Fixed Income

Curve-Based and Empirical Fixed-Income Risk Measures

Exam Focus

Detail

- Effective duration
- Effective convexity

Background

- Key rate duration
- Empirical duration

Effective Duration

Reminder:

Previously covered:

- Macaulay duration (measure of **time**)
- Modified duration (**% change** in price)
- Money duration (**$ change** in price)

All of these assume the bond is option free ("straight")
(i.e., future cash flows and their timings are known)

Effective Duration

Effective duration should be used for bonds with embedded options:

- Callable bonds (at choice of issuer/borrower)
- Putable bonds (at choice of investor)
- Mortgage-backed securities (repayment at choice of borrower)

Reminder:

There are various yields for bonds with embedded options:
yield to maturity; yield to first call; yield to second call; yield to worst

Effective Duration

Reminder:

$$\text{Approximate modified duration} = \frac{V_{-} - V_{+}}{2V_0 \, \Delta\text{YTM}}$$

$$\text{Effective duration} = \frac{V_{-} - V_{+}}{2V_0 \, \Delta\textbf{curve}}$$

Effective Convexity

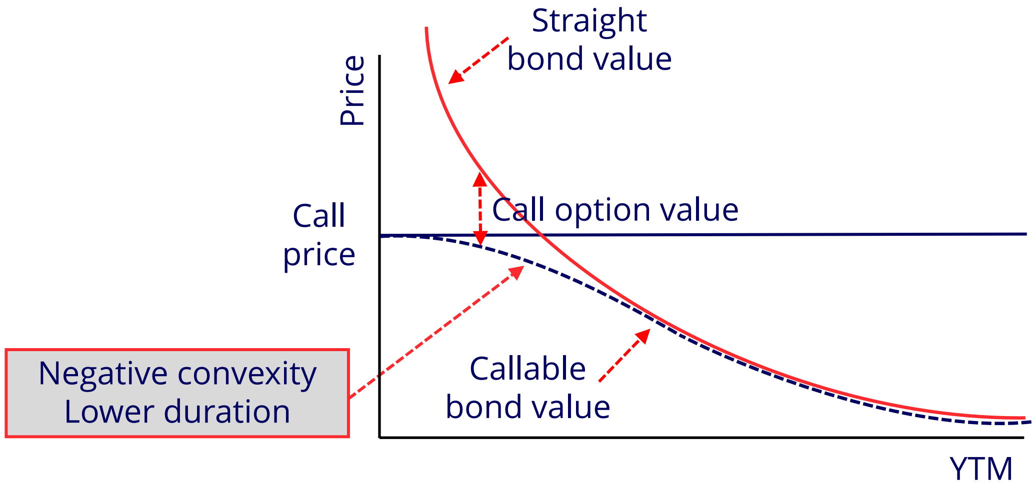
Reminder:

$$\text{Approximate convexity} = \frac{V_{-} - V_{+} - 2V_0}{(\Delta YTM)^2 \, V_0}$$

$$\text{Effective convexity} = \frac{V_{-} - V_{+} - 2V_0}{(\Delta \textbf{curve})^2 \, V_0}$$

Effective convexity: can be **negative** at low yields for call options

Callable Bonds



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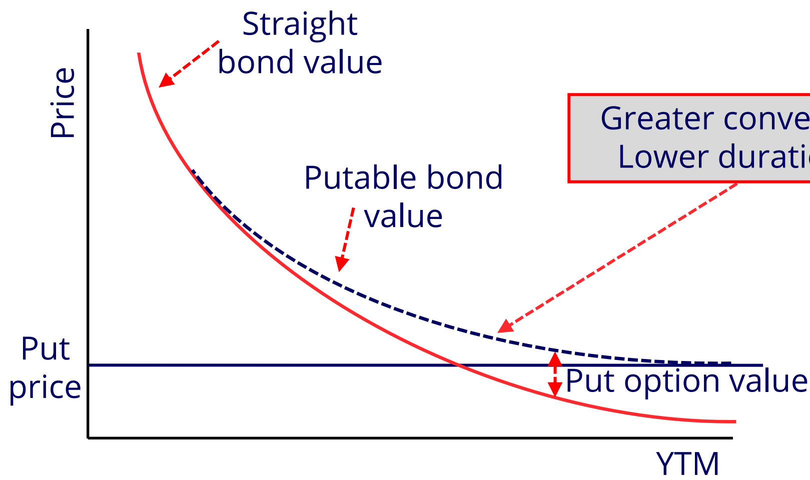
Putable Bonds



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Effective Duration: Example

Calculate the **effective duration** for a callable bond, which has a current price of $101.06. When the curvature changes by 25 bps, the prices are:

V+ = 99.050

V– = 102.891

$$\text{Effective duration} = \frac{V_{-} - V_{+}}{2V_0\,\Delta\textbf{curve}}$$

Effective duration = ________________ =

-1

Source: CFA Institute. Used with permission.

Effective Convexity: Example

Calculate the **effective convexity** for a callable bond, which has a current price of \$101.06. When the curvature changes by 25 bps, the prices are:

$V_+ = 99.050$

$V_- = 102.891$

$$\text{Effective convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta \textbf{curve})^2\, V_0}$$

Effective convexity = ________________ =

-1

Effective Duration & Convexity: Example

BRWA's five year 3.2% semi-annual bond priced at par has a duration figure of 4.816 and a convexity of 40. Compute the change in price for a 100 basis point increase and decrease in the benchmark curve.

$\%\Delta$ in price = – effective duration(Δcurve) + ½ annual convexity (Δcurve)^2

Increase in curve:

$\%\Delta$ in price = + =

Decrease in curve:

$\%\Delta$ in price = + =

-6

Key Rate Duration

- Effective duration assumes parallel shifts in the benchmark yield curve, regardless of maturity.
- Nonparallel shifts can be measured using **key rate duration**.
- This is the sensitivity of the value of a bond to changes in the benchmark yield for a **specific** maturity.
- Each cash flow has its own unique key rate duration measure.

Empirical Duration

- All duration measures seen so far are **analytical durations**.
- **Empirical durations** use observed historical relationships instead.

Duration: Example

When comparing analytical duration and empirical duration, which of the following statements is correct?

A. Empirical duration and convexity are estimated duration and convexity statistics using mathematical formulas.

B. Analytical duration and convexity are estimated using historical data in non-statistical models that incorporate various factors affecting bond prices.

C. Neither A nor B.

-1

Source: CFA Institute. Used with permission.

Solutions

Effective Duration: Example

Calculate the **effective duration** for a callable bond, which has a current price of $101.06. When the curvature changes by 25 bps, the prices are:

$V_+ = 99.050$

$V_- = 102.891$

$$\text{Effective duration} = \frac{V_- - V_+}{2V_0 \, \Delta\textbf{curve}}$$

$$\text{Effective duration} = \frac{102.891 - 99.050}{2 \times 101.06 \times 0.0025} = 7.601$$

-1

Source: CFA Institute. Used with permission.

Effective Convexity: Example

Calculate the **effective convexity** for a callable bond, which has a current price of $101.06. When the curvature changes by 25 bps, the prices are:

$V_+ = 99.050$

$V_- = 102.891$

$$\text{Effective convexity} = \frac{V_- + V_+ - 2V_0}{(\Delta\textbf{curve})^2 \, V_0}$$

$$\text{Effective convexity} = \frac{102.891 + 99.050 - (2 \times 101.06)}{0.0025^2 \times 101.06} = -283$$

-1

Source: CFA Institute. Used with permission.

Effective Duration & Convexity: Example

BRWA's five year 3.2% semi-annual bond priced at par has a duration figure of 4.816 and a convexity of 40. Compute the change in yield for a 100 basis point increase and decrease in the benchmark curve.

%Δ in price = – effective duration(ΔYTM) + ½ annual convexity $(\Delta YTM)^2$

Increase in yield:

%Δ in price = (–4.816 × 0.01) + (0.5 × 40 × 0.01^2) = –4.616%

Decrease in yield:

%Δ in price = (–4.816 × -0.01) + (0.5 × 40 × $–0.01^2$) = 5.016%

Duration: Example

When comparing analytical duration and empirical duration, which of the following statements is correct?

A. Empirical duration and convexity are estimated duration and convexity statistics using mathematical formulas.

B. Analytical duration and convexity are estimated using historical data in non-statistical models that incorporate various factors affecting bond prices.

C. Neither A nor B.