Lògica en la Informàtica

Definició de Lògica de Primer Ordre
Deducció en Lògica de Primer Ordre

José Miguel Rivero   Robert Nieuwenhuis

Dept. Ciències de la Computació
Facultat de Informàtica
Universitat Politècnica de Catalunya (UPC)

Tardor 2022

Deducció en Lògica de Primer Ordre

Continguts:

Examen final de 2017 tardor. Exercici 6

6) Consider the following Prolog program and its well-known behaviour:

```
animals([dog,lion,elephant]).
bigger(lion,cat).
faster(lion,cat).
better(X,Y):- animals(L), member(X,L), bigger(X,Y), faster(X,Y).

    ?- better(U,V).
    U = lion
    V = cat
```

In Prolog, a list like `[dog,lion,elephant]` is in fact represented as a term `f(dog,f(lion,f(elephant,emptylist)))`.

Therefore, we assume that the program also contains the standard clauses for `member` like this:

```
member( E, f(E,_) ).
member( E, f(_,L) ):- member(E,L).
```


Examen final de 2017 tardor. Exercici 6

6) Consider the following Prolog program and its well-known behaviour:

```
animals([dog,lion,elephant]).
bigger(lion,cat).
faster(lion,cat).
better(X,Y):- animals(L), member(X,L), bigger(X,Y), faster(X,Y).

member( E, f(E,_) ).
member( E, f(_,L) ):- member(E,L).
```

Express the program as a set of first-order clauses P and prove that $\exists u\,\exists v\; better(u,v)$ is a logical consequence of P. Which values did the variables u and v get (by unification) in your proof? **Only write the steps and values. No explanations.**

Examen final de 2017 tardor. Exercici 6

`better(X,Y):- animals(L), member(X,L), bigger(X,Y), faster(X,Y).`

$better(X,Y) \leftarrow animals(L) \wedge member(X,L) \wedge bigger(X,Y) \wedge faster(X,Y)$

$a \rightarrow b \;\equiv\; \neg a \vee b$

$better(X,Y) \;\vee\; \neg(\,animals(L) \wedge member(X,L) \wedge bigger(X,Y) \wedge faster(X,Y)\,)$

$better(X,Y) \vee \neg animals(L) \vee \neg member(X,L) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$

Examen final de 2017 tardor. Exercici 6

Les clàusulas de P són les següents:

1. $animals(f(dog, f(lion, f(elephant, emptylist))))$
2. $bigger(lion, cat)$
3. $faster(lion, cat)$
4. $better(X,Y) \vee \neg animals(L) \vee \neg member(X,L) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
5. $member(E, f(E, \_))$
6. $member(E, f(\_, L)) \vee \neg member(E, L)$

La negació de $\exists u\,\exists v\; better(u,v)$ és:

$\neg\exists u\,\exists v\; better(u,v)$

$\forall u\,\neg\exists v\; better(u,v)$

$\forall u\,\forall v\,\neg better(u,v)$

que en forma clausal (ometent els $\forall u\,\forall v$) és:

7. $\neg better(u,v)$

Examen final de 2017 tardor. Exercici 6

Resolució en LPO:

$$\frac{A \vee C \qquad \neg B \vee D}{(C \vee D)\sigma}$$

A,B són àtoms
si $\sigma = mgu(A,B)$
(*most general unifier*)

He d'obtenir la clàusula buida $\square$ mitjançant resolució a partir d'aquestes 7 clàusules.

1. $animals(f(dog, f(lion, f(elephant, emptylist))))$
2. $bigger(lion, cat)$
3. $faster(lion, cat)$
4. $better(X,Y) \vee \neg animals(L) \vee \neg member(X,L) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
5. $member(E, f(E, \_))$
6. $member(E, f(\_, L)) \vee \neg member(E, L)$
7. $\neg better(u,v)$

Examen final de 2017 tardor. Exercici 6

1. $animals(f(dog, f(lion, f(elephant, emptylist))))$
2. $bigger(lion, cat)$
3. $faster(lion, cat)$
4. $better(X,Y) \vee \neg animals(L) \vee \neg member(X,L) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
5. $member(E, f(E, \_))$
6. $member(E, f(\_, L)) \vee \neg member(E, L)$
7. $\neg better(u,v)$

res entre **mgu**
4+7 $\{u=X, v=Y\}$ obtenim:
8. $\neg animals(L) \vee \neg member(X,L) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
1+8 $\{L=f(dog, f(lion, f(elephant, emptylist)))\}$ obtenim:
9. $\neg member(X, f(dog, f(lion, \ldots) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
6+9 $\{E=X, L=f(lion, f(elephant, emptylist))\}$ obtenim:
10. $\neg member(X, f(lion, \ldots)) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
5+10 $\{X=lion, E=lion, \_=f(elephant, emptylist)\}$ obtenim:
11. $\neg bigger(lion, Y) \vee \neg faster(lion, Y)$
2+11 $\{Y=cat\}$ obtenim:
12. $\neg faster(lion, cat)$
3+12 $\{\ \}$ obtenim:
13. $\square$

Examen final de 2017 tardor. Exercici 6

...

7. $\neg better(u,v)$

res entre **mgu**
4+7 $\{u=X, v=Y\}$ obtenim:
8. $\neg animals(L) \vee \neg member(X,L) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
1+8 $\{L=f(dog, f(lion, f(elephant, emptylist)))\}$ obtenim:
9. $\neg member(X, f(dog, f(lion, \ldots) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
6+9 $\{E=X, L=f(lion, f(elephant, emptylist))\}$ obtenim:
10. $\neg member(X, f(lion, \ldots)) \vee \neg bigger(X,Y) \vee \neg faster(X,Y)$
5+10 $\{X=lion, E=lion, \_=f(elephant, emptylist)\}$ obtenim:
11. $\neg bigger(lion, Y) \vee \neg faster(lion, Y)$
2+11 $\{Y=cat\}$ obtenim:
12. $\neg faster(lion, cat)$
3+12 $\{\ \}$ obtenim:
13. $\square$

$u = X = lion$
$v = Y = cat$

Hem vist que no sols hem demostrat que $P \models \exists u\,\exists v\; better(u,v)$, sinó que fins i tot hem calculat dos valors concrets d'u i v.

4) (3 points) For 4a and 4b, just write the simplest and cleanest possible formula F. Use no more predicate or function symbols than just p. Give no explanations.

4a) Write a satisfiable first-order formula F, using only a *binary* predicate p, such that all models I of F have an infinite domain D_I.

4b) Write a satisfiable formula F of first-order logic with equality, using only a *unary* predicate p, such that F expresses that there is a single element satisfying p, that is, all models I of F have a single (unique) element e in its domain D_I such that $p_I(e) = 1$.

4a) Write a satisfiable first-order formula F, using only a *binary* predicate p, such that all models I of F have an infinite domain D_I.

Resposta:

$\forall x\, \neg p(x,x)$ (irreflexivitat)

$\wedge$

$\forall x\, \forall y\, \forall z\, (\, p(x,y) \wedge p(y,z) \leftarrow p(x,z)\,)$ (transitivitat)

$\wedge$

$\forall x\, \exists\, y\, p(x,y)$ ("existència de successors")

4b) Write a satisfiable formula F of first-order logic with equality, using only a *unary* predicate p, such that F expresses that there is a single element satisfying p, that is, all models I of F have a single (unique) element e in its domain D_I such that $p_I(e) = 1$.

Resposta:

$\exists\, x\, (\ \ p(x) \ \ \wedge \ \ \forall y\, (\, \neg eq(x,y) \rightarrow \neg p(y)\,)\ \ )$

5) (3 points) Let F be the first-order formula $\exists x \forall y \exists z\ (\ p(z,y) \wedge \neg p(x,y)\ )$.

5a) Give a model I of F with $D_I = \{a,b,c\}$.

5b) Is it true that $F \models \forall x\ p(x,x)$?

5c) Is there any model of F with a single-element domain?

5) (3 points) Let F be the first-order formula $\exists x \forall y \exists z\ (\ p(z,y) \wedge \neg p(x,y)\ )$.

5a) Give a model I of F with $D_I = \{a,b,c\}$.

Solució:

$D_I = \{a,b,c\}$

$p_I(a,a) = 0$

$p_I(a,b) = 0$

$p_I(a,c) = 0$

$p_I(b,a) = 1$

$p_I(b,b) = 1$

$p_I(b,c) = 1$

$p_I(c,a) =$ no importa

$p_I(c,b) =$ no importa

$p_I(c,c) =$ no importa

5) (3 points) Let F be the first-order formula $\exists x \forall y \exists z\ (\ p(z,y) \wedge \neg p(x,y)\ )$.

5b) Is it true that $F \models \forall x\ p(x,x)$?

Solució:

No. perquè existeix una I tal que I és model de F, és a dir $I \models F$, però I no és model de $\forall x\, p(x,x)$.
I aquesta I és la de l'apartat **5a)**.

5) (3 points) Let F be the first-order formula $\exists x \forall y \exists z\ (\ p(z,y) \wedge \neg p(x,y)\ )$.

5c) Is there any model of F with a single-element domain?

Solució:

No. Si tinguéssim $D_I = \{e\}$, hauríem de definir
$p_I(e,e) = 1$
o bé
$p_I(e,e) = 0$

En tots dos casos, si $x{=}e$, $y{=}e$, i $z{=}e$, no es compleix $p_I(e,e) \wedge \neg p_I(e,e)$

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

A: All people that have electric cars are ecologists.
B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.
C: A person is an ecologist if his/her mother is an ecologist.
D: Mary is John's grandmother.
E: Mary has an electric car.
F: John is an ecologist.

Resposta:

$hasEcar(x) \equiv$ "x has an electric car"
$isEcologist(x) \equiv$ "x is an ecologist"
$mother(x,y) \equiv$ "y is the mother of x"
$grandma(x,y) \equiv$ "y is the grandmother of x"

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

A: All people that have electric cars are ecologists.

$hasEcar(x) \equiv$ "x has an electric car"
$isEcologist(x) \equiv$ "x is an ecologist"
$mother(x,y) \equiv$ "y is the mother of x"
$grandma(x,y) \equiv$ "y is the grandmother of x"

A: $\forall x\, (hasEcar(x) \rightarrow isEcologist(x))$

A $\neg hasEcar(x) \vee isEcologist(x)$

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.

$\mathit{hasEcar}(x)$	$\equiv$	"x has an electric car"
$\mathit{isEcologist}(x)$	$\equiv$	"x is an ecologist"
$\mathit{mother}(x,y)$	$\equiv$	"y is the mother of x"
$\mathit{grandma}(x,y)$	$\equiv$	"y is the grandmother of x"

NO és correcte:

$\forall x \;\; (\, \exists y\, (\, \mathit{grandma}(x,y) \rightarrow \exists z\, (\mathit{mother}(x,z) \wedge \mathit{mother}(z,y)\,)\;)\;)$

$\forall x \;\; (\, \exists y\, (\, \neg \mathit{grandma}(x,y) \vee \exists z\, (\mathit{mother}(x,z) \wedge \mathit{mother}(z,y)\,)\;)\;)$

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.

NO és correcte:

$\forall x \;\; (\, \exists y\, (\, \neg \mathit{grandma}(x,y) \vee \exists z\, (\mathit{mother}(x,z) \wedge \mathit{mother}(z,y)\,)\;)\;)$

Aquesta formalització del llenguatge natural no és adequada: si tenim una situació I amb persones $D_I = \{p1, p2, \mathit{avia}\}$ i on l'àvia de $p1$ es avia: $\quad \mathit{grandma}_I(p1, \mathit{avia}) = 1$
i on tota la resta és fals (ningú és mare de ningú, etc.) llavors I satisfà la fòrmula, perquè $\forall x\, \exists y\, \neg \mathit{grandma}(x,y)$. De fet, amb aquesta formalització no és possible obtenir la clàusula buida.

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

B: If someone has a grandmother, then that someone has a mother whose mother is that grandmother.

El que sí és correcte és:

$\forall x\, \forall y\; (\, \mathit{grandma}(x,y) \rightarrow \exists z\, (\, \mathit{mother}(x,z) \wedge \mathit{mother}(z,y)\,)\,)$

➤ Elim. $\rightarrow$

$\forall x\, \forall y\; (\, \neg \mathit{grandma}(x,y) \vee \exists z\, (\mathit{mother}(x,z) \wedge \mathit{mother}(z,y)\,)\,)$

➤ Skolem:

$\forall x\, \forall y\; (\, \neg \mathit{grandma}(x,y) \vee (\, \mathit{mother}(x, f_z(x,y)) \wedge \mathit{mother}(f_z(x,y), y)\,)\;)$

➤ Distrib:

B1 $\neg \mathit{grandma}(x,y) \vee \mathit{mother}(x, f_z(x,y))$
B2 $\neg \mathit{grandma}(x,y) \vee \mathit{mother}(f_z(x,y), y))$

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

C: A person is an ecologist if his/her mother is an ecologist.

$\mathit{hasEcar}(x)$	$\equiv$	"x has an electric car"
$\mathit{isEcologist}(x)$	$\equiv$	"x is an ecologist"
$\mathit{mother}(x,y)$	$\equiv$	"y is the mother of x"
$\mathit{grandma}(x,y)$	$\equiv$	"y is the grandmother of x"

C: $\forall x\, (\;\exists y\, (\, \mathit{mother}(x,y) \wedge \mathit{isEcologist}(y)\,) \rightarrow \mathit{isEcologist}(x)\,)$

$\forall x\, (\, \neg \exists y\, (\, \mathit{mother}(x,y) \wedge \mathit{isEcologist}(y)\,) \vee \mathit{isEcologist}(x)\,)$

$\forall x\, (\;\forall y\; \neg\, (\, \mathit{mother}(x,y) \wedge \mathit{isEcologist}(y)\,) \vee \mathit{isEcologist}(x)\,)$

$\forall x\, (\;\forall y\, (\, \neg\, \mathit{mother}(x,y) \vee \neg\, \mathit{isEcologist}(y)\,) \vee \mathit{isEcologist}(x)\,)$

C $\neg\, \mathit{mother}(x,y) \vee \neg\, \mathit{isEcologist}(y) \vee \mathit{isEcologist}(x)$

Examen final de 2020 tardor. Exercici 6

6) (4 points) Formalize and prove by resolution that sentence F is a logical consequence of the first five:

$\mathit{hasEcar}(x)$	$\equiv$	"x has an electric car"
$\mathit{isEcologist}(x)$	$\equiv$	"x is an ecologist"
$\mathit{mother}(x,y)$	$\equiv$	"y is the mother of x"
$\mathit{grandma}(x,y)$	$\equiv$	"y is the grandmother of x"

D: Mary is John's grandmother.

D $\mathit{grandma}(\mathit{john}, \mathit{mary})$

E: Mary has an electric car.

E $\mathit{hasEcar}(\mathit{mary})$

F: John is an ecologist.
$\neg$ F: John is not an ecologist.

$\neg$ F $\neg\, \mathit{isEcologist}(\mathit{john})$

Examen final de 2020 tardor. Exercici 6

Volem demostrar que $A \wedge B \wedge C \wedge D \wedge E \models F$.

I això passa ssi $A \wedge B \wedge C \wedge D \wedge E \wedge \neg F$ és insatisfactible.

A	$\neg \mathit{hasEcar}(x) \vee \mathit{isEcologist}(x)$
B1	$\neg \mathit{grandma}(x,y) \vee \mathit{mother}(x, f_z(x,y))$
B2	$\neg \mathit{grandma}(x,y) \vee \mathit{mother}(f_z(x,y), y))$
C	$\neg \mathit{mother}(x,y) \vee \neg \mathit{isEcologist}(y) \vee \mathit{isEcologist}(x)$
D	$\mathit{grandma}(\mathit{john}, \mathit{mary})$
E	$\mathit{hasEcar}(\mathit{mary})$
$\neg$F	$\neg \mathit{isEcologist}(\mathit{john})$

He d'obtenir la $\square$ mitjançant resolució a partir d'aquestes 7 clàusules.

Examen final de 2020 tardor. Exercici 6

A	$\neg \mathit{hasEcar}(x) \vee \mathit{isEcologist}(x)$
B1	$\neg \mathit{grandma}(x,y) \vee \mathit{mother}(x, f_z(x,y))$
B2	$\neg \mathit{grandma}(x,y) \vee \mathit{mother}(f_z(x,y), y))$
C	$\neg \mathit{mother}(x,y) \vee \neg \mathit{isEcologist}(y) \vee \mathit{isEcologist}(x)$
D	$\mathit{grandma}(\mathit{john}, \mathit{mary})$
E	$\mathit{hasEcar}(\mathit{mary})$
$\neg$F	$\neg \mathit{isEcologist}(\mathit{john})$

	res entre	**mgu**	
	E+A	$\{x = \mathit{mary}\}$	obtenim:
1.	$\mathit{isEcologist}(\mathit{mary})$		
	D+B1	$\{x = \mathit{john}, y = \mathit{mary}\}$	obtenim:
2.	$\mathit{mother}(\mathit{john}, f_z(\mathit{john}, \mathit{mary}))$		
	D+B2	$\{x = \mathit{john}, y = \mathit{mary}\}$	obtenim:
3.	$\mathit{mother}(f_z(\mathit{john}, \mathit{mary}), \mathit{mary})$		
	2+C	$\{x = \mathit{john}, y = f_z(\mathit{john}, \mathit{mary})\}$	obtenim:
4.	$\neg \mathit{isEcologist}(f_z(\mathit{john}, \mathit{mary})) \vee \mathit{isEcologist}(\mathit{john})$		
	4+$\neg$F	$\{\,\}$	obtenim:
5.	$\neg \mathit{isEcologist}(f_z(\mathit{john}, \mathit{mary}))$		

Examen final de 2020 tardor. Exercici 6

A	$\neg \mathit{hasEcar}(x) \vee \mathit{isEcologist}(x)$
B1	$\neg \mathit{grandma}(x,y) \vee \mathit{mother}(x, f_z(x,y))$
B2	$\neg \mathit{grandma}(x,y) \vee \mathit{mother}(f_z(x,y), y))$
C	$\neg \mathit{mother}(x,y) \vee \neg \mathit{isEcologist}(y) \vee \mathit{isEcologist}(x)$
D	$\mathit{grandma}(\mathit{john}, \mathit{mary})$
E	$\mathit{hasEcar}(\mathit{mary})$
$\neg$F	$\neg \mathit{isEcologist}(\mathit{john})$

	res entre	**mgu**	
1.	$\mathit{isEcologist}(\mathit{mary})$		
2.	$\mathit{mother}(\mathit{john}, f_z(\mathit{john}, \mathit{mary}))$		
3.	$\mathit{mother}(f_z(\mathit{john}, \mathit{mary}), \mathit{mary})$		
4.	$\neg \mathit{isEcologist}(f_z(\mathit{john}, \mathit{mary})) \vee \mathit{isEcologist}(\mathit{john})$		
5.	$\neg \mathit{isEcologist}(f_z(\mathit{john}, \mathit{mary}))$		
	3+C	$\{x = f_z(\mathit{john}, \mathit{mary}), y = \mathit{mary}\}$	obtenim:
6.	$\neg \mathit{isEcologist}(\mathit{mary}) \vee \mathit{isEcologist}(f_z(\mathit{john}, \mathit{mary}))$		
	6+5	$\{\;\;\}$	obtenim:
7.	$\neg \mathit{isEcologist}(\mathit{mary})$		
	1+7	$\{\,\}$	obtenim:
8.	$\square$		

Deducció en Lògica de Primer Ordre

Per al proper dia de classe:

- Per a estudiar teoria de LI:
  - repassa els materials del que hem estudiat, i
  - FÉS ELS EXÀMENS PENJATS, començant pels últims, cap als anteriors, treballant sempre primer l'enunciat SENSE resoldre, i després l'examen resolt.
- Continueu fent els exercicis del tema 5. Pròxima classe els farem, i també els d'examen que em proposeu!!!