$$V(r) = -g^2 \frac{e^{-\mu r}}{r} + \frac{Zq^2}{r}$$

$$V'(r) = -g^2\left[\frac{-\mu e^{-\mu r} r - e^{-\mu r}}{r^2}\right] - \frac{Zq^2}{r^2} = 0$$

$$= -g^2\left[\frac{-\mu e^{-\mu r}}{r} - \frac{e^{-\mu r}}{r^2}\right] - \frac{Zq^2}{r^2}$$

$$= -g^2\left[-\mu e^{-\mu r} - \frac{e^{-\mu r}}{r}\right] - \frac{Zq^2}{r} = g^2 e^{-\mu r}\left[\mu + \frac{1}{r}\right] - \frac{Zq^2}{r} = 0$$

$$\frac{Zq^2}{g^2} = r e^{-\mu r}\left[\mu + \frac{1}{r}\right] \to \frac{-Zq^2}{g^2 e} = [-\mu r + 1][e^{-\mu r - 1}]$$

$$\cancel{x} = W(x)\, e^{W(x)}$$

$$\to W\left(\frac{-Zq^2}{g^2 e}\right) = [-\mu r - 1]$$

$$\to \frac{W\left(\frac{-Zq^2}{g^2 e}\right) + 1}{-\mu} = r_{max}$$

Sea a $\frac{-Zq^2}{g^2 e} = az$

donde $a = -\frac{q^2}{g^2 e}$

$$V(r) = -g^2 \frac{e^{-\mu r}}{r} + \frac{z g^2}{r}$$

$$\rightarrow V(r_{max}) = -g^2 \frac{e^{-\mu r_{max}}}{r_{max}} + \frac{z g^2}{r_{max}}$$

$$= -g^2 e^{-(W(az)+1)\cdot\mu/\mu} \cdot \frac{-\mu}{W(az)+1} + z g^2 \cdot \frac{-\mu}{W(az)+1} = V(z)$$

para maximizar $V(r)$, maximizo $V(z)$

$$V'(z) = \left(-g^2 - az\, e) \cdot \frac{-\mu}{W(az)+1}\right)' - \left(\frac{z g^2 \mu}{W(az)+1}\right)'$$

Para el primer término:

$-g^2 - aze \cdot \frac{-\mu}{W(az)+1}$ es máxima en el valor

mínimo del dominio de $W(az)$. Sabemos que $a = \frac{-g^2}{g^2} < 0$, y que $z \geq 1$. Entonces $az < 0$. Por probiedad de Lambert, el mínimo de $W(az) = \frac{-1}{e}$.

$kz = -\frac{1}{e} \rightarrow z = \frac{-1}{ae} > 0$.

$z = \frac{-1}{e} \cdot \frac{g^2}{-g^2 e} = 82,18 \approx 82.$

$r\,max = \frac{W(kz)+1}{-\mu} = 0.02342\, eV^{-1}$

5) $$T^{-1} = \frac{1+V_0^2}{4E(E+V_0)} \sin^2\left[\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right]$$

$$\rightarrow T = \frac{4E^2+V_0 E}{1+V_0^2} \sin^2\left[\frac{2a}{\hbar}\sqrt{2m(E+V_0)}\right]$$