Derivatives

Pricing and Valuation: The Role of Arbitrage

Exam Focus

- Arbitrage and replication
  - Futures/forwards and the cost of carry
  - Forward rate agreements (FRAs)
  - Short-term interest rate futures
  - Swaps
  - Options: put-call parity, binomial option pricing

Arbitrage, Replication, and the Cost of Carry in Pricing Derivatives

Arbitrage

- For two assets that have the same future payoffs, regardless of future events, but different prices, buying the lower-priced asset and selling the higher-priced asset provides a riskless arbitrage profit.

- The actions of arbitrageurs will reduce the price difference to zero (prices converge to the no-arbitrage price).

Replication

- We can **replicate** a derivative by creating a portfolio that has future payoffs identical to those of a derivative.
- Consider a long forward contract to buy a share of Acme at 31.50 in one year when Acme is trading at 30.
- Compare to borrowing 30 at 5% to buy a share of Acme, and holding it for one year (no dividends).
- The up-front cost of each is zero; the payoff at *T* of both is (S_T – 31.50).

Replication

- **Forward**: at settlement, the long forward has a payoff of (S_T – 31.50)
- **Borrow and buy**: at settlement, one-year loan is repaid for 30(1.05) = 31.50, so payoff is (S_T – 31.50)
- 31.50 is the no-arbitrage 1-year forward price of an Acme share, $F_0(T)$, when R_f = 5% and there are no costs or benefits of holding the Acme share

No-Arbitrage Forward Price

$F_0(T) = S_0(1 + R_f)^T$ or $F_0(T) / S_0 = (1 + R_f)^T$

Sell asset forward for $F_0(T)$ at time = 0	Buy asset now, hold until time = T	Buy asset now, sell forward at $F_0(T)$	Earn R_f until time = T

If $F_0(T)$ is 32.00, *sell forward*, *borrow 30* at 5%, *and buy Acme share*. At settlement, pay back 31.50 and receive 32.00 for Acme share → 0.50 riskless gain. Cash and carry arbitrage.

If $F_0(T)$ is 31.00, *buy forward*, *short Acme* for 30, and *lend* at 5%. At settlement, receive 31.50 on loan, buy Acme share at 31 to cover short → 0.50 riskless gain. Reverse cash and carry arbitrage.

Arbitrage: Example

Which of the following is closest to the arbitrage profit available to an investor who is able to buy an asset for a spot price of £50 at t = 0 and simultaneously sell a six-month forward commitment on the same asset at a forward price of £52.50? The risk-free rate of interest is 4%, and the asset has no additional costs or benefits.

A. £0.99.

B. £0.48.

C. £1.51.

Arbitrage-free forward price:

FP > arbitrage-free value =

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Benefits and Costs

No-arbitrage forward price with costs and benefits

$F_0(T) = [S_0 - PV_0(\text{ben}) + PV_0(\text{cost})] \times (1 + Rf)^T$

S_t	Asset price at time *t*
Rf	Opportunity cost of funds
PV_0(cost)	PV of storage/insurance costs (monetary costs)
PV_0(ben)	PV of cash flows (monetary benefits) and **convenience yield** (nonmonetary benefits)

Rf is also a cost (opportunity cost) of holding (carrying) the asset

Benefits: Example

Which of the following statements best defines a convenience yield?

A. Convenience yield reflects the preference that market participants exhibit for buying forward contracts to avoid having to pay cash up front.

B. Convenience yield reflects the preference that market participants exhibit for buying in the spot market to avoid having to pay for storage.

C. Convenience yield reflects the preference that market participants exhibit for buying in the spot market for non-cash reasons, including low inventories in the underlying cash market.

-1

Cost of Carry: Example

Assume Hightest Capital agrees to deliver 1,000 Unilever (UL) shares at an agreed-upon price to a financial intermediary in six months under a forward contract. Assume that UL has a spot price (S_0) of €50 and assume a risk-free rate (r) of 5%. UL pays a quarterly dividend of €0.30, which occurs in exactly three months and again at time T. Solve for $F_0(T)$ in six months.

Present value of dividends	
3 months	
6 months	
PVD	=

$F_0(T)$ =

$F_0(T)$ =

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Continuous Compounding

PV and FV with continuous compounding

For one year: $FV = Se^{r}$ $PV = Se^{-r}$

For T years: $FV = Se^{rT}$ $PV = Se^{-rT}$

With r = 3% and continuous compounding

FV of S_0 in 2 years $S_2 = S_0\ e^{0.03(2)} = S_0(1.0618)$

PV of S_2 $S_0 = S_2\ e^{-0.03(2)} = S_2(0.9418)$

Continuous Compounding

With annual continuously compounded % costs (c) and benefits (i), the no-arbitrage one-year forward price is:

$$F_0(T) = S_0\ e^{(Rf + c - i)T}$$

Continuous Compounding: Example

The Viswan Family Office (VFO) would like to enter into a three-month forward commitment contract to purchase the NIFTY 50 benchmark Indian stock market index traded on the National Stock Exchange. The spot NIFTY 50 index price is INR15,200, the continuously compounded index dividend yield is 2.2%, and the Indian rupee continuous risk-free rate is 4%. Use Equation 6 (with c = 0) to solve for the forward price:

$$F_0(T) = S_0 e^{(Rf - i)T}$$

F_0

=

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Forward Exchange Rate

From Econ, the no-arbitrage forward exchange rate:

$$\text{forward}(A/B) = \left[\frac{1 + Rf_A}{1 + Rf_B}\right] \times \text{spot}(A/B)$$

Covered interest parity

Spot USD/EUR = 1.10, Rf_{USD} = 1.98%, Rf_{EUR} = 2.96% (continuously compounded annual rates)

$F_0(1)$ =

EUR (base) has the higher interest rate: $F_0(T) < S_0$ = base currency trading at a discount

-1

Spot and Forward Relationship: Question

Identify which example corresponds to each of the following relationships between the spot and forward rate:

A. $F_0(T) > S_0$

B. $F_0(T) < S_0$

C. Not enough information to determine the relationship between $F_0(T)$ and S_0

1. A fixed-coupon bond priced at par whose coupon is above the risk-free rate
2. A foreign currency forward where the domestic risk-free rate is greater than the foreign risk-free rate
3. A commodity with a convenience yield as well as storage and insurance costs

Solutions

Arbitrage: Example

Which of the following is closest to the arbitrage profit available to an investor who is able to buy an asset for a spot price of £50 at t = 0 and simultaneously sell a six-month forward commitment on the same asset at a forward price of £52.50? The risk-free rate of interest is 4%, and the asset has no additional costs or benefits.

A. £0.99.

B. £0.48.

C. £1.51.

Arbitrage-free forward price: $£50(1.04)^{0.5} = £50.99$

FP > arbitrage-free value = cash and carry arbitrage

Today	
Sell forward	
Borrow at risk-free rate	£50
Purchase underlying	£50
Net	0

6 Months	
Deliver U/L	
Receive forward price	£52.50
Pay off borrowing	£50.99
Net	£1.51

Benefits: Example

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Present Value of Dividends		
3 months	$€0.30 \times \frac{1}{(1.05)^{3/12}}$	= €0.2964
6 months	$€0.30 \times \frac{1}{(1.05)^{6/12}}$	= €0.2928
	PVD	= €0.5892

$$F_0(T) = (€50 - €0.5892)(1.05)^{6/12}$$

$$F_0(T) = €50.6310$$

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Continuous Compounding: Example

The Viswan Family Office (VFO) would like to enter into a three-month forward commitment contract to purchase the NIFTY 50 benchmark Indian stock market index traded on the National Stock Exchange. The spot NIFTY 50 index price is INR15,200, the continuously compounded index dividend yield is 2.2%, and the Indian rupee continuous risk-free rate is 4%. Use Equation 6 (with c = 0) to solve for the forward price:

$$F_0(T) = S_0 e^{(Rf - i)T}$$

$$F_0(3 \text{ month}) = 15{,}200 e^{(0.04 - 0.022)3/12}$$

$$= \text{INR } 15{,}268.55$$

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Forward Exchange Rate

From Econ, the no-arbitrage forward exchange rate:

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Covered interest parity

Spot USD/EUR = 1.10, Rf_{USD} = 1.98%, Rf_{EUR} = 2.96% (continuously compounded annual rates)

$F_0(1) = 1.10e^{(0.0198 - 0.0296)} = 1.0893$

EUR (base) has the higher interest rate: $F_0(T) < S_0$ = base currency trading at a discount

-1

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2. A foreign currency forward where the domestic risk-free rate is greater than the foreign risk-free rate
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