$$dG = a\,dt + b\,dX$$

$$\mathbb{E}[dG] = \mathbb{E}[a\,dt] + \mathbb{E}[b\,dX]$$

$$= a\,dt\,\mathbb{E}[1] + b\,\mathbb{E}[\underbrace{dX}_{=0}]$$

$$= a\,dt$$

$$\mathbb{V}[dG] = \underbrace{\mathbb{V}[a\,dt]}_{=0} + b^2\,\underbrace{\mathbb{V}[dX]}_{=dt}$$

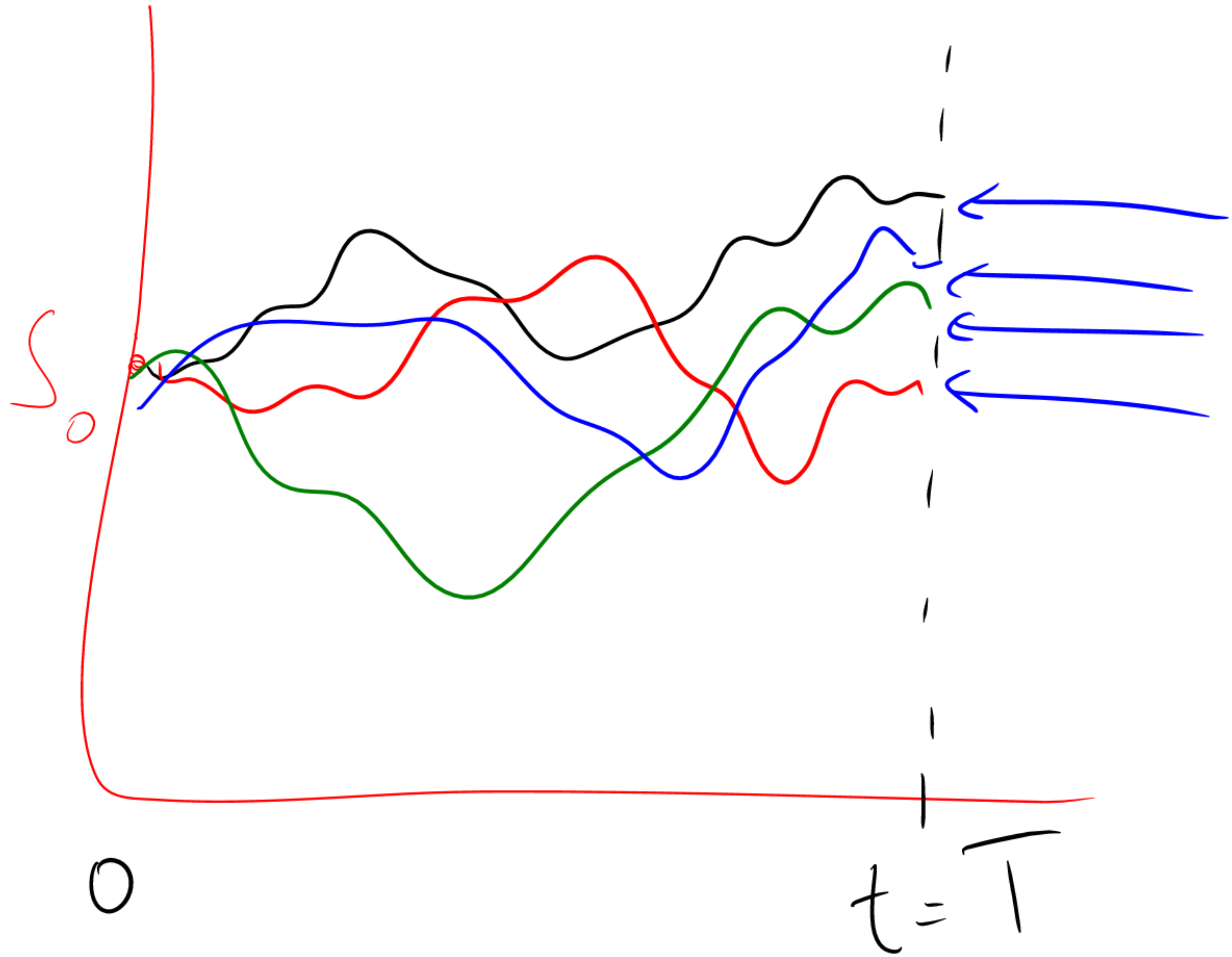
$$= b^2\,dt$$

$$dG = A\,dt + B\,dX$$

$$\boxed{dG^2 = B^2\,dt}$$

$$V = V(G, t)$$

Itô: $$dV = \left(\frac{\partial V}{\partial t} + A \frac{\partial V}{\partial G} + \frac{1}{2} B^2 \frac{\partial^2 V}{\partial G^2} \right) dt + B \frac{\partial V}{\partial G}\, dX$$



S_0
0
t = T

$$du = -\gamma u\, dt + \sigma\, dX \qquad u(0) = x$$

(Ornstein - Uhlenbeck process)

$$e^{\gamma t}(du + \gamma u\, dt) = e^{\gamma t}\sigma\, dX$$

$$d(ue^{\gamma t}) = \sigma e^{\gamma t}\, dX$$

integrate over $[0, t]$

$$\int_0^t d(ue^{\gamma s}) = \sigma \int_0^t e^{\gamma s}\, dX$$

$$u(t)\underline{\underline{e^{\gamma t}}} - u(0) = \text{- - - - - -}$$

$$\frac{1}{2}\sigma^2 \frac{dp}{dr} = -\gamma(r-\bar{r})p$$

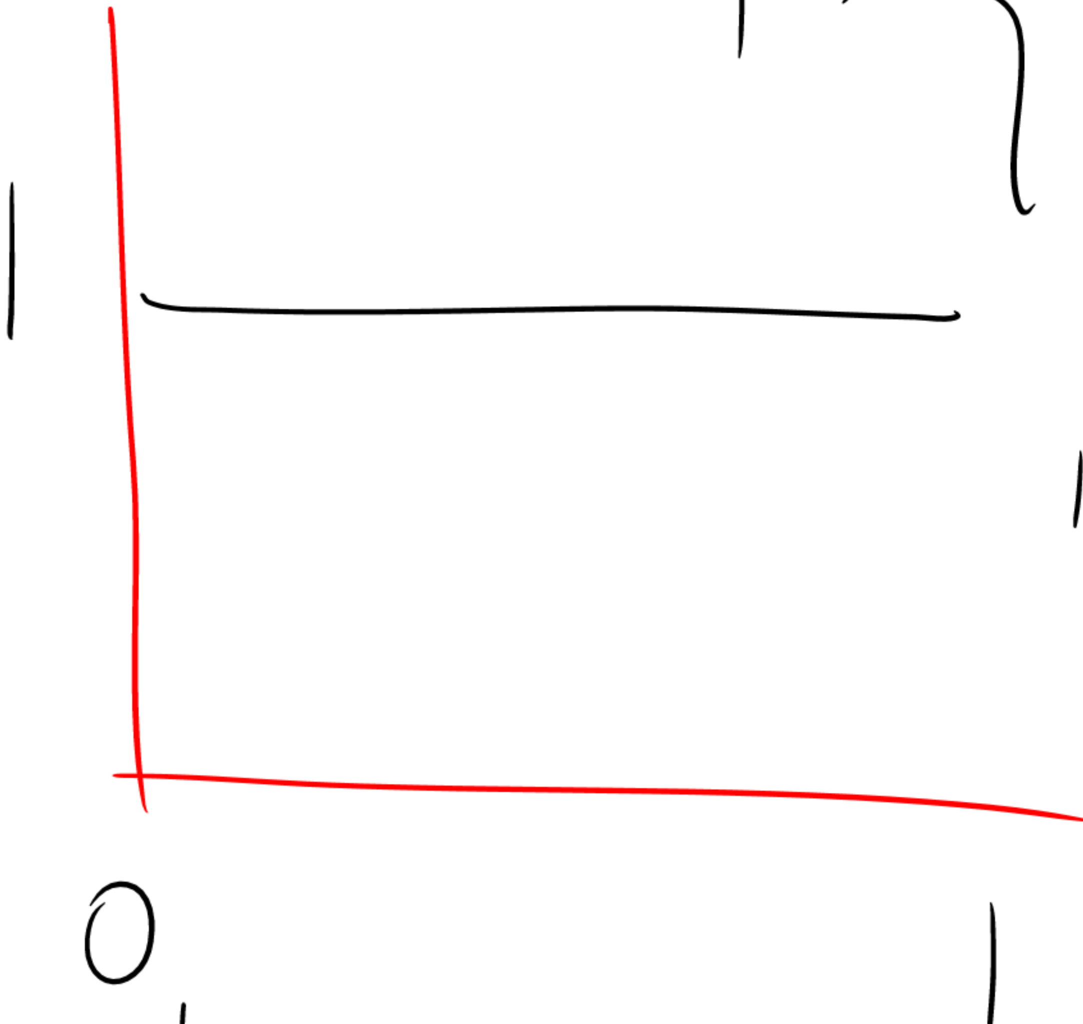
$$\int \frac{dp}{p} = -\frac{2\gamma}{\sigma^2}\int \underbrace{(r-\bar{r})}_{\frac{1}{2}\frac{d}{dr}(r-\bar{r})^2} dr$$

$$\delta S = \mu S\, \delta t + \sigma S \phi\, \delta t^{1/2}$$

$$S_{i+1} - S_i = S_i \left[\mu\, \delta t + \sigma \phi\, \delta t^{1/2} \right]$$

$$S_{i+1} = S_i \left[1 + \mu\, \delta t + \sigma \phi\, \delta t^{1/2} \right]$$

$$p(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$



$$\mathbb{E}[x] = \int_{\mathbb{R}} x \, p(x) \, dx$$

$$= \int_0^1 x \, dx$$

$$= 1/2$$

$$\mathbb{V}(x) = \int_0^1 x^2 p(x) - \left(\frac{1}{2}\right)^2 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

$$\mathbb{E}\left[\sum_{i=1}^{N} \text{RAND}()\right] = \sum_{i=1}^{N} \underbrace{\mathbb{E}\left(\text{RAND}()\right)}_{1/2} = \frac{N}{2}$$

$$\therefore \quad \sum_{i=1}^{N} \text{RAND}() - \frac{N}{2}$$

$$\mathbb{V}\left[\sum_{i=1}^{N} \text{RAND}() - \frac{N}{2}\right] = \mathbb{V}\left(\sum_{i=1}^{N} \text{RAND}()\right] = \sum_{i=1}^{N} \underbrace{\mathbb{V}\left(\text{RAND}()\right)}_{1/12}$$

$$= \frac{N}{12} \neq 1 \qquad \mathbb{V}\left[\alpha\left(\sum_{i=1}^{N} \text{RAND}() - \frac{N}{2}\right)\right] = 1$$

$$\alpha^2 \, \mathbb{V}\underbrace{\left[\qquad\right)}_{N/12} = 1 \qquad \alpha^2 \frac{N}{12} = 1 \quad \therefore \quad \alpha = \sqrt{\frac{12}{N}}$$

$$dr = -\gamma(r-\bar{r})\,dt + \sigma\,dX$$

Put $\sigma = 0$

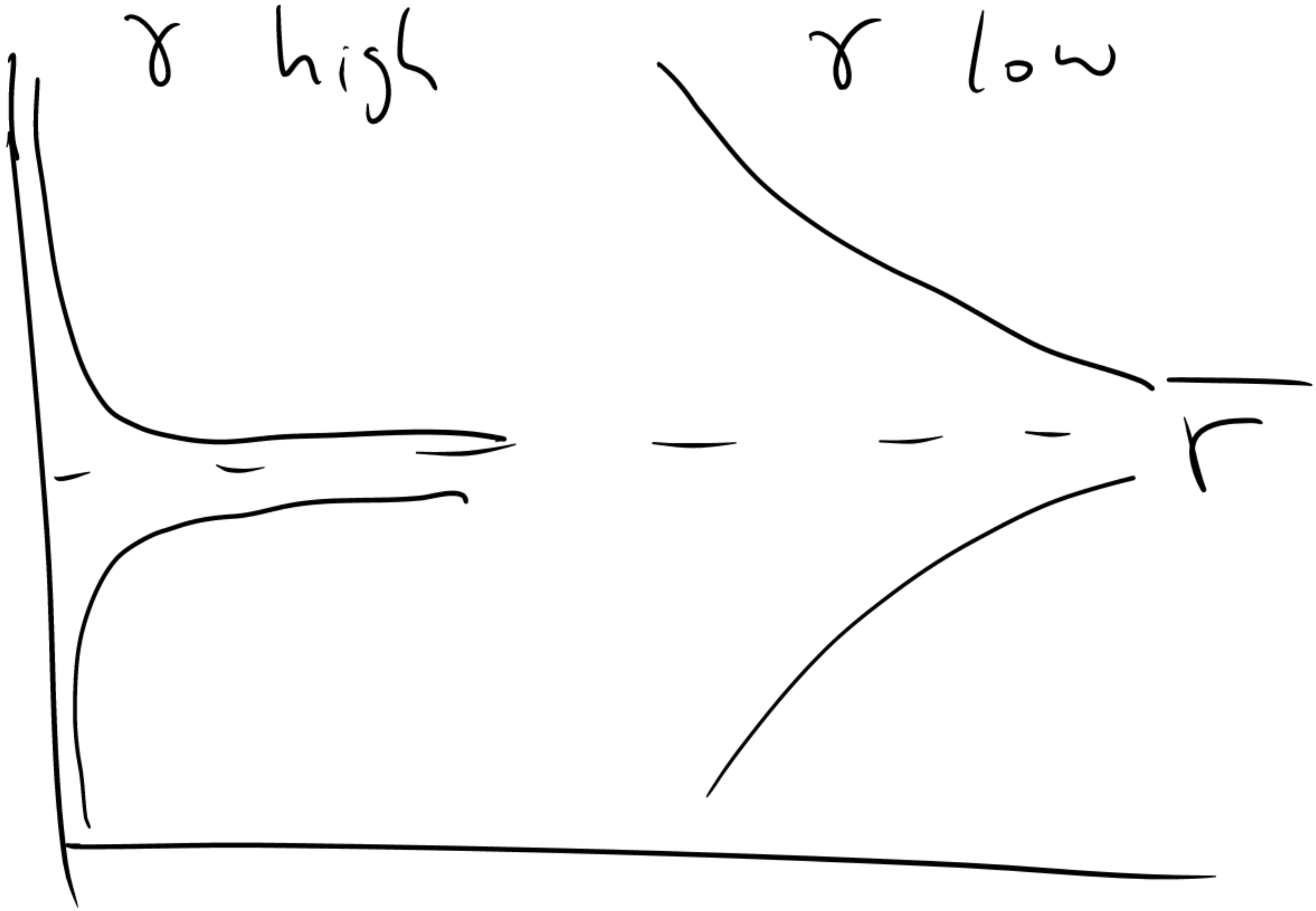
$$dr = -\gamma(r-\bar{r})\,dt$$

$$\int \frac{dr}{r-\bar{r}} = -\gamma \int dt$$

$$\log(r-\bar{r}) = -\gamma t + A$$

$$r-\bar{r} = Ce^{-\gamma t}$$

$$r = \bar{r} + Ce^{-\gamma t}$$


γ high
γ low
$\bar{r}$

$$\varepsilon_1, \varepsilon_2 \sim N(0,1) \quad \text{but} \quad \mathbb{E}[\varepsilon_1 \varepsilon_2] = 0$$

$$\text{Set} \quad \phi_1 = \varepsilon_1 \quad \text{but} \quad \phi_2 = \alpha \varepsilon_1 + \beta \varepsilon_2$$

$$\text{s.t.} \quad \boxed{\mathbb{E}[\phi_1 \phi_2] = \rho}$$

1 $$\mathbb{E}[\phi_1 \phi_2] = \rho = \mathbb{E}\left[\varepsilon_1 (\alpha \varepsilon_1 + \beta \varepsilon_2)\right]$$

$$= \alpha \underbrace{\mathbb{E}[\varepsilon_1^2]}_{1} + \beta \underbrace{\mathbb{E}[\varepsilon_1 \varepsilon_2]}_{0} = \rho \Rightarrow \boxed{\alpha = \rho}$$

2 $$\mathbb{E}[\phi_2^2] = 1 \quad \therefore \quad \mathbb{E}\left[\alpha^2 \varepsilon_1^2 + \beta^2 \varepsilon_2^2 + 2\alpha\beta \varepsilon_1 \varepsilon_2\right] = 1$$

$$\alpha^2 \mathbb{E}[\varepsilon_1^2] + \beta^2 \mathbb{E}[\varepsilon_2^2] + 2\alpha\beta \mathbb{E}[\varepsilon_1 \varepsilon_2] = 1$$

$$\rho^2 + \beta^2 = 1 \Rightarrow \beta = \sqrt{1-\rho^2}$$

$$\therefore \quad \alpha = \rho \qquad \beta = \sqrt{1-\rho^2}$$

$$\phi_1 = \varepsilon_1$$

$$\phi_2 = \rho\,\varepsilon_1 + \sqrt{1-\rho^2}\,\varepsilon_2$$

$$r_{i+1} = r_i - \gamma(r_i - \bar{r})\,\delta t + \sigma \phi \sqrt{\delta t}$$

$$\text{rate} = 5.5\%$$

$$\gamma = 1$$

$$\sigma = 0.015$$

$$\bar{r} = \frac{?}{\gamma} = 5.65\%$$