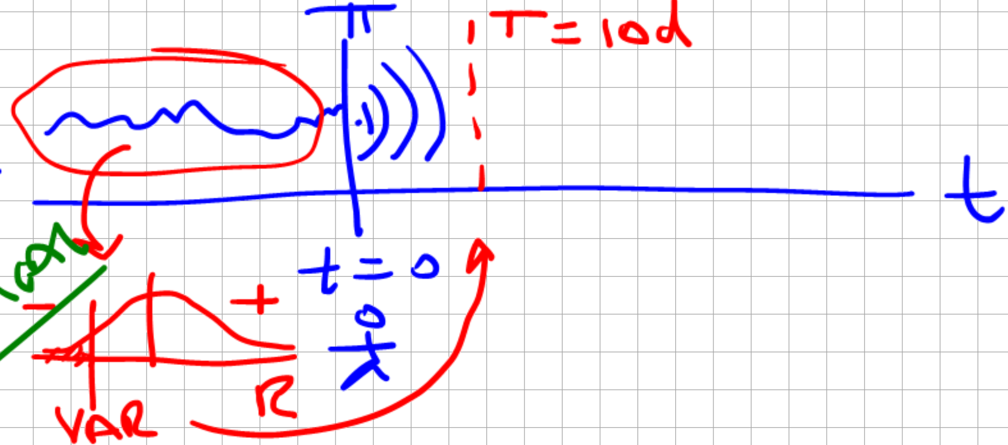
VaR

Methods VAR

Historical

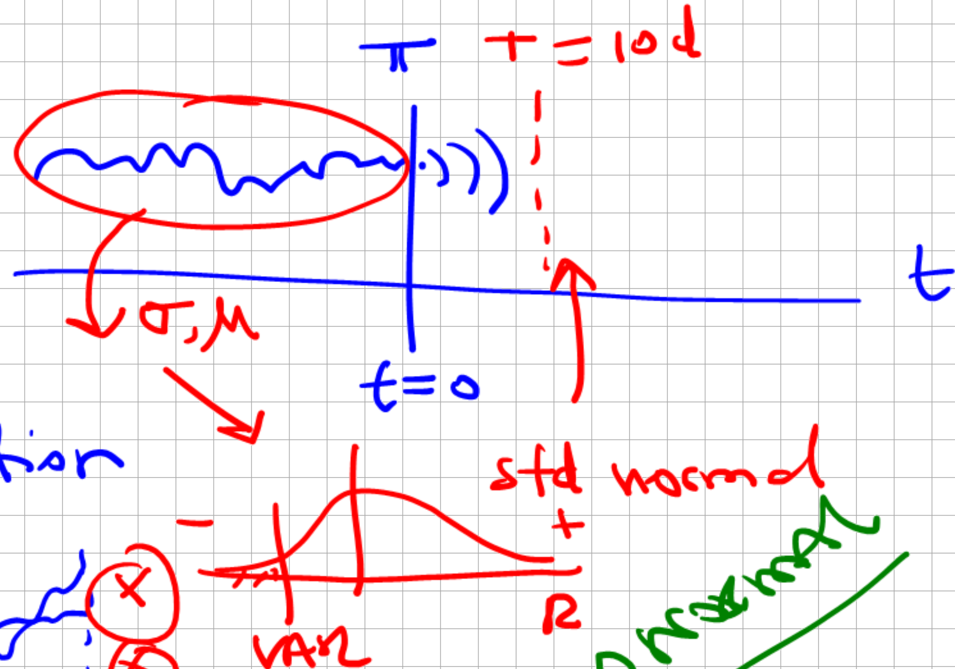
Non-Parametric

Empirical



Analytic

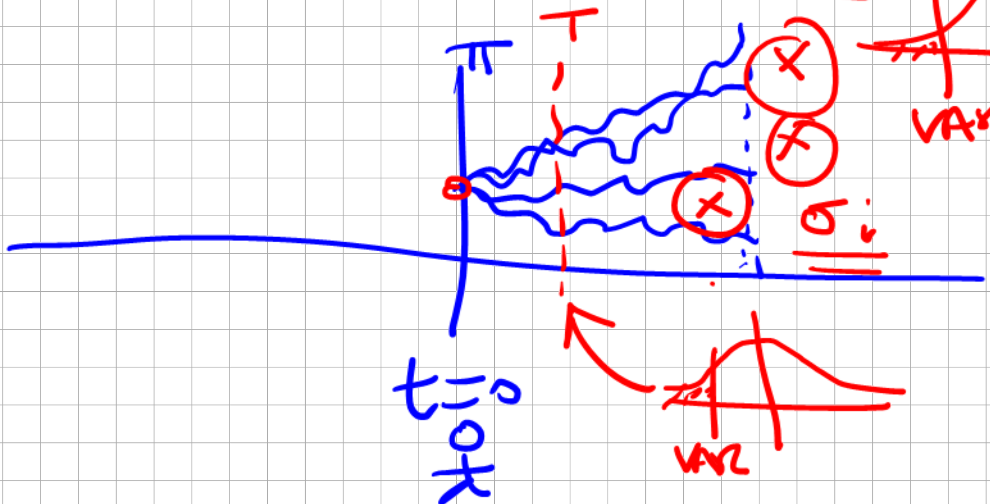
parametric



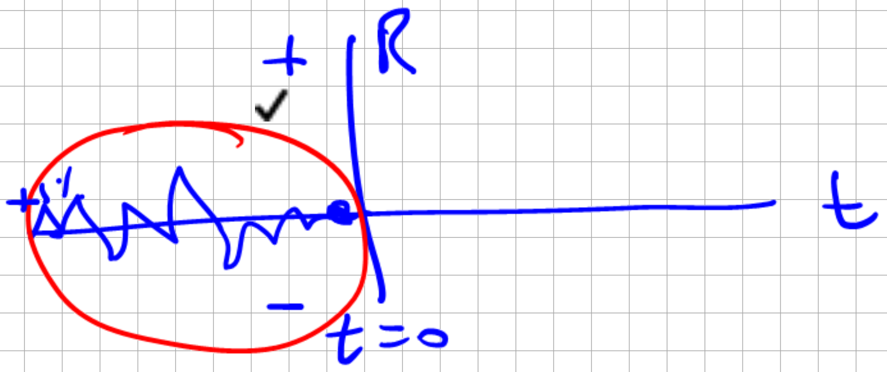
std normal

STD normal

MC simulation



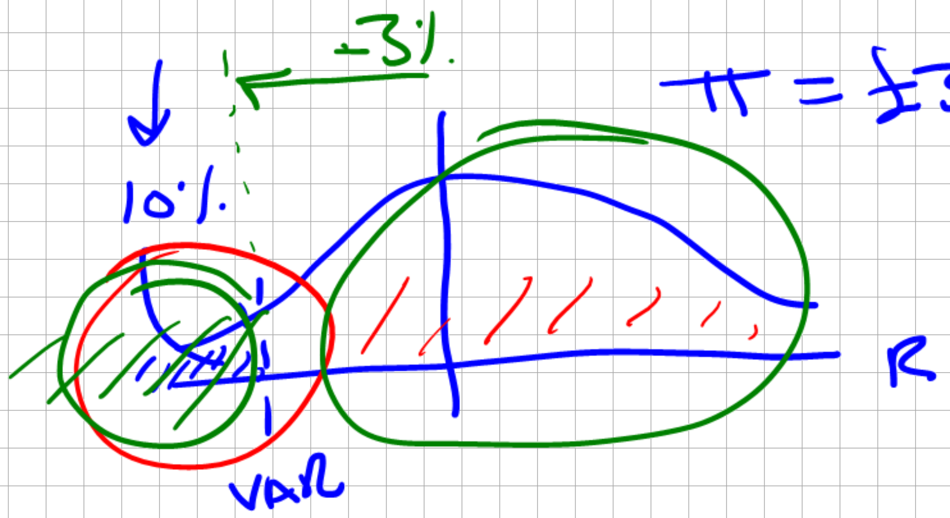
①



n = 10

VAR(1d, 90%)

π = £3m



(daily) Returns	sorted returns
+1%	−3%
0%	−2%
−1%	−1%
−2%	−1%
+1%	0%
+3%	0%
−1%	0%
0%	+1%
−3%	+1%
0%	+3%

10

1

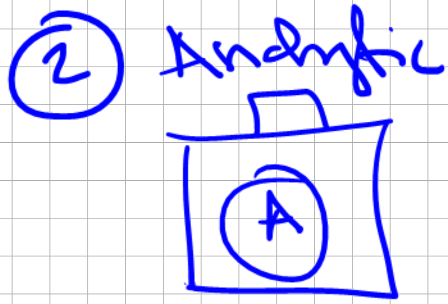
9

VAR(1d, 90%) = −3%   [%]

VAR(1d, 90%) = −3% (£3m) = −900,000 £

Problem 1: Historical Simulation
Compute the 1-day VAR at 90% confidence
(both in percent and monetary terms)
for a portfolio of £3 million whose recent
daily returns have been: **+1%, 0%, -1%, -2%, +1%, +3%, -1%, 0%, -3%, 0%**

② Analytic



$\Pi = £1m$

$\sigma_{1d} = 1\%$

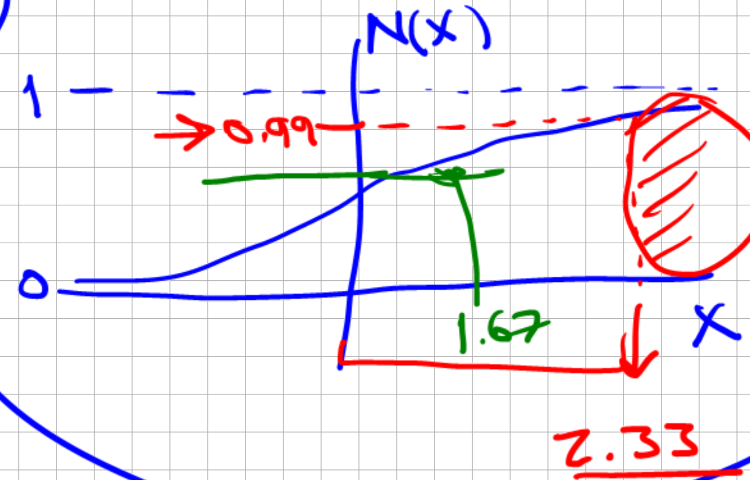
$(\%)\,VAR(1d, 99\%) = \sigma_{1d}\, N^{-1}(x)$

$= (1\%)(N^{-1}(0.99))$

$= (1\%)(2.33)$

$= 2.33\%$

N(x): cumulative std normal distribution



Problem 2: Analytic VAR
Compute
(a) the 1-day VAR at 99% confidence,
(b) the 10-day VAR at 99% significance,
for a portfolio composed of a single asset whose value is £1 million,
a volatility sigma_daily=1%

$(£)\,VAR(1d, 99\%) = (2.33\%)(£1m)$

$= £23,300$

$(£)\,VaR(10d, 99\%) = (£23,300)(\sqrt{10})$

$= £73,681$

Boost

$T\text{-}VaR = 1\text{-}VaR \times \sqrt{T}$

$10d\ VAR = 1d\ VAR \times \sqrt{10}$

$\sigma + \sigma + \cdots = \sigma\sqrt{T}$

(3)

$\boxed{A + B}$

£100 £100

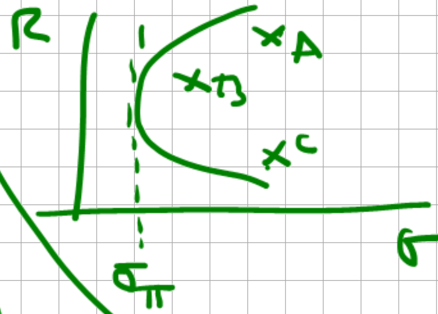
$\sigma_A = 1\%$ $\sigma_B = 1\%$

$\underbrace{\sigma_A = £1 \quad \sigma_B = £1}$

$\rho_{AB} = 0.5$

$\boxed{VAR(T, x) = \sigma_T\, N^{-1}(x)}$?

Markowitz (1951): MPT



$$R_\pi = \sum_i w_i R_i$$

$$\sigma_\pi^2 = \sigma_A^2 + \sigma_B^2 + 2\sigma_A\sigma_B\rho_{AB}$$

$$\sigma_\pi^2 = (1)^2 + (1)^2 + (2)(1)(1)(0.5)$$

$$\sigma_\pi^2 = 3 \rightarrow \sigma_\pi = \sqrt{3} = £1.73$$

$\therefore$

$$VAR(1d, 99\%) = (£1.73)\underbrace{N^{-1}(0.99)}_{2.33}$$

$$= £4.03$$

Problem 3: Analytic VAR for Portfolio
Compute the 1-day VAR at 99%,
for a portfolio composed of two assets,

whose values are
A=£100
B=£100.

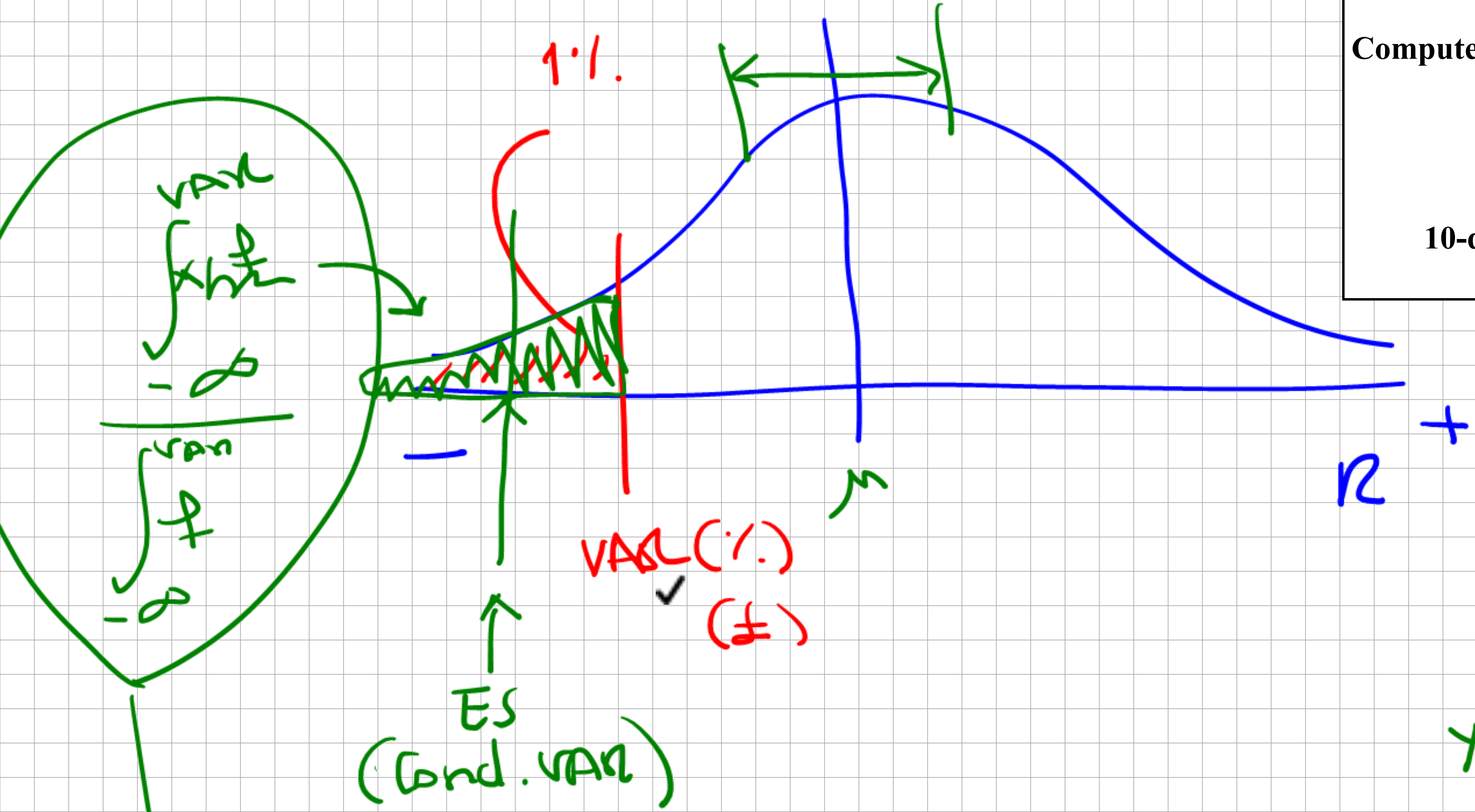
Their volatilities are
sigma_A=1%
sigma_B=1%,

correlation rho_AB=50%

Problem 4: Expected Shortfall (ES)

Compute the 10-day Expected Shortfall at 99%

ES (10d ,99%) for a portfolio whose 10-day mean expected return mu=0 10-day volatility sigma = £30 million.



Y : xth percentile of the std normal distribution

$$ES(T,X)=\mu+\sigma\frac{\exp\left(-\frac{Y^2}{2}\right)}{\sqrt{2\pi}\,(1-X)}$$

$\mu = 0$

$\sigma = £30m$

$ES(10d, 99\%) =$

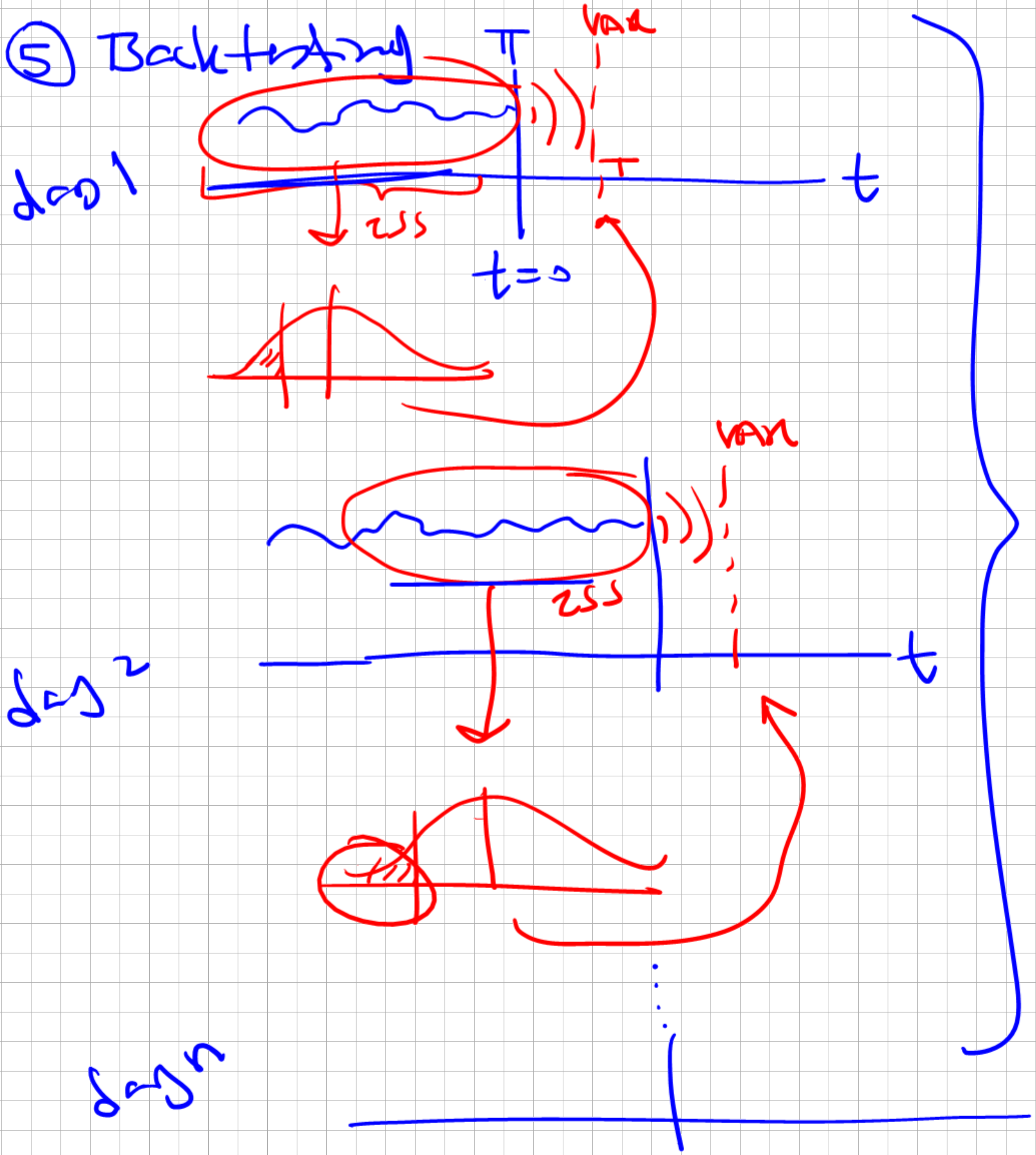
$$= 0 + (£30m) \underbrace{\frac{\exp\left(-\frac{\overset{\text{y: \# std deviations from mean}}{2.326^2}}{2}\right)}{\sqrt{2\pi}\,(1 - \underset{\text{significance level}}{0.99})}}_{2.66}$$

$= (30)(2.66)$

$= £79.8m$

Problem 5: Backtesting
For a portfolio where VAR(T,X), where T is the day time horizon and X the significance level, assume that T=1 day and X=99%, has been calculated daily for the last one hundred days, what is the theoretical probability that VAR will be exceeded m days during the period? If the empirical number of observed excedences in the period has been m=3, should we reject the VAR model?

5 Backtesting

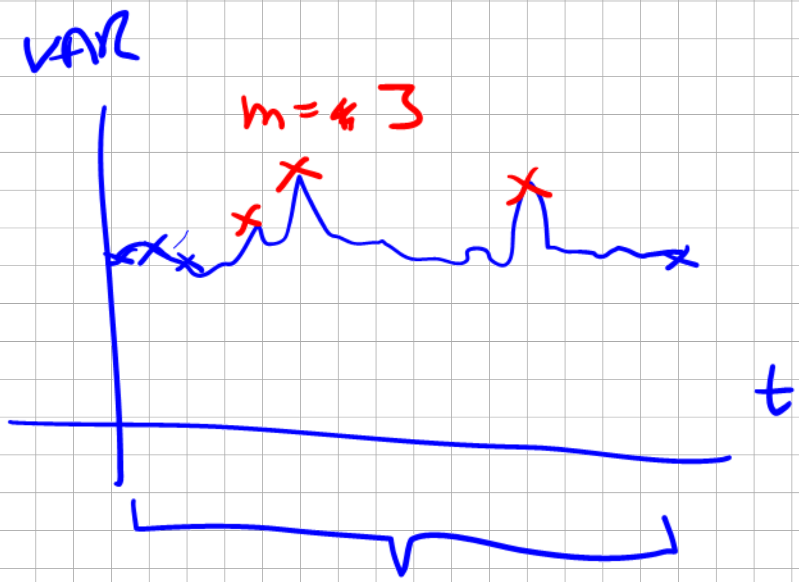


THEORETICAL

VAR(1d, X)

99%

1%



1%

theory 95% → m=1

empirical → m=3

3%

n: dag
m: exedances

Exedances VAR

$$\sum_{k=m}^{n} \frac{n!}{k!\,(n-k)!} p^{k} (1-p)^{n-k}$$

binomial model