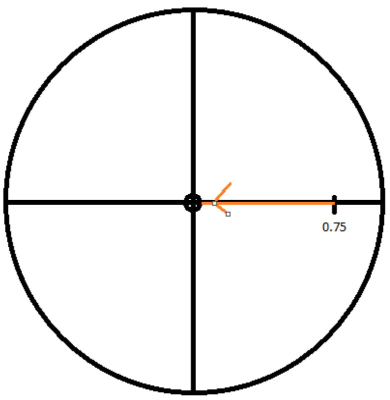
Recu 2020-2021

2C-3



(a)

(b)

Forma 1:

$$G_{fb}(z)=\frac{Kz}{z-0.75+Kz}\rightarrow\frac{Kz}{(1+K)z-0.75}$$

$$T_s=0.05\,s\rightarrow\sigma=\frac{4}{0.05}=80\rightarrow\rightarrow z_p=e^{-80*0.01}=0.45$$

$$0.45+0.45K-0.75=0\rightarrow K=\frac{0.3}{0.45}=0.67$$

Forma 2:

$$1+KG(z)=0\rightarrow KG(z)=-1\rightarrow\begin{array}{c}|KG(z)|=1\\ \sphericalangle G(z)=(2i+1)\pi\end{array}$$

$$K|G(z)|=K\frac{m_z}{m_p}\rightarrow K\frac{0.45}{0.75-0.45}=1\rightarrow K=\frac{0.3}{0.45}=0.67$$

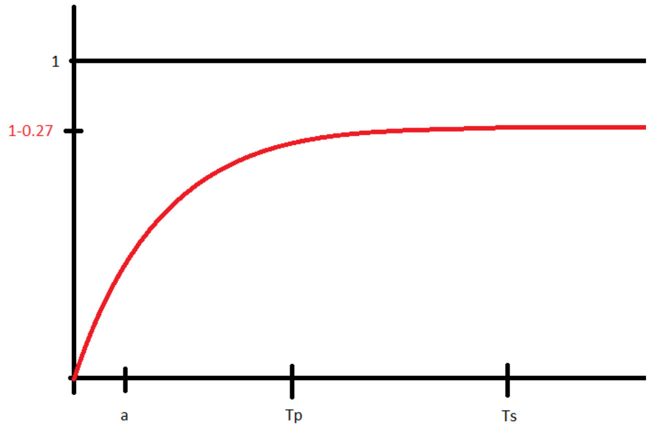
(c) $K_p=\lim_{z\rightarrow 1}\frac{0.67z}{z-0.75}=\frac{0.67}{0.25}=2.68\rightarrow\rightarrow e_s(\infty)=\frac{1}{1+2.68}=0.27$

(d) $H(z)=G_{fb}(z)*\frac{z}{z-1}=\frac{k_0}{z-1}+\frac{k_1}{(1+K)z-0.75}$

$c(t)=[k_0+k_1e^{-80*0.01}]u(t)$

$c[n]=[k_0+k_1*0.45^n]u[n]$

(e)



(f)

$G_{fb}(z)=\frac{Kz}{z-0.75+Kz}\rightarrow\frac{Kz}{(z-1)((1+K)z-0.75)}$

Prueba 2 2021-2022

3B-1

(a) $G_{OL}(s)=\frac{K}{s(s+5)(s+6)}$

(b) $G_I(s)=\frac{K}{s}\quad G(s)=\frac{1}{(s+5)(s+6)}$

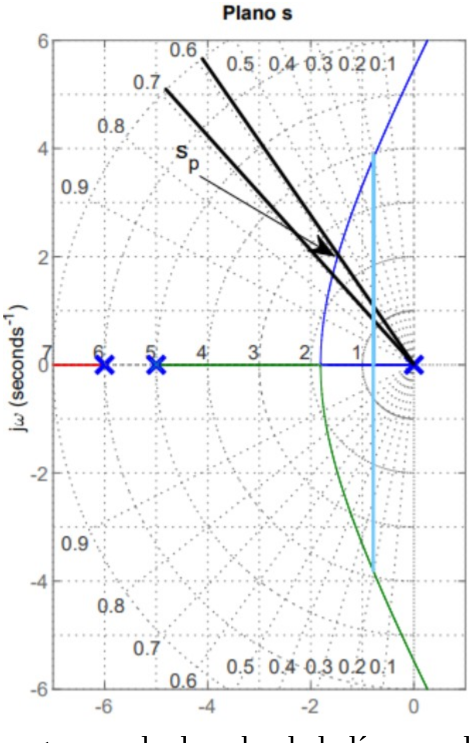
(c) $T_{sMAX}=5s=\frac{4}{\sigma_d}\rightarrow\sigma_d\geq 0.8$

$OS\%=[5;10]=100e^{-\pi\zeta/\sqrt{1-\zeta^2}}\rightarrow\ln\left(\frac{[5;10]}{100}\right)=\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}\rightarrow\ln\left(\frac{[5;10]}{100}\right)^2-\ln\left(\frac{[5;10]}{100}\right)^2\zeta^2=\pi^2\zeta^2\rightarrow$

$\ln\left(\frac{[5;10]}{100}\right)^2=\left(\ln\left(\frac{[5;10]}{100}\right)^2+\pi^2\right)\zeta^2\rightarrow\zeta=\sqrt{\ln\frac{\left(\frac{[5;10]}{100}\right)^2}{\pi^2+\ln\left(\frac{[5;10]}{100}\right)^2}}\rightarrow\begin{matrix}\zeta_5=0.69\\ \zeta_{10}=0.59\end{matrix}$

$\omega_{n5}=\frac{0.8}{0.69}=1.16\; OS\%=5\rightarrow s_{1,2}=-0.8\pm 0.84j$

$\omega_{n10}=\frac{0.8}{0.59}=1.36\; OS\%=10\rightarrow s_{1,2}=-0.8\pm 1.1j$



Sí habrá siempre que el polo se encuentre entre las líneas negras y a la derecha de la línea azul claro

(d) $T_s=1s=\frac{4}{\sigma_d}\rightarrow\sigma_d=4$

No puedes asegurar un polo en 4 porque no sería el dominante en ningún caso ya que hay un polo más pequeño que él

(e)

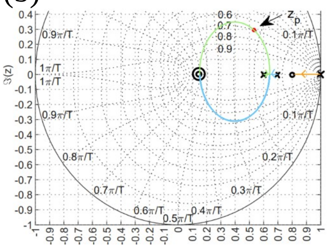
$$|G_{OL}(s_p)|=1 \rightarrow \rightarrow K=m_{p1}*m_{p2}*m_{p3}=\sqrt{1.5^2+2^2}*\sqrt{(5-1.5)^2+2^2}*\sqrt{(6-1.5)^2+2^2}=49.5$$

3B-2

$$T_m=\frac{1}{10}=0.1\,s \quad G_{OL}(z)=\frac{0.12\,z^2(z-0.8)K}{(z-0.6)(z-0.7)(z-1)}$$

(a) Debe tener su conjugado en z_p= 0.5-j0.3 y $\begin{array}{c} |G_{pi}(z)G(z)|=1 \\ \measuredangle(G_{pi}(z)G(z))=(2i+1)\pi \text{ con } i\in R \end{array}$

(b)



5B-5

$$G_{pd}=K\frac{z-a}{z} \quad G_p(z)=\frac{z^2}{(z-0.5)(z-0.7)} \quad z_{CL}=0.39\pm 0.17\,j \quad T_m=0.1\,s$$

$$0.1=\frac{\pi}{\omega_d}\rightarrow \omega_d=10\pi$$

$$\theta_{OL}(z=z_{CL})=2\theta_{c1}-(\theta_{p1}+\theta_{p2})=2atan(\frac{0.17}{0.39})-(atan(\frac{0.17}{0.39-0.5})+atan(\frac{0.17}{0.39-0.7}))=2.32\,rad$$

$$\theta_p=atan(\frac{0.17}{0.39})=0.41\,rad$$

$$\theta_a=\pi-\theta_{OL}+\theta_p=\pi-2.33+0.41=1.22\,rad$$

$$a=0.39-\frac{0.17}{\tan(1.22)}=0.33$$

$$|G_{OL}|=1\rightarrow K\frac{m_{ca}*m_{c1}^2}{m_{pa}*m_{p1}*m_{p2}}=1\rightarrow K=\frac{m_{pa}*m_{p1}*m_{p2}}{m_{ca}*m_{c1}^2}\rightarrow\rightarrow$$

$$\rightarrow\rightarrow K=\frac{\sqrt{0.39^2+0.17^2}*\sqrt{(0.39-0.5)^2+0.17^2}*\sqrt{(0.39-0.7)^2+0.17^2}}{\sqrt{(0.39-0.33)^2+0.17^2}*(\sqrt{0.39^2+0.17^2})^2}=0.93$$

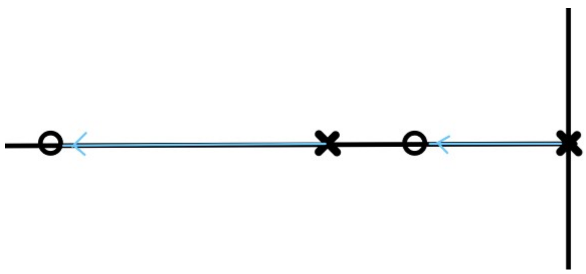
$$G_{pd}=0.93\frac{z-0.33}{z}$$

Recu 2020-2021

2C-1

$$G(s)=\frac{150}{(s+3)(s+5)(s+10)} \quad G_{PID}(s)=K\frac{(s+6)(s+2)}{s}$$

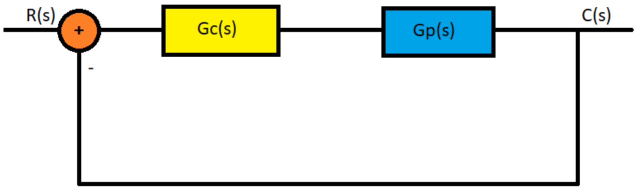
$$G_{OL}(s)=\frac{150K(s+6)(s+2)}{s(s+3)(s+5)(s+10)}\rightarrow\rightarrow \text{polos dominantes}\rightarrow\rightarrow\frac{150K(s+6)(s+2)}{s(s+3)}$$



(b)
a)
b)
c)

2C-2

(a) $G_{OL}(z)=\frac{K(z+0.5)(z-0.8)}{(z-1)(z-0.7)(z-0.2)}$



(b)
(c) 2 ceros y 3 polos
(d) $k_1 < K < k_2$
(e) $0 < K < k_1$ porque a partir de k_1 hay polos complejos conjugados
(f) $K_v = \lim_{z \to 1} \frac{1}{T}(z-1) G_{OL}(z) = \frac{1.5*0.2K}{0.3*0.8T} = \frac{0.3K}{0.24T} = 1.25\frac{K}{T}$

$$e_s(\infty) = \frac{1}{K_v} = \frac{1}{\frac{5K}{4T}} = \frac{4T}{5K}$$

5A-2 → EZ

5A-3 → Matlab
5A-4

$$G_{fb}(s) = \frac{K}{s^2 + 12s + 20 + K}$$

(a) $OS\% = 3 = 100e^{-\pi\zeta/\sqrt{1-\zeta^2}} \rightarrow \zeta = \sqrt{\frac{\ln\left(\frac{3}{100}\right)^2}{\pi^2 + \ln\left(\frac{3}{100}\right)^2}} = 0.745$

$$12 = 2*0.745*\omega_n \rightarrow \omega_n = 8.05$$
$$20 + K = \omega_n^2 = 64.8 - 20 = 44.8 \approx 45$$

$$T_s = \frac{4}{\sigma_d} = \frac{4}{\zeta\omega_n} = \frac{4}{0.745*8.05} = \frac{4}{6} = 0.67\,s \quad T_p = \frac{\pi}{\omega_d} = \frac{\pi}{8.05\sqrt{1-0.745^2}} = 0.59\,s$$
$$G_{OL}(s) = \frac{K}{(s+2)(s+10)}$$
$$K_p = \lim_{s \to 0} G_{OL}(s) = \frac{45}{20} = 2.25$$
$$e_s(\infty) = \frac{1}{1+K_p} = \frac{1}{3.25} = 0.31$$

(b) $T_p = 0.3\,s = \frac{\pi}{\omega_d} \rightarrow \omega_d = \frac{\pi}{0.3} = 10.47$

$$12 = 2\zeta\omega_n \rightarrow \omega_n = \frac{6}{\zeta} \rightarrow\rightarrow\rightarrow \omega_n = \frac{6}{0.49} = 12.24$$

$$\omega_n\sqrt{1-\zeta^2} = 10.47 \rightarrow\rightarrow \frac{6\sqrt{1-\zeta^2}}{\zeta} = 10.47 \rightarrow 36 - 36\zeta^2 = 109.62\zeta^2 \rightarrow \zeta = \sqrt{\frac{36}{109.62+36}} = 0.49$$

$20 + K = 12.24^2 = 129.8$ La diferencia es por redondear mucho

$$T_s = \frac{4}{0.49*12.24} = 0.67\,s$$
$$OS\% = 100e^{-0.49\pi/\sqrt{1-0.49^2}} = 17.1\%$$

$$K_p = \lim_{s \to 0} G_{OL}(s) = \frac{129.8}{20} = 6.49$$
$$e_s(\infty) = \frac{1}{1+K_p} = \frac{1}{7.49} = 0.13$$

(c) $e_s(\infty) = 0.05 = \frac{1}{1+K_p} = K_p = \frac{0.95}{0.05} = 19$

$$K_p = 19 = \lim_{s \to 0} G_{OL}(s) = \frac{K}{20} \rightarrow K = 20 * 19 = 380$$

$$G_{fb}(s) = \frac{380}{s^2 + 12s + 400}$$
$$\omega_n^2 = 400 \rightarrow \omega_n = 20 \quad 12 = 40\zeta \rightarrow \zeta = \frac{12}{40} = 0.3$$
$$T_s = \frac{4}{\sigma_d} = \frac{4}{0.3 * 20} = 0.67\,s$$
$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{20\sqrt{1-0.3^2}} = 0.16\,s$$
$$OS\% = 100\,e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 37.23\%$$
$$K_p = \lim_{s \to 0} G_{OL}(s) = \frac{380}{20} = 19$$
$$e_s(\infty) = \frac{1}{1+K_p} = \frac{1}{20} = 0.05$$

$$\frac{(2*0+1)\pi}{3} = \frac{\pi}{3}$$
$$\frac{(2*1+1)\pi}{3} = \pi$$
$$\frac{(2*2+1)\pi}{3} = \frac{5\pi}{3} = \frac{-\pi}{3}$$