$$M\underline{x} = \underline{y}$$

$$\underline{x} = M^{-1}\underline{y}$$

$$L(\underbrace{U\underline{x}}_{z}) = \underline{y}$$

$$U\underline{x} = \underline{z} \rightarrow \boxed{\underline{x}}$$

$$L\underline{z} = \underline{y} \rightarrow \underline{z}$$

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

$$\underbrace{\left[\frac{\partial}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2}{\partial S^2} + rS\frac{\partial}{\partial S} - r\right]}_{\text{Differential Operator}} V = 0$$

B-S operator

$$\frac{dy}{dx} \qquad \boxed{\frac{d}{dx}}\, y$$

operator

$$\mathcal{L} \equiv \frac{\partial}{\partial t} + \cdots\cdots\cdots - rV$$

$$\mathcal{L}V = 0$$

$$D \equiv \frac{\partial}{\partial S}$$

$$\frac{f(x+\delta x)-f(x)}{\delta x}$$

$\delta x = h$

$$\frac{\partial V}{\partial S} = \frac{V(S+h)-V(S-h)}{2h}$$

Centred 1st order derivative.

$$\frac{\partial^2 V}{\partial S^2} \equiv \frac{V(S-\delta S, t) - 2V(S, t) + V(S+\delta S, t)}{\delta S^2}$$

B.V.P

$$a(x)\,y'' + b(x)\,y' + c(x)\,y = g(x)$$

subject to $y(a) = \alpha$

$y(b) = \beta$

for some $a(x)$, $b(x)$, $c(x)$

$$\delta x = \frac{b-a}{N} \longrightarrow x_i = a + i\,\delta x$$

$$\therefore \quad y(x_i) = y_i$$

$$y(x+\delta x) = y + y'\delta x + \frac{1}{2!}y''\delta x^2 + O(\delta x^3) \quad ①$$

$$y(x-\delta x) = y - y'\delta x + \frac{1}{2!}y''\delta x^2 \; O(\delta x^3) \quad ②$$

① − ②

$$y(x+\delta x) - y(x-\delta x) = 2y'\delta x + O(\delta x^3)$$

$$y' = \frac{1}{2\delta x}\left[y(x+\delta x) - y(x-\delta x)\right] + O(\delta x^2)$$

$$y' \approx \frac{1}{2\delta x}\left[y_{i+1} - y_{i-1}\right]$$

① + ②

$$y(x+\delta x)+y(x-\delta x)=2y+y''\delta x^2+O(\delta x^4)$$

$$y''=\frac{y(x-\delta x)-2y+y(x+\delta x)}{\delta x^2}+O(\delta x^2)$$

$$y''\approx\frac{y_{i-1}-2y_i+y_{i+1}}{\delta x^2}$$

$$a(x)y'' + b(x)y' + c(x)y = g(x) \xrightarrow{\text{discrete}}$$

$y(a) = \alpha$

$y(b) = \beta$

$$\frac{a_i}{\delta x^2}\left[y_{i-1} - 2y_i + y_{i+1}\right] + \frac{b_i}{2\delta x}\left[y_{i+1} - y_{i-1}\right] + c_i y_i = g_i$$

Subject $y_0 = \alpha$

$y_N = \beta$

A_i B_i C_i

$$y_{i-1} : \boxed{\frac{a_i}{\delta x^2} - \frac{b_i}{2\delta x}} \; ; \; y_i : \boxed{-\frac{2a_i}{\delta x^2} + c_i} \; ; \; y_{i+1} : \boxed{\frac{a_i}{\delta x^2} + \frac{b_i}{2\delta x}}$$

$$A_i y_{i-1} + B_i y_i + C_i y_{i+1} = g_i \quad \text{for } 1 \leq i \leq N-1$$

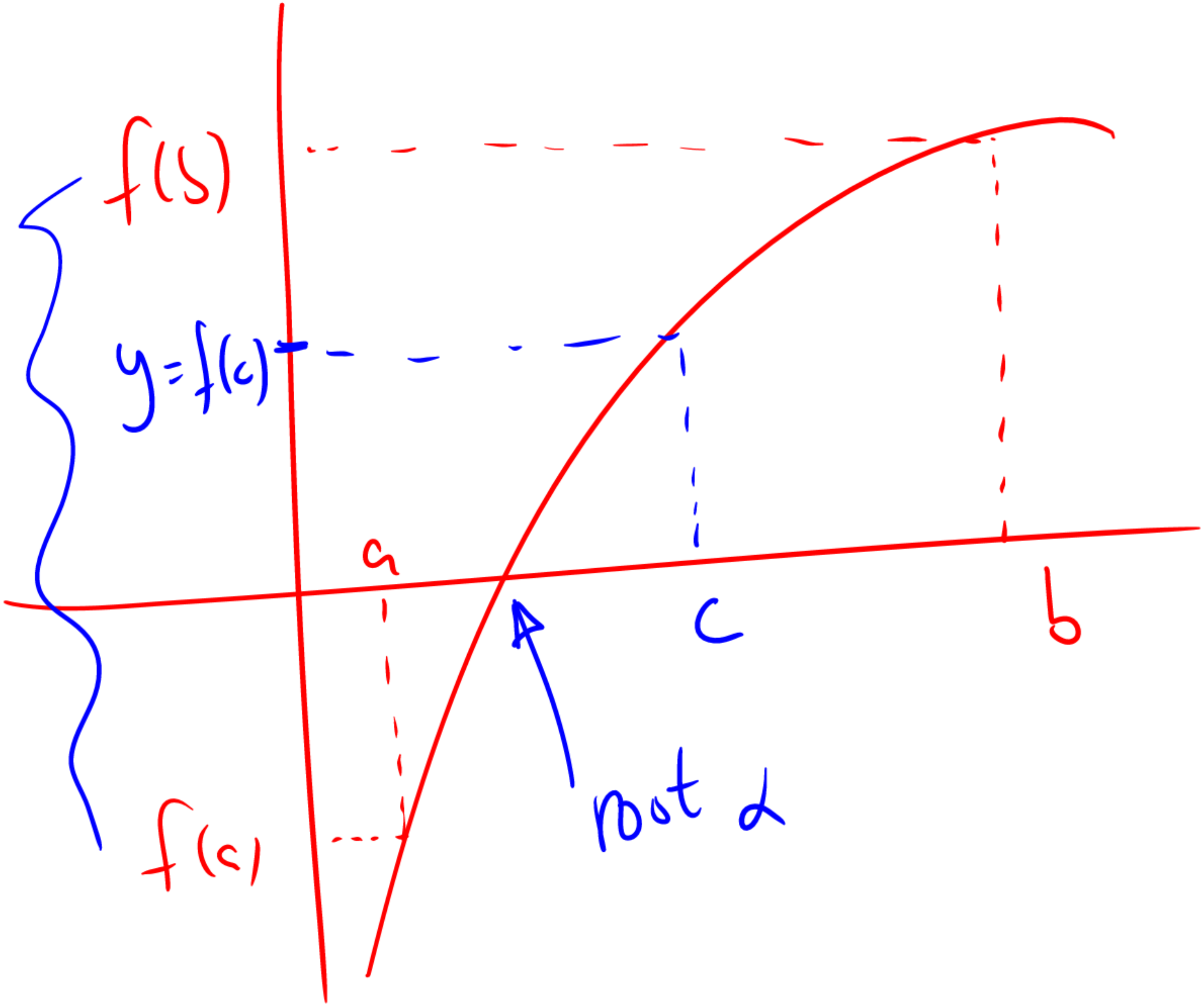
If V is the co-eff. matrix

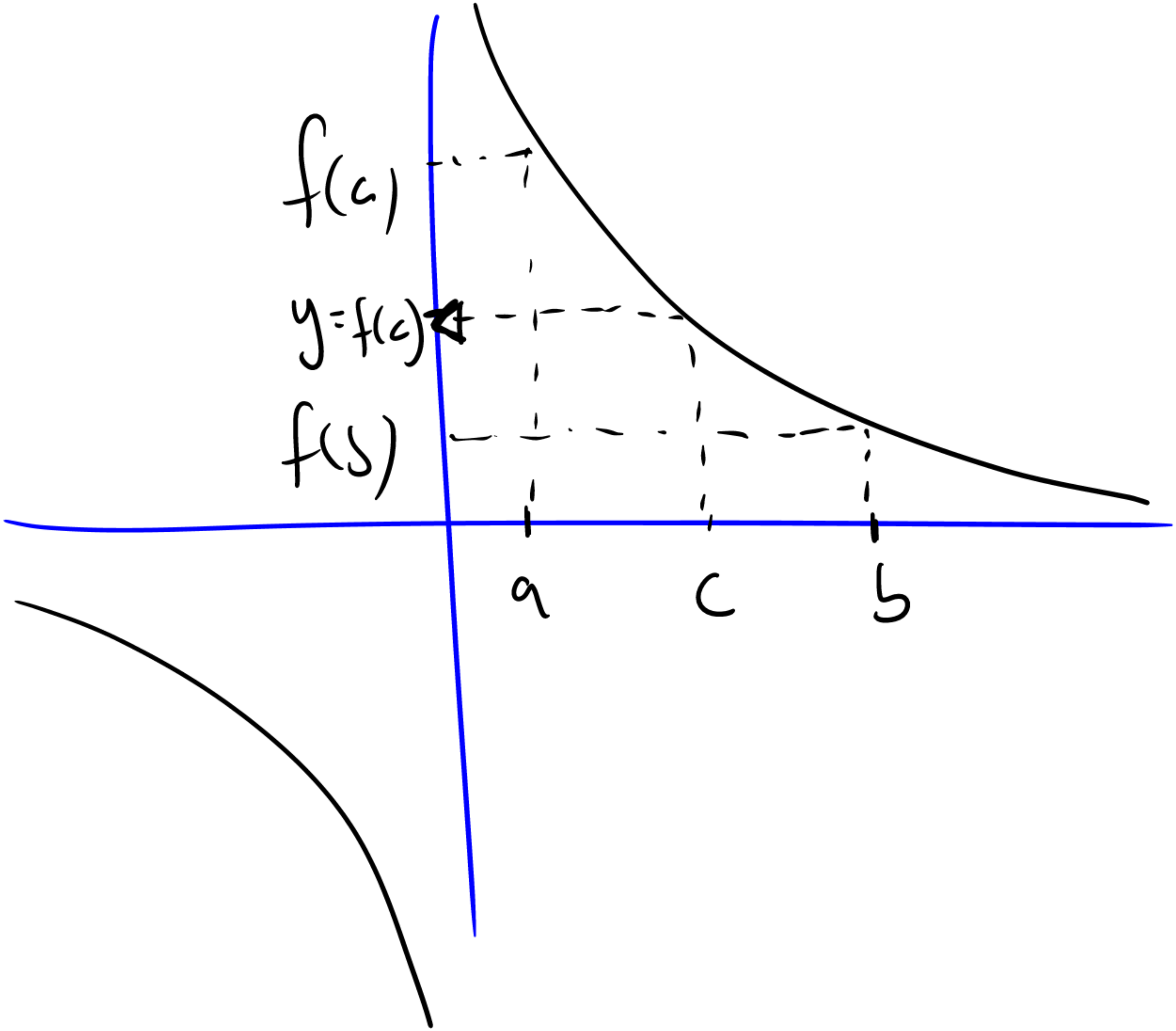
$$V\underline{y} = \underline{G} \longrightarrow \text{linear system for the unknown vector } \underline{y}$$

$$
\begin{bmatrix}
a & 0 & \cdots & & & & 0 \\
A_1 & B_1 & C_1 & 0 & \cdots & & 0 \\
0 & A_2 & B_2 & C_2 & 0 & \cdots & 0 \\
\vdots & & \ddots & \ddots & \ddots & & \vdots \\
 & & & & & & 0 \\
 & & & & A_{N-1} & B_{N-1} & C_{N-1} \\
0 & \cdots & & & & 0 & b
\end{bmatrix}
\begin{bmatrix}
y_0 \\ y_1 \\ y_2 \\ \vdots \\ y_{N-2} \\ y_{N-1} \\ y_N
\end{bmatrix}
=
\begin{bmatrix}
\alpha \\ g_1 \\ g_2 \\ \vdots \\ g_{N-1} \\ \beta
\end{bmatrix}
$$

$$V = LU$$

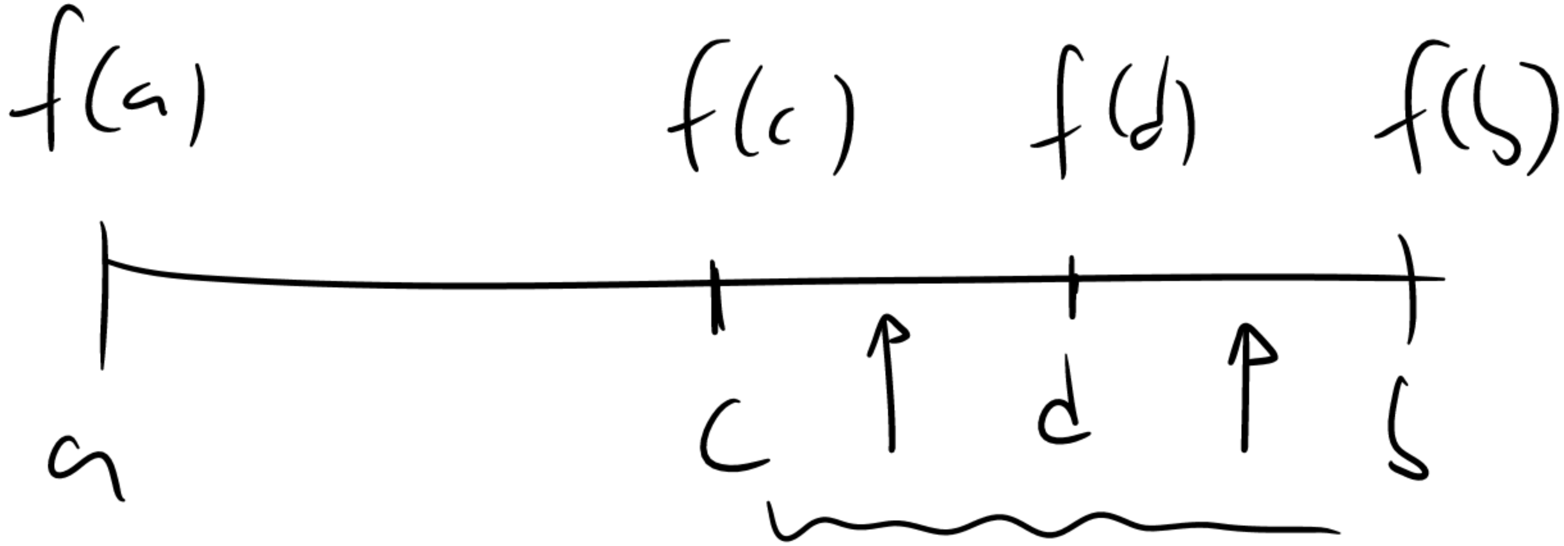
$f(a)f(b)<0$





f(a)
y=f(c)
f(b)
a
c
b

$f(a)f(b) < 0 \Rightarrow \exists\ \alpha$ s.t $\alpha \in [a,b]$



$f(a)f(c) > 0$

$f(c)f(b) < 0$

$$x_2 = \frac{x_0 f_1 - f_0 x_1}{f_1 - f_0} = \frac{(1 \times 5) - (2 \times -1)}{5 - (-1)} = \frac{7}{6} = 1.166667$$

$f(x_2) = f_2 = -0.577 \quad f_2 f_1 < 0 \quad \therefore x_3 \in [x_2, x_1]$

i.e. root $\in [1.167, 2]$ Reapply formula

$$x_3 = \frac{1.167 f(2) - 2f(1.167)}{f(2) - f(1.167)} = 1.253 \quad 2^{nd} \text{ approx}$$

4^{th} approx $= 1.311 \rightarrow f(1.311) = -0.06$

$$x_{n+1} = 2\ln(2e - x_n)$$

$$f(x) = x^2 - e^x \log x$$

$$f(x) = \boxed{x^2 \quad e^x \log x}$$

Solving $f(x) = 0$ $\quad x^2 = e^x \log x$

$$x_{n+1} = \sqrt{e^x \log x_n}$$

$$x_{n+1} = g(x_n) = \sqrt{e^x \log x}$$

i	0	1	2	3
x_i				
y_i				

Construct $l_i(x)$

$$l_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}$$

$$l_3(x) = \frac{(x-x_0)\cdots(x-x_2)}{(x_3-x_0)\cdots(x_3-x_2)}$$

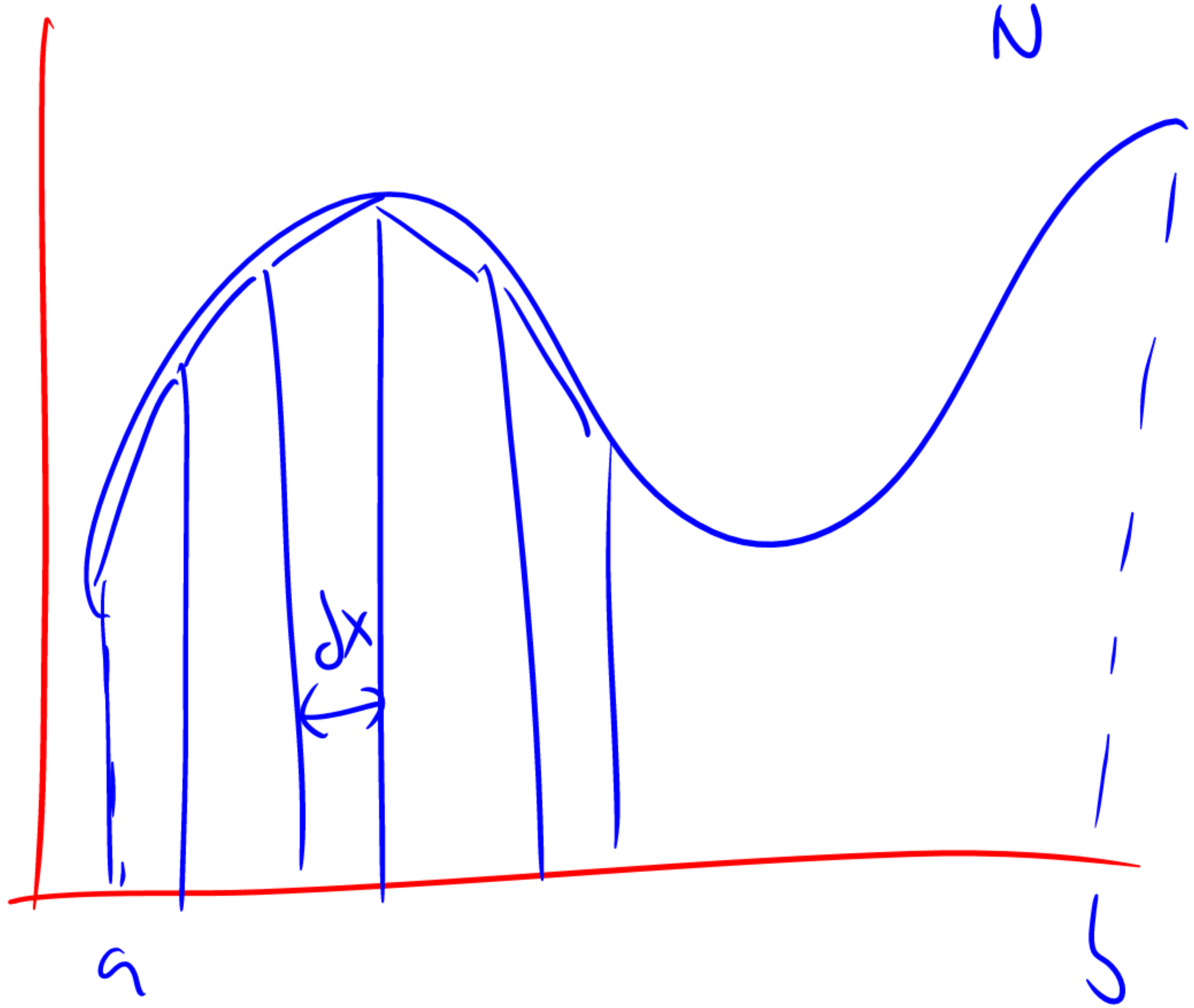
$$\boxed{P_3(x) = \sum_{i=0}^{3} y_i \, l_i(x)}$$

1.150251198

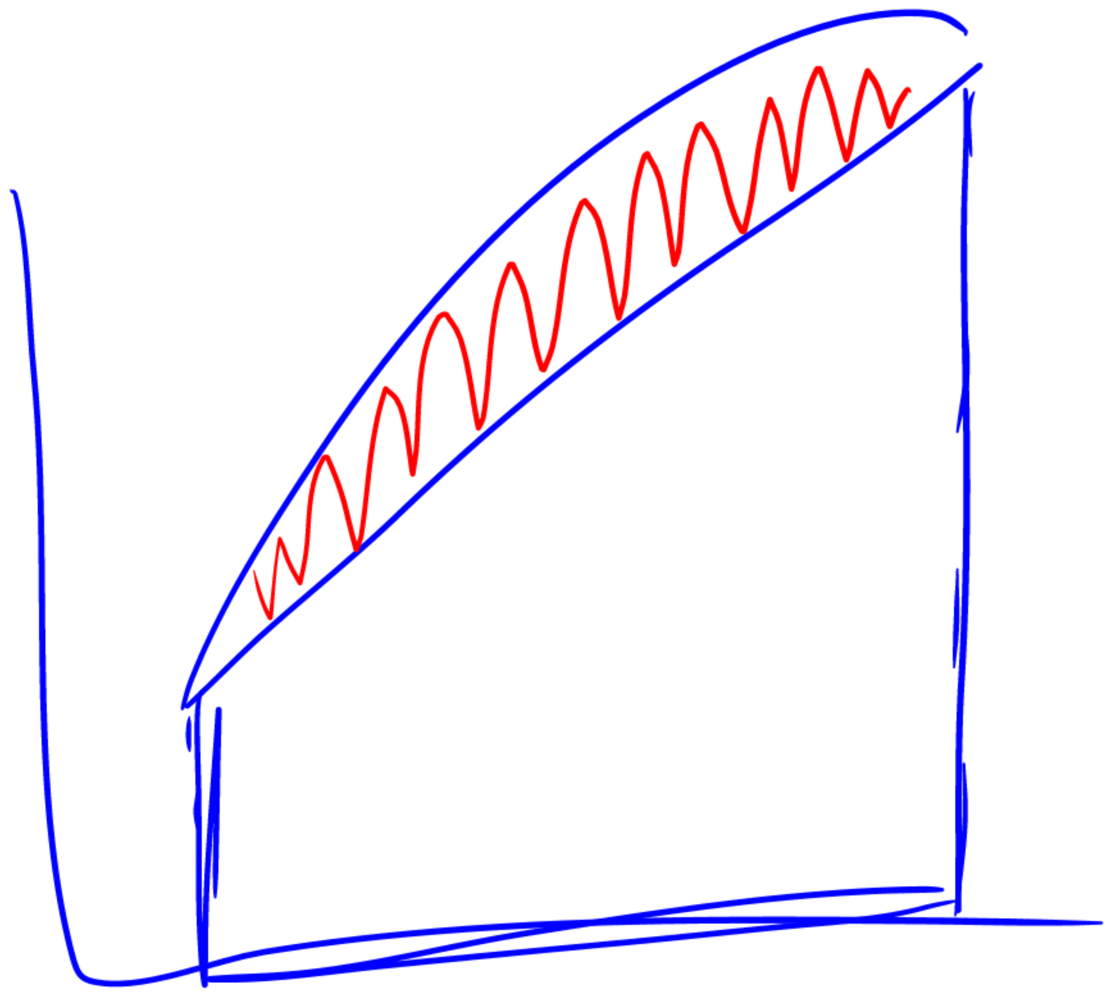
$e^{0.14} = ?$

$\boxed{0.18276)))}$ No. 4

$$\delta x_i = \frac{b-a}{N} \qquad x_i = a + i\,\delta x$$



$$\lim_{N \to \infty} \longrightarrow \int_a^b f(x)\,dx$$



$$\int_0^{\pi/4} \sin x \, dx = \frac{h}{2}\left(f(0) + f(\pi/4)\right) = 0.27768$$

$$+$$

$$\int_{\pi/4}^{\pi/2} \sin x \, dx = \frac{h}{2}\left[f(\pi/4) + f(\pi/2)\right] = 0.670379$$

$$\int_0^{\pi/2} \sin x \, dx = 0.94806$$

Actual Error $= 1 - 0.94806 = 0.05194$

$h = \pi/2$ 1 strip

$h = \pi/4$ 2 strips

0 90

① ② ③

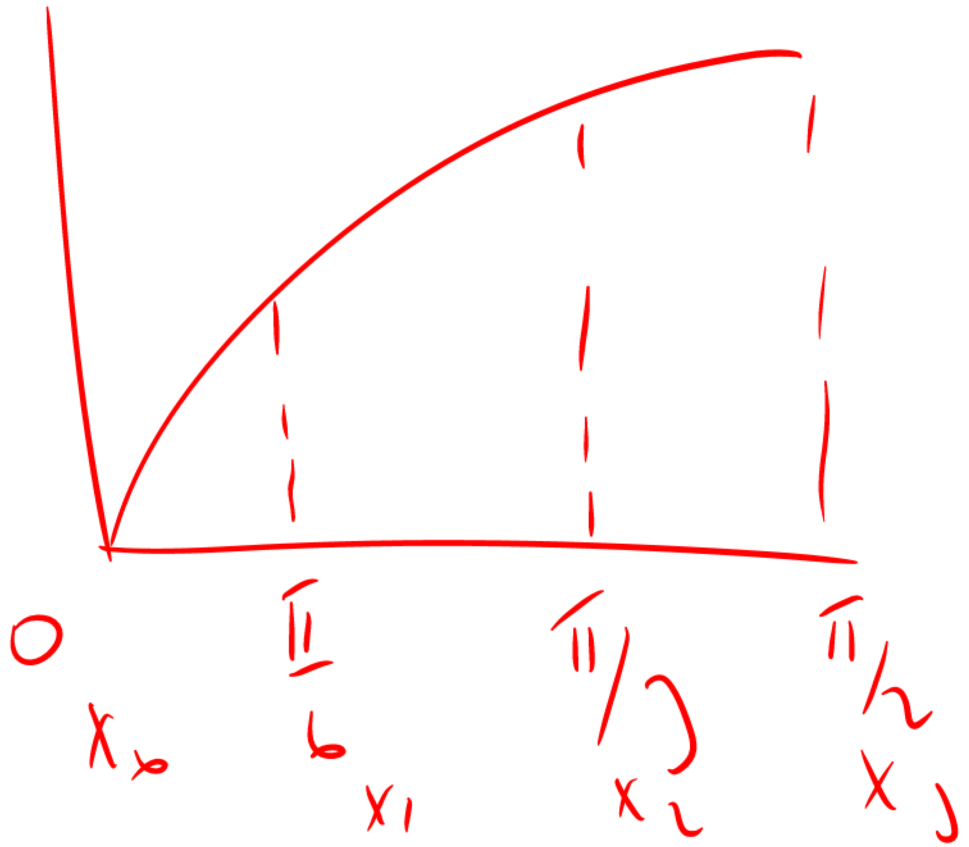
$0 \quad \frac{\pi}{6} \quad \frac{\pi}{3} \quad \frac{\pi}{2}$

$$\int_0^{\pi/2} \sin x \, dx = 1$$

$h = \frac{\pi}{6}$

$h = \frac{\pi/2}{3} = \frac{\pi}{6}$

$$\int_0^{\pi/2} \sin x \, dx = \frac{\pi/6}{2}\left[f_0 + f_3 + 2\left[f_1 + f_2\right]\right]$$



0.977

0.023

then $\exists$ a real value c such that

$a < c < b$ and

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{OR}$$

$$f(b) - f(a) = (b-a) f'(a) \qquad \boxed{\text{M.V.T}}$$

$$\frac{1}{\sqrt{2\pi}} \boxed{\int_{-\infty}^{x} e^{-\frac{1}{2}x^2}\, dx}$$

$$\int_{\infty} \frac{1}{x}$$

$$\int_{-\infty}^{3} x^2 + dx$$

$$a\boxed{x_1} + b x_2 + c x_3 = A \rightarrow x_1^{(k+1)} = \frac{1}{a}\left(-b x_2^{(k)} - c x_3^{(k)}\right) + \frac{A}{a}$$

$$d x_1 + e\boxed{x_2} + f x_3 = B \quad x_2^{(k+1)} = \frac{1}{e}\left[-d x_1^{(k)} - f x_3^{(k)}\right] + \frac{B}{e}$$

$$g x_1 + h x_2 + i x_3 = C \quad x_3^{(k+1)} = \frac{1}{i}\left[-g x_1^{(k)} - h x_2^{(k)}\right] + \frac{C}{i}$$

$$k = 0 \quad \underline{x}^{(0)} = \left\{ x_1^{(0)}, x_2^{(0)}, x_3^{(0)} \right]$$

$$\underline{x}^{(1)} = \left\{ x_1^{(1)}, x_2^{(1)}, x_3^{(1)} \right]$$

$\underline{x}^{(n)}$ and $\underline{x}^{(n+1)}$

$$\boxed{\frac{\| \underline{X}^{(k+1)} - \underline{X}^{(k)} \|_\infty}{\| \underline{X}^{(k)} \|_\infty}} < \underset{\sim\sim\sim\sim\sim\sim}{\text{tolerance}}$$

$$\| \cdot \| \qquad \| \cdot \|_\infty$$

cm^2

Seydel Computational Finance