Physics 116B
Mathematical Methods in Physics
Small Group Tutoring

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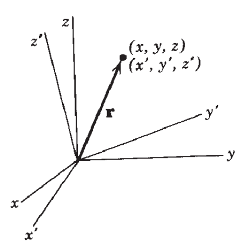
Week 2

1 Preliminary Questions

- Explain qualitatively how the following matrix equation relates to transformations.



$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} l_1 & m_1 & n_1 z \\ l_2 & m_2 & n_2 z \\ l_3 & m_3 & n_3 z \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

- Write out each summation:

$$\sum_{i=1}^{3} a_{ii} \tag{1}$$

$$\sum_{i=1}^{5} x_i x_i \tag{2}$$

$$\sum_{j=1}^{3} \sum_{i=1}^{3} u_{ij} v_{ji} \tag{3}$$

- For a rotating rigid body, the torque is calculated using $\boldsymbol{\tau} = \frac{d\boldsymbol{L}}{dt}$. Explain what it is meant by each term in the equation $\boldsymbol{L} = I\boldsymbol{w}$. Under what conditions are $\boldsymbol{L}$ and $\boldsymbol{w}$ parallel?

- Write out the mathematical definitions for the Kronocker Delta δ_{ii} and the Levi-Civita Symbol ϵ_{ijk}.

2 Group Problems

Work together as a group for the following problems. Once solved, prepare a presentation to explain the problems in an organized manner.

2.1 Problem 1

Rotations can be described by using the nine angles between the two coordinates systems, (x, y, z) and (x', y', z'). Show that such angles in matrix form form an orthonormal set.

Hint: Use linear algebra properties.

2.2 Problem 2

Show that the contracted tensor $T_{ijk}V_k$ is a 2^{nd} rank tensor. T_{ijk} and V_k are defined as follows:

$$V_k = \sum_{i=1}^{3} a_{\tau k} V'_\tau \tag{4}$$

$$T_{ijk} = \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=1}^{3} a_{\alpha i} a_{\beta j} a_{\gamma k} T'_{\alpha\beta\gamma} \tag{5}$$

Hint: Use that fact that $a_{ij}a_{kj} = \delta_{ik}$ and check how many free indices you have left.

2.3 Problem 3

Evaluate the following products:

1. $\delta_{ij}\delta_{jk}\delta_{km}\delta_{im}$
2. $\epsilon_{jk2}\epsilon_{k2j}$
3. $\epsilon_{23i}\epsilon_{2i3}$

Hint for 1: Set $j = 1$, then $k = i$ and finally $m = i$.

Hint for 2: Take advantage of the fact that $\epsilon_i jk$ is anti-symmetric and $\delta_j k$ is symmetric. Interchange k and j in the final step that allows you to evaluate.

Hint for 3: The product of two permutations can be evaluated using a 3x3 matrix of the constituent Kronocker Delta functions.