Radioactivity — The Absorption of Gamma Rays by Matter

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Physics 133 - Gregory Kaminsky

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Abstract

We explore many properties of gamma radiation and the equipment used to detected the gamma ray radiation. A Geiger-Mueller tube is used to detect gamma rays, and this device is used to count the gamma ray emissions from a cesium-137 source. The Geiger-Mueller tube was connected to an oscilloscope, so that a spike in voltage,as shown on the oscilloscope, represents an incident ray on the Geiger-Mueller tube. This configuration has an intrinsic resistance of 4.95x10$^5\Omega$ and intrinsic capacitance of 8.83x10^{-10}F. We also confirmed that the emission counts are a random process by the Poisson distribution. We investigate the dead time of the G-M tube that is being used for the experiments, and calculate it to be $348.3 \pm 250.0\mu s$. We determined the absorption coefficient of lead by varying the thickness of lead between the gamma ray emitting source, cesium-137, and the Geiger-Mueller tube, which was found to be $\mu = 1.08 \pm 0.30cm^{-1}$ for the mono-energetic gamma rays. Finally, we will investigate the effects of a graded-Z shield and how the effectiveness of a graded-Z shield depends on the configuration of the materials that make it up. Mainly, we will show that a graded-Z shield is most effective when gamma rays pass through it in the order of decreasing atomic number (Z).

1 Introduction

1.1 Cesium-137 Source

Cesium-137 is the radio-isotope source that is used in this experiment, since it emits radiation in the forms of mono-energetic gamma rays (high-energy photons) and high speed electrons (beta particles). It is a relatively safe source to be around. Cesium-137 has a half life of approximately 30 years, which makes the intensity of the radioactivity stable during the course of the experiment. Cesium-137 decays by beta ray emission to the element Barium-137 (see figure 1). This decay process occurs in two ways. For about 92% of cases, Cesium-137 will decay into Barium-137 which exists at a higher energy that will release a gamma ray and then transition to the ground state of Barium-137. For about 8% of the cases, Cesium-137 will decay straight to the ground state, and stable state, of Barium-137. Due to the gamma rays only being produced by the higher energy state Barium-137, the resulting gamma rays are mono-energetic. However, since Cesium-137 undergoes beta decay, beta rays are also produced, which can be detected by the Geiger-Mueller tube that is used in the experiment. Because of this, a special capsule was designed. When the Cesium-137 source in enclosed with the special capsule, one side (with the label) allows only beta rays to be emitted, while the other side only allows gamma rays to be emitted without interference from the beta rays. For the experiments, except for the dead-time experiment, the gamma ray emission side will be the only side that will be used.

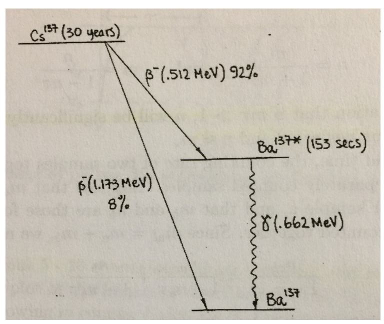

Figure 1: This shows the beta decay model for Cesium-137. When Cesium-137 decays into Barium-137, there are two paths that may take place. For about 92% of cases, the Cesium-137 will decay into Barium-137, that will exist at a higher energy, and thus will emit a gamma ray(half-life of 153 second). For about 8%, Cesium-137 will directly decay into the ground state of Barium-137. (Figure 7.5 from Physics 133 reader for University of California, Santa Cruz)

The decay of a radioactive nucleus, which is unstable, is a random process. This means that the exact moment in which the radioactive nucleus will decay is not predictable. If we have N nuclei, where each has a probability p of decaying in a time interval Δt, then there will be a number ΔN that will decay. Thus we expect that the probability will be proportional to Δt, such that $p = \beta \Delta t$, where β is a constant of proportionality, and that the number of nuclei that will decay is proportional to the probability of a nuclei decaying, such that $\Delta N = pN$. From this, it is expected that $\frac{\Delta N}{N} = -\beta \Delta t$. There is a minus sign because ΔN is a decrease in the value given by N. If Δt becomes infinitesimally small, then the previous equation has a known solution by integration:

$$N(t) = N_0 e^{-\beta \Delta t} \tag{1}$$

where N_0 is the number of nuclei at time $t = 0$, and β is the decay rate. When a nucleus decays, an ionizing particle is detected by the Geiger-Mueller tube, so the rate of which these ionizing particles are detected is equal to the rate of decay, which is:

$$-\frac{d}{dt}N(t) = \beta N_0 e^{-\beta t} = \beta N(t) \tag{2}$$

Since the process of decay is completely random, a different number of emissions will be counted in a each corresponding time intervals Δt. Thus, a Poisson distribution (equation 3) will result in the probability of a number of particles m being detected:

$$P(m) = e^{-\lambda} \frac{\lambda^m}{m!} \tag{3}$$

where λ is the mean number of particles detected, which serves as a constant.

1.2 Geiger-Mueller Tube

A Geiger-Mueller tube (shown in figure 2), or G-M tube, is used in the experiments to detect the gamma rays that are emitted by Cesium-137. A G-M tube is a conducting cylinder that has a thin wall that is filled with a rare gas (argon), and also has a thin, fine wire that runs down the center of the cylinder. The medium in the cylinder is a gas due to its poor ability to stop gamma rays, although gamma rays do have a small probability of interacting with the gas. A high voltage is applied between the central fine wire and the wall of the cylinder, in order to create a positive potential difference with respect to the wall. This will make the fine central wire the anode and the wall of the cylinder the cathode.

A G-M tube is triggered when an ionizing particle, in this case it is a gamma ray, passes through the cylinder wall and ionized the gas, which frees electrons, of which are attracted to the fine central wire in the cylinder. The electrons that are approaching the central wire may gain enough energy to create addition ionization by collisions, of which also accelerate to the central wire and may also create additional ionization. At a low value for the high voltage, the G-M tube behaves like a proportional counter. This means that the number of electrons that reach the central wire in the G-M tube is proportional to the initial energy that the gamma ray has. As the high voltage is increased, the number of electrons the reach the central wire also increases. However, if the high voltage gets too high, positive ions surround the central wire and reduce out the anode's electric field. This results in a saturation of electrons, which may cause multiple pulses or an electrical breakdown.

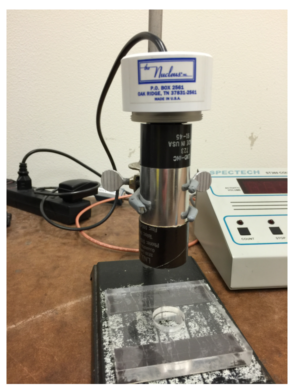

Figure 2: This shows the Geiger-Mueller tube that was used in the experiments.

1.3 Pulse Shape Observation, Internal Capacitance and Internal Resistance, Charge, and Number of Electrons

A Geiger-Mueller tube, which is in a circuit with an oscilloscope and a SPECTECH (see figure 3), was used to detect the emitted gamma ray from the beta decay of the Cesium-137 source. This circuit configuration has an intrinsic resistance and an intrinsic capacitance. When the Geiger-Mueller tube detects a gamma ray, the SPECTECH counter will trigger and start to count the detections, but only if the negative pulses exceed about -0.40 volts, and the pulse will be displayed on the screen of the oscilloscope. The purpose of this experiment is to understand the relationship between amplitude of the pulses and the count rate to the voltage of the Geiger-Mueller tube, or high voltage, that ranged from 400 volts to 900 volts, as well as to find the intrinsic resistance, the intrinsic capacitance, the total charge and the number of electrons that is collected by the anode of the Geiger-Mueller tube by attaching a known capacitor or a known resistor in parallel to the circuit with the G-M tube, SPECTECH and oscilloscope. The high voltage did not exceed 900 volts in order to prevent a continuous discharge of the Geiger-Mueller tube.

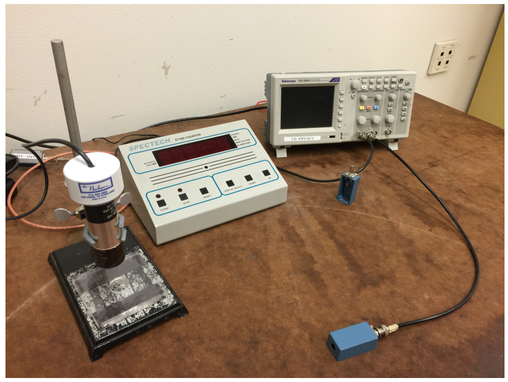

Figure 3: This is the experimental set up that was used for the experiments. To the left is the Geiger-Mueller tube. In the middle in the SPECTECH device, which is used to measure the counts that are detected by the Geiger-Mueller tube, as well as to set the voltage to the Geiger-Mueller tube. To the right is the oscilloscope, which shows the voltage of the detected gamma rays from the Geiger-Mueller tube.

1.4 Absorption of Gamma Rays

Gamma rays may pass through some materials without gradually losing energy, until the strongly interact with one atoms or electron at random. It is expected that an individual gamma ray may be scattered from its path, completely absorbed, or will continue along its path unaffected. If a gamma ray is scattered, it may either increase or decrease the count rate in the detector that is being used. However, gamma rays lose energy that is dependent on the type of material it is passing through, and the density of that material. The change in intensity ΔI of a gamma ray that passes through a material of thickness Δx is $\Delta I = -\mu I \Delta x$, where μ is the absorption coefficient. The gamma rays in this experiment are mono-energetic gamma rays, so Δx is $\Delta I = -\mu I \Delta x$ may be integrated to yield:

$$I = I_0 e^{-\mu x} \tag{4}$$

On top of that, we will use a Graded Z Shield (see figure 4) and observe the absorption qualities of this material. The graded Z shield is composed of tantalum, tin and stainless steel. When a gamma ray is incident upon one of the materials of the graded Z shield, there is a probability that it may be absorbed by an atom and emit a photo-electron, which does not travel to far since all of its energy transforms to heat. However, when a photo-electron is emitted , an electron in an orbital state of higher energy replaces it, which causes a photon of lower energy than the incident gamma ray to be produced and may go in any direction. This fluorescence photon has a small probability of being absorbed by the same material, but it will encounter a new material with a lower atomic number and has a higher probability of being absorbed, which repeats the process again. The result of this process creates photons of lower energy than the incident gamma rays. The graded Z shield we will use does this process stated above three times, with the resulting emitted photons with a lower energy than the incident gamma ray and scattered in many directions. This will reduce the count rate that is detected.
The purpose of this experiment is to determine the absorption coefficient μ of lead and to determine the dependence of the absorption coefficient μ on the gamma ray energy, as well as to observe what happens to the count rate when the gamma ray that are emitted from Cesium-137 interact with a graded Z shield.

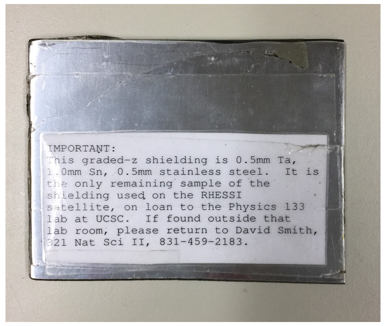

Figure 4: This is an image of the graded Z shield that was used in the experiment.

2 Procedure

2.1 Preliminary Experiments

2.1.1 Pulse Shape

The Geiger-Mueller tube, oscilloscope and the SPECTECH was connected in a circuit configuration that is shown in figure 5. The Cesium-137 source was placed underneath the Geiger-Mueller tube with the label side pointing away from the G-M tube, which results in only gamma rays being emitted and detected by the G-M tube. Whenever the G-M tube detects a gamma ray, a pulse will be shown on the oscilloscope (see figure 6). The amplitude of the pulse was measured with the measure tools that are available on the oscilloscope. Also, when a gamma ray is detected by the G-M tube, the SPECTECH counter will trigger, but only if the negative pulses exceed about -0.40 volts. When the SPECTECH counter is triggered, it will start to count the total number of detected gamma rays that are picked up by the G-M tube. The high voltage value started at 400 volts and went to 900 volts, with increments of 20 volts, and the amplitude of each corresponding pulse and the corresponding count rate at each high voltage, which was obtained by taking the count on the SPECTECH and divided it by the time of the interval the SPECTECH measured the counts for, were measured. Two graphs were then created to help compare the amplitude of the resulting pulses and the count rate to the high voltages. One graph was the amplitude of the pulse on the y-axis with the high voltage values on the x-axis. The other graph was the count rate on the y-axis with the high voltage values on the x-axis.

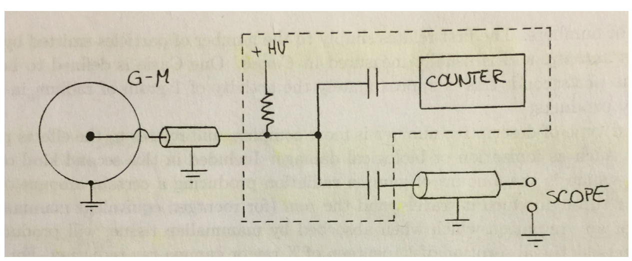

Figure 5: This circuit diagram resembles the configuration that the Geiger-Mueller tube (G-M), SPECTECH (counter), and the oscilloscope (scope) were in during the experiment (Figure 7.2 from Physics 133 reader for University of California, Santa Cruz).

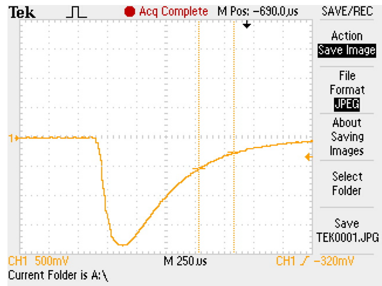

Figure 6: This shows a pulse on the screen of the oscilloscope. When the gamma ray penetrates the Geiger-Mueller tube's wall and ionizes the gas to free an electron, the freed electron is detected by the G-M tube, which is shown when there is a sharp change in voltage on the oscilloscope to a local minimum, which is the amplitude of the pulse, and then is followed by an exponential decay of the detected electron. This is known as the "dead time", which is the time that is needed for the high voltage of the Geiger-Mueller tube to be re-established. A particle passing through the G-M tube during this time frame will not be detected.

2.1.2 Intrinsic Properties

The Geiger-Mueller tube and the oscilloscope have capacitance's C_G and C_o, respectively, and the oscilloscope has a 1 megaohm resistance. In figure 7, the circuit diagram is simplified to an equivalent circuit of the G-M apparatus, where R is effective parallel resistance and C is the effective parallel capacitance.

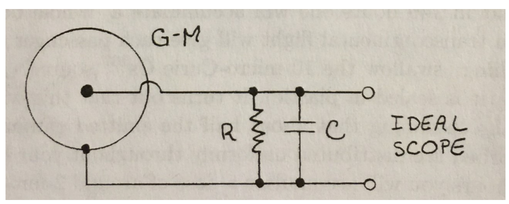

Figure 7: This circuit is equivalent to that of which is shown in figure 5 for the G-M apparatus. R is the effective parallel resistance that is given by the parallel configuration of the charging resistance and input resistance of the oscilloscope. C is the effective parallel capacitance that is given by the parallel configurations of the Geiger-Mueller tube, the two coaxial cables, the input capacitance to ground of the SPECTECH, and the input capacitance of the oscilloscope. This configuration will help make calculations easier to find the intrinsic resistance and the intrinsic capacitance of the G-M apparatus (Figure 7.3 from Physics 133 reader for University of California, Santa Cruz).

When the G-M tube detects a pulse, the anode gains a charge Q, which charges the various parallel capacitors, whose sum is C. The charge then discharges through the parallel resistors, whose sum is R. The detected pulse from the G-M tube creates a voltage that decays exponentially, with a time constant γ equal to RC and a peak amplitude V_0 equal to $\frac{Q}{C}$. However, the equation $V(t) = V_0 e^{-\frac{t}{\gamma}}$ was used to find the time constant γ. We chose a voltage V_0 on the line shown on the oscilloscope that represents the exponential decay of the detected pulse from the G-M tube by using the cursors in the measurement feature of the oscilloscope to obtain the value. By taking that selected V_0 value and dividing it by e^1, it results in a second voltage point on the experiential decay line. Thus the formula now looks like $\frac{V_0}{e^1} = V_0 e^{-\frac{t}{\gamma}}$. Through algebra, we found that:

$$\begin{aligned}
\frac{V_0}{e^1} &= V_0 e^{-\frac{t}{\gamma}} \\
\frac{V_0}{e^1 V_0} &= e^{-\frac{t}{\gamma}} \\
ln(\frac{1}{e^1}) &= ln(e^{-\frac{t}{\gamma}}) \\
-1 &= -\frac{t}{\gamma} \\
\gamma &= t
\end{aligned}$$

The time constant is equivalent to the time the is between V_0 and $\frac{V_0}{e^1}$, which is measured by the cursors in the oscilloscope. A known capacitance C_K of $1.71 \pm 0.01nF$ was connected in parallel with the output of the G-M apparatus. This will result in the time constant γ_C to be equal to $R(C + C_K)$, and the amplitude V_{0C} will be equal to $\frac{Q}{(C+C_K)}$. The known capacitor was then replaced with a known resistor R_K of value $9.90 \pm 0.01k\Omega$, which was also in parallel with the output of the G-M apparatus. This will result in the time constant γ_R to be equal to $C(\frac{RR_K}{R+R_K})$, while the amplitude V_{0R} will remain as $\frac{Q}{C}$. With these equations, and some of the values, known to us, we are able to find the intrinsic resistance, the intrinsic capacitance, the charge Q, and the number of electrons that are in the G-M tube. *The known resistor was kept attached for the rest of the experiment*

2.1.3 Background Count Rate

With the Cesium-137 source underneath the Geiger-Mueller tube, the high voltage was adjusted so that the pulses are at least -0.8 volts in amplitude, which turned out to be a high voltage of 820 volts. The amplitude of the pulse was measured by the measuring tools available on the oscilloscope. Then the Cesium-137 was moved far away from the Geiger-Mueller tube so that the only detections the G-M tube will pick up will be mainly from cosmic rays and other background radiation. With the high voltage still at 820 volts, we set the timer on the SPECTECH to run for 600 seconds. In that 600 second interval, the SPECTECH will collect all of the counts from the backgrount radiation. The background count rate will be calculated by taking the value of the total number of counts divided by 600 seconds. This value will be of significance for later experiments.

2.1.4 Count Distribution

The Cesium-137 was placed at a distance so that 5 to 10 counts per second were detected. The counts were recorded for one-hundred one second intervals, and the frequency of each detected emission was recorded. This frequency distribution was compared to a Poisson distribution. A Chi Squared analysis was performed on the data by:

$$\chi^2 = \Sigma_{i=1}^{\nu} \frac{(x_i - \mu_i)^2}{\sigma_i^2} \tag{5}$$

A χ^2 "goodness of fit" test was used to determine if the Poisson distribution is reasonable for collected data.

2.1.5 Dead Time

There is a small amount of time after a detection, where the G-M tube does not release a voltage pulse after a detection, as shown in figure 8, which is known as "dead time" and it can be measured in two ways. One way is with the help of the oscilloscope. The Cesium-137 was placed really close to the Geiger-Mueller tube, in order to detect as many emissions as possible. The oscilloscope was then set to record and persist and patterns that show on the screen, which means that the oscilloscope will keep every pulse detection on its screen.

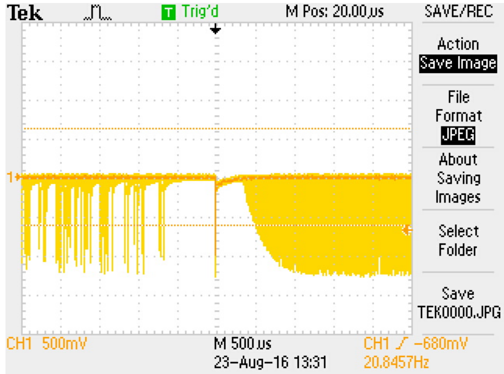

Figure 8: This it the dead time of the Geiger-Mueller tube that is shown by the oscilloscope. The spike in the center is a detected gamma ray. As you can see, there is a small amount of time after that detection where the G-M tube does not pick up an emissions from the cesium-137 source.

Also, dead time can be measured by seeing how many emissions are not detected by the Geiger-Mueller tube. The dead time τ is calculated by:

$$\tau = \frac{1}{n_{ab}}\left(1 - \left(1 - \frac{n_{ab}(n_a + n_b - n_{ab})}{n_a n_b}\right)^{\frac{1}{2}}\right) \tag{6}$$

where n_a is the count rate of one cesium-137 source, call it cs_1, n_b is the count rate of another cesium-137 source, call it cs_2, and n_{ab} is the count rate of both cs_1 and cs_2 together. n_a was measured by placing cs_1 underneath the Geiger-Mueller tube, such that half of the Geiger-Mueller tube covered half of it. The SPECTECH was set so that it counted the detected emissions for ten seconds, which allowed us to calculate the count rate, Then n_{ab} was measured by keeping the cs_1 in the same place, but adding cs_2 next to cs_1, so the the other half of the Geiger-Mueller tube covered half of cs_2. The SPECTECH was set so that it counted the detected emissions for ten seconds, which allowed us to calculate the count rate. n_b was measured by removing cs_1 and keeping cs_2 in the same place, and then setting the SPECTECH to count the detected emissions for ten seconds, which allowed us to calculate the count rate.

2.2 Absorption of Gamma Rays

2.2.1 Lead

In order to determine the absorption coefficient μ of lead, we measured the count rate while varying the thickness of lead that is between the source, cesium-137, and the Geiger-Mueller tube. The thickness of lead varied from 0mm to about 30mm, with increasing increments of about 3mm, but one measurement with 1mm of thickness taken. To reduce error, our measurement were taken by waiting for the count on the SPECTECH to reach approximately 1000, and then the time was recorded, which will give us a count rate. A graph of the natural logarithm of intensity versus the thickness of lead was created, in which the slope of this graph should yield the absorption coefficient.

2.2.2 Graded Z Shield

The absorption of the graded Z shield was testing in multiple configurations to see how each configuration interacted with the gamma rays that were being emitted by the cesium-137. Four configurations were used: one with the graded Z shield place on top of the 1mm of lead with the shiny side of the graded Z shield facing the G-M tube, one with the graded Z shield place on top of the 1mm of lead with the shiny side of the graded Z shield facing away from the G-M tube, one with the graded Z shield by itself with the shiny side facing the G-M tube, and one with the graded Z shield by itself with the shiny side facing away from the G-m tube. It should be noted that the 1mm thick piece of lead and the graded Z shield were placed in-between the cesium-137 and the G-M tube. Our hypothesis is that the graded Z shield with the shiny side facing the G-M tube will result in a lower count rate since the gamma rays that are emitted from the cesium-137 will travel through the graded Z shield in order of decreasing atomic number (Z).

3 Results

3.1 Error Analysis

Recall that an experimental result is expresed in the form $\bar{x} \pm \sigma_{\bar{x}}$, where $\bar{x}$ is the measurement and $\sigma_{\bar{x}}$ represents the error. Error propagation for a function f with a variable x that has an uncertainty is defined to be $\sigma = |\frac{\partial f}{\partial x}\sigma x|$. Suppose there is a function f with multiple independent variables, such as $a = f(x, y, ...)$. The error propagation is found through the equation

$$\sigma_a^2 = (\frac{\partial f}{\partial x})^2\sigma_x^2 + (\frac{\partial f}{\partial y})^2\sigma_y^2 + \dots \tag{7}$$

Since we have error in our measurements, we need to propagate the error through our calculations.

While using the lead sheets, we kept in mind the uncertainty in the measurements associated with them There are uncertainties in the length, width, thickness, and mass of the lead sheets. The density of lead is given as $11.32\frac{g}{cm^3}$, and density ρ is equal to mass divided by volume of the object. Since the lead sheets are rectangular, the volume of the lead sheets may be approximated by the formula $V = lw\Delta x$, where l is the length, w is the width, and Δx is the thickness. Thus we may equate this with the density, resulting in $\rho = \frac{m}{lw\Delta x}$. The thickness of the lead may be found by $\Delta x = \frac{m}{lw\rho}$ with the error propagation, with use of equation 7, being:

$$\sigma_{\Delta x} = \sqrt{\left(\frac{\sigma_m}{lw\rho}\right)^2 + \left(\left(\frac{m^2}{l^4w^2\rho^2}\right)\sigma_l^2\right)^2 + \left(\left(\frac{m^2}{l^2w^4\rho^2}\right)\sigma_w^2\right)} \tag{8}$$

Since the decay of a particle is random, the uncertainty of a count N is $\sqrt{N}$. The count rate, or counts per time, yield an intensity I. Later on, there will be two intensities; one intensity I_0 is without lead sheets between the cesium-137 and the G-M tube, while the other intensity I is with lead sheets between the cesium-137 and the G-M tube. If we call N_0 the number of counts without lead sheets in a time interval t, then $I_0 = \frac{N_0}{t}$, and the error propagation using equation 7 will be:

$$\sigma_{I_0} = \frac{\sqrt{N_0}}{t} \tag{9}$$

While on the other hand, if we let N be the number of counts with lead sheets in a time frame t, then $I = \frac{N}{t}$, and the error propagation using equation 7 will be:

$$\sigma_I = \frac{\sqrt{N}}{t} \tag{10}$$

The intensity after a beam passes through a material of thickness x is given by $I = I_0e^{-\mu x}$. By use of algebra, the absorption coefficient is calculated by $\mu = \frac{ln(I_0)-ln(I)}{x}$, with the resulting error propagation being calculated by:

$$\sigma_\mu = \sqrt{\left(\frac{\sigma_{I_0}}{I_0x}\right)^2 + \left(\frac{\sigma_I}{Ix}\right)^2 + \left(\frac{ln(I)-ln(I_0)}{x^2}\right)^2\sigma_x^2} \tag{11}$$

Since multiple absorption constants will be calculated, along with the corresponding error through error propagation, the weighted mean and its error propagation is calculated by:

$$\bar{x} = \frac{\Sigma_{k=1}^n \frac{x_k}{\sigma_k^2}}{\Sigma_{k=1}^n \frac{1}{\sigma_k^2}} \qquad \sigma_{\bar{x}} = \sqrt{\frac{1}{\Sigma_{k=1}^n \frac{1}{\sigma_k^2}}} \tag{12}$$

3.2 Preliminary Experiments

3.2.1 Pulse Shape

The count rate and the amplitude of the pulses detected by the G-M tube are shown in table 5 (see appendix) as the high voltage was increased from 400 volts to 900 volts by increments of 20 volts. The graph of the amplitude of the pulse versus the high frequency is shown in figure 9. No pulses were detected until approximately $740V$. This is due to the counter on the SPECTECH, which will trigger only if the negative pulses exceed about -0.40 volts. This means that even though the G-M tube is detecting emitted gamma rays from the cesium-137, those detected pulses are less that -0.40 volts, which will not trigger the SPECTECH to start counting the detections.

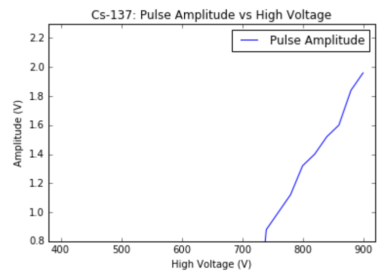

Figure 9: The high voltage measurements is shown on the x-axis. The amplitude of the corresponding pulse is shown on the y-axis. No pulse amplitude, or change in voltage in the G-M tube, was detected until around $740V$ for the high voltage. This figure shows that there is a linear relationship, such that when the high voltage is increases, the amplitude of the detected pulse is also increased.

The graph of the count rate versus the high voltage is shown in figure 10. Again, no pulses were detected until approximately 740 volts. The graph shows that the count rate increases as the high voltage, but around 820 volts for the high voltage and on, the count rate seems to flatten out. Also, it is shown that the count rate is not precise; that it is random. This is shown especially around the 840V through 900V interval, where the count rate fluctuates.

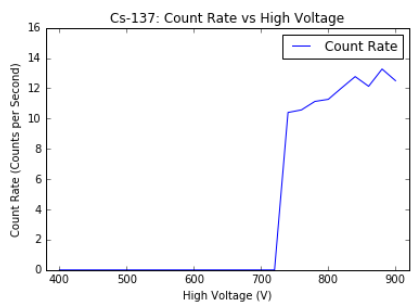

Figure 10: The high voltage is shown on the x-axis. The count rate is shown on the y-axis. No pulse amplitude, or change in voltage in the G-M tube, was detected until around $740V$ for the high voltage. The count rate seems to flatten out around $820V$ to 900, while the count rate seems to increase from $740V$ to $820V$.

3.2.2 Intrinsic Properties

We had a capacitor C_K and a resistor R_K, whose values were known. Their values were measured with a multimeter. The capacitor had a capacitance of $1.71 \pm 0.01nF$ and the resistor had a resistance of $9.90 \pm 0.01k\Omega$. The time constant(s) (γ, γ_C, γ_R) were measured with the formula $\frac{V_0}{e^1} = V_0 e^{-\frac{t}{\gamma}}$. The measured values V_0, $\frac{V_0}{e^1}$, t, and the pulse amplitude V_A are shown in table 6 (see appendix). However, it was found that:

$$\begin{aligned}
\frac{V_0}{e^1} &= V_0 e^{-\frac{t}{\gamma}} \\
\frac{V_0}{e^1 V_0} &= e^{-\frac{t}{\gamma}} \\
ln(\frac{1}{e^1}) &= ln(e^{-\frac{t}{\gamma}}) \\
-1 &= -\frac{t}{\gamma} \\
\gamma &= t
\end{aligned}$$

Thus, the time constant (γ, γ_C, γ_R) is equivalent to the time that is between V_0 and $\frac{V_0}{e^1}$. With the values of each time constants known, as well as the known values for both the known capacitor and resistor, the intrinsic capacitance by $C = \frac{\gamma_C}{R} - C_K$ and $C = \frac{\gamma}{R}$, the intrinsic resistance was calculated by $R = \frac{\gamma_C - \gamma}{C_K}$, the charge was calculated by $Q = V_0 C$ and $Q = A_C(C + C_K)$, and the number of electrons were found by dividing the charge the the charge of an electron e, which is 1.602x10^{-19} C. Then the average for each value was taken, since multiple values for the intrinsic resistance, intrinsic capacitance, charge, and number of electrons were calculated. The calculated values are shown in table 1.

Table 1: Intrinsic Resistance, Intrinsic Capacitance, Charge, and Number of Electrons

	Intrinsic Resistance (Ω)	Intrinsic Capacitance (F)	Charge (C)	Number of Electrons
Average Value	4.95x10^{5}	8.83x10^{-10}	1.83x10^{-9}	1.42x10^{10}

3.2.3 Background Count Rate

The cesium-137 was placed underneath the G-M tube and the high voltage was set to 820 volts in order to obtain pulse amplitudes of around -0.80 volts. The cesium-137 was then moved far away from the G-M tube. The timer on the SPECTECH was set to go to 600 seconds in order to get a reasonable reading of the background radiation. After the 600 seconds, the SPECTECH counted a total of 264 counts. The count rate was calculated by dividing 264 counts by 600 seconds. The background count rate was determined to be 0.44 $\frac{counts}{second}$.

3.2.4 Count Distribution

The frequency distribution of the counts during one second intervals are shown in table 2.

Table 2: This table shows the counts m that were recorded in 100 one second intervals and the number of times each count occurred $F(m)$.

Count (m)	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Number of One Second Intervals Showing m Counts	3	3	6	10	16	13	17	8	4	9	7	3	0	1

A graph of the number of occurrences a count was recorded $F(m)$ versus the count m was made. The result, shown in figure 11, looks similar to a Poisson distribution, which was calculated using equation 3, where λ is the average count $\bar{m}$ and m the count. The value λ in the Poisson distribution was calculated to be 10.61. The comparison between $F(m)$ vs m and the Poisson distribution is shown in figure 11.

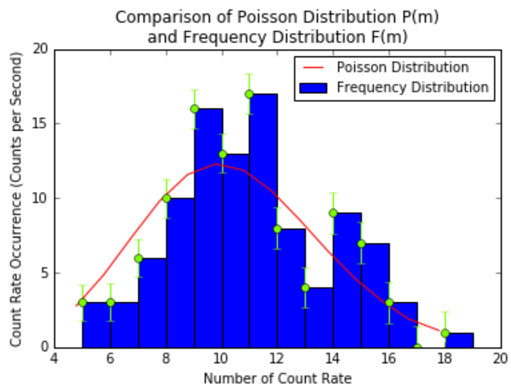

Figure 11: This graph is a comparison between the frequency distribution of the counts that were recorded in 100 one second intervals and a Poisson distribution of the same data. As you can see, the frequency distribution of the counts looks similar to a Poisson distribution, with a couple notable differences. Also, you can see that the average count in around 10.

A χ^2 "goodness of fit" was then performed to compare the observed frequency distribution $(F(m) vs m)$ to the expected frequency distribution $100P(m)$. The χ^2 value was found using equation 5 and was calculated to be equal to 12.15, with $\nu = 14$ degrees of freedom. The chi-square distribution was then calculated, shown in figure 12, of which we can see that as the degrees of freedom increase, the chi-square distribution tends to look more Gaussian. The area under the chi-square distribution, which is found by $\int_{\chi^2_{\nu,\alpha}}^{\infty} f(\chi^2) d\chi^2 = \alpha$, represents the probability α that a particular value of χ^2 falls between χ^2 and $\chi^2 + d(\chi^2)$, where χ^2_ν is median value.

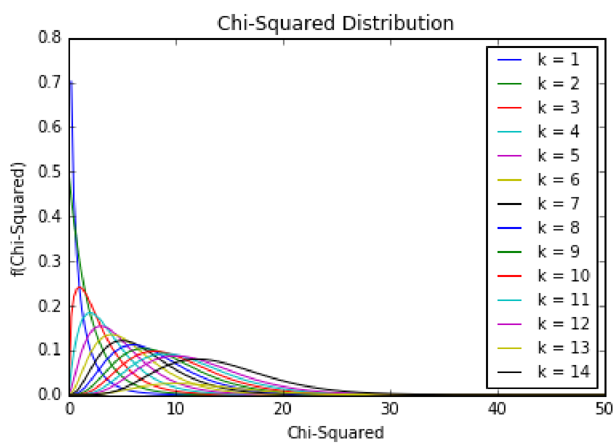

Figure 12: This shows the chi-square distribution of all of the degrees of freedom ν up to $\nu = 14$. As ν increases, the shape of the function tends to look more Gaussian. The area under the curve represents the probability α of a value showing up in that region.

Using the previous equation with $\chi^2_{14} = 12.15$ as our lower limit resulted in $\alpha = 0.59$, which is near $\alpha = 0.50$. This means that our observed frequency distribution is reasonable and is indeed similarly resembled by the Poisson distribution. Furthermore, we set $\alpha = 0.15$ and calculated the lower limit of the equation $\int_{\chi^2_{\nu,\alpha}}^{\infty} f(\chi^2) d\chi^2 = \alpha$, in which it turned out to be 19.41. This value is greater than our calculated value $\chi^2 = 12.15$. Also, $\frac{\chi^2}{\nu} = 0.87$ is close to 1. Based on the χ^2 "goodness of fit" test, we accept that our observed frequency distribution of our recorded counts in 100 one second intervals is similar to the Poisson distribution of the same recorded data.

3.2.5 Dead Time

We brought up the voltage pulse on the screen of the oscilloscope and measured the dead time of the G-M tube by using the measuring tools on the oscilloscope. The dead time was measured to be around $480.0 \pm 250\mu s$. Two sources of cesium-137, #4 and #6 (from now on referred as A and B, respectively), were used to help calculate the dead time by equation 6. The count rate, with the background count rate subtracted, for each source and both combined is found in table 3.

Table 3: Count Rates for A, B, and AB

Source	Count (for 10 seconds)	Count Rate (Counts per Second)
A	2257	225.26 ± 4.75
B	1880	187.56 ± 4.34
AB	3858	385.36 ± 6.21

By using equation 6 and the values in table 3, the dead time τ was calculated to be $348.4 \pm 250\mu s$. Our calculated value of τ and the measured value of τ from the oscilloscope differ in value by 27.4%. However, the measured value for dead time is within its uncertainty of the calculated value for dead time. This is due to inaccurate measurements of the dead time on the oscilloscope and the error in the measurements for both the calculated dead time and the measured dead time, and for the count rate of A, B, and AB.

However, the experiments are not significantly affect by the dead time. To show this, the highest count and count rate was used from table 2, which was data that was taken from the examining the count distribution. This value has a count of 17.0 in one second, which is a count rate of $17.00\frac{counts}{second}$. For an one second, the G-M tube will be dead for $n\tau$ seconds, where n is the pulse per second and τ is the dead time. Then the number of counts that are detected during this time interval is $n = (1 - n\tau)m$, where m is the particles per second. We may find m by the equation $m = \frac{n}{1-n\tau}$. In this case, $n = 17.00\frac{pulses}{second}$, which results in $m = 17.10$. The error corresponding to n and m is ± 4.12 and ± 4.13 respectively. As you can see, these values are so similar that they might be rounded due to significant figures. Keep in mind that this for a large count and count rate. With lower count and count rates, these values result in an even less effect from the dead time. None of the experiments that involve count and count rate will be significantly affected by the dead time.

3.3 Absorption of Gamma Rays

3.3.1 Lead

The mass, length and width of various sheets of lead were measured, which is shown in table 7, in order to find the thickness of the lead sheet being measure. The thickness was calculated by $\Delta x = \frac{m}{lw\rho}$, where ρ is the density of lead ($11.34\frac{g}{cm}$). The error propagation for the thickness of each lead sheet is calculated by equation 8.

The count rates, or intensity, at various thickness of lead are shown in table 8. There was an increase in the count rate when we added a $1mm$ thick sheet of lead between the cesium-137 and the G-M tube, which can bee seen in figure 13. This is due to Compton scattering. Compton scattering is when a photon passes through a material and are scattered in random directions. If more of the photons are scattered toward the detector of the G-M tube, the count rate will increase. If less of the photons are scattered away from the detector of the G-M tube, the count rate will decrease. Also, Compton scattering could also release a high energy beta ray towards the detector of the G-M tube, which would also help increase the count rate. However, as the thickness of the lead increases between the cesium-137 and the G-M tube, the Compton scattering affect will decrease due to the exponential decay of the intensity, which in turn prevents the emission of the high energy beta rays.

The absorption coefficient μ of lead is found by the equation $I = I_0 e^{-\mu x}$, where I_0 is the intensity without any lead sheets between the cesium-137 and the G-M tube, and x is the thickness of all of the lead sheets between the cesium-137 and the G-M tube. However, the background count rate was subtracted from the intensity, or count rate, in order to make our calculations more accurate. By taking the natural logarithm of both side, the previous because becomes

$$ln(I) = ln(I_0) - \mu x \tag{13}$$

When plotting a graph of $ln(I)$ versus the thickness of the lead sheets, $-\mu$ will be the slope and $ln(I_0)$ will become the y-intercept. By using the data in table 7, we graphed $ln(I) vs x$, as shown in figure 13. The natural log of the adjusted count rates were graphed with respect to the thickness of lead, and a linear regression was formed.

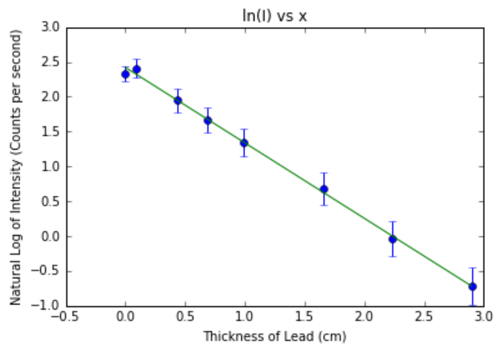

Figure 13: A linear function is formed when a graph of the natural log of the count rate (intensity) versus the lead thickness, as given by $ln(I) = ln(I_0) - \mu x$ from $I = I_0 e^{-\mu x}$. The slope of the linear regression is the negative of the absorption coefficient $-\mu$, and the y-intercept is the natural log of the count rate with no lead sheets between the cesium-137 and the G-M tube. In this figure, the lead thickness x is measured in cm, so the absorption coefficient in measured in cm^{-1}.

The absorption coefficient was found to be $1.08 \pm 0.30 cm^{-1}$. The theoretical gamma ray energy from a Cesium-137 atom is $0.662 MeV$. We then graphed a set of a few known values of μ and their related photon energies, given in table 7.1 that is shown in figure 15 (in the appendix), with the value of μ we measured and its photon energy of $0.662 MeV$, we were able to compare our measured μ to the expected value of μ at $0.662 Mev$. This comparison is show in figure 14.

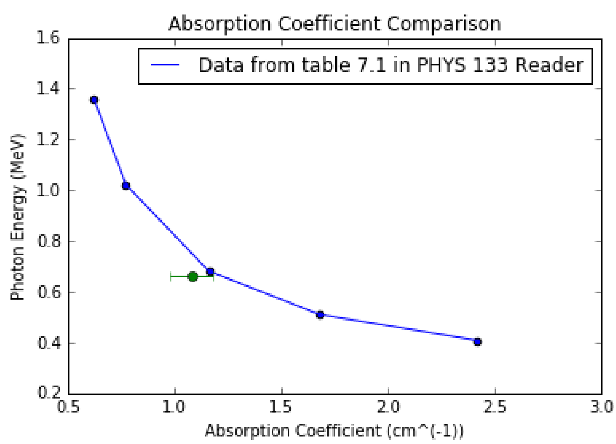

Figure 14: The points along the blue curve represent the few known values of μ and their corresponding photon energy for cesium-137, which is given in table 7.1 provided for us in the Physics 133 Reader, which is shown in figure 15. The green point is our measured value of μ with the theoretical photon energy of $0.662 MeV$. Our measured value is not on the theoretical line of photon energy , but it within its error, or uncertainty.

Our measured value of the absorption constant μ is not on the line that represents the theoretical values of photon energies of cesium-137 compared to the absorption constant. However, our measured value of μ

should be closer to a value of around $1.20cm^{-1}$, according to figure 14, but our measured value is withing its uncertainty from the theoretical value. This resulting error is due to inaccurate measurements of count rates and uncertainties, and the background radiation might have changed during the time of our experiment.

By visual inspection, we calculated the energy release by a photon released from the cesium-137 source we used to have 0.66 ± 0.01 MeV. The range of gamma ray energy that is accepted by our data is indeed 0.66 ± 0.01, which shows that the theory of the photon energy of table 7.1 in figure 15 is correct.

3.3.2 Graded-Z Shield

Four configurations for the graded-Z shield were used. These measurements are shown in table 4

Table 4: Graded-Z Shield Count Rates

Graded-Z Shield Configuration	Count Rate (counts per second)
Shiny Side Up with 1mm Lead Sheet	8.34 ± 0.26
Shiny Side Down with 1mm Lead Sheet	8.81 ± 0.28
Shiny Side Up with no Lead Sheet	8.27 ± 0.26
Shiny Side Down with no Lead Sheet	8.48 ± 0.26

The graded-Z shield is more effective, or has a lower count rate, when the shiny side of the graded-Z shield is facing up. This agrees with the theory of how graded-Z shields work, since the gamma rays are passing through it in order of decreasing atomic number, which significantly reduces the energy of the gamma rays, as well as scattering them in multiple directions.

The count rate is larger when the shiny side of the graded-Z shield is up and when the $1mm$ thick lead sheet is in place, compared to when the same lead sheet is not in place. This can be due to Compton scattering, even though this effect is reduced compared to just having the $1mm$ thick lead sheet in place. Also, the high energy beta rays that are produced from the Compton scattering after passing through the $1mm$ thick lead sheet are being absorbed by the graded-Z shield.

When the graded-Z shield has its shinny side down, the count rate is larger than when its shinny side is up. This is because when the gamma ray interacts with the graded-Z shield of increasing atomic number, the gamma rays to not go through the process of fluorescence three times, as it happens when the graded-Z shield has its shinny side up.

4 Conclusion

By attaching a known resistor and a known capacitor in parallel to the G-M apparatus, and know the respective time constants, the intrinsic resistance and the intrinsic capacitance of the G-M apparatus was calculated to be $4.95 \text{x} 10^5 \Omega$ and $8.83 \text{x} 10^{-10} F$, respectively. Also, the charge in the G-M tube was able to be determined to be $1.83 \text{x} 10^{-9}$, which results in $1.42 \text{x} 10^{10}$ electrons.

We are able to show that the number of particles that were detected due to the random gamma ray emissions in a time interval is describe by the Poisson distribution. A χ^2 "goodness of fit" test was performed, which indicated that our results are reasonable and accepted, even though there were some differences.

We were able to calculate some properties of the Geiger-Mueller tube that was used for the experiments. The dead time was estimated by measuring the time interval with no detection, shown in figure 8, on the oscilloscope with the use of the measuring tools available on the oscilloscope. Two sources of cesium-137 were used, by using the count rate of each source and them combines, in order to calculate the dead time to compare to the measured value on the oscilloscope. The calculated value of dead time was found to be $348.3 \pm 250.0 \mu s$.

We observed how the count rates are reduced when the thickness of lead between the cesium-137 source and the G-M tube is increased. By measuring the count rates and taking the natural logarithm of them, a linear regression was formed by a graph of $ln(I) vs x$, where x is the thickness of lead between the cesium-137 and the G-M tube. The slope of this linear regression is the absorption coefficient μ of lead for photons of $0.662 MeV$, which was found to be $\mu = 1.08 \pm 0.30 cm^{-1}$. Table 7.1 in the Physics 133 Reader for UCSC, which is shown in figure 15, gives known absorption coefficients for different photon energies. Using this information, we constructed a graph with the photon energy on the y-axis and the absorption coefficients on the x-axis. By comparing our measured value of the absorption coefficient and theoretical photon energy of $0.662 MeV$ to these values, it was found that our cesium-137 source had an energy of $0.66 \pm 0.01 MeV$, which the theoretical energy is withing. Our measurement of μ, however, is not exactly by the absorption coefficients the are predicted, but they are withing our measured absorption coefficient's uncertainty in measurement. This, therefore, confirms our measured value of the absorption coefficient μ is reasonable.

Finally, the graded-Z shield was observed to be more effective lowering the energy of the gamma rays and scattering them when placed with the shinny side up, or in decreasing atomic order, thus confirming our hypothesis. The effectiveness of the graeded-Z shield is reduced when the shinny side of the graded-Z shield is faced down. Also, the addition of the $1 mm$ thick lead sheet below the graded-Z shield did not significantly affect the count rate.

5 Appendix

Table 5: This table shows the measurements of the amplitude of a pulse and the count rate at each value of high voltage.

High Voltage (Volts)	Pulse Amplitude (Volts)	Count Rate (Counts per Second)
400	0	0
420	0	0
440	0	0
460	0	0
480	0	0
500	0	0
520	0	0
540	0	0
560	0	0
580	0	0
600	0	0
620	0	0
640	0	0
660	0	0
680	0	0
700	0	0
720	0	0
740	0.88	10.4
760	1.00	10.57
780	1.12	11.13
800	1.32	11.27
820	1.40	12.03
840	1.52	12.77
860	1.60	12.13
880	1.84	13.27
900	1.96	12.50

Table 6: This table shows the measurements taken to help find the time constant of a detected pulse, which is used to calculate the intrinsic resistance, the intrinsic capacitance of the G-M apparatus, the charge built up on the anode of the G-M tube, and the corresponding number of electrons around that anode.

Type	First Voltage Point (V_0, volts)	Second Voltage Point ($\frac{V_0}{e^1}$, volts)	Δt (seconds)	Pulse Amplitude (V_A, volts)
No external Resistor or Capacitor Connected	-1.44 ± 0.20 V	-0.53 ± 0.08 V	(4.55 ± 2.00)x10^{-6}s	-1.84 ± 0.20 V
	-1.00 ± 0.20 V	-0.37 ± 0.08 V	(4.30 ± 2.00)x10^{-6}s	-1.80 ± 0.20 V
	-1.04 ± 0.20 V	-0.38 ± 0.08 V	(4.15 ± 2.00)x10^{-6}s	-1.96 ± 0.20 V
	-1.44 ± 0.20 V	-0.53 ± 0.08V	(4.55 ± 2.00)x10^{-6}s	-1.84 ± 0.20 V
	-1.20 ± 0.20 V	-0.44 ± 0.08 V	(4.15 ± 2.00)x10^{-6}s	-1.78 ± 0.20 V
Capacitor Connected	-0.50 ± 0.20 V	-0.18 ± 0.08 V	(1.28 ± 0.02)x$10^{-3}s$	-0.84 ± 0.20 V
	-0.50 ± 0.20 V	-0.18 ± 0.08 V	(1.32 ± 0.02)x$10^{-3}s$	-0.76 ± 0.20 V
	-0.50 ± 0.20 V	-0.18 ± 0.08 V	(1.24 ± 0.02)x$10^{-3}s$	-0.78 ± 0.20 V
	-0.50 ± 0.20 V	-0.18 ± 0.08 V	(1.32 ± 0.02)x$10^{-3}s$	-0.80 ± 0.20 V
	-0.50 ± 0.20 V	-0.18 ± 0.08 V	(1.24 ± 0.02)x$10^{-3}s$	-0.74 ± 0.20 V
Resistor Connected	-0.82 ± 0.20 V	-0.30 ± 0.08 V	(5.60 ± 2.00)x10^{-6}s	-1.20 ± 0.20 V
	-0.80 ± 0.20 V	-0.29 ± 0.08 V	(6.00 ± 2.00)x10^{-6}s	-1.24 ± 0.20 V
	-0.80 ± 0.20 V	-0.29 ± 0.08 V	(6.20 ± 2.00)x10^{-6}s	-1.30 ± 0.20 V
	-0.80 ± 0.20 V	-0.29 ± 0.08 V	(6.40 ± 2.00)x10^{-6}s	-1.32 ± 0.20 V
	-0.80 ± 0.20 V	-0.29 ± 0.08 V	(5.60 ± 2.00)x10^{-6}s	-1.24 ± 0.20 V

Table 7: Length, Width, Mass and Calculated Thickness of each Lead Sheet that was used

Lead Sheet	Length (cm)	Width (cm)	Mass (g)	Thickness Δx (cm)
# 16	7.50 ± 0.05	7.50 ± 0.05	58.9 ± 0.1	$0.09 \pm (0.88x10^{-3})$
# 18	10.12 ± 0.05	10.11 ± 0.05	200.1 ± 0.1	$0.17 \pm (0.12x10^{-2})$
# 25	10.18 ± 0.05	10.19 ± 0.05	202.6 ± 0.1	$0.17 \pm (0.12x10^{-2})$
# 6	7.54 ± 0.05	7.54 ± 0.05	114.4 ± 0.1	$0.18 \pm (0.17x10^{-2})$
# 9	7.33 ± 0.05	7.33 ± 0.05	54.5 ± 0.1	$0.09 \pm (0.87x10^{-3})$
# 15	10.06 ± 0.05	10.05 ± 0.05	96.9 ± 0.1	$0.08 \pm (0.60x10^{-3})$
# 16*	10.09 ± 0.05	10.09 ± 0.05	200.5 ± 0.1	$0.17 \pm (0.12x10^{-2})$
# 6(JT10)	9.95 ± 0.05	7.44 ± 0.05	39.8 ± 0.1	$0.05 \pm (0.42x10^{-3})$
# 4	10.12 ± 0.05	10.05 ± 0.05	776.9 ± 0.1	$0.67 \pm (0.47x10^{-3})$
# 2	10.32 ± 0.05	10.31 ± 0.05	1495 ± 0.1	$1.24 \pm (0.85x10^{-2})$

Table 8: Count measured in a time interval with a certain thickness of lead.

Thickness (cm) and Lead Sheet Configuration	Count N (error is $\sqrt{N}$)	Time (s)	Background Count Rate ($\frac{counts}{second}$)	Accounted for Intensity ($\frac{counts}{second}$)
0 cm No Lead Sheets	1070	100	0.44	10.26 ± 0.33
$0.09 \pm (0.88x10^{-3})$ # 16	1018	88	0.44	11.12 ± 0.36
$0.44 \pm (0.19x10^{-2})$ # 16, # 18, # 25	1087	146	0.44	7.01 ± 0.22
$0.69 \pm (0.27x10^{-2})$ # 16, # 18, # 25, # 6, # 9	1092	190	0.44	5.31 ± 0.17
$0.99 \pm (0.30x10^{=2})$ # 16, # 18, # 25, # 6, # 9, # 15, # 16*, # 6(JT10)	1002	235	0.44	3.82 ± 0.13
$1.66 \pm (0.31x10^{-2})$ # 16, # 18, # 25, # 6, # 9,# 15, # 16*, # 6(JT10), # 4	1001	416	0.44	1.97 ± 0.08
$2.23 \pm (0.90x10^{-2})$ # 16, # 18, # 25, # 6, # 9,# 15, # 16*, # 6(JT10), # 2	1000	716	0.44	0.96 ± 0.04
$2.90 \pm (0.90x10^{-2})$ # 16, # 18, # 25, # 6, # 9,# 15, # 16*, # 6(JT10), # 4, # 2	1000	1076	0.44	0.49 ± 0.03

Photon energy MeV	Photo-electric σ_{PE}	Comp-ton σ_C	Pair form-ation σ_{PF}	Total σ_T	Coefficient per cm, μ, cm^{-1}	Mass coefficient μ/ρ cm^2/gm
0.1022	1782	40.18		1822	59.9	5.30
0.1277	985	38.01		1023	33.6	2.97
0.1703	465	35.04		500	16.4	1.45
0.2554	161	30.70		192	6.31	0.558
0.3405	75.7	27.63		103.3	3.39	0.300
0.4086	47.8	25.74		73.5	2.42	0.214
0.5108	27.7	23.50		51.2	1.68	0.149
0.6811	14.5	20.73		35.2	1.16	0.102
1.022	6.31	17.14		23.45	0.771	0.0682
1.362	3.86	14.81	0.1948	18.87	0.620	0.0549
1.533		13.91	0.3313			
2.043	2.08	11.86	1.247	15.19	0.499	0.0442
2.633						
3.065		9.313	3.507			
4.086	0.369	7.761	5.651	14.28	0.469	0.0415
5.108	0.675	6.698	7.560	14.93	0.491	0.0434
6.130		5.917	9.119			
10.22	0.316	4.115	14.04	18.47	0.607	0.0537
15.32	0.206	3.042	18.00	21.25	0.698	0.0618
25.54	0.122	2.044	23.24	25.41	0.835	0.0739

Table 7.1: Values of the Absorption Coefficient for Lead ($10^{-24} cm^2/atom$).

Figure 15: This is table 7.1 on page 102 in the Physics 133 Reader for this class. This table was used in order to plot the theoretical photon energy with the corresponding absorption coefficient. Our measured absorption coefficient and its theoretical photon energy of $0.662 MeV$ was compared to a few of these values that were close to our photon energy of $0.662 MeV$ to compare the measured absorption coefficient to the theoretical absorption coefficient (Table 7.1 from Physics 133 reader for University of California, Santa Cruz).

References

[1] *Physics 133 Reader* for University of California, Santa Cruz