8. $\Omega = \{(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2))\}$

a. $\sum_{i=0}^{n} f(x_i) L(x_i) = P_n(x)$, $L_i(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}$

$$P(x) = \underbrace{f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}}_{Pt1} + \underbrace{f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}}_{Pt2} + \underbrace{f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}}_{Pt3}$$

b. $\frac{d}{dx} P(x)\Big]_{x_0}$ ¡Sea: $x_1 = x_0 + h$, $x_2 = x_0 + 2h$ Para discretización equidistante.

Parte 1: $\frac{d}{dx}\left[f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}\right]_{x_0} = \left[f(x_0)\frac{(x-x_2)+(x-x_1)}{(x_0-x_1)(x_0-x_2)}\right]_{x_0}$

$$= f(x_0)\frac{(x_0-x_2)+(x_0-x_1)}{(x_0-x_1)(x_0-x_2)} = f(x_0)\frac{-3h}{2h^2} = -3f(x_0)\cdot\frac{1}{2h}$$

Parte 2: $\frac{d}{dx}\left[f(x_1)\frac{(x-x_0)\cdot(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right]_{x_0} = \left[f(x_1)\frac{(x-x_0)+(x-x_2)}{(x_1-x_0)(x_1-x_2)}\right]_{x_0}$

$$= f(x_1)\frac{x_0-x_2}{(x_1-x_0)(x_1-x_2)} = f(x_0+h)\frac{-2h}{-h^2} = 2f(x_0+h)\frac{1}{h} = 4f(x_0+h)\cdot\frac{1}{2h}$$

Parte 3: $\frac{d}{dx}\left[f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right]_{x_0} = \left[f(x_2)\frac{(x-x_0)+(x-x_1)}{(x_2-x_0)(x_2-x_1)}\right]_{x_0}$

$$= f(x_2)\frac{x_0-x_1}{(x_2-x_0)(x_2-x_1)} = f(x_0+2h)\frac{-h}{2h^2} = -f(x_0+2h)\cdot\frac{1}{2h}$$

$\hookrightarrow f'(x_0) \approx \frac{4f(x_0+h) - 3f(x_0) - f(x_0+2h)}{2h}$ → discretización equidistante.

$\hookrightarrow f'(x_0) \approx \frac{4f(x_1) - 3f(x_0) - f(x_2)}{2h}$

e. $f(x) = \sqrt{\tan(x)}$, $f'(x) = \dfrac{\sec^2(x)}{2\sqrt{\tan(x)}} = \dfrac{1}{2 \cdot \cos^2(x) \cdot \sqrt{\tan x}}$

$f'(x) = Df(x) = \dfrac{1}{2 \cdot \cos^2(x) \cdot \sqrt{\tan(x)}}$ → Función de derivada usada para derivada exacta.