$\int$ Derivatives

power rule
product rule / quotient rule
logarthmic / expontential differentiation
↓
ln/e differentiation   chain rule
u - subsition

$\cancel{y}\,|\,t \sim y_{1+} \sim y_{+}$

$y_0 \quad y_1$

QUESTION #1

$\frac{d}{dx} = e^{(-x)} + x \cdot -e^{(-x)}$

$$\boxed{\frac{d}{dx}\, e^{(-x)} + -xe^{(-x)}}$$

$\boxed{-e^{-x}} + (-x)' \cdot e^{(-x)} + (-x) \cdot e^{(-x)'}$

$-1 \cdot e^{(-x)} + (-x) \cdot -e^{(-x)}$

$-e^{(-x)} - e^{(-x)} + xe^{(-x)}$

$-2e^{(-x)} + xe^{(-x)}$

question 2

(2) Assume $\xrightarrow{\text{Given}} \ln|x-1|$

$$\frac{dy}{dx} + x\left[\frac{d^2y}{dx^2}\right]$$

$$\boxed{\begin{array}{c} x = 0 \\ \hookrightarrow 2 \end{array}}$$

$$\frac{dy}{dx}(f(x)) = \frac{dy}{dx}(\ln|1-x|)$$

the first derivative of $F(x) = F'(x)$

Chain rule

$$f'(x) = \frac{\frac{dy}{dx}(|1-n|)}{|1-n|} = \left[\frac{\frac{(1-n)\frac{dy}{dx}(1-n)}{|1-n|}}{|1-n|}\right]$$

$\frac{d|u|}{du}$ 0; where $u=(1-n)$ \$ $\frac{d}{du}(|u|) = \frac{u}{|u|}$

$$\left(x \cdot \frac{d^2y}{dx^2}\right)^{\nearrow 0} \ln|1-n|$$

$$\frac{dx}{dx}$$

$y''(x) = 0$ or $-\frac{1}{(x-1)^2}$

∞

$y'(x) = \frac{1}{x-1}$

Question 3

Question 3 What is the value of the limiting problem $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

Step 1 – Simplify : $\frac{(x+h)^3 - x^3}{h}$: $\frac{3x^2 + 3xh + h^2}{h}$

Expansion

TSE

1 $(x+h)^3 - x^3 = x^3 + 3x^2h + 3xh^2 + h^3$

2 $(x+h)^3 : x^3 + 3x^2h + 3xh^2 + h^3$

$= x^3 + 3xh^2 + h^3 - x^3$

Simplified

Group like terms

$= (-x^3 + x^3) + 3x^2h + 3xh^2 + h^3$

$= \left(\frac{3x^2h + 3xh^2 + h^3}{h}\right)$

Apply Exponent Rule

$a^{b+c} = a^b a^c$

$= 3hx^2 + 3hhx + hh^2$

Factor out h term

$= \left[\frac{h(3x^2 + 3xh + h^2)}{h}\right]$

Cancel Factor h

$= 3x^2 + 3xh + h^2$

Note $\lim h = \emptyset$

$\lim_{h \to 0} (3x^2 + 3xh + h^2)$

$= (3x^2 + 3x\emptyset + \emptyset^2)$

Final Simplification

$= 3x^2$ (D) Final Answer

QUESTION 04

Q Calculate $\lim_{x \to 0} \left(\frac{2x + \sin x}{x - 1} \right)$

divide by x
highest denom. power

$$= \left[\frac{\frac{2x}{x} + \frac{\sin x}{x}}{\frac{x(x-1)}{x}} \right]$$

$$= \frac{2 + \frac{\sin(x)}{x}}{x - 1}$$

$$= \frac{\lim_{x \to 0} \left(2 + \frac{\sin x}{x}\right)}{\lim_{x \to 0} (x-1)}$$

pt A

$$= \left[2 + \boxed{\left[\lim_{x \to 0} \left(\frac{\sin x}{x} \right) \right] = 1} \right]$$

$2 + 1 = 3$

pt B $x = 0$

$$\lim_{x \to 0} (x - 1)$$
$$= (0 - 1) = -1$$

$$= \frac{3}{-1} = \boxed{-3}$$

Final ANSWER is (E) -3

QUESTION 05

IF $\quad I = \int_0^\infty x \exp(-x^2)\, dx$

Then $\quad I = ?$

compute the indefinite integral

$$= \int x \exp(-x^2)\, dx = \frac{\exp(-x^2)}{2} + C$$

Compute the Boundaries

$x = 0$

Ⓐ $\lim_{x \to 0^+} \left(-\frac{\exp(-x^2)}{2} \right) = -\frac{1}{2}$

Ⓑ $\lim_{x \to \infty} \left(-\frac{\exp(-x^2)}{2} \right)$

$= -\lim_{x \to \infty} \left(\frac{\exp(-x^2)}{2} \right)$

$$= -\left(\frac{\lim_{x \to \infty} [\exp(-x^2)]}{\lim_{x \to \infty} (2)} \right) \sim \left(-\frac{0}{2} \right)$$

Apply limit & chain Rule

$$\boxed{0 - \left(-\frac{1}{2}\right) = \frac{1}{2} \quad \text{or} \quad E}$$

Final Answer =

Question #6

Evaluate the integral $\int_0^1 \frac{dx}{x^2+3x+2}$

$$\int \frac{1}{x^2+3x+2}\,dx$$

Sum Rule

$$= \int \frac{1}{x+1}\,dx - \int \frac{1}{x+2}\,dx$$

add constant

$$= \ln|x+1| - \ln|x+2| + C$$

Boundary Computation

$$\int_0^1 \frac{1}{x^2+3x+2}\,dx$$

A

$$\lim_{x \to 0^+}\left(\ln|x+1| - \ln|x+2|\right) = -\ln(2)$$

B

$$\lim_{x \to 1^-}\left(\ln|x+1| - \ln|x+2|\right) = -\ln\left(\frac{3}{2}\right)$$

$$\boxed{= -\ln\left(\frac{3}{2}\right)} \longleftrightarrow$$ Final Answer D

Question #9

Question 09

9

$$P(x) = \begin{cases} K(1-x^3) \\ 0 \end{cases}$$

$$\int_0^1 K\left(1-\frac{3}{x}\right) = 1$$

$$K\left[x - \frac{x^4}{4}\right]_0^1 = 1$$

$$K\left[1-\frac{1}{4}\right] = 1$$

$$K \cdot \left(\frac{3}{4}\right) = 1$$

$$\boxed{K = \frac{4}{3}}$$

C

Question 11

Question 11

11

$$V(x) = \sigma^2 = \left[\sum_D x^2 \cdot p(x)\right] - \mu^2 = E(x^2) - [E(x)]^2$$

$16 + 4 = 20$

B

Question 7

Question 07

7

$$T(x) = f(a) + f'(a)(x-G) + \frac{f''(a)\,(x-G)^n}{n!}$$

$$\sum_{j-1}^{\infty} = f^n(a)\frac{(x-a)^n}{n!}$$

$$f(x) = e^x \quad F^n(x) = e^x \quad F^n(a) = e^a$$

$$\frac{e^{-3}(x+3)^n}{n!} = \frac{e^{-3}(x+3)^n}{n!} = \frac{e^{-3}}{n!}(x+3)^n$$

A

Question #8

Question 08

do anti derivative of all the problems

$$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \quad \frac{(-1)^{n+1}x^n}{n}$$

$$\sum_1^{\infty} \frac{(-1)^{n+1}x^n}{n} = \sum_0^{\infty} \frac{(-1)^n x^{n+1}}{(N+1)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$

C

Question 10

Question 10

$$P(x) = \begin{cases} \frac{3}{4}(1-x^2) & \text{if } |x| \le 1 \\ 0 \end{cases}$$

$$\int_0^{\frac{1}{2}} \frac{3}{4} - (1-x^2)\,dx = \frac{3}{4}\left[x - \frac{x^3}{3}\right]_0^{\frac{1}{2}}$$

$$\frac{3}{4}\left(\frac{1}{2} - \frac{\left(\frac{1}{2}\right)^3}{3}\right)$$

$$\frac{3}{4}\left(\frac{1}{2} - \frac{1}{24}\right)$$

$$\frac{3}{4}\left(\frac{12}{24} - \frac{1}{24}\right) = \frac{3}{4}\left(\frac{11}{24}\right) = \boxed{\frac{11}{32}}$$

8

Question

Question 13

$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\left(-\frac{x-\mu^2}{\sigma^2}\right)}$$

$$\frac{\gamma}{\sqrt{2\pi\sigma}} = \gamma = \frac{\sqrt{\cancel{2\pi}\sigma^2}}{} =$$

$$\cancel{2}\sigma^2 = \sqrt[2]{\frac{\sigma}{\gamma^2}} = \boxed{\frac{\sqrt{\sigma}}{\gamma}}$$

Normal standard σ

Question 14

Question 14

$$f(x,y) = x^2 - 4xy + 3$$

$$f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x} f\right)$$

(Fx)

$$f_x = 2x - 4y + 0 + 0 = 2x - 4y$$

$$f_y = 0 - 4x + 3y^2 + 4$$

$$f_{yy} = \frac{\partial}{\partial y}(-4x + 3y^2 + 4) = 6y$$

$$f_{xx} + f_{yy} = 2 + 6y$$

$\boxed{E}$ none of abov

Question 15

Question 15

(15) $\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$

$\tan^{-1}(y) + C = \tan^{-1}(x) + C$

$y = \tan(\tan^{-1}(x+C))$

$C = \cancel{0}$ $\quad \tan(\tan^{-1}) = x$

Replace

$\boxed{y = \frac{x+C}{1-Cx}}$

~~Stop~~ derivative?

$\frac{dy}{dx} = \frac{(1-Cx)(1) - (x-C)(-C)}{(1-Cx)^2}$

$= 1 - Cx + Cx - C^2 = \frac{\left(\frac{1-C^2}{1-Cx}\right)^2}{1+x^2}$

$\frac{1-C^2}{(1-Cx)^2} = \frac{1+y^2}{1+x^2} = \frac{1+\left(\frac{x+C}{1-Cx}\right)^2}{1+x^2} = \frac{x+C^x}{1-C^x}$

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(19)

Question 19

Question 19

$\begin{vmatrix} K & K \\ 8 & 4K \end{vmatrix}$ $(K \cdot 4K) - (K \cdot 8)$

$4K^2 - 8K = 0$

$(4K) \cdot (K-2) = 0$

$2-2 = 0$

$2-2 = 0$

Answer is A

Question 16

Question 16

$$\frac{dy}{dx} - xy = x$$

$$x + xy = x(1+y)$$

$$\frac{dy}{1+y} = x\,dx$$

$$\ln(1+y) = \frac{1}{2}x^2 + C$$

$$1+y = e^{(\frac{1}{2}x^2 + c)} = c' e^{\frac{1}{2}x^2}$$

$$y = c' e^{\frac{1}{2}x^2} - 1$$

$$A = \nearrow$$

$$\boxed{A e^{(\frac{1}{2}x^2)}}$$

$\boxed{D}$

Question 17

Question 17

$y'' - 4y' + 13y = 0$ ~~to obtain y =~~

Second order Homogenius linear Second order ODE → Laplace transform Eigenval Eigenvalue

A ~~Answer~~

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + qy = 0$$

Quadratic Formula on constants

m_1 + | m_2 −

$$y(x) = m^2 - 4m + 13 = 0$$

$$m = \frac{4 \pm \sqrt{16 - 4 \cdot 13 \cdot 1}}{2} = \mathbb{C}$$

$$(a^2 + bi^3)$$

$$\frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

$$y = e^{2x}(c_1(\cos 3x) + c_2(\sin 3x))$$

$$\boxed{e^{2x^2}(A\cos 3x + B\sin 3x)}$$

Second order ODE

Non linear differential Equation

Integrating factor

Homogenous

Question 18

(18) Solve $x^2 y'' - 4xy' + 6y = 0$ to obtain $y =$

integrating factor for second order ODE

QUESTION 18

$$x^2 \frac{du}{dx} - 4xu + 6y = 0$$

$I = e^{\int P dx}$

let $\boxed{u = y'}$ then $\boxed{\frac{du}{dx} = y''}$ — this reduces 2nd order equation to a first order equation w/ seperate variables

the dependent variable is u

the independent variable is x

$$x^2 \frac{du}{dx} = 2x^{2-1} = 2x$$

$$2x \frac{du}{dx} - 4xu + 6y = 0$$

$$\frac{d^2 y}{dx^2} x^2 - 4 \frac{dy}{dx} x + 6y = 0$$

→ Assume constant / Euler Cauchy $= x^c$

Assume $y(x) = x^m$

$$\frac{d^2}{dx^2}(x^m) - 4x \frac{d}{dx}(x^m) + 6(x^m) = 0$$

Substitute

$$\frac{d^2}{dx^2}(x^m) = (u-1)\, u\, x^{(m-1)}$$

$$\frac{d}{dx}(x^m) = u\, x^{(m-1)}$$

$$\hookrightarrow u^2 x^m - 5u x^m + 6x^m = 0$$

Simplify factor out (x^m)

$$(u^2 - 5u + 6)x^m = 0$$

Assume $x \neq 0$

$$u^2 - 5u + 6 = 0$$

$$(u-3)(u-2) = 0$$

Solve for u: $u = 2$ or $u = 3$

$$\boxed{y(x) = c_2 x^3 + c_1 x^2}$$

Assume C_1 & C_2 are Arbitrary Constants

Replace with ↗ $\boxed{C_1 = A \quad C_2 = B}$

the root of $u=2$ gives $y_1(x) = C_1 x^2$

" " " $u=3$ " $y_2(x) = C_2 x^3$

$$y_1(x) + y_2(x) = \boxed{y(x) = C_1 x^2 + C_2 x^3}$$

Final Answer B → $y(x) = Ax^2 + Bx^3$

Question 20

X,Y,Z

Gauss Jordan elimination

Reduced Row Echelon form

R-Ref

Question 20

$$\left|\begin{matrix} 2 & 1 & -1 & 1 \\ 1 & 0 & -2 & -5 \\ 1 & -2 & 3 & 6 \end{matrix}\right|$$

$$\begin{matrix} -R_1 \\ -R_2 \\ \end{matrix} \left|\begin{matrix} 1 & 0 & -2 & -5 \\ 2 & 1 & -1 & 1 \\ 1 & -2 & 3 & 6 \end{matrix}\right|$$

$$\left|\begin{matrix} 1 & 0 & -2 & -5 \\ 0 & 1 & 3 & 11 \\ 0 & -2 & 5 & 16 \end{matrix}\right|$$

$$+2R2 \left|\begin{matrix} 1 & 0 & -2 & -5 \\ 0 & 1 & 3 & 11 \\ 0 & 0 & 11 & 38 \end{matrix}\right|$$

C

$$\boxed{\begin{matrix} X = 1 \\ y = 2 \\ z = 3 \end{matrix}}$$

$$\left|\begin{matrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{matrix}\right|$$

INNER Product = 0 if perpendicular

22

Question 22

Question 22

$$(-1 \quad 6) + (0 \; (-4)) + K \cdot 2 = 0$$

$-6 \qquad 0 \qquad (3 \cdot 2)$ 6

$\boxed{3} = K$

partial Chain Rule

Question 23

$$2x \frac{df}{d\theta} + 2y \cdot \frac{dy}{d\theta}$$

$x = \sin\theta$

$y = \cos\theta$

$$\frac{dx}{d\theta} = 2\cos\theta \qquad \frac{dy}{d\theta} = -2\sin\theta$$

$$\frac{df}{d\theta} = 4x\cos\theta + 2y(-2y \cdot -2\sin\theta)$$

$$= 4\sin 2\theta \cos 2\theta - 4\cos\theta \, 2\sin\theta$$

$$= 0$$

Question 23

B

Q 21

Question 21

Question 21

Obtain determinate of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 3 & 2 & -3 \\ -1 & -3 & 5 \end{bmatrix}$

$$= 2\det\underbrace{\begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix}}_{A} - 0\det\underbrace{\begin{bmatrix} 3 & -3 \\ -1 & 5 \end{bmatrix}}_{B} + 1\det\underbrace{\begin{bmatrix} 3 & 2 \\ -1 & -3 \end{bmatrix}}_{C}$$

$2 \cdot 5 - (-3)(-3) = 1$ $\quad$ $3 \cdot 5 - (-3)(-1) = 12$ $\quad$ $3(-3) - 2(-1) = -7$

$$= 2 \cdot 1 - 0 \cdot 12 + 1(-7)$$

$$\boxed{= -5}$$

Question 21

Q. 25

Question 25

Question #25

Find the largest eigenvalue of matrix $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$

Where: A = square matrix / v = vector / λ = eigenvalue of Eigenvector v of A

Note:
the eigenvalues of of A are the roots of following equation
↳ $\det(A - \lambda I) = 0$

$$\det\left[\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right] = \det\begin{pmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{pmatrix} = (\lambda - 4)(\lambda - 1) = \emptyset$$

$\lambda = 1$, $\lambda = 4$ → $\boxed{4 \text{ is the largest Eigenvalue}}$

Question 25

Question 24

24

24 If $x = 2 - 3i$ is a complex number, then it's size is?

$$|2 - 3i| = \sqrt{(2)^2 + (3)^2}$$

$$\sqrt{4 + 9} = \boxed{\sqrt{13}} \approx 3.6055$$

$z = a + bi$

Note the size of Complex # is denoted $|z|$ if it's plotted (a, b) it's distance to origin is $\sqrt{a^2 + b^2}$

Question 26

Question 26

Evaluate $\int_{-2}^{1} |x|\, dx$

A $\int_{-2}^{0} (-x)\, dx = 2$

B $\int_{0}^{1} (x)\, dx = \frac{1}{2}$ ← Power Rule

$$2 + \frac{1}{2} = \boxed{5/2}$$

$$\boxed{\frac{5}{2}} = \text{Final}$$