6) Para minimizar $\chi^2(a_0, a_1)$, se calculan $\partial\chi^2/\partial a_0$ y $\partial\chi^2/\partial a_1$, para igualarlos a cero y hallar los mínimos.

· Para a_0:

$$\Rightarrow \frac{\partial \chi^2}{\partial a_0} = \frac{\partial}{\partial a_0}\sum_{i=1}^{n}(y_i-(a_0+a_1x_i))^2 = \sum_{i=1}^{n}\frac{\partial}{\partial a_0}(y_i-(a_0+a_1x_i))^2$$

$$= \sum_{i=1}^{n} 2(y_i-(a_0+a_1x_i))\cdot -1 = -2\sum_{i=1}^{n}(y_i-(a_0+a_1x_i))$$

$$\Rightarrow 0 = \sum_{i=1}^{n}(y_i-(a_0+a_1x_i)) = \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} a_0 - \sum_{i=1}^{n} a_1 x_i$$

$$= \sum_{i=1}^{n} y_i - n a_0 - a_1\sum_{i=1}^{n} x_i$$

$$\Rightarrow n a_0 = \sum_{i=1}^{n} y_i - a_1\sum_{i=1}^{n} x_i \Rightarrow a_0 = \frac{\sum_{i=1}^{n} y_i}{n} - \frac{a_1}{n}\sum_{i=1}^{n} x_i$$

$$\Rightarrow a_0 = \bar{y} - a_1\bar{x}$$

· Para a_1:

$$\Rightarrow \frac{\partial \chi^2}{\partial a_1} = \frac{\partial}{\partial a_1}\sum_{i=1}^{n}(y_i-(a_0+a_1x_i))^2 = \sum_{i=1}^{n}\frac{\partial}{\partial a_1}(y_i-(a_0+a_1x_i))^2$$

$$= \sum_{i=1}^{n} 2(y_i-(a_0+a_1x_i))\cdot x_i = 2\sum_{i=1}^{n}(y_ix_i - a_0x_i - a_1x_i^2)$$

$$\Rightarrow 0 = \sum_{i=1}^{n}(y_ix_i - a_0x_i - a_1x_i^2) = \sum_{i=1}^{n} y_ix_i - \sum_{i=1}^{n} a_0x_i$$

$$- \sum_{i=1}^{n} a_1x_i^2 = \sum_{i=1}^{n} y_ix_i - a_0\sum_{i=1}^{n} x_i - a_1\sum_{i=1}^{n} x_i^2$$

$$= \sum_{i=1}^{n} y_ix_i - (\bar{y} - a_1\bar{x})\sum_{i=1}^{n} x_i - a_1\sum_{i=1}^{n} x_i^2$$

$$= \sum_{i=1}^{n} y_ix_i - \frac{1}{n}\sum_{i=1}^{n} y_i\sum_{i=1}^{n} x_i + a_1\frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 - a_1\sum_{i=1}^{n} x_i^2$$

$$= \sum_{i=1}^{n} y_i x_i - \frac{1}{n} \sum_{i=1}^{n} y_i \sum_{i=1}^{n} x_i - a_1 \left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n} \left(\sum_{i=1}^{n} x_i \right)^2 \right]$$

$$= \sum y_i x_i - \frac{\sum y_i \sum x_i}{n} - a_1 \left[\sum x_i^2 - \frac{(\sum x_i)^2}{n} \right]$$

$$\Rightarrow a_1 = \frac{\sum y_i x_i - \frac{1}{n} \sum y_i x_i}{\sum x_i^2 - \frac{1}{n} (\sum x_i)^2}$$

* Se hace lo mismo para $\chi^2 (a_0, a_1, a_2)$

- Para a_0:

$$\Rightarrow \frac{\partial \chi^2}{\partial a_0} = \frac{\partial}{\partial a_0} \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2$$

$$= \sum_{i=1}^{n} \frac{\partial}{\partial a_0} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2 = -2 \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))$$

$$\Rightarrow 0 = \sum_{i=1}^{n} y_i - (a_0 + a_1 x_i + a_2 x_i^2) = \sum_{i=1}^{n} y_i - \sum_{i=1}^{n} a_0$$

$$- \sum_{i=1}^{n} a_1 x_i - \sum_{i=1}^{n} a_2 x_i^2 \Rightarrow \sum_{i=1}^{n} [y_i = a_0 + a_1 x_i + a_2 x_i^2]$$

- Para a_1:

$$\Rightarrow \frac{\partial \chi^2}{\partial a_1} = \frac{\partial}{\partial a_1} \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2))^2 = \sum_{i=1}^{n} \frac{\partial}{\partial a_1} (y_i$$

$$- (a_0 + a_1 x_i + a_2 x_i))^2 = -2 \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2)) x_i$$

$$\Rightarrow 0 = \sum_{i=1}^{n} (y_i x_i - a_0 x_i - a_1 x_i^2 - a_2 x_i^3)$$

$$\Rightarrow \sum_{i=1}^{n} [y_i x_i = a_0 x_i + a_1 x_i^2 + a_2 x_i^3]$$

- Para a_2:

$$\Rightarrow \frac{\partial \chi^2}{\partial a_2} = \frac{\partial}{\partial a_2} \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i))^2 = \sum_{i=1}^{n} \frac{\partial}{\partial a_2} (y_i$$

$$- (a_0 + a_1 x_i + a_2 x_i^2))^2 = -2 \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2)) x_i^2$$

$$\Rightarrow 0 = \sum_{i=1}^{n} (y_i - (a_0 + a_1 x_i + a_2 x_i^2)) x_i^2$$

$$\Rightarrow \sum_{i=1}^{n} [y_i x_i^2 = a_0 x_i^2 + a_1 x_i^3 + a_2 x_i^4]$$