Portfolio Management

Portfolio Risk and Return: Part 1

Exam Focus

- Risk vs. return
  - Expected return and variance
  - Covariance and correlation
  - CAL and indifference curves
- Differences between equity and non-equity indices

U.S. Asset Class Risk and Return (%)

Asset Class		1930s	1940s	1950s	1960s	1970s	1980s	1990s	2000s	2010-2017	1926–2017
Large stocks	Return	–0.1	9.2	19.4	7.8	5.9	17.6	18.2	–1.0	13.9	10.2
	Risk	41.6	17.5	14.1	13.1	17.2	19.4	15.9	16.3	13.6	19.8
Small stocks	Return	1.4	20.7	16.9	15.5	11.5	15.8	15.1	6.3	14.8	12.1
	Risk	78.6	34.5	14.4	21.5	30.8	22.5	20.2	26.1	19.4	31.7
LT corp. bonds	Return	6.9	2.7	1.0	1.7	6.2	13	6.4	7.7	8.3	6.1
	Risk	5.3	1.8	4.4	4.9	8.7	14.1	6.9	11.7	8.8	8.3
LT gov't bonds	Return	4.9	3.2	–0.1	1.4	5.5	12.6	8.8	7.7	6.8	5.5
	Risk	5.3	2.8	4.6	6	8.7	16	8.9	12.4	10.9	9.9
Treasury bills	Return	0.6	0.5	1.9	3.9	6.3	8.9	4.9	2.8	0.2	3.4
	Risk	0.2	0.1	0.2	0.4	0.6	0.9	0.4	0.6	0.1	3.1
Inflation	Return	–2.0	5.4	2.2	2.5	7.4	5.1	2.9	2.5	1.7	2.9
	Risk	2.5	3.1	1.2	0.7	1.2	1.3	0.7	1.6	1.1	4.0

Source: CFA Institute. Used with permission.

Source: 2018 SBBI Yearbook

Measuring Return

United States: Real Returns and Risk Premiums (1900–2017)				
	Asset	**GM%**	**AM%**	**SD%**
Real returns	Equities	6.5	8.4	20.0
	Bonds	2.0	2.5	10.4
Premiums	Equities vs. bonds	4.4	6.5	20.7

Source: 2018 Credit Suisse Global Investment Returns Sourcebook

Measuring Return

World: Real Returns and Risk Premiums (1900–2017)				
	Asset	**GM%**	**AM%**	**SD%**
Real returns	Equities	5.2	6.6	17.4
	Bonds	2.0	2.5	11.0
Premiums	Equities vs. bonds	3.2	4.4	15.3

Source: 2018 Credit Suisse Global Investment Returns Sourcebook

Measuring Return

World (excl. U.S.): Real Returns and Risk Premiums (1900–2017)				
	Asset	**GM%**	**AM%**	**SD%**
Real returns	Equities	4.5	6.2	18.9
	Bonds	1.7	2.7	14.4
Premiums	Equities vs. bonds	2.8	3.8	14.4

Source: 2018 Credit Suisse Global Investment Returns Sourcebook

Source: CFA Institute. Used with permission.

Other Investment Characteristics

- Skewness
- Kurtosis
- Liquidity
  - Bid-ask spread
  - Price impact of trades

Risk Aversion

Risk-averse investors dislike risk (uncertainty about an outcome).

Consider two alternatives:

- Receive $2 if heads and $0 if tails; expected payoff = $1

or

- Receive $1 (with certainty)
  - **Risk averse** prefers $1 payment
  - **Risk neutral** is indifferent between the two
  - **Risk seeking** would prefer the gamble

Risk Aversion: Example

Abner is risk averse. Which of the following statements is *most likely* correct?

A. He will choose relatively safe investments.

B. He may hold some very risky investments.

C. His risk tolerance is relatively low.

-1

Investor Utility

Utility function: $U = E(r) - 0.5A\sigma^2$

Utility increases with higher expected return

A is coefficient of risk aversion

A > 0 for risk averse, = 0 for risk neutral, < 0 for risk seeking

U.S. large stocks utility = $0.102 - (0.5 \times A \times 0.198^2)$

Shows the tradeoff between risk and expected return

Greater risk aversion (A); more expected return to compensate for risk

Risk Aversion and Utility: Example

Which investment will a risk aversion coefficient of 2 choose?

U =

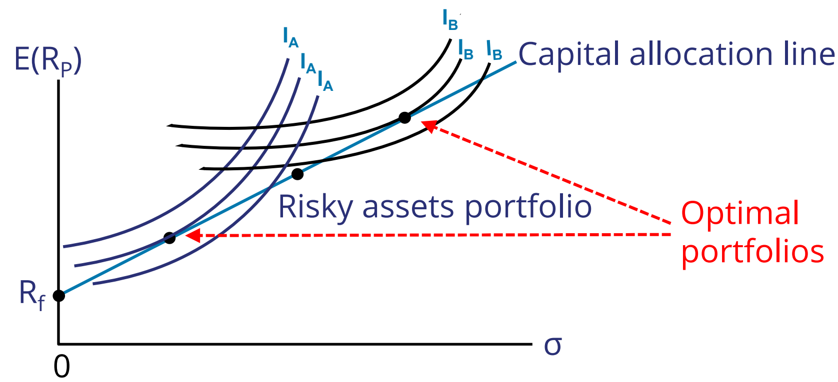
Investment	Expected Return E(r) %	Standard Deviation σ %	Utility A = 2
1	12	30	
2	15	35	0.0275
3	21	40	0.0500
4	24	45	0.0374

-6

Source: CFA Institute. Used with permission.

Investor's Optimal Portfolio

Investor A is more risk averse than Investor B (steeper indifference curves).



-7

Arithmetic Mean Return: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%

Mean Return A: (11 + 4 – 3) / 3 = 4%

Mean Return B: (6 – 2 + 14) / 3 = 6%

-2

Variance: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%

$$Var_A = \frac{(11-4)^2 + (4-4)^2 + (-3-4)^2}{3-1} =$$

$$Var_B = \frac{(6-6)^2 + (-2-6)^2 + (14-6)^2}{3-1} =$$

-4

Covariance: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%
Mean	4%	6%

Covariance of returns for Assets A and B:

$$\frac{(11-4)(6-6) + (4-4)(-2-6) + (-3-4)(14-6)}{3-1} =$$

-1

Correlation: Example

$Cov_{AB} = -0.0028;\ \sigma_A = 0.07;\ \sigma_B = 0.08$

$$\rho_{A,B} = \frac{Cov_{A,B}}{\sigma_A \sigma_B}$$

$$\rho_{A,B} = \frac{-0.0028}{(0.07)(0.08)} =$$

-1

Portfolio Returns Variance

$$\sigma_{Rp} = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B Cov_{A,B}}$$

Note: $Cov_{A,B} = \rho_{A,B} \sigma_A \sigma_B$

$$\sigma_{Rp} = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \rho_{A,B} \sigma_A \sigma_B}$$

Correlation = 1: $\sigma_{Rp} = w_A \sigma_A + w_B \sigma_B$

Correlation < 1: $\sigma_{Rp} < w_A \sigma_A + w_B \sigma_B$

Return and Risk of a Two-Asset Portfolio: Example

Asset	Weight%	E(r)%	σ%	Covariance (Decimal)
S&P 500 Index	80	9.93	16.21	0.005
MSCI Index	20	18.20	33.11	

Compute the two-asset portfolio's expected return and risk:

R_P = + =

σ_P^2 = + + =

σ_P = = or

-3

Return and Risk of a Two-Asset Portfolio: Example

Asset	Weight %	E(r) %	σ%	Correlation
FTSE 100 Index	60	5.5	13.2	−0.01
Gilts	40	0.7	4.2	

Compute the two-asset portfolio's expected return and risk:

R_P = + =

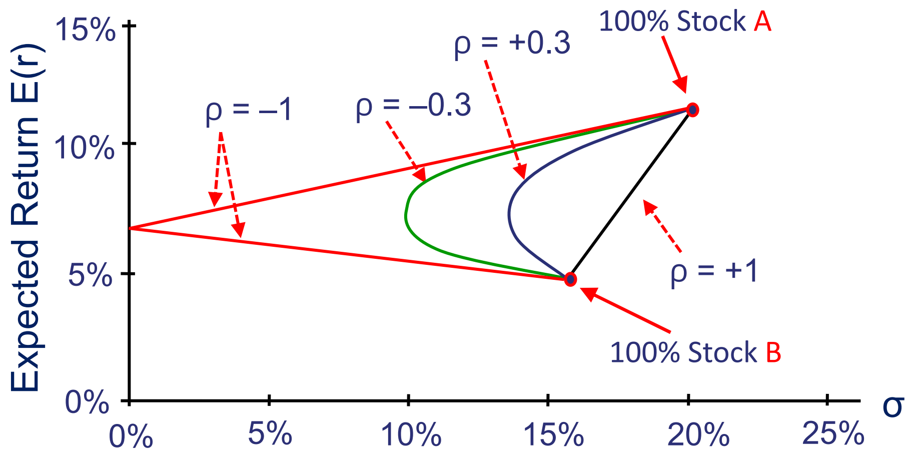
σ_P^2 = + +

=

σ_P = = or

-3

Correlation and Risk Reduction



-4

Portfolio Diversification: Example

A portfolio manager adds a new stock that has the same standard deviation of returns as the existing portfolio but has a correlation coefficient with the existing portfolio that is less than +1. Adding this stock will *most likely* have what effect on the standard deviation of the revised portfolio's returns? The standard deviation will:

A. increase.

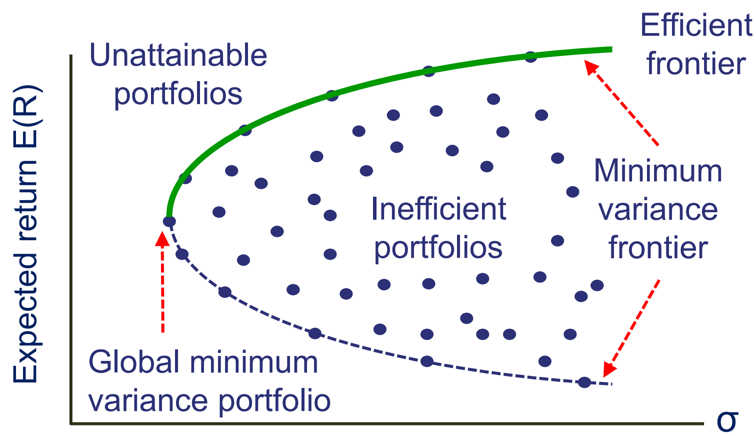
B. decrease.

C. remain the same.

-1

Efficient Frontier

This is computed from expectations of assets' (or asset classes') risk, returns, and correlations:



Efficient Frontier: Example

Which of the following portfolios falls below the Markowitz efficient frontier?

Portfolio	Expected Return	Standard Deviation
A	7%	14%
B	9%	26%
C	12%	22%

A. Portfolio A.

B. Portfolio B.

C. Portfolio C.

Solutions

Risk Aversion and Utility: Example

Which investment will a risk aversion coefficient of 2 choose?

$$U = 0.12 - \left(0.5 \times 2 \times 0.3^2\right)$$

Investment	Expected Return E(r) %	Standard Deviation σ %	Utility A = 2
1	12	30	0.0300
2	15	35	0.0275
3	21	40	0.0500
4	24	45	0.0374

-6

Source: CFA Institute. Used with permission.

Risk Aversion: Example

Abner is risk averse. Which of the following statements is *most likely* correct?

A. He will choose relatively safe investments.

B. He may hold some very risky investments.

C. His risk tolerance is relatively low.

-1

Variance: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%

$$\text{Var}_A = \frac{(11-4)^2 + (4-4)^2 + (-3-4)^2}{3-1} = 49 \text{ (or 0.0049)} \quad \text{St. Dev.} = 7\%$$

$$\text{Var}_B = \frac{(6-6)^2 + (-2-6)^2 + (14-6)^2}{3-1} = 64 \text{ (or 0.0064)} \quad \text{St. Dev.} = 8\%$$

-4

Covariance: Example

Year	Asset A	Asset B
1	+11%	+6%
2	+4%	–2%
3	–3%	+14%
Mean	4%	6%

Covariance of returns for Assets A and B:

$$\frac{(11-4)(6-6)+(4-4)(-2-6)+(-3-4)(14-6)}{3-1} = -28 \text{ or } -0.0028$$

-1

Correlation: Example

$Cov_{AB} = -0.0028$; $\sigma_A = 0.07$; $\sigma_B = 0.08$

$$\rho_{A,B} = \frac{Cov_{A,B}}{\sigma_A \sigma_B}$$

$$\rho_{A,B} = \frac{-0.0028}{(0.07)(0.08)} = -0.5$$

-1

Portfolio Returns Variance

$$\sigma_{Rp} = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B Cov_{A,B}}$$

Note: $Cov_{A,B} = \rho_{A,B}\sigma_A\sigma_B$

$$\sigma_{Rp} = \sqrt{\sigma_A^2 w_A^2 + \sigma_B^2 w_B^2 + 2 w_A w_B \rho_{A,B}\sigma_A\sigma_B}$$

Correlation = 1: $\sigma_{Rp} = w_A\sigma_A + w_B\sigma_B$

Correlation < 1: $\sigma_{Rp} < w_A\sigma_A + w_B\sigma_B$

Return and Risk of a Two-Asset Portfolio: Example

Asset	Weight%	E(r)%	σ%	Covariance (decimal)
S&P 500 index	80	9.93	16.21	0.005
MSCI index	20	18.20	33.11	

Compute the two-asset portfolio's expected return and risk:

$$R_P = (0.8 \times 0.0993) + (0.2 \times 0.1820) = 11.58\%$$

$$\sigma_P^2 = \left(0.8^2 \times 0.1621^2\right) + \left(0.2^2 \times 0.3311^2\right) + (2 \times 0.8 \times 0.2 \times 0.005) = 0.022802$$

$$\sigma_P = \sqrt{0.22802} = 0.151 \text{ or } 15.1\%$$

-3

Source: CFA Institute. Used with permission.

Return and Risk of a Two-Asset Portfolio: Example

Asset	Weight %	E(r) %	σ%	Correlation
FTSE 100 index	60	5.5	13.2	–0.01
Gilts	40	0.7	4.2	

Compute the two-asset portfolio's expected return and risk:

$R_P = (0.6 \times 0.055) + (0.4 \times 0.007) = 3.58\%$

$\sigma_P^2 = (0.6^2 \times 0.132^2) + (0.4^2 \times 0.042^2) + (2 \times 0.6 \times 0.4 \times -0.01 \times 0.132 \times 0.042)$

$= 0.006528$

$\sigma_P = \sqrt{0.006528} = 0.081$ or 8.1%

-3

Source: CFA Institute. Used with permission.

Portfolio Diversification: Example

A portfolio manager adds a new stock that has the same standard deviation of returns as the existing portfolio but has a correlation coefficient with the existing portfolio that is less than +1. Adding this stock will *most likely* have what effect on the standard deviation of the revised portfolio's returns? The standard deviation will

A. increase.

B. decrease.

C. remain the same.

-1

Efficient Frontier: Example

Which of the following portfolios falls below the Markowitz efficient frontier?

Portfolio	Expected Return	Standard Deviation
A	7%	14%
B	9%	26%
C	12%	22%

A. Portfolio A.

B. Portfolio B.

C. Portfolio C.

-1