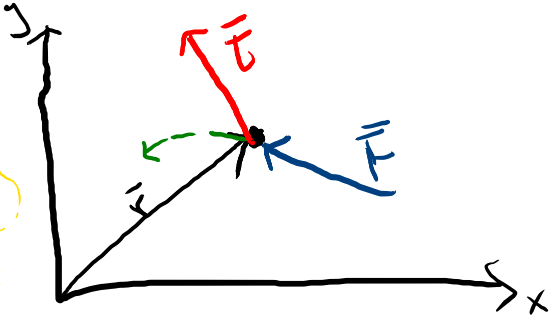
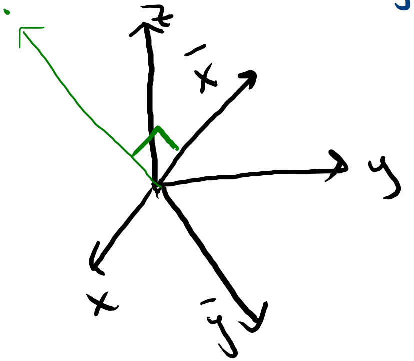
Equilibrio rotacional



$\bar r = (r_x, r_y)$

$\bar F = (F_x, F_y)$

$\bar\tau = \bar z$



$a+b$

$a\bar b$

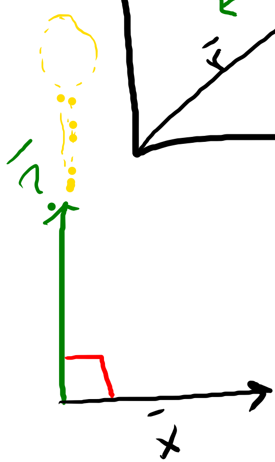
$a \cdot \frac{1}{b}$

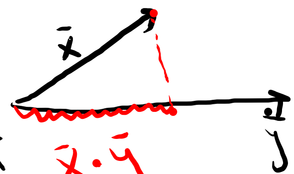
$\bar x = (x_1, x_2)$

$\bar y = (y_1, y_2)$

$\bar x \pm \bar y = (x_1 \pm y_1, x_2 \pm y_2)$

$a\bar x = (ax_1, ax_2)$





$\bar F = m\bar a$

$\bar x \cdot \bar y$

Proyección

$\bar x = (x_1, x_2, x_3)$

$\bar y = (y_1, y_2, y_3)$

$\bar x \cdot \bar y = x_1 y_1 + x_2 y_2$

$\bar\tau = \bar r \times \bar F$, $\bar L = I\bar\omega$

I masa inercial

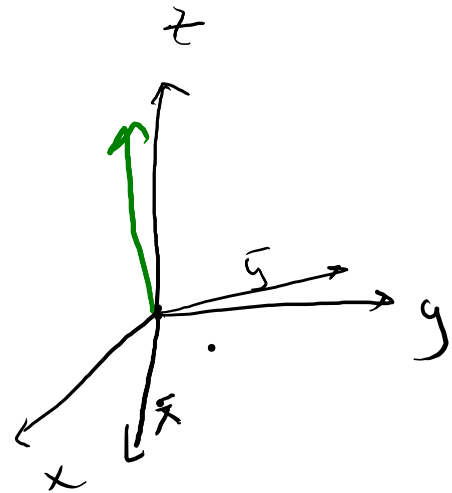
$\bar\omega$: aceleración angular

$\bar\tau = \frac{d\bar L}{dt}$

$\bar z = (z_1, z_2, z_3) \neq \bar 0 = (0, 0, 0)$

$$\bar{z} = \bar{x} \times \bar{y} = \begin{vmatrix} +\hat{i} & -\hat{j} & +\hat{k} \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix} =$$

$\times$: producto cruz



$\bar{x}, \bar{y}$ son vectores en el plano y por lo tanto su tercera coordenada es cero.

$\bar{x} = (x_1, x_2, 0)$ $\quad$ $\bar{y} = (y_1, y_2, 0)$

$$\bar{z} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & x_2 & 0 \\ y_1 & y_2 & 0 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(x_1 y_2 - x_2 y_1) = (0, 0, x_1 y_2 - x_2 y_1)$$

$$\bar{\tau} = \frac{d\,\bar{I}\bar{\omega}}{dt} = \bar{I}\frac{d\bar{\omega}}{dt} + \cancel{\frac{d\bar{I}}{dt}}^{\,0}\bar{\omega} = \bar{I}\frac{d\bar{\omega}}{dt} = \bar{I}\bar{\alpha}$$

sup. que $\bar{I}$ no cambia

Para tener equilibrio,

1. $\bar{\alpha} = \frac{d\bar{\omega}}{dt} = 0$
2. $\bar{F}_{total} = 0$

$\bar{\tau} = \bar{r} \times \bar{F}$

$\|\bar{\tau}\| = \|\bar{r} \times \bar{F}\| = \|\bar{r}\|\|\bar{F}\|\sin\theta$ ← magnitud de la torque

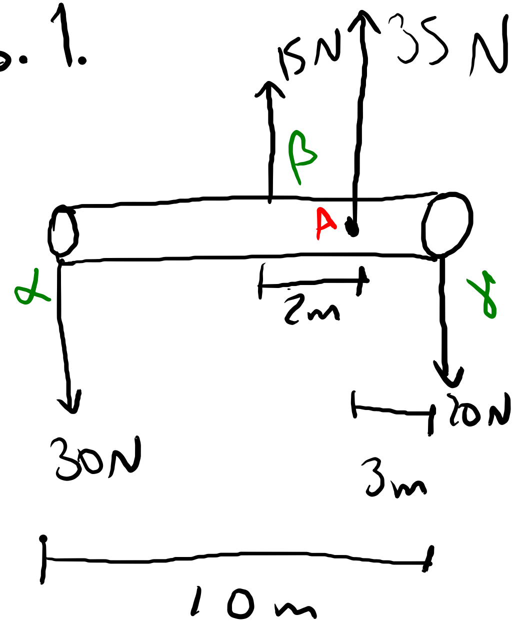
$\|\bar{\tau}\| = \tau = rF\sin\theta$

$$\boxed{\tau = F\,d}$$

$\theta = 90°$, $\sin(90°) = \sin\left(\frac{\pi}{2}\right)$

$\sin(90°) = 1 = \frac{\sqrt{4}}{2}$

Prob. 1.



$\sum_{i=1} \bar{F}_i = \bar{0}$ Por equilibrio rotacional

$\sum_{i=1} \bar{\tau}_i = \bar{0}$

$\bar{F}_\alpha = (0, -30N)$ $\bar{F}_\gamma = (0, -20N)$

$\bar{F}_\beta = (0, 15N)$ $\bar{F}_A = (0, F_A)$

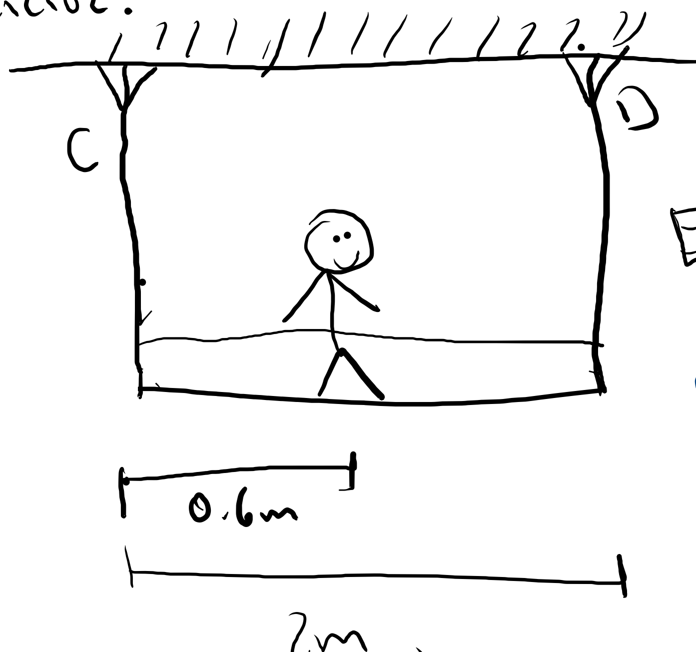
$\bar{F}_\alpha + \bar{F}_\beta + \bar{F}_\gamma + \bar{F}_A = \bar{0}$

$-30N + 15N - 20N + F_A = 0$

$\boxed{F_A = 35N}$

$\tau_A = F_A d = 35N\,(2m) = 70\,N.m$

Ejercicio 2.



Tip: Siempre pararse en un extremo.

$W = 500\,N$

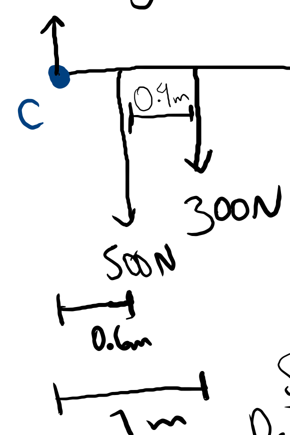
$W_T^p = 300\,N$

$F_D = \dfrac{\tau_D}{2m} = 300\,N$

$\bar{F}_C - 500N - 300N + 300N = 0$

$\boxed{\bar{F}_C = 500N}$

Estrategia



$\sum_{i=1}^{4} \bar{\tau}_i = 0 \quad (1)$

$\sum_{i=1}^{4} \bar{F}_i = 0 \quad (2)$

Sol.

$D_c\,(1),\ \sum_i \bar{\tau}_i = 0$

$\bar{\tau}_c + \bar{\tau}_t + \bar{\tau}_p + \bar{\tau}_D = 0$

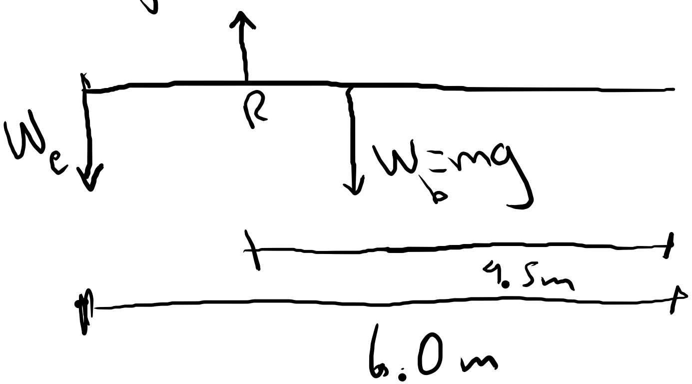
$D_c\,(2),\ \sum_i \bar{F}_i = 0$

$F_c(0) - 500N(0.6m) - 300N(1m) + \tau_D = 0$

$\tau_D = 300\,N\cdot m + 300\,N\cdot m$

$\tau_D = 600\,N\cdot m$

Ejercicio 3.



Estrategia

1) Pararse sobre el extremo en cuestión
2) Aplicar cond. de eq.

Solución

① $\sum_{i}^{3} \bar{\tau}_i = 0$, ② $\sum_{i}^{3} \bar{F}_i = 0$

De ① $\bar{\tau}_1 + \bar{\tau}_2 + \bar{\tau}_3 = 0$

$W_e \cancel{0}^{0} + F_R(1.5\,m) - (200\,kg)(9.81\,m/s^2)3\,m = 0$

$1.5\,m\,F_R \approx 2000\,N \cdot 3\,m$

$F_R = \frac{2000\,N \cdot 3\,m}{1.5\,m} = \underline{4000\,N}$

De ② $\bar{F}_1 + \bar{F}_2 + \bar{F}_3 = 0$

$W_e + 4000\,N - 2000\,N = 0$

$W_e = -2000\,N$

$t = 0$ seg

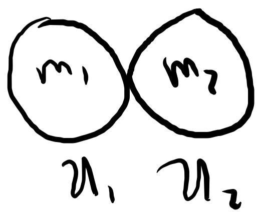
1D

$$\sum P_i = 0$$

$$m_1 u_1 + m_2 u_2$$

$$= m_1 V_1 + m_2 V_2$$

$t = t_1$ seg



$t = t_2$ seg

$$(2500\,kg)(75\,km/h) + (\overset{70}{\cancel{3\,kg}})(0) = (2500\,kg)V + 3\,kg\,V$$

$$187{,}500\,kg \cdot km/h = V[2503\,kg]$$

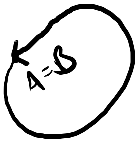
$$V = \frac{187{,}500\,\cancel{kg} \cdot km/h}{2503\,\cancel{kg}} = 74.9\,km/h$$

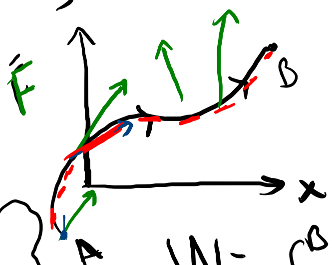
Energía y trabajo en campos gravitacionales

$$\bar{F} = m\bar{a} = m\frac{d\bar{v}}{dt} = m\frac{\Delta v}{\Delta t}$$

$$x = vt \quad , \quad v = \frac{x}{t}$$

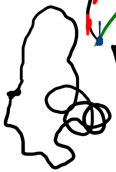
$$d\bar{r} = \bar{v}\,dt$$





$$W = \int_A^B \bar{F} \cdot d\bar{r}$$

$$\underbrace{\text{si } A=B}_{W=0.}$$



$$W = \int_A^B m\frac{d\bar{v}}{\cancel{dt}} \cdot \bar{v}\,\cancel{dt} = \int_A^B m\, d\bar{v} \cdot \bar{v}$$

$$\int x\,dx = \frac{x^2}{2} + C$$

$$= m\int_A^B \bar{v} \cdot d\bar{v} = \frac{m}{2} v^2 \Big|_A^B = \frac{m}{2}\left[v_B^2 - v_A^2\right]$$

$$= \frac{mv_B^2}{2} - \frac{mv_A^2}{2}$$

$$= E_c^B - E_c^A = \Delta E$$

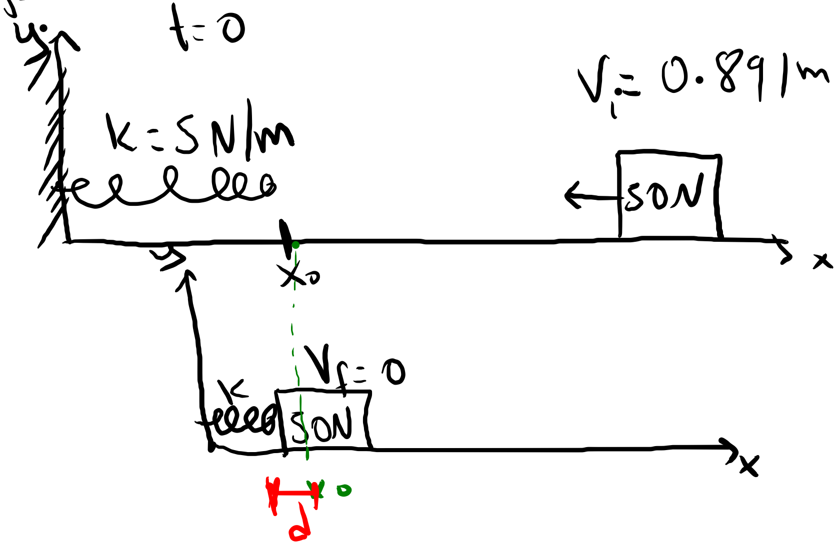
Energía: La capacidad que tiene un cuerpo de realizar un trabajo.

La energía total en un sistema

$$E_k + E_p = K + V.$$

$$W = \Delta K + \Delta V$$

Ejercicio 1.



$$\Delta E_c + \Delta E_p = 0$$

$$\Delta E_c = \frac{1}{2}m\left(V_f^2 - V_i^2\right)$$

$$\Delta E_p = \frac{1}{2}kx^2 \leftarrow$$ para un resorte es siempre

$$\bar{F} = -k\bar{x}$$

$$F = -kx$$

$$\bar{F} = -\frac{d}{dx}\Delta E_p = -\frac{k}{2}2x = -kx$$

$$m = \frac{50N}{10m/s^2} = 5kg$$

$$\frac{1}{2}m\left(0 - [0.891 m/s]^2\right) + \frac{1}{2}kx^2 = 0$$

$.792 m^2/s^2$

$$-\frac{1}{2}mV_i^2 + \frac{1}{2}kx^2 = 0 \Rightarrow kx^2 = V_i^2 m$$

$$x^2 = \frac{m}{k}V_i^2, \quad x = \sqrt{\frac{m}{k}}V_i = 0.89 m$$