

6.5 Ejercicios: Teorema de Bayes

$$P(A_m/B) = \frac{P(B/A_m) P(A_m)}{\sum_{i=1}^n P(B/A_i) P(A_i)}$$

1.

$$a). P(M) = \sum_{i=1}^n P(M/F_i) P(F_i) = P(M/F_1) P(F_1) + P(M/F_2) P(F_2)$$

$$F_1: \text{no Fuma}, P(F_1) = \frac{6}{10} = \frac{3}{5}, F_2: \text{Fuma}, P(F_2) = \frac{4}{10} = \frac{2}{5}$$

$$P(M/F_1) = \frac{6}{10} = \frac{3}{5}, P(M/F_2) = \frac{25}{100} = \frac{1}{4}$$

$$P(M) = \left(\frac{3}{5}\right) \left(\frac{3}{5}\right) + \left(\frac{1}{4}\right) \left(\frac{2}{5}\right) = \frac{9}{25} + \frac{1}{10} = \frac{90}{250} + \frac{25}{250} = \frac{115}{250}$$

$$P(M) = \frac{115}{250} = \frac{23}{50}$$

$$b). P(H/F_2) = \frac{P(H \cap F_2)}{P(F_2)} = \frac{P(F_2 \cap H)}{P(F_2)}$$

$$P(F_2 \cap H) = P(H/F_2) P(F_2) \parallel P(H/F_2) = \frac{25}{100} = \frac{1}{4}$$

$$P(F_2) = \frac{4}{10} = \frac{2}{5}$$

$$P(F_2 \cap H) = \left(\frac{1}{4}\right) \left(\frac{2}{5}\right) = \frac{2}{20} = \frac{1}{10}$$

$$c). P(F_2/M) = \frac{P(M/F_2) P(F_2)}{\sum_{i=1}^n P(M/F_i) P(F_i)} = \frac{P(M/F_2) P(F_2)}{P(M)}$$

$$P(F_2) = \frac{2}{5}, P(M) = \frac{23}{50}, P(M/F_2) = \frac{25}{100} = \frac{1}{4}$$

$$P(F_2/M) = \frac{\left(\frac{1}{4}\right) \left(\frac{2}{5}\right)}{\left(\frac{23}{50}\right)} = \frac{50}{23} \cdot \frac{2}{20} = \frac{5}{23} \cdot \frac{2}{2} = \frac{5}{23}$$