Quantitative Methods

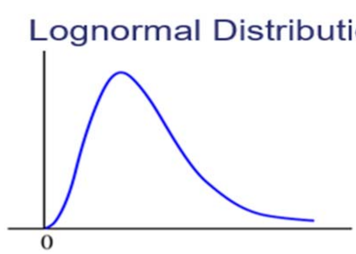
Simulation Methods

Intro and Exam Focus

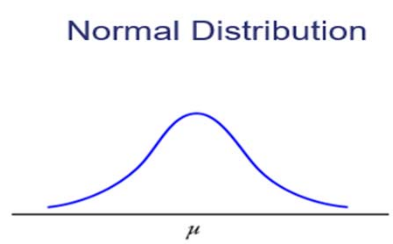
- Lognormal distribution for modeling prices
  - Link to continuously compounded returns
- Simulation methods
  - Monte Carlo
  - Bootstrapping

Lognormal Distribution

- Random variable Y is **lognormal** if ln(Y) is normal
- Lognormal is always positive and positively skewed
- Used to model price "relatives": P_T / P_0



$Y = P_T/P_0$



$\ln(Y) = \ln(P_T/P_0)$

Continuous Compounding

Recall that for a quoted continuously compounded rate R_{cc}:

$$P_T = P_0 e^{Rcc}$$

$$\rightarrow P_T \, / \, P_0 = e^{Rcc}$$

Also, recall that natural logarithm function is inverse of exponential function:

$$\rightarrow \ln(P_T \, / \, P_0) = \ln(e^{Rcc}) = R_{cc}$$

Key takeaway: **P_T / P_0 lognormal ←→ R_{cc} normal**

Scaling Volatility: Example

- Assuming identical independent returns (i.i.d), to scale from a short time period *t* to longer time period *T:*

$$\sigma_T^2 = \sigma_t^2 \left(T / t \right)$$

$$\sigma_T = \sigma_t \sqrt{T / t}$$

Daily volatility of FTSE 100 Index returns is estimated to be 0.86%. Calculate the annualized estimated volatility of FTSE 100 returns assuming 250 trading days in the year.

Solution:

$$\sigma_{250} = \sigma_1 \sqrt{250 / 1} = \qquad\qquad =$$

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Monte Carlo Simulation

- A stock is currently priced at $50.
- Each quarter, the stock price could rise by $6 or fall by $5 with equal likelihood.
- What is the value of the right to buy the stock (call) at $55 in one year's time?

Using the *Monte Carlo* approach:

1. *Specify the value to be modeled:*
   - Option value = PV of payoff at expiry (S_T – X)
   - Variables = S_T and R_f

Monte Carlo Simulation (cont.)

2. *Construct a time grid:*
   - Time horizon is next year split into 4 quarters
3. *Specify distributional assumptions for risk factors and draw random numbers:*
   - Δstock price = +6 if B = 1 and –5 if B = 0, where B is a random binary variable with 50% chance of B = 1 and 50% chance of B = 0
   - First random draw gives result 0, 1, 0, 1

Monte Carlo Simulation (cont.)

4. *Use the simulated values to generate a path for stock prices:*

Time	**0**	**1**	**2**	**3**	**4**
Stock price ($)	$50	$45	$51	$46	$52

5. *Value the call option under the stock path:*
   - *Value of call option = $0 because it would not be exercised*
   - *Discount future value of option back to today*
6. *Repeat simulation trials to generate multiple paths and multiple values.* ***The average option value is the Monte Carlo estimate of the value of the option****.*

Monte Carlo Simulation (cont.)

- Over 100 simulations, option value frequencies were as follows:

Option Value ($)	Frequency
0	70
8	20
19	10

Average option value = $3.50 (undiscounted)

Monte Carlo Simulation: Features

- Process generates a **range** of values, not one single value
  - Useful for both return and risk analysis
- Useful for complex securities with no neat "analytical" formula for pricing
- Model assumptions can be changed to assess sensitivity of output

Bootstrapping

- Large sample is treated like a population, and many samples are drawn from it with replacement ("resampling")
- Sampling distributions for key sample statistics (mean, variance, skew, kurtosis) as estimators for population parameters can be directly observed
- Can also be used in valuation
  - Process same as for Monte Carlo simulation except Steps 3 and 4: data is drawn from sample from existing data rather than generated from estimated distributions

Solutions

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Solution:

$$\sigma_{250} = \sigma_1 \sqrt{250/1} = 0.86\% \sqrt{250} = 13.6\%$$

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