6.3 Ejercicios: Probabilidad condicional y total.

1. a). Sea $\mathbb{P}(N)$ la probabilidad de no usar gafas.

$$\mathbb{P}(H) = \sum_{i=1}^{n} \mathbb{P}(H/A_i)\mathbb{P}(A_i) \;\|\; A_1 = N,\ A_2 = G$$

$$\mathbb{P}(H) = \mathbb{P}(H/N)\mathbb{P}(N) + \mathbb{P}(H/G)\mathbb{P}(G)$$

$$\mathbb{P}(H) = \frac{415}{415+285} \cdot \frac{415+285}{1000} + \frac{185}{185+115} \cdot \frac{185+115}{1000}$$

$$\mathbb{P}(H) = \frac{415}{1000} + \frac{185}{1000} = \frac{600}{1000} = \underline{\underline{3/5}}$$

b). $H \cup M = \Omega \rightarrow 1 = \mathbb{P}(\Omega) = \mathbb{P}(H \cup M) = \mathbb{P}(H) + \mathbb{P}(M)$

$$\mathbb{P}(H) + \mathbb{P}(M) = 1 \longrightarrow \mathbb{P}(M) = 1 - \mathbb{P}(H) = 1 - \frac{3}{5} = \underline{\underline{2/5}}$$

$$\mathbb{P}(M) = \mathbb{P}(M/N)\mathbb{P}(N) + \mathbb{P}(M/G_1)\mathbb{P}(G_1)$$

$$\mathbb{P}(M) = \frac{285}{415+285} \cdot \frac{415+285}{1000} + \frac{115}{115+185} \cdot \frac{115+185}{1000}$$

$$\mathbb{P}(M) = \frac{285}{1000} + \frac{115}{1000} = \frac{400}{1000} = \underline{\underline{2/5}}$$

c). $\mathbb{P}(G_1) = \mathbb{P}(G_1/H)\mathbb{P}(H) + \mathbb{P}(G_1/M)\mathbb{P}(M)$

$$\mathbb{P}(G_1) = \frac{185}{185+415} \cdot \frac{3}{5} + \frac{115}{115+285} \cdot \frac{2}{5} = \frac{185}{600} \cdot \frac{600}{1000} + \frac{115}{400} \cdot \frac{400}{1000}$$

$$\mathbb{P}(G) = \frac{185}{1000} + \frac{115}{1000} = \frac{300}{1000} = \underline{\underline{3/10}}$$

d). $P(G/M) = \frac{P(G \cap M)}{P(M)}$ || $P(M) = \frac{2}{5}$

$P(G \cap M)$: Probabilidad de que use gafas y sea mujer

$$P(G \cap M) = \frac{115}{1000}$$

$$P(G/M) = \frac{P(G \cap M)}{P(M)} = \left(\frac{115}{1000}\right) \cdot \left(\frac{5}{2}\right) = \frac{115}{200} \cdot \left(\frac{1}{2}\right) = \frac{115}{400}$$

$$P(G/M) = \frac{115}{200} = \frac{23}{80}$$

2). U_1: Urna #1, U_2: Urna #2, R: bolas rojas, N: bolas negras, V: bolas verdes

a). $P(R) = \sum_{i=1}^{n} P(R/U_i) P(U_i) = P(R/U_1)P(U_1) + P(R/U_2)P(U_2)$

$$P(R) = \frac{3}{3+1+6} \cdot \frac{2}{6} + \frac{6}{6+2+2} \cdot \frac{4}{6} = \frac{1}{10} + \frac{4}{10} = \frac{5}{10}$$

$$P(R) = 1/2$$

b). $P(N) = \sum_{i=1}^{n} P(N/U_i) P(U_i) = P(N/U_1)P(U_1) + P(N/U_2)P(U_2)$

$$P(N) = \frac{1}{3+1+6} \cdot \frac{2}{6} + \frac{2}{6+2+2} \cdot \frac{4}{6} = \frac{1}{30} + \frac{2}{15} = \frac{1}{30} + \frac{4}{30} = \frac{5}{30}$$

$$P(N) = 1/6$$

c). $P(U_1/N) = \frac{P(U_1 \cap N)}{P(N)}$ || $P(N) = 1/6 \rightarrow \frac{1}{P(N)} = 6$

$P(U_1 \cap N) = P(U_1) \cdot P(N/U_1) = \frac{1}{3} \cdot \frac{1}{10}$

$$P(U_1/N) = \frac{1}{P(N)} \cdot P(U_1 \cap N) = (6) \cdot \left(\frac{1}{3} \cdot \frac{1}{10}\right) = \frac{6}{30}$$

$$P(U_1/N) = \frac{1}{5}$$

d). $P(U_2/N) = \frac{P(U_2 \cap N)}{P(N)}$ || $P(N) = \frac{1}{6} \rightarrow \frac{1}{P(N)} = 6$

$P(U_2 \cap N) = P(U_2) \cdot P(N/U_2) = \frac{2}{3} \cdot \frac{1}{5}$

$$P(U_2/N) = \frac{1}{P(N)} P(U_2 \cap N) = (6) \cdot \frac{2}{3} \cdot \frac{1}{5} = (6) \cdot \frac{2}{15} = \frac{12}{15} = \frac{4}{5}$$

$$P(U_2/N) = \frac{4}{5}$$

3). $P(F/F) = \frac{P(F \cap F)}{P(F)}$ || $P(F/F)$: Probabilidad de que se de F dado que se dio F anteriormente. ($P(F/F) = \frac{2-1}{5-1} = \frac{2}{4} = \frac{1}{2}$)

$P(F)$: Probabilidad de que se de F la primera vez. ($P(F) = \frac{3}{5}$)

$P(F \cap F) = P(F) P(F/F)$

$$P(F \cap F) = \left(\frac{3}{5}\right) \cdot \left(\frac{1}{2}\right) = \frac{3}{10}$$