$$\Pi = V_1 - \Delta V_2$$

$$t \to t + dt \qquad d\Pi = dV_1 - \Delta\, dV_2$$

$$dV_i = \frac{\partial V_i}{\partial t}\, dt + \frac{\partial V_i}{\partial r}\, dr + \frac{1}{2}\frac{\partial^2 V_i}{\partial r^2}\, dr^2$$

$i = 1, 2$

$$\boxed{dr^2 = \omega^2 dt}$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(x,t) = X(x)T(t)$$

$$XT' = X''T$$

$$\underbrace{\frac{T'}{T}}_{\text{indep of } x} = \underbrace{\frac{X''}{\cancel{X}}}_{\text{indep of } t}$$

MPOR 2

BPE: $$\frac{\partial V}{\partial t} + \frac{1}{2}\omega^2 \frac{\partial^2 V}{\partial r^2} + u \frac{\partial V}{\partial r} = \underline{rV + \lambda\omega \frac{\partial V}{\partial r}}$$

$$dV = \omega \frac{\partial V}{\partial r} dX + \left(\underline{\underline{r}}V + \lambda\omega \frac{\partial V}{\partial r} \right) dt$$

$$dV - rV\,dt = \lambda\omega \frac{\partial V}{\partial r} dt + \omega \frac{\partial V}{\partial r} dX$$

dV : unhedged bond

$rV\,dt$: risk-free return

$$= \omega \frac{\partial V}{\partial r} \left[dX + \lambda\,dt \right]$$

$$\frac{\partial V}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2} - a(r,t)\frac{\partial V}{\partial r} - rV = 0$$

$$\frac{\partial V}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2} + u\frac{\partial V}{\partial r} = \boxed{a\frac{\partial V}{\partial r} + rV + u\frac{\partial V}{\partial r}}$$

Unhedged bond

$$dV = w\frac{\partial V}{\partial r}\,dX + \left(\frac{\partial V}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2} + u\frac{\partial V}{\partial r}\right)dt$$

$$dV = w\frac{\partial V}{\partial r}\,dX + \left(a\frac{\partial V}{\partial r} + rV + u\frac{\partial V}{\partial r}\right)dt$$

$$dV - rV\,dt = w\frac{\partial V}{\partial r}\,dX + \left(a\frac{\partial V}{\partial r} + u\frac{\partial V}{\partial r}\right)dt$$

$$= \omega \frac{\partial v}{\partial r}\left[dx + \underbrace{\left(\frac{a+u}{\omega}\right)}_{\lambda} dt \right]$$

$$\lambda = \frac{a+u}{\omega} \longrightarrow \boxed{\lambda\omega - u = a}$$

Equity

$$dS = \mu S\,dt + \sigma S\,dX$$

$$\boxed{\mu - \lambda\sigma}$$

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (\mu - \lambda\sigma) S \frac{\partial V}{\partial S} - rV = 0$$

$V = S$ is a sol^{n}

$$0 + 0 + (\mu - \lambda\sigma) S - rS = 0$$

$$\boxed{r = \mu - \lambda\sigma} \Rightarrow \lambda = \frac{\mu - r}{\sigma}$$

$$dV = \mathcal{L}(V)\,dt + \frac{\partial V}{\partial r}\,dr + \frac{\partial V}{\partial l}\,dl$$

$$dr = \left(\eta(t) - \gamma(t)\,r\right)dt + \sqrt{\alpha(t)\,r + \beta(t)}\;dX$$

$$dV = \left(\frac{\partial V}{\partial t} + \frac{1}{2}w^2\frac{\partial^2 V}{\partial r^2} + \frac{1}{2}q^2\frac{\partial^2 V}{\partial l^2} + \rho w q\frac{\partial^2 V}{\partial r\,\partial l}\right)dt$$

$$\mathcal{L}$$

$$+ \frac{\partial V}{\partial r}\,dr + \frac{\partial V}{\partial l}\,dl$$

$$dV = \left[\left(\frac{\partial}{\partial t} + \frac{1}{2}w^2\frac{\partial^2}{\partial r^2} + \frac{1}{2}q^2\frac{\partial^2}{\partial l^2} + \rho w q\frac{\partial^2}{\partial r\,\partial l}\right)\right]V\,dt$$

$$+ \partial V/\partial r\,dr + \frac{\partial V}{\partial l}\,dl$$

$$\left(\quad \right) + r \left(\quad \right)$$

$$= 0 + r \times 0$$