Dynamic response of a 3D structure

Project, Structural Dynamics, MW 2136

Prof. Daniel J. Rixen

Technische Universität München

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In this exercise, we will use a simple Finite Element code to model a hangar structure (see Figure at the end of the assignment).

PART I: Beam and bar finite element model

1. **Building a Finite Element model.**

   A Matlab code is provided. In the main file, called FEM_struct_dyn.m, the geometrical parameters, the table of nodes and elements and the assembled stiffness and mass matrices are computed. Some simple visualization routines were also written.

   In this part, read the code to understand the different steps. A good way to understand what happens in the code is to look at the intermediate results, or change some parts of the code for instance to add an element to the model.

2. **Checking the model.**

   Verify that, if no degrees of freedom are fixed,

   $$\boldsymbol{K}\boldsymbol{u}_{trans} = \boldsymbol{0}$$

   where $\boldsymbol{u}_{trans}$ is a translation rigid body mode with unit amplitude. Then you can also check that the total mass of your structure by computing

   $$\boldsymbol{u}_{trans}^T \boldsymbol{M} \boldsymbol{u}_{trans} = m_{total}$$

   where $\boldsymbol{M}$ is the mass matrix of the non-constrained system.

PART II: Numerical methods for dynamic analysis

The dynamics of the structure modeled in part I will now be analyzed according the following steps:

1. **Free vibration.**

   Compute the first 10 eigenmodes and eigenfrequencies of the system (do not forget to apply the boundary conditions from now on). For that, program and apply the inverse iteration technique with deflation. Verify your algorithm by comparing the eigensolutions obtained with the eigensolutions computed from the *eig* or *eigs* function of Matlab. In particular verify that the accuracy of higher modes is deteriorating due to the successive application of deflation.

2. **Transient dynamics.**

   Consider now a vertical force applied to the front portal of the hangar (see figure). Its time variation is a triangular function af total length equal to half the period of the fourth eigenfrequency and has an amplitude of 100 N.

   For that excitation, compute the transient time response using direct time integration, applying an **implicit** Newmark scheme. Observe the cconvergence of the solution when the time step is decreased.

   Apply then an **explicit** Newmark scheme. In particular observe the computational cost of this method, and check that the solution becomes numerically unstable for a time-step larger than the critical limit.

Writing the report

For writing the report, you should do as if the report is intended for an engineering office that has to understand how you did the analysis (assumptions in the model, analysis methods ...) and that must, as clearly as possible, comprehend the result of your analysis.

It is suggested to shortly describe the model used and the analysis procedure followed. For example think of discussing the assumption of the model, the dynamic behavior found from the free vibration modes, the adequacy of the model to compute the modes, the choice of time step ... Have a clear and critical discussion of the results found.

Show only the graphs and picture you think are necessary to illustrate your discussion. Be aware that the director of the engineering office (your client) has *maximum 30 minutes* to read your report! So the main text of the report should not exceed 10 pages. Any complementary material that is not essential for the director should be put in appendix. Please list also your matlab files in the appendix.

Appendix: example of a hangar structure

A hangar made out of steel is depicted in Fig. 1. The dimensions are (in meters):

$$
\begin{aligned}
L &= 80 \\
l_1 &= 4 \\
l_2 &= 5 \\
l_3 &= 5 \\
l_4 &= 4 \\
l_5 &= 6 \\
l_6 &= 4 \\
l_7 &= 10 \\
l_8 &= 3 \\
l_9 &= 9 \\
l_{10} &= 10 \\
h = 25 \qquad H &= 25
\end{aligned}
$$

and all beams and bars have a circular hollow cross section (in meter):

$$
\begin{aligned}
D1 &= 0.20 & \qquad d1 &= 0.195 \\
D2 &= 0.20 & d2 &= 0.195 \\
D3 &= 0.25 & d3 &= 0.244 \\
D4 &= 0.05 & d4 &= 0.046 \\
D5 &= 0.40 & d5 &= 0.380 \\
D6 &= 0.08 & d6 &= 0.075 \\
D7 &= 0.25 & d7 &= 0.24
\end{aligned}
$$

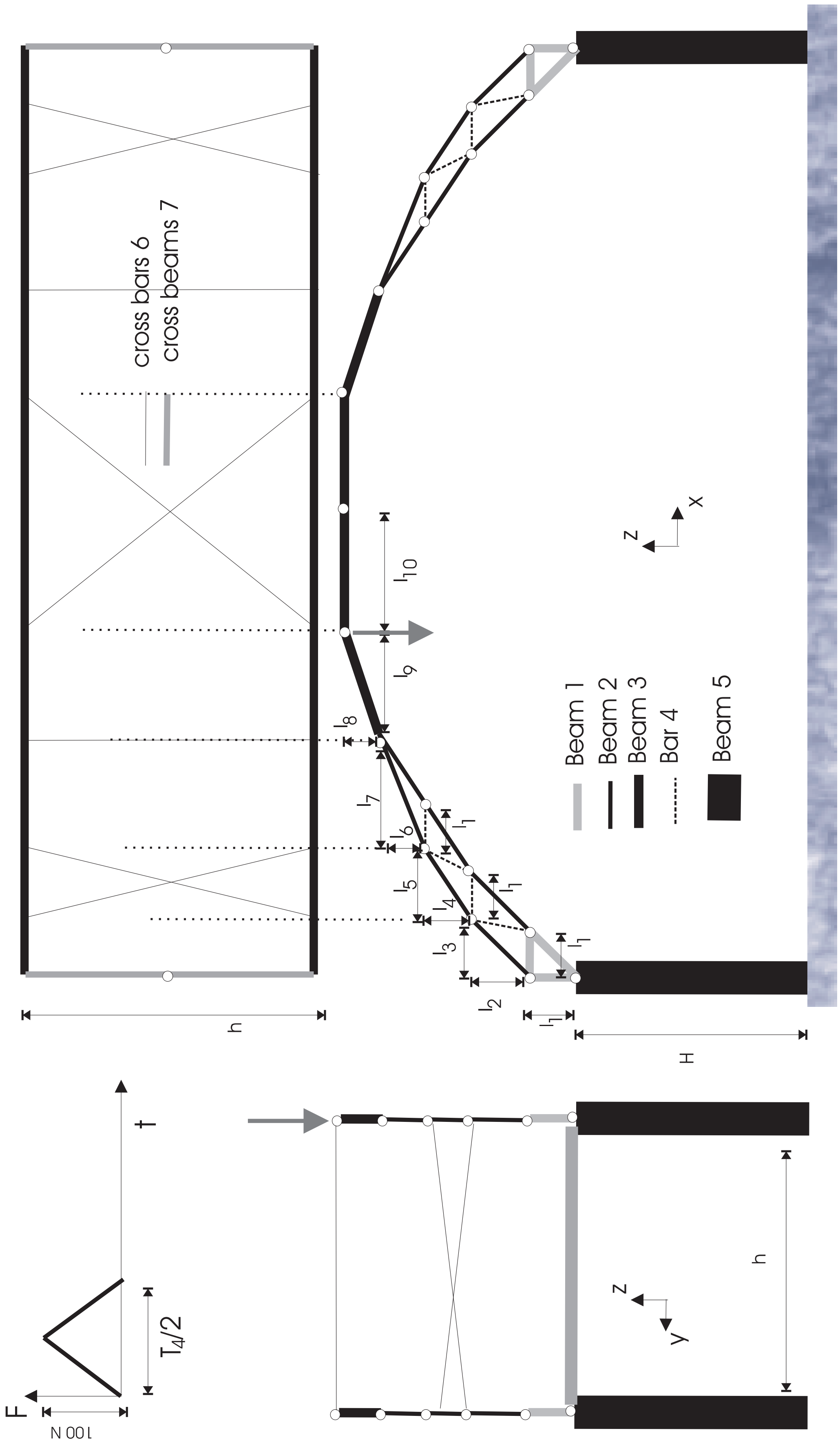

Figure 1: The structure