Fun With Natural Numbers

In this problem we will have some fun with the Natural numbers (1, 2, 3, 4, …). You are to implement four methods in the `FunWithNaturalNumbers` class. The four methods are `summation(int n)`, `prod(int n)`, `groupEm(int n)`, and `sumThoseGroups()`.

The `summation(int n)` method returns the largest value m such that 1+2+3+…+m <= n. For example:

```
summation(1+2+3) = summation(6) = 3:  since 1+2+3 = 6
summation(50) = 9:                    since 1+2+3+4+5+6+7+8+9 = 45
                                      and 1+2+3+4+5+6+7+8+9+10 = 55
summation(91) = 13:                   since 1+2+3+4+5+6+7+8+9+10+11+12+13 = 91
```

The following code shows the results of the `summation(n)` method.

The following code	Returns
`FunWithNaturalNumbers.summation(1+2+3);`	3
`FunWithNaturalNumbers.summation(50);`	9
`FunWithNaturalNumbers.summation(91);`	13

The `prod(int n)` method returns the largest value m such that $1*2*3*4*m <= n(n+1)$. For example:

```
prod(24) = 5:          1*2*3*4*5 = 120 <= 600 = 24*25
                   and 1*2*3*4*5*6 = 720 > 600 = 24*25
prod(99) = 7:          1*2*3*…*6*7 = 5040 <= 9900 = 99*100
                   and 1*2*3*…*6*7*8 = 40320 > 9900 = 99*100
prod(2318) = 10:       1*2*3*…*9*10 = 3628800 <= 5375442 = 2318*2319
                   and 1*2*3*…*10*11 = 3991680 > 5375442 = 2318*2319
```

The following code shows the results of the `prod(n)` method.

The following code	Returns
`FunWithNaturalNumbers.prod(24);`	5
`FunWithNaturalNumbers.prod(99);`	7
`FunWithNaturalNumbers.prod(2318);`	10

Motivation found in: Number5 Are For3ver: mathematical fats and Curiosities by Liz Strachan

The `groupEm(int n)` method is used to help demonstrates just how amazing Natural numbers are. Write out the Natural Numbers:

1, 2 , 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, ….

Now arrange them into groups of 1 number, 2 numbers, 3 numbers, and so on:

1, 2 , 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, ….

And cross out the 2nd, 4th, 6th and all other even numbered groups.

1, ~~2 , 3~~, 4, 5, 6, ~~7, 8, 9, 10~~, 11, 12, 13, 14, 15, ~~16, 17, 18, 19, 20, 21~~, 22, 23, 24, 25, 26, 27, 28, ~~29, 30,~~ ….

So you are left with:

1, 4, 5, 6, 11, 12, 13, 14, 15, 22, 23, 24, 25, 26, 27, 28, 37, 38, 39, 40, 41, 42, 43, 44, 45, …..

The `groupEm(int n)` method should return a `List` containing the corresponding group of numbers. For example:

groupEm(1) returns the `List` containing the `Integer` 1.
groupEm(2) returns the `List` containing the `Integer`s 4, 5, and 6.
groupEm(3) returns the `List` containing the `Integer`s 11, 12, 13, 14, and 15

The following code shows the results of the `groupEm(n)` method.

The following code	Returns
`List<Integer> ans = FunWithNaturalNumbers.groupEm(2);`	
`ans.size();`	3
`ans.contains(new Integer(4));`	true
`ans.contains(new Integer(5));`	true
`ans.contains(new Integer(6));`	true

The following code shows the results of the `groupEm(n)` method.

The following code	Returns
`List<Integer> ans = FunWithNaturalNumbers.groupEm(3);`	
`ans.size();`	5
`ans.contains(new Integer(11));`	true
`ans.contains(new Integer(12));`	true
`ans.contains(new Integer(13));`	true
`ans.contains(new Integer(14));`	true
`ans.contains(new Integer(15));`	true

Motivation found in: Number5 Are For3ver: mathematical fats and Curiosities by Liz Strachan

The `sumThoseGroups(int n)` method demonstrates just how amazing Natural numbers are. The `sumThoseGroups(int n)` method returns the sum all the `ints` from group 1 up to and including group `n` as defined by the `groupEm` method. For example:

sumThoseGroups(1) returns the value 1.

sumThoseGroups(2) returns the value 16 = groupEm(1) + groupEm(2) = 1 + (4 + 5 +6).

sumThoseGroups(3) returns the value 81 = groupEm(1) + groupEm(2) + groupEm(3)
= 1 + (4 + 5 +6) + (11 + 12 + 13 + 14 + 15)

Oh yea, how does this demonstrate that Natural numbers are amazing. I leave that up to you to figure out ☺.

The following code shows the results of the `sumThoseGroups(n)` method.

The following code	Returns
`FunWithNaturalNumbers.sumThoseGroups(1);`	`1`
`FunWithNaturalNumbers.sumThoseGroups(2);`	`16 = 1+(4+5+6)`
`FunWithNaturalNumbers.sumThoseGroups(3);`	`81 = 1 + (4+5+6) + (11+12+13+14+15)`

Motivation found in: Number5 Are For3ver: mathematical fats and Curiosities by Liz Strachan