Physics 116B
Mathematical Methods in Physics
Small Group Tutoring

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1 Preliminary Questions

- What condition allows us to describe a matrix as "orthogonal"? What does it mean graphically to take the determinant of a transformation matrix? Under what situations is such determinant negative?
- The right hand rule is conventionally used to describe the Cartesian coordinate system (x, y, x). Does this rule still hold when one, two or three axis are reversed? What is the determinant sign in each case?
- What is a polar vector and a pseudovector? Explain what type of vectors linear and angular velocity are when considering transformations.
- When is a coordinate system curvilinear? State two examples of these types of coordinates systems.

2 Group Problems

Work together as a group for the following problems. Once solved, prepare a presentation to explain the problems in an organized manner.

2.1 Problem 1

In a Cartesian coordinate system (x, y, z), show that reversing x and y such that $x' = -x$ and $y' = -y$ is equivalent to a 180 rotation of the (x, y) plane with respect ot the z axis.

The rotation matrix is often expressed as:

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

Hint: Remember that the imaginary number $i = \sqrt{-1}$ can be thought of a rotation of 90 degrees! (conventionally clockways).

2.2 Problem 2

Using the equation

$$V_i = \frac{1}{2}\epsilon_{ijk}T_{jk} \tag{1}$$

show that if T_{jk} is a polar vector then V_i is a pseudovector. What if T_{jk} is a pseudovector?

Hint: Differentiate the expression and use the relationships $T'_{jk} = a_{j\alpha}a_{k\beta}a_{\alpha\beta}$ and $\epsilon'_{ijk} = det(A)a_{im}a_{jn}a_{kp}\epsilon_{mnp}$

2.3 Problem 3

For Cartesian (rectangular) coordinates, the linear element is expressed as $ds^2 = dx^2 + dy^2 + dz^2$. Find the line element ds^2 for cylindrical coordinates.

Hint: draw a circle of radius r in the $x - y$ plane to derive a relationship between rectangular and cylindrical coordinates.

2.4 Problem 4

Find the linear element ds^2, the vector $d\boldsymbol{s}$, the volume element dV and the base vectors $\boldsymbol{a}$ and $\boldsymbol{e}$ for parabolic cylinder coordinates (u, v, z), where:

$$x = \frac{1}{2}(u^2 + v^2) \tag{2}$$

$$y = uv \tag{3}$$

$$z = z \tag{4}$$

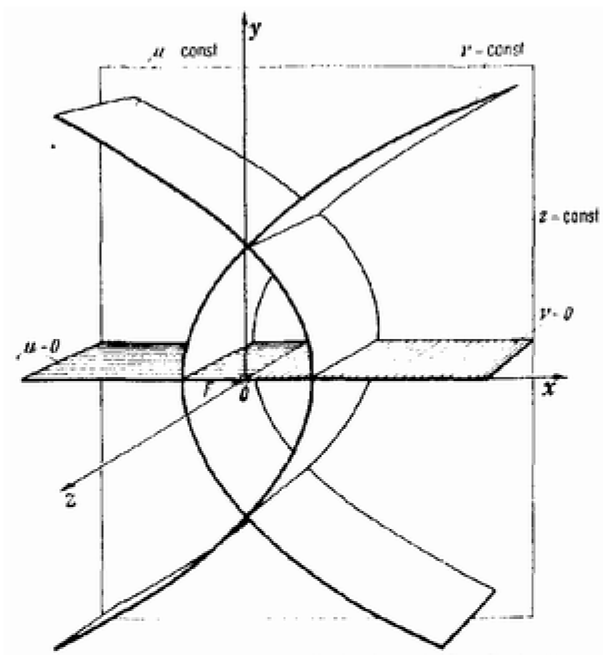

Figure 1: Parabolic cylinder coordinates