

1. $f(x) = xe^{-x}$ then using the product rule

$$f'(x) = \left(\frac{d}{dx}x\right)e^{-x} + x\left(\frac{d}{dx}e^{-x}\right)$$

$$f'(x) = e^{-x} - xe^{-x}$$

differentiating once more we have

$$f''(x) = \frac{d}{dx}(e^{-x} - xe^{-x})$$

$$= \frac{d}{dx}(e^{-x}) - \frac{d}{dx}(xe^{-x})$$

using the product rule in the second term

$$= -e^{-x} - (e^{-x} - xe^{-x})$$

$$= -2e^{-x} + xe^{-x}$$

$$= f(x) - 2e^{-x}$$

2. $y = \ln|1-x|$ we need to find an expression for $\frac{dy}{dx} + x\frac{d^2y}{dx^2}$

Note that $\frac{dy}{dx} = -\frac{1}{1-x}$ (whatever happens to the absolute value?) and $\frac{d^2y}{dx^2} = -\frac{1}{(1-x)^2}$
 this implies that

$$\begin{aligned}\frac{dy}{dx} + x\frac{d^2y}{dx^2} &= -\frac{1}{1-x} - \frac{x}{(1-x)^2} \\ &= \frac{-1+x-x}{(1-x)^2} = -\frac{1}{(1-x)^2}\end{aligned}$$