Physics 116B
Mathematical Methods in Physics
Small Group Tutoring

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1 Preliminary Questions

- Differential equations of the form:

$$a_2 \frac{d^2y}{dx^2} + a_1 \frac{dy}{dx} + a_0 y = 0 \tag{1}$$

  are often used in physics to model free oscillations of many types of systems, such as a pendulum in a constant gravitational field. How would you modify this ordinary differential equation (ODE) to include an external force? In the pendulum case, think of adding a purely horizontal force in the x direction.

- Consider the following equation:

$$y'' - 4y' + 3y = 5 \tag{2}$$

  What is a particular solution y_p to this ODE and what is the corresponding complimentary function y_c? The solutions of y_c are related and can be used to construct the solutions of function y, but how? Also, will solutions obtained this way be general?

- Sometimes, ODE's can have more than one term in the right hand side, such that:

$$(D^2 + aD + b)y = f(x) + g(x) + h(x) + etc... \tag{3}$$

  How can you solve such equations? What is the required principle called and under what conditions can it be used?

2 Group Problems

Work together as a group for the following problems. Once solved, prepare a presentation to explain the problems in an organized manner.

2.1 Problem 1

Solve the following second order differential equations:

1. $(D-3)^2 y = 6e^3 x$
2. $(D^2+4D+12)y = 80\sin 2x$
3. $(D^2+2D+17)y = 60e^{-4x}\sin 5x$

2.2 Problem 2

Using the principle of superposition, find the general solution to the following second order differential equation:

$$(D^2+1)y = 7+4x\cos 4x \tag{4}$$

2.3 Problem 3

Sometimes, second order ODE's can have missing linear terms x and/or y. The way you deal with this is by making the substitution $y' = p$ to obtain a simpler form in which other methods for solving ODE's can be used. Use this trick to solve the following equation:

$$2yy'' = y'^2 \tag{5}$$

2.4 Problem 4

It is important to pay attention to equations of the following form:

$$y'' + f(y) = 0 \tag{6}$$

These equations arise very frequently in physics applications, such as the motion of a simple pendulum. In order to solve these, one can multiply each term by y' and then integrate the whole expression. This can also be used when considering a particle of mass m moving along the x axis under the action of a force $F(x)$, with a resulting equation of motion:

$$m\frac{d^2x}{dt^2} = F(x) \tag{7}$$

Consider the driving force to be $F(x) = \frac{m}{x^3}$. If the mass starts out at rest at a location of $x = 1$, what is $v(x)$ and how can you obtain $x(t)$ from it?