Derivatives

Pricing and Valuation of Options

Intrinsic Value

- **Intrinsic value** of European **call** = Max{0, (S_t – X)}
- **Intrinsic value** of European **put** = Max{0, (X – S_t)}
- Exercise value is amount **in the money**
- Intrinsic value = payoff if exercised
- ITM: call S_t > X, put S_t < X Intrinsic value positive
- ATM: call S_t = X, put X = S_t Intrinsic value zero
- OTM: call S_t < X, put S_t > X Intrinsic value zero

Put Option Exercise Value: Example

Consider the case of a one-year put option with an exercise price (X) of €1,000 and a risk-free rate of 1%. What is the exercise value of the option in six months if the spot price (S_t) equals €950?

Put-option exercise value =

Forwards vs. Contingent Claims

Forward commitments have:

- Zero value at initiation
- Symmetric payoffs
- Unlimited gains/losses (except by zero asset price)

Contingent claims have:

- Positive value at issuance (premium)
- Asymmetric payoffs
- Max loss = option price for long puts and calls
- Max gain = option price for short puts and calls

Values of Contingent Claims

Arbitrage puts limits on the minimum and maximum values of options:

Option	Minimum Value	Maximum Value
Call	$c_t \geq \text{Max}[0, S_t - X(1 + Rf)^{-(T-t)}]$	S_t
Put	$p_t \geq \text{Max}[0, X(1 + Rf)^{-(T-t)} - S_t]$	$X(1 + Rf)^{-(T-t)}$

Option price (premium) = options current payoff + time value

Put Option Time Value: Example

The prior example showed that a one-year put option with an exercise price (X) of €1,000 had an exercise value of €45.04 with six months remaining to maturity when the spot price (S_t) was €950. If we observe a current put option price (p_t) of €50, what is the time value of the put option?

Option Value: Example

A European call option with three months remaining to maturity on an underlying stock with no additional cash flows has an exercise price (X) of £50, a risk-free rate of 2%, and a current underlying price (St) of £57.50. If the current call option price is £10, which response below most closely shows the correct exercise value and the time value of the option?

A. Exercise value = GBP 7.50; time value = GBP 2.50
B. Exercise value = GBP 7.75; time value = GBP 2.25
C. Exercise value = GBP 0; time value = GBP 10

Time value =

Exercise value =

Exercise value =

Factors That Affect Option Values

Factor	Calls	Puts
Asset price	+	–
Exercise price	–	+
Volatility	+	+
Risk-free rate	+	–
Time to expiry	+	+*
Benefits of holding asset	–	+
Costs of holding asset	+	–

*Except for some European puts with long time to expiration

Factors: Question

Match the following changes in factors affecting option value (holding other factors constant) with their corresponding option value change:

A. A lower exercise price (X)

B. A lower underlying price (S_T)

C. A rise in the volatility of the underlying price

1. Increases the value of both a call option and a put option
2. Increases the value of a call option
3. Increases the value of a put option

Solutions

Put Option Exercise Value: Example

Consider the case of a one-year put option with an exercise price (X) of €1,000 and a risk-free rate of 1%. What is the exercise value of the option in six months if the spot price (S_t) equals €950?

Put-option exercise value = $1{,}000(1.01)^{-0.5} - 950$ = EUR 45.04

Put Option Time Value: Example

The prior example showed that a one-year put option with an exercise price (X) of €1,000 had an exercise value of €45.04 with six months remaining to maturity when the spot price (S_t) was €950. If we observe a current put option price (p_t) of €50, what is the time value of the put option?

Put price (premium) = (PV(X) – S_t) + time value

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Ignore negative values

€50 = €45.04 + time value

€50 – €45.04 = time value = €4.96

Option Value: Example

A European call option with three months remaining to maturity on an underlying stock with no additional cash flows has an exercise price (X) of £50, a risk-free rate of 2%, and a current underlying price (St) of £57.50. If the current call option price is £10, which response below most closely shows the correct exercise value and the time value of the option?

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C. Exercise value = GBP 0; time value = GBP 10

Exercise value = S_t – PV(X) or 0

Exercise value = £57.50 – £50(1.02) $^{-(3/12)}$ ≈ £7.75

Time value = option price – exercise value = £10 – £7.75 = £2.25

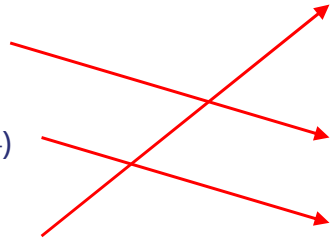
Factors: Question

Match the following changes in factors affecting option value (holding other factors constant) with their corresponding option value change:

A. A lower exercise price (X)

B. A lower underlying price (S_T)

C. A rise in the volatility of the underlying price



1. Increases the value of both a call option and a put option
2. Increases the value of a call option
3. Increases the value of a put option

Derivatives

Option Replication Using Put-Call Parity

Put-Call Parity (European Options)

$$S + P = C + X / (1 + R_f)^T$$

Protective put = Fiduciary call

Both have same payoffs at expiration, so must have same values before expiry to prevent arbitrage:

Expiration		$S_T \geq X$	$S_T \leq X$
Fiduciary call	Long call		
	Long R_f bond		
	Net		
Protective put	Long put		
	Long stock		
	Net		

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Synthetics

- Put-call parity: $c_0 + PV(X) = p_0 + S_0$
- Call = $p_0 + S_0 - PV(X)$
- Put = $c_0 + PV(X) - S_0$
- Stock = $c_0 + PV(X) - p_0$
- Risk-free bond = $p_0 + S_0 - c_0$
- For short positions, swap +/– signs

Put-Call Parity: Example

The Viswan Family Office (VFO) held non-dividend-paying Biomian shares currently priced (S_0) at INR295 per share. VFO is considering the purchase of a six-month put on Biomian shares at an exercise price, X, of INR265. If VFO's chief investment officer observes a traded six-month call option price of INR59 per share for the same INR265 exercise price, what should he expect to pay for the put per share if the relevant risk-free rate is 4%?

p_0 =

p_0 =

Put-Call-Forward Parity

We can replicate the underlying asset with a forward contract and a risk-free bond that pays the forward price at expiration:

$F_0(T) = S_0 (1 + R_f)^T$

$S_0 = F_0(T) / (1 + R_f)^T$

Same relationships hold:

Put-call parity: $S_0 + p_0 = c_0 + X / (1 + R_f)^T$

Put-call forward parity:
$F_0(T) / (1 + R_f)^T + p = c + X / (1 + R_f)^T$

Put-Call Parity: Question

Identify which of the following positions has the same no-arbitrage value as which portfolio under put-call parity:

A. Long call option (c_0)

B. Short risk-free bond ($-X(1 + r)^{-T}$)

C. Short put option ($-p_0$)

1. Long underlying, short risk-free bond, and short call option
2. Long underlying, long put option, and short risk-free bond
3. Short underlying, long call option, and short put option

Firm Value

Firm value $V_0 = E_0 + PV(D)$

Assumption = firms debt finance is a ZCB

Solvency $V_T > D$

Value of equity $E_T = V_T - D$

Debt value = D

Insolvency $V_T < D$

Value of equity = 0

Debt value = $D = V_T$

Call option at expiry: $S_T - X$ or zero

Equity value at expiry: $V_T - D$ or zero

Conclusion owning equity is equivalent to a long call on the firm's assets

Put option at expiry: $X - S_T$ or zero

Debt value at expiry: V_T or D

Debt = long bond + short put

Solutions

Put-Call Parity (European Options)

$S + P$	=	$C + X / (1 + R_f)^T$
Protective put		Fiduciary call

Both have same payoffs at expiration, so must have same values before expiry to prevent arbitrage:

Expiration		$S_T \geq X$	$S_T \leq X$
Fiduciary call	Long call	$S_T - X$	0
	Long R_f bond	X	X
	Net	S_T	X
Protective put	Long put	0	$X - S_T$
	Long stock	S_T	S_T
	Net	S_T	X

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Put-Call Parity: Example

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$p_0 = c_0 + PV(X) - S_0$

$p_0 = 59 + 265/(1.04)^{0.5} - 295 =$ INR 23.85

Put-Call Parity: Question

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C. Short put option ($-p_0$)

1. Long underlying, short risk-free bond, and short call option
2. Long underlying, long put option, and short risk-free bond
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