Ayudantía 2. Física Estadística.

2.2 Pathria

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(1) $d\omega$ del espacio fase de una partícula es invariante bajo una transformación de coordenadas cartesianas (x, y, z, p_x, p_y, p_z) a coordenadas esféricas.

Solución

$x = r\cos\theta\sin\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

$$|M| = \left|\frac{\partial(x,y,z,p_x,p_y,p_z)}{\partial(r,\theta,\phi,p_r,p_\theta,p_\phi)}\right| = \left|\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}\right|\left|\frac{\partial(p_x,p_y,p_z)}{\partial(p_r,p_\theta,p_\phi)}\right|$$

$$= r^2\sin\theta\,\frac{\partial(p_x,p_y,p_z)}{\partial(p_r,p_\theta,p_\phi)}$$

$$p_x = m\dot{x} = m\dot{r}\sin\theta\cos\phi + mr\cos\theta\cos\phi\,\dot{\theta} - mr\sin\theta\sin\phi\,\dot{\phi}$$

$$p_y = m\dot{y} = m\dot{r}\sin\theta\sin\phi + mr\cos\theta\sin\phi\,\dot{\theta} + mr\sin\theta\cos\phi\,\dot{\phi}$$

$$p_z = m\dot{z} = m\dot{r}\cos\theta - mr\sin\theta\,\dot{\theta}$$

Recordamos que en esféricas:

$$p_r = m\dot{r} , \quad p_\theta = mr^2\dot{\theta} , \quad p_\phi = mr^2\sin^2\theta\,\dot{\phi}$$

$$p_x = \sin\theta\cos\phi\, p_r + \frac{\cos\theta\cos\phi}{r} p_\theta - \frac{\sin\phi}{r\sin\theta} p_\phi$$

$$p_y = \sin\theta\sin\phi\, p_r + \frac{\cos\theta\sin\phi}{r} p_\theta + \frac{\cos\phi}{r\sin\theta} p_\phi$$

$$p_z = \cos\theta\, p_r - \frac{\sin\theta}{r} p_\theta$$

$$\frac{\partial(p_x,p_y,p_z)}{\partial(p_r,p_\theta,p_\phi)} = \begin{vmatrix} \frac{\partial p_x}{\partial p_r} & \frac{\partial p_x}{\partial p_\theta} & \frac{\partial p_x}{\partial p_\phi} \\ \frac{\partial p_y}{\partial p_r} & \longrightarrow & \\ \frac{\partial p_z}{\partial p_r} & \longrightarrow & \end{vmatrix} = \frac{1}{r^2\sin\theta} \Rightarrow \underline{|M| = 1}$$

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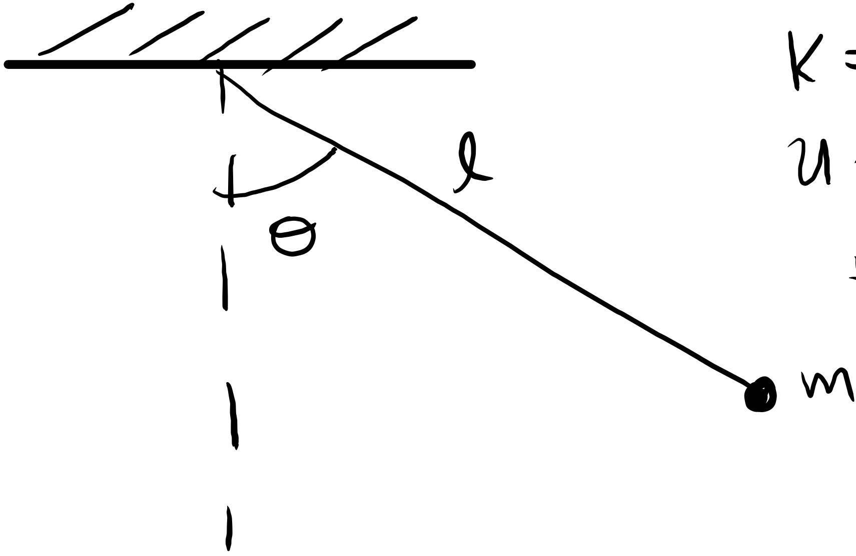

$$K = \frac{1}{2} m l^2 \dot{\theta}^2$$

$\theta << 1.$

$$U = mgl(1-\cos\theta) \simeq mgl\frac{\theta^2}{2}$$

$$H = E = \frac{p_\theta^2}{2ml^2} + mgl\frac{\theta^2}{2}$$

$$\boxed{1 = \frac{p_\theta^2}{a^2} + \frac{\theta^2}{b^2}}, \quad a^2 \equiv 2ml^2E, \quad b^2 \equiv 2E/mgl$$

$\frac{1}{\omega}$

$$A(\bigcirc) = \pi ab = 2\pi E\sqrt{\frac{l}{g}} = E\tau, \quad \boxed{\tau = 2\pi\sqrt{\frac{l}{g}}}$$

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$$x(t) = A\cos(\omega_0 t + \phi)$$

$$P(x)\,dx = P(x, x+dx)$$

Si $T = \frac{2\pi}{\omega_0}$ es el tiempo requerido para que la partícula complete un ciclo,

$P(x)\,dx = 2\frac{dt}{T}$: fracción del periodo que la partícula pasa en el intervalo $(x, x+dx)$

Supongamos que $\phi = \pi/2$, entonces $x(t) = A\sin(\omega_0 t)$

$v = \frac{dx}{dt} \Rightarrow dt = \frac{dx}{v}$, $v = \omega_0 A\cos(\omega_0 t) = \omega_0\sqrt{A^2 - x^2(t)}$

$$P(x)\,dx = 2\frac{dt}{T} = \frac{2}{T}\frac{dx}{\omega_0\sqrt{A^2-x^2(t)}} = \frac{2\cancel{\omega_0}}{2\pi\,\cancel{\omega_0}}\frac{dx}{\sqrt{A^2-x^2(t)}} = \underbrace{\frac{1}{\pi}\frac{dx}{\sqrt{A^2-x^2(t)}}}$$

$$\int_{-A}^{A} P(x)\,dx \overset{!}{=} 1.$$

$$\frac{1}{\pi}\int_{-A}^{A}\frac{dx}{\sqrt{A^2-x^2(t)}} \overset{!}{=} 1$$

$$= \frac{1}{\pi}\arctan\left(\frac{x}{\sqrt{A^2-x^2(t)}}\right)\Bigg|_{-A}^{A} = \frac{1}{\pi}\left(\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right) = 1.$$

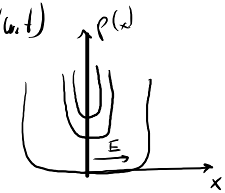
De la conservación de la energía,

$$E = \frac{mv^2}{2} + \frac{1}{2}m\omega_0^2 x^2$$

$$= \frac{1}{2}m\omega_0^2 A^2\cos^2(\omega_0 t) + \frac{1}{2}m\omega_0^2 A^2\sin^2(\omega_0 t)$$

$$\underbrace{E = \frac{1}{2}m\omega_0^2 A^2}$$

$$\underbrace{A^2 = \frac{2E}{m\omega_0^2}}$$



$$P(x) = \frac{1}{\pi}\left(\frac{k}{2E - kx^2}\right)^{1/2} \qquad \underbrace{k = m\omega_0^2}$$

$$P_n(x) = \frac{1}{\pi}\left(\frac{k}{2E_n - kx^2}\right)^{1/2}$$

$$E_n = \hbar\omega\left(n + 1/2\right)$$

③ $\bar{P} = \langle \Psi_k | \bar{\sigma} | \Psi_k \rangle$, $\bar{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ Pathria

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$|\Psi_k\rangle = a_+ \phi_{k,+1/2} + a_- \phi_{k,-1/2}$

$$P_x = (a^*_+, a^*_-) \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} a_+ \\ a_- \end{pmatrix} = (a^*_+ a_- + a^*_- a_+) \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$P_y = (a^*_+, a^*_-) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a_+ \\ a_- \end{pmatrix} = i(a^*_- a_+ - a^*_+ a_-) \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$P_z = (a^*_+, a^*_-) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a_+ \\ a_- \end{pmatrix} = (a^*_+ a_+ - a^*_- a_-)$$

$$|\bar{P}| = \sqrt{P_x^2 + P_y^2 + P_z^2} = 1 .$$ (a)

(b) $\bar{P} = \rho_+ \langle \phi_{k,1/2} | \bar{\sigma} | \phi_{k,+1/2} \rangle + \rho_- \langle \phi_{k,-1/2} | \bar{\sigma} | \phi_{k,-1/2} \rangle$

$$= \rho_+ \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \rho_- \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \rho_+ - \rho_- \end{pmatrix}, \quad \rho_+ = \rho_- = \frac{1}{2}.$$

$$\bar{P} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad |\bar{P}| = 0.$$

$(\bar{\sigma}, \mathbb{1})$: forman una base.

$$\rho_{ss'} = \begin{pmatrix} \rho_+ & 0 \\ 0 & \rho_- \end{pmatrix}, \quad \rho_+ + \rho_- = 1$$

$$|\Psi_{k,s}\rangle\langle \Psi_{k,s'}| = \begin{pmatrix} a^*_+ a_+ & a^*_- a_+ \\ a^*_+ a_- & a^*_- a_- \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix}.$$

5 (a) $\langle \hat{O} \rangle = tr[\hat{\rho}(t)\hat{O}]$ $\hat{\rho}(t) = \sum_i p_i |\psi_i(t)\rangle\langle\psi_i(t)|$

$\sum_i p_i = 1$

$\rho(t)$ $\{|\bar{r}\rangle\}$

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$\hat{\rho}_{\bar{r}}(t) \longrightarrow \langle \bar{r}'|\hat{\rho}(t)|\bar{r}\rangle$

$\hat{\phi}(p_1,p_2) = \hat{\rho}_{\bar{p}}(t) \longrightarrow \langle \bar{p}'|\hat{\rho}(t)|\bar{p}\rangle$

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(b) $\hat{\phi}(p_1,p_2) = f(p_1)\,\delta_{p_1p_2}$

$$\langle \bar{r}'|\hat{\rho}(t)|\bar{r}\rangle = \sum_{\bar{p}'\bar{p}} \langle \bar{r}'|\bar{p}\rangle\langle\bar{p}'|\hat{\rho}(t)|\bar{p}\rangle\langle\bar{p}|\bar{r}\rangle$$

$$= \frac{1}{V}\sum_{\bar{p}'\bar{p}} \phi(\bar{p}',\bar{p})\exp\{i(\bar{r}'\cdot\bar{p}' - \bar{r}\cdot\bar{p})\}$$

$$= \frac{1}{V}\sum_{\bar{p}'\bar{p}} f(p)\delta_{pp'}\exp\{i(\bar{r}'\cdot\bar{p}' - \bar{r}\cdot\bar{p})\}$$

$$= \frac{1}{V}\sum_{\bar{p}} f(p)\exp\{i(\bar{r}'-\bar{r})\cdot\bar{p}\}$$

$\langle \bar{r}|\hat{\rho}(t)|\bar{r}\rangle = \frac{1}{V}\sum_p f(p)$ es una constante.