Fixed Income

Yield-Based Bond Convexity and Portfolio Properties

Exam Focus

Detail

- Convexity
- Adjustment to modified duration calculation

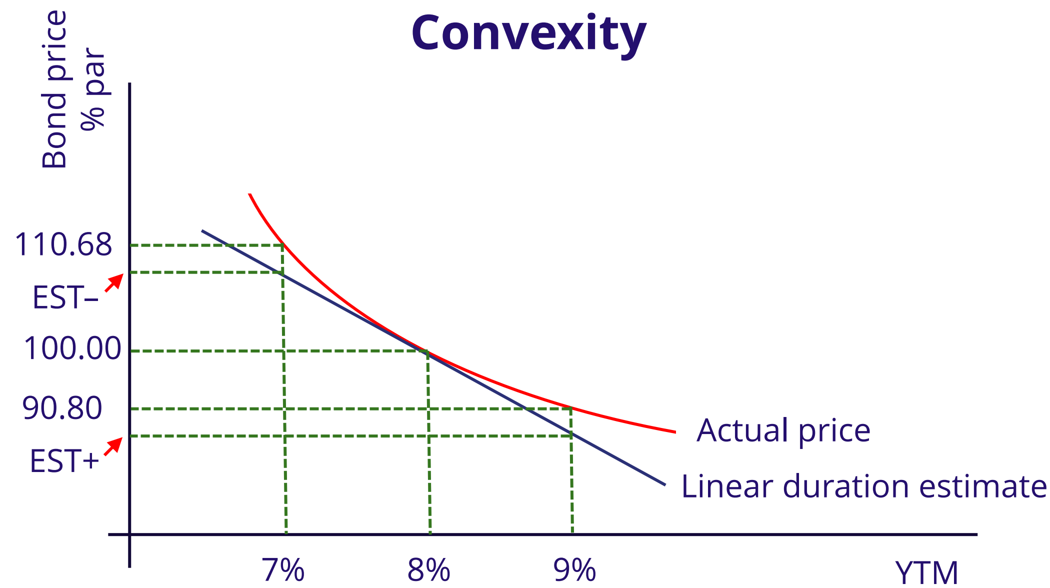
Background

- Money convexity
- Portfolio duration

Convexity

- **Reminder:** modified duration finds the % change in price for a given change in yield
- This formula is **linear**, but we know price yield relationship is **curved** (convex)—and therefore, ModDur will underestimate the price of the bond

Convexity



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Convexity Calculation

Convexity measures the curvature:

convexity of a single cash flow at period *t* = $\frac{t \times (t+1)}{(1+r)^2}$

t = time period

r = YTM/periodicity

Convexity Calculation: Example

Calculate the convexity of a 3-year, 3.2% annual-pay bond, issued at par:

Time Period	Cash Flow	PV of Cash Flow (YTM 3.2%)	PV Weighting	Numerator: t × (t+1)	Denominator: $(1+r)^2$	Weighted Convexity
T1	3.2				1.0650	
T2	3.2				1.0650	
T3	103.2				1.0650	
Totals						

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Approximate Convexity

Approximate convexity: $\dfrac{V_- + V_+ - 2V_0}{(\Delta YTM)^2 \, V_0}$

Reminder:

Approximate ModDur formula was quite similar: $\dfrac{V_- - V_+}{2V_0 \, \Delta YTM}$

Approximate Convexity: Example

Approximate convexity: $\dfrac{V_{-} - V_{+} - 2V_0}{(\Delta YTM)^2 V_0}$

Calculate the approximate convexity of a 3-year, annual-pay, 3.20% bond, issued at par value $100.

1. Calculate the change in price (PV) for a change in yield—we'll use 50 bps.

2. Plug the numbers into the formula!

Approximate Convexity: Example

1. Change in price (PV) for a 50 bps change in yield

<u>0.5% increase in yield = 3.70%</u>

N = 3; I/Y = 3.7; PMT = 3.2; FV = 100; **PV CPT =**

<u>0.5% decrease in yield = 2.70%</u>

N = 3; I/Y = 2.7; PMT = 3.2; FV = 100; **PV CPT =**

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Approximate Convexity - Example

2. Compute convexity:

$$\text{Approximate convexity} = \frac{V_{-} + V_{+} - 2V_0}{(\Delta YTM)^2\, V_0}$$

Approximate convexity =

Approximate convexity =

-2

Modified Duration and Convexity

What do we do with the convexity figure?

Modified duration = % change in price of the bond for 1% change in yield

We can improve this by including a convexity adjustment:

% change in price for given change in yield =

–annual modified duration (ΔYTM) + ½ annual convexity $(\Delta YTM)^2$

Modified Duration and Convexity: Example

- Calculate the % change in price of the following bond for a 0.05% increase/decrease in yield.
  - 30-year, 4.625% annual-pay bond, with a current YTM of 4.75%.

1. Find ModDur (we'll use approximate).
2. Find Convexity (we'll use approximate).
3. Plug into formula!

Modified Duration and Convexity: Example

- 30-year, 4.625% annual-pay bond, with a current YTM of 4.75%.

1) Approximate ModDur = $\dfrac{V_{-} - V_{+}}{2V_0 \,\Delta YTM}$

<u>Current price (**V0**):</u>

N = 30; I/Y = ; PMT = 4.625; FV = 100; PV CPT =

<u>**V–** (YTM falls by 0.05%)</u>

N = 30; I/Y = ; PMT = 4.625; FV = 100; PV CPT =

<u>**V+** (YTM rises by 0.05%)</u>

N = 30; I/Y = ; PMT = 4.625; FV = 100; PV CPT =

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Modified Duration and Convexity: Example

- 30-year, 4.625% annual-pay bond, with a current YTM of 4.75%.

1. Approximate ModDur = $\dfrac{V_{-} - V_{+}}{2V_0 \, \Delta YTM}$

$\dfrac{\quad}{\quad}$

=

(i.e., a 1% change in yield would cause a 15.9% change in price)

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Modified Duration and Convexity: Example

- 30-year, 4.625% annual-pay bond, with a current YTM of 4.75%.

2. Approximate convexity = $\dfrac{V_{-} + V_{+} - 2V_0}{(\Delta YTM)^2 \, V_0}$

$\mathbf{V_0}$ = 98.0224

V– = 98.8066

V+ = 97.2474

$\dfrac{\quad}{\quad}$ =

-2

Modified Duration and Convexity: Example

Calculate the % change in price of the bond for a 0.05% increase/decrease in yield.

3. % change in price for 0.05% increase in yield =

–annual modified duration (ΔYTM) + ½ annual convexity $(\Delta YTM)^2$

=

=

=

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Modified Duration and Convexity: Example

Calculate the % change in price of the bond for a 0.05% increase/decrease in yield.

3. % change in price for 0.05% decrease in yield =

–annual modified duration (ΔYTM) + ½ annual convexity $(\Delta YTM)^2$

=

=

=

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Modified Duration and Convexity: Conclusions

- Convexity adjustment is always **positive**
- Modified duration by itself underestimates the price of the bond
- Prices rise more quickly than they fall for the same % change in yield

Money Convexity

Reminder:

Money duration = annual ModDur × full price of bond position

So,

Money convexity = annual convexity × full price of bond position

Portfolio Duration

- Weighted average of the individual asset durations =

$W_1D_1 + W_2D_2 + W_3D_3 ... + W_ND_N$

Could also recalculate the duration by taking all bonds as “one” and compute the present values and weightings of every cash flow

Portfolio Duration: Example

An investor purchases EUR10 million par value of a 5-year, zero-coupon bond and a 10-year, fixed-rate semiannual coupon bond. Details of the bonds are shown below.

Bond	Maturity (Yrs)	Coupon (%)	Price	YTM (%)	Duration	Convexity
Zero	5	0.00	83.1877	3.750	4.81928	27.87052
Semi-ann	10	5.50	105.91556	4.750	7.71210	72.54897

Based on rising inflation and tightening monetary policy, the investor expects interest rates to rise. Given that view, which bond should the investor consider replacing the 10-year bond with?

A. A 20-year bond.

B. A 15-year floating-rate bond.

C. A 10-year bond with a lower coupon.

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Source: CFA Institute. Used with permission.

Solutions

Portfolio Duration: Example

An investor purchases EUR10 million par value of a 5-year, zero-coupon bond and a 10-year, fixed-rate semiannual coupon bond. Details of the bonds are shown below.

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Based on rising inflation and tightening monetary policy, the investor expects interest rates to rise. Given that view, which bond should the investor consider replacing the 10-year bond with?

A. A 20-year bond. *higher duration than current 10-year bond*

B. A 15-year floating-rate bond *Floating-rate bonds have low interest rate risk because coupon payments adjust to changing interest rates.*

C. A 10-year bond with a lower coupon. *higher duration than current 10-year bond*

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