Relación 1

sábado, 23 de septiembre de 2023 16:52

1. De las siguientes afirmaciones, indicar cuales son ciertas y cuáles no:

a) $n^2 \in O(n^3)$
b) $n^3 \in O(n^2)$
c) $2^{(n+1)} \in O(2^n)$
d) $(n+1)! \in O(n!)$
e) $f(n) \in O(n) \Rightarrow 2^{f(n)} \in O(2^n)$
f) $3^n \in O(2^n)$
g) $\log n \in O(n^{1/2})$
h) $n^{1/2} \in O(\log n)$
i) $n^2 \in \Omega(n^3)$
j) $n^3 \in \Omega(n^2)$
k) $2^{(n+1)} \in \Omega(2^n)$

1

a) $n^2 \in O(n^3) \to V$

b) $n^3 \in O(n^2) \to F$

c) $2^{(n+1)} \in O(2^n) \to F$

d) $(n+1)! \in O(n!) \to F$

e) $f(n) \in O(n) \Rightarrow 2^{f(n)} \in O(2^n) \to V$

f) $3^n \in O(2^n) \to F$

g) $\log_2 n \in O(\sqrt{n}) \to V$

$\log_{10} n \in O(\sqrt{n}) \to V$

h) $n^{1/2} \in O(\log n) \to \sqrt{n} \in O(\log n) \to F$

i) $n^2 \in \Omega(n^3) \to F$

j) $n^3 \in \Omega(n^2) \to V$

k) $2^{n+1} \in \Omega(2^n) \to V$

2. En los siguientes segmentos de código, determinar su tiempo de ejecución en función de n:

a)
```
c=1;
while (c <n) {
  algo_de_O(1);
  c= 2*c;
}
```

$c=1$; 1, $(c<n)$ 4, algo_de_O(1); 1, c= 2*c; 2

$\to t(n) = \left(\sum_{i=1}^{k} 4\right) + 1 + 1 = 4k + 2 = 4 \cdot \log_2 n + 2$

$\boxed{2^k \geq n \ ; \ \log_2 2^k \geq \log_2 n \ ; \ k \geq \log_2 n}$

b)
```
c=n;
while (c >1) {
  algo_de_O(1);
  c= c/2;
}
```

$\to t(n) = \left(\sum_{i=1}^{k} 4\right) + 1 + 1 = 4k + 2 = 4\log_2 n + 2$

$\boxed{n/2^k \leq 1 \ ; \ n \leq 2^k \ ; \ \log_2 n \leq \log_2 2^k \ ; \ \log_2 n \leq k}$

c)
```
for (int i= 0; i <n; i++) {
  c=n;
  while (c >1) {
    algo_de_O(1);
    c= c/2;
  }
}
```

( ) → Fin de bucle

$\to t(n) = 2 + \sum_{i=0}^{n-1}(3 + 1 + T_b(n)) = 2 + n(4 + T_b(n)) = 2 + n(4 + (4\log_2 n + 2)) =$

$= \boxed{2 + 6n + 4n\log_2 n}$ Preguntar profe

$n/2^k \leq 1 \to \log_2 n \leq k$

d)
```
for (int i= 1; i <= n; i++) {
  c=i;
  while (c >1) {
    algo_de_O(1);
    c= c/2;
  }
}
Pista: para n suficientemente grande, log(n!) ≈ nlogn − n
```

3

$T(n) = 2T(n-1) + 3^n(n+5)^{(*)_1}; \quad n > 0$

$T(0) = 0 \leftarrow$ Caso base

$(*)_1 \quad T(n) - 2T(n-1) = 3^n(n+5)$

$a_1 = -2 \ ; \ k = 1 \ ; \ h(n) = 3^n(n+5)$

$3^n(n+5) = \sum p(n)^{(m_i)} \cdot S_i^n \to \sum_1^1 (n+5) \cdot 3^n \to m_1 = 1 \ ; \ S_1 = 3$

$c_1(x) = x^k + a_1 x^{k-1} + \ldots + a_k$

$c_2(x) = (x - S_1)^{m_1+1} + \ldots + (x - S_i)^{m_i+1}$

$c_1(x) = x - 2 \cdot x^{1-1} = x - 2$

$c_2(x) = (x-3)^2$

$\to c_1 \cdot c_2 = (x-2) \cdot (x-3)^2 \to R = \{2, 3, 3\}^{(*)_2}$

$m_1 = 1 \quad m_2 = 2$

$(*)_2$ Raíces múltiples $\to B = \{\underline{2^n, 3^n, n \cdot 3^n}\} \to T(n) = \lambda_1 \cdot 2^n + \lambda_2 \cdot 3^n + \lambda_3 \cdot n \cdot 3^n \Rightarrow T(n) = -9 \cdot 2^n + 9 \cdot 3^n + 3n \cdot 3^n = -9 \cdot 2^n + 3^{n+2} + n \cdot 3^{n+1}$

Sea $T(0) = 0 \to T(0) = \lambda_1 + \lambda_2 = 0 \Rightarrow \lambda_1 = -\lambda_2$

Sea $T(1) = 2T(0) + 3^1(1+5) = 18$

Sea $T(2) = 2T(1) + 3^2(2+5) = 2 \cdot 18 + 9 \cdot 7 = 36 + 63 = 99$

Sea $T(3) = 2 \cdot T(2) + 3^3(3+5) = 2 \cdot 99 + 27 \cdot 8 = 198 + 216 = 414$

$\to$ Sistema: $\begin{cases} \lambda_1 + \lambda_2 = 0 \\ \lambda_1 2 + \lambda_2 \cdot 3 + \lambda_3 \cdot 1 \cdot 3 = 18 \\ \lambda_1 \cdot 4 + \lambda_2 \cdot 9 + \lambda_3 \cdot 2 \cdot 9 = 99 \end{cases}$ $(*)_3$

$(*)_3$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 2 & 3 & 3 & 18 \\ 4 & 9 & 18 & 99 \end{array}\right) \xrightarrow{F_3 \to F_3 - 2F_2} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 2 & 3 & 3 & 18 \\ 0 & 3 & 12 & 63 \end{array}\right) \xrightarrow{F_2 \to F_2 - 2F_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 18 \\ 0 & 3 & 12 & 63 \end{array}\right) \longrightarrow$$

$\xrightarrow{F_3 \to F_3 - 3F_2} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 3 & 18 \\ 0 & 0 & 3 & 9 \end{array}\right) \longrightarrow \begin{cases} \lambda_1 + \lambda_2 = 0 \to \boxed{\lambda_1 = -9} \\ \lambda_2 + 3\lambda_3 = 18 \to \lambda_2 + 3 \cdot 3 = 18 ; \boxed{\lambda_2 = 9} \\ 3\lambda_3 = 9 ; \boxed{\lambda_3 = 3} \end{cases}$

(4)

a) $T(n) = 3T(n-1) + 4T(n-2)$ si $n > 1$; $T(0) = 0$; $T(1) = 1$

$T(n) - 3T(n-1) - 4T(n-2) = 0 \to a_1 = -3 ; a_2 = 4 ; K = 2$

$x^2 - 3x - 4 = 0 \quad ; \quad x = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-4)}}{2 \cdot 1} = \frac{3 \pm \sqrt{9+16}}{2} = \frac{3 \pm 5}{2} = \left\langle \begin{array}{l} x_1 = 4 \\ x_2 = -1 \end{array} \right.$

$R = \{4, -1\} \to c(x) = (x-4) \cdot (x+1) \to m_1 = 1 ; m_2 = 1$ (Raíces simples)

$B = \{4^n, (-1)^n\} \to T(n) = \lambda_1 \cdot 4^n + \lambda_2 (-1)^n \to T(n) = \frac{4^n}{5} - \frac{(-1)^n}{5} = \frac{1}{5} \cdot (4^n - (-1)^n) \in \Theta(4^n)$

$\begin{cases} T(0) = \lambda_1 \cdot 4^0 + \lambda_2 \cdot (-1)^0 \to \lambda_1 + \lambda_2 = 0 \\ T(1) = \lambda_1 \cdot 4^1 + \lambda_2 \cdot (-1)^1 \to 4\lambda_1 - \lambda_2 = 1 \end{cases} \to \lambda_1 = -\lambda_2 ; -4\lambda_2 - \lambda_2 = 1 ; -5\lambda_2 = 1 ; \boxed{\lambda_2 = -\frac{1}{5}} \Rightarrow \boxed{\lambda_1 = \frac{1}{5}}$

b) $T(n) = 4T(n/2) + n^2 ; n > 4 ; T(1) = 1 ; T(2) = 8$

$T(n) - 4T(n/2) = n^2 ; T(2^m) - 4T(2^m/2) = 2^{2m} ; T(2^m) - 4T(2^{m-1}) = 2^{2m} \longrightarrow$

$\xrightarrow[1]{\text{cambio}}$ Si $T(2^m) = t(m) \to t(m) - 4t(m-1) = 4^m \overset{(*)}{} 4 \to a_1 = -4 ; k = 1 ; h(m) = 4^m = \sum_{1}^{1} n^0 \cdot 4^m ; m_1 = 0 ; S_1 = 4$

$\left.\begin{array}{l} c_1(x) = x - 4 \\ c_2(x) = (x-4)^{0+1} = x - 4 \end{array}\right\} \to c_1 \cdot c_2 = (x-4)^2 = c(x)$, $\to$ multiplicidad $= 2$ { Raíces múltiples o simples

Raíces múltiples $\to \underbrace{B = \{4^m, m4^m\}}_{\text{Sacamos base}} \to \underbrace{T(m) = \lambda_1 \cdot 4^m + \lambda_2 \cdot m \cdot 4^m}_{\text{Ec. general}}$

$\xrightarrow[\lambda_2]{\text{cálculo}}$ $t(m) = \lambda_2 m \cdot 4^m \xrightarrow{(*) 4} \lambda_2 \cdot m \cdot 4^m - 4 \cdot \lambda_2 (m-1) \cdot 4^{(m-1)} = 4^m ; \lambda_2 m 4^m - \lambda_2 (m-1) 4^m = 4^m ;$

$\lambda_2 m - \lambda_2 (m-1) = 1 ; \boxed{\lambda_2 = 1}$

$\xrightarrow[\text{queda}]{t(m)}$ $t(m) = \lambda_1 \cdot 4^m + m \cdot 4^m \xrightarrow[\text{el cambio}]{\text{Deshago}} t(m) = T(2^m) ; n = 2^m ; \log_2 n = m \longrightarrow T(n) = \lambda_1 \cdot 4^{\log_2 n} + \log_2 n \cdot 4^{\log_2 n}$

$T(n) = \lambda_1 \cdot n^2 + \log_2 n \cdot n^2$

$T(1) = 1 = \lambda_1 \cdot 1^2 + \log_2 1^{\,0} \cdot 1^2 \quad ; \boxed{\lambda_1 = 1}$ $\xrightarrow[\text{te queda así}]{\text{Finalmente}}$ $T(n) = n^2 + n^2 \log_2 n \in \Theta(n^2 \log n)$

c) $T(n) = 2T(n/2) + n \cdot \log n ; n > 1, n = 2^m$

$\xrightarrow[1]{\text{cambio}}$ $T(2^m) = 2T(2^{m-1}) + 2^m \cdot \log 2^m ; T(2^m) = 2T(2^{m-1}) + 2^m \cdot m$

$\to T(2^m) = t(m) \longrightarrow t(m) = 2t(m-1) + 2^m \cdot m ; t(m) - 2t(m-1) = 2^m \cdot m$

$a_1 = -2 ; k = 1 ; h(n) = 2^m \cdot m \xrightarrow[\Sigma]{\text{como}} 2^m \cdot m = \sum_{i=1}^{1} m \cdot 2^m$

$\Rightarrow m_1 = 1 , S_1 = 2$

$\xrightarrow[c_1 y c_2]{\text{saco}}$ $\left.\begin{array}{l} c_1 = x - 2 \\ c_2 = (x-2)^2 \end{array}\right\} c_1 \cdot c_2 = (x-2) \cdot (x-2)^2 = (x-2)^3 \to m_2 = 3 \to R = \{2, 2, 2\}$

$\xrightarrow[\text{múltiples}]{\text{raíces}}$ $B = \{2^m, m \cdot 2^m, m^2 \cdot 2^m\} \to t(m) = \lambda_1 \cdot 2^m + \lambda_2 \cdot m \cdot 2^m + \lambda_3 m^2 \cdot 2^m$

raíces múltiples $B=\{2^m,\ m\cdot 2^m,\ m^2\cdot 2^m\} \to t(m)=\lambda_1\cdot 2^m+\lambda_2\cdot m\cdot 2^m+\lambda_3 m^2\cdot 2^m$

Deshago cambios $t(m)=T(2^m)$; $2^m=n$; $\log 2^m=\log n$; $m=\log n \Rightarrow T(n)=\lambda_1 n+\lambda_2\cdot n\cdot\log n+\lambda_3 n\cdot\log^2 n \in\Theta(n\log^2 n)$

d) $T(n)=3T(n/2)+5n+3$ si $n>1, n=2^m$

$\to n=2^m \to T(2^m)=3T(2^{m-1})+5\cdot 2^m+3$; $T(2^m)-3T(2^{m-1})=5\cdot 2^m+3$;

$\to T(2^m)=t(m) \to t(m)-3T(m-1)=5\cdot 2^m+3$; $a_1=-3$; $k=1$; $h(m)=$ ¿?

$\to \sum_{i=1}^{1} p(m)\, S_i^m \to$

e) $T(n)=2T(n/2)+\log n$ si $n>1$, $n=2^m \to T(2^m)=2T(2^{m-1})+\log 2^m$; $T(2^m)-2T(2^{m-1})=m$

$\to T(2^m)=t(m) \to t(m)-2t(m-1)=m$; $a_1=-2$; $k=1$; $h(m)=m$

$\to \sum p(m)\cdot S^m \to \sum_{i=1}^{1} m^1\cdot 1^m \to m_1=1$; $S_1=1$; $\to$ $\left.\begin{array}{l} c_1(x)=x-2 \\ c_2(x)=(x-1)^2 \end{array}\right\} c_1\cdot c_2=(x-2)(x-1)^2$

$R=\{2,1,1\} \to m_1=1$; $m_2=2$; $B=\{2^m, 1, m\}$

$t(m)=\lambda_1\cdot 2^m+\lambda_2+\lambda_3\cdot m$ Deshago cambio $T(2^m)=\lambda_1\cdot 2^m+\lambda_2+\lambda_3\cdot m \xrightarrow[2^m=n]{m=\log n} T(n)=\lambda_1\cdot n+\lambda_2+\lambda_3\cdot\log(n)\in\Theta(\log(n))$ [si $\lambda_1=0$]

$\in\Theta(n)$ [si $\lambda_1\neq 0$]

f) $T(n)=T(n-1)+2T(n-2)-2T(n-3)$; $n>2$; $T(n)=9n^2-15n+106$ si $n=0,1,2$

$T(n)-T(n-1)-2T(n-2)+2T(n-3)=\emptyset$; $a_1=-1, a_2=-2, a_3=2, k=3, h(n)=\emptyset$

$x^3-x^2-2x+2=0$;

$$\begin{array}{c|cccc} & 1 & -1 & -2 & 2 \\ 1 & & 1 & 0 & -2 \\ \hline & 1 & 0 & -2 & 0 \end{array}$$

$\longrightarrow x^2-2=0$; $x^2=2$; $\boxed{x=\pm\sqrt{2}}$

$R=\{1,\sqrt{2},-\sqrt{2}\} \to B=\{1, \sqrt{2}^n, (-\sqrt{2})^n\} \to T(n)=\lambda_1+\lambda_2\cdot(\sqrt{2})^n+\lambda_3\cdot(-\sqrt{2})^n$

$T(0)=9\cdot 0^2-15\cdot 0+106=106$

$T(1)=9\cdot 1^2-15\cdot 1+106=9-15+106=100$

$T(2)=9\cdot 2^2-15\cdot 2+106=36-30+106=112$

$$\left\{\begin{array}{l} 106=\lambda_1+\lambda_2\cdot(\sqrt{2})^0+\lambda_3\cdot(-\sqrt{2})^0;\ 106=\lambda_1+\lambda_2+\lambda_3 \\ 100=\lambda_1+\lambda_2\cdot(\sqrt{2})^1+\lambda_3(-\sqrt{2})^1;\ 100=\lambda_1+\sqrt{2}\cdot\lambda_2-\sqrt{2}\cdot\lambda_3 \\ 112=\lambda_1+\lambda_2\cdot(\sqrt{2})^2+\lambda_3(-\sqrt{2})^2;\ 112=\lambda_1+2\lambda_2+2\lambda_3 \end{array}\right\} \longrightarrow$$