Physics 116B
Mathematical Methods in Physics
Small Group Tutoring

Pablo Sevilla

Week 4 - April 23/25 2018

1 Preliminary Questions

- What is a differential equation, and when is it ordinary vs. partial? What is referred as the order of such equation? What constitutes as a solution to these equations?
- State the order of such differential equations and provide context on their application to real problems.

$$\boldsymbol{F} = m\boldsymbol{a} \tag{1}$$

$$\frac{dQ}{dt} = kA\frac{dT}{dx} \tag{2}$$

$$\frac{dV}{dt} = \frac{d}{dt}\left(L\frac{dI}{dt} + RI + \frac{q}{c}\right) \tag{3}$$

- By inspection, find one solution for each of the differential equations $y'' = -\cos x$ and $y'' = -y$.
- When are solutions of an n order differential equation general? What information is required to obtain a particular solution from a general solution?
- Find the distance which an object falls under gravity in t seconds using the relationship $\frac{d^2x}{dt^2} = g$. By algebra manipulation, find a solution to such equation and state whether it is a general or a particular solution.
- Separation of variables is one of the most effective methods to find solutions to differential equations. For the equations $xy' = y + 1$ and $xy'' = y' + 1$, is it possible to use such method and why? If it is possible, do it.

2 Group Problems

Work together as a group for the following problems. Once solved, prepare a presentation to explain the problems in an organized manner.

2.1 Problem 1

Find the position x of a particle at time t if its acceleration is $\frac{d^2x}{dt^2} = B \cos wt$.

2.2 Problem 2

Find the distance which an object moves in time t if it starts from rest and has an acceleration $\frac{d^2x}{dt^2} = -ge^{-kt}$.

2.3 Problem 3

Find the general solution to the following differential equation:

$$(yx^2 - 6y)dy = 0 \tag{4}$$

The solution passes through the point $(4, 2)$. Use this information to collapse the general solution into the particular solution of this specific boundary condition.

2.4 Problem 4

The growth of a bacteria population $\frac{dN}{dt}$ is proportional to the square root of the number of bacteria present at that time. If I start with a population of 10 bacteria, how much will this population have grown after 5 seconds? Take the constant of proportionality to be 2.

2.5 Problem 5 ***CHALLENGING***

Linear first order differential equations can be written in the form:

$$y' + P(x)y = Q(x) \tag{5}$$

Show that the general solution to this equation is:

$$y = e^{-I}\left(\int Q(x)e^{I}dx + y_o\right) \tag{6}$$

where $I = \int P(x)dx$.

Hint: First consider the case where $Q(x) = 0$ and obtain a solution for this case. For tidyness, remember that you can write $e^C = A$, where C is the constant obtained after taking an indefinite integral. Once this specific solution is obtained, differentiate with respect to x and see if you can relate your answer to the general case where $Q(x) \neq 0$.

Use this property to find the general solutions to the following linear first order differential equations:

- $x^2y' + 3xy = 1$
- $y' + y\cos x = \sin 2x$
- $dy + (y - e^x)dx = 0$