Quantitative Methods

Probability Trees and Conditional Expectations

Intro and Exam Focus

- Discrete probability distribution: calculating mean and standard deviation
- Using conditional probabilities in probability trees
- Bayes' formula

Expected Value of Discrete Probability Distribution: CFA Institute Example

Expected value: $E(X) = \Sigma P(x_i)x_i$

Probability Distribution: BankCorp's EPS		
$P(x_i)$	EPS \$ (x_i)	$P(x_i)x_i$
0.15	2.60	0.39
0.45	2.45	1.1025
0.24	2.20	0.528
0.16	2.00	0.32
1		**E(X) =**

Variance of a Discrete Probability Distribution

Much quicker on the BAII+

Variance: $\sigma^2_X = \Sigma P(x_i)[x_i - E(X)]^2$

$P(x_i)$	EPS \$ (x_i)	$P(x_i)x_i$	$P(x_i)[x_i - E(X)]^2$
0.15	2.60	0.39	
0.45	2.45	1.1025	0.005445
0.24	2.20	0.528	0.004704
0.16	2.00	0.32	0.018496
1		E(X) = 2.34	$= \sigma^2$

Standard deviation: square root of σ^2 =

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Dependent/Independent Events

Independent events: occurrence of one event does not change the probability of other event

$$P(A|B) = P(A)$$

Example: flipping a fair coin: P(3 heads) = $0.5 \times 0.5 \times 0.5 = 0.5^3 = 0.125$

Dependent events: knowing the outcome of one event *changes* the probability of another event occurring

$$P(A|B) \neq P(A)$$

Example: picking cards from a pack without replacement

P(2 aces) = (4 / 52) × **(3 / 51)** = 0.0045

Probability Trees: Example

- P (interest rate increase) = P(I) = 70%
- P (recession | increase) = P(R|I) = 60%
- P (recession | no increase) = $P(R|I^C)$ = 20%

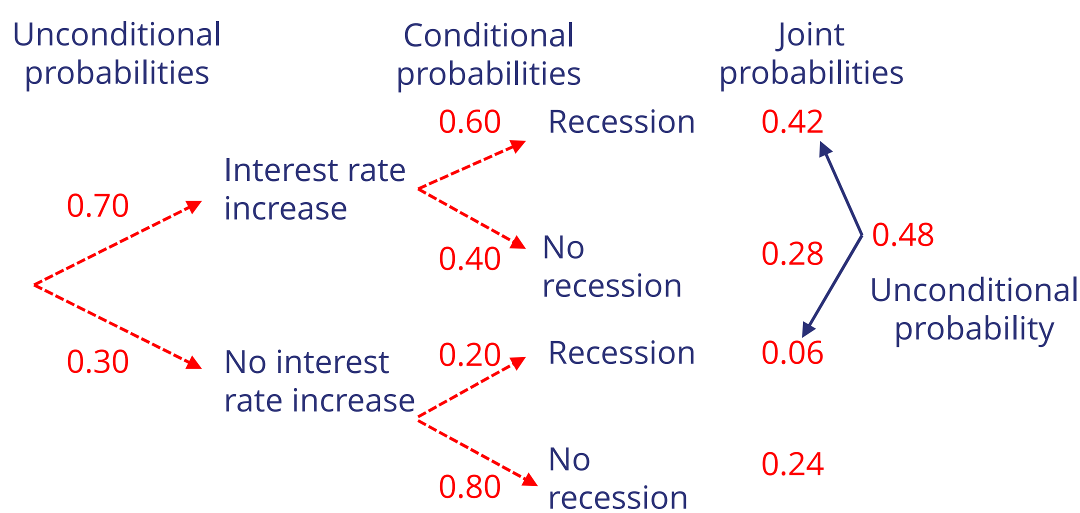
What is the (unconditional) probability of recession?

$P(R) = [P(R|I) \times P(I)] + [P(R|I^C) \times P(I^C)]$

$= P(RI) + P(RI^C)$

= + =

Probability Trees (cont.)



Probability Tree: CFA Institute Example

BankCorp's Earnings Per Share Part 2:

- Probability of declining interest environment = 0.60
- Probability of stable interest environment = 0.40
- In a declining interest environment, there is 25% probability a company's EPS will be \$2.60, and a 75% probability EPS will be \$2.45.
- In a stable interest environment, there is 60% probability a company's EPS will be \$2.20, and a 40% probability EPS will be \$2.00

Calculate:

- The expected EPS of the company, E(EPS)
- The conditional expected EPS given a declining interest environment
- The conditional variance of EPS in a declining interest environment

Probability Trees (cont.)

Unconditional probabilities		Conditional probabilities		Joint probabilities	Expected EPS
0.60	declining interest environment	0.25	EPS = \$2.60	0.15	0.39
		0.75	EPS = \$2.45	0.45	1.1025
0.40	stable interest environment	0.60	EPS = \$2.20	0.24	0.528
		0.40	EPS = \$2.00	0.16	0.32

EV =

-5

Probability Tree: Solution

Expected EPS = $2.34

= 0.15($2.60) + 0.45($2.45) + 0.24($2.20) + 0.16($2.00)

Conditional expectations of EPS:

E(EPS) | Declining interest rates = + =

E(EPS) | Stable interest rates = + =

Expected EPS = + =

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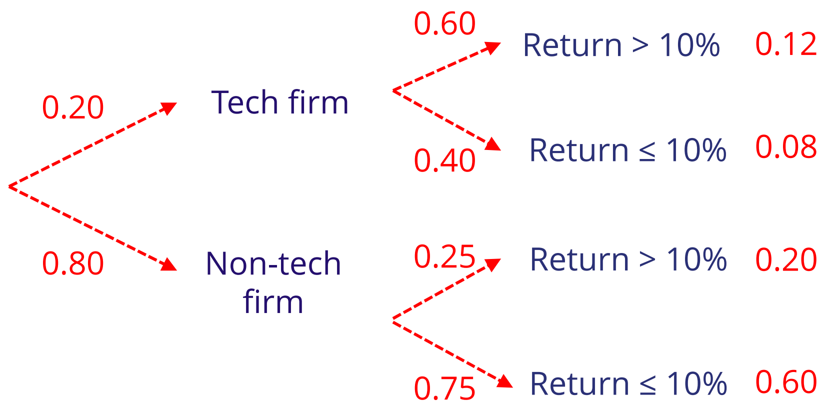
Conditional Variances: Solution

Condition	X_i	$P(X_i)$	$P(X_i)X_i$	$(X_i - \overline{X})^2$	$P(X_i)(X_i - \overline{X})^2$
Declining interest rates	$2.60	0.25	0.65		
	$2.45	0.75	1.8375	0.001406	0.001055
			$\overline{X}$ = 2.4875	σ^2 =	
Stable interest rates	$2.20	0.60	1.32	0.0064	0.00384
	$2.00	0.40	0.80	0.0144	0.00576
			$\overline{X}$ = 2.12	σ^2 =	

-5

Bayes’ Formula: CFA Institute Example

Given the following probability tree, what is the probability that a randomly selected firm that has returns > 10% was also a tech stock?



of all firms had earnings >10%

= or

Bayes’ Formula

“Prior probability”

$$P(A|B)=\frac{P(AB)}{P(B)}=\frac{P(B|A)\times P(A)}{P(B)}=\frac{0.12}{0.32}=0.375 \text{ or } 37.5\%$$

“Posterior probability”

Solutions

Expected Value of Discrete Probability Distribution: CFA Institute Example

Expected value: $E(X) = \Sigma P(x_i)x_i$

Probability Distribution: BankCorp's EPS		
$P(x_i)$	**EPS \$ (x_i)**	**$P(x_i)x_i$**
0.15	2.60	0.39
0.45	2.45	1.1025
0.24	2.20	0.528
0.16	2.00	0.32
1		**E(X) = 2.34**

-2

Variance of a Discrete Probability Distribution

Much quicker on the BAII+

Variance: $\sigma^2_X = \Sigma P(x_i)[x_i - E(X)]^2$

$P(x_i)$	EPS \$ (x_i)	$P(x_i)x_i$	$P(x_i)[x_i - E(X)]^2$
0.15	2.60	0.39	0.01014
0.45	2.45	1.1025	0.005445
0.24	2.20	0.528	0.004704
0.16	2.00	0.32	0.018496
1		E(X) = 2.34	**0.038785 = σ^2**

Standard deviation: square root of σ^2 = \$0.1969

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Probability Trees: Example

- P (interest rate increase) = P(I) = 70%
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- P (recession | no increase) = $P(R|I^C)$ = 20%

What is the (unconditional) probability of recession?

$P(R) = [P(R|I) \times P(I)] + [P(R|I^C) \times P(I^C)]$

$= P(RI) + P(RI^C)$

$= [0.60 \times 0.70] + [0.20 \times 0.30] = 48\%$

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Probability Trees (cont.)

Unconditional probabilities		Conditional probabilities		Joint probabilities	Expected EPS
0.60	declining interest environment	0.25	EPS = $2.60	0.15	0.39
		0.75	EPS = $2.45	0.45	1.1025
0.40	stable interest environment	0.60	EPS = $2.20	0.24	0.528
		0.40	EPS = $2.00	0.16	0.32

EV = 2.34

-5

Probability Tree: Solution

Expected EPS = $2.34

= 0.15($2.60) + 0.45($2.45) + 0.24($2.20) + 0.16($2.00)

Conditional expectations of EPS:

E(EPS) | Declining interest rates = 0.25($2.60) + 0.75($2.45) = $2.4875

E(EPS) | Stable interest rates = 0.60($2.20) + 0.40($2.00) = $2.12

Expected EPS = 0.60($2.4875) + 0.40($2.12) = $2.34

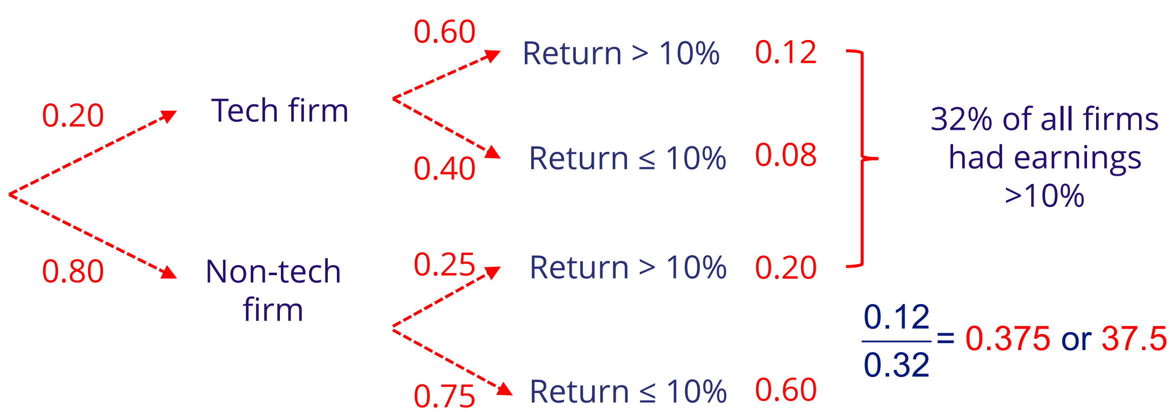
-3

Conditional Variances: Solution

Condition	X_i	$P(X_i)$	$P(X_i)X_i$	$(X_i - \bar{X})^2$	$P(X_i)(X_i - \bar{X})^2$
Declining interest rates	\$2.60	0.25	0.65	0.012656	0.003164
	\$2.45	0.75	1.8375	0.001406	0.001055
			$\bar{X}$ = 2.4875		σ^2 = 0.004219
Stable interest rates	\$2.20	0.60	1.32	0.0064	0.00384
	\$2.00	0.40	0.80	0.0144	0.00576
			$\bar{X}$ = 2.12		σ^2 = 0.0096

Bayes' Formula: CFA Institute Example

Given the following probability tree, what is the probability that a randomly selected firm that has returns > 10% was also a tech stock?



$$\frac{0.12}{0.32} = 0.375 \text{ or } 37.5\%$$