

Punto #3. Hacer pasos intermedios para encontrar la regla de Simpson Simple. (Ecuación 4.107)

Ecuación 4.107: $I = \int_a^b f(x) dx = \int_a^b P_2(x) dx = \frac{h}{3} (f(a) + 4f(x_m) + f(b))$
 cambiar $[f(x)]$ por polinomio interpolador $[P_2(x)]$ con el conjunto de soporte $[\Omega = \{(a, f(a)), (b, f(b)), (x_m, f(x_m))\}]$.
 $[h = \frac{b-a}{2}]$ y $[x_m = \frac{a+b}{2}]$. $P_2(x) = Ax^2 + Bx + C$

$$f(a) = P_2(a) = Aa^2 + Ba + C \quad // \quad f(b) = P_2(b) = Ab^2 + Bb + C$$

$$f(x_m) = P_2(x_m) = Ax_m^2 + Bx_m + C$$

$$I = \int_a^b f(x) dx = \int_a^b P_2(x) dx = A \int_a^b x^2 dx + B \int_a^b x dx + C \int_a^b 1 dx$$

$$= \frac{A(b^3 - a^3)}{3} + \frac{B(b^2 - a^2)}{2} + C(b - a) = \frac{(b-a)}{6} [2A(b^2 + ba + a^2) + 3B(b+a) + 6C]$$

$$= \frac{h}{3} [2Ab^2 + 2Aba + 2Aa^2 + 3Bb + 3Ba + 6C]$$

$$= \frac{h}{3} [\underbrace{Aa^2 + Ba + C}_{f(a)} + \underbrace{Ab^2 + Bb + C}_{f(b)} + Ab^2 + Aa^2 + 2Aba + 2Bb + 2Ba + 4C]$$

$$= \frac{h}{3} [f(a) + f(b) + A(b^2 + 2ba + a^2) + 2B(b+a) + 4C] \quad // \quad \begin{aligned} x_m &= \frac{a+b}{2} \\ 4x_m &= 2(a+b) \\ x_m^2 &= \frac{(a+b)^2}{4} \\ 4x_m^2 &= (a+b)^2 \end{aligned}$$

$$= \frac{h}{3} [f(a) + f(b) + A(a+b)^2 + B(2(a+b)) + 4C]$$

$$= \frac{h}{3} [f(a) + f(b) + \underbrace{4Ax_m^2 + 4Bx_m + 4C}_{4f(x_m)}]$$

$$\hookrightarrow I = \frac{h}{3} [f(a) + 4f(x_m) + f(b)]$$