

1. Which of the following are correct?

- a.  $(A \wedge B) \models (A \Leftrightarrow B)$
- b.  $A \Leftrightarrow B \models \neg A \vee B$
- c.  $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$ .
- d.  $(A \vee B) \wedge \neg(A \Rightarrow B)$  is satisfiable

$KB \models \alpha$  if and only if  $M(KB) \subseteq M(\alpha)$

- a.  $(A \wedge B) \models (A \Leftrightarrow B)$  is true because the left-hand side has exactly one model that is one of the two models of the right-hand side.  $\square$
- b.  $A \Leftrightarrow B \models \neg A \vee B$  is true because the RHS is  $A \Rightarrow B$ , one of the conjuncts in the definition of  $A \Leftrightarrow B$ .  $\square$
- c.  $(C \vee (\neg A \wedge \neg B)) \equiv ((A \Rightarrow C) \wedge (B \Rightarrow C))$  is true; proof by truth table enumeration, or by application of distributivity (Fig 7.11).  $\square$

$$\begin{aligned} \text{LHS} &= (C \vee (\neg A \wedge \neg B)) \\ &= (C \vee \neg A) \wedge (C \vee \neg B) \\ &= (\neg A \vee C) \wedge (\neg B \vee C) \\ &= (A \Rightarrow C) \wedge (B \Rightarrow C) \end{aligned}$$

- d. If a sentence  $\alpha$  is true in model  $m$ , we say that  $m$  satisfies  $\alpha$  or sometimes  $m$  is a model of  $\alpha$ .

$$\begin{aligned} &(A \vee B) \wedge \neg(A \Rightarrow B) \\ &= (A \vee B) \wedge \neg(\neg A \vee B) \\ &= (A \vee B) \wedge (A \wedge \neg B) \\ &= (A \wedge A \wedge \neg B) \vee (B \wedge A \wedge \neg B) \\ &= A \wedge \neg B \end{aligned}$$

$(A \vee B) \wedge \neg(A \Rightarrow B)$  is satisfiable; model has  $A$  and  $\neg B$ .

2. Consider a vocabulary with only four propositions, A, B, C, and D. How many models are there for the following sentences?
- $B \vee C$
  - $\neg A \vee \neg B \vee \neg C \vee \neg D$
  - $(A \Rightarrow B) \wedge A \wedge \neg B \wedge C \wedge D$

These can be computed by counting the rows in a truth table that come out true, but each has some simple property that allows a short-cut:

- Sentence is false only if B and C are false, which occurs in 4 cases for A and D, leaving 12.  $\square$
- Sentence is false only if A, B, C, and D are false, which occurs in 1 case, leaving 15.  $\square$
- The last four conjuncts specify a model in which the first conjunct is false, so 0.  $\square$