# Derivatives

# Pricing and Valuation of Futures Contracts

# Futures Contracts

- With no costs or benefits of holding the asset, the no-arbitrage price of a futures contract is the same as that of a forward:

$$f_0(T) = S_0 \times (1 + Rf)^T$$

- With costs and/or benefits of holding the asset:

$$f_0(T) = [S_0 - [PV_0(Ben) - PV_0(Cost)]] \times (1 + Rf)^T$$



# Futures vs. Forward Prices

- After initiation, forward price does not change. Value changes as asset price changes (when there are no MTM cash flows).

- Because futures have MTM cash flows, price resets to the settlement price and value returns to zero daily as MTM gains and losses are settled.

## Futures Price and Value: Example

- Consider a long futures contract on 100 ounces of gold initiated at \$1,800 per ounce.
- The following illustrates mark-to-market cash flows and their effects on the price and value of a long futures position.



## Futures Price and Value

**Day 0:** price = settlement price 1,800 MTM value = 0

**Day 1:** Settlement price = 1,810 MTM v
\$1,000

\$1,000 added to ma

New futures price = 1,810

**Day 2:** nent price = 1,790 MTM value = –\$2,000

\$2,000 deduction fr rgin

New futures price = 1,790

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## Forward Prices vs. Futures Prices

- Because futures have daily MTM cash flows, if interest rates are **positively correlated** with underlying asset value, a long futures contract is **preferred** to a forward without MTM cash flows
- Higher rate when "lending" than when "borrowing"
- In practice, no significant difference in prices/values



## STIRs

- STIR = short-term interest rate future
- Exchanged-traded version of an FRA (standardized and liquid)
- Implied forward rate (forward MRR) computed the same way as an FRA
- Priced differently:
  - FRA price = annualized implied forward rate
  - STIR price = 100 – (100 × implied forward rate)
- Impact:
  - STIR long party gains when price increases (implied forward rate falls)
  - FRA long party gains when implied forward rate increases

## Forward Prices vs. Futures Prices

- Consider a long future, $1mm on six-month MRR priced at 97.50 = (1 – annualized MRR of 2.5%) × 100
- Each basis point change in MRR changes the futures contract basis point value by 0.0001 × 6 / 12 × 1mm = $50; payoff is linear
- If MRR = 2.44% at settlement, futures price is 100 – 2.44 = 97.56
- $97.56 – $97.50 = +6 basis points (0.025% – 0.0244%) = + 0.0006 or 6 bp
- Payment to long is (2.50% – 2.44%) × 6 / 12 × 1mm = $300 (6 bp × $50)
- The change in interest cost on a future 6-month loan



## Convexity of Forward Payoffs

- Consider an equivalent FRA:
  - At settlement with MRR = 2.56%, payment to the long is:

$$\$300 \Big/ \left(1+\frac{0.0256}{2}\right) = \$296.21$$

Convexity

  - At settlement with MRR = 2.44%, long pays:

$$\$300 \Big/ \left(1+\frac{0.0244}{2}\right) = \$296.38$$

## Convexity of Forward Payoffs

- The gain from an interest rate decrease is larger than the loss from an interest rate increase.
- As with bond convexity, **forward convexity bias** has value to the investor.
- The difference in payoffs is small for short-dated FRAs but significant for long-dated FRAs.



## Solutions

# Futures Price and Value

**Day 0:** price = settlement price 1,800 MTM value = 0

**Day 1:** Settlement price = 1,810 MTM value = $1,000

$1,000 added to margin

New futures price = 1,810

**Day 2:** settlement price = 1,790 MTM value = –$2,000

$2,000 deduction from margin

New futures price = 1,790

-2

# Derivatives

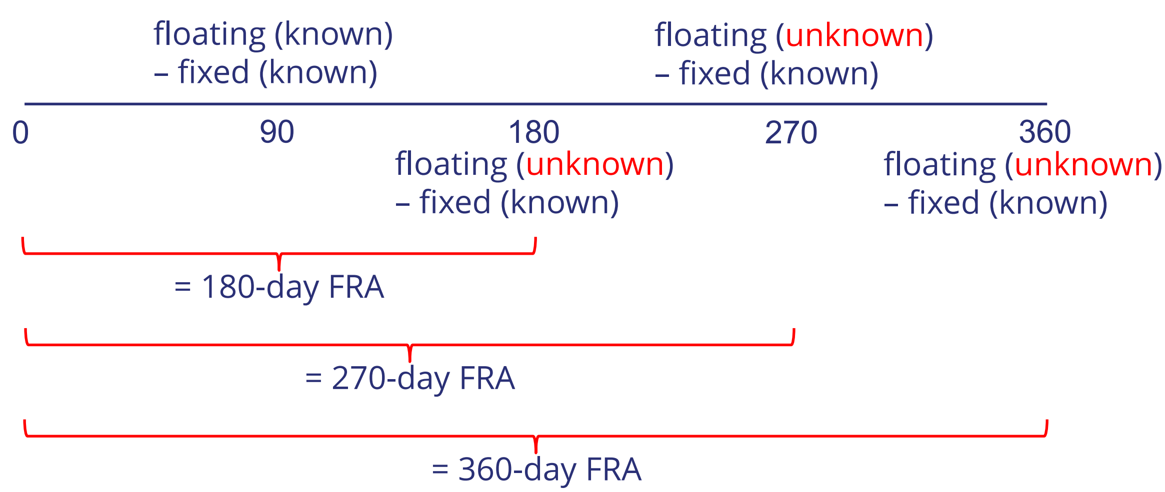
# Pricing and Valuation of Interest Rates and Other Swaps

# Interest Rate Swap

- Fixed-rate swap payer pays fixed rate and receives MRR times notional principal on each payment date
- Each payment is equivalent to an FRA at the fixed rate
- Swap is equivalent to a series of FRAs at swap (fixed) rate
- At initiation, swap has zero value, but individual FRAs at fixed rate may have positive or negative values
- <u>Fixed rate payer</u>: equivalent to issuing a fixed-coupon bond and using the proceeds to buy a FRN
- <u>Floating rate payer</u>: equivalent to issuing a FRN and using the proceeds to buy a fixed-coupon bond



# One-Year Quarterly Pay Swap



## Interest Rate Swaps

- Swap price is the fixed rate
- At initiation, swap value is zero
  - PV of fixed payments = PV of floating payments
- Pay-floating swap + fixed-rate debt = floating-rate debt
- A pay-floating swap loses value when forward rate curve (expectations) shifts upward
- Forward curve shifts up = higher fixed rate on a new swap with the same remaining settlement dates as the original swap



## Pricing a Swap

Three recently issued annual fixed-coupon government bonds had the following coupons, prices, yields to maturity, and zero (or spot) rates:

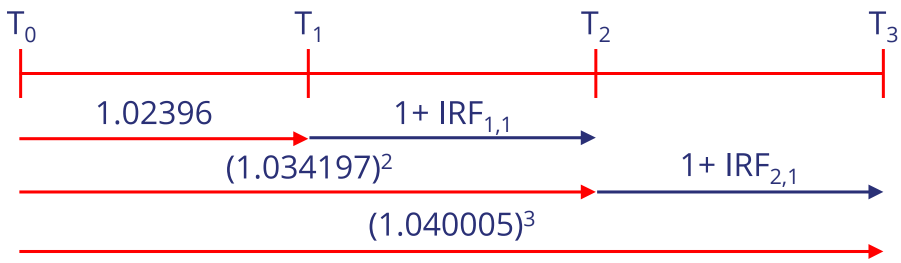
| Years to Maturity | Annual Coupon | PV (Per 100 FV) | YTM % | Spot (Z) % |
|---|---|---|---|---|
| 1 | 1.5% | 99.125 | 2.3960 | 2.3960 |
| 2 | 2.5% | 98.275 | 3.4068 | 3.4197 |
| 3 | 3.25% | 98.000 | 3.9703 | 4.0005 |

Assume a notional principal on the swap of $100.

Compute the fixed rate for a 3-year annual settlement interest rate swap.

# Pricing a Swap

*Step 1:* Compute implied forward rates for each settlement period.




$IFR_{0,1}$ = 2.396%

$IRF_{1,1} = (1 + S_2)^2 / (1 + S_1) - 1 = 1.034197^2 / 1.02396 - 1 =$ 4.4536%

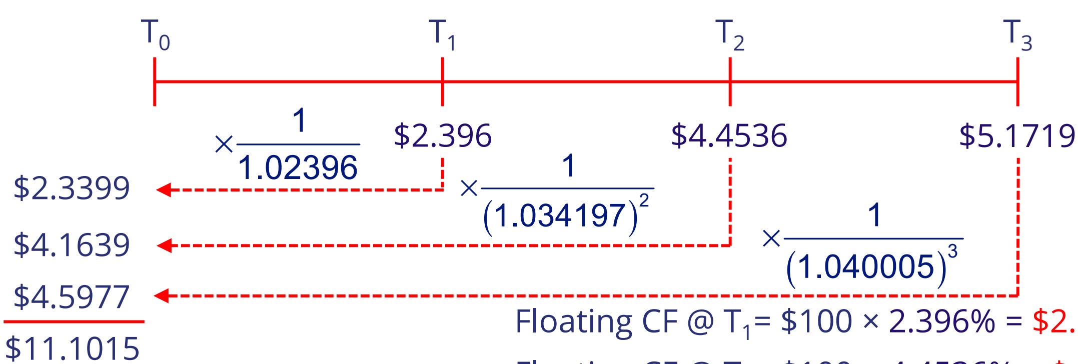
$IFR_{2,1} = (1 + S_3)^3 / (1 + S_2)^2 - 1 = 1.040005^3 / 1.034197^2 - 1 =$ 5.1719%





# Pricing a Swap

*Step 2:* Compute floating payments and discount using spot rates.



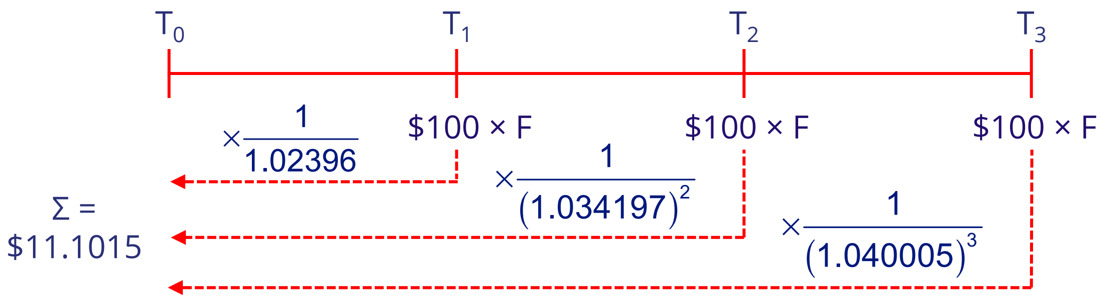

Floating CF @ $T_1$= \$100 × 2.396% = \$2.396

Floating CF @ $T_2$= \$100 × 4.4536% = \$4.4536

Floating CF @ $T_2$= \$100 × 5.1719% = \$5.1719

# Pricing a Swap

*Step 3:* Compute the zero-arbitrage fixed rate.




$2.800545 \times 100F = 11.1015$

$= 2.800545 \times F = 0.111015$

$F = \frac{0.111015}{2.800545} = 0.03964$ or $3.964\%$

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# Valuing a Swap

At t = 1, fixed-rate payer makes a payment of

(0.02396 – 0.03964) × notional principal

Floating rate (received)

Fixed rate (paid)

- The mark to market for a fixed-rate payer at *t* = 1 is the PV of remaining MRR payments – PV of remaining fixed payments.
- An increase in expected MRRs increases the value to the fixed-rate payer.

## Swaps: Question 1

Identify which of the following benefits of using swaps over forwards are most applicable to which derivative end users:

A. Swaps allow these end users to match the periodic cash flows of a specific balance sheet liability to transform their interest rate exposure profile.
B. Swaps enable these end users to actively adjust their interest rate exposure profile without buying or selling underlying securities.
C. Swaps involving a series of cash flows enable these end users to avoid the administrative burden of entering into and managing multiple forward contracts.

1. Both issuers and investors
2. Issuers
3. Investors

-3





## Swaps: Question 2

Identify which of the following statements is associated with which position in an interest rate swap contract:

A. Establishes a set of certain net future cash flows on a swap contract at inception
B. Realizes an MTM gain on a swap contract if the expected future floating-rate payments increase
C. An investor may increase portfolio duration by entering this position in a swap contract

1. Fixed-rate payer (floating-rate receiver)
2. Fixed-rate receiver (floating-rate payer)
3. Neither a fixed-rate payer nor a fixed-rate receiver

-3

# Solutions

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