

$$S_2 \left[D(0, T_1) P(T_1) \Delta t_1 + D(0, T_2) P(T_2) \Delta t_2 \right] = \underbrace{(1-R)}_L \left[D(0, T_1) \left(\cancel{P(T_0)}^1 - P(T_1) \right) + D(0, T_2) \left(P(T_1) - P(T_2) \right) \right]$$

$$S_2 D(0, T_1) P(T_1) \Delta t_1 + S_2 D(0, T_2) P(T_2) \Delta t_2 = L D(0, T_1) (1 - P(T_1)) + L D(0, T_2) \times (P(T_1) - P(T_2))$$

$$S_2 D(0, T_1) P(T_1) \Delta t_1 + S_2 D(0, T_2) \underline{P(T_2)} \Delta t_2 = L D(0, T_1) - L D(0, T_1) P(T_1) + L D(0, T_2) P(T_1) - L D(0, T_2) \underline{P(T_2)}$$

$$S_2 D(0, T_2) P(T_2) \Delta t_2 + L D(0, T_2) P(T_2) = L D(0, T_1) - L D(0, T_1) P(T_1) + L D(0, T_2) P(T_1) - S_2 D(0, T_1) P(T_1) \Delta t_1$$

$$P(T_2) \left[S_2 D(0, T_2) \Delta t_2 + L D(0, T_2) \right] = \dots$$

(A)

$$P(T_2) \left[D(0, T_2) (s_2 \Delta t_2 + L) \right] = D(0, T_1) \left(L - L P(T_1) - s_2 P(T_1) \Delta t_1 \right) + D(0, T_2) L P(T_1)$$

$$P(T_2) \left[D(0, T_2) (s_2 \Delta t_2 + L) \right] = D(0, T_1) \left[L - P(T_1) (L + s_2 \Delta t_1) \right] + D(0, T_2) L P(T_1)$$

$$P(T_2) = \frac{D(0, T_1) [L - P(T_1) (L + s_2 \Delta t_1)]}{D(0, T_2) (s_2 \Delta t_2 + L)} + \frac{\cancel{D(0, T_2)} L P(T_1)}{\cancel{D(0, T_2)} (s_2 \Delta t_2 + L)}$$

$$P(T_2) = \frac{D(0, T_1) [L - P(T_1) (L + s_2 \Delta t_1)]}{D(0, T_2) (L + s_2 \Delta t_2)} + \frac{P(T_1) L}{L + s_2 \Delta t_2}$$

$$\underline{\underline{N=3}}$$

$$P(T_N) = \frac{\sum_{n=1}^{N-1} D(0, T_n) [LP(T_{n-1}) - (L + \Delta t_n S_N) P(T_n)]}{D(0, T_N)(L + \Delta t_N S_N)} + \frac{P(T_{N-1}) L}{(L + \Delta t_N S_N)}$$

$$P(T_3) = \frac{\sum_{n=1}^2 D(0, T_n) [LP(T_{n-1}) - (L + \Delta t_n S_3) P(T_n)]}{D(0, T_3)(L + \Delta t_3 S_3)} + \frac{P(T_2) L}{(L + \Delta t_3 S_3)}$$

Note: The bootstrapping formulas above are implemented in the XLS file:
ImprovedBootstrappingExample.xls

PROBABILITY OF DEFAULT

Recovery Rate 50%

$$N=1$$

$$P(T_1) = \frac{L}{S_1 \Delta t_1 + L}$$

TIME (Years)	dt	MARKET SPREAD	DF	IMPLIED SURVIVAL PROB	first term	second term	third term	fourth term	sum	quotient	first term	last term
0				100.00%								
1	1	29.00	0.9803	99.42%								
2	1	39.00	0.9514	98.45%	- 0.0010				- 0.0010	0.4794	- 0.0020	0.9865
3	1	46.00	0.9159	97.26%	- 0.0017	0.0003			- 0.0013	0.4622	- 0.0029	0.9755
4	1	52.00	0.8756	95.88%	- 0.0022	- 0.0002	0.0008		- 0.0017	0.4424	- 0.0038	0.9626
5	1	57.00	0.8328	94.37%	- 0.0027	- 0.0007	0.0004	0.0013	- 0.0018	0.4211	- 0.0043	0.9480

$$N=5$$

$$P(T_5) = \sum_{n=1}^4 (\text{shaded squares})$$

$$+ \frac{P(T_4) L}{L + \Delta t_5 S_5}$$