Fixed Income

The Term Structure of Interest Rates:

Spot, Par, and Forward Curves

Exam Focus

- Spot, par, and forward rates
- Yield curves

Spot Rates

- Yield to maturity is weighted average of each period's spot rate
- Spot rates can be used to discount a cash flow from when it is paid back to T_0
- Spot rates can be observed from zero-coupon bonds
- Expect a different spot rate each for each period
- Can find price (PV) of a bond by discounting each CF by its relevant spot rate
- Denoted as Z or S

Spot Rates: Example

Calculate the price of a $1,000 par value five-year Canadian government bond, with a 1.00% annual-pay coupon using the following spot rates:

Maturity	Gov. Spot Rate
1 yr	0.31%
2 yrs	0.57%
3 yrs	0.80%
4 yrs	0.96%
5 yrs	1.11%

Source: CFA Institute. Used with permission.

Spot Rates: Example

T	CF	Discount Factor	PV
1	10		
2	10		
3	10		
4	10		
5	1,010		
Total			

Maturity	Gov. Spot Rate
1 yr	0.31%
2 yrs	0.57%
3 yrs	0.80%
4 yrs	0.96%
5 yrs	1.11%

PV =

Calculating YTM:
FV = 1,000; PMT = 10; PV = –995.01; N = 5; I/Y CPT =

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Source: CFA Institute. Used with permission.

Par Rates

- Fixed coupon rate a bond would have to offer to be priced at par
  - In other words, YTM for specific maturity

- Discounting each PMT by its appropriate spot rate must give a PV of 100
  - Solve to calculate the (fixed) PMT required to give PV = 100

$$100 = \frac{PMT}{1+Z_1} + \frac{PMT}{(1+Z_2)^2} + \frac{PMT}{(1+Z_3)^3} + \ldots\ldots + \frac{100+PMT}{(1+Z_N)^N}$$

Par Rates: Example

Calculate the par rate for a $100 three-year Canadian annual-pay government bond, using the following spot rates:

Maturity	Gov. Spot Rate
1 yr	0.31%
2 yrs	0.57%
3 yrs	0.80%

A. 0.3814%.

B. 0.5798%.

C. 0.7974%.

Par Rates: Example

T	CF	Discount Factor	DF
1	X	1/1.0031	0.99691
2	X	$1/(1.0057)^2$	0.98870
3	100 + X	$1/(1.0080)^3$	0.97638
			2.96199

Maturity	Gov. Spot Rate
1 yr	0.31%
2 yrs	0.57%
3 yrs	0.80%

Total PV must = 100

$$100 = \frac{X}{1.0031} + \frac{X}{(1.0057)^2} + \frac{X}{(1.0080)^3} + \frac{100}{(1.0080)^3}$$

$$100 = (X \times 0.99691) + (X \times 0.98870) + (X \times 0.97638) + (100 \times 0.97638)$$

Par Rates: Example

$$100 = (X \times 0.99691) + (X \times 0.98870) + (X \times 0.97638) + (100 \times 0.97638)$$

$$100 = (X \times 2.96199) + (100 \times 0.97638)$$

$$100 - 97.638 = (X \times 2.96199)$$

$$\frac{2.362}{2.96199} = X = 0.79744$$

T	CF	DF	PV
1	0.79744	1/1.0031	0.79498
2	0.79744	$1/(1.0057)^2$	0.78843
3	100.79744	$1/(1.0080)^3$	98.41650
			100

Forward Rates

- Spot rate = rate starting today
- Forward rate = rate starting at some future date

Notation:

- 2y1y = 1 year loan, starting two years from now
- 1y2y = 2 year loan, starting one year from now

Forward Rates

- No-arbitrage between spot rate and forward rates:

$$(1 + Z_3)^3 = (1 + Z_1)(1 + 1y1y)(1 + 2y1y)$$

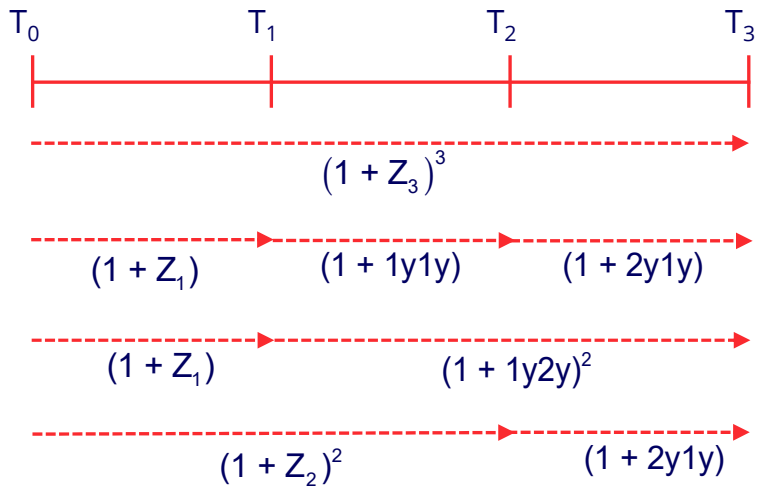
$$(1 + Z_3)^3 = (1 + Z_1)(1 + 1y2y)^2$$

$$(1 + Z_3)^3 = (1 + Z_2)^2(1 + 2y1y)$$

The cost of borrowing for three years at Z_3 should equal cost of borrowing for:

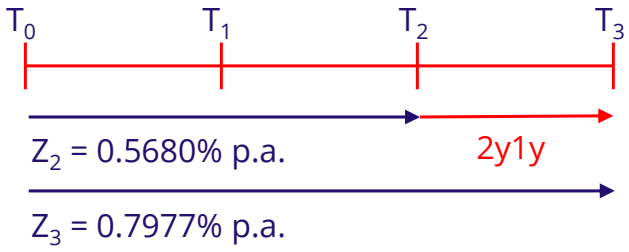
- One year at Z_1, one year at 1y1y, and one year at 2y1y
- One year at Z_1 and for two years at 1y2y
- Two years at Z_2 and for one year at 2y1y

Implied Forward Rates



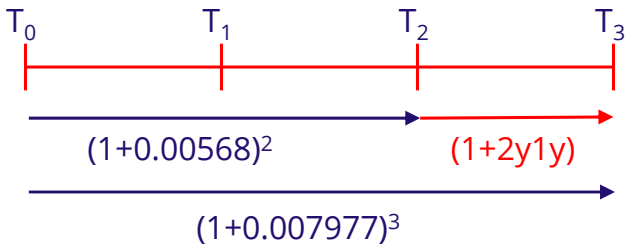
Forward Rates: Example

Calculate the 2y1y forward rate from the following spot rates (assume annual compounding):



Maturity	Gov. Spot Rate
2 yrs	0.5680%
3 yrs	0.7977%

Forward Rates: Example



=

2y1y = ——— =

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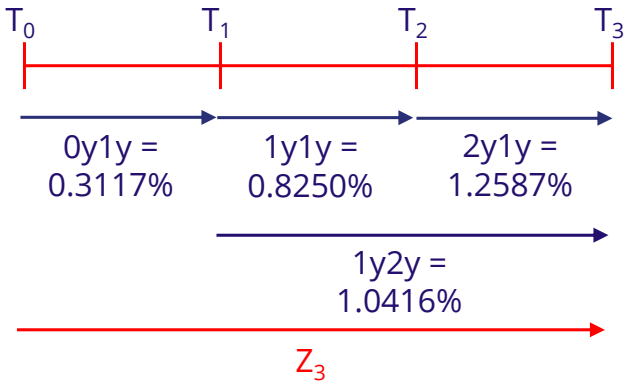
Forward Rates: Example

- Calculate the 3-year spot rate from the following forward rates:



Forward Tenor	Rates
0y1y	0.3117%
1y1y	0.8250%
2y1y	1.2587%
1y2y	1.0416%

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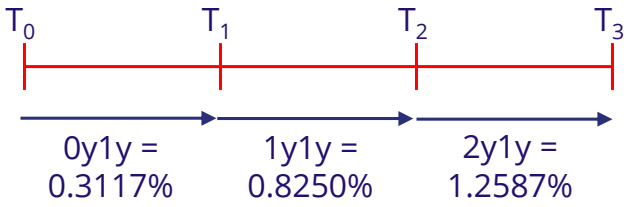
Forward Rates: Example



$(1+Z_3)^3 = (1+0y1y)(1+1y1y)(1+2y1y)$

$(1+Z_3)^3 =$ =

$(1+Z_3) =$ =

$Z_3 =$ = or

Or.....

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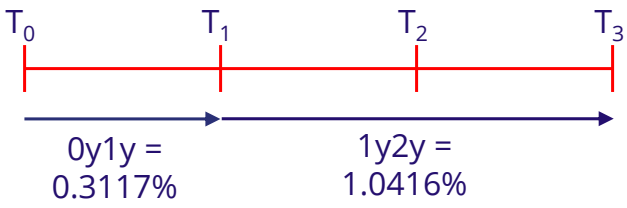
Forward Rates: Example



$(1+Z_3)^3 = (1+0y1y)(1+1y2y)^2$

$(1+Z_3)^3 =$ =

$1+Z_3 =$ =

$Z_3 =$ = or

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Yield Curves

- **Spot curve** shows spot rates versus maturity for a particular bond or issuer
- **Par curve** shows YTM of a hypothetical bond trading at par for each maturity
- **Forward yield curve** shows forward rates for bonds for annual periods in future (typically *q*-year securities for each future year)
- Spot/par/forward rates all connected, so all have similar shape—generally upward sloping but could be downward sloping

Solutions

Spot Rates: Example

T	CF	Discount Factor	PV
1	10	1.0031	9.9691
2	10	1.0057^2	9.8870
3	10	1.0080^3	9.7638
4	10	1.0096^4	9.6250
5	1,010	1.0111^5	955.7643
Total			995.01

Maturity	Gov. Spot Rate
1 yr	0.31%
2 yrs	0.57%
3 yrs	0.80%
4 yrs	0.96%
5 yrs	1.11%

PV = 995.01

Calculating YTM:
FV = 1,000; PMT = 10; PV = –995.01; N = 5; I/Y CPT = 1.10%

Par Rates: Example

Calculate the par rate for a $100 three-year Canadian annual-pay government bond, using the following spot rates:

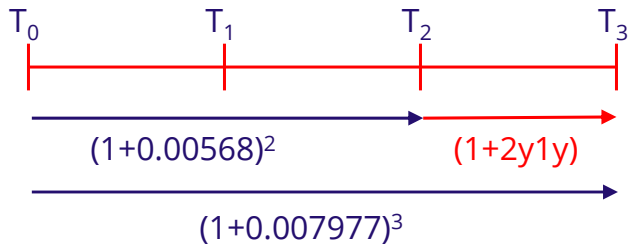
Maturity	Gov. Spot Rate
1 yr	0.31%
2 yrs	0.57%
3 yrs	0.80%

A. 0.3814%.

B. 0.5798%.

C. 0.7974%.

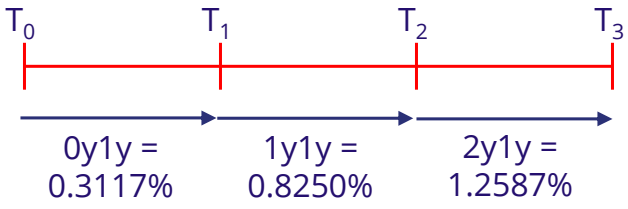
Forward Rates: Example



$1.007977^3 = 1.00568^2 \times (1+2y1y)$

$$2y1y = \frac{1.007977^3}{1.00568^2} - 1 = 1.2587\%$$

Forward Rates: Example



$(1+Z_3)^3 = (1+0y1y)(1+1y1y)(1+2y1y)$

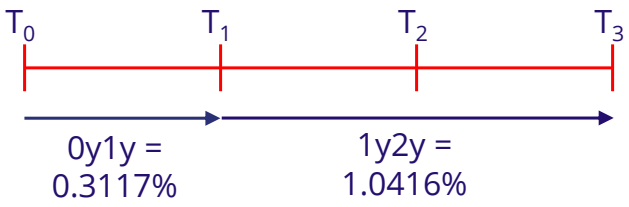
$(1+Z_3)^3 = 1.003117 \times 1.008250 \times 1.012587 = 1.024123$

$(1+Z_3) = (1.024123)^{1/3} = 1.007977$

$Z_3 = 1.007977 - 1 = 0.007977$ or 0.7977%

Or.....

Forward Rates: Example



$(1+Z_3)^3 = (1+0y1y)(1+1y2y)^2$

$(1+Z_3)^3 = 1.003117 \times 1.010416^2 = 1.024123$

$1+Z_3 = (1.024123)^{1/3} = 1.007977$

$Z_3 = 1.007977 - 1 = 0.007977$ or 0.7977%