

# Ejercicios-Complejidad-T1-Resuel...



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**Análisis y diseño de algoritmos****2º Grado en Ingeniería Informática****Escuela Técnica Superior de Ingeniería Informática  
Universidad de Málaga**

**Que no te escriban poemas de amor  
cuando terminen la carrera** ►►►►►►►

☺  
(a nosotros por  
suerte nos pasa)

**WUOLAH**

# IMPRIME TUS APUNTES APRUEBA SIN GASTAR UN DINERAL.



## Relación Ejercicios TI

① a)  $n^2 \in O(n^3)$

$$\lim_{n \rightarrow \infty} \frac{n^2}{n^3} = \frac{1}{n} < \infty \text{ pertenece}$$

b)  $n^3 \in O(n^2)$

$$\lim_{n \rightarrow \infty} \frac{n^3}{n^2} = n = \infty \text{ no pertenece}$$

c)  $2^{n+1} \in O(2^n)$

$$\lim_{n \rightarrow \infty} \frac{2^{n+1}}{2^n} = \frac{2^n \cdot 2}{2^n} = 2 < \infty \text{ pertenece}$$

d)  $(n+1)! \in O(n!)$

$$\lim_{n \rightarrow \infty} \frac{(n+1)!}{n!} = \infty \text{ no pertenece}$$

e)  $f(n) \in O(n) \Rightarrow 2^{f(n)} \in O(2^n)$

Contraejemplo: Sea  $f(n) = 3n$

$$\lim_{n \rightarrow \infty} \frac{3n}{n} = 3 < \infty \text{ pertenece}$$

$$\lim_{n \rightarrow \infty} \frac{2^{3n}}{2^n} = \lim_{n \rightarrow \infty} 2^{3n-n} = \infty \text{ no pertenece}$$

f)  $3^n \in O(2^n)$

$$\lim_{n \rightarrow \infty} \frac{3^n}{2^n} = \infty \text{ no pertenece}$$

g)  $\log n \in O(n^{1/2})$

$$\lim_{n \rightarrow \infty} \frac{\log n}{\sqrt{n}} = 0 \text{ pertenece}$$

h)  $n^{1/2} \in O(\log n)$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\log n} = \infty \text{ no pertenece}$$

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i)  $n^2 \in \Omega(n^3)$

$$\lim_{n \rightarrow \infty} \frac{n^2}{n^3} = \frac{1}{n} = 0 \quad \text{no pertenece}$$

j)  $n^3 \in \Omega(n^2)$

$$\lim_{n \rightarrow \infty} \frac{n^3}{n^2} = \infty \quad \text{pertenece}$$

k)  $2^{n+1} \in \Omega(2^n)$

$$\lim_{n \rightarrow \infty} \frac{2^{n+1}}{2^n} = 2 \quad \text{pertenece}$$

②  $T(n) = 2T(n-1) + 3^n(n+5)^1$  con  $n \geq 1$ ,  $T(0) = 0$

$$T(n) - 2T(n-1) = -(n+5)^1 3^n$$

$$(x-2)(x-3)^2 = 0$$

$$T(n) = c_1 2^n + c_2 3^n + c_3 n 3^n$$

$$T(1) = 2T(0) - 6 \cdot 3 = -18$$

$$T(2) = 2T(1) - 7 \cdot 9 = -99$$

$$0 = T(0) = c_1 2^0 + c_2 3^0 + c_3 0 \cdot 3^0 = c_1 + c_2$$

$$-18 = T(1) = c_1 2^1 + c_2 3^1 + c_3 1 \cdot 3^1 = 2c_1 + 3c_2 + 3c_3$$

$$-99 = T(2) = c_1 2^2 + c_2 3^2 + c_3 2 \cdot 3^2 = 4c_1 + 9c_2 + 18c_3$$

$$\cancel{c_1} - 2c_2 - 2c_3 = 0$$

$$2c_1 + 3c_2 + 3c_3 = -18$$

$$\hline c_2 + 3c_3 = -18$$

$$\cancel{c_1} - 4c_2 - 6c_3 = 36$$

$$4c_1 + 9c_2 + 18c_3 = -99$$

$$3c_2 + 12c_3 = -63$$

$$\begin{cases} c_2 + 3c_3 = -18 \\ 3c_2 + 12c_3 = -63 \end{cases} \Rightarrow \begin{cases} c_2 + 3c_3 = -18 \\ -c_2 - 4c_3 = 21 \end{cases}$$

$$\hline -c_3 = 3$$

$$c_3 = -3$$

EL PRIMER NÚMERO  
QUE VEAS, SERÁ  
TU NOTA EN  
EL PRÓXIMO EXAMEN



O	L	G	S	N	R	W	B	F	Q	L	Y	Q	E
S	U	T	M	W	T	C	U	A	T	R	O	O	H
E	P	G	R	R	R	J	S	E	A	N	L	M	R
A	N	G	J	E	E	P	V	Q	T	F	N	O	L
Y	R	P	E	Y	S	P	P	M	J	G	Z	M	L
M	A	T	R	I	C	U	L	A	V	A	A	F	C
Y	S	Y	C	L	G	K	K	E	F	H	X	S	L
V	N	M	I	U	Y	G	A	J	J	L	Z	C	O
X	U	D	O	S	R	Q	V	Y	N	E	O	R	Y
B	E	S	A	M	K	D	I	E	S	S	C	T	B
S	V	I	V	O	B	H	S	V	E	C	H	G	A
W	E	E	E	V	T	I	J	I	I	G	O	U	J
N	D	T	C	I	N	C	O	J	S	Z	F	F	P
E	N	E	A	U	U	N	O	J	J	O	W	S	D

Comparte por rr.ss. la nota que hayas visto y etiquéтанos

BeReal.



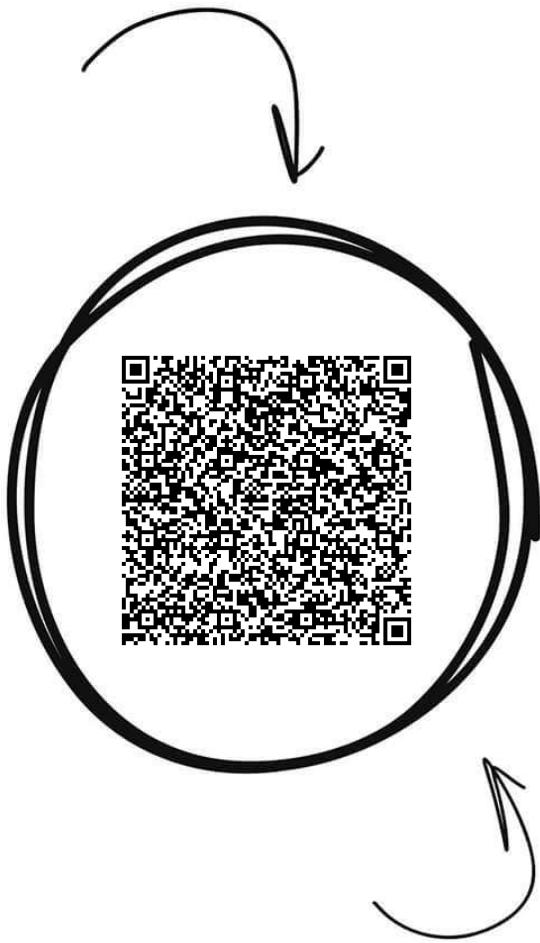
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# Análisis y diseño de algoritmos



Comparte estos flyers en tu clase y consigue más dinero y recompensas

- 1 Imprime esta hoja
- 2 Recorta por la mitad
- 3 Coloca en un lugar visible para que tus compis puedan escanear y acceder a apuntes
- 4 Llévate dinero por cada descarga de los documentos descargados a través de tu QR



## Banco de apuntes de la



$$c_2 + 3c_1 = -18$$

$$c_2 = -9$$

$$c_1 = -c_2 \Rightarrow c_1 = 9$$

$$T(n) = 92^n - 93^n - 3n3^n \in \Theta(n3^n)$$

$$\textcircled{3} \quad \textcircled{a} \quad T(n) = 3T(n-1) + 4T(n-2) \quad \text{si } n > 1; T(0) = 0; T(1) = 1$$

$$T(n) - 3T(n-1) - 4T(n-2) = 0$$

$$x^2 - 3x - 4 = 0$$

$$\frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{3 \pm \sqrt{25}}{2} \begin{cases} 4 \\ -1 \end{cases}$$

$$T(n) = c_1 4^n + c_2 (-1)^n$$

$$\left. \begin{array}{l} 0 = T(0) = c_1 4^0 + c_2 (-1)^0 = c_1 + c_2 \\ 1 = T(1) = c_1 4^1 + c_2 (-1)^1 = 4c_1 - c_2 \end{array} \right\} \Rightarrow \begin{array}{l} c_2 = -c_1 \Rightarrow c_1 = \frac{1}{5} \\ -4c_2 - c_2 = 1 \\ -5c_2 = 1 \\ c_2 = -\frac{1}{5} \end{array}$$

$$T(n) = \frac{1}{5}(4^n - (-1)^n) \in \Theta(4^n)$$

$$\textcircled{b} \quad T(n) = 4T(n/2) + n^2 \quad \text{si } n > 4, n \text{ potencia de 2}; T(1) = 1; T(2) = 8$$

$$n = 2^k \Rightarrow k = \log n$$

$$T(2^k) = 4T(2^{k-1}) + 2^{2k}$$

$$F(k) = T(2^k) \Rightarrow t_k = 4t_{k-1} + 4^k$$

$$t_k - 4t_{k-1} = 4^k \times 0 \Rightarrow t_k = c_1 4^k + c_2 k 4^k$$

$$(x - 4)(x - 4)$$

Deshacemos el cambio:

$$\left. \begin{array}{l} n = 2^k \\ T(2^k) = c_1 2^{2k} + c_2 k 2^{2k} \\ \Rightarrow T(n) = c_1 n^2 + c_2 n^2 \log n \end{array} \right.$$

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$$\begin{aligned} 1 &= T(1) = c_1 \cdot 1^2 + c_2 \cdot 1 \cdot 0 = c_1 \\ 8 &= T(2) = c_1 \cdot 2^2 + c_2 \cdot 2^2 \cdot 1 = 4c_1 + 4c_2 \end{aligned} \quad \left. \begin{array}{l} c_1 = 1 \\ 4 + 4c_2 = 8 \\ 4c_2 = 4 \\ c_2 = 1 \end{array} \right\}$$

$$T(n) = n^2 + n^2 \log n \in \Theta(n^2 \log n)$$

$$\textcircled{D} \quad T(n) = 2T\left(\frac{n}{2}\right) + n \log n \quad \text{si } n > 1, n \text{ potencia de 2}, T(1) = 1$$

$$n = 2^k \rightarrow k = \log n \rightarrow T(2^k) = 2T(2^{k-1}) + k \cdot 2^k$$

$$t_k = T(2^k) \rightarrow t_k = 2t_{k-1} + k \cdot 2^k$$

$$(x-2)(x-2)^2 = 0$$

$$F(k) = a \cdot 2^k + b \cdot k \cdot 2^k + c \cdot k^2 \cdot 2^k$$

Deshacemos el combio:

$$T(n) = an + bn \log n + cn \log^2 n \in \Theta(n \log^2 n)$$

$$T(1) = 1 \quad F(0) = 1 \quad a = 1$$

$$T(2^k) = F(k) \quad F(1) = 4 \quad 2a + 2b + 2c = 4$$

$$T(2^0) = F(0) \quad F(2) = 16 \quad 4a + 8b + 16c = 16$$

$$\begin{cases} 2b + 2c = 2 \\ 8b + 16c = 12 \end{cases} \rightarrow \begin{array}{r} -8b - 8c = -8 \\ 8b + 16c = 12 \\ \hline 8c = 4 \\ c = \frac{1}{2} \end{array} \quad \begin{array}{l} 2 + 2b + 1 = 4 \\ 2b = 1 \\ b = \frac{1}{2} \end{array}$$

$$\textcircled{D} \quad T(n) = 3T\left(\frac{n}{2}\right) + 5n + 3 \quad \text{si } n > 1, n \text{ potencia de 2}, T(1) = 1$$

$$n = 2^k \rightarrow T(2^k) = 3T(2^{k-1}) + 5 \cdot 2^k + 3$$

$$t_k = T(2^k) \rightarrow t_k = 3t_{k-1} + 5 \cdot 2^k + 3$$

$$(x-3)(x-2)(x-1) = 0$$

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$$F(k) = a3^k + b2^k + c$$

Deshacemos el cambio:

$$T(n) = a3^{\log n} + bn + c = an^{\log 3} + bn + c$$

$$T(1) = 1 \quad F(0) = 1 \Rightarrow a + b + c = 1$$

$$T(2^k) = F(k) \quad F(1) = 16 \Rightarrow 3a + 2b + c = 16$$

$$T(2^0) = F(0) \quad F(2) = 71 \Rightarrow 9a + 4b + c = 71$$

Resolvemos el sistema de ecuaciones:

$$\begin{array}{l} a + b + c = 1 \\ -3a - 2b - c = -16 \\ \hline -2a - b = -15 \end{array}$$

$$\begin{array}{r} a + b + c = 1 \\ -9a - 4b - c = -71 \\ \hline -8a - 3b = -70 \end{array}$$

$$\begin{array}{l} -2a - b = -15 \\ -8a - 3b = -70 \\ \hline b = -10 \end{array} \rightarrow \begin{array}{r} 8a + 4b = 60 \\ -8a - 3b = -70 \\ \hline b = -10 \end{array}$$

$$\begin{array}{l} -2a + 10 = -15 \\ -2a = -25 \\ a = \frac{25}{2} \end{array}$$

$$\begin{array}{l} \frac{25}{2} - 10 + c = 1 \\ c = 1 - \frac{5}{2} = \frac{2}{2} - \frac{5}{2} = -\frac{3}{2} \end{array}$$

$$F(k) = \frac{25}{2} 3^k - 10 2^k - \frac{3}{2} \in \Theta(3^{\log n})$$

$$\textcircled{2} T(n) = 2T\left(\frac{n}{2}\right) + \log n, n \geq 1, n \text{ potencia de } 2, T(1) = 1$$

$$n = 2^k \rightarrow T(2^k) = 2T(2^{k-1}) + k \quad k = \log n$$

$$t_k = T(2^k) \rightarrow t_k = 2t_{k-1} + k$$

$$(x-2)(x-1)^2$$

$$F(k) = a2^k + b + ck$$

Deshacemos el cambio:

$$T(n) = an + b + c \log n$$

$$T(1) = 1 \quad F(0) = 1 \Rightarrow a + b = 1$$

$$T(2^k) = F(k) \quad F(1) = 3 \Rightarrow 2a + b + c = 3$$

$$T(2^0) = F(0) \quad F(2) = 8 \Rightarrow 4a + b + 2c = 8$$

Resolvemos el sistema:

$$\begin{array}{r} a + b = 1 \\ -2a - b - c = -3 \\ \hline -a - c = -2 \end{array} \quad \begin{array}{r} 2a + b + c = 3 \\ -4a - b - 2c = -8 \\ \hline -2a - c = -5 \end{array}$$

$$\begin{cases} -a - c = -2 \Rightarrow -3 - c = -2 & 3 + b = 1 \\ 2a + c = +5 & -c = 1 \\ \hline a = 3 & c = -2 \end{cases} \quad b = -2$$

$$F(k) = 3n - 2 - k \in \mathbb{H}(n)$$

D)  $T(n) = T(n-1) + 2T(n-2) - 2T(n-3)$  si  $n > 2$

$$T(n) = 9n^2 - 15n + 106 \quad \text{si } n = 0, 1, 2$$

$$x^3 - x^2 - 2x + 2 = 0$$
$$\begin{array}{r} 1 - 1 - 2 \quad 2 \\ \hline 1 \quad 1 \quad 0 \quad -2 \\ \hline 1 \quad 0 \quad -2 \quad 0 \end{array}$$
$$x^2 - 2 = 0 \Rightarrow x = \pm \sqrt{2}$$

$$T(n) = a + b(\sqrt{2})^n + c(-\sqrt{2})^n$$

$$T(0) = 0 - 0 + 106 = 106 \Rightarrow a + b + c = 106$$

$$T(1) = a + b\sqrt{2} - c\sqrt{2} = 100$$

$$T(2) = a + 2b + 2c = 112$$

Resolvemos el sistema:

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$$\begin{array}{r} a + b + c = 106 \\ -a - b\sqrt{2} + c\sqrt{2} = -100 \\ \hline b - b\sqrt{2} + c + c\sqrt{2} = 6 \end{array}$$

$$\begin{array}{r} a + b + c = 106 \Rightarrow a = 100 \\ -a - 2b - 2c = -112 \\ \hline -b - c = -6 \end{array}$$

$$b + c = 6$$

$$b = 6 - c$$

$$6 - c - (6 - c)\sqrt{2} + c + c\sqrt{2} = 6$$

$$b = 3$$

$$6 - c - (6\sqrt{2} - \sqrt{2}c) + c + c\sqrt{2} = 6$$

$$2c\sqrt{2} - 6\sqrt{2} = 0$$

$$2c\sqrt{2} = 6\sqrt{2}$$

$$2c = 6$$

$$c = 3$$

$$T(n) = 100 + 3(\sqrt{2})^n + 3(-\sqrt{2})^n \in \Theta(2^{n/2})$$

②  $T(n) = 2T\left(\frac{n}{4}\right) + n^{3/2}$  si  $n \geq 4$ ,  $n$  potencia de 4;  $T(1) = 1$

$$n = 4^k \rightarrow k = \log_4 n \quad 4^{3/2k} = 2^k$$

$$T(4^k) = 2T(4^{k-1}) + 2^k$$

$$F(k) = 2F(k-1) + 2^k$$

$$(x-2)(x-2)$$

$$F(k) = a \cdot 2^k + b \cdot k \cdot 2^k \rightarrow a = 1$$

$$T(4^k) = 1 = F(0)$$

$$2a + 2b = 4 \Rightarrow 2 + 2b = 4$$

$$2b = 2$$

$$b = 1$$

$$F(1) = 2 \cdot F(0) + 2 = 4$$

$$F(k) = 2^k + 2^k k = T(4^k)$$

$$T(n) = n^{3/2} + n^{3/2} \cdot \log n$$

$$T(n) = \sqrt{n} + \sqrt{n} \cdot \log n \in \Theta(\log n \cdot \sqrt{n})$$

③  $T(n) = 4T(n/3) + n^2$  si  $n \geq 3$ ,  $n$  potencia de 3;  $T(1) = 1$

$$n = 3^k \rightarrow k = \log_3 n$$

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$$T(3^k) = 4T(3^{k-1}) + 3^{2k}$$

$$F(k) = 4F(k-1) + 9^k$$

$$(x-4)(x-9)$$

$$F(k) = a \cdot 4^k + b \cdot 9^k \rightarrow a + b = 1; b = 1 - a \rightarrow b = 1 + \frac{4}{5}$$

$$4a + 9b = 13$$

$$T(3^0) = 1 = F(0)$$

$$4a + 9(1-a) = 13$$

$$F(1) = 13$$

$$4a + 9 - 9a = 13$$

$$-5a = 4$$

$$a = -\frac{4}{5}$$

$$b = \frac{5}{5} + \frac{4}{5}$$

$$b = \frac{9}{5}$$

$$F(k) = -\frac{4}{5}4^k + \frac{9}{5}9^k$$

$$T(n) = -\frac{4}{5}4^{\log n} + \frac{9}{5}n^2 \in \Theta(n^2)$$

$$\textcircled{i} \quad T(n) = T(n-1) + 6T(n-2) + n; T(0) = 0, T(1) = 1$$

$$(x^2 - x - 6)(x - 1)^2 = 0$$

$$\frac{1 \pm \sqrt{1-4 \cdot 1 \cdot (-6)}}{2 \cdot 1} = \frac{1 \pm \sqrt{25}}{2}$$

$$T(n) = a3^n + b(-2)^n + c1^n + dn \cdot 1^n$$

$$T(2) = T(1) + 6T(0) + 2 = 3$$

$$T(3) = T(2) + 6T(1) + 3 = 12$$

$$\left. \begin{array}{l} a + b + c = 0 \\ 3a - 2b + c + d = 1 \\ 9a + 4b + c + 2d = 3 \\ 27a - 8b + c + 3d = 12 \end{array} \right\} \Rightarrow \begin{array}{l} a = \frac{9}{20} \\ b = -\frac{4}{45} \\ c = -\frac{13}{36} \\ d = -\frac{1}{6} \end{array}$$

$$T(n) = \frac{9}{20}3^n - \frac{4}{45}(-2)^n - \frac{13}{36} - \frac{1}{6}n \in \Theta(3^n)$$