Punto #21. (a).

$$f(x)=\sum_{n=0}^{N} C_n P_n(x) \rightarrow f(x)P_m(x)=\sum_{n=0}^{N} C_n P_n(x)P_m(x)$$

$$\int_{-1}^{1} f(x)P_m(x)\,dx = \int_{-1}^{1}\sum_{n=0}^{N} C_n P_n(x)P_m(x)\,dx = \sum_{n=0}^{N}\int_{-1}^{1} C_n P_n(x)P_m(x)\,dx$$

$$= \sum_{n=0}^{N} C_n \int_{-1}^{1} P_n(x)P_m(x)\,dx$$ //

Por definición $\int_{-1}^{1} P_n(x)P_m(x)\,dx = \frac{2}{2n+1}\delta_{nm} \rightarrow \begin{cases}\delta_{nm}=1, & m=n\\ \delta_{nm}=0, & m\neq n\end{cases}$

↳ todos los elementos de la suma se van a ves multiplicados por cero menos cuando [n=m].

↳ $\int_{-1}^{1} f(x)P_m(x)\,dx = C_m\frac{2}{2m+1} \rightarrow \underline{C_m = \frac{2m+1}{2}\int_{-1}^{1} f(x)P_m(x)\,dx}$

↳ $m=0,1,\ldots,N$