Derivatives

Pricing and Valuation of Forward Contracts and for an Underlying With Varying Maturities

Forward Price and Value

With no costs of storage or benefits from holding the underlying, the no-arbitrage forward price is:

$$F_0(T) = S_0 \times (1 + R_f)^T$$

The no-arbitrage price is the forward price that ensures the **forward has a zero value at initiation**:

$$F_0(T) / (1 + R_f)^T = S_0$$

Forward Contract Value

Value of forward at settlement, t = T:

$$V_T(T) = S_T - F_0(T)$$

Value of forward at time *t* (during contract life):

$$V_t(T) = S_t - \underbrace{[F_0(T) / (1 + R_f)^{(T-t)}]}_{\text{Present value of forward price at valuation date}}$$

Note: value = value to long counterparty
Short counterparty = equal and opposite

Forward Contract Value: Example

The Viswan Family Office (VFO) currently owns 10,000 common non-dividend-paying shares of Biomian Limited, a Mumbai-based biotech company, at a spot price of INR 295 per share. VFO agrees to sell forward 1,000 shares of Biomian stock to a financial intermediary for INR 300.84 per share in six months. Calculate the contract value at maturity, $V_T(T)$, from both the buyer's and the seller's perspective if the spot price at maturity (S_T) is:

(1) S_T = INR 287

(2) S_T = INR 312

Forward Contract Value: Solution

1. S_T = INR 287 and $F_0(T)$ = INR 300.84. The contract value per share at maturity equals its settlement value from the perspective of both the financial intermediary (buyer) and VFO (seller), as follows:

   Buyer (long position): $V_T(T) = S_T - F_0(T) =$ =

   Seller (short position):

2. S_T = INR 312 and $F_0(T)$ = INR 300.84.

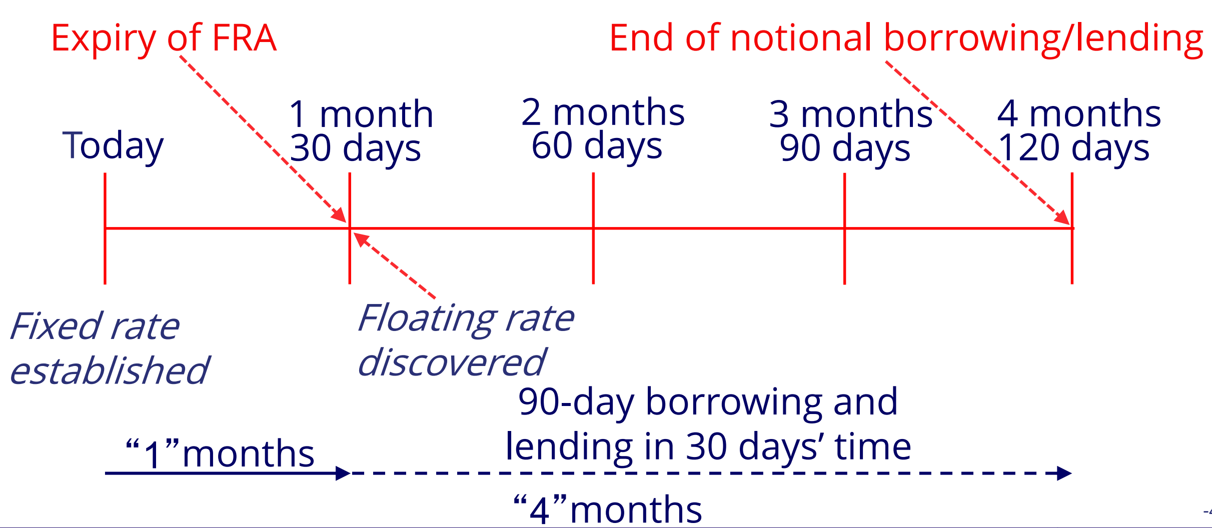
   Buyer (long position): $V_T(T) = S_T - F_0(T) =$ =

   Seller (short position):

Forward Rate Agreement (FRA)

- Exchange fixed-rate for floating-rate payment
- At settlement (long is fixed-rate payer):
  - If MRR > fixed: long receives PV of [MRR – fixed] × NA
  - If MRR < fixed: long pays PV of [fixed – MRR] × NA
- Note: contract settled at FRA expiry, and not end of notional borrowing/lending period

FRA $F_{1,3}$



Replicating an FRA $F_{1,3}$

Borrow for 90 days, starting
30 days from now

0 30 60 90 120

Replicate FRA in cash market

Borrow for 120 days

0 30 60 90 120

Lend for 30 days

FRA Payoffs

Given $F_{3,6}$ = 1.3% and notional amount = $1mm:

If 6-month MRR 3 months from now is 1.5%:

T_0 90 – days T_{90} 180 – days T_{270}

Fixed rate = 1.3% Floating rate = 1.5%

<u>Settlement T_{90}</u>:

Discounted to FRA expiry =

<u>Floating-rate payer</u> payoff

1mm

-5

FRA Payoffs

Given $F_{3,6}$ = 1.3% and notional amount = \$1mm:

If 6-month MRR 3 months from now is 1.0%:

T_0 90 – days T_{90} 180 – days T_{270}

Fixed rate = 1.3% Floating rate = 1.0%

Settlement T_{90}:

Discounted to FRA expiry =

Fixed-rate payer pays

1mm

-5

FRA Uses

A corporation that expects to **borrow** 1mm for 6 months in three months can fix borrowing cost with pay-fixed position in an FRA with $F_{3,6}$ = 1.3%:

Fixed rate 1.3%

Corporation FRA

~~Unknown floating rate~~

~~Unknown floating rate~~

Borrow for 6 months commencing in 3 months

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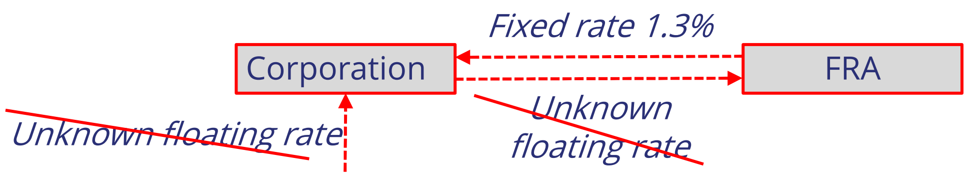
FRA Uses

A corporation that expects to **lend** 1mm for 6 months in three months can fix lending rate with pay-floating position in an FRA with $F_{3,6}$ = 1.3%:



Lend for 6 months commencing in 3 months

Bootstrapping Spot Rates

Assume we observe three most recently issued <u>annual</u> fixed-coupon government bonds, with coupons and prices as follows:

Years to Maturity	Annual Coupon	PV (Per 100 FV)	YTM %
1	1.5%	99.125	2.3960
2	2.5%	98.275	3.4068
3	3.25%	98.000	3.9703

The 1-year spot rate = 2.3960%. A bond with one year to maturity only has one set of cash flows (therefore, YTM = spot).

Bootstrapping Spot Rates

2-year annual bond:

$$98.275 = \frac{2.5}{1.02396} + \frac{102.5}{(1+Z_2)^2}$$

$$98.275 - \frac{2.5}{1.02396} = \frac{102.5}{(1+Z_2)^2}$$

$$95.833 = \frac{102.5}{(1+Z_2)^2}$$

$$(1+Z_2)^2 = \frac{102.5}{95.833} = 1.06956$$

$$Z_2 = (1.06956)^{1/2} - 1 = 0.034197 \text{ or } 3.4197\%$$

Bootstrapping Spot Rates

3-year annual bond:

$$98.000 = \frac{3.25}{1.02396} + \frac{3.25}{(1.034197)^2} + \frac{103.25}{(1+Z_3)^3}$$

$$98.000 - \frac{3.25}{1.02396} - \frac{3.25}{(1.034197)^2} = \frac{103.25}{(1+Z_3)^3}$$

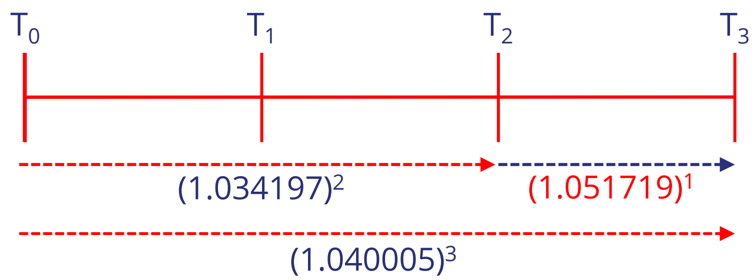
$$91.787 = \frac{103.25}{(1+Z_3)^3}$$

$$(1+Z_3)^3 = \frac{103.25}{91.787} = 1.12489$$

$$Z_3 = (1.12489)^{1/3} - 1 = 0.040005 \text{ or } 4.0005\%$$

Implied Forward Rates

Using the spot rates from the previous example, compute the $IFR_{2,1}$:



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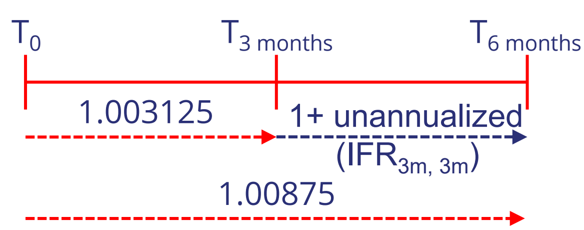
Implied Forward Rates: MRR

Assume that today we observe a current three-month MRR of 1.25% and a current six-month MRR of 1.75%. Solve for the three-month implied forward MRR in three months' time ($IFR_{3m,\,3m}$).

Unannualize MRR:

3 month = $1.25\% \times \dfrac{3}{12} = 0.3125\%$

6 month = $1.75\% \times \dfrac{6}{12} = 0.875\%$



$$\frac{1.00875}{1.003125} = 1+\text{unannualized }(IFR_{3m,3m}) = 1.0056075$$

$$\text{Annualized}(IFR_{3m,3m}) = \left[(1.0056075)-1\right] \times \frac{12}{3} = 0.0224299 \text{ or } 2.24299\%$$

Solutions

Forward Contract Value: Solution

1. S_T = INR 287 and $F_0(T)$ = INR 300.84. The contract value per share at maturity equals its settlement value from the perspective of both the financial intermediary (buyer) and VFO (seller), as follows:

   Buyer (long position): $V_T(T) = S_T - F_0(T) = 287 - 300.84 = -$ INR 13.84

   Seller (short position): + INR 13.84

2. S_T = INR 312 and $F_0(T)$ = INR 300.84.

   Buyer (long position): $V_T(T) = S_T - F_0(T) = 312 - 300.84 = +$ INR 11.16

   Seller (short position): − INR 11.16

FRA Payoffs

Given $F_{3,6}$ = 1.3% and notional amount = \$1mm:

If 6-month MRR 3 months from now is 1.5%

T_0 90 – days T_{90} 180 – days T_{270}

Fixed rate = 1.3% Floating rate = 1.5%

Settlement T_{90}:

Discounted to FRA expiry = \$1,000/(1.0075) = \$992.56

Floating-rate payer payoff

1mm × (0.015 – 0.013) × 6/12 = \$1,000

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FRA Payoffs

Given $F_{3,6}$ = 1.3% and notional amount = \$1mm:

If 6-month MRR 3 months from now is 1.0%:

T_0 90 – days T_{90} 180 – days T_{270}

Fixed rate = 1.3% Floating rate = 1.0%

Settlement T_{90}:

Discounted to FRA expiry =

\$1,500/(1.005) = \$1,492.54

Fixed-rate payer pays

1mm × (0.013 – 0.010) × 6/12 = \$1,500

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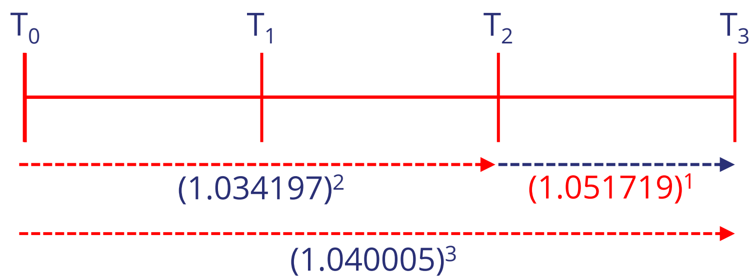
Implied Forward Rates

Using the spot rates from the previous example, compute the $IFR_{2,1}$:



$$\left(\frac{1.040005^3}{1.034197^2}\right)^{1/1} - 1 = 5.1719\%$$

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