Fixed Income

Yield-Based Bond Duration Measures and Properties

Exam Focus

- Modified duration
- Money duration
- Factors affecting interest rate risk

Different Types of Duration

- **Macaulay duration**: measure of **time periods** (e.g., 5.85 years)
- **Modified duration**: **% change in bond's price** for a 1% change in YTM (e.g., if the YTM increases by 1% [from 6% to 7%], bond price will decrease by 4.5%)
- **Money duration**: measure of **$ change in bond's price**

Modified Duration

$$\text{ModDur} = \frac{\text{MacDur}}{(1 + \text{YTM})}$$

For a semiannual-pay bond with YTM quoted on semiannual bond basis:

$$\text{ModDur}_{\text{SEMI}} = \frac{\text{MacDur}_{\text{SEMI}}}{1 + (\text{YTM}/2)}$$

Modified Duration: Example

Calculate the modified duration of a 5-year, semiannual-pay, 3.20% bond, issued at par value $100. Macaulay duration is 9.3203.

$$\text{ModDur}_{\text{SEMI}} = \frac{\text{MacDur}_{\text{SEMI}}}{(1 + \text{YTM} / 2)}$$

$$\text{ModDur}_{\text{SEMI}} = \text{\_\_\_\_\_}$$

$$\text{ModDur}_{\text{SEMI}} =$$

$$\text{ModDur} =$$

Approximate Modified Duration

$$\text{Approximate ModDur} = \frac{V_- - V_+}{2V_0 \Delta YTM}$$

- What is this doing?

- Looks at the **average movement** in price for a given change in yield (both increases and decreases) and calculates this movement as a %

Approximate Modified: Example

Calculate the approximate modified duration of a 5-year, semiannual-pay, 3.20% bond, issued at par value $100.

1. Calculate the change in price (PV) for a change in yield—we'll use 5 bps.
2. Plug the numbers into the formula!

$$\frac{V_- - V_+}{2V_0 \Delta YTM}$$

Approximate Modified Duration: Example

1. Change in price (PV) for a 5 bps change in yield:

0.05% increase in yield = 3.25%.

N = 10; I/Y = 3.25/2; PMT = 1.6; FV = 100; **PV CPT =**

0.05% decrease in yield = 3.15%.

N = 10; I/Y = 3.15/2; PMT = 1.6; FV = 100; **PV CPT =**

Approximate Modified Duration: Example

2. Compute duration:

$$\text{Approximate ModDur} = \frac{V_- - V_+}{2V_0\,\Delta YTM}$$

Approximate ModDur = _____

Approximate ModDur = 4.59

Modified Duration: Example

Suppose a 4%, semi-annual coupon bond has an annualized Macaulay duration of 3.589. The anticipated percentage change in the bond's full price if the bond's yield rises from 5% to 6% is *closest* to:

A. –3.485%.

B. –3.502%.

C. –3.589%.

-1

Source: CFA Institute. Used with permission.

Money Duration

- Money duration = annual ModDur × full price of bond position
- Money duration will give the $ change in price for a **100% change in yield** (i.e., will be a large number most likely!)
- Then, multiply this by actual change in YTM desired, to give a $ change in bond value

Money Duration: Example

Calculate the money duration of a 5-year, semiannual-pay, 3.20% bond, issued at par value $100, with an annual modified duration of 4.58676. The investor holds $1m par value in the bond.

Money duration = annual ModDur × full price of bond position.

Money duration =

Then, find the expected change in price for example a 2% rise in yield:

(*fall* in price).

-2

Price Value of a Basis Point (PVBP)

- 100 basis points (bps) = 1%
- 1 basis point = 0.01%
- PVBP is the movement in price for a 0.01% change in yield
- In other words, PVBP = money duration × 0.01%

Price Value of a Basis Point (PVBP): Example

Calculate the PVBP for the previous example.

Money duration = $4,586,760

PVBP =

For the $1m par value of the bond, each 0.01% increase/decrease in yield with decrease/increase the bond value by

-1

Factors Affecting Interest Rate Risk

Reminder: interest rate risk is the risk a bond faces from a combination of reinvestment risk and price risk, due to changing interest rates.

Key factors affecting interest rate risk:

- Time to maturity
- Coupon rate
- YTM

Factors Affecting Interest Rate Risk

- **Time to maturity**
  - Longer the time to maturity, the higher the interest rate risk (usually)
- **Coupon rate**
  - Lower the coupon, the higher the interest rate risk
- **YTM**
  - Lower YTM will increase interest rate risk
    - In other words, a 1% absolute change in interest rate when YTM is 2% will have a larger impact than a 1% absolute change in interest rate when YTM is 7%, all else equal

Interest Rate Risk: Example

A bond analyst is comparing Bond D, a perpetual bond with a coupon of 5%, with Bond E, a zero-coupon bond maturing in five years. If both bonds are priced to yield 6%, Bond D has:

A. lower interest rate risk.

B. the same interest rate risk.

C. higher interest rate risk.

-1

Source: CFA Institute. Used with permission.

Macaulay Duration: Example

Between coupon payments, if the yield-to-maturity does not change, the Macaulay duration of a bond:

A. decreases throughout the coupon period.

B. is constant throughout the coupon period.

C. increases throughout the coupon period.

-1

Source: CFA Institute. Used with permission.

Solutions

Modified Duration: Example

Calculate the modified duration of a 5-year, semiannual-pay, 3.20% bond, issued at par value $100. Macaulay duration is 9.3203.

$$\text{ModDur}_{\text{SEMI}} = \frac{\text{MacDur}_{\text{SEMI}}}{(1 + \text{YTM} / 2)} \qquad \text{ModDur}_{\text{SEMI}} = \frac{9.3203}{1 + (3.2\% / 2)}$$

$\text{ModDur}_{\text{SEMI}} = 9.17351$

ModDur = 9.17351 / 2 = 4.58676

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Approximate Modified Duration: Example

1. 0.05% increase in yield = 3.25%

   N = 10; I/Y = 3.25/2; PMT = 1.6; FV = 100; **PV CPT =** 99.771

2. 0.05% decrease in yield = 3.15%

   N = 10; I/Y = 3.15/2; PMT = 1.6; FV = 100; **PV CPT =** 100.230

-4

Approximate Modified Duration: Example

$$\text{Approximate ModDur} = \frac{V_- - V_+}{2V_0 \Delta YTM}$$

$$\text{Approximate ModDur} = \frac{100.230 - 99.771}{2 \times 100 \times 0.0005}$$

Approximate ModDur = 4.59

-2

Modified Duration: Example

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A. –3.485%.

B. –3.502%.

C. –3.589%.

-1

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Money Duration: Example

Calculate the money duration of a 5-year, semiannual-pay, 3.20% bond, issued at par value $100, with an annual modified duration of 4.58676. The investor holds $1m par value in the bond.

Money duration = annual ModDur × full price of bond position.

Money duration = 4.58676 × $1m = $4,586,760.

Then, find the expected change in price for example a 2% rise in yield:

$4,586,760 × 2% = $91,735.20 (*fall* in price).

-2

Price Value of a Basis Point (PVBP): Example

Calculate the PVBP for the previous example.

Money duration = $4,586,760

PVBP = $4,586,760 × 0.01% = $458.68

For the $1m par value of the bond, each 0.01% increase/decrease in yield with decrease/increase the bond value by $458.68.

-1

Interest Rate Risk: Example

A bond analyst is comparing Bond D, a perpetual bond with a coupon of 5%, with Bond E, a zero-coupon bond maturing in five years. If both bonds are priced to yield 6%, Bond D has:

A. lower interest rate risk.

B. the same interest rate risk.

C. higher interest rate risk.

-1

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Between coupon payments, if the yield-to-maturity does not change, the Macaulay duration of a bond:

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