5.7.1

• Parte #1(a): $x_{n+1} = 4x_n - x_n^2, \quad x_0 = 4\,\text{Sen}^2\theta$

Caso base:

$$X_1 = 4\,\text{Sen}^2\theta\,(4 - 4\,\text{Sen}^2\theta) = 16\,\text{Sen}^2\theta\,\cos^2\theta$$

$\text{Sen}(2\varphi) = 2\,\text{Sen}\,\varphi\,\cos\varphi, \quad \text{Sen}^2(2\varphi) = 4\,\text{Sen}^2(\varphi)\,\cos^2(\varphi)$

$$= 4\,\text{Sen}^2(2\theta) = 4\,\text{Sen}^2(2^{0+1}\theta)$$

Paso inductivo:

Si $[x_n = 4\,\text{Sen}^2(2^n\theta)]$ es cierto, entonces:

$$x_{n+1} = 4x_n - x_n^2 = x_n(4 - x_n) = 4\,\text{Sen}^2(2^n\theta)(4 - 4\,\text{Sen}^2(2^n\theta))$$

$$= 16\,\text{Sen}^2(2^n\theta)\,(1 - \text{Sen}^2(2^n\theta)) = 16\,\text{Sen}^2(2^n\theta)\,\cos^2(2^n\theta)$$

$\text{Sen}(2\varphi) = 2\,\text{Sen}(\varphi)\cos(\varphi) \rightarrow \text{Sen}^2(2\varphi) = 4\,\text{Sen}^2(\varphi)\cos^2(\varphi) \rightarrow 16\,\text{Sen}^2(\varphi)\cos^2(\varphi) = 4\,\text{Sen}^2(2\varphi)$

$$\hookrightarrow = 4\,\text{Sen}^2(2[2^n\theta]) = \underline{4\,\text{Sen}^2(2^{n+1}\theta)} \quad \blacksquare$$

• Parte #2(b): $x_{n+1} = 4x_n - 4x_n^2, \quad x_0 = \text{Sen}^2\theta$

Caso base:

$$x_1 = 4\,\text{Sen}^2\theta\,(1 - \text{Sen}^2\theta) = 4\,\text{Sen}^2\theta\,\cos^2\theta = \text{Sen}^2(2\theta)$$

$$= \text{Sen}^2(2^{0+1}\theta)$$

Paso inductivo:

Si $[x_n = \text{Sen}^2(2^n\theta)]$ es cierto, entonces:

$$x_{n+1} = 4x_n - 4x_n^2 = 4x_n(1 - x_n) = 4\,\text{Sen}^2(2^n\theta)\,(\underbrace{1 - \text{Sen}^2(2^n\theta)}_{\cos^2(2^n\theta)})$$

$$= 4\,\text{Sen}^2(2^n\theta)\,\cos^2(2^n\theta) = \text{Sen}^2(2[2^n\theta]) = \underline{\text{Sen}^2(2^{n+1}\theta)} \quad \blacksquare$$