2 Linear Algebra Problem Sheet

1. Find the transpose A^{T} of the matrix:

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 2 & 3 & 4 & 5 \\ 4 & 4 & 4 & 4 \end{pmatrix}$$

2. Let $A = \begin{pmatrix} 1 & 2 \\ 4 & -3 \end{pmatrix}$; Find $2A$; A^2; A^3

3. Calculate $(2A - BC)^{\mathrm{T}}$ for

$$A = \begin{pmatrix} 2 & 0 \\ 1 & 1 \\ 3 & 1 \end{pmatrix};\ B = \begin{pmatrix} 1 & 1 & -1 & 1 \\ 2 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{pmatrix} C = \begin{pmatrix} 1 & 1 \\ -1 & 2 \\ 0 & 1 \\ 1 & 1 \end{pmatrix}$$

4. Calculate all possible products between the following matrices

$$(1, -1, 2, 0);\ \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix};\ \begin{pmatrix} 1 & 2 \\ 2 & 1 \\ -1 & 1 \\ 0 & 3 \end{pmatrix};\ \begin{pmatrix} 1 & -1 & 0 & 1 \\ 2 & 1 & -1 & 2 \end{pmatrix}$$

5. Calculate all the minors and cofactors of $A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 0 & 1 \\ 3 & 2 & 1 \end{pmatrix}$

6. Evaluate the determinant $|A|$ of

$$A = \begin{pmatrix} t-2 & 4 & 3 \\ 1 & t+1 & -2 \\ 0 & 0 & t-4 \end{pmatrix}.$$

Determine those values of t for which $|A| = 0$.

7. Reduce to echelon form where

$$A = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}; \qquad A = \begin{pmatrix} 0 & 1 & 3 & -2 \\ 0 & 4 & -1 & 3 \\ 0 & 0 & 2 & 1 \\ 0 & 5 & -3 & 4 \end{pmatrix}$$

8. Solve the linear system

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 8 \\ 9 \end{pmatrix}$$

9. What is the condition on a, b, c so that the following linear system has a solution

$$\begin{aligned} x + 2y - 3z &= a \\ 2x + 6y - 11z &= b \\ x - 2y + 7z &= c \end{aligned}$$

10. A matrix A is orthogonal if $A^{-1} = A^T$. Show that A is orthogonal where

$$A = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

11. Show that

$$\begin{vmatrix} y-z & z-x & x-y \\ z-x & x-y & y-z \\ x-y & y-z & z-x \end{vmatrix} = 0;$$

$$\begin{vmatrix} yz & x & x^2 \\ zx & y & y^2 \\ xy & z & z^2 \end{vmatrix} = \begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z^2 & z^3 \end{vmatrix}$$

12. Solve the following linear system for all values of λ

$$\begin{aligned} 4x_1 - 2x_2 - 7x_3 &= \lambda^2 - 1 \\ x_1 + x_2 - 4x_3 &= \lambda^2 + 2 \\ -5x_1 + 3x_2 + 8x_3 &= \lambda \end{aligned}$$