$$\sigma^2 \longrightarrow \frac{1}{T-t}\int_t^T \sigma_\tau^2 \, d\tau$$

$$r \longrightarrow \frac{1}{T-t}\int_t^T r_\tau \, d\tau$$

$$d\Pi = \underline{\underline{dV}} - \Delta dS$$

GBM

dV?

$$V = V(S, t) \quad (*)$$

Itô: $(*)$

$$dV = \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} dS^2$$

$$dS^2 = \sigma^2 S^2 dt$$

Linear?

① fn. V ; const $K \Rightarrow kV$ also is a sol^n of the BSE ⑥

② fn. V_1, $V_2 \Rightarrow (V_1 + V_2)$ is also a sol^n of B.S.E (6)

Call	Put
$S \to 0 \quad C \sim 0$	$S \to \infty \quad P \sim 0$
$S \to \infty \quad C \sim S$	$S \to 0$ Put-Call Parity $\Rightarrow C - P = S - Ee^{-r(T-t)}$ $P = Ee^{-r(T-t)}$
$C(S,T) = \max[S_T - E, 0]$	$P(S,T) = \max[E - S_T, 0]$

Solving the eqⁿ

① transfⁿ & substⁿ →
1D heat eqⁿ

② Solve the heat eqⁿ for the fundamental solⁿ

③ Reverse the transfⁿ

Given $C = S N(d_1) - E e^{-r(T-t)} N(d_2)$

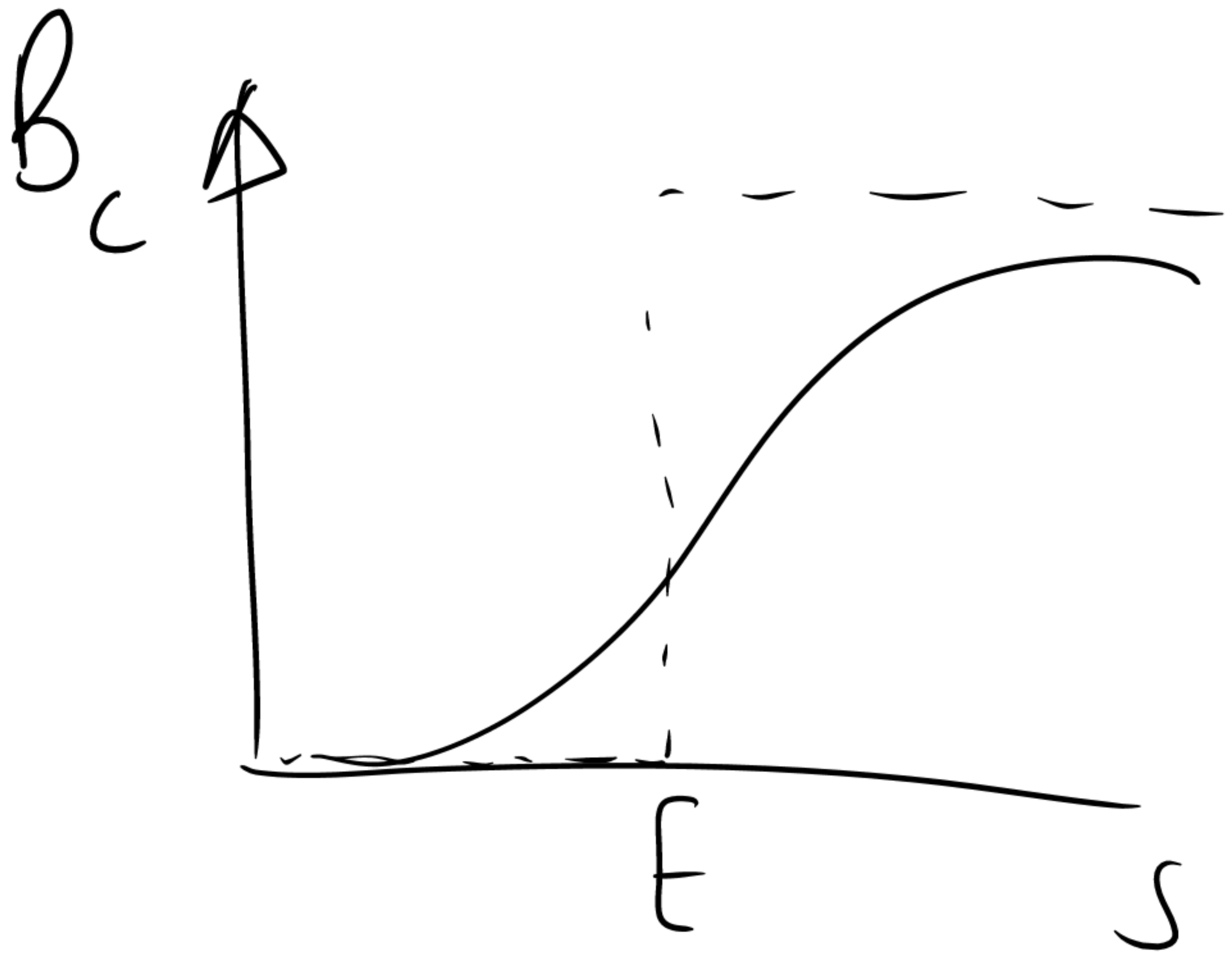
$$C - P = S - E e^{-r(T-t)}$$

$$P = \underline{\underline{C}} - S + E e^{-r(T-t)}$$

$$= S N(d_1) - E e^{-r(T-t)} N(d_2) - S + E e^{-r(T-t)}$$

$$= -S\Big[\overbrace{1 - N(d_1)}^{N(-d_1)}\Big] + E e^{-r(T-t)} \Big[\overbrace{1 - N(d_2)}^{N(-d_2)}\Big]$$

$$N(x) + N(-x) = 1 \Rightarrow 1 - N(x) = N(-x)$$


B_c
E
S

$$\boxed{dS_i = \mu_i S_i \, dt + \sigma_i S_i \, dX_i}$$

$i = 1, 2$

$V(S_1, S_2, t)$

2 sources of randomness

$\Rightarrow$ 2 instruments for hedging

$$\Pi = V(t, S_1, S_2) - \Delta_1 S_1 - \Delta_2 S_2$$

$$d\Pi = dV - \Delta_1 dS_1 - \Delta_2 dS_2$$

$$dX_1\, dX_2 = \rho\, dt$$

$$\left.\begin{aligned} dS_1^2 &= \sigma_1^2 S_1^2\, dt \\ dS_2^2 &= \sigma_2^2 S_2^2\, dt \end{aligned}\right\} \; dS_1\, dS_2 = \rho\,\sigma_1\sigma_2 S_1 S_2\, dt$$

$$d\Pi = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} + \rho\,\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2} \right) dt + \left(\Delta_1 - \frac{\partial V}{\partial S_1} \right) dS_1$$

$$+\left(\Delta_2 - \frac{\partial V}{\partial S_2}\right) dS_2$$

$$\Rightarrow \quad \Delta_i = \frac{\partial V}{\partial S_i} \qquad i = 1, 2$$

No A.S.

$$d\Pi = r\Pi\, dt$$

$$\left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma_1^2 S_1^2 \frac{\partial^2 V}{\partial S_1^2} + \frac{1}{2}\sigma_2^2 S_2^2 \frac{\partial^2 V}{\partial S_2^2} + \rho\sigma_1\sigma_2 S_1 S_2 \frac{\partial^2 V}{\partial S_1 \partial S_2}\right) dt$$

$$= r\left[V - S_1 \frac{\partial V}{\partial S_1} - S_2 \frac{\partial V}{\partial S_2}\right] dt$$

$$V(S, t) \qquad S \to S + \delta S$$

$$V(S+\delta S, t) = V(S,t) + \frac{\partial V}{\partial S}\delta S + \frac{1}{2}\frac{\partial^2 V}{\partial S^2}\delta S^2$$

$$+ \frac{1}{6}\frac{\partial^3 V}{\partial S^3}\delta S^3 + \cdots$$

$$\frac{V(S+\delta S, t) - V(S,t)}{\delta S} = \frac{\partial V}{\partial S} = \Delta$$