A.2 AI For Investment Management Team 4

Data Exploration

In [2]:
```
# Package Installation
# https://anaconda.org/conda-forge/hmmlear
# pip% install ____
```

In [1]:
```
# Importing the required Dataset and Libraries
from pypfopt import EfficientFrontier
from pypfopt import risk_models
from pypfopt import expected_returns
from pypfopt import HRPOpt
from pypfopt import plotting
from sklearn.model_selection import train_test_split
from hmmlearn import hmm
import copy
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
```

In [3]:
```
# read asset prices dataset from Github into a pandas dataframe
# (MODIFY file name for your group!)
fcsv = "https://raw.githubusercontent.com/multidis/hult-ai-investment-management/main/
df = pd.read_csv(fcsv, index_col=[0], parse_dates=['Date'])
df
```

Out[3]:

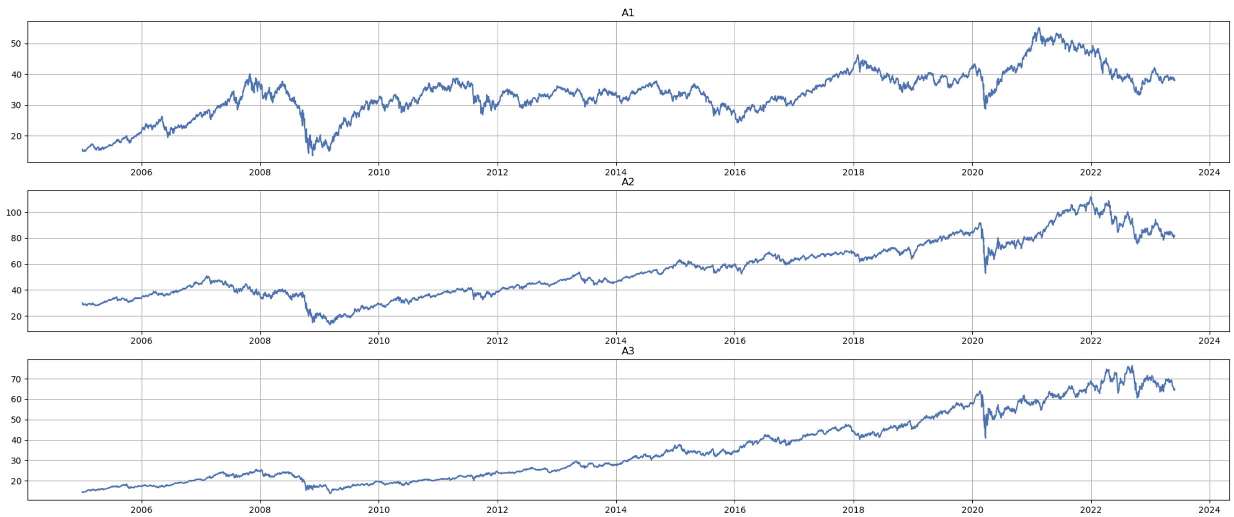
Date	A1	A2	A3
2005-01-03	15.559469	30.156376	14.526597
2005-01-04	15.080416	29.742468	14.426337
2005-01-05	14.895803	28.629606	14.199438
2005-01-06	14.885677	28.874975	14.273320
2005-01-07	14.914498	28.872492	14.273320
...	...	...	...
2023-05-24	38.101505	80.131714	65.470001
2023-05-25	38.022125	80.141663	64.589996
2023-05-26	38.696838	81.067291	64.589996
2023-05-30	38.250336	81.415657	64.339996
2023-05-31	37.893135	81.843636	64.930000

4634 rows × 3 columns

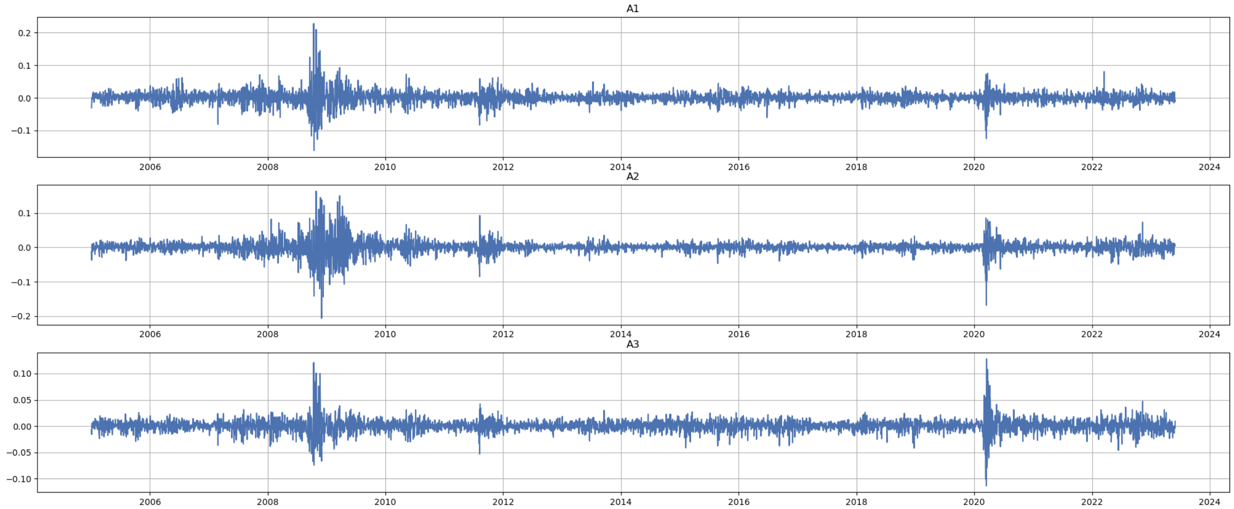
Analyzing the market

In [4]:
```
tickers = list(df.columns)
```

In [5]:
```
# asset prices
plt.figure(figsize = (25, 10))
plt.subplot(3,1,1)
plt.plot(df.index, df[tickers[0]])
plt.title(tickers[0])
plt.grid(True)
plt.subplot(3,1,2)
plt.plot(df.index, df[tickers[1]])
plt.title(tickers[1])
plt.grid(True)
plt.subplot(3,1,3)
plt.plot(df.index, df[tickers[2]])
plt.title(tickers[2])
plt.grid(True)
plt.show()
```



In [6]:
```
# daily returns
plt.figure(figsize = (25, 10))
plt.subplot(3,1,1)
plt.plot(df.index, df[tickers[0]].pct_change())
plt.title(tickers[0])
plt.grid(True)
plt.subplot(3,1,2)
plt.plot(df.index, df[tickers[1]].pct_change())
plt.title(tickers[1])
plt.grid(True)
plt.subplot(3,1,3)
plt.plot(df.index, df[tickers[2]].pct_change())
plt.title(tickers[2])
plt.grid(True)
plt.show()
```



In [7]:
```
def retcolumn(ticker):
    return df[ticker].pct_change().dropna().values
```

In [8]:
```
# excract numerical values only from pandas dataframe (numpy array)
X = np.column_stack((retcolumn(tickers[0]), retcolumn(tickers[1]), retcolumn(tickers[2
X
```

Out[8]:
```
array([[-0.03078855, -0.01372539, -0.00690181],
       [-0.01224186, -0.03741659, -0.01572812],
       [-0.0006798 ,  0.00857046,  0.00520317],
       ...,
       [ 0.01774528,  0.01154991,  0.        ],
       [-0.01153848,  0.00429724, -0.00387057],
       [-0.0093385 ,  0.00525671,  0.0091701 ]])
```

Modelling

In [9]:
```
# Learning the hidden states (HMM model fit)
model = hmm.GaussianHMM(n_components = 2, covariance_type = "diag", n_iter = 50, rando
model
```

Out[9]:
```
GaussianHMM(n_components=2, n_iter=50, random_state=15)
```

In [10]:
```
model.fit(X)
```

Out[10]:
```
GaussianHMM(n_components=2, n_iter=50, random_state=15)
```

In [11]:
```
# hidden states corresponding to observed X
Z = model.predict(X)
Z
```

Out[11]:
```
array([0, 0, 1, ..., 1, 1, 1], dtype=int64)
```

In [12]:
```
# distinct states
states = pd.unique(Z)
states
```

Out[12]:
```
array([0, 1], dtype=int64)
```

In [13]:
```
# hidden state transition probabilities
model.transmat_
```

Out[13]:
```
array([[0.76311961, 0.23688039],
       [0.04299079, 0.95700921]])
```

Defining status labels

High Volatility State: Market condition with large and frequent price fluctuations, indicating increased risk and uncertainty.

Low Volatility State: Market condition with small and infrequent price movements, indicating stability and reduced risk.

In [14]:
```
# Define status labels
states_dict = {0:'High volatility', 1:'Low volatility', 2:'Original'}
```

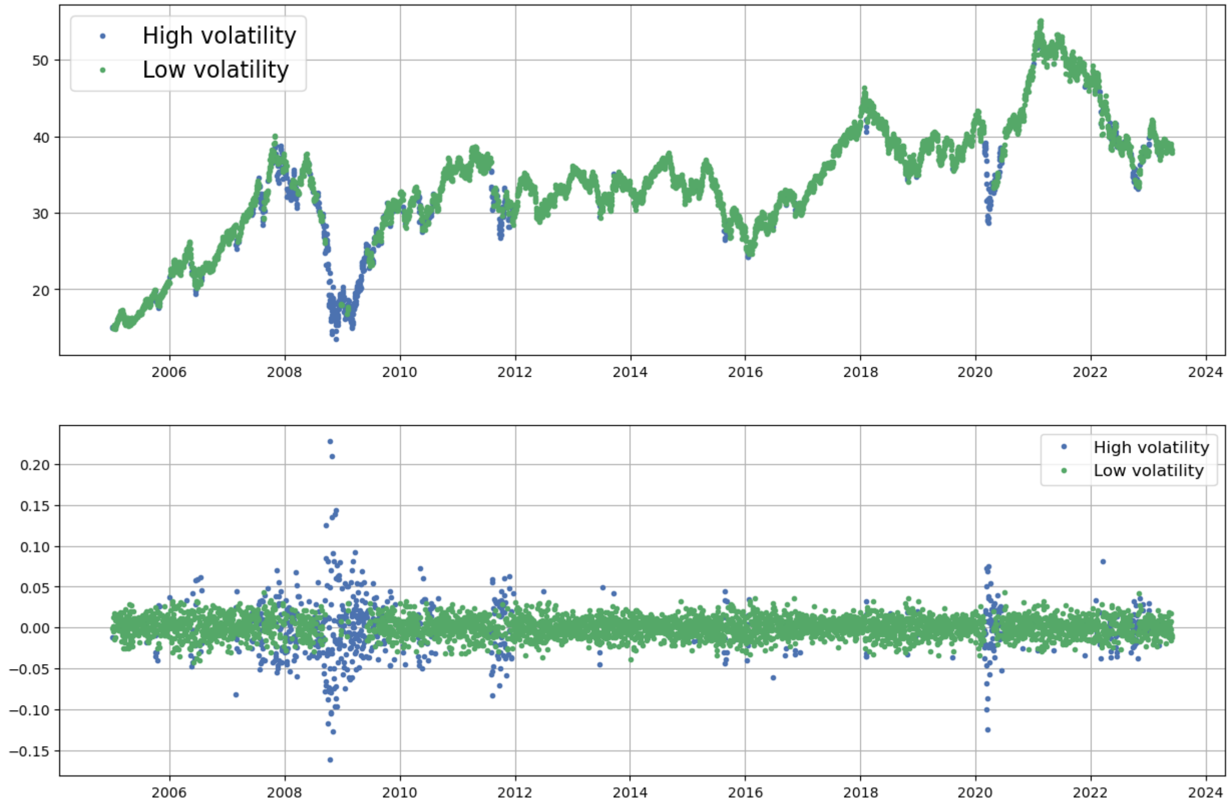
In [15]:
```
pd.DataFrame(data=model.means_, index=[states_dict[0], states_dict[1]], columns=['A1',
```

Out[15]:

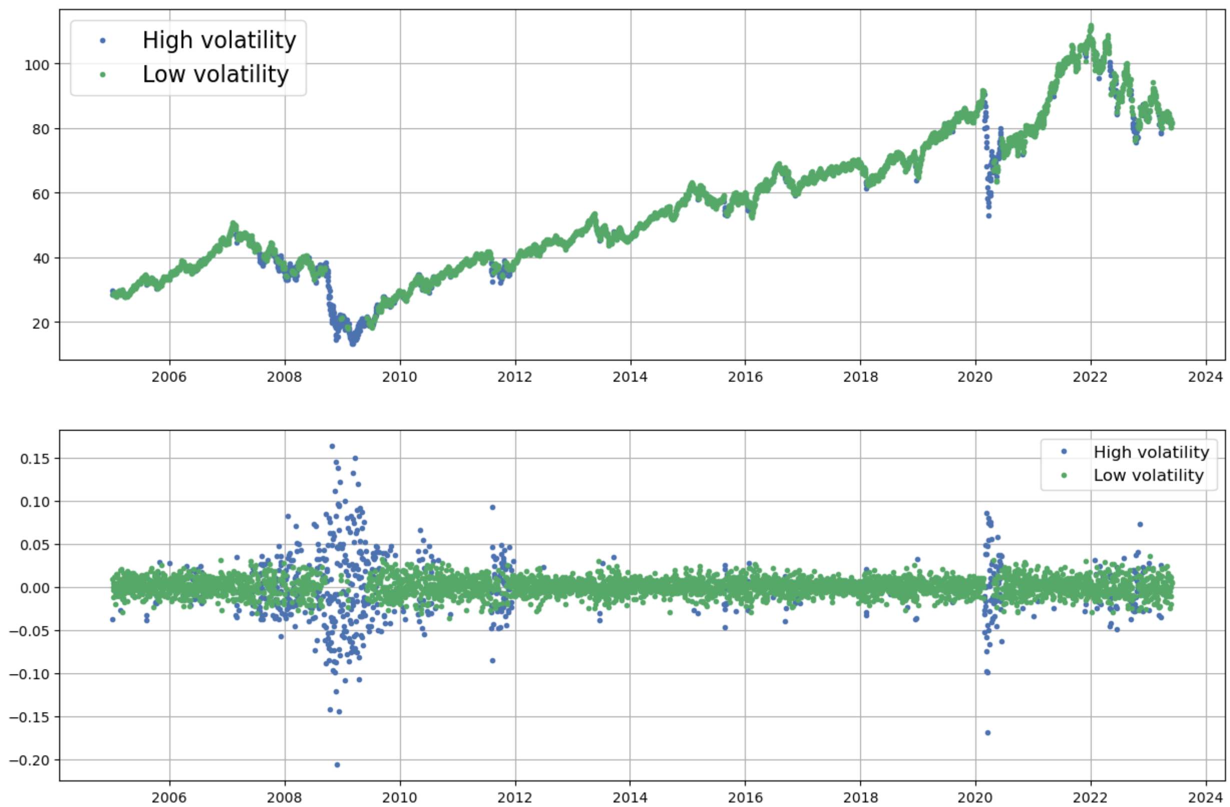
	A1	A2	A3
High volatility	-0.001679	-0.002035	-0.002186
Low volatility	0.000728	0.000826	0.000866

In [16]:
```
# regime plotting function (will be called repeatedly for each asset)
def regime_plot(df, nasset):
    plt.figure(figsize=(15, 10))
    plt.subplot(2, 1, 1)

    for i in states:
        s = (Z == i)
        x = df.index[s]
        y = df[tickers[nasset]].iloc[s]
        plt.plot(x, y, '.', label=states_dict[i])  # Add label based on regime
    plt.legend(fontsize=16)
    plt.grid(True)

    plt.subplot(2, 1, 2)
    for i in states:
        s = (Z == i)
        x = df.index[s]
        y = df[tickers[nasset]].pct_change().iloc[s]
        plt.plot(x, y, '.', label=states_dict[i])  # Add label based on regime
    plt.legend(fontsize=12)
    plt.grid(True)

    return plt
```

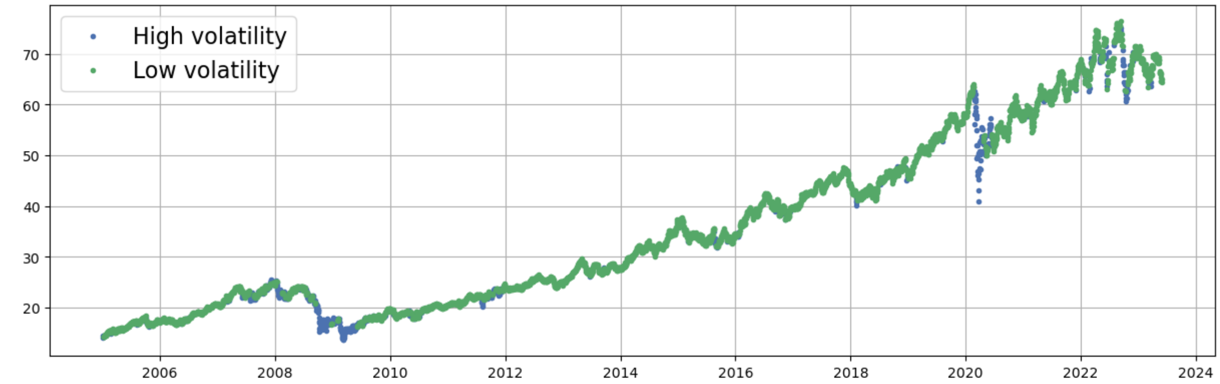
In [17]:
```
# removing the first row of the data that has NA-return
regime_plot(df.iloc[1: , :], 0).show()
```

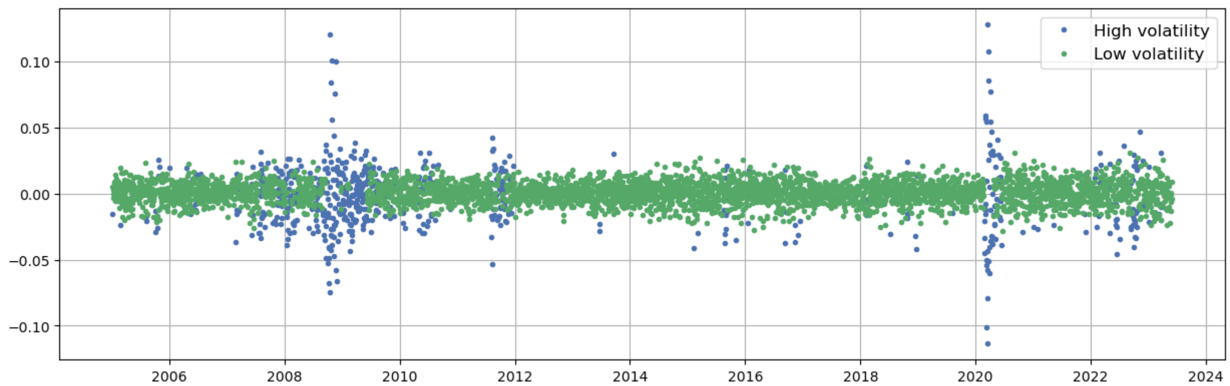


In [18]:
```
regime_plot(df.iloc[1: , :], 1).show()
```



In [19]:
```
regime_plot(df.iloc[1: , :], 2).show()
```





In [20]:

```
# Calculating the returnso of the market
states_df = df.copy()
states_returns_df = df.copy()
states_returns_df = states_df.pct_change().dropna()

# Adding a column that specifies the State for each date
states_df = states_df.iloc[1:]
states_df['state'] = Z
states_returns_df['state'] = Z

states_df, states_returns_df
```

Out[20]:
```
(                   A1         A2         A3  state
 Date
 2005-01-04  15.080416  29.742468  14.426337      0
 2005-01-05  14.895803  28.629606  14.199438      0
 2005-01-06  14.885677  28.874975  14.273320      1
 2005-01-07  14.914498  28.872492  14.273320      1
 2005-01-10  14.933196  28.855146  14.347193      1
 ...               ...        ...        ...    ...
 2023-05-24  38.101505  80.131714  65.470001      1
 2023-05-25  38.022125  80.141663  64.589996      1
 2023-05-26  38.696838  81.067291  64.589996      1
 2023-05-30  38.250336  81.415657  64.339996      1
 2023-05-31  37.893135  81.843636  64.930000      1

 [4633 rows x 4 columns],
                   A1        A2        A3  state
 Date
 2005-01-04 -0.030789 -0.013725 -0.006902      0
 2005-01-05 -0.012242 -0.037417 -0.015728      0
 2005-01-06 -0.000680  0.008570  0.005203      1
 2005-01-07  0.001936 -0.000086  0.000000      1
 2005-01-10  0.001254 -0.000601  0.005176      1
 ...              ...       ...       ...    ...
 2023-05-24 -0.007495 -0.020441 -0.006525      1
 2023-05-25 -0.002083  0.000124 -0.013441      1
 2023-05-26  0.017745  0.011550  0.000000      1
 2023-05-30 -0.011538  0.004297 -0.003871      1
 2023-05-31 -0.009338  0.005257  0.009170      1

 [4633 rows x 4 columns])
```

In [21]:
```
# EXAMPLE: average returns in the first market state (Z == 0)
df.pct_change().dropna().iloc[(Z == 0)].cov()
```

Out[21]:

	A1	A2	A3
A1	0.001500	0.001269	0.000685
A2	0.001269	0.001811	0.000694
A3	0.000685	0.000694	0.000608

In [22]:
```
# EXAMPLE: covariance matrix for the second market state (Z == 1)
df.pct_change().dropna().iloc[(Z == 1)].cov()
```

Out[22]:

	A1	A2	A3
A1	0.000134	0.000049	0.000026
A2	0.000049	0.000088	0.000040
A3	0.000026	0.000040	0.000064

Regime Analysis

In [23]:
```
# Separating the original dataframe into high volatility and loow bolatility DataFrame
low_vol_df = states_df.loc[states_df['state'] == 1]
```

```
high_vol_df = states_df.loc[states_df['state'] == 0]

# Separate df into low volatility and high volatility
low_vol_returns_df = states_returns_df.loc[states_returns_df['state'] == 1]
high_vol_returns_df = states_returns_df.loc[states_returns_df['state'] == 0]

# Print Stats
low_vol_stats = low_vol_returns_df.describe()
high_vol_stats = high_vol_returns_df.describe()
original_stats = states_returns_df.describe()

low_vol_stats, high_vol_stats, original_stats
```

Out[23]:

```
(                A1           A2           A3   state
 count  3968.000000  3968.000000  3968.000000  3968.0
 mean      0.000709     0.000789     0.000842     1.0
 std       0.011559     0.009377     0.007990     0.0
 min      -0.042443    -0.036101    -0.028064     1.0
 25%      -0.006707    -0.004860    -0.004189     1.0
 50%       0.000962     0.000995     0.001128     1.0
 75%       0.008043     0.006583     0.005984     1.0
 max       0.043037     0.036343     0.031288     1.0,
               A1          A2          A3  state
 count  665.000000  665.000000  665.000000  665.0
 mean    -0.001748   -0.002032   -0.002276    0.0
 std      0.038725    0.042555    0.024658    0.0
 min     -0.161662   -0.206111   -0.113577    0.0
 25%     -0.026299   -0.027551   -0.017362    0.0
 50%     -0.003048   -0.007849   -0.001825    0.0
 75%      0.020044    0.025109    0.012732    0.0
 max      0.227699    0.163253    0.127934    0.0,
                A1           A2           A3        state
 count  4633.000000  4633.000000  4633.000000  4633.000000
 mean      0.000357     0.000384     0.000395     0.856464
 std       0.018170     0.018327     0.011959     0.350656
 min      -0.161662    -0.206111    -0.113577     0.000000
 25%      -0.008052    -0.006152    -0.005169     1.000000
 50%       0.000901     0.000811     0.000966     1.000000
 75%       0.008983     0.007457     0.006457     1.000000
 max       0.227699     0.163253     0.127934     1.000000)
```

In [24]:

```
# Joining std and mean in the same graph to show market behavior
market_behavior = pd.DataFrame(original_stats.loc[["mean", 'std']])
market_behavior
```

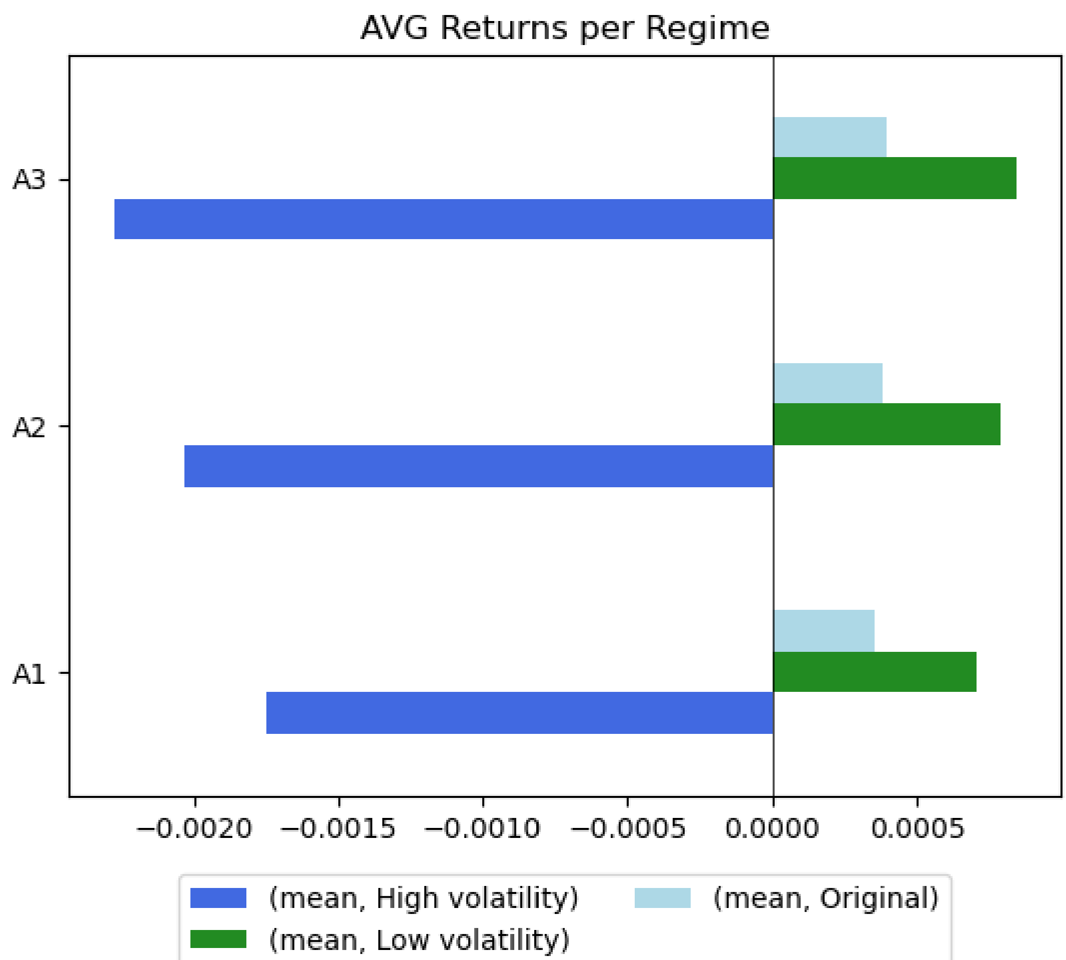
Out[24]:

	A1	A2	A3	state
mean	0.000357	0.000384	0.000395	0.856464
std	0.018170	0.018327	0.011959	0.350656

In [25]:

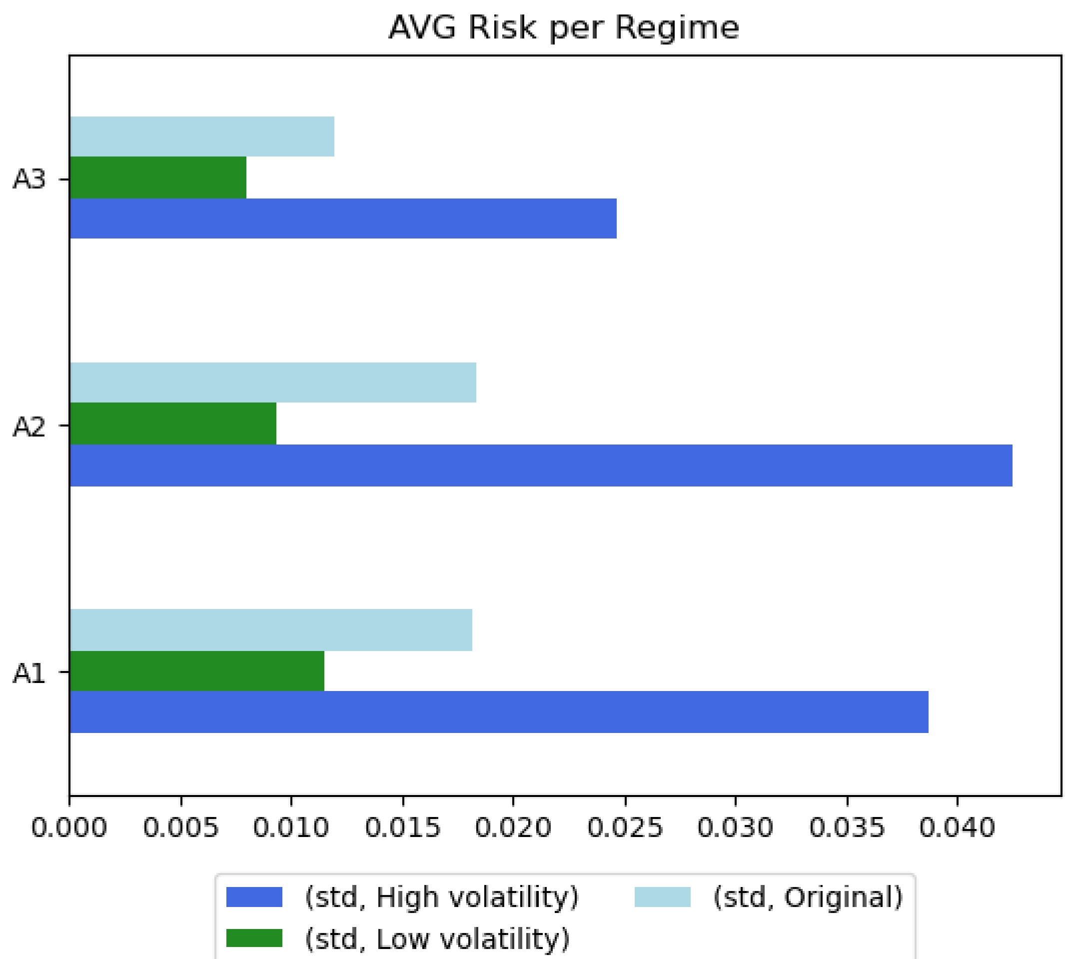
```
# Separate the mean into a different DataFrame for each state of the market
low_vol_avg_returns = pd.DataFrame(low_vol_stats.loc["mean"])
low_vol_avg_returns["state"] = states_dict[1]

# Separate the mean into a different DataFrame for each state of the market
high_vol_avg_returns = pd.DataFrame(high_vol_stats.loc["mean"])
high_vol_avg_returns["state"] = states_dict[0]
```

```
# Separate the mean into a different DataFrame for each state of the market
original_avg_returns = pd.DataFrame(original_stats.loc["mean"])
original_avg_returns["state"] = states_dict[2]

# Concatenate in the same table for plotting
avg_returns_df = pd.concat([low_vol_avg_returns, high_vol_avg_returns,original_avg_ret
avg_returns_df.drop("state", inplace = True)

# Plot the graph
ax = avg_returns_df.pivot(columns='state').plot.barh(color=['royalblue', 'forestgreen'
plt.title('AVG Returns per Regime')

# Add a line at 0 and place the legend outside of the graph
ax.axvline(0, color='black', linewidth=0.5)
ax.legend(loc='lower center', bbox_to_anchor=(0.5, -0.25), ncol=len(avg_returns_df.co

# Add a line at each maximum value
plt.show()
```



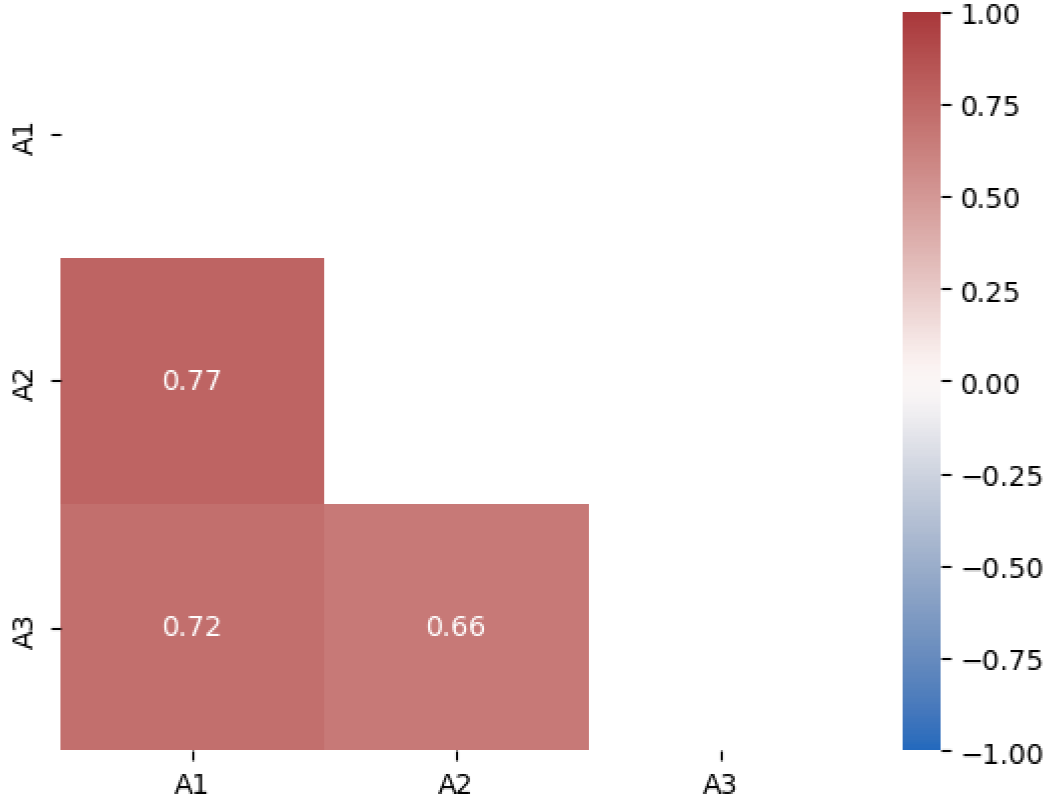
In [26]:
```
# Separate the standard deviation into a different DataFrame for each state of the mar
low_vol_avg_returns = pd.DataFrame(low_vol_stats.loc["std"])
low_vol_avg_returns["state"] = states_dict[1]

# Separate the standard deviation into a different DataFrame for each state of the mar
high_vol_avg_returns = pd.DataFrame(high_vol_stats.loc["std"])
high_vol_avg_returns["state"] = states_dict[0]

# Separate the standard deviation into a different DataFrame for each state of the mar
```

```
original_avg_returns = pd.DataFrame(original_stats.loc["std"])
original_avg_returns["state"] = states_dict[2]

# Concatenate in the same table for plotting
avg_returns_df = pd.concat([low_vol_avg_returns, high_vol_avg_returns,original_avg_ret
avg_returns_df.drop("state", inplace = True)

# Plot the graph
ax = avg_returns_df.pivot(columns='state').plot.barh(color=['royalblue', 'forestgreen'
plt.title('AVG Risk per Regime')

# Add a line at 0 and place the legend outside of the graph
ax.axvline(0, color='black', linewidth=0.5)
ax.legend(loc='lower center', bbox_to_anchor=(0.5, -0.25), ncol=len(avg_returns_df.col

# Add a line at each maximum value
plt.show()
```

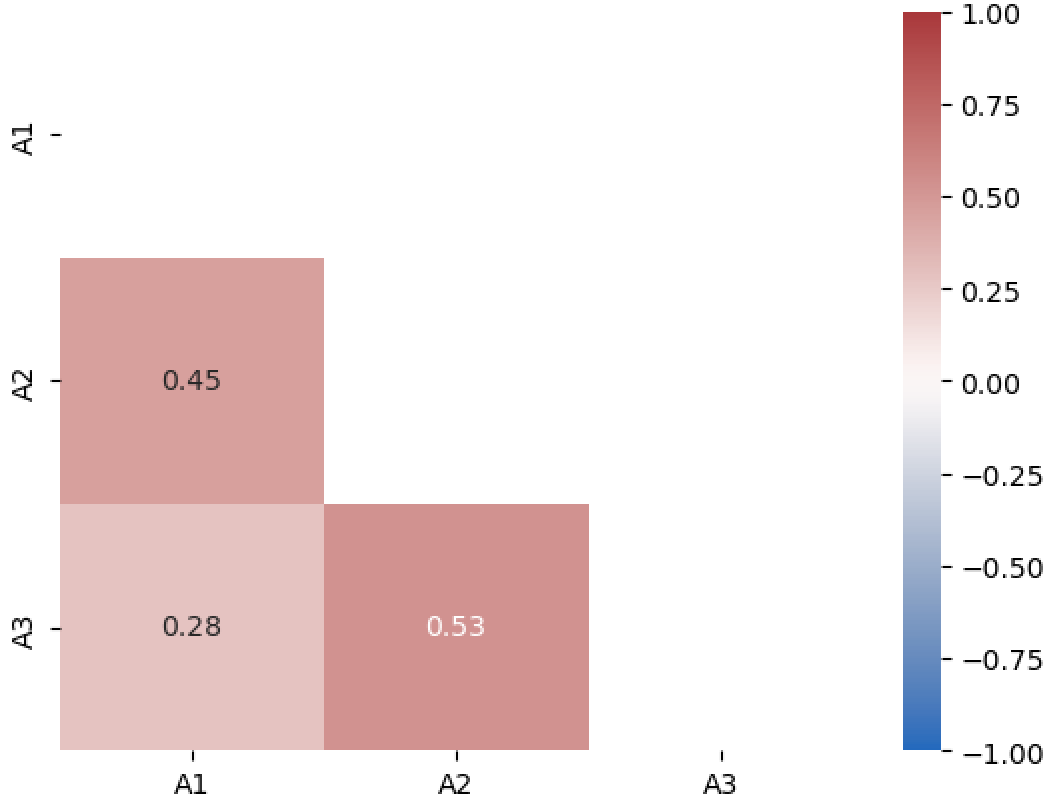


In [27]:

```
# Calculating correlation between Assets
corr_matrix = high_vol_returns_df[["A1","A2","A3"]].corr()
mask = np.triu(np.ones_like(corr_matrix, dtype=bool))

# Plot the correlation matrix as a heatmap
sns.heatmap(corr_matrix, annot=True, vmin=-1, vmax=1, cmap="vlag", mask=mask)
plt.show()
```

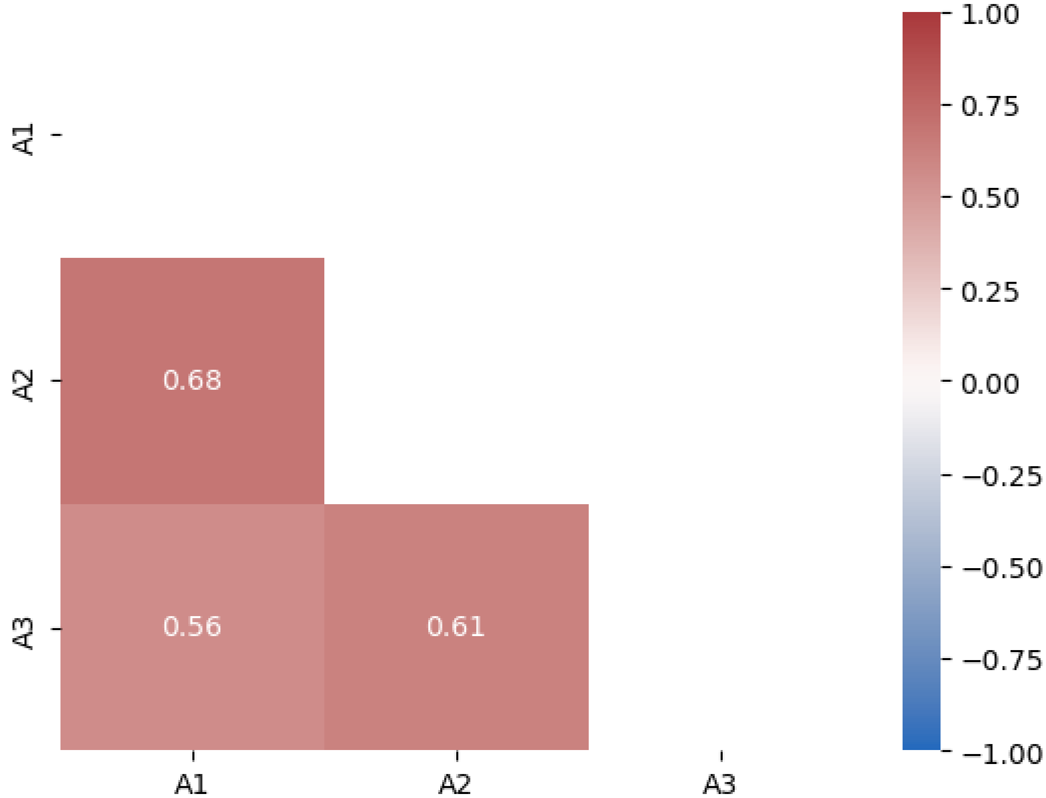


In [28]:
```
# Calculating correlation between Assets
corr_matrix = low_vol_returns_df[["A1","A2","A3"]].corr()
mask = np.triu(np.ones_like(corr_matrix, dtype=bool))

# Plot the correlation matrix as a heatmap
sns.heatmap(corr_matrix, annot=True, vmin=-1, vmax=1, cmap="vlag", mask=mask)
plt.show()
```



In [29]:

```
# Calculating correlation between Assets
corr_matrix = states_returns_df[["A1","A2","A3"]].corr()
mask = np.triu(np.ones_like(corr_matrix, dtype=bool))

# Plot the correlation matrix as a heatmap
sns.heatmap(corr_matrix, annot=True, vmin=-1, vmax=1, cmap="vlag", mask=mask)
plt.show()
```



Portfolio Creation and Testing

Both states combined

In [30]:
```
# Create Dataframes containing the prices and the returns of each asset and in each st
# Low Volatility state, prices
low_vol_df_s = low_vol_df.copy()
low_vol_df_s = low_vol_df_s[['A1','A2','A3']]

# Low Volatility state, returns
low_vol_returns_df_s = low_vol_returns_df.copy()
low_vol_returns_df_s = low_vol_returns_df_s[['A1','A2','A3']]

# high Volatility state, prices
high_vol_df_s = high_vol_df.copy()
high_vol_df_s = high_vol_df_s[['A1','A2','A3']]

# high Volatility state, returns
high_vol_returns_df_s = high_vol_returns_df.copy()
high_vol_returns_df_s = high_vol_returns_df_s[['A1','A2','A3']]

# Original, both states combined, prices
states_df_s = states_df.copy()
states_df_s = states_df_s[['A1','A2','A3']]

# Original, both states combined, returns
states_returns_df_s = states_returns_df.copy()
states_returns_df_s = states_returns_df_s[['A1','A2','A3']]
```

In [31]:
```
# Splitting low_vol_df into train and test sets
low_vol_train, low_vol_test = train_test_split(low_vol_df_s, test_size=0.2, shuffle=Fa
low_vol_returns_train, low_vol_returns_test = train_test_split(low_vol_returns_df_s, t

# Splitting high_vol_df into train and test sets
high_vol_train, high_vol_test = train_test_split(high_vol_df_s, test_size=0.2, shuffle
high_vol_returns_train, high_vol_returns_test = train_test_split(high_vol_returns_df_s

# Splitting states_df into train and test sets
states_train, states_test = train_test_split(states_df_s, test_size=0.2, shuffle=False
states_returns_train, states_returns_test = train_test_split(states_returns_df_s, test
```

In [32]:
```
# average historical returns
mu = expected_returns.mean_historical_return(states_train)
mu.sort_values(ascending=False)
```

Out[32]:
```
A3    0.099094
A2    0.073264
A1    0.064398
dtype: float64
```

In [33]:
```
# historical covariance matrix
S = risk_models.sample_cov(states_train)
S
```

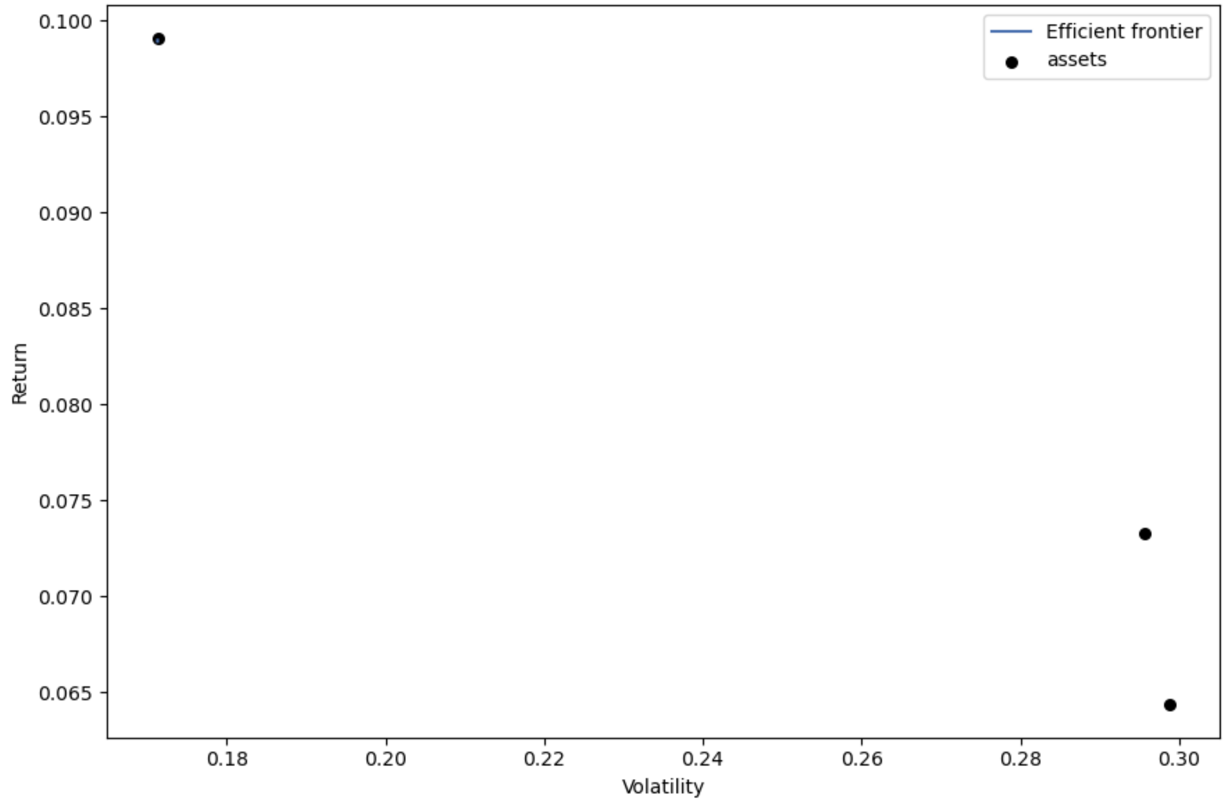
Out[33]:

	A1	A2	A3
A1	0.089234	0.060272	0.030934
A2	0.060272	0.087383	0.028964
A3	0.030934	0.028964	0.029349

In [34]:
```
# find efficient frontier
ef = EfficientFrontier(mu, S)
```

In [35]:
```
# save object copies for further calculations and plotting (auxiliary step)
ef_cp = ef.deepcopy()
ef_tangent = ef.deepcopy()
```

In [36]:
```
# plot efficient frontier: see PyPortfolioOpt docs
# https://pyportfolioopt.readthedocs.io/en/latest/Plotting.html
fig, ax = plt.subplots(figsize=(9,6))
plotting.plot_efficient_frontier(ef, ax=ax, show_assets=True)
plt.show()
```



In [37]:
```
# find the tangency (max. Sharpe ratio) portfolio
ef_tangent.max_sharpe()
ret_tangent, std_tangent, _ = ef_tangent.portfolio_performance(verbose=True)
```

```
Expected annual return: 9.9%
Annual volatility: 17.1%
Sharpe Ratio: 0.46
```

In [38]:
```
# tangency portfolio weights
tangent_weights = ef_tangent.clean_weights()
tangent_weights
```

Out[38]:
```
OrderedDict([('A1', 0.0), ('A2', 0.0), ('A3', 1.0)])
```

In [39]:
```
# generate random portfolios for visualization
n_samples = 10000
w = np.random.dirichlet(np.ones(ef.n_assets), n_samples)
rets = w.dot(ef.expected_returns)
stds = np.sqrt(np.diag(w @ ef.cov_matrix @ w.T))
sharpes = rets / stds
sharpes
```

Out[39]:
```
array([0.41725959, 0.34277297, 0.33911361, ..., 0.31644514, 0.29831892,
       0.37016377])
```

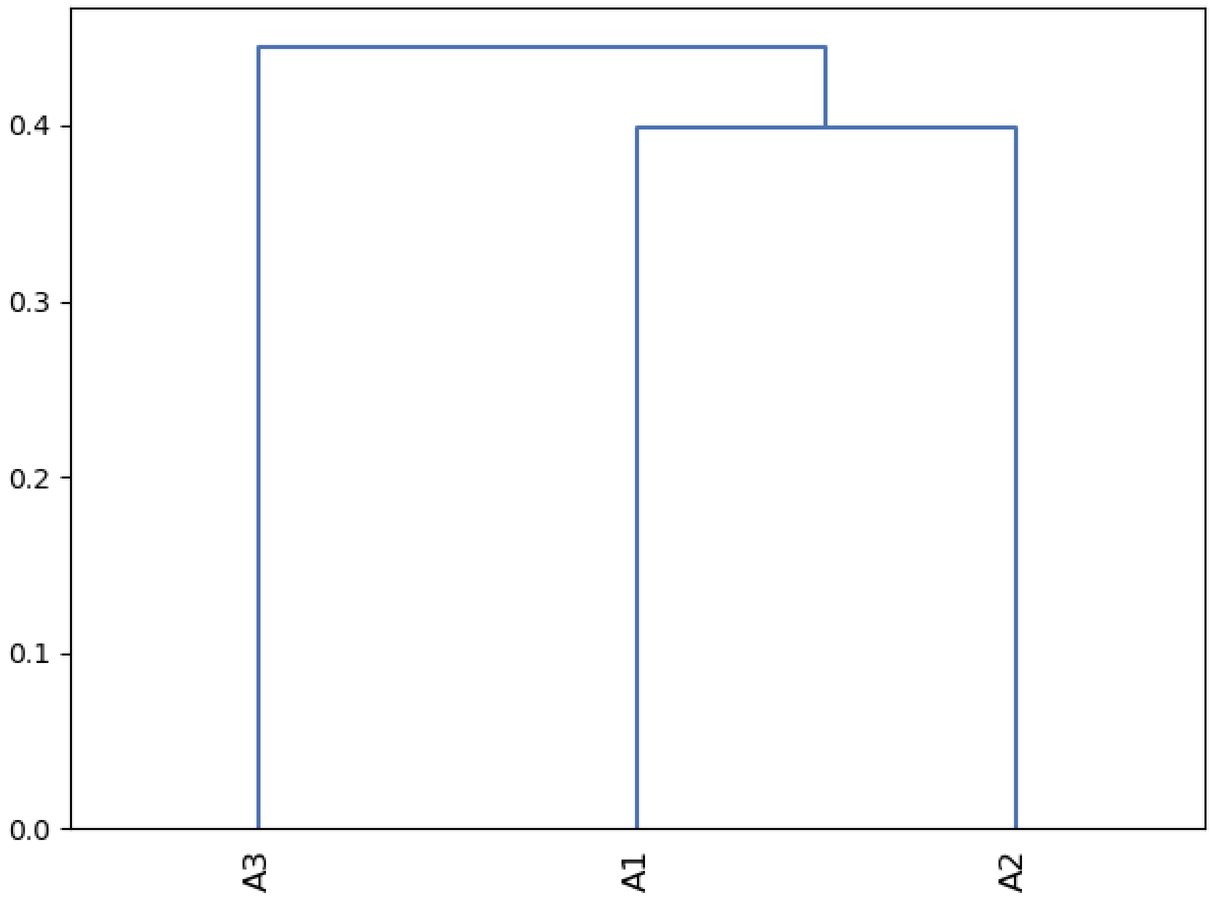
In [40]:
```
# Hierarchical Risk Parity (HRP) portfolio
hrp = HRPOpt(states_returns_train)
hrp_weights = hrp.optimize()
hrp_weights
```

Out[40]:
```
OrderedDict([('A1', 0.14004440170405413),
             ('A2', 0.14309429590602324),
             ('A3', 0.7168613023899226)])
```

In [41]:
```
# Calculating Metrics
ret_hrp, std_hrp, _ = hrp.portfolio_performance(verbose=True)
```

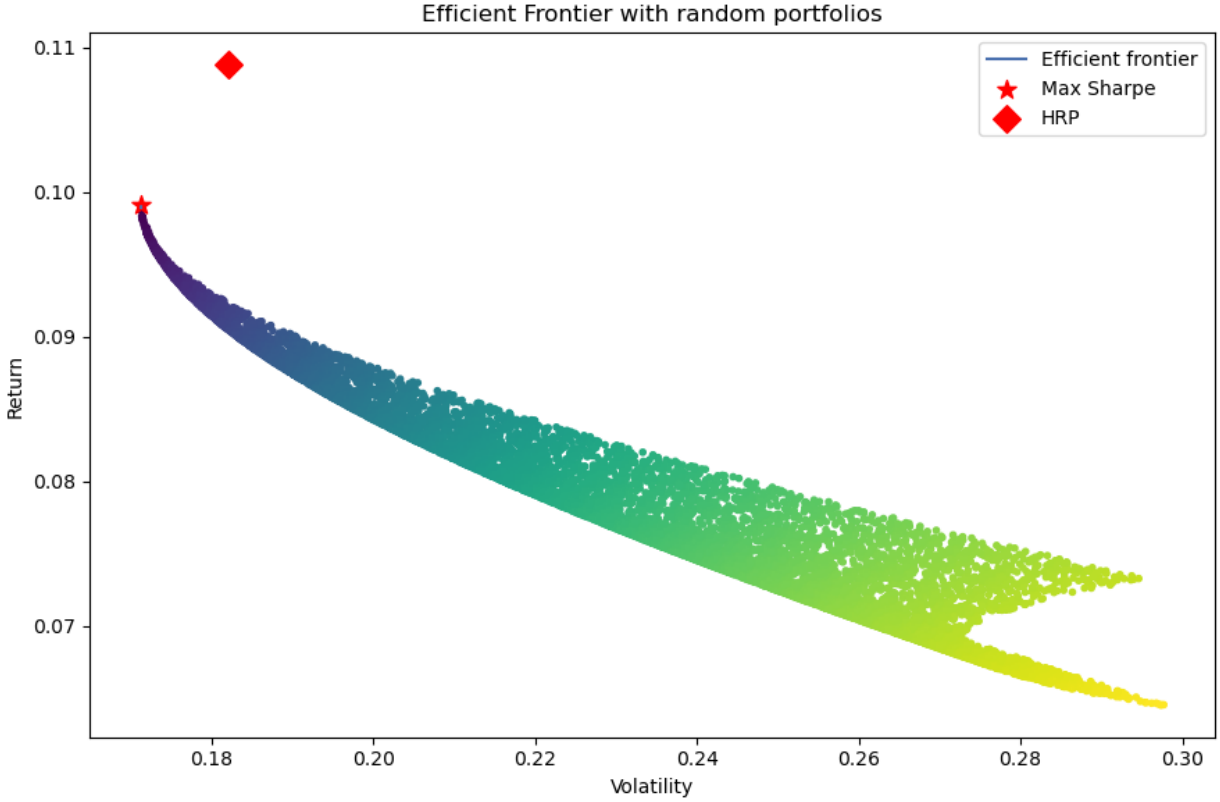
```
Expected annual return: 10.9%
Annual volatility: 18.2%
Sharpe Ratio: 0.49
```

In [42]:
```
# Hierarchical Clustering dendrogram
plotting.plot_dendrogram(hrp, showfig = True)
```



Out[42]: <AxesSubplot:>

In [43]:
```
# Plotting both Portfolio compositions
fig, ax = plt.subplots(figsize=(9,6))
plotting.plot_efficient_frontier(ef_cp, ax=ax, show_assets=False)

ax.scatter(stds, rets, marker=".", c=sharpes, cmap="viridis_r")

ax.scatter(std_tangent, ret_tangent, marker="*", s=100, c="r", label="Max Sharpe")

ax.scatter(std_hrp, ret_hrp, marker="D", s=100, c="r", label="HRP")

ax.set_title("Efficient Frontier with random portfolios")
ax.legend()
plt.tight_layout()
plt.show()
```



In [44]:
```
# construct portfolio returns: testing time period
states_returns_test['port_tangent'] = 0
for ticker, weight in tangent_weights.items():
    states_returns_test['port_tangent'] += states_returns_test[ticker]*weight
```

In [45]:
```
states_returns_test['port_hrp'] = 0
for ticker, weight in hrp_weights.items():
    states_returns_test['port_hrp'] += states_returns_test[ticker]*weight
```

In [46]:
```
# cumulative equity curve (recall from the financial data practice earlier)
port_equity_tangent = (1 + states_returns_test['port_tangent']).cumprod() - 1
port_equity_hrp = (1 + states_returns_test['port_hrp']).cumprod() - 1
port_equity = port_equity_tangent.to_frame().join(port_equity_hrp)
port_equity
```

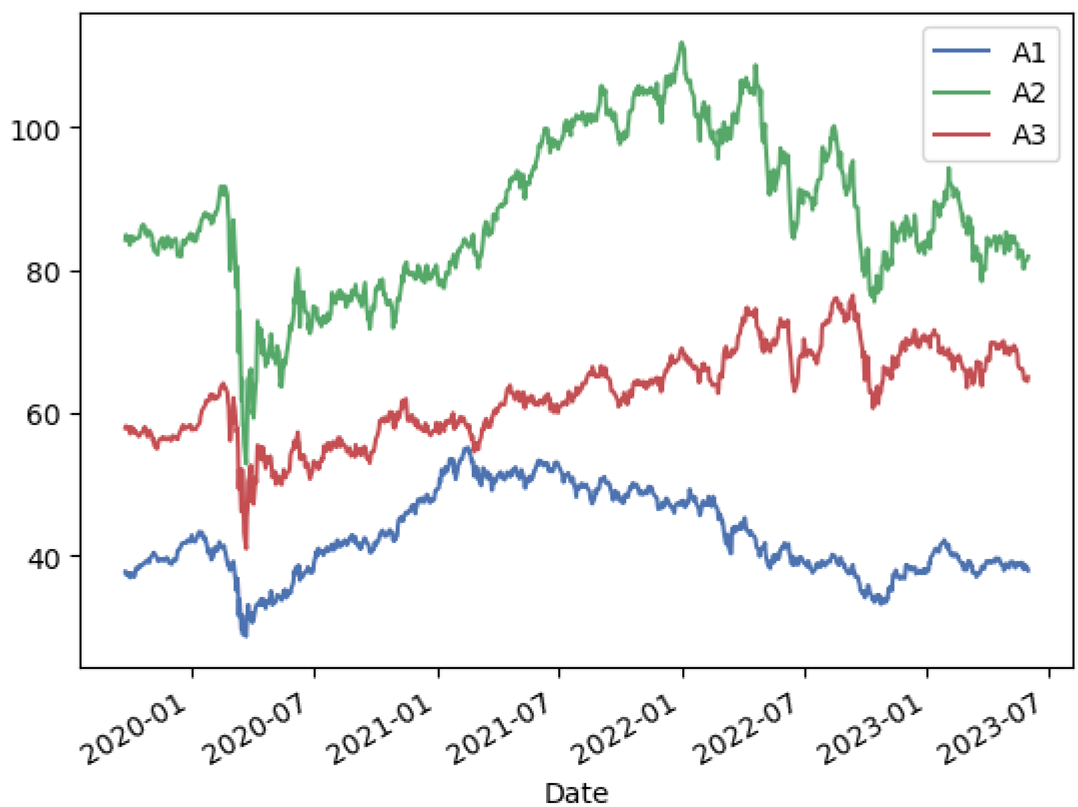
Out[46]:

Date	port_tangent	port_hrp
2019-09-25	-0.000619	-0.000438
2019-09-26	0.004642	0.004469
2019-09-27	0.001083	-0.000766
2019-09-30	0.001702	0.001005
2019-10-01	-0.000619	-0.003168
...	...	...
2023-05-24	0.131302	0.107121
2023-05-25	0.116096	0.096150
2023-05-26	0.116096	0.100685
2023-05-30	0.111776	0.096529
2023-05-31	0.121971	0.103128

927 rows × 2 columns

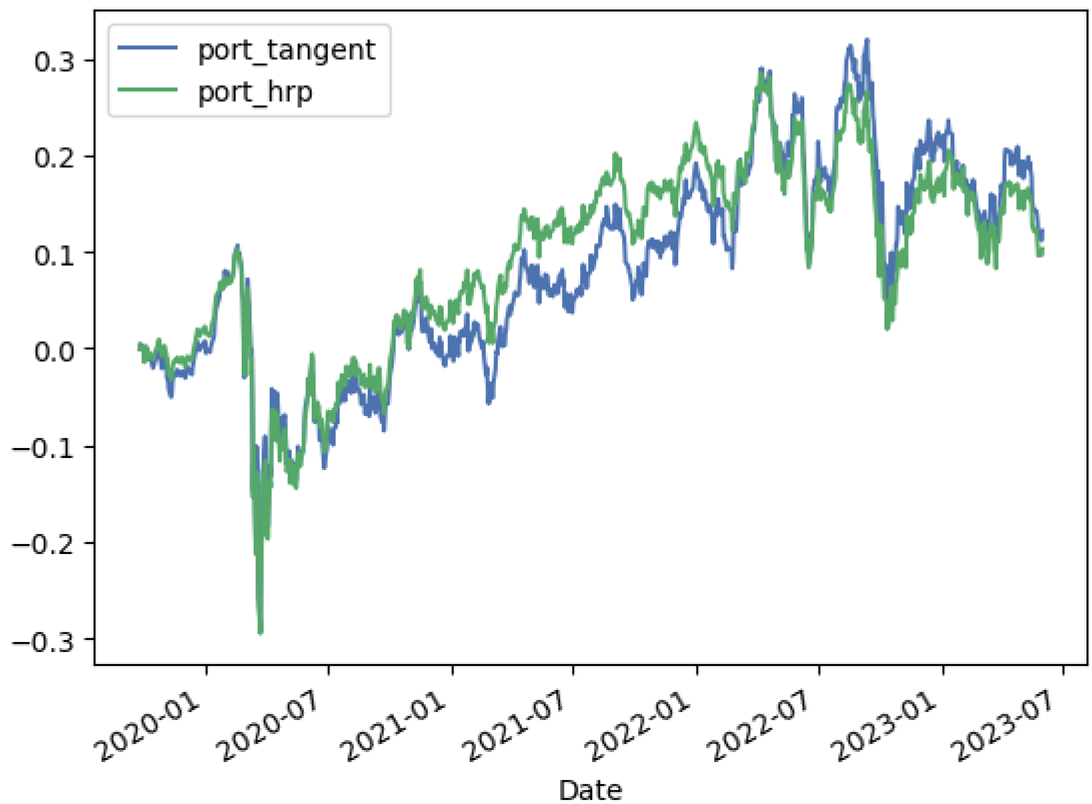
In [47]:
```
states_test.plot()
```

Out[47]:
```
<AxesSubplot:xlabel='Date'>
```



In [48]:
```
# out-of-sample performance
port_equity.plot()
```

Out[48]: <AxesSubplot:xlabel='Date'>



In [49]:
```
# out-of-sample volatilities
port_equity.std()
```

Out[49]:
```
port_tangent    0.107861
port_hrp        0.100904
dtype: float64
```

Low Volatility State

In [50]:
```
# average historical returns
mu = expected_returns.mean_historical_return(low_vol_train)
mu.sort_values(ascending=False)
```

Out[50]:
```
A3    0.116555
A2    0.088726
A1    0.077613
dtype: float64
```

In [51]:
```
# historical covariance matrix
S = risk_models.sample_cov(low_vol_train)
S
```

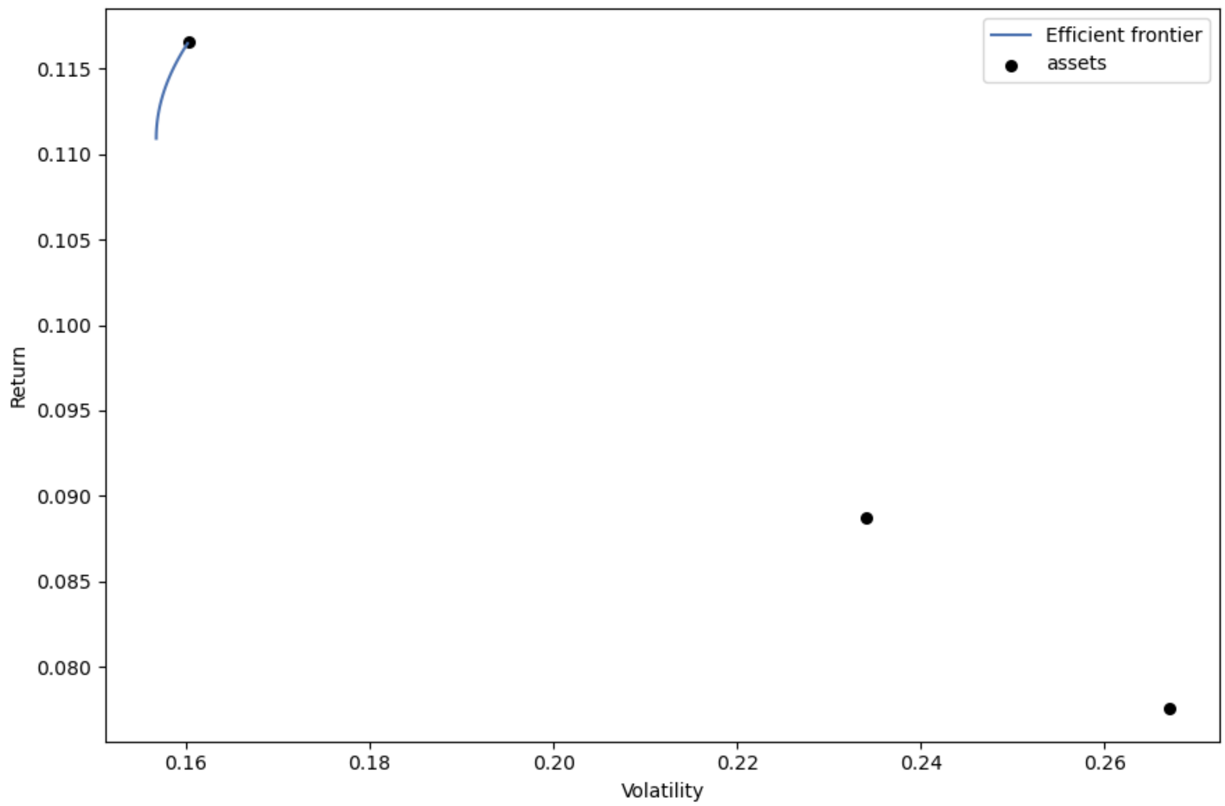
Out[51]:

	A1	A2	A3
A1	0.071284	0.040395	0.017892
A2	0.040395	0.054772	0.020280
A3	0.017892	0.020280	0.025695

In [52]:
```
# find efficient frontier
ef = EfficientFrontier(mu, S)
```

In [53]:
```
# save object copies for further calculations and plotting (auxiliary step)
ef_cp = ef.deepcopy()
ef_tangent = ef.deepcopy()
```

In [54]:
```
# plot efficient frontier: see PyPortfolioOpt docs
# https://pyportfolioopt.readthedocs.io/en/latest/Plotting.html
fig, ax = plt.subplots(figsize=(9,6))
plotting.plot_efficient_frontier(ef, ax=ax, show_assets=True)
plt.show()
```



In [55]:
```
# find the tangency (max. Sharpe ratio) portfolio
ef_tangent.max_sharpe()
ret_tangent, std_tangent, _ = ef_tangent.portfolio_performance(verbose=True)
```

```
Expected annual return: 11.7%
Annual volatility: 16.0%
Sharpe Ratio: 0.60
```

In [56]:
```
# tangency portfolio weights
tangent_weights = ef_tangent.clean_weights()
tangent_weights
```

Out[56]:
```
OrderedDict([('A1', 0.0), ('A2', 0.0), ('A3', 1.0)])
```

In [57]:
```
# generate random portfolios for visualization
n_samples = 10000
w = np.random.dirichlet(np.ones(ef.n_assets), n_samples)
rets = w.dot(ef.expected_returns)
stds = np.sqrt(np.diag(w @ ef.cov_matrix @ w.T))
```

```
sharpes = rets / stds
sharpes
```

Out[57]:
```
array([0.45115376, 0.40700269, 0.44006153, ..., 0.46581966, 0.31518937,
       0.51648571])
```

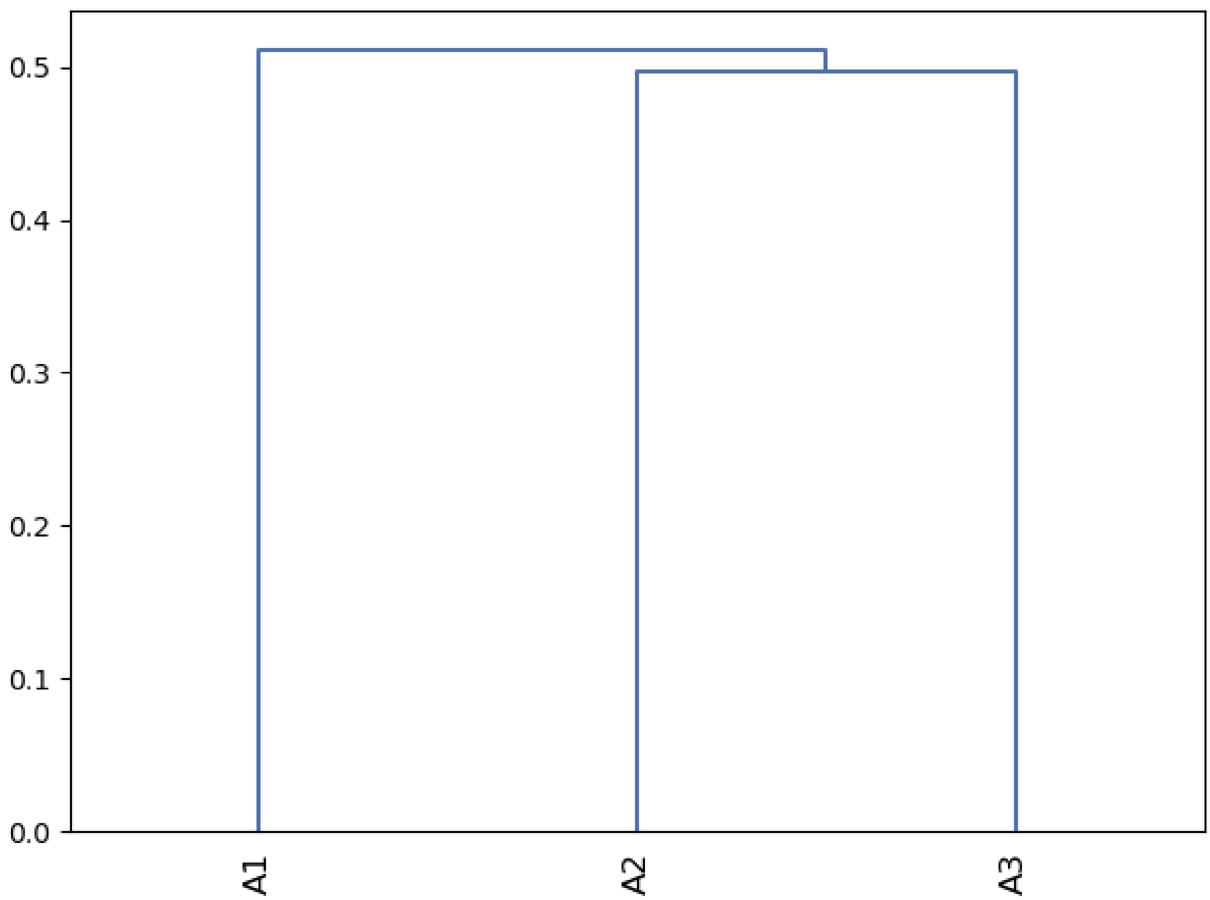
In [58]:
```
# Hierarchical Risk Parity (HRP) portfolio
hrp = HRPOpt(low_vol_returns_train)
hrp_weights = hrp.optimize()
hrp_weights
```

Out[58]:
```
OrderedDict([('A1', 0.27260424258482474),
             ('A2', 0.299567256653504),
             ('A3', 0.4278285007616713)])
```

In [59]:
```
ret_hrp, std_hrp, _ = hrp.portfolio_performance(verbose=True)
```

```
Expected annual return: 20.5%
Annual volatility: 11.5%
Sharpe Ratio: 1.60
```

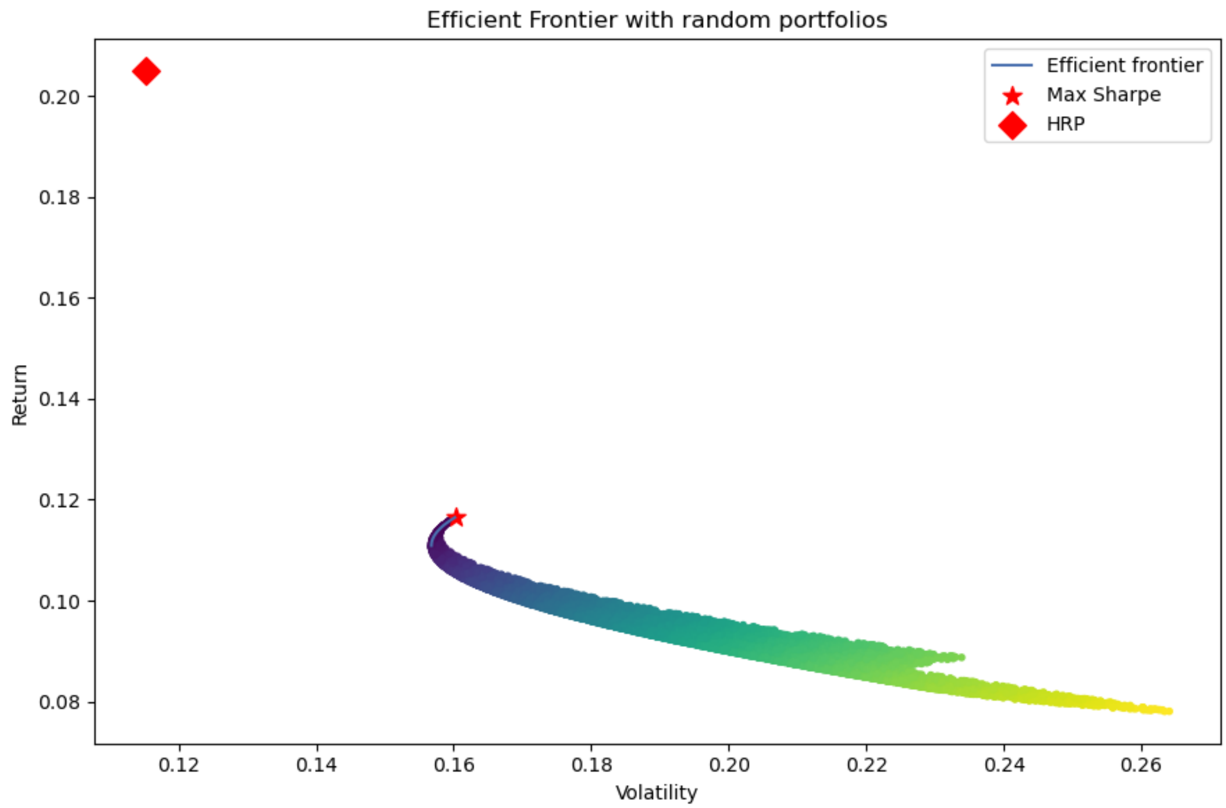
In [60]:
```
# Hierarchical Clustering dendrogram
plotting.plot_dendrogram(hrp, showfig = True)
```



Out[60]:
```
<AxesSubplot:>
```

In [61]:
```
# Plotting both Portfolio compositions
fig, ax = plt.subplots(figsize=(9,6))
plotting.plot_efficient_frontier(ef_cp, ax=ax, show_assets=False)

ax.scatter(stds, rets, marker=".", c=sharpes, cmap="viridis_r")
```

```
ax.scatter(std_tangent, ret_tangent, marker="*", s=100, c="r", label="Max Sharpe")

ax.scatter(std_hrp, ret_hrp, marker="D", s=100, c="r", label="HRP")

ax.set_title("Efficient Frontier with random portfolios")
ax.legend()
plt.tight_layout()
plt.show()
```



In [62]:
```
# construct portfolio returns: testing time period
low_vol_returns_test['port_tangent'] = 0
for ticker, weight in tangent_weights.items():
    low_vol_returns_test['port_tangent'] += low_vol_returns_test[ticker]*weight
```

In [63]:
```
low_vol_returns_test['port_hrp'] = 0
for ticker, weight in hrp_weights.items():
    low_vol_returns_test['port_hrp'] += low_vol_returns_test[ticker]*weight
```

In [64]:
```
# cumulative equity curve (recall from the financial data practice earlier)
port_equity_tangent = (1 + low_vol_returns_test['port_tangent']).cumprod() - 1
port_equity_hrp = (1 + low_vol_returns_test['port_hrp']).cumprod() - 1
port_equity = port_equity_tangent.to_frame().join(port_equity_hrp)
port_equity
```

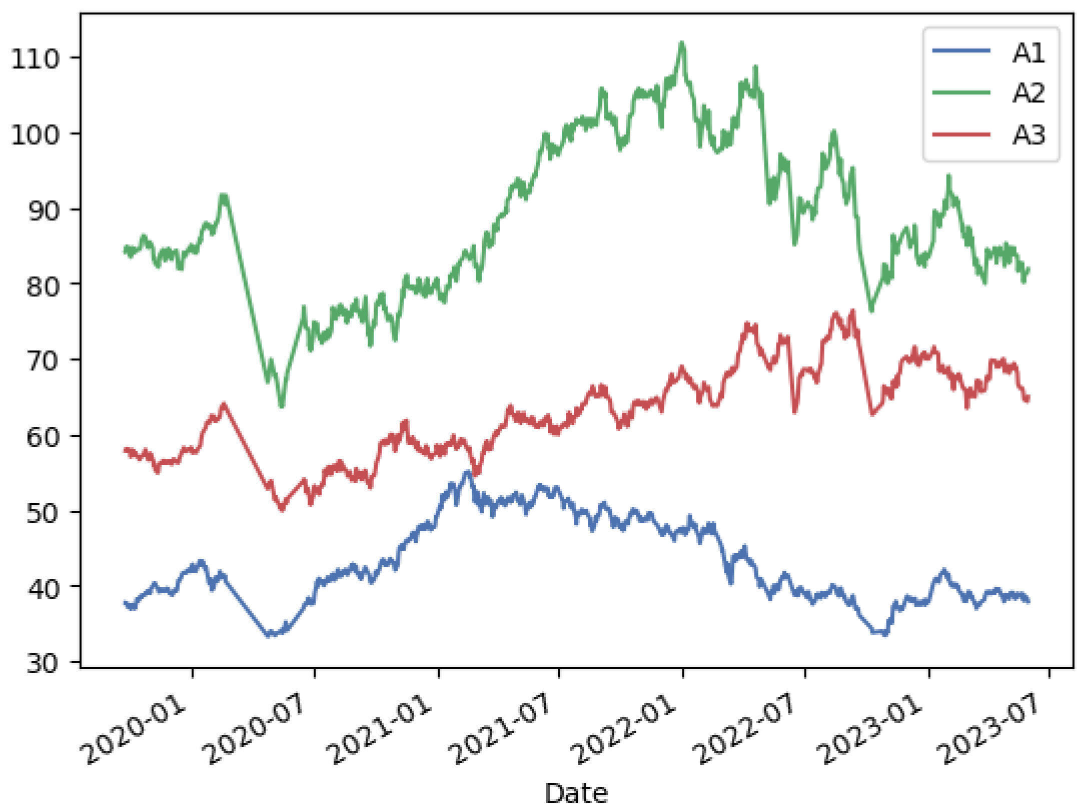
Out[64]:

Date	port_tangent	port_hrp
2019-09-24	0.011741	0.001770
2019-09-25	0.011114	0.001532
2019-09-26	0.016437	0.006173
2019-09-27	0.012837	-0.000716
2019-09-30	0.013463	0.002215
...	...	...
2023-05-24	0.882769	0.698477
2023-05-25	0.857462	0.687808
2023-05-26	0.857462	0.701812
2023-05-30	0.850273	0.695832
2023-05-31	0.867240	0.700839

794 rows × 2 columns

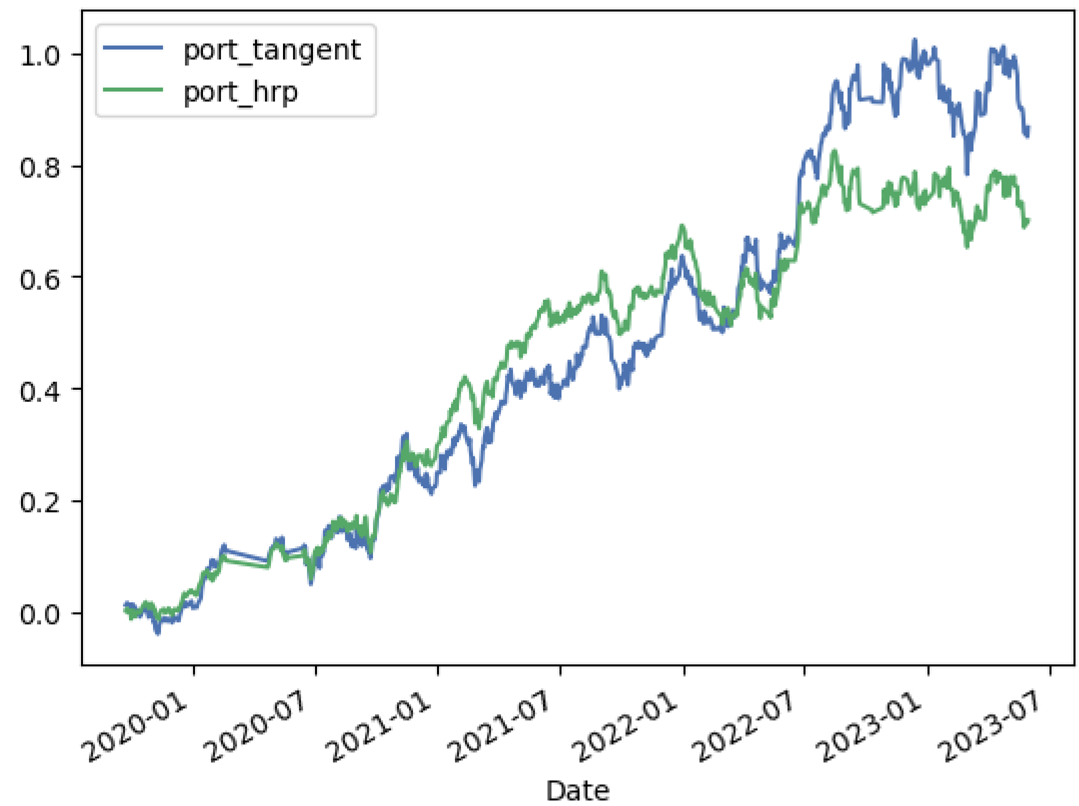
In [65]:
```
low_vol_test.plot()
```

Out[65]: <AxesSubplot:xlabel='Date'>



In [66]:
```
# out-of-sample performance
port_equity.plot()
```

Out[66]: <AxesSubplot:xlabel='Date'>



In [67]:
```
# out-of-sample volatilities
port_equity.std()
```

Out[67]:
```
port_tangent    0.317391
port_hrp        0.257587
dtype: float64
```

High Volatility State

In [68]:
```
# average historical returns
mu = expected_returns.mean_historical_return(high_vol_train)
mu.sort_values(ascending=False)
```

Out[68]:
```
A3    1.008731
A2    0.694519
A1    0.569492
dtype: float64
```

In [69]:
```
# historical covariance matrix
S = risk_models.sample_cov(high_vol_train)
S
```

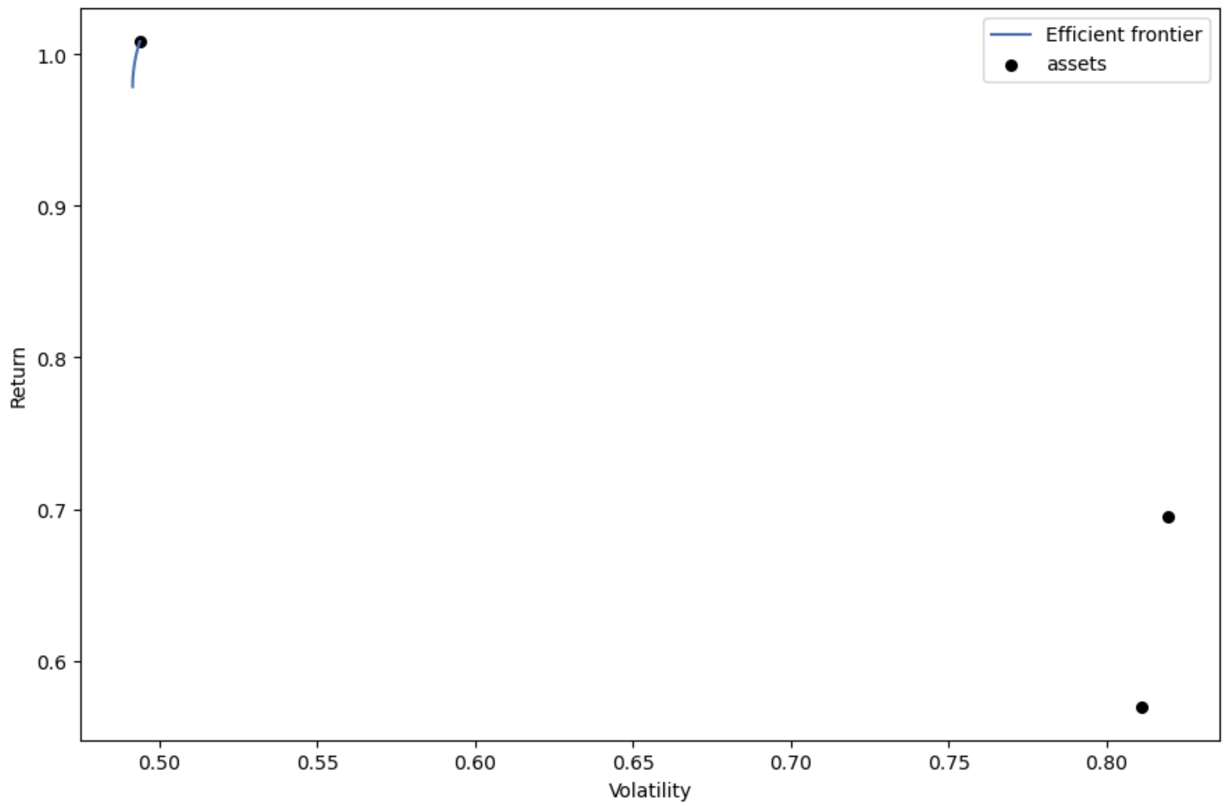
Out[69]:

	A1	A2	A3
A1	0.657958	0.418443	0.211039
A2	0.418443	0.671379	0.267919
A3	0.211039	0.267919	0.243797

In [70]:
```
# find efficient frontier
ef = EfficientFrontier(mu, S)
```

In [71]:
```
# save object copies for further calculations and plotting (auxiliary step)
ef_cp = ef.deepcopy()
ef_tangent = ef.deepcopy()
```

In [72]:
```
# plot efficient frontier: see PyPortfolioOpt docs
# https://pyportfolioopt.readthedocs.io/en/latest/Plotting.html
fig, ax = plt.subplots(figsize=(9,6))
plotting.plot_efficient_frontier(ef, ax=ax, show_assets=True)
plt.show()
```



In [73]:
```
# find the tangency (max. Sharpe ratio) portfolio
ef_tangent.max_sharpe()
ret_tangent, std_tangent, _ = ef_tangent.portfolio_performance(verbose=True)
```

```
Expected annual return: 100.9%
Annual volatility: 49.4%
Sharpe Ratio: 2.00
```

In [74]:
```
# tangency portfolio weights
tangent_weights = ef_tangent.clean_weights()
tangent_weights
```

Out[74]:
```
OrderedDict([('A1', 0.0), ('A2', 0.0), ('A3', 1.0)])
```

In [75]:
```
# generate random portfolios for visualization
n_samples = 10000
w = np.random.dirichlet(np.ones(ef.n_assets), n_samples)
rets = w.dot(ef.expected_returns)
stds = np.sqrt(np.diag(w @ ef.cov_matrix @ w.T))
```

```
sharpes = rets / stds
sharpes
```

Out[75]:
```
array([0.93120036, 0.94000871, 1.41155692, ..., 0.95558678, 0.98009046,
       0.98403315])
```

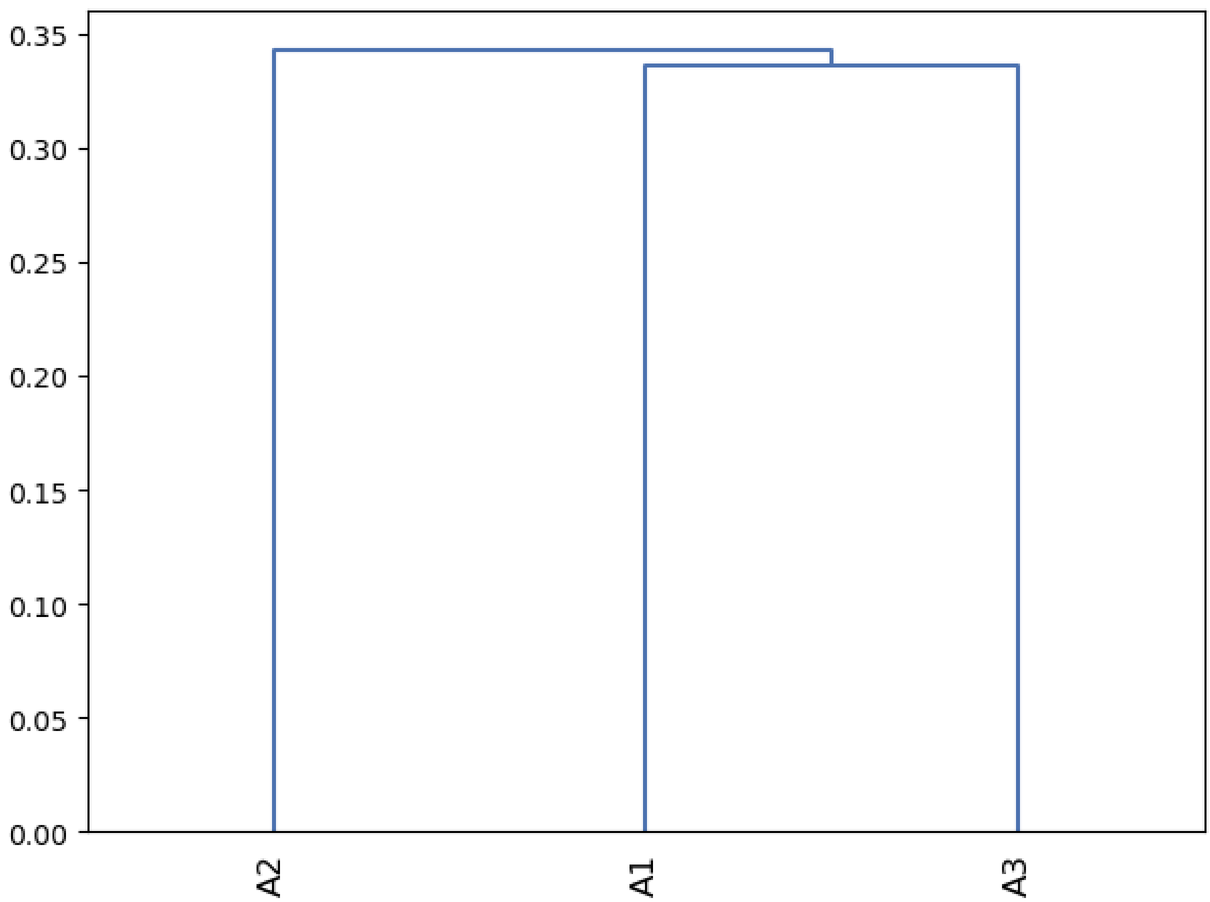
In [76]:
```
# Hierarchical Risk Parity (HRP) portfolio
hrp = HRPOpt(high_vol_returns_train)
hrp_weights = hrp.optimize()
hrp_weights
```

Out[76]:
```
OrderedDict([('A1', 0.1660694837609595),
             ('A2', 0.23403797056583764),
             ('A3', 0.5998925456732028)])
```

In [77]:
```
ret_hrp, std_hrp, _ = hrp.portfolio_performance(verbose=True)
```

```
Expected annual return: -48.7%
Annual volatility: 42.6%
Sharpe Ratio: -1.19
```

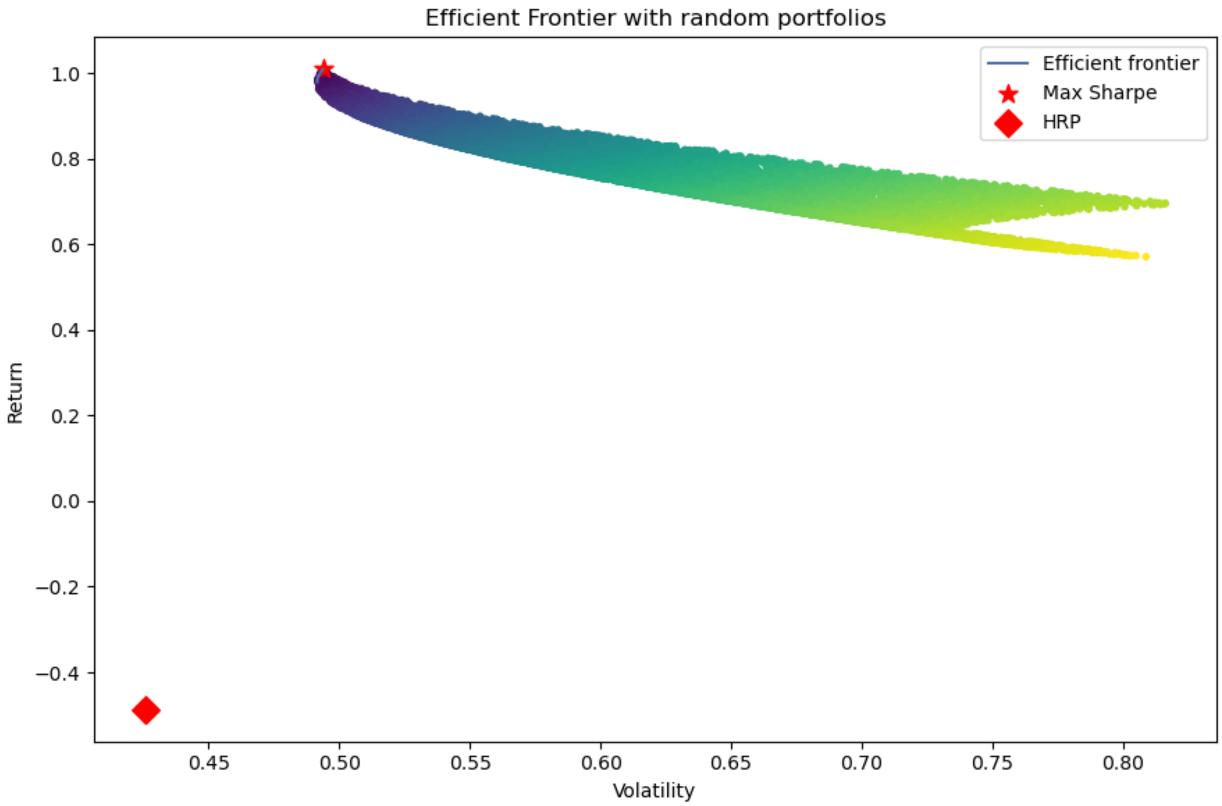
In [78]:
```
# Hierarchical Clustering dendrogram
plotting.plot_dendrogram(hrp, showfig = True)
```



Out[78]:
```
<AxesSubplot:>
```

In [79]:
```
# Plotting both Portfolio compositions
fig, ax = plt.subplots(figsize=(9,6))
plotting.plot_efficient_frontier(ef_cp, ax=ax, show_assets=False)

ax.scatter(stds, rets, marker=".", c=sharpes, cmap="viridis_r")
```

```
ax.scatter(std_tangent, ret_tangent, marker="*", s=100, c="r", label="Max Sharpe")

ax.scatter(std_hrp, ret_hrp, marker="D", s=100, c="r", label="HRP")

ax.set_title("Efficient Frontier with random portfolios")
ax.legend()
plt.tight_layout()
plt.show()
```



In [80]:
```
# construct portfolio returns: testing time period
high_vol_returns_test['port_tangent'] = 0
for ticker, weight in tangent_weights.items():
    high_vol_returns_test['port_tangent'] += high_vol_returns_test[ticker]*weight
```

In [81]:
```
high_vol_returns_test['port_hrp'] = 0
for ticker, weight in hrp_weights.items():
    high_vol_returns_test['port_hrp'] += high_vol_returns_test[ticker]*weight
```

In [82]:
```
# cumulative equity curve (recall from the financial data practice earlier)
port_equity_tangent = (1 + high_vol_returns_test['port_tangent']).cumprod() - 1
port_equity_hrp = (1 + high_vol_returns_test['port_hrp']).cumprod() - 1
port_equity = port_equity_tangent.to_frame().join(port_equity_hrp)
port_equity
```

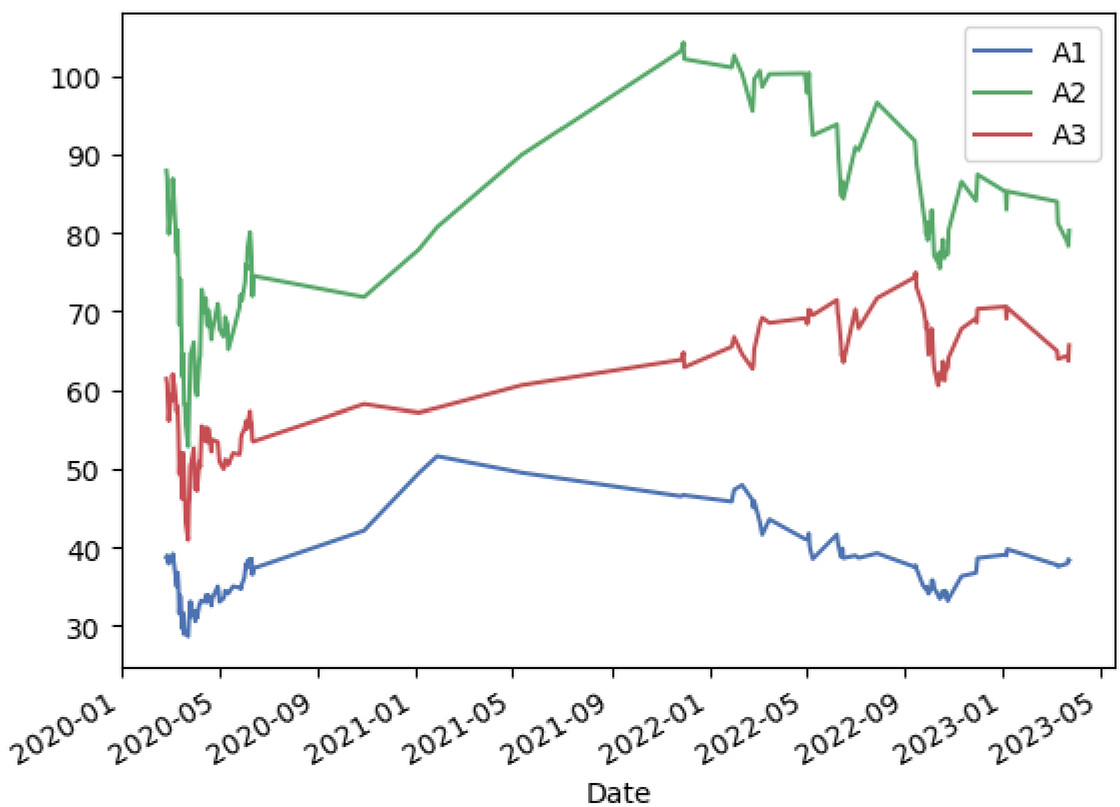
Out[82]:

Date	port_tangent	port_hrp
2020-02-25	-0.021004	-0.020188
2020-02-26	-0.030931	-0.027256
2020-02-27	-0.074522	-0.069314
2020-02-28	-0.105453	-0.095352
2020-03-02	-0.052798	-0.049947
...	...	...
2023-03-09	-0.375448	-0.358054
2023-03-10	-0.385277	-0.369613
2023-03-22	-0.397356	-0.381762
2023-03-23	-0.403536	-0.384999
2023-03-24	-0.384902	-0.370154

133 rows × 2 columns

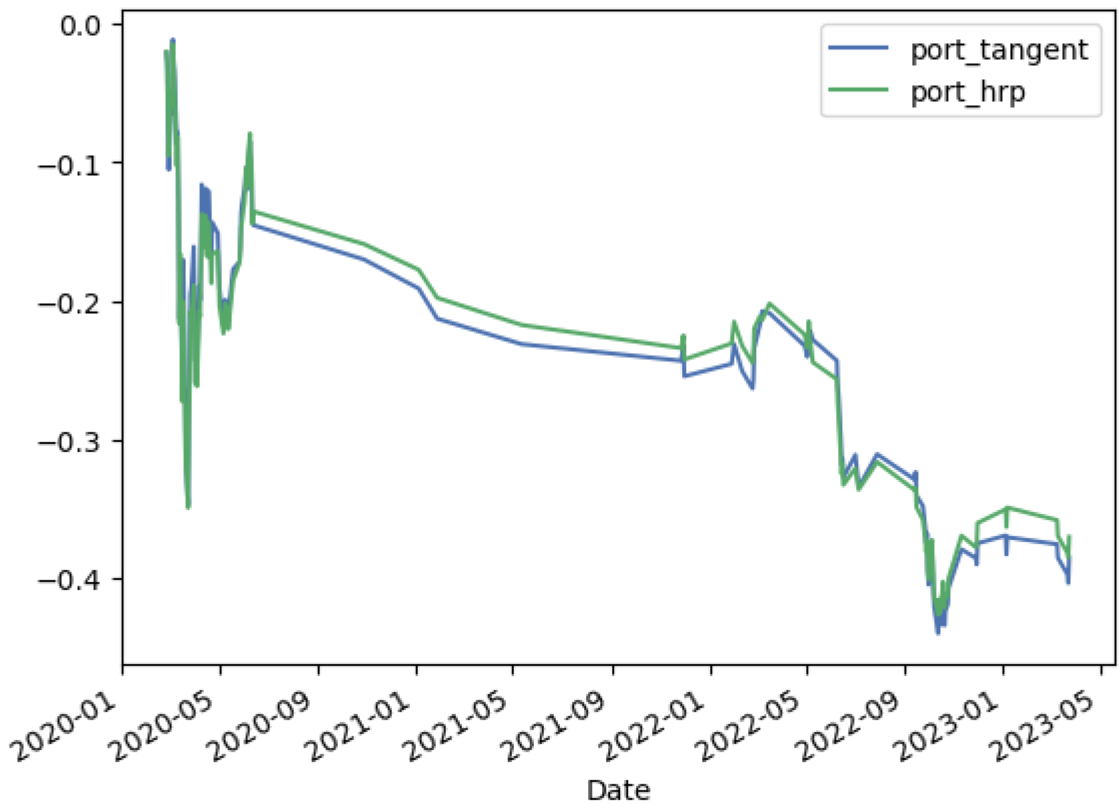
In [83]:
```
high_vol_test.plot()
```

Out[83]: <AxesSubplot:xlabel='Date'>



In [84]:
```
# out-of-sample performance
port_equity.plot()
```

Out[84]: <AxesSubplot:xlabel='Date'>



In [85]:
```
# out-of-sample volatilities
port_equity.std()
```

Out[85]:
```
port_tangent    0.112450
port_hrp        0.108273
dtype: float64
```

Recommendations

In both Low-volatility and Original state the Maximum Sharpe portfolio outperforms the HRP portfolio strategy. Therefore, the algorithm suggests investing only in A3. However, in scenarios such the High-volatility state, the HRP portfolio strategy reveals to perform slightly better than the Maximum Sharpe strategy. In this case, the portfolio allocation should be the following: A1 16.6%, A2 23.4%, A3 59.9%