Low Frequency Impedances

University of California, Santa Cruz

Physics 133 - Gregory Kaminsky

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Abstract

By measuring the resistance and voltage of a voltage source and a load resistor in series, we are able to determine the output voltage and the output resistance of the voltage source. Then we assembled a circuit in order to measure the impedance of an unknown two-terminal black box over a wide range of frequencies. We analyzed a total of three unknown two-terminal black boxes. Finally, we examined a current vs. voltage graph for a diode and a Zener diode to determine when the diodes would start conducting, as well as to find the Zener voltage for the Zener diode.

1 Introduction

1.1 Output Impedance

When you draw a current from a battery, the voltage at the terminals of the battery will be less than the stated voltage. This is due to the battery having an internal resistance R_o. Thevenin's theorem of linear circuit analysis tells us that any two-terminal network of resistors and voltage sources will be equivalent to a single resistor R_o in series with a single voltage source V_o. With no current flowing, the voltage at the two terminals of the battery is called the output voltage V_o. Then if you assemble a load resistor R_L to the two-terminal battery(see figure 1), a current I will start to flow through the circuit and cause the output voltage to be reduced to V, since the internal resistance drop the voltage. We may represent V as $V = V_o - IR_o$. In this experiment, we will figure out the output voltage and output resistance of a battery.

1.2 Unknown Linear Black Boxes

Resistors, inductors and capacitors are examined in different and unknown configurations, each with its own value of impedance. Given that the current I may be expressed in a complex form of $I = I_o e^{j\omega t}$, where $j = \sqrt{-1}$ and ω is the angular frequency, we may find the complex impedance Z by $V = IZ$ for the components that have a current through it and a voltage across it. We constructed a circuit, as shown in figure 2, which allowed us to observe the impedance as a function of frequency.

If a current goes through a resistor that is connected to a voltage source, we may find its impedance by $V = IR$. Thus $Z = R$ for a resistor.

The voltage across an inductor is $V = L\frac{dI}{dt}$. Since $I = I_o e^{j\omega t}$, we can say $V = L\frac{d}{dt}(I_o e^{j\omega t}) = (j\omega L)I$, which is of the form $V = IZ$. Thus $Z = j\omega L$ for an inductor.

The voltage across a capacitor is $V = \frac{Q}{C}$, where $Q = \int I dt = \int I_o e^{j\omega t} dt = \frac{I_o}{j\omega} e^{j\omega t} = \frac{1}{j\omega} I$. Then $V = \frac{1}{j\omega C} I$, which is of the form $V = IZ$. Thus $Z = \frac{1}{j\omega C}$ for a capacitor.

1.3 Nonlinear Elements

A diode is a two terminal semiconductor that usually allows current to flow only in one direction. We will look at a diode (box J) and a Zener diode (box K) and how the current and voltage do not have a linear relationship. This means that when you look at a current (y-axis) vs. voltage (x-axis) graph, the resulting figure will not be a line. A diode will only allow a current to flow through it in one direction due to a p-n junction. The current I that results from when a voltage V is applied to the p-n junction is $I = I_S(e^{eV/kT} - 1)$. We are conducting the experiment at room temperature, so for a voltage greater than +50 millivolts, the logarithm of the current should have a linear relationship with the voltage.

A Zener diode behaves like a diode for a voltage greater than a specific negative voltage. If the voltage is less than that specific negative voltage, the diode begins to conduct again and has a "Zener breakdown" effect.

2 Procedure

2.1 Output Impedance

We took our battery, or the voltage source, and measured the output voltage with no load resistance attached with a voltmeter. Then we constructed a circuit, as seen in figure 1, with a load resistor and recorded the voltage over a range of load resistances, primarily from 500 ohms to $3,000$ ohms. Then we plotted a voltage vs. current graph from the acquired data to find the value of the output voltage and the output resistance of the battery.

2.2 Unknown Linear Black Boxes

We had to find out what components were in three black boxes without peaking inside of them. First we measured the DC resistance of each black box with a multimeter. This can provide helpful information about what will be inside, since an inductor will have a small DC resistance and a capacitor will have an infinite

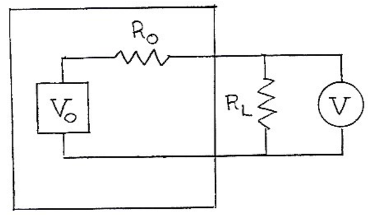

Figure 1: Voltage source with an output voltage of V_o and output resistance R_o connected to a load resistance R_L, while a voltmeter V is attached to measure the voltage across the load resistance(Figure 6.1 from Physics 133 reader for University of California, Santa Cruz).

resistance.
Afterwards, we assembled a circuit shown in figure 2, which allowed us to measure the voltage across the sensing resistor and the voltage across the unknown black box with the help of an oscilloscope over the range from a few tens of hertz to around 100 kilohertz. The channel attached to the unknown black box will show voltage and the channel attached to the sensing resistor will show current on the oscilloscope. We changed

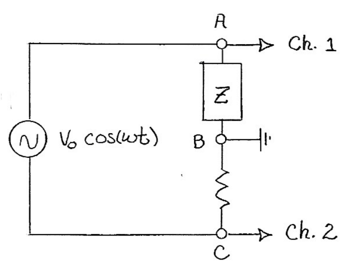

Figure 2: This circuit consists of the unknown black box (Z), the sensing resistor, a voltage source, and an oscilloscope. This circuit is useful to help find Z as a function of frequency(Figure 6.9 from Physics 133 reader for University of California, Santa Cruz)

the sensing resistor to different resistances in order to keep the voltages across the sensing resistor and the unknown black box within a factor of ten of each other. We also inverted the channel of the oscilloscope that is showing the current of the sensing resistor since the voltage across the sensing resistor is proportional to the negative of the current through the unknown black box. The two waveforms on the oscilloscope will show us if one is lagging or leading the other while going through the range of frequencies. Knowing this information, we were able to find the complex impedance Z and its magnitude $|Z|$, as well as to determine the unknown circuit configuration of the unknown black box that is being measured at the time of the experiment, and to find the values of the components of that unknown black box.

2.3 Nonlinear Elements

We analyzed two more boxes, this time knowing the components. Box J contained a diode and box K contained a Zener diode. In order to analyze the two boxes, we switched the oscilloscope into XY mode, which resulted the readings to be place on an x-y axis. We used the same circuit configuration as shown in figure 2, with replacing the unknown black box Z with the known box, either box J or box K. This allowed us to view the value of current and voltage through the diode, with current on the y-axis and voltage on the x-axis. We then analyze the resulting graph on the oscilloscope to figure out the properties of each box.

3 Results

3.1 Output Impedance

We measured 8.79 volts for battery with no load resistance. Then we attached the load resistor to the battery and recorded the voltage with a voltmeter. The resulting data is shown in table 1. With the use of Ohm's law, $V = IR$, we found the current that was going through the circuit for each load resistance. We then made a plot of voltage vs. current and found that the data resembled a linear form so we used a linear regression. The plots fit the function $V = V_o - IR_o$, where the output resistance R_o is the slope and the output voltage V_o is the y-intercept. From our graph, shown in figure 3, we found the output voltage to be 9.03 ± 0.01 volts and the output resistance to be 949 ± 1 ohms. This tells us that the battery is a 9.0 volt battery with an internal resistance of 949 ± 1 ohms. Looking at our measurement of the voltage of the battery with no load resistance, we see that it is lower than 9.0 volts. This is could be due to the battery draining while being used and due to the internal resistance of the battery which lowers the output voltage.

Table 1: Battery Measurements

R_L (Ohms)	V (Volts)	I(Amps)
500	3.226	0.006
999	4.71	0.0047
1499	5.56	0.0037
1999	6.12	0.0031
2499	6.51	0.0026
2.99 kΩ	6.80	0.0023

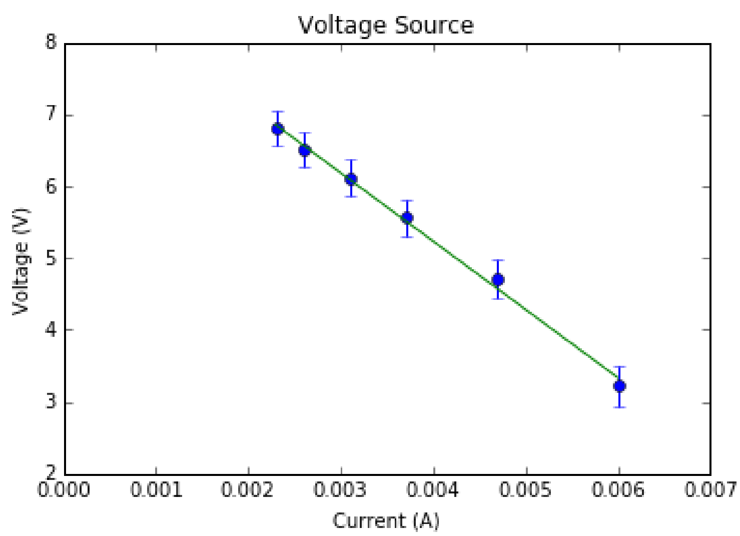

Figure 3: Voltage(y-axis) was measured by a voltmeter. Current(x-axis) was measured by Ohm's law. The error bars represent the standard deviation of uncertainty, or how far the recorded value is from the actual value. The line represents the linear regression of the data points, which helps to determine the slope and intercepts of the data points.

3.2 Unknown Linear Black Boxes

3.2.1 Box B

We connected box B to the circuit configuration shown in figure 2. By using the measurement tools on the oscilloscope, we measured the peak-to-peak voltage of channel one and the peak-to-peak voltage of channel 2, the frequency, the resistance from the sensing resistor, and the change in time between the two waveforms(this was to measure the phase θ using $\theta = (\omega)(\Delta t)$). The data is shown in table 2. With Box

Table 2: Box B

Trial	f (Hz)	R (label)(Ohms)	Phase (Degrees)	V_Z(Volts)	V_R(Volts)	Δt (s)	R (measured)(Ohms)
1	4.98 ± 0.01	100	5.98	2.20 ± 0.02	3.96 ± 0.04	$3.00ms \pm 5\mu s$	100±1
2	16.03 ± 0.01	100	2.31	2.20 ± 0.02	3.96 ± 0.04	$400\mu s \pm 0.1ms$	100±1
3	54.01 ± 0.01	100	5.83	2.20 ± 0.02	3.92 ± 0.04	$300\mu s \pm 0.1ms$	100±1
4	180.48 ± 0.01	100	14.29	2.22 ± 0.02	3.96 ± 0.04	$220\mu s \pm 0.1ms$	100±1
5	592 ± 0.01	100	27.28	2.40 ± 0.02	3.84 ± 0.04	$128\mu s \pm 20\mu s$	100±1
6	1.951 ± 0.001kHz	100	70.24	3.48 ± 0.02	3.48 ± 0.04	$100 \pm 10\mu s$	100±1
7	6.400 ± 0.03kHz	200	82.94	4.64 ± 0.04	3.44 ± 0.04	$36 \pm 2\mu s$	200.3 ± 1
8	21.31 ± 0.004kHz	1000	93.59	4.96 ± 0.04	5.24 ± 0.04	$12.2 \pm 1\mu s$	999 ± 1
9	8.21 ± 0.01	100	1.18	2.54 ± 0.02	4.60 ± 0.04	$400\mu s \pm 2ms$	100±1
10	13.43 ± 0.01	100	0.97	2.54 ± 0.02	4.60 ± 0.04	$200\mu s \pm 1ms$	100±1
11	36.18 ± 0.01	100	1.30	2.56 ± 0.02	4.64 ± 0.04	$100\mu s \pm 0.5ms$	100±1
12	97.36 ± 0.01	100	2.80	2.56 ± 0.02	4.64 ± 0.04	$80\mu s \pm 0.2ms$	100±1
13	261.2 ± 0.1	100	13.16	2.60 ± 0.02	4.60 ± 0.04	$140 \pm 50\mu s$	100±1
14	701.8 ± 0.1	100	38.40	2.86 ± 0.02	4.52 ± 0.04	$152 \pm 20\mu s$	100±1
15	1.808 ± 0.01kHz	200	57.28	2.62 ± 0.02	5.44 ± 0.04	$88 \pm 10\mu s$	200.3±1
16	5.003 ± 0.001kHz	400	77.45	3.22 ± 0.02	5.76 ± 0.04	$43 \pm 5\mu s$	400±1
17	13.406 ± 0.001kHz	1000	94.59	3.60 ± 0.02	6.08 ± 0.04	$19.6 \pm 2\mu s$	999±, 1
18	36.102 ± 0.01kHz	3000	93.58	3.64 ± 0.02	6.36 ± 0.04	$7.2 \pm 1\mu s$	2.99±1kΩ
19	100.332 ± 0.001kHz	20000	141.59	6.08 ± 0.02	9.68 ± 0.04	$3.92 \pm 0.2\mu s$	$19.99 \pm 1k\Omega$
20	525.14 ± 0.01kHz	300	249.55	1.20 ± 0.02	6.00 ± 0.04	$1.32 \pm 20\mu s$	299.9 ± 1

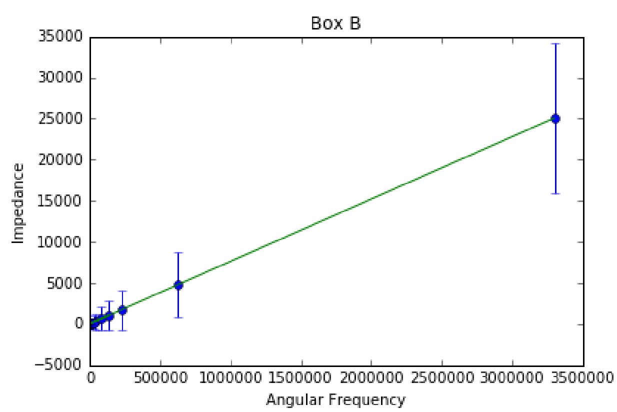

Figure 4: The impedance (y-axis) was measured by $|Z| = (R^2 + (\omega)^2(L)^2)^{1/2}$, with the values R and L known. Angular frequency (x-axis) was found by using the frequencies from table 2, with the help of the known equation $\omega = 2\pi f$. The error bars represent the standard deviation of uncertainty, or how far the recorded value is from the actual value. The line represents the linear regression of the data points, which helps to determine the slope and intercepts of the data points.

B by itself, we measured its resistance with a multimeter and we recorded $54.5 \pm 1.0\Omega$. This tells us that a capacitor is not in series, since that will result in an infinite resistance. As we went through the range of frequencies, the voltage was almost in phase with the current, as show in in table 2 and figure 4. However, as we increased the frequency, the voltage started to lead the current, approaching a phase shift of 90 degrees, or $\frac{\pi}{2}$. When we got to frequencies of about 50kHz and above, we started to get a lot of noise, which altered

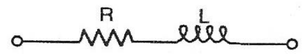

Figure 5: This shows a circuit configuration of a resistor and an inductor in series. The impedance of this circuit will be $Z = R + j\omega L$, with the magnitude of the impedance being $|Z| = (R^2 + (\omega)^2(L)^2)^{1/2}$ (Figure 6.5 from Physics 133 reader for University of California, Santa Cruz).

our data results, mainly seen in trial 19 and trial 20 in table 2. This might have been due to resistance and capacitance of the wires.
The phase shift suggests an inductor, so we hypothesized Box B to have an inductor and resistor in series, as shown in figure 5. The impedance (Z) of a resistor and inductor in series is $Z = R + j\omega L$. From this hypothesized equation, a graph of $|Z| vs. \omega$ will be linear. The phase angle will then be found by $tan(\theta) = \frac{\omega L}{R}$ according to our hypothesis. Also, by Kirchhoff's Laws, we know that $\Sigma I = 0$, so then $I_Z + I_R = \frac{V_Z}{Z} + \frac{V_R}{R} = 0$. Then $Z = -R\frac{V_Z}{V_R}$, where R is the resistance of the sensing resistor.
At $\omega = 0$, $Z = R$. Hence Z is the resistance of $54.5 \pm 1.0\Omega$. At high ω, the resistance becomes negligible since the inductor dominates the circuit. Thus we may ignore the R and say $Z = \omega L$, where $Z = -R\frac{V_Z}{V_R}$. We then found $L = 0.0076$H through calculations. We then found the magnitude of the complex impedance (Z), which is $|Z| = (R^2 + (\omega)^2(L)^2)^{1/2}$. Now knowing what R and L is, we plotted $|Z|$ and a function of angular frequency ω, as shown in figure 4. The result gave us a linear relationship, with the slope (L) being 0.0076H and the y-intercept (R) being 31.3Ω. The inductance found to be the slope of the linear regression of the data corresponds well to our calculated value of inductance 0.0076H. However, the resistance found by the y-intercept of the linear regression of the data is off by a percent error of 43.6% from our measured value of resistance, which is $54.5 \pm 1.0\Omega$. This error is due to the fact that the assumed model of inductance and resistor in series does not fit at high frequencies, as which you can see in both table 2 and figure 4 by the error bars. This resulted in error in our final calculations. With excluding the high frequency data points,

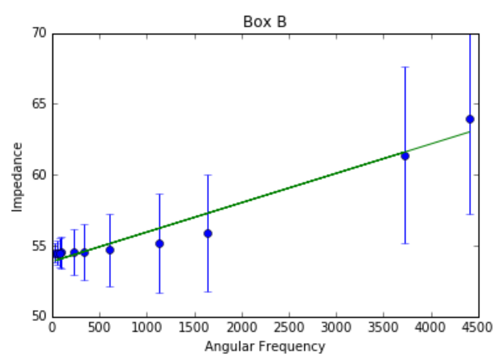

Figure 6: The impedance (y-axis) was measured by $|Z| = (R^2 + (\omega)^2(L)^2)^{1/2}$, with the values R and L known. Angular frequency (x-axis) was found by using the frequencies from table 2, with the help of the known equation $\omega = 2\pi f$. However, this figure only shows low frequency data. The error bars represent the standard deviation of uncertainty, or how far the recorded value is from the actual value. The line represents the linear regression of the data points, which helps to determine the slope and intercepts of the data points.

namely trials 6 through 8 and trials 15 through 20, the y-intercept of the resulting graph is 53.9Ω, which is shown in figure 6. This concurs with our measured value of the resistance of 54.5Ω and proves that at high frequencies, the assumed circuit does not work. The results we obtained from our data, which is mentioned above, proves our hypothesis that box B contains an inductor and a resistor in series since the graph of $|Z| vs \omega$ is linear with the slope and y-intercept of our data corresponding with our measured values.

3.2.2 Box C

We connected box C to the circuit configuration shown in figure 2. By using the measurement tools on the oscilloscope, we measured the peak-to-peak voltage of channel one and the peak-to-peak voltage of channel 2, the frequency, the resistance from the sensing resistor, and the change in time between the two waveforms(this was to measure the phase θ using $\theta = (\omega)(\Delta t)$). The data is shown in table 3.

Table 3: Box C

Trial	f (Hz)	R (label)(Ohms)	Phase (Degrees)	V_Z(Volts)	V_R(Volts)	Δt (s)	R (measured)(Ohms)
1	5.028 ± 0.01	100	1.81	3.80 ± 0.02	3.40 ± 0.04	$1.0ms \pm 5ms$	100±1
2	8.097 ± 0.01	100	1.16	3.80 ± 0.01	3.40 ± 0.04	$400\mu s \pm 2ms$	100±1
3	13.32 ± 0.01	100	1.92	3.80 ± 0.01	3.36 ± 0.04	$400\mu s \pm 1ms$	100±1
4	22.24 ± 0.01	100	3.20	3.80 ± 0.01	3.36 ± 0.04	$400\mu s \pm 1ms$	100±1
5	36.17 ± 0.01	100	3.64	3.80 ± 0.01	3.36 ± 0.04	$280\mu s \pm 1ms$	100±1
6	59.58 ± 0.01	100	2.57	3.80 ± 0.01	3.36 ± 0.04	$120\mu s \pm 0.2ms$	100±1
7	97.06 ± 0.01	100	2.80	3.80 ± 0.04	3.40 ± 0.04	$80\mu s \pm 0.2ms$	100±1
8	159.27 ± 0.01	100	8.03	3.84 ± 0.04	3.36 ± 0.04	$140\mu s \pm 100\mu s$	100±1
9	259.38 ± 0.01	100	13.07	3.84 ± 0.04	3.36 ± 0.04	$140 \pm 100\mu s$	100±1
10	430.366 ± 0.01	100	18.59	3.88 ± 0.04	3.32 ± 0.04	$120 \pm 50\mu s$	100±1
11	702.05 ± 0.01	100	27.80	4.00 ± 0.04	3.28 ± 0.04	$100 \pm 50\mu s$	100±1
12	1.151 ± 0.01kHz	100	39.78	4.36 ± 0.04	3.12 ± 0.04	$96 \pm 10\mu s$	100±1
13	1.849 ± 0.01kHz	200	54.58	3.00 ± 0.04	3.44 ± 0.01	$82 \pm 10\mu s$	200.3±1
14	3.123 ± 0.001kHz	200	63.00	3.80 ± 0.04	3.00 ± 0.01	$56 \pm 5\mu s$	200.3±1
15	5.070 ± 0.001kHz	500	76.66	3.32 ± 0.04	3.92 ± 0.04	$42.4 \pm 2.0\mu s$	500±1
16	8.366 ± 0.001kHz	900	72.28	3.48 ± 0.04	4.00 ± 0.04	$24 \pm 2.0\mu s$	901±1
17	13.150 ± 0.001kHz	2000	86.16	3.64 ± 0.04	3.76 ± 0.04	$18.2 \pm 1.0\mu s$	1999±1
18	15.330 ± 0.001kHz	3500	75.06	3.56 ± 0.04	3.40 ± 0.04	$13.6 \pm 1.0\mu s$	$3.49 \pm 1k\Omega$
19	17.287 ± 0.001kHz	8500	51.03	2.96 ± 0.04	3.12 ± 0.04	$8.2 \pm 1\mu s$	8500±1
20	18.499 ± 0.001kHz	8500	-67.93	2.88 ± 0.04	3.24 ± 0.04	$-10.2 \pm 1\mu s$	8500±1
21	20.133 ± 0.001kHz	4000	-72.48	3.68 ± 0.04	3.24 ± 0.04	$-10 \pm 1\mu s$	4000±1
22	40.34 ± 0.001	700	-108.92	4.00 ± 0.04	4.20 ± 0.04	$-7.5 \pm 0.5\mu s$	700±1
23	28.59 ± 0.01kHz	1300	-88.51	8.40 ± 0.04	8.96 ± 0.04	$-8.6 \pm 1\mu s$	1300±1
24	33.79 ± 0.01kHz	900	-92.45	8.72 ± 0.04	8.96 ± 0.04	$-7.6 \pm 1\mu s$	901±1
25	36.26 ± 0.01kHz	700	-93.99	9.20 ± 0.04	8.48 ± 0.04	$-7.2 \pm 1\mu s$	700±1
26	39.19 ± 0.01kHz	700	-95.86	8.72 ± 0.04	8.96 ± 0.04	$-6.8 \pm 1\mu s$	700±1
27	21.12 ± 0.01kHz	3400	-82.11	7.84 ± 0.04	8.08 ± 0.04	$-10.8 \pm 1\mu s$	3400±1
28	23.75 ± 0.01kHz	2000	-90.63	8.32 ± 0.04	8.48 ± 0.04	$-10.6 \pm 1\mu s$	1999±1
29	19.35 ± 0.01kHz	7000	-68.27	7.04 ± 0.04	7.44 ± 0.04	$-9.8 \pm 1\mu s$	7000±1
30	20.26 ± 0.01kHz	4000	-67.10	7.76 ± 0.04	7.76 ± 0.04	$-9.2 \pm 1\mu s$	4000±1

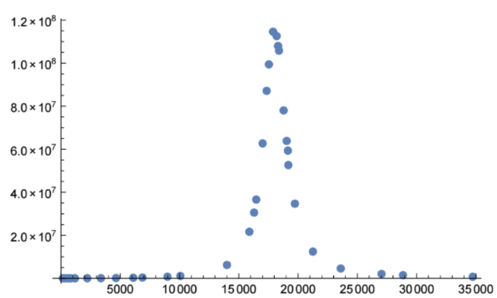

Figure 7: Impedance(y-axis) vs. Angular Frequency(x-axis) for Box C

With Box C by itself, we measured its resistance with a multimeter and we recorded $111.76 \pm 1.0\Omega$. This is taken when the angular frequency ω is zero. However, the effective resistance of an inductor will change directly with the frequency; when frequency increases, the effective resistance of the inductor will also increase. This information tells us that there is not a capacitor in series, since that will result in an infinite resistance. As we varied in ranged of frequencies, we saw that the voltage was in phase with the current at low frequencies. As we approached around $250Hz$, according to our data that is shown in table 3, we saw

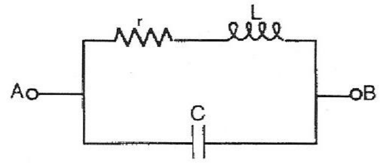

Figure 8: This shows a circuit configuration of an inductor and a capacitor in parallel. The inductor will have its own resistance r, as shown. The impedance of this circuit, written in terms of the complex admittance will be $Y = \frac{1}{Z} = j\omega C + \frac{1}{r+j\omega L}$, with the magnitude being $|Z| = (\frac{r^2C^2}{L^2} + (\omega C - \frac{1}{\omega L}^2)^{-\frac{1}{2}}$ (Figure 6.16 from Physics 133 reader for University of California, Santa Cruz).

that the voltage started to lead the current and was approaching a phase shift of about 90 degrees, or $\frac{\pi}{2}$. With the results that we saw while going through the range in frequencies, we hypothesized that there is an inductor and a capacitor in parallel, as seen in figure 8. However, the current started to decrease to almost

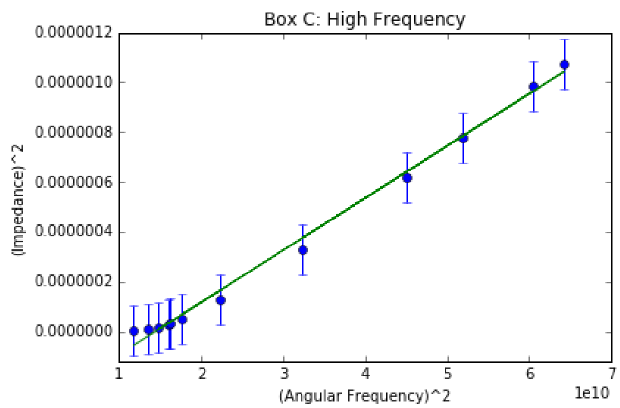

Figure 9: The impedance (y-axis) was measured by $|Y|^2 = \frac{r^2C^2}{L^2} + (\omega C - \frac{1}{\omega L})^2$, with only using high frequencies, with the values r, L and C known. Angular frequency squared (x-axis) was found from squaring the angular frequencies by using the frequencies found in table 3, with the help of the known equation $\omega = 2\pi f$. The error bars represent the standard deviation of uncertainty, or how far the recorded value is from the actual value. The line represents the linear regression of the data points, which helps to determine the slope and intercepts of the data points.

zero when we got to around 17500 kHz, and then the voltage started to lag the current while approaching a phase shift of about $-\frac{\pi}{2}$. With these findings, we hypothesized that there is a capacitor and an inductor in parallel in Box C. The complex impedance, or complex admittance $Y = \frac{1}{Z}$, of an inductor and capacitor in parallel is
$Y = \frac{1}{Z} = j\omega C + \frac{1}{r+j\omega L}$. At $\omega = 0$, $Z = r$ so then $r = 111.76\Omega$, as stated earlier. At high frequency, the resistance r becomes negligible so we have $Y = \frac{1}{Z} = j\omega C + \frac{1}{\omega L}$. We know that the circuit has resonance, which is given by $\omega_o^2 = \frac{1}{LC}$. Using this property of resonance and the equation for impedance, we found $L = 0.017H$ and $C = 4.6x10^{-9}F$. The magnitude of the complex impedance of our hypothesized circuit will be $|Z| = (\frac{r^2C^2}{L^2} + (\omega C - \frac{1}{\omega L}^2)^{-\frac{1}{2}}$. With now knowing the values of the capacitance C, inductance L and the resistance r, we may plot $|Z|$ as a function of ω, which shown in figure 7. We know that the complex admittance, $|Y|$, is the inverse of the complex impedance $|Z|$. Thus $|Y|^2 = \frac{r^2C^2}{L^2} + (\omega C - \frac{1}{\omega L})^2$. For high frequencies, we plotted $|Y|^2$ vs ω^2, as seen in figure 9. The slope will be C^2 and the y-intercept will be $\frac{r^2C^2}{L^2}$. For low frequencies, we plotted $|Y|^2$ vs $\frac{1}{\omega^2}$, as seen in figure 10. The slope will be $\frac{1}{L^2}$ and the y-intercept will be $\frac{r^2C^2}{L^2}$ as well. From the graph shown in figure 9, the capacitance C was found to be $C = 4.57x10^{-9}F$. From the graph shown in figure 10, the inductance L was found to be $L = 0.017H$.

The results that we obtained from our data coincides with the our theoretical predictions of the circuit and values of the components, and proves our hypothesis that box C has a circuit configuration of an inductor and a capacitor in parallel

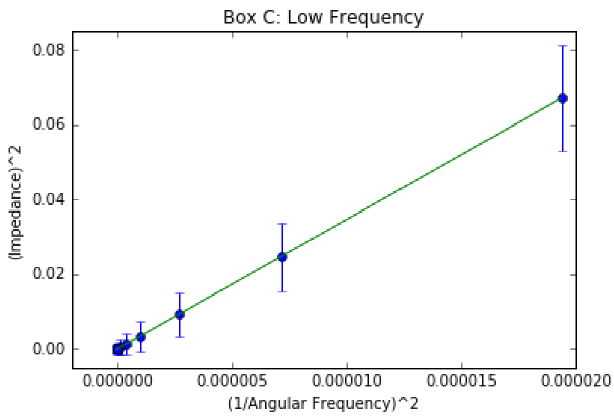

Figure 10: The impedance (y-axis) was measured by $|Y|^2 = \frac{r^2C^2}{L^2} + (\omega C - \frac{1}{\omega L})^2$, with only using low frequencies, with the values r, L and C known. The inverse of the angular frequency squared (x-axis) was found from squaring the angular frequencies by using the frequencies found in table 3, with the help of the known equation $\omega = 2\pi f$. The error bars represent the standard deviation of uncertainty, or how far the recorded value is from the actual value. The line represents the linear regression of the data points, which helps to determine the slope and intercepts of the data points

3.2.3 Box D

We connected box D to the circuit configuration shown in figure 2. By using the measurement tools on the oscilloscope, we measured the peak-to-peak voltage of channel one and the peak-to-peak voltage of channel 2, the frequency, the resistance from the sensing resistor, and the change in time between the two waveforms(this was to measure the phase θ using $\theta = (\omega)(\Delta t)$). The data is shown in table 4. With box

Table 4: Box D

Trial	f (Hz)	R (label)(Ohms)	Phase (Degrees)	V_Z(Volts)	V_R(Volts)	Δt (s)	R (measured)(Ohms)
1	$4.941 \pm 0.001 Hz$	$1M\Omega$	-61.19	3.12 ± 0.04	3.08 ± 0.04	$-34.4 \pm 2ms$	1.003 ± 1 MΩ
2	8.418 ± 0.001	$700k\Omega$	-77.58	3.00 ± 0.04	3.64 ± 0.04	$-25.6 \pm 2ms$	702±1 kΩ
3	13.548 ± 0.01	$300k\Omega$	-86.83	3.36 ± 0.04	3.60 ± 0.04	$-17.8 \pm 1ms$	300 ± 1 kΩ
4	22.383 ± 0.001	$140k\Omega$	-86.21	3.68 ± 0.04	3.52 ± 0.04	$-10.7 \pm 0.5ms$	140.3±1 kΩ
5	36.201 ± 0.001	$70k\Omega$	-88.62	4.00 ± 0.04	3.24 ± 0.04	$-6.8 \pm 0.5ms$	70.1±1 kΩ
6	60.787 ± 0.001	$50k\Omega$	-91.91	3.80 ± 0.04	3.64 ± 0.04	$-4.2 \pm 0.2ms$	50±1 kΩ
7	100.703 ± 0.001	$30k\Omega$	-92.08	3.72 ± 0.04	3.72 ± 0.04	$-2.54ms \pm 100\mu s$	30±1 kΩ
8	160.287 ± 0.001	$17k\Omega$	-96.94	3.88 ± 0.04	3.52 ± 0.04	$-1.68ms \pm 100\mu s$	17.03 ± 1 kΩ
9	262.186 ± 0.001	$12k\Omega$	-90.61	3.36 ± 0.04	3.52 ± 0.04	$-960 \pm 50\mu s$	12.02 ± 1 kΩ
10	430.758 ± 0.001	$7k\Omega$	-85.77	3.40 ± 0.04	3.44 ± 0.04	$-600 \pm 50\mu s$	7 ± 1 kΩ
11	700.745 ± 0.001	$5k\Omega$	-85.77	3.16 ± 0.04	3.68 ± 0.04	$-340 \pm 50\mu s$	4.99 ± 1 kΩ
12	1.183 ± 0.001kHz	$3k\Omega$	-97.10	3.08 ± 0.04	3.68 ± 0.04	$-228 \pm 20\mu s$	2.99 ± 1 kΩ
13	1.496 ± 0.001	$1.4k\Omega$	-73.24	3.56 ± 0.04	3.16 ± 0.04	$-136 \pm 10\mu s$	1.4 ± 1 kΩ
14	3.122 ± 0.001kHz	900	-85.42	3.52 ± 0.04	3.28 ± 0.04	$-76 \pm 10\mu s$	901 ± 1
15	5.096 ± 0.001kHz	500	-82.56	3.00 ± 0.04	3.00 ± 0.04	$-45 \pm 5\mu s$	500±1
16	8.338 ± 0.001kHz	340	-79.24	3.36 ± 0.04	3.04 ± 0.04	$-26.4 \pm 2\mu s$	340±1
17	13.631 ± 0.001kHz	240	-74.59	3.08 ± 0.04	3.00 ± 0.04	$-15.2 \pm 2\mu s$	240.6±1
18	22.108 ± 0.001kHz	170	-59.09	2.92 ± 0.04	2.84 ± 0.04	$-7.5 \pm 0.5\mu s$	170.5±1
19	36.353 ± 0.001kHz	130	-49.73	2.72 ± 0.04	2.68 ± 0.04	$-3.8 \pm 0.5\mu s$	130.2±1
20	60.061 ± 0.001kHz	100	-35.03	2.76 ± 0.04	2.40 ± 0.04	$-1.64 \pm 0.2\mu s$	100±1

D by itself, the resistance was measured by the multimeter and was found to be too high to record. This

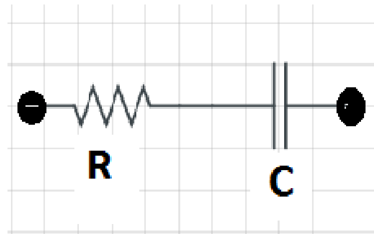

Figure 11: This shows a circuit configuration of a resistor and a capacitor in series. The impedance of this circuit will be $Z = R + \frac{1}{j\omega C}$, with the magnitude of the impedance being $|Z| = (R^2 + \frac{1}{(\omega C)^2})^{-1/2}$.

strongly indicated a capacitor in series. As we varied through the range of frequencies, we noticed that the voltage lagged the current, as well as the voltage tending towards a minimum as the current increased. We also did not notice any resonance, which indicated that there is no inductor in the circuit. This led us to hypothesize that a resistor and a capacitor is in series, as seen in figure 11. The complex impedance of the hypothesized circuit is $Z = R + \frac{1}{j\omega C}$, with the magnitude being $|Z| = (R^2 + \frac{1}{(\omega C)^2})^{-1/2}$. The phase of this circuit is found by $tan(\theta) = -\frac{1}{R\omega C}$ according to our hypothesis. At high frequencies, the capacitor will dominate the circuit. Using the information given in table 4 and knowing Kirchhoff's Laws, we found the capacitance of the circuit to be $C = 3.3x10^{-7}$F. Also, knowing the phase from table 4 and now knowing the capacitance, by using $tan(\theta) = -\frac{1}{R\omega C}$, we found the resistance to be $1324.72 \pm 1\Omega$. We then graphed $|Z| vs. \omega$, as seen in figure 12. We can see that the total impedance decreases as the angular frequency increase. Since the voltage source supplies a current voltage and by Ohm's Law $V = IZ$, the current I increases. Since the $|Z| vs. \omega$ is not linearly represented, by graphing $|Z|^2 vs. \frac{1}{\omega^2}$ we obtain a linear regression, as seen in figure 13. The slope will represent $\frac{1}{C^2}$ and the y-intercept will be R^2. We found the capacitance to be $3.3x10^{-7}$F and the resistance to be 1324.75Ω, just as we predicted with out calculations. Thus the data we recorded agrees with our hypothesis that box D contains a resistor and a capacitor in series.

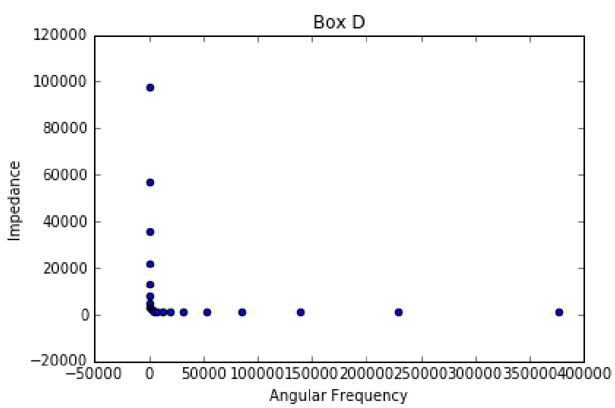

Figure 12: The impedance (y-axis) was measured by $|Z| = (R^2 + \frac{1}{(\omega C)^2})^{-1/2}$. Angular frequency (x-axis) was found by using the frequencies from table 4, with the help of the known equation $\omega = 2\pi f$. The error bars represent the standard deviation of uncertainty, or how far the recorded value is from the actual value. The line represents the linear regression of the data points, which helps to determine the slope and intercepts of the data points.

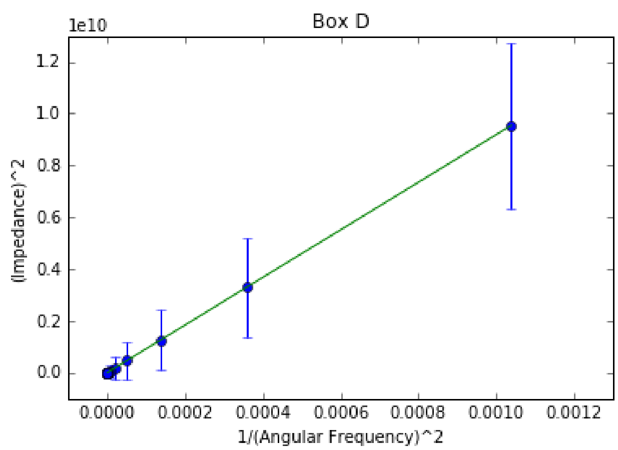

Figure 13: The magnitude of the impedance squared (y-axis) was measured by $|Z| = (R^2 + \frac{1}{(\omega C)^2})^{-1/2}$. The inverse of the angular frequency squared (x-axis) was found by using the frequencies from table 4, with the help of the known equation $\omega = 2\pi f$. The error bars represent the standard deviation of uncertainty, or how far the recorded value is from the actual value. The line represents the linear regression of the data points, which helps to determine the slope and intercepts of the data points.

3.3 Nonlinear Elements

We used the same circuit configuration as shown in figure 2 where the diode and Zener diode are in place of the box Z in the figure. When the voltage is negative for box J, which is a diode, there is no current flows through the diode. This can be seen in figure 14, which is a graph of current vs voltage. When the

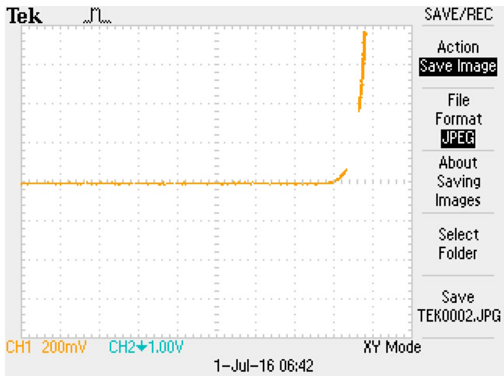

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Figure 14: Current (y-axis) vs Voltage (x-axis) of Diode

voltage for the diode reaches around $600mV$, the current starts to flow through the diode. The relationship of current and voltage is given by $I = I_S(e^{eV/kT} - 1)$.
Box K contains a Zener diode, which uses the same circuit as box J. The Zener diode behaves just like a normal diode, but after a certain negative voltage the Zener Diode begins to conduct again, which is known as a "Zener breakdown" effect. The graph of current vs voltage of the Zener diode (Box K) is shown in figure 15. Just like the diode, the Zener diode begins to have a current flow through it around $+600mV$.

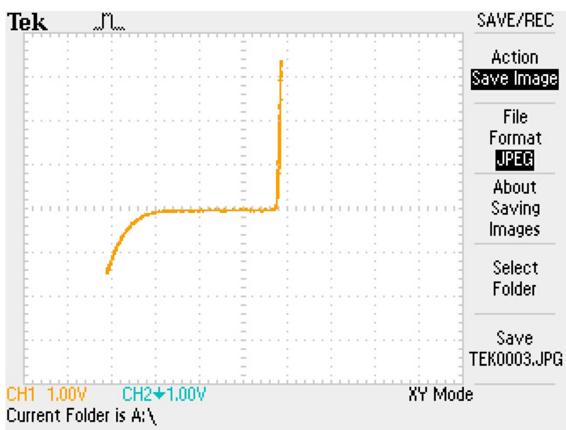

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Figure 15: Current (y-axis) vs Voltage (x-axis) of Zener Diode

However, at around -2.2 ± 0.2V the Zener diode begins to conduct again, which sets the Zener voltage around -2.2 ± 0.2V.

4 Conclusion

4.1 Output Impedance

Using a voltmeter, we were able to find the output voltage of a voltage source, which was a battery. To verify this, we placed a load resistor in series with the battery. By recording multiple values of Resistance and the corresponding voltage and with Ohm's Law, $V = IR$, the current was able to be determined. Seeing how a voltage vs current graph results is nearly linear, $V = V_o - IR_o$ was used, where V_o is the y-intercept and R_o is the slope. The imperfections in the data could be attributed to heat loss in the circuit and to an imperfect battery.

4.2 Unknown Linear Black Boxes

Box B was determined to contain an inductor and resistor in series. The resistor has a resistance of $54.5 \pm 1\Omega$ and the inductor had an inductance of $0.0076 \pm 0.0003H$. The data was expected to fit $Z = R + j\omega L$, with the magnitude being $|Z| = (R^2 + \omega^2 L^2)^{\frac{1}{2}}$. The phase angle can be found by $tan(\theta) = \frac{\omega L}{R}$. However, the higher frequencies deviated from a linear regression line while the lower frequencies fit well. The deviation of the higher frequencies may be due to a capacitance in the wires because of the high frequencies.

Box C was determined to contain an inductor and a capacitor in parallel. The complex admittance is given by $|Y| = (\frac{r^2C^2}{L^2} + (\omega C - \frac{1}{\omega L})^2)^{-1/2}$. The inductor has an inductance of $0.017H$ and a resistance of 111.76Ω, and the capacitor has a capacitance of $5.0x10^{-9}F$. The circuit had resonance that occurred around $17500kHz$.

Box D was determined to contain a resistor and a capacitor in series. The complex resistance is given by $Z = R + \frac{1}{j\omega C}$, with its magnitude being $|Z| = (R^2 + \frac{1}{(\omega C)^2})^{-1/2}$. The phase angle is given by $tan(\theta) = -\frac{1}{R\omega C}$. The capacitor has a capacitance of $3.3x10^{-7}$F. The resistor has a resistance of 1324.78Ω. The circuit did not have any resonance, indicated there is no inductor.

4.3 Nonlinear Elements

The diode and the Zener diode that were observed both allowed current to flow through it when the voltage reached around $+600mV$. The Zener diode differed from the diode since the Zener diode started to rapidly conduct after reaching $-2.2V$. The diode would not allow current to flow at all for any negative voltage.

References

- Physics 133 Reader for University of California, Santa Cruz