Physics 116B
Mathematical Methods in Physics
Small Group Tutoring

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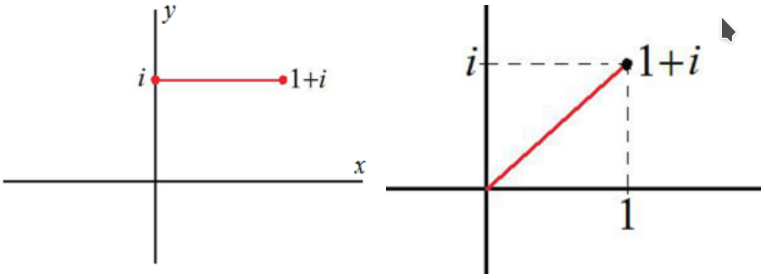
1 Preliminary Questions

- A line integral is an integral where the function to be integrated is evaluated along a curve. The terms path integral, curve integral, and curvilinear integral are also used. One of the many applications in physics is calculating the work done by a particle in a field travelling along a path C. Contour integral are typically reserved for line integrals in the complex plane. Find the following Contour Integrals:



- Cauchy's Theorem states that

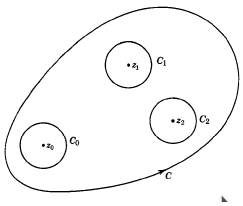
$$\oint_C f(z)dz = 0 \tag{1}$$

along a closed curve C. Under what condition does this hold? If this is true, how can you obtain the value of $f(z)$ at a point $z = a$ inside C? Refer to the textbook.

- A Laurent series of a complex function $f(z)$ is a representation of that function as a power series which includes terms of negative degree. It may be used to express complex functions in cases where a Taylor series expansion cannot be applied. They are expressed as:

$$f(z) = \sum_{n=-\infty}^{\infty} a_n(z - z_0)^n \tag{2}$$

When all the negative n terms are zero, what can we say about $f(z)$ at $z = z_o$? And what if one, a few or all these terms are non-zero? This brings us to the Residue Theorem. This is extremely useful when taking a Contour integral of a closed curve C that is analytic except of several isolated singularities $z_0, z_1, z_2, ...$, as shown in the figure below. What is the value of the Contour integral in this case?



2 Group Problems

Work together as a group for the following problems. Once solved, prepare a presentation to explain the problems in an organized manner.

2.1 Problem 1

Show that by differentiating Cauchy's formula n times, you obtain the following relationship:

$$\frac{d^n}{dz^n}\left(f(a)\right) = \frac{d^n}{dz^n}\left(\frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}dz\right) = \frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-a)^{n+1}}dz \tag{3}$$

2.2 Problem 2

Find all the residues of the following complex functions:

1. $\frac{1}{(1-2z)(5z-4)}$
2. $\frac{1}{z(1-z)}$
3. $\frac{z}{1-z^4}$
4. $\frac{cosz}{1-2sinz}$
5. $\frac{z}{(z^2+1)^2}$

2.3 Problem 3

The following integrals are often found in integral tables used in many areas of physics, such as Quantum Mechanics. Evaluate the following integrals by the methods you have recently learned:

1. $\int_0^{2\pi} \frac{1}{13+5\sin\theta}d\theta$
2. $\int_0^{2\pi} \frac{\sin^2\theta}{5+3\cos\theta}d\theta$