

$$\text{Sumatoria}(n) = \sum_{i=1}^n i =$$

$$1 + 2 + 3 + \dots + (n-1) + n$$

$\text{Sumatoria}(n-1)$

$$\text{Sumatoria}(0) = \sum_{i=1}^0 i = 0$$

$$\text{Sumatoria}(n) = n(n+1)/2$$

$$f_1(n) = \sum_{i=0}^n 2^i =$$

$$2^0 + 2^1 + \dots + 2^{n-1} + 2^n$$

$f_1(n-1)$

$$f_1(0) = \sum_{i=0}^0 2^i = 2^0 = 1$$

$$f_1(n) = \sum_{i=0}^n 2^i = 2^{n+1} - 1$$

$$f_2(n, q) = \sum_{i=1}^n q^i =$$

$$q^1 + q^2 + \dots + q^{n-1} + q^n$$

$f_2(n-1, q)$

$$f_2(0, n) = \sum_{i=1}^0 q^i = 0$$

$$f_3(n, q) = \sum_{i=1}^{2n} q^i =$$

$$q^1 + q^2 + \dots + q^{2n-2} + q^{2n-1} + q^{2n}$$

$f_3(n-1, q)$

$$f_3(0, q) = 0$$

Otra forma $f_3(n, q) = f_2(2 \cdot n, q)$

$$f_4(n, q) = \sum_{i=n}^{2n} q^i =$$

$$q^n + q^{n+1} + \dots + q^{2n-2} + q^{2n-1} + q^{2n}$$

$f_4(n-1, q)$

$$f_4(0, q) = \sum_{i=0}^0 q^i = q^0 = 1$$

$$f_4(n, q) = \sum_{i=n}^{2n} q^i =$$

$$(q^1 + q^2 + \dots + q^{n-1} + q^n + q^{n+1} + \dots + q^{2n-2} + q^{2n-1} + q^{2n}) - (q^1 + \dots + q^{n-1})$$

$f_3(n, q) - f_2(n-1, q)$

$$\hat{e}(n) = \sum_{i=0}^n \frac{1}{i!} =$$

$$\frac{1}{0!} + \frac{1}{1!} + \dots + \frac{1}{(n-1)!} + \frac{1}{n!}$$

$\hat{e}(n-1)$

$$\hat{e}(0) = \sum_{i=0}^0 \frac{1}{0!} = 1$$

$$f(n, m) = \begin{matrix} 1^1 + 1^2 + \dots + 1^m + \\ 2^1 + 2^2 + \dots + 2^m + \\ \vdots \\ (n-1)^1 + (n-1)^2 + \dots + (n-1)^m + \\ n^1 + n^2 + \dots + n^m \end{matrix}$$

$f(n-1, m)$
+
 $f_2(n, m)$

$$f(0, m) = 0$$

$$SP(q, m, m) = \sum_{a=1}^m \sum_{b=1}^m q^{a+b} =$$

$$\begin{matrix} q^{1+1} + q^{1+2} + \dots + q^{1+(m-1)} + q^{1+m} \\ q^{2+1} + q^{2+2} + \dots + q^{2+(m-1)} + q^{2+m} \\ \vdots \\ q^{m+1} + q^{m+2} + \dots + q^{m+(m-1)} + q^{m+m} \end{matrix}$$

$SP(q, m, m-1)$

$$SP(q, m, 0) = 0$$

$$\sum_{i=1}^m q^{i+m} = q^m \sum_{i=1}^m q^i = q^m f_2(m, q)$$

$$SR(m, m) = \sum_{p=1}^m \sum_{q=1}^m p/q =$$

$$\begin{matrix} \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{m-1} + \frac{1}{m} \\ \frac{2}{1} + \frac{2}{2} + \dots + \frac{2}{m-1} + \frac{2}{m} \\ \vdots \\ \frac{m}{1} + \frac{m}{2} + \dots + \frac{m}{m-1} + \frac{m}{m} \end{matrix}$$

$\sum_{i=1}^m \frac{i}{m} = \frac{1}{m} \sum_{i=1}^m i = \frac{\text{Sumatoria}(m)}{m}$

$$SR(m, 0) = 0$$

$$SR(m, m-1)$$

$$\text{Sumatoria}(m)$$