Fixed Income

Fixed-Income Bond Valuation: Prices and Yields

Exam Focus

- Calculating value and yield of bonds
- Accrued interest, flat and full prices
- Bond relationships
- Matrix pricing

Par, Discount, and Premium Bonds: Example

Value a 10-year bond, annual coupon of 1.6%, par $1,000:

YTM of 1.6%. Price = $1,000 (coupon = YTM)

YTM of 1.2%. Price > $1,000 (coupon > YTM)

YTM of 2.0%. Price < $1,000 (coupon < YTM)

Source: CFA Institute. Used with permission.

Par, Discount, and Premium Bonds: Example

To calculate the price, discount the coupons and redemption value at YTM:

YTM of 1.2%; Price > $1,000 (coupon > YTM)

FV = $1,000

PMT = $16

N = 10

I/Y = 1.2

PV CPT =

-5

Source: CFA Institute. Used with permission.

Value of Semiannual Coupon Bonds

Previous examples had annual coupons; most bonds will have semiannual payments:

N = number of years × number of coupons per year

I/Y = YTM/number of coupons per year

PMT = cash flow per coupon

Semiannual Coupon Bonds: Example

10-year bond, coupon of 1.6% (semiannual payment), par $1,000:

YTM of 1.2%; Price > $1,000 (coupon > YTM)

FV = $1,000

PMT = $8

N = 20

I/Y = 0.6

PV CPT =

-5

Source: CFA Institute. Used with permission.

Bond Relationships

- Inverse relationship between YTM and price
- All else equal, the price of a bond with a lower coupon rate is more sensitive to a change in yield
- All else equal, the price of a bond with a longer maturity is more sensitive to a change in yield
- % decrease in price when YTM increases is smaller than % increase in value when YTM decreases (convex relationship)

Convex Relationship: Example

- Bond with face value $1,000; coupon payment 1.6% (annual pay), 10 years remaining, YTM 1.6%
- Current price: $1,000 (as coupon = YTM)
- Calculate price if YTM increases or decreases by 0.4%

Convex Relationship: Example

- Bond with face value $1,000; coupon payment 1.6% (annual pay), 10 years remaining, YTM 1.6%.

YTM increases by 0.4% to 2.0%.

FV = $1,000; PMT = $16; I/Y = 2.0; N = 10

PV CPT =

Fall in value of

-3

Convex Relationship: Example

- Bond with face value $1,000; coupon payment 1.6% (annual pay), 10 years remaining, YTM 1.6%.

YTM decreases by 0.4% to 1.2%.

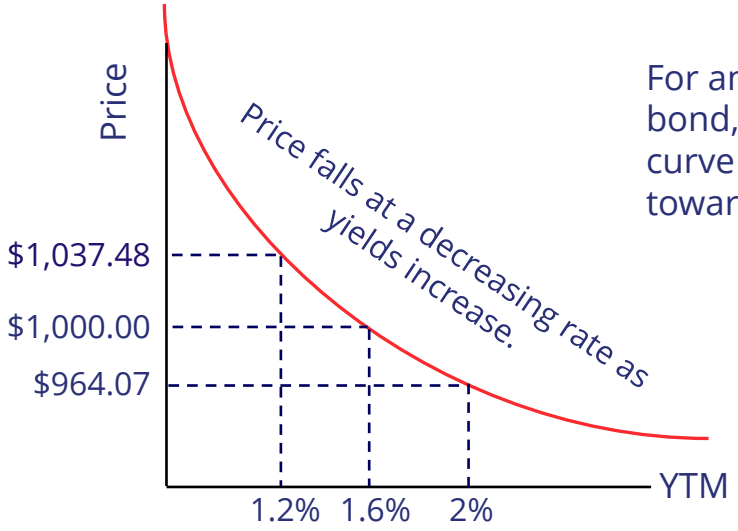
FV = $1,000; PMT = $16; I/Y = 1.2; N = 10

PV CPT =

Rise in value of

-3

Par, Discount, and Premium Bonds: Example



For an option-free bond, the price-yield curve is convex toward the origin.

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Calculating the YTM: Example

Find the YTM of 5-year bond, coupon of 3.2% (paid annually), priced at 108.15.

FV = $1,000

PMT = $32

N = 5

PV = –$1,081.50

I/Y CPT =

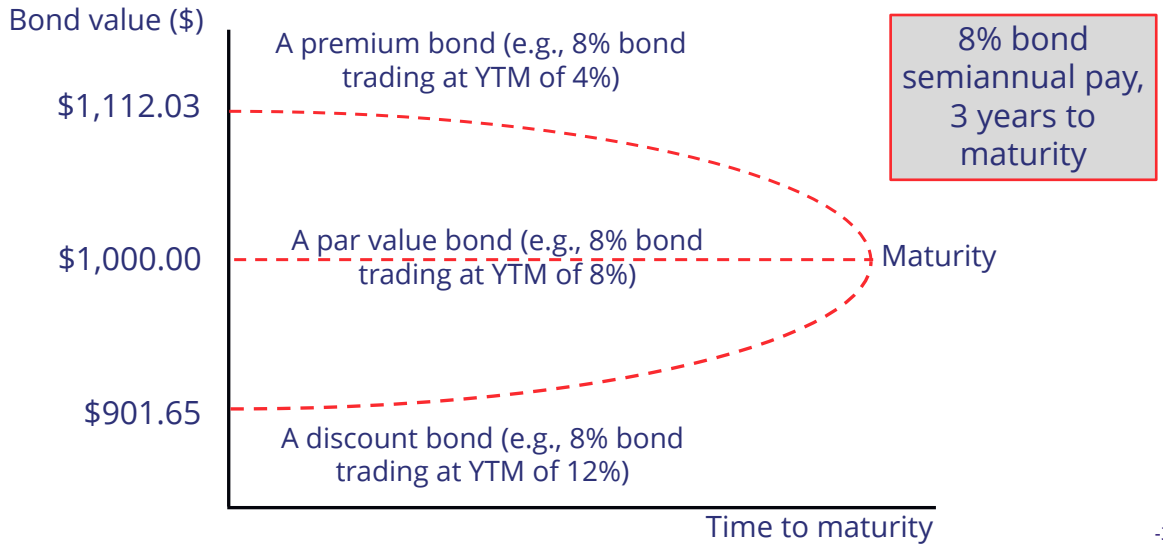
YTM assumes:

1. Held to maturity
2. All payments made
3. Coupon payments reinvested at YTM

-5

Source: CFA Institute. Used with permission.

Price on a Constant Yield Trajectory



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Accrued Interest

- Calculations so far have been on the date of issuance
- If valued between coupon dates, price needs to reflect portion of coupon owed to seller:

$$\text{Accrued interest} = \text{coupon payment} \times \frac{\text{days from last coupon to settlement}}{\text{days in coupon period}}$$

May use actual/actual to calculate days, or 30/360

Accrued Interest: Example

A government bond pays 4.625% semiannual interest on 15 May and 15 November, using actual/actual.

Calculate accrued interest for 27 June settlement date.

Accrued interest = ×———— =

-3

Source: CFA Institute. Used with permission.

Accrued Interest: Example

A corporate bond pays 4.625% semiannual interest on 15 May and 15 November, using 30/360.

Calculate accrued interest for 27 June settlement date.

Accrued interest = $\left(\frac{\$46.25}{2}\right) \times$ ——— =

-2

Source: CFA Institute. Used with permission.

Flat and Full Prices

- **Flat price** = *clean* price = quoted price = without accrued interest
- **Full price** = *dirty* price = invoice price = flat price + accrued interest
- Need to calculate full price between coupon dates and deduct accrued interest to find flat price

Flat and Full Prices: Example

$$\text{Full price} = \text{PV on last coupon date} \times \left(1 + \frac{\text{YTM}}{\text{periods per year}}\right)^{\frac{\text{days since last coupon}}{\text{days in coupon period}}}$$

4.625% EUR annual pay, actual/actual coupon bond issued by Romania, matures on 3 April 2049, YTM 3.5%.

Calculate **full** and **flat price** on 15 December 2031.

1) PV on last coupon date = 3 April 2031.

FV = €1,000; PMT = €46.25; I/Y = 3.5; N = 18; **PV CPT =**

Flat and Full Prices: Example

$$\text{Full price} = \text{PV on last coupon date} \times \left(1 + \frac{\text{YTM}}{\text{periods per year}}\right)^{\frac{\text{days since last coupon}}{\text{days in coupon period}}}$$

PV on last coupon date = €1,148.38

2) Full price on 15 December 2031=

Full price =

Flat and Full Prices: Example

3) Accrued interest = coupon payment × $\frac{\text{days from last coupon to settlement}}{\text{days in coupon period}}$

= × ——— =

4) Flat price = full price – accrued interest = €1,176.35 – =

-3

Matrix Pricing

- Used to estimate required YTM or price of bonds that are not traded
- Use YTMs of traded bonds with same credit quality, and ideally similar maturity and coupon

Matrix Pricing: Example

Value a 3-year, 4% semi-annual coupon payment bond which is not actively traded. Bonds with similar credit quality are as follows:

Bond	Tenor	Coupon	Price	YTM
A	2 years	3.00%	98.5	3.786%
B	2 years	5.00%	102.25	3.821%
C	5 years	2.00%	90.25	4.181%
D	5 years	4.00%	99.125	4.196%

Source: CFA Institute. Used with permission.

Matrix Pricing: Example

1) Calculate average YTM for each maturity of similar credit ratings.

Bond	Tenor	Coupon	Price	YTM
A	2 years	3.00%	98.5	3.786%
B	2 years	5.00%	102.25	3.821%
C	5 years	2.00%	90.25	4.181%
D	5 years	4.00%	99.125	4.196%

2 years: 3.8035%

5 years: 4.1885%

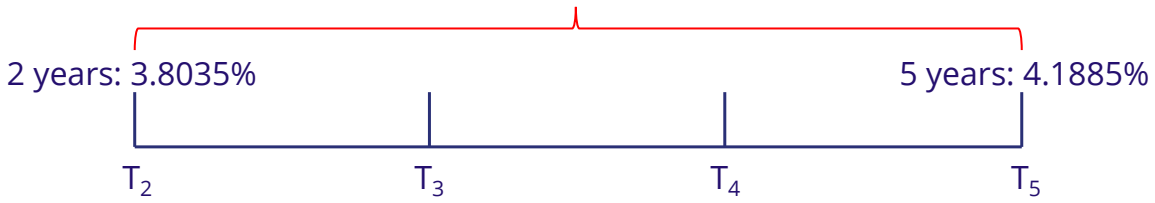
-2

Source: CFA Institute. Used with permission.

Matrix Pricing: Example

2) Estimate 3-year YTM based on linear interpolation.

Total increase in YTM =



1-year increase in YTM = =

3-year YTM = 3.8035% + =

-3

Source: CFA Institute. Used with permission.

Matrix Pricing: Example

3) Use interpolated YTM to calculate price of untraded bond.

Reminder: Value a 3-year, 4% semiannual coupon payment bond which is not actively traded.

FV = $1,000

PMT = $40 / 2 = $20

I/Y = 3.9318 / 2 = 1.9659

N = 6

PV CPT =

-5

Source: CFA Institute. Used with permission.

Matrix Pricing Using Spreads

- Previous example interpolates the full YTM
- Instead, matrix pricing could be used to interpolate the spread over a benchmark rate

Matrix Pricing Using Spreads: Example

A corporation has a 4-year, 3% annual coupon trading at $102.40.
The YTM is 2.36%. The corporation is considering issuing a 5-year bond.

Three and five year government bonds have YTMs of 0.75% and 1.45%.

The four year estimated YTM for a government bond = ———————— =

Corporate 4-year bond has spread of over benchmark.

Estimated YTM on a new 5-year bond = + =

-3

Source: CFA Institute. Used with permission.

Solutions

Par, Discount, and Premium Bonds: Example

To calculate the price, discount the coupons and redemption value at YTM:

YTM of 1.2%; Price > $1,000 (coupon > YTM)

FV = $1,000

PMT = $16

N = 10

I/Y = 1.2

PV CPT = $1,037.48

-5

Source: CFA Institute. Used with permission.

Semiannual Coupon Bonds: Example

10-year bond, coupon of 1.6% (semiannual payment), par $1,000:

YTM of 1.2%; Price > $1,000 (coupon > YTM)

FV = $1,000

PMT = $8

N = 20

I/Y = 0.6

PV CPT = $1,160.00

-5

Source: CFA Institute. Used with permission.

Convex Relationship: Example

- Bond with face value $1,000; coupon payment 1.6% (annual pay), 10 years remaining, YTM 1.6%.

YTM increases by 0.4% to 2.0%.

FV = $1,000; PMT = $16; I/Y = 2.0; N = 10

PV CPT = $964.07

Fall in value of $35.93

-3

Convex Relationship: Example

- Bond with face value $1,000; coupon payment 1.6% (annual pay), 10 years remaining, YTM 1.6%.

YTM decreases by 0.4% to 1.2%.

FV = $1,000; PMT = $16; I/Y = 1.2; N = 10

PV CPT = $1,037.48

Rise in value of $37.48

-3

Calculating the YTM: Example

Find the YTM of 5-year bond, coupon of 3.2% (paid annually), priced at 108.15.

FV = $1,000

PMT = $32

N = 5

PV = –$1,081.50

I/Y CPT = 1.50%

YTM assumes:

1. Held to maturity
2. All payments made
3. Coupon payments reinvested at YTM

-5

Source: CFA Institute. Used with permission.

Accrued Interest: Example

A government bond pays 4.625% semiannual interest on 15 May and 15 November, using actual/actual.

Calculate accrued interest for 27 June settlement date.

$$\text{Accrued interest} = \left(\frac{\$46.25}{2}\right) \times \frac{43 \text{ days}}{184 \text{ days}} = \$5.4042$$

-3

Source: CFA Institute. Used with permission.

Accrued Interest: Example

A corporate bond pays 4.625% semiannual interest on 15 May and 15 November, using 30/360.

Calculate accrued interest for 27 June settlement date.

$$\text{Accrued interest} = \left(\frac{\$46.25}{2}\right) \times \frac{42 \text{ days}}{180 \text{ days}} = \$5.3958$$

-2

Source: CFA Institute. Used with permission.

Flat and Full Prices: Example

$$\text{Full price} = \text{PV on last coupon date} \times \left(1 + \frac{\text{YTM}}{\text{periods per year}}\right)^{\frac{\text{days since last coupon}}{\text{days in coupon period}}}$$

4.625% EUR annual pay, actual/actual coupon bond issued by Romania, matures on 3 April 2049, YTM 3.5%.

Calculate **full** and **flat price** on 15 December 2031.

1) PV on last coupon date = 3 April 2031.

FV = €1,000; PMT = €46.25; I/Y = 3.5; N = 18; **PV CPT =** €1,148.38

Flat and Full Prices: Example

$$\text{Full price} = \text{PV on last coupon date} \times \left(1 + \frac{\text{YTM}}{\text{periods per year}}\right)^{\frac{\text{days since last coupon}}{\text{days in coupon period}}}$$

PV on last coupon date = €1,148.38

2) Full price on 15 December 2031= $€1{,}148.38 \times \left(1 + \frac{0.035}{1}\right)^{256/366}$

Full price = €1,176.35

Flat and Full Prices: Example

3) Accrued interest = coupon payment × $\frac{\text{days from last coupon to settlement}}{\text{days in coupon period}}$

= €46.25 × $\frac{256}{366}$ = €32.35

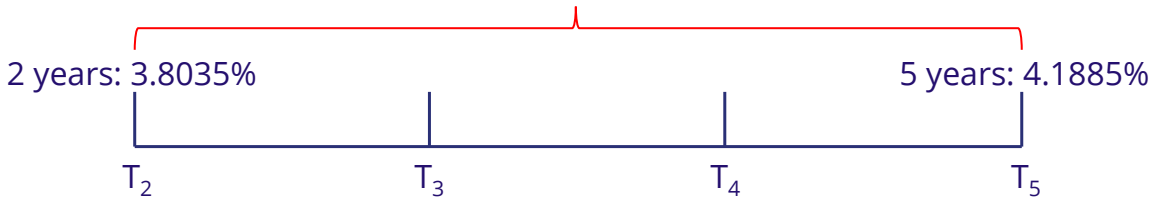
4) Flat price = full price – accrued interest = €1,176.35 – €32.35 = €1,144.00

-3

Matrix Pricing: Example

2) Estimate 3-year YTM based on linear interpolation:

Total increase in YTM = 0.385%



1-year increase in YTM = (0.385% / 3) = 0.1283%

3-year YTM = 3.8035% + 0.1283% = 3.9318%

-3

Source: CFA Institute. Used with permission.

Matrix Pricing: Example

3) Use interpolated YTM to calculate price of untraded bond.

Reminder: Value a 3-year, 4% semiannual coupon payment bond which is not actively traded.

FV = $1,000

PMT = $40 / 2 = $20

I/Y = 3.9318 / 2 = 1.9659

N = 6

PV CPT = $1,001.91

-5

Source: CFA Institute. Used with permission.

Matrix Pricing Using Spreads: Example

A corporation has a 4-year, 3% annual coupon trading at $102.40.
The YTM is 2.36%. The corporation is considering issuing a 5-year bond.

Three and five year government bonds have YTMs of 0.75% and 1.45%.

The four year estimated YTM for a government bond = $\frac{0.75 + 1.45}{2} = 1.1\%$

Corporate 4-year bond has spread of 1.26% over benchmark.

Estimated YTM on a new 5-year bond = 1.45% + 1.26% = 2.71%.

-3

Source: CFA Institute. Used with permission.