt{f_j(x_i)}$ as uncertainity
- Compute residuals as:

$$Res_{i,j} = \frac{\Delta_{i,j}}{\sigma_{i,j}} \rightarrow \chi^2_j = \sum_i Res^2_{i,j}$$

- Verify $\chi^2_j \approx \#\{x_i\}$ and residuals mostly between -2 and 2.

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model

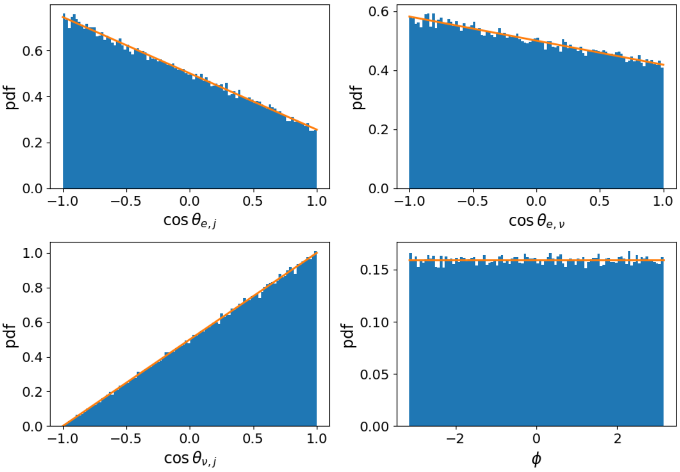


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Standard Model

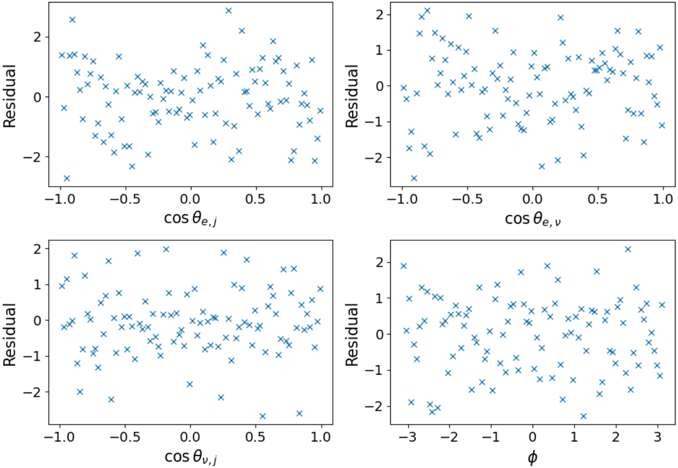


Figure: Residuals from the comparison between CRADLE simulation and theory of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ.

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Real Positive

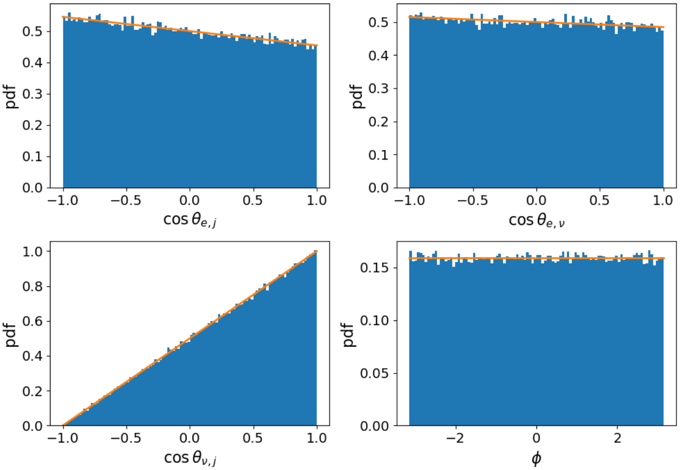


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Real Positive

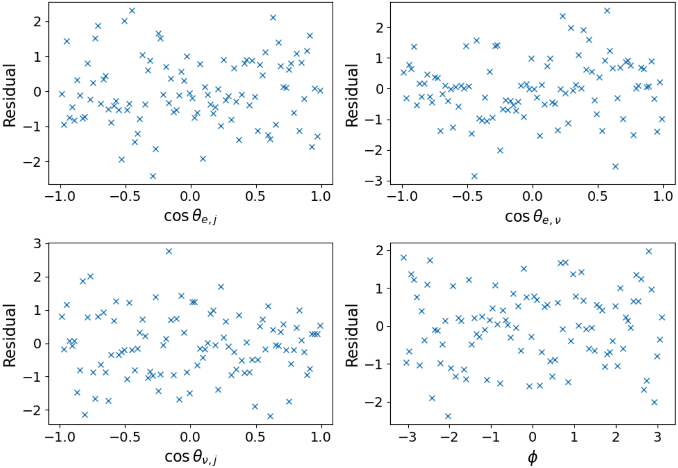


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $C_T = C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Real Negative

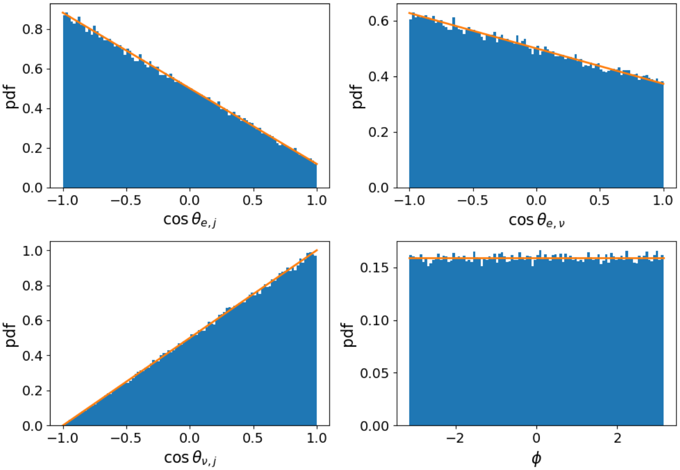


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = -1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Real Negative

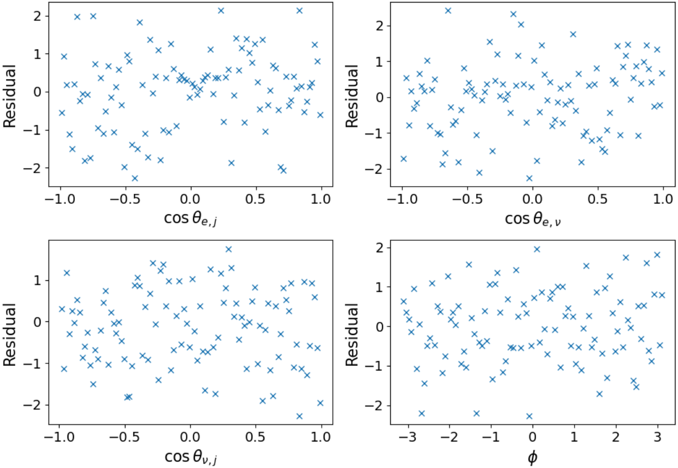


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $C_T = C_T' = -1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C_T'$ Imaginary Positive

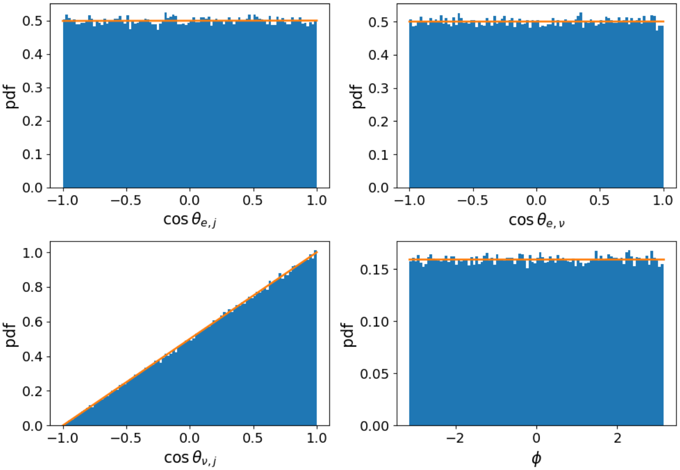


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C_T' = i/\sqrt{2}$

Gamow-Teller Decay: ^{60}Co

$C_T = C_T'$ Imaginary Positive

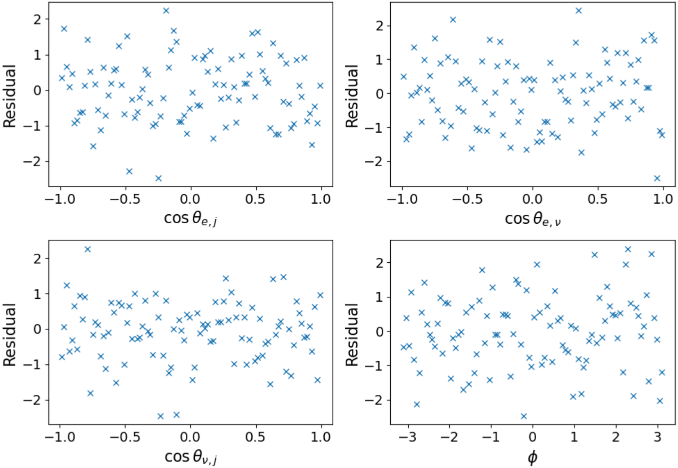


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $C_T = C_T' = i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Imaginary Negative

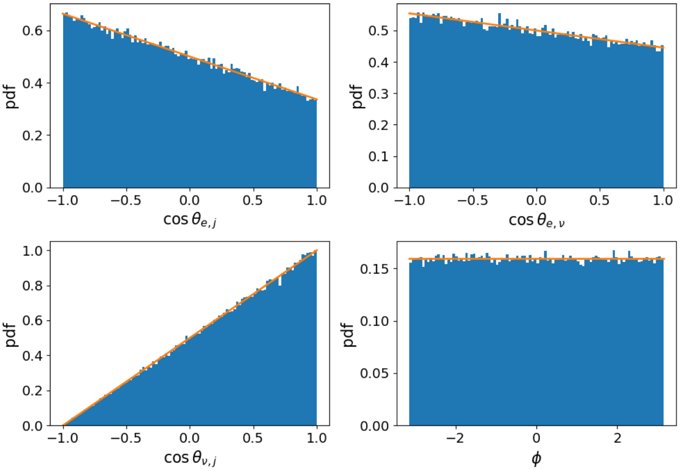


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = C'_T = -i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = C'_T$ Imaginary Negative

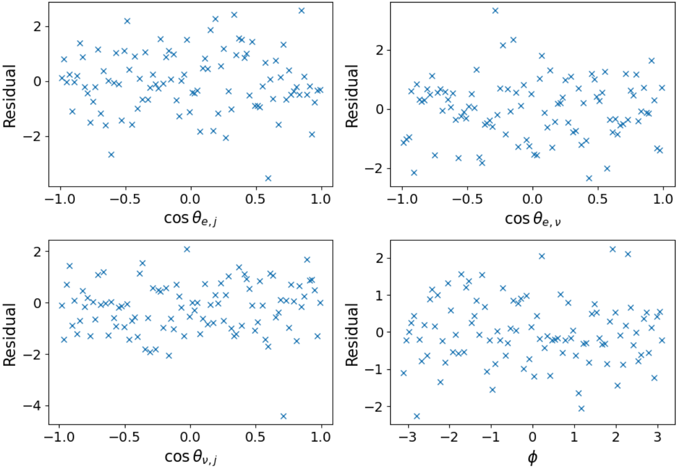


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $C_T = C'_T = -i/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = -C'_T$

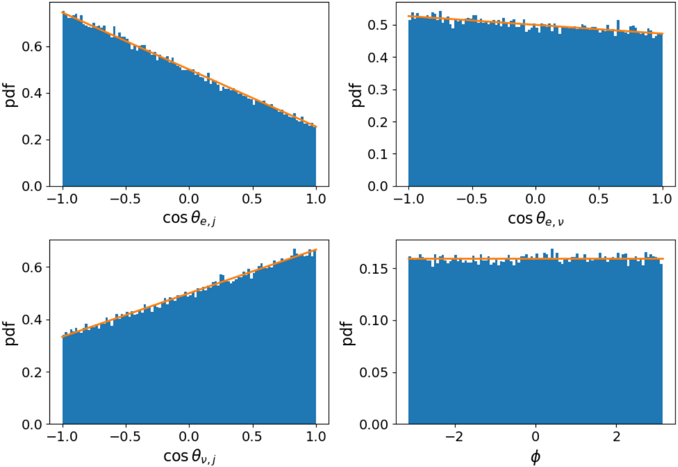


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value with $C_T = -C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

$C_T = -C'_T$

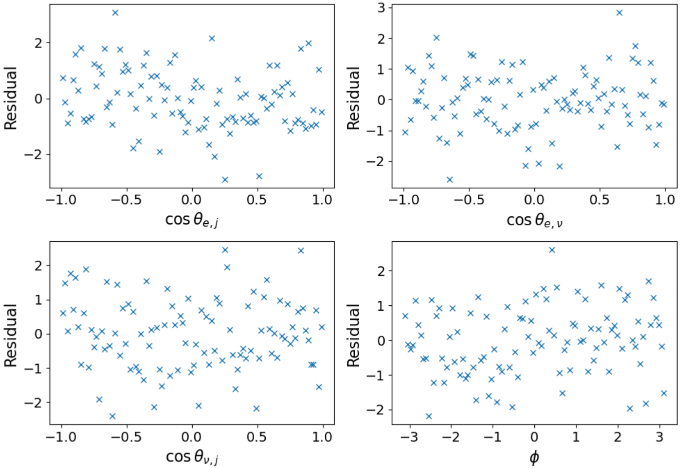


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $C_T = -C'_T = 1/\sqrt{2}$

Gamow-Teller Decay: $^{60}\mathrm{Co}$

Summary

We present here the χ^2 values.

	$\chi^2(z_e)$	$\chi^2(z_\nu)$	$\chi^2(z_{e,\nu})$	$\chi^2(\phi)$
SM	1.253	0.864	1.047	1.026
$Re(C_T) > 0$	0.915	0.915	0.944	0.936
$Re(C_T) < 0$	1.000	0.856	0.982	0.859
$Im(C_T) > 0$	0.826	0.696	0.959	1.066
$Im(C_T) < 0$	1.208	1.051	1.030	0.739
$C_T + C_T' = 0$	1.149	1.051	0.905	0.915

Table: Values of $\chi^2/100$ for each distribution for each of the tests performed. The first one is the Standard Model values, rows 2 to 5 correspond to tests where $C_T = C_T'$ and the last one features $C_T = -C_T'$. For all tests $M_{GT} = 1$, $C_A = C_A' = 1.2754$ and $|C_T| = |C_T'| = 1/\sqrt{2}$

All close to 1

Mixed Decays: $^{39}\mathrm{Ca}$

Now we consider a mixed β^+ decay: $^{39}\mathrm{Ca}\left(\frac{3}{2}\right)^+ \rightarrow {}^{39}\mathrm{K}\left(\frac{3}{2}\right)^+$
Properties:

- Q = 5502.5 keV (a bit inconvenient, $\langle\gamma_e^{-1}\rangle = 0.2$ and $\alpha Z = 0.14$)
- No γ produced

First, do tests with 2 non-zero coupling for pairs with $N = 10^6$ decays. Shown those with non-zero correlations A, B and D that are not present in a Gamow-Teller decay. We use $|M_{GT} C_{A,T}^{(\prime)}| = |M_F C_{V,S}^{(\prime)}| = 1$ for convenience

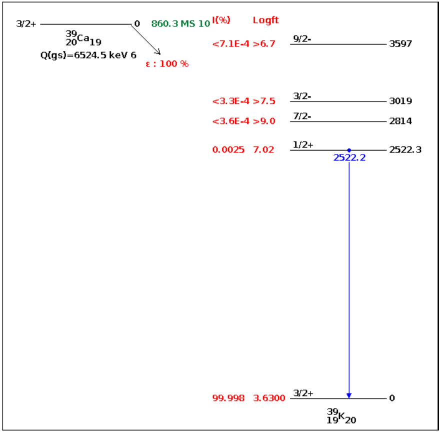


Figure: Decay Scheme of $^{39}\mathrm{Ca}$ into $^{39}\mathrm{K}$.

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_T$

Non zero D proportional to $\langle\beta_e\rangle$

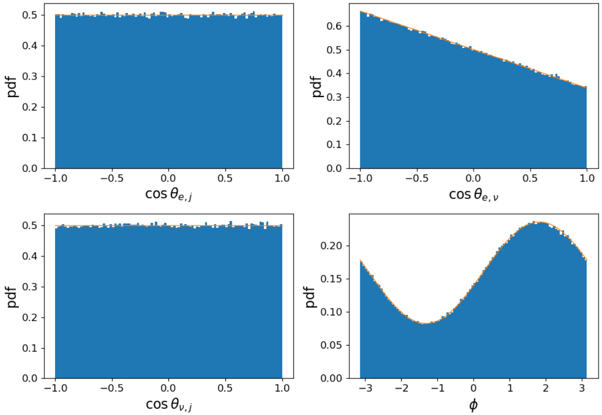


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_T = 0.6 - 0.8i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_T$

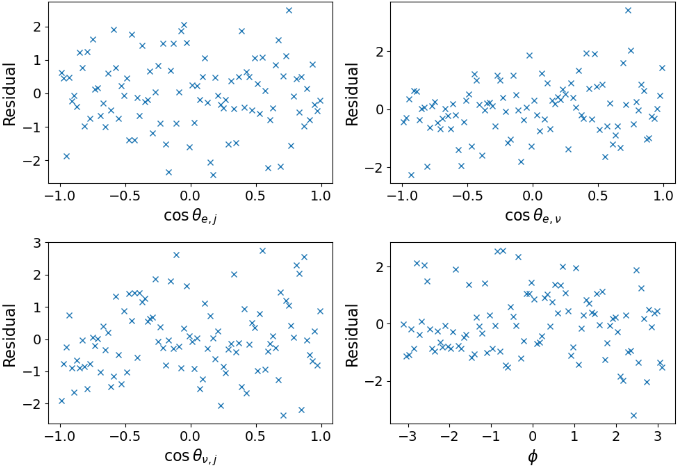


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_T = 0.6 - 0.8i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_T'$

Non zero $A > 0, B < 0$ proportional to $\langle \beta_e \rangle$

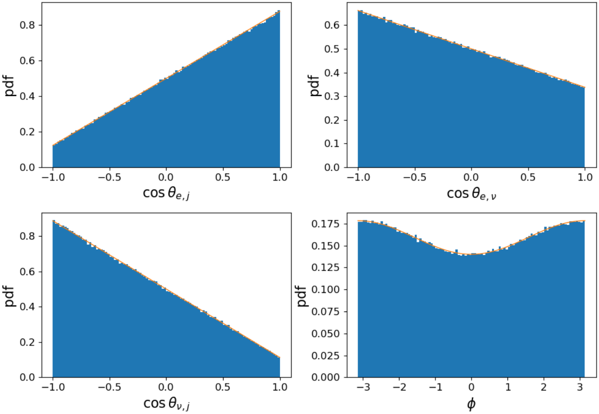


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 1, M_{GT} C_T' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_T'$

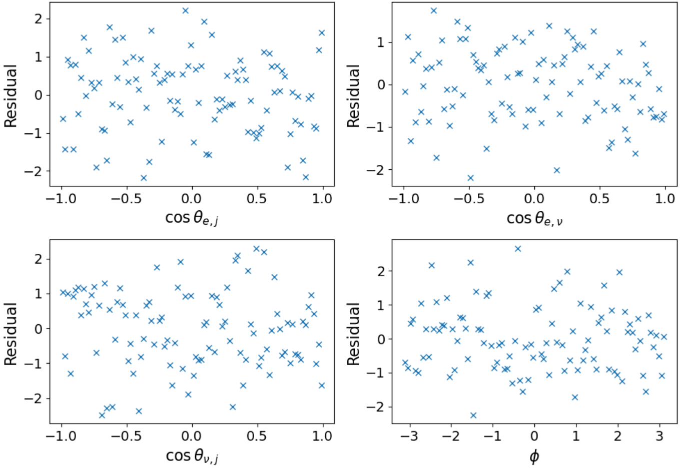
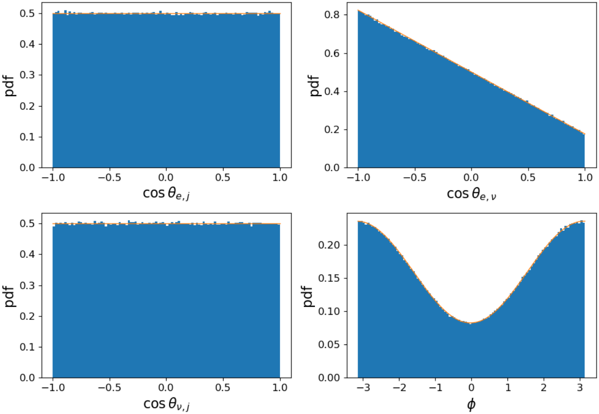


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_S = 1, M_{GT} C_T' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A$

Non zero $D > 0$ proportional to $\langle \alpha Z \gamma_e^{-1} \rangle$: very hard to see



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Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A$

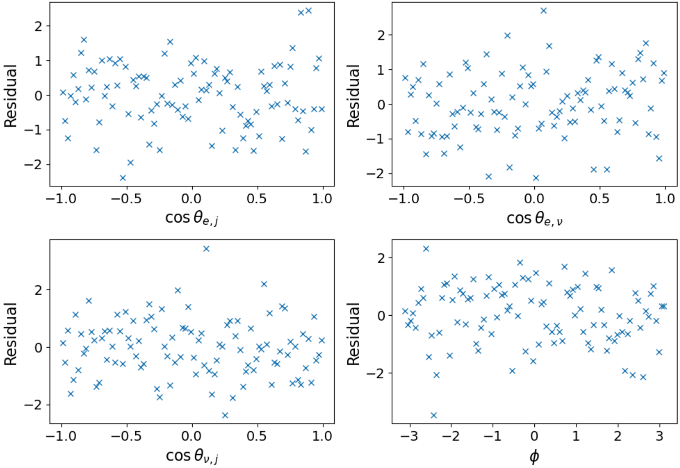


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_S = 1, M_{GT} C_A = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A$: D Component

To see the D component: increase N (number of decays) from 10^6 to $3 \cdot 10^6$

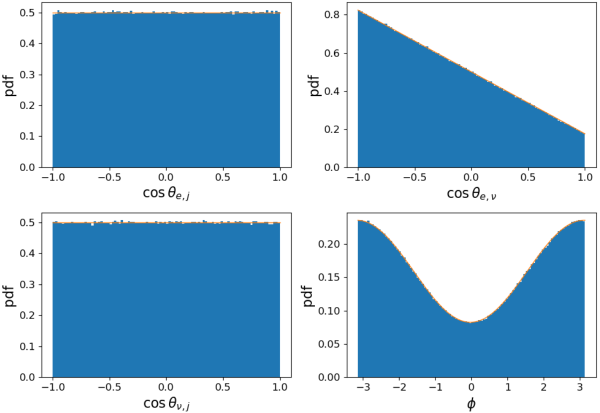


Figure: Distribution of various relevant angles,z_e, z_ν, $z_{e,\nu}$ and ϕ and the theoretical value for $M_F C_S = 1, M_{GT} C_A = 1$

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A$: D Component

To see the D component:

- Increase N
- Keep only the sine component ($b = 0$)

$$f_4(\phi) \propto \left(a + \frac{c}{3}\right) \langle \beta_e \rangle \frac{\pi^2}{16} \cos\phi$$

$$+ \langle D\beta_e \rangle \frac{\pi^2}{16} \sin\phi + 1$$

- Reduce number of bins ($100 \rightarrow 50$)

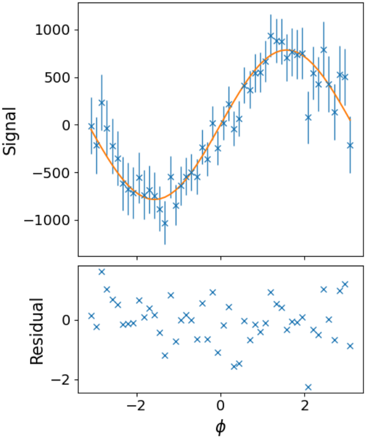


Figure: In top, cosine component of the ϕ distribution (blue) and its theoretical vaule (orange). At the bottom, residuals.

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A'$

Non zero $B > 0$ proportional to $\langle \gamma_e^{-1} \rangle$

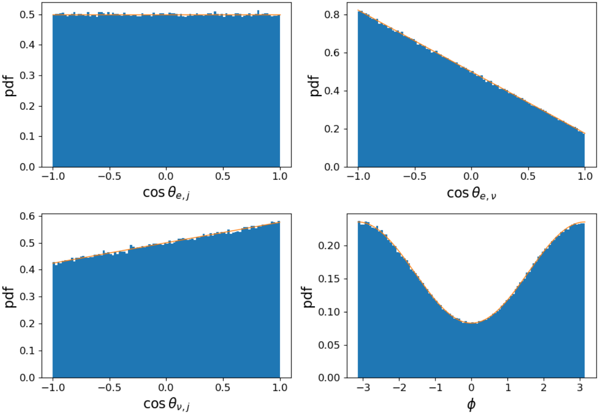


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 1, M_{GT} C_A' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_S C_A'$

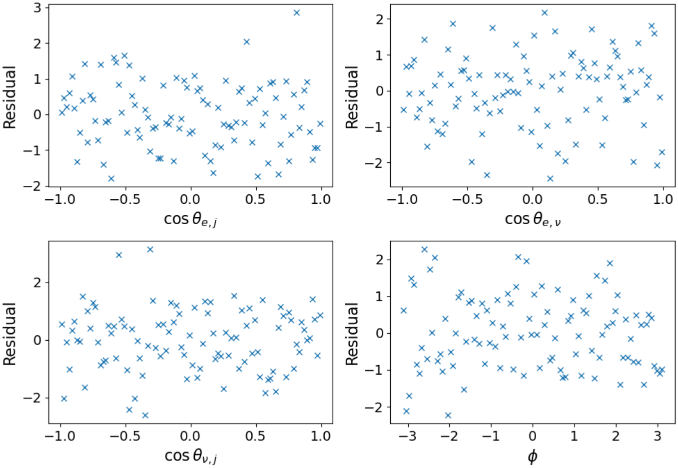


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_S = 1, M_{GT} C_A' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_A'$

Non zero $A < 0$ proportional to $\langle \alpha Z \gamma_e^{-1} \rangle$: hard to see

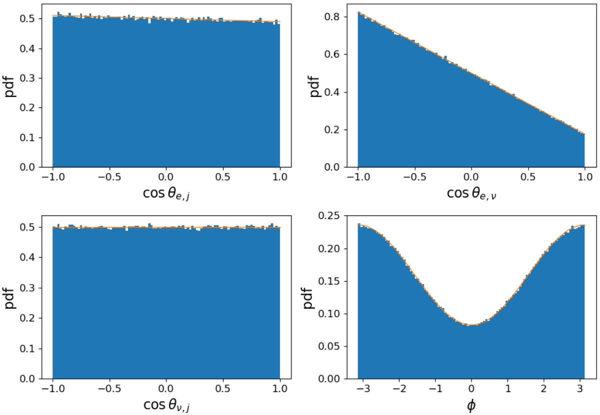


Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_A' = 0.6 - 0.8i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_A'$

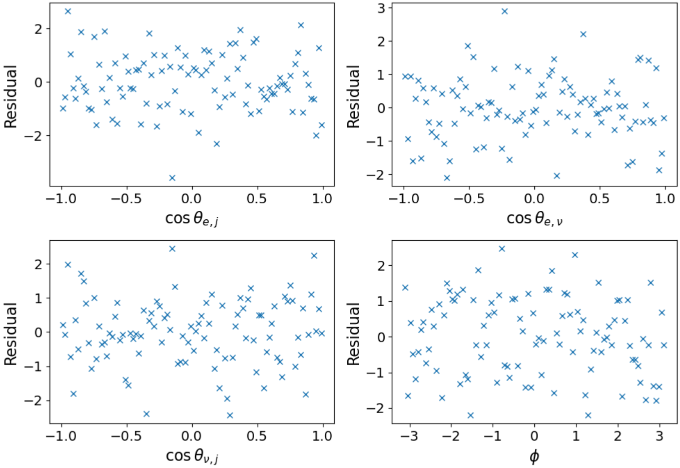
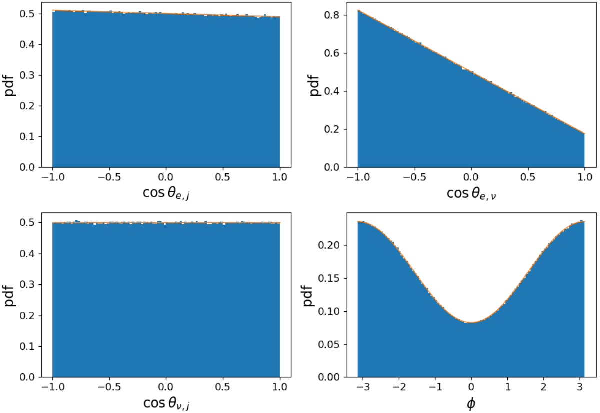


Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_S = 0.8 + 0.6i$, $M_{GT} C_A' = 0.6 - 0.8i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_A'$: A Component

To see the A component: increase N (number of decays) from 10^6 to $3 \cdot 10^6$



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Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_S C_A'$: A Component

To see the A component:

- Increase N
- Keep only the linear component ($b = 0$)

$$f_1(z_e) \propto 1 + \langle A\beta \rangle\, z_e$$

- Reduce number of bins ($100 \to 50$)

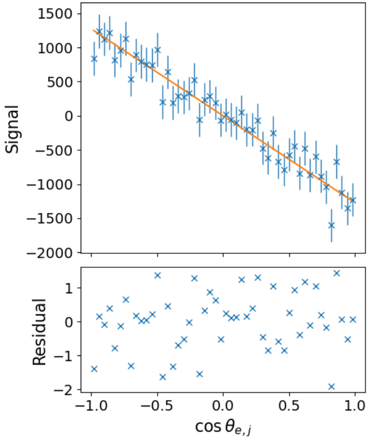
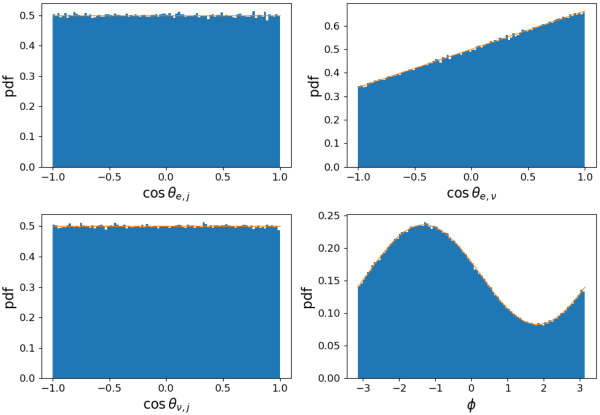


Figure: In top, linear component of the z_e distribution (blue) and its theoretical vaule (orange). At the bottom, residuals.

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_V C_A$

Non-zero $D < 0$ proportional to $\langle \beta_e \rangle$



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Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_V C_A$

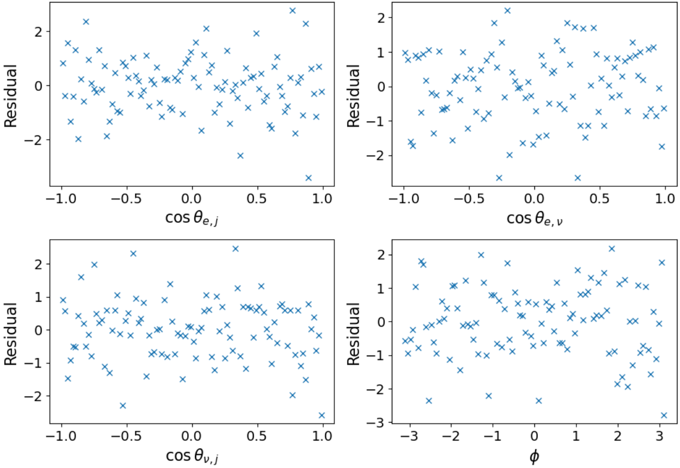

Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_V = 0.8 + 0.6i$, $M_{GT} C_A = 0.6 - 0.8i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_A'$

Non-zero A, $B < 0$ proportional to $\langle \beta_e \rangle$

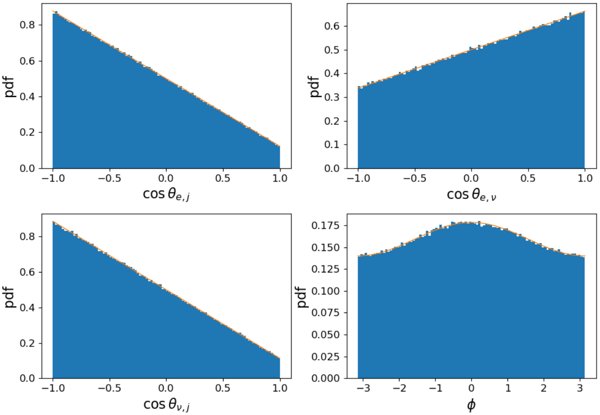

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 1, M_{GT} C_A' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_A'$

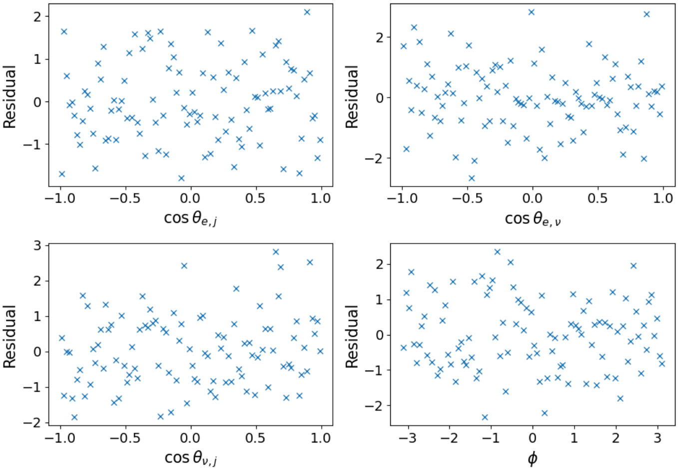

Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_V = 1, M_{GT} C_A' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_T$

Non-zero $D < 0$ proportional to $\langle \alpha Z \gamma_e^{-1} \rangle$: very hard to see

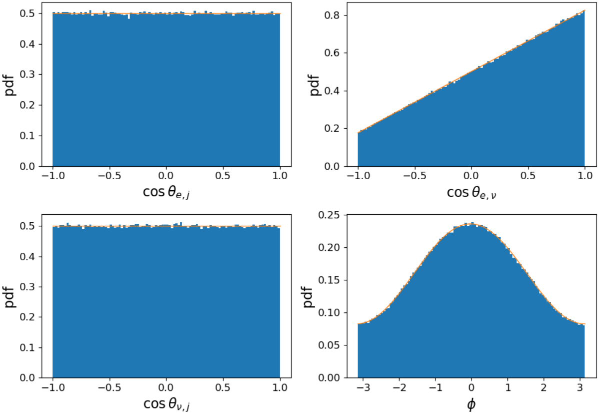

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 1, M_{GT} C_T = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_T$

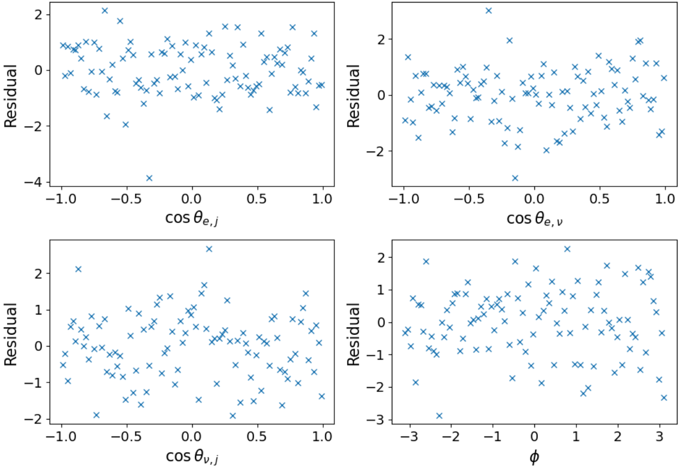

Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_V = 1, M_{GT} C_T = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_T'$

Non-zero $B > 0$ proportional to $\langle \gamma_e^{-1} \rangle$

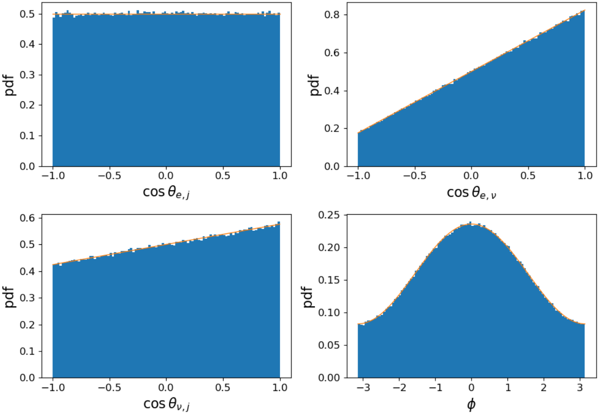

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 1, M_{GT} C_T' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Real $C_V C_T'$

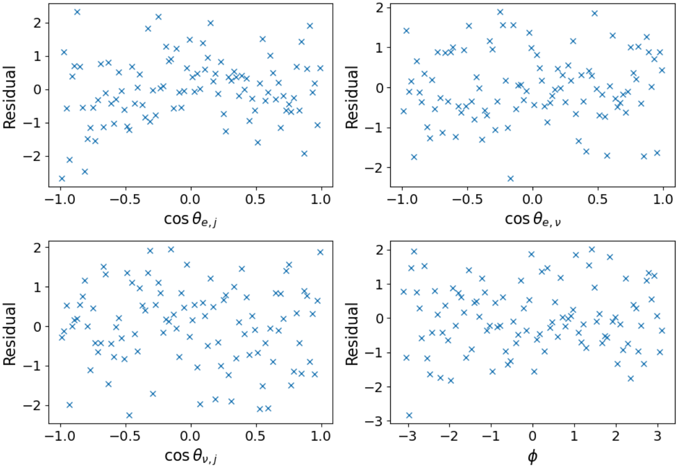

Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_V = 1, M_{GT} C_T' = 1$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_V C_T'$

Non-zero $A < 0$ proportional to $\langle \alpha Z \gamma_e^{-1} \rangle$: hard to see

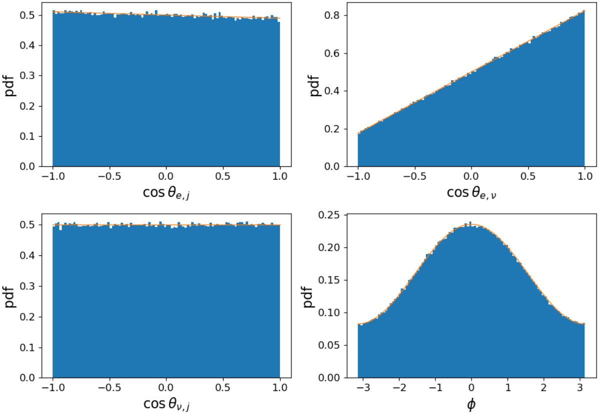

Figure: Distribution of various relevant angles, z_e, z_ν, $z_{e,\nu}$ and ϕ, each with a well-known distribution, and the theoretical value for $M_F C_V = 0.6 - 0.8i, M_{GT} C_T' = 0.8 + 0.6i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Imaginary $C_V C_T'$

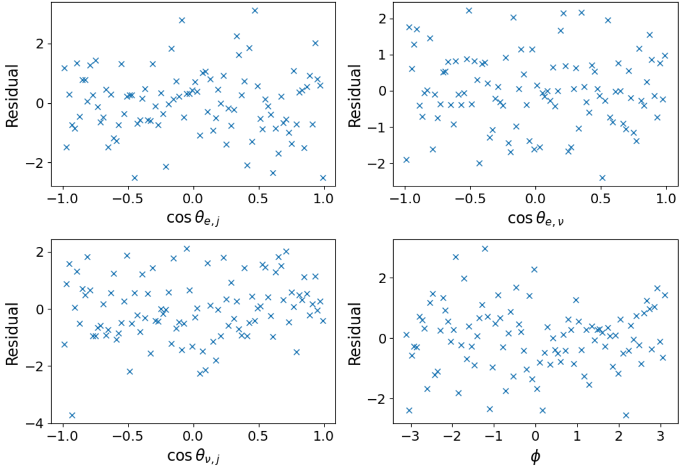

Figure: Residuals from the comparison between CRADLE simulation and theory for z_e, z_ν, $z_{e,\nu}$ and ϕ distributions with $M_F C_V = 0.6 - 0.8i$, $M_{GT} C_T' = 0.8 + 0.6i$ and rest of couplings 0

Mixed Decay: $^{39}\mathrm{Ca}$

Summary

Table: Values of $\chi^2/100$ for each distribution for each of the tests

	$\chi^2(z_e)$	$\chi^2(z_\nu)$	$\chi^2(z_{e,\nu})$	$\chi^2(\phi)$
Re $C_S C_S'$	1.409	1.207	0.840	1.111
Im $C_S C_S'$	0.997	1.109	0.981	1.229
Im $C_S C_T$	1.117	1.219	0.924	1.256
Re $C_S C_T'$	0.975	1.185	0.727	0.908
Re $C_S C_A$	0.820	0.951	0.877	1.057
Re $C_S C_A'$	0.822	1.127	1.046	0.963
Im $C_S C_A'$	1.162	0.905	0.869	1.133
Re $C_S' C_T$	0.891	1.086	0.974	1.003
Im $C_S' C_T'$	0.930	1.179	1.360	1.268
Re $C_S' C_A'$	0.895	0.873	0.893	1.225
Im $C_S' C_A$	1.024	0.898	0.844	0.908
Im $C_S' C_A$	1.121	0.912	0.972	0.733
Re $C_T C_T'$	1.301	1.069	0.966	1.130
Im $C_T C_T'$	1.015	1.087	0.981	1.051

Mixed Decay: $^{39}\mathrm{Ca}$

Summary

Re $C_T C_V$	0.843	0.801	0.952	1.096
Re $C_T C'_V$	0.942	0.911	0.947	1.262
Im $C_T C'_V$	0.832	0.809	1.093	0.877
Re $C'_T C'_V$	0.833	0.882	1.107	1.118
Re $C'_T C_V$	0.933	0.997	0.757	0.962
Im $C'_T C_V$	1.163	1.169	1.017	1.147
Re $C_T C'_A$	0.724	0.832	1.062	1.026
Im $C_T C'_A$	1.181	1.243	1.001	0.711
Re $C_V C'_V$	0.947	0.962	0.957	1.092
Im $C_V C'_V$	0.980	1.304	0.930	1.124
Re $C_A C'_A$	1.089	0.834	0.963	1.239
Im $C_A C'_A$	0.965	1.019	0.841	0.954
Re $C_V C'_A$	0.848	0.992	1.147	0.988
Im $C_V C_A$	1.097	0.817	1.043	1.058
	$\chi^2(z_e)$	$\chi^2(z_\nu)$	$\chi^2(z_{e,\nu})$	$\chi^2(\phi)$

Table: Values of $\chi^2/100$ for each distribution for each of the tests

Plausible scenarios with non-zero D

Looking at D in terms of C_X,C'_X

$$D = 2\delta_{J_i,J_f} M_F M_{GT} \sqrt{\frac{J_i}{J_i+1}} \Big[Im\left(C_S C_T{}^* + C'_S C'_T{}^* - C_V C_A{}^* - C'_V C'_A{}^*\right)$$

$$\mp \frac{\alpha Z m_e}{p_e} Re\left(C_S C_A{}^* + C_S C'_A{}^* + C_T C_V{}^* + C'_T C'_V{}^*\right)\Big]$$

Multiple simple options:

- $C_A = C'_A$ with an imaginary component, $C_V = C'_V = 1$
- Keep $C_A^{(\prime)}$ and $C_V^{(\prime)}$, and add either a $C_S = C'_S$ or a $C_T = C'_T$ coupling with a real component (simpler C_S)
- Keep $C_A^{(\prime)}$ and $C_V^{(\prime)}$, add add $C_S = -C'_S$ and $C_T = -C'_T$ couplings, one real and other imaginary
- Keep $C_A^{(\prime)}$ and $C_V^{(\prime)}$, and add $C_S = C'_S$ and $C_T = C'_T$ couplings, one real and other imaginary (not so simple)

Imaginary $C_V C_A$

Idea: add just a phase shift between C_A and C_V, value θ.

- Global symmetry to a phase shift of all coupling constants $\rightarrow$ C_V chosen 1
- $|C_A|$ untouched $\rightarrow \xi, a, b = 0, c$ constant
- $\delta D \propto \sin(\theta) \approx \theta$ and $\delta D \propto \langle \beta_e \rangle$ (*good*)
- $\delta A, \delta B \propto 1 - \cos(\theta) = O(\theta^2)$

$$A\xi = 2|M_{GT}|^2 \lambda_{JJ'} \left[\pm Re \left(C_T C_T'^* - C_A C_A'^* \right) + \frac{\alpha Z m_e}{p_e} Im \left(C_T C_A'^* + C_T' C_A^* \right) \right]$$

$$+2\delta_{J'J} M_F M_{GT} \sqrt{\frac{J}{J+1}} \left[Re(C_S C_T'^* + C_S' C_T^* - C_V C_A'^* - C_V' C_A^*) \right.$$

$$\left. \pm \frac{\alpha Z m}{p_e} Im \left(C_S C_A'^* + C_S' C_A^* - C_V C_T'^* - C_V' C_T^* \right) \right]$$

Imaginary $C_V C_A$

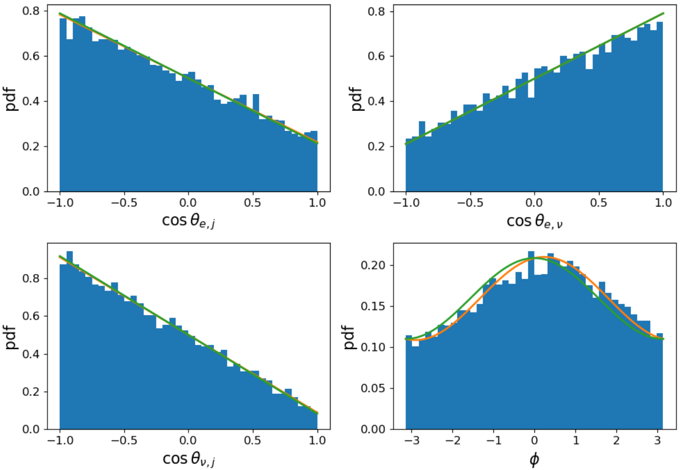

Figure: Distribution of z_e, z_ν, $z_{e,\nu}$ and ϕ for a simulation with $\mathrm{N} = 10000$ decays of ${}^{39}\mathrm{Ca}$ with $C_V = C_V' = 1,\ C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (green) imaginary component of C_A

Imaginary $C_V C_A$

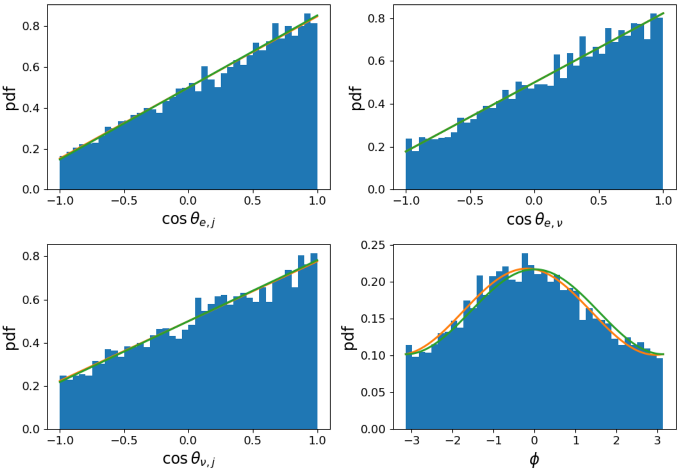

Figure: Distribution of z_e, z_ν, $z_{e,\nu}$ and ϕ for a simulation with N $= 10000$ decays of $^{23}\mathrm{Mg}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (green) imaginary component of C_A

Imaginary $C_V C_A$

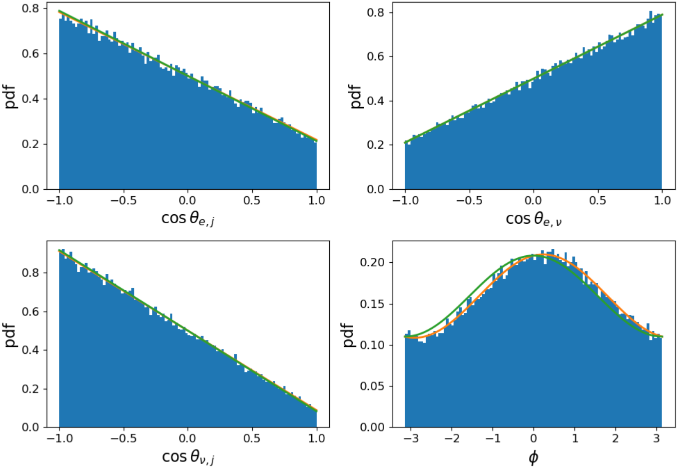

Figure: Distribution of z_e, z_ν, $z_{e,\nu}$ and ϕ for a simulation with N = 100000 decays of $^{39}\mathrm{Ca}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (green) imaginary component of C_A

Imaginary $C_V C_A$

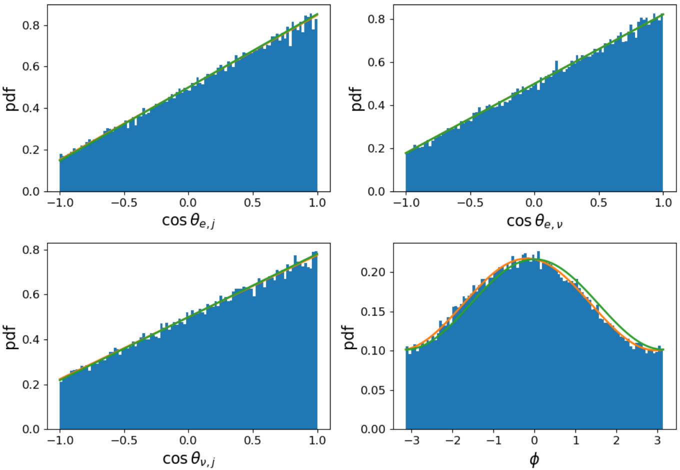

Figure: Distribution of z_e, z_ν, $z_{e,\nu}$ and ϕ for a simulation with N $=$ 100000 decays of $^{23}\mathrm{Mg}$ with $C_V = C'_V = 1$, $C_A = C'_A = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (green) imaginary component of C_A

Imaginary $C_V C_A$

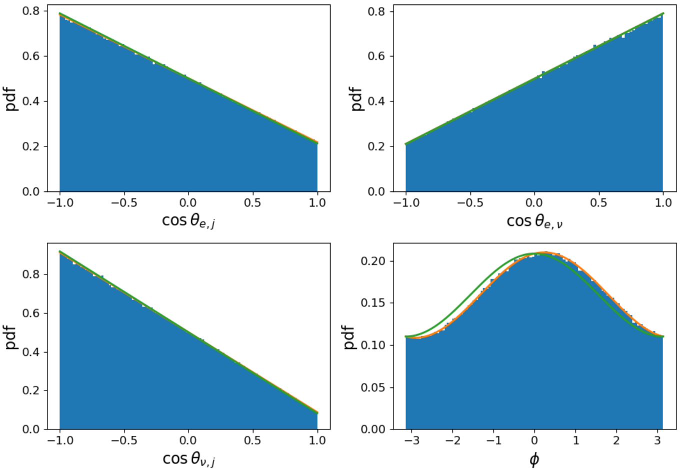

Figure: Distribution of z_e, z_ν, $z_{e,\nu}$ and ϕ for a simulation with N = 1000000 decays of $^{39}\mathrm{Ca}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (green) imaginary component of C_A

Imaginary $C_V C_A$

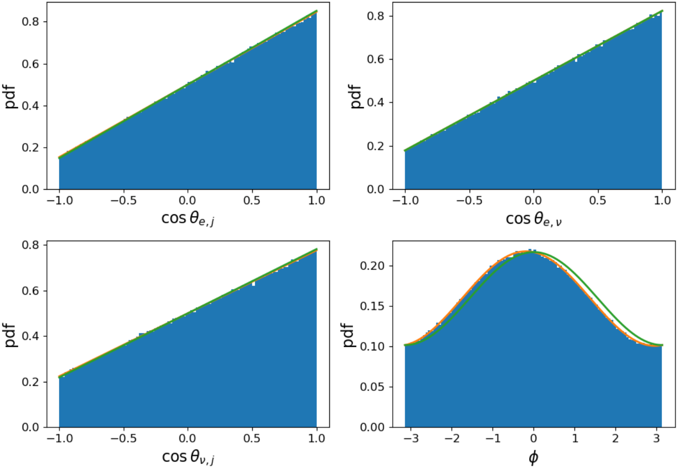

Figure: Distribution of z_e, z_ν, $z_{e,\nu}$ and ϕ for a simulation with N $=$ 1000000 decays of ${}^{23}\mathrm{Mg}$ with $C_V = C_V' = 1$, $C_A = C_A' = \exp(\pi i/9)$ and rest of couplings 0, and theoretical values with (orange) and without (green) imaginary component of C_A

Imaginary $C_V C_A$

For theory: using $|C_V| = |C_A| = 1$ and θ the angle between C_V and C_A

$$D\left\langle \beta_e \right\rangle = \frac{4 M_{GT} \sin\theta \sqrt{\frac{2}{3}}}{2 + 2|M_{GT}|^2} \left\langle \beta_e \right\rangle$$

For experiment: noting $b = 0$, we can relate $N(\phi > 0)$ and $N(\phi < 0)$ to the distribution

$$\frac{N(\phi > 0)}{N} = \int_0^{\pi} f_4(\phi) = \int_0^{\pi} \frac{1 + \left\langle \left(a + \frac{c}{3}\right) \beta_e \right\rangle \frac{\pi^2}{16} \cos\phi + \left\langle D\beta_e \right\rangle \frac{\pi^2}{16} \sin\phi}{2\pi}$$

$$\frac{N(\phi > 0)}{N} = \frac{1}{2} + D\left\langle \beta_e \right\rangle \frac{\pi}{16} \rightarrow D\left\langle \beta_e \right\rangle = \frac{N(\phi > 0) - N(\phi < 0)}{N(\phi > 0) + N(\phi < 0)} \frac{8}{\pi}$$

Imaginary $C_V C_A$

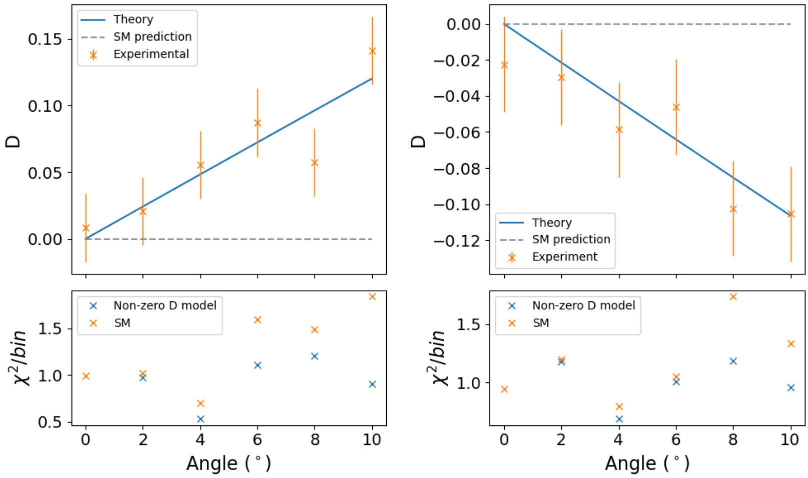

Figure: Experimental values of D and χ^2 of the distribution of ϕ for $^{39}\mathrm{Ca}$ and $^{23}\mathrm{Mg}$ for N $=$ 10000 decays

Imaginary $C_V C_A$

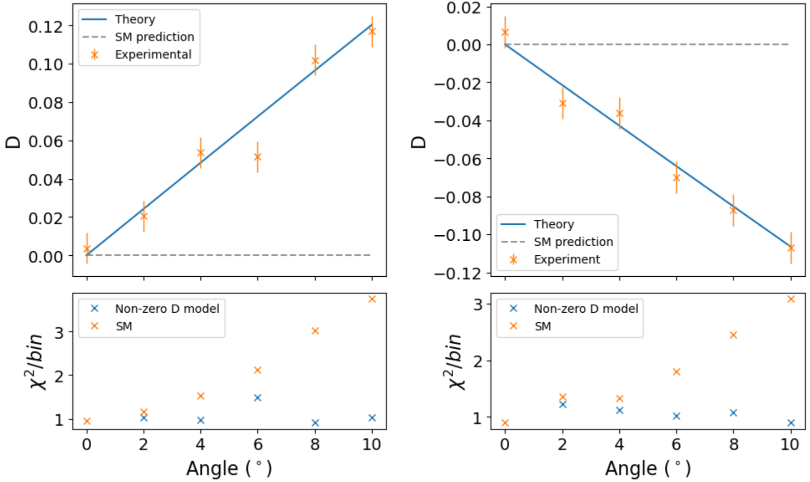

Figure: Experimental values of D and χ^2 of the distribution of ϕ for $^{39}\mathrm{Ca}$ and $^{23}\mathrm{Mg}$ for for N $= 100000$ decays

Imaginary $C_V C_A$

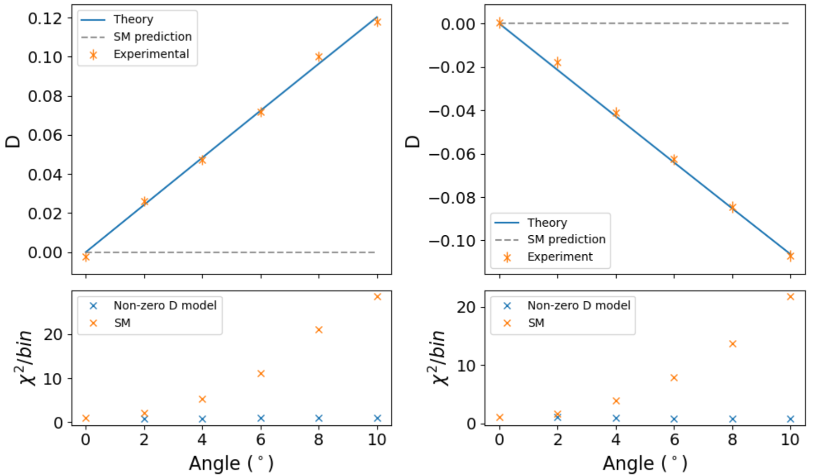

Figure: Experimental values of D and χ^2 of the distribution of ϕ for $^{39}\mathrm{Ca}$ and $^{23}\mathrm{Mg}$ for for N $=$ 1000000 decays

Imaginary $C_A C_V$

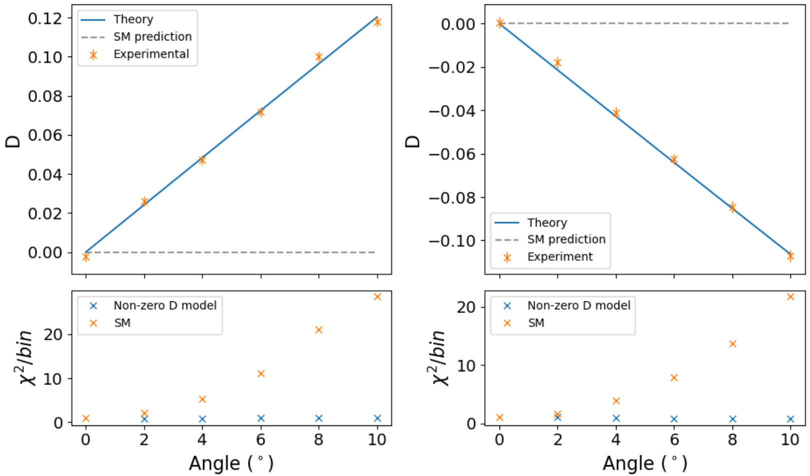

Figure: Experimental values of D and χ^2 of the distribution of ϕ for $^{39}\mathrm{Ca}$ and $^{23}\mathrm{Mg}$ for for N $=$ 1000000 decays