Credit Risk: Structural Models

Abstract

In this lecture we will study credit risk and its mathematical modelling. After reviewing the main concepts associated with credit and credit risk, we will follow a guided tour of the main modelling techniques existing in the literature. We will then focus on a particular class of credit risk models known as structural models. We will conclude by creating a structural model of credit risk in an practical Workshop.

Instructor:

Alonso Peña, PhD, CQF

SDA Professor
Banking and Insurance Department
SDA Bocconi School of Management
Milan, Italy

alonso.pena@sdabocconi.it

Contents of this Lecture

Part 1

Introduction: From Credit to Credit Risk

1.1 The Word Credit

1.2 Some History

1.3 Types of Credit

1.4 Bonds & Risky Bonds

1.5 Defaults: Who? Why? How?

1.6 Credit Derivatives

1.7 What is Credit Risk?

The Word Credit: Definition

credit, *noun*, the facility of being able to obtain goods or services before payment, based on the trust that payment will be made in the future.

From the Latin *creditum*, from *credere* believe, trust.

Oxford English Dictionary (2007)

The Word Credit: Qualitative Aspects

"...based on the trust that payment will be made in the future."

- trust: how do we quantify confidence, faith?

- payment: how much will actually be paid?

- future: how do we estimate the time of payment?

The Word Credit: Quantitative Aspects

"...based on the trust that payment will be made in the future."

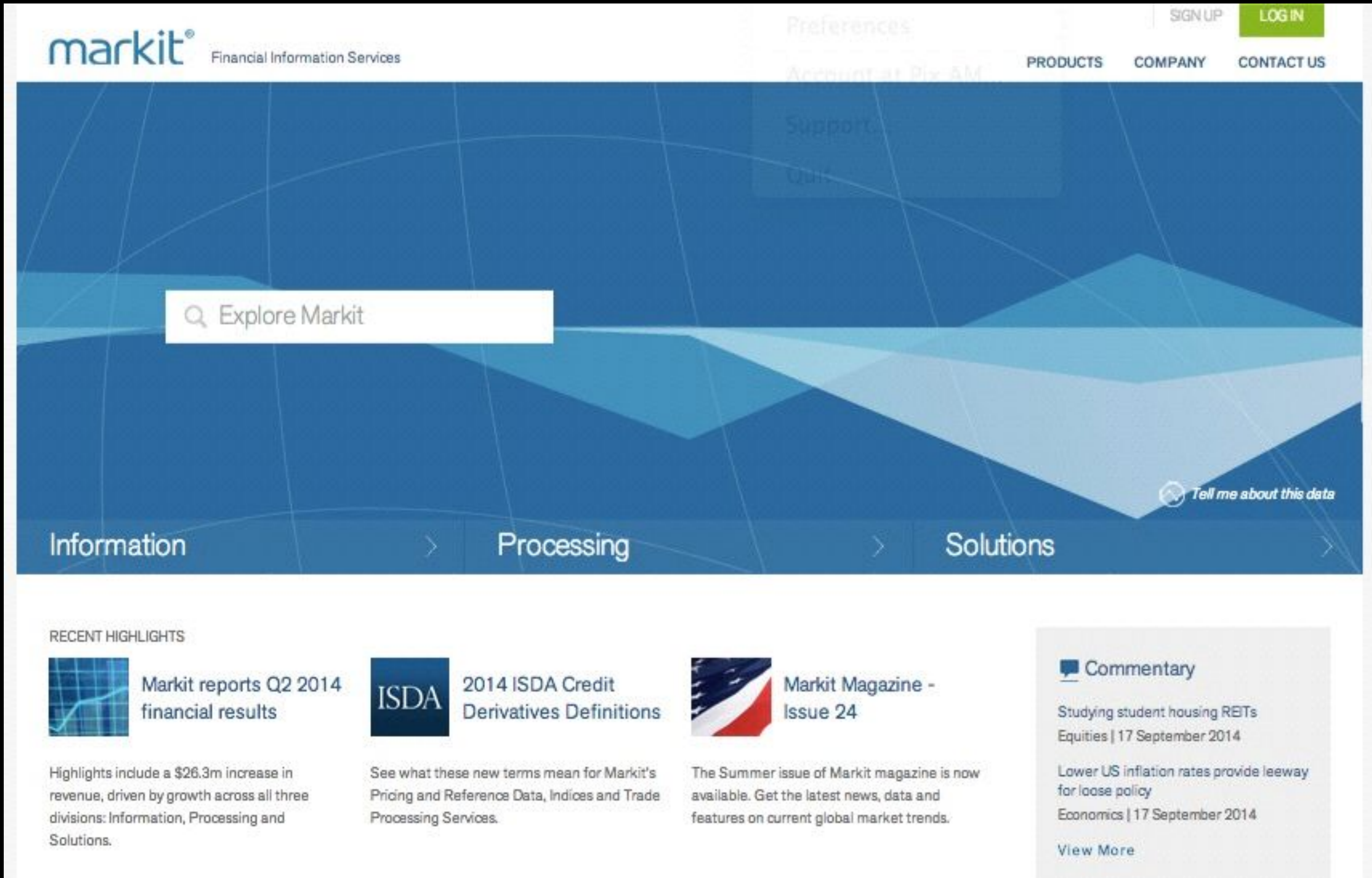
- trust: Probability of Default $\rightarrow P(T)$

- payment: Recovery Rate $\rightarrow R$

- future: Default time $\rightarrow \tau$

In order to go forward, we need to go backward and see how these issues have been approached in the past...

The credit markets


markit
Financial Information Services
SIGN UP
LOG IN
PRODUCTS
COMPANY
CONTACT US
Explore Markit
Tell me about this data
Information
Processing
Solutions
RECENT HIGHLIGHTS
Markit reports Q2 2014 financial results
Highlights include a $26.3m increase in revenue, driven by growth across all three divisions: Information, Processing and Solutions.
ISDA
2014 ISDA Credit Derivatives Definitions
See what these new terms mean for Markit's Pricing and Reference Data, Indices and Trade Processing Services.
Markit Magazine - Issue 24
The Summer issue of Markit magazine is now available. Get the latest news, data and features on current global market trends.
Commentary
Studying student housing REITs
Equities | 17 September 2014
Lower US inflation rates provide leeway for loose policy
Economics | 17 September 2014
View More

DERIVATIVES | SOVEREIGN CREDIT-DEFAULT SWAPS

CNBC

Search Quotes, News & Video GO

Symbol	Price		Change	%Change
* AUT CDS 5YR	25.135	▼	-0.865	-3.33%
* BEL CDS 5YR	35.55	▼	-1.45	-3.92%
* CHN CDS 5YR	138.00	▲	16.00	13.11%
* DEN CDS 5YR	16.675	▼	-0.325	-1.91%
* DUBAI CDS 5YR	197.00	---	UNCH	0%
* EGY CDS 5YR	390.50	---	UNCH	0%
* FIN CDS 5YR	19.675	▼	-0.325	-1.63%
* FRA CDS 5YR	29.80	▼	-1.20	-3.87%
* GER CDS 5YR	13.13	▲	0.13	1.00%
* GRE CDS 5YR	1114.08	---	UNCH	0%
* HUN CDS 5YR	162.73	▲	0.73	0.45%
* INA CDS 5YR	221.00	▼	-2.00	-0.90%
* IRE CDS 5YR	46.61	▼	-0.39	-0.83%
* ITA CDS 5YR	103.00	---	UNCH	0%
* JPN CDS 5YR	43.50	---	UNCH	0%
* KOR CDS 5YR	60.645	▼	-0.355	-0.58%

http://www.cnbc.com/sovereign-credit-default-swaps/

markit

Indices | Calculators

Indices :: Total Return Indices :: Credit Volatility Indices :: CDS Bond Basis Indices

Indices

Credit Index Annexes

Date (e.g. 01Jan05) 28Oct15 Family Show all Sector Show all Category Show all Term Show all

On Run only

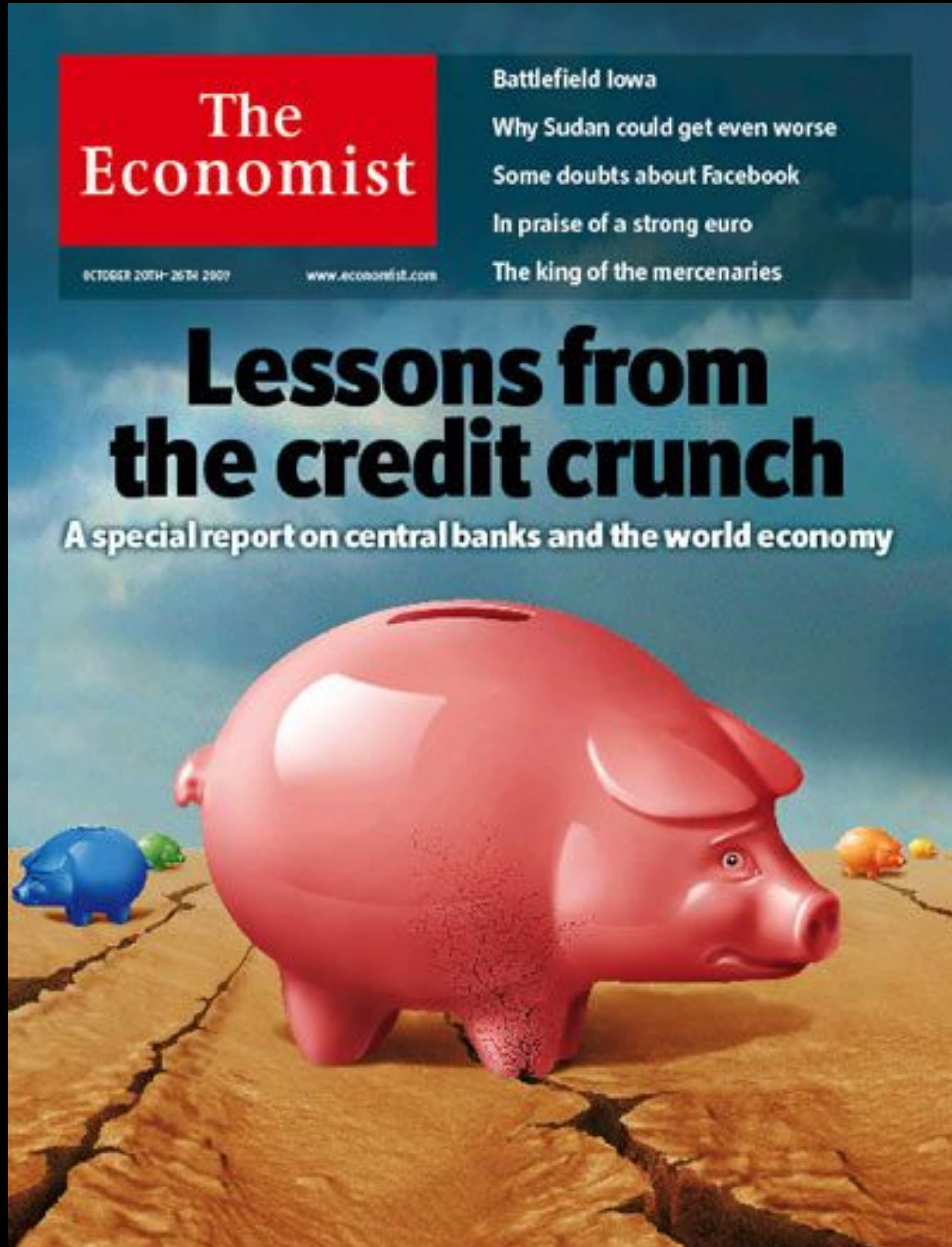
Submit | Reset to defaults

Pricing for 28-Oct-15 On The Run indices (Default list)

Index	Series	Version	Term	RED Id	On Run	Coupon	Maturity	Date	Time	Comp Price	Theo Price	Comp Spread	Theo Spread	Mid Day Spread Change
CDX.NA.HY	25	1	5Y	2I65BRNA4	Y	500	20Dec20	28Oct15	NY1930	103.219%	101.813%	426.06	457.90	
CDX.NA.HY.B	25	1	5Y	2I65BSGJ1	Y	500	20Dec20	28Oct15	NY1930	103.375%	102.663%	422.57	438.57	
CDX.NA.HY.BB	25	1	5Y	2I65BVCL3	Y	500	20Dec20	28Oct15			106.780%		348.45	
CDX.EM	24	2	5Y	2I65BZCJ9	Y	100	20Dec20	28Oct15	NY1930	89.840%	89.761%	323.38	325.27	
CDX.EM.DIVERSIFIED	12	3	5Y	2165EKAW9	Y	100	20Dec15	28Oct15			99.800%		235.96	
CDX.NA.IG	25	1	5Y	2I65BYDJ1	Y	100	20Dec20	28Oct15	NY1930	101.104%	100.639%	77.50	86.93	
CDX.NA.IG.HVOL	25	1	5Y	2I65B3CB7	Y	100	20Dec20	28Oct15			94.387%		221.53	
CDX.NA.XO	11	3	10Y	1D764IBG1	Y	340	20Dec18	28Oct15			103.061%		236.51	
iTraxx Europe	24	1	5Y	2I666VBE4	Y	100	20Dec20	28Oct15	LN1930	101.446%	101.132%	71.37	77.52	
iTraxx Europe HiVol	20	1	5Y	2I667LAX4	Y	100	20Dec18	28Oct15	LN1930	100.698%	100.567%	77.67	81.86	
iTraxx Europe Crossover	24	1	5Y	2I667KEA2	Y	500	20Dec20	28Oct15	LN1930	109.313%	108.243%	297.13	318.82	
iTraxx Japan	24	1	5Y	2I668HBX1	Y	100	20Dec20	28Oct15	LN1200	101.738%	101.650%	65.71	67.41	
iTraxx Australia	24	1	5Y	2I668IAX0	Y	100	20Dec20	28Oct15	LN1200	99.197%	99.320%	116.65	114.07	
iTraxx Asia ex-Japan IG	24	1	5Y	4ABCAMAQ9	Y	100	20Dec20	28Oct15	LN1200	98.591%	99.057%	129.35	119.57	
iTraxx SDI-75	4	3	10Y	4ABCAIAK1	Y	35	20Jun17	28Oct15			99.764%		49.38	
iTraxx SovX Western Europe	8	1	5Y	5C769MAO9	Y	100	20Dec17	28Oct15	LN1930	101.771%	101.771%	17.66	17.68	
iTraxx SovX CEEMEA	11	2	5Y	5C769NAV1	Y	100	20Jun19	28Oct15	LN1930	97.825%	97.810%	162.29	162.72	
iTraxx SovX Global Liquid Investment Grade	14	1	5Y	5C769KAQ8	Y	100	20Dec20	28Oct15			99.007%		120.62	
iTraxx SovX G7	14	1	5Y	5C769JAN8	Y	100	20Dec20	28Oct15			103.226%		35.42	
iTraxx SovX BRIC	14	1	5Y	4ABCAPAL3	Y	100	20Dec20	28Oct15			93.515%		241.58	
iTraxx SovX Asia Pacific	10	1	5Y	4ABCANAH7	Y	100	20Dec18	28Oct15			100.820%		73.43	
LCDX.NA	21	3	5Y	5F199GJA6	Y	250	20Dec18	28Oct15	N1600	103.333%		137.73		
iTraxx LevX Senior	6	18	5Y	4ABCAJMK6	Y	500	20Jun15	20Jan15	L1630	100.250%				
iTraxx LevX Subordinated	3	21	5Y	4ABCAKFH8	Y	1,200	20Dec13							
MCDX.NA	25	1	5Y	5A79DPAP2	Y	100	20Dec20	28Oct15	NY1930	100.034%		99.25		

markit

http://www.markit.com/markit.jsp


The Economist
OCTOBER 20TH-26TH 2007
www.economist.com
Battlefield Iowa
Why Sudan could get even worse
Some doubts about Facebook
In praise of a strong euro
The king of the mercenaries
Lessons from the credit crunch
A special report on central banks and the world economy


ΓΣΕΕ
ΠΑΛΙ ΕΜΕΙΣ


REVOLUTION
WE ARE THE 99

ISDA Safe, Efficient Markets

markit

ISDA CDS Standard Model℠

ISDA CDS Standard Model

The ISDA CDS Standard Model is a source code for CDS calculations and can be downloaded freely through this website.

The source code is copyright of ISDA and available under an Open Source license.

Background

As the CDS market evolves to trade single name contracts with a fixed coupon and upfront payment, it is critical for CDS investors to match the upfront payment amounts and to be able to translate upfront quotations to spread quotations and vice versa in a standardized manner.

One of the primary goals in making the code available is to enhance transparency and to optimize use of standard technology for CDS pricing. Implementing the ISDA CDS Standard Model and using the agreed standard input parameters will allow CDS market participants to tie out calculations and thus improve consistency and reduce operational differences downstream.

Download

Source Code and Excel add-in

Related Links

- Documentation
- ISDA Standard Model Settings for Fee Computations
- Test Grids

Contact us | Secure area

LCH.CLEARNET Trusted Risk Management for Cleared Markets

Home | Asset Classes | Risk & Collateral Management | Rules & Regulations | Membership | News & Events | About Us

- Listed Commodities
- OTC Commodities & Freight
- **OTC CREDIT DEFAULT SWAPS**
  - Products
  - How it Works
  - Fees
  - Volumes
  - Pricing Data
  - Team Contacts
- OTC Foreign Exchange
- OTC Interest Rate Derivatives

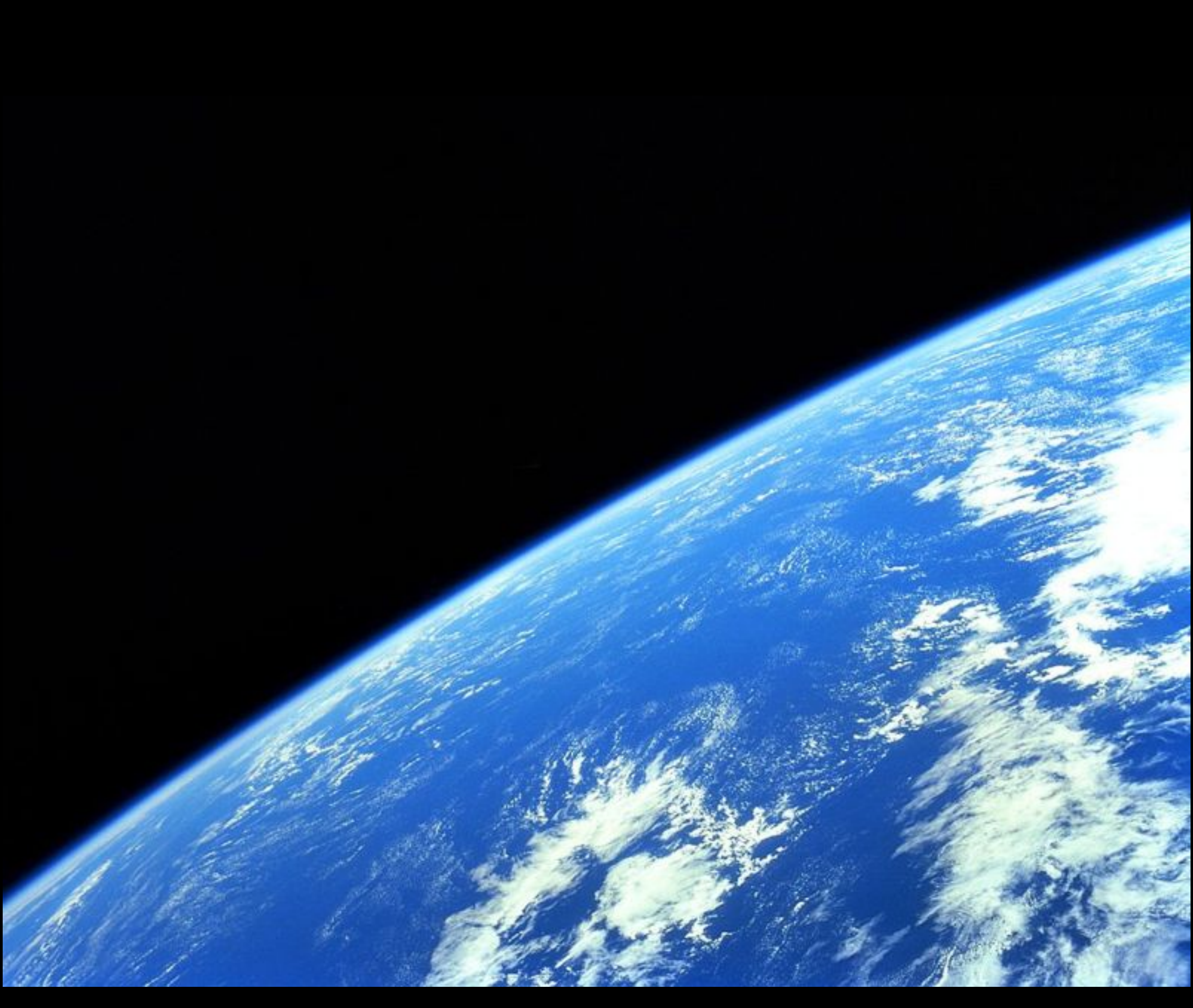
Why CDSClear?

RELATED INFORMATION

- Acceptable Collateral
- CDS Clearing Rulebook
- FCM CDS Clearing Rulebook
- CDS Clearing Supplement
- S.A. Treasury Fees
- LCH.Clearnet S.A. Membership Criteria
- CDSClear Portal
- EMIR Account Structures

WHAT'S NEW

CDSClear Publishes Whitepaper on 2014 ISDA Credit Derivative Definitions



Some History

Hammurabi's Code (circa 1750 BC)

Interest was rarely charged on advances by the temple or wealthy landowners for pressing needs. Merchants (and even temples, in some cases) made ordinary business loans, charging from 20 percent, for loans on silver, and 33 percent, for loans on grain.

"If any one **fails to meet a claim for debt**, and sell himself, his wife, his son, and daughter for money or gives them away to forced labor: they shall work for three years in the house of the man who bought them and in the fourth year they shall be set free."

"If any one **owes a debt for a loan**, and a storm prostrates the grain, or the harvest fail, or the grain does not grow for lack of water; in that year he need not give his creditor any grain, he washes his debt-tablet in water and pays no rent for this year."

Some History

Leviticus 25:854 The Old Testament prescribes that one "Holy Year" or "Jubilee Year"' should take place every 50 years, when all debts are eliminated among Jews and all debt-slaves are freed, due to the heavenly command.

Ancient Rome (326 BC)"Nexum" was a debt bondage contract in Ancient Rome where the debtor pledged his person as collateral should he default on his loan. It was abolished by the *Lex Poetelia Papiria* in 326 BC.

Genghis Khan (1162-1227 AD) The Yassa (the principal law) of the Mongol Empire contained a provision that mandated the death penalty for anyone who became bankrupt three times.

Some History

Medieval Europe The word bankruptcy is formed from the ancient Latin *bancus* (a bench or table), and *ruptus* (broken). A "bank" originally referred to a bench, which the first bankers had in the public places, in markets, fairs, etc. Hence, when a banker failed, he broke his bank, to advertise that he was no longer in business.

Usury (from the Latin *usura* "interest") originally meant the charging of interest on loans. This included charging a fee for the use of money. After interest became acceptable, usury came to mean the interest above the rate allowed by law. In common usage today, the word means the charging of unreasonable or relatively high rates of interest. The term is largely derived from Abrahamic religious principles; Riba is the corresponding Arabic term and ribbit the Hebrew one.

Some History

Spain (1527-1598 AD) Philip II of Spain had to declare four state bankruptcies in 1557, 1560, 1575 and 1596. Spain became the first sovereign nationin history to declare bankruptcy.

Britain (1800's) A debtors' prison is a prison for those who are unable to pay a debt. Prior to the mid 19th century debtors' prisons were a common way to deal with unpaid debt.The father of Charles Dickens was sent to one of these prisons (Marshalsea Prison), which were often described in Dickens' novels.

Moody's Investors Service (1900) John Moody & Company published *Moody's Manual of Industrial and Miscellaneous Securities* which provided information and statistics on stocks and bonds of financial institutions, government agencies, manufacturing, mining, utilities, and food companies. By 1924, Moody's ratings covered nearly 100 percent of the US bond market.

Types of Credit

Loans. In the most typical case, the lender (a bank, a company, or an individual) gives money or property to a borrower. The borrower agrees to return the fund or property at some future point in time, and for the use of the money, the borrower has to pay the lender a fee or interest.

Currency. There is another much more common type of credit that we all enjoy often without thinking about it: currency. We all seem to agree, for instance, that a 100 USD bill is worth exactly 100 USD. However, the value of the bill is backed by the country's credit.

Bonds. Debt obligations are a way for both companies and governments to raise money. The bond issuer promise to return the initial sum or principal at a determined future date. The issuer also normally agrees to pay interest or a coupon at a fixed or floating rate on various dates.

Bonds & Risky Bonds

"Risk-free" Zero-coupon bond: no credit spread

$$Z(t,T) = e^{-R(t,T)(T-t)}$$

"Risky" Zero-coupon bond: with credit spread

$$\tilde{Z}(t,T) = e^{-[R(t,T)+S(t,T)](T-t)}$$

$$\tilde{Z}(t,T) = Z(t,T)e^{-S(t,T)(T-t)}$$

Rating Agencies: NRSRO

A Nationally Recognized Statistical Rating Organization is a credit rating agency which issues credit ratings that the U.S. Securities and Exchange Commission (SEC) permits other financial firms to use for certain regulatory purposes. As of 25 September 2008, ten organizations were designated as NRSROs. The three most important being:

- Moody's Investors Service
- Standard & Poor's
- Fitch Ratings

Rating Agencies: the big three

Moody's Investors Service was founded in 1909 by John Moody, beginning with Analyses of Railroad Investments, "a book about railroad securities, using letter grades to assess their risk".

Standard & Poor's Standard & Poor's traces its history back to 1860, with the publication by Henry Varnum Poor of History of Railroads and Canals in the United States. In 1906 Luther Lee Blake founded the Standard Statistics Bureau, with the view to providing financial information on non-railroad companies.

Fitch Ratings The firm was founded by John Knowles Fitch on 1913 in New York as the Fitch Publishing Company. Fitch is the smallest of the "big three" NRSROs, frequently positions itself as a "tie-breaker" when the other two agencies have ratings similar, but not equal.

Rating Agencies: Recovery Rates

Senior secured	54.44
Senior unsecured	38.39
Senior subordinated	32.85
Subordinated	31.61
Junior subordinated	24.47

Recovery rates on corporate bonds as a percentage of face value, 1982-2003.

Source: Moody's.

Rating Agencies: Ratings

Long-term obligation ratings (source Moody's)

Aaa	"highest quality", "smallest degree of risk".
Aa	"very low credit risk", "susceptibility to long-term risks
A	"upper-medium grade", subject to "low credit risk"
Baa	"moderate credit risk", "protective elements may be lacking"
Ba	"questionable credit quality"
B	"subject to high credit risk", "poor credit quality"
C	"poor standing, subject to very high credit risk"

Investment grade ratings (Aaa, Aa, A, Baa), and Non-investment grade (Ba, B, C).

Rating Agencies: Default Probabilities

	1Y	2Y	3Y	4Y	5Y
	0.000	0.000	0.000	0.026	0.099
Aa	0.008	0.019	0.042	0.106	0.177
A	0.021	0.095	0.220	0.344	0.472
Baa	0.181	0.506	0.930	1.434	1.938
Ba	1.205	3.219	5.568	7.958	10.215
B	5.236	11.296	17.043	22.054	26.794
C	19.476	30.494	39.717	46.904	52.622

Average cumulative default rates (in percent), 1970-2006.

Source: Moody's.

Credit Derivatives

Credit derivatives are a derivative security that has a payoff which is conditioned on the occurrence of a *credit event*.

The credit event is defined with respect to a *reference credit*, and the *reference credit assets* issued by the reference credit.

If the credit event has occurred, the *default payment* has to be made by one of the counterparties. Besides the default payment, a credit derivative can have further payoffs that are not default contingent.

Credit Derivatives

The market for credit derivatives was created in the early 1990s in London and New York. The largest share in the market is taken up by the credit default swaps (CDSs) and their variations such as first-to-default swaps (FtDs). The second largest group are portfolio-related credit derivatives like collateralized loan obligations (CLOs), portfolio tranche protection and synthetic collateralized debt obligations (CDOs). Finally, there are more exotic credit derivatives like credit spread options and hybrid instruments.

Credit Derivatives

Credit default swap (CDS)

Total return swap (TRS)

Constant maturity credit default swap (CMCDS)

First to Default Credit Default Swap (F2D)

Credit Spread Option (CSO)

CDS index products (iTraxx, CDX)

Credit linked note (CLN)

Collateralized Debt Obligation (CDO)

Constant Proportion Debt Obligation (CPDO)

Constant Proportion Portfolio Insurance (CPPI)

Defaults: Who?

Individuals. Excessive credit card use, poor investment management, and lowered real estate value are common causes for bringing individuals to the brink of personal bankruptcy.

Companies. Just like individuals, companies file for bankruptcy when their costs exceed their revenues and available capital. One of the sure signs of an upcoming corporate default is when a firm stops paying coupons on the bonds they have issued. Unable to meet this financial obligation, the actual default is not far away.

Countries. For the most part, bonds issued by governments are immune from. However, sometimes countries do occasionally default on their debt. Examples: Mexico (1914, 1982), Russia (1998), Turkey (2001), and Argentina (2002).

Defaults: Why?

There are several so-called credit events that might lead to default. Typical credit events include:

Bankruptcy, when a company or organization is dissolved or becomes insolvent and is unable to pay its debts.

Failure to pay within a reasonable amount of time after the due date and after reminders from the receiver.

Significant **downgrading** of credit rating.

Credit event after **merger**, which renders the new merged entity financially weaker than the original entity.

Default: How?

Although a country can default on selected loans without declaring bankruptcy (as Argentina did in 2002) most companies that default on a bond almost automatically go into full bankruptcy.

When a company declares bankruptcy and defaults on all its due loans and credits, the **liquidation process** gathers whatever can be saved in the form of financial assets.

How much that can be gathered relative to all outstanding debt is known as the **recovery rate**. All debt bank loans, bonds, credit lines, and so on is then ranked by seniority to decide which debtors to pay back first.

Default: How?

The debt is traditionally broken up into two major parts: **senior and junior debt**, with senior debt ranked ahead of junior.

For any new debt contract, such as a bond, a company is required to indicate if the new debt is junior or senior to already outstanding debt.

Creditors with junior debt do not get paid until the senior debt holders have been paid in full.

Senior corporate bonds thus carry less risk for investors than junior bonds, but also have a lower profit potential.

Default: How?

If bankruptcy actually takes place, debt holders have priority over stock and equity holders.

The company's suppliers and providers should be paid first, and only after that should the company's owners be given whatever might be left.

The seniority of debt and the resulting cascade of cash flow are often referred to as the **debt waterfall**.

What is Credit Risk?

Using the terminology we have developed in this lecture we can say that:

Credit risk is the risk of loss arising from a potential credit event with a counterparty in the future.

The question now is:

How do we model credit risk?

Part 2

Modeling Credit Risk: A Guided Tour

2.1 Traditional Approaches

2.1.1 Expert systems

2.1.2 Rating systems

2.1.3 Credit scoring models

2.2 Modern Approaches

2.2.1 Structural Models

2.2.2 Intensity Models

Modeling Credit Risk: A Guided Tour

Traditional Approaches

Traditional methods try to estimate the probability of default (denoted PD), rather than the potential losses in the event of default (denoted LGD). Furthermore, these models typically specify bankruptcy filing, default, or liquidation, thereby ignoring consideration of the downgrades and upgrades in credit quality that are measured in mark to market models. The three broad categories of traditional models used to estimate the probability of default are:

Modeling Credit Risk: A Guided Tour

Expert Systems. Historically, bankers have relied on expert systems to assess credit quality. These are based on, the Character (reputation), the Capital (leverage), the Capacity (earnings volatility), the Collateral, and the Cycle (macroeconomic) conditions.

Rating Systems. External credit ratings provided by firms specializing in credit analysis were first offered in the U.S. by Moody's in 1909. Agency ratings are opinions based on extensive human analysis of both the quantitative and qualitative performance of a firm.

Credit scoring models. The most commonly used traditional credit risk measurement methodology is the multiple discriminant credit scoring analysis pioneered by Altman (1968). This model is a multivariate approach built on the values of both ratio-level and categorical univariate measures.

Modeling Credit Risk: A Guided Tour

Altman's Z-Score model was constructed using **multiple discriminant analysis**, a multivariate technique. From the original set of 22 variables the final Z-Score model chosen was the following discriminant function of five variables:

$$Z = 0.012X_1 + 0.014X_2 + 0.033X_3 + 0.006X_4 + 0.999X_5$$

where: X_1 = working capital/total assets, X_2 = retained earnings/total assets,X_3 = earnings before interest and taxes/total assets, X_4 = market value equity/book value of total liabilities, X_5 = sales/total assets, and Z = overall index.

Modern Approaches

Modern methodologies of credit risk measurement can be divided in two alternative approaches with respect to their relationship with the asset pricing literature of academic finance: the structural approach pioneered by Merton (1974) and a reduced form approach utilizing intensity-based models to estimate stochastic hazard rates, pioneered by Jarrow and Turnbull (1995), Jarrow, Lando, and Turnbull (1997), and Duffie and Singleton (1998, 1999). These two schools of thought propose differing methodologies to accomplish the estimation of default probabilities. The structural approach models the economic process of default, whereas reduced form models decompose risky debt prices in order to estimate the random intensity process underlying default.

Modeling Credit Risk: A Guided Tour

Structural Models. Merton (1974) models equity in a firm as a call option on the firm's assets (V) with a strike price equal to the liabilities of the firm (D). If at expiration (coinciding to the maturity of the firm's liabilities - the firms liabilities are assumed to be comprised of pure discount debt instruments) the market value of the firm's assets is greater than the value of its debt, then the firm's shareholders will exercise the option to repurchase the company's assets by repaying the debt. However, if the market value of the firm's assets is less than the value of its debt ($V < D$), then the option will not be exercised and the firm's shareholders will default. Thus, the probability of default until expiration is equal to the likelihood that the option will expire unexercised.

Modeling Credit Risk: A Guided Tour

Intensity models. Default occurs after adequate early warning in Merton's structural model. That is, default occurs only after a gradual descent (diffusion) in asset values to the default point (equal to the debt level). This process implies that the probability of default steadily approaches zero as the time to maturity declines, something not observed in empirical term structures of credit spreads. More realistic credit spreads are obtained from reduced form or intensity-based models. That is, whereas structural models view default as the outcome of a gradual process of deterioration in asset values, intensity-based models view default as a sudden, unexpected event, thereby generating probability of default estimates that are more consistent with empirical observations.

Modeling Credit Risk: A Guided Tour

In contrast to structural models, intensity-based models do not specify the economic process leading to default. Default is modelled as a point process. Defaults occur randomly with a probability determined by the intensity of a hazard function. Intensity-based models decompose observed credit spreads on defaultable debt to ascertain both the probability of default (conditional on there being no default prior to time t) and the LGD (which is 1 minus the recovery rate). Thus, intensity-based models are fundamentally empirical, using observable risky debt prices (and credit spreads) in order to ascertain the stochastic jump process governing default.

See: Jarrow and Turnbull (1995), Duffie and Singleton (1998).

Part 3
Structural Models: Basic

3.1 Merton (1974)

3.2 Merton (1974): advantages and disadvantages

3.3 Black and Cox (1976): flat barrier

Introduction

Structural models use the evolution of firms' structural variables, such as asset and debt values, to determine the time of default. Merton's model (1974) was the first modern model of default and is considered the first structural model. In Merton's model, a firm defaults if, at the time of servicing the debt, its assets are below its outstanding debt. A second approach, within the structural framework, was introduced by Black and Cox (1976). In this approach defaults occur as soon as firm's asset value falls below a certain threshold. In contrast to the Merton approach, default can occur at any time. In the following we analyse both models.

Merton Model (1974)

Merton (1974) makes use of the Black and Scholes (1973) option pricing model to value corporate liabilities. This is an straightforward application only if we adapt the firm's capital structure and the default assumptions to the requirements of the Black-Scholes model.

Let us assume that the capital structure of the firm is comprised by equity and by a zero-coupon bond with maturity T and face value of D, whose values at time t are denoted by E_t and D_t respectively. The firm's asset value V_t is simply the sum of equity and debt values,

$$V_t = E_t + D_t$$

Merton Model (1974)

Under these assumptions, it can be shown that equity represents a call option on the firm's assets with maturity T and strike price D. Why?

If at maturity T the firm's asset value V_T is enough to pay back the face value of the debt D, the firm does not default and shareholders receive $V_T - D$.

Otherwise if $V_T < D$ the firm defaults, bondholders take control of the firm, and shareholders receive nothing.

Implicit in this argument is the fact that the firm can only default at time T. This assumption is important to be able to treat the firm's equity as a vanilla European call option, and therefore apply the Black-Scholes pricing formula.

Merton Model (1974)

Other assumptions Merton (1974) adopts are (a) the inexistence of transaction costs, (b) bankruptcy costs, (c) taxes, (d) problems with fractioning of assets, (e) continuous time trading (f) unrestricted borrowing and lending at a constant interest rate r, (g) no restrictions on the short selling of the assets, (h) the value of the firm is invariant under changes in its capital structure (Modigliani-Miller Theorem) and (i) that the firm's asset value follows a Geometric Brownian Motion type diffusion process.

Merton Model (1974): firm's process

The firm's asset value is assumed to follow a diffusion process given by

$$dV_t = rV_t dt + \sigma_V V_t dW_t$$

where

σ_V is the asset volatility, and

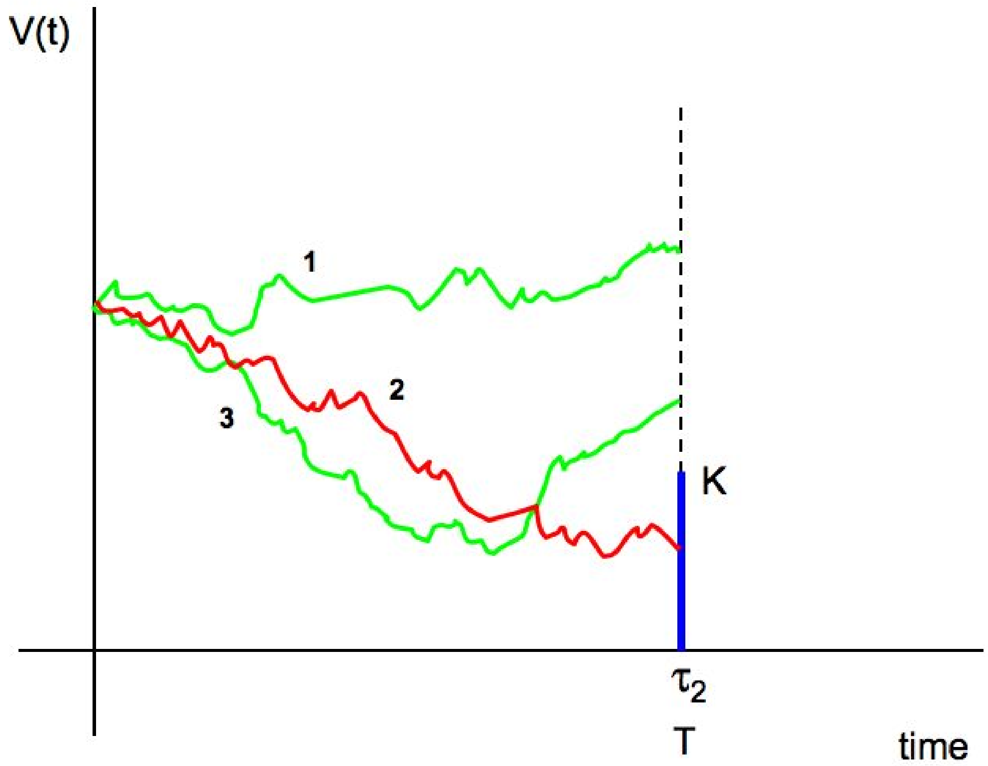
W_t is a Brownian motion.

Merton Model (1974): payoffs

The payoffs to equity holders and bondholders at time T under the assumptions of this model are respectively:

$$E_T = \max(V_T - D, 0)$$

$$D_T = V_T - E_T$$



Merton (1974) Model. Three possible paths for the evolution of the firm. Paths 1 and 3 do not default as they are above the face value of debt D at maturity. Path 2 defaults with default time $\tau_2 = T$

Merton Model (1974): equity value

Applying the Black-Scholes pricing formula, the value of equity at time $t = 0$ is given by

$$E_0 = V_0 N(d_1) - D \exp(-rT) N(d_2)$$

where $N()$ is the distribution function of a standard normal random variable and d_1 and d_2 are given by

$$d_1 = \frac{1}{\sigma_V \sqrt{T}} \left[\log\left(\frac{V_0}{D}\right) + \left(r + \frac{1}{2}\sigma_V^2\right) T \right]$$

$$d_2 = d_1 - \sigma_V \sqrt{T}$$

Merton Model (1974): default probability

The probability of default at time T is given by

$$P\left[V_T < D\right] = N\left(-d_2\right)$$

At time $t = 0$ the value of the debt is $D_0 = V_0 - E_0$.

Which in the equity world is the probability for a call option to finish out of the money.

Merton Model (1974): implementation

In order to implement Merton's model we have to estimate the initial firm's asset value V_0, its volatility σ_V (both unobservable processes), and we have to transform the debt structure of the firm into a zero-coupon bond with maturity T and face value D.

The maturity T of the zero-coupon bond can be chosen either to represent the maturity structure of the debt or simply as a required time horizon (for example, in case we are pricing a credit derivative with some specific maturity).

Merton Model (1974): advantages and disadvantages

Advantage: The main advantage of Merton's model is that it allows to directly apply the theory of European options pricing developed by Black and Scholes (1973).

But to do so the model needs to make the necessary assumptions to adapt the dynamics of the firm's asset value process, interest rates, and capital structure to the requirements of the Black-Scholes model.

There is a trade-off between realistic assumptions and ease of implementation, and Merton's model opts for the latter one.

Merton Model (1974): advantages and disadvantages

Disadvantage: One problem of Merton's model is the restriction of default time to the maturity of the debt, ruling out the possibility of an early default, no matter what happens with the firm's value before the maturity of the debt. If the firm's value falls down to minimal levels before the maturity of the debt but it is able to recover and meet the debt's payment at maturity, the default would be avoided in Merton's approach.

Merton Model (1974): advantages and disadvantages

Disadvantage: The usual capital structure of a firm is much more complicated than a simple zero-coupon bond. Geske (1977, 1979) considers the debt structure of the firm as a coupon bond, in which each coupon payment is viewed as a compound option and a possible cause of default. At each coupon payment, the shareholders have the option either to make the payment to bondholders, or to not make it, in which case the firm defaults. Geske also extends the model to consider characteristics such as sinking funds, safety covenants, debt subordination, and payout restrictions.

Merton Model (1974): advantages and disadvantages

Disadvantage: The assumption of a constant and flat term structure of interest rates is another limitation. Stochastic interest rates allow to introduce correlation between the firm's asset value and the short rate, and have been considered, among others, by Ronn and Verma (1986), Kim, Ramaswamy and Sundares an (1993), Nielsen et al. (1993), Longstaff and Schwartz (1995), Briys and de Varenne (1997), and Hsu, Saa-Requejo and Santa-Clara (2004).

Merton Model (1974): advantages and disadvantages

Disadvantage: Another characteristic of Merton's model is the predictability of default. Since the firm's asset value is modelled as a geometric Brownian motion and default can only happen at the maturity of the debt, it can be predicted with increasing precision as the maturity of the debt comes near.

As a result, in this approach default does not come as a surprise, which makes the models generate very low short-term credit spreads. Introducing jumps in the process followed by the asset value has been one of the solutions considered to this problem.

Black and Cox (1976)

In order to overcome some of this limitations, Black and Cox (1976) extended the Merton model to the case when the firm may default at any time, not only at the maturity date of the debt.

Consider, as in the previous section, that the dynamics of the firm's asset value under the risk neutral probability measure **P** are given by the diffusion process

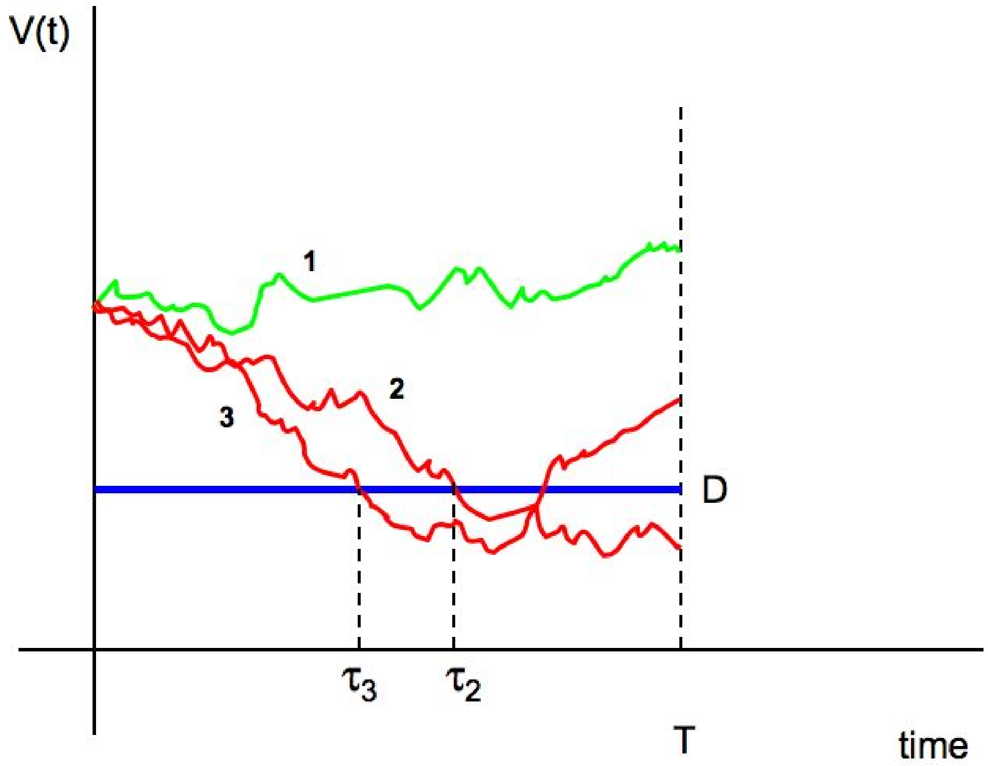
$$dV_t = rV_t dt + \sigma_V V_t dW_t$$

and that there exists a lower level of the asset value such that the firm defaults once it reaches this level.

Black and Cox (1976)

Although Black and Cox (1976) considered a time dependent default threshold, let us assume first a constant default threshold $K > 0$. If we are at time $t \leq 0$ and default has not been triggered yet and $V_t > K$, then the time of default τ is given by

$$\tau = \inf\left(s \geq t \middle| V_s \leq K\right)$$



Black-Cox (1976) Model, flat barrier. Three possible paths for the evolution of the firm. Path 1 do not default as its always above the lower barrier K during the life of the option. Paths 2 and 3 default when they touch the barrier at τ_2 and τ_3, respectively.

Black and Cox (1976)

Using the properties of Brownian motion, it can be shown that the default probability from time t to time T is

$$P\left[\tau \leq T \middle| \tau > t\right] = N\left(h_1\right) + \exp\left\{2 \quad r - \frac{\sigma_V^2}{2}\right) \ln \quad \frac{K}{V_0}\right) \frac{1}{\sigma_V^2}\right\} N\left(h_2\right)$$

where

$$h_1 = \frac{\ln\left(\frac{K}{e^{rT}V_0}\right) + \frac{\sigma_V^2}{2}T}{\sigma_V\sqrt{T}}$$

$$h_2 = h_1 - \sigma_V\sqrt{T}$$

Part 4

Structural Models: Examples

Example 1: Initial Public Offering

ABC Corp. is a privately held company which is about to go public and start selling equity shares to the general public. Because it is still a private company with no traded equity, its debt and equity are impossible to observe. We do know, however, for the sake of this example, the value of its assets. Because the assets are made up of a portfolio of various equity type securities that are publicly traded, their value is easy to observe. We can therefore estimate the value of these assets to be 100 million USD. Let's further imagine that we are the investment bank that is preparing to take this company public. Our job is to set a fair value for its shares. To do this, we split the assets into two parts, or tranches. One tranche is a zero-coupon bond with a four-year maturity and a face value of 70 million USD. The other tranche corresponds to the equity. Our job as bankers is then to value the company's debt and equity now (i.e. $t = 0$).

Example 1: Initial Public Offering

The following table summarizes what we know about the company so far. It is not much; in fact, we only know the asset value. To be able to use the Merton model, we make a few more assumptions, namely that the average asset price volatility is 20 percent (which we have calculated based on the traded securities in the asset portfolio) and that the risk-free interest rate is 5 percent.

Asset	100	Debt	?
		Equity	?
	100		100

Example 1: Initial Public Offering

Our ultimate goal is to determine what is the capital structure of the firm today, conditioned on (a) the liabilities that have to be paid at maturity ($t = T$) and (b) some assumptions about the future behaviour of the firm value (i.e. the stochastic differential equation that describes the firm).

Using these information we can calculate what are the implied debt value today (D_0) and its associated credit spread.

Example 1: Initial Public Offering

Finding the Debt by Calculating the Equity. As we know, we view equity as a call option on a portfolio of assets, and we value it using the Black-Scholes formula for a call option. The debt is simply going to be what is left after the equity value we calculate has been taken from the assets. Using what we know about the company from earlier, we can compute variables d_1 and d_2:

$$d_1 = \frac{1}{\sigma_V \sqrt{T}} \left[\log\left(\frac{V_0}{D}\right) + \left(r + \frac{1}{2}\sigma_V^2\right) T \right]$$

$$d_1 = \frac{1}{(0.20)\sqrt{4}} \left[\log\left(\frac{100}{70}\right) + \left(0.05 + \frac{1}{2}(0.2)^2\right) 4 \right] = 1.592$$

$$d_2 = d_1 - \sigma_V \sqrt{T} = 1.592 - (0.20)\sqrt{4} = 1.192$$

Example 1: Initial Public Offering

We then use a normal distribution to find the values of $N(d_1)$ and $N(d_2)$

$$N(d_1) = N(1.592) = 0.944$$

$$N(d_2) = N(1.192) = 0.883$$

With these two numbers, we can then make use of the baseline call option equation.

$$E_0 = V_0 N\left(d_1\right) - D\exp\left(-rT\right) N\left(d_2\right)$$

$$E_0 = (100)(0.944) - (70)\exp\left(-(0.05)(4)\right)(0.883)$$

$$E_0 = 43.79$$

Example 1: Initial Public Offering

Thus the estimated equity value or market value of ABC Corp today is 43.79 million USD. Because we know that the assets are worth 100 million USD, the debt must be worth the difference, or 56.21 million USD.

Example 1: Initial Public Offering

Arriving at the Credit Spread. Of course, the goal of the preceding exercise is not only to calculate the debt value. We need the debt value as a base from which to calculate the credit spread, which you will recall is a measurement not only of price, but also of the company's default risk. Let us therefore calculate the credit spread for this particular debt, which we have established to be 56.21 million USD.

The credit spread is the difference between a bond's yield and the yield of a risk-free government bond such as a U.S. Treasury Bond. Recall also that to calculate the yield to maturity, you take the face value of the debt (which is what it is worth at expiration to its holder) and use a discount value to arrive at today's market value. That discount value is the yield to maturity.

Example 1: Initial Public Offering

If we apply this to our current ABC Corp. example, we have a face value of 70 million USD, a newly computed market debt value of 56.21 million USD and a time to maturity of 4 years. However, we do not know the yield to maturity, y. This gives us the following equation:

$$56.21 = 70 \exp(-4y)$$

Solving for y gives us

$$0.0548501 = 5.49\%$$

Example 1: Initial Public Offering

To arrive at the credit spread, we then take the yield we just calculated and subtract the corresponding Treasury rate. We have assumed throughout this example that the risk-free rate is 5 percent, so let us use that value. This gives us a credit spread of

$$5.49\% - 5\% = 0.49\%$$

In a Black-Scholes economy in which ABC Corp. plans to go public, the credit spread for this particular debt is therefore 0.49 percent or 49 basis points.

Example 2: Recapitalization

In the previous example, we assumed you could observe the volatility of assets. In reality, this is not possible. This time, let's consider a more realistic situation. The more complete balance sheet of another company, XYZ Ltd., is shown in the following table.

Asset	100 million USD	Debt	40 million USD
		Equity	60 million USD

Example 2: Recapitalization

This time, the balance sheet shows that both the debt and the equity are fully traded in the open markets. This allows us to assign them market values, and in turn come up with the asset value. As the figure shows, the debt has a market value of 40 million and the equity has a market value of 60 million USD, giving us assets of 100 million USD. It should also be noted that the debt has a face value of 50 million USD, the outstanding debt has a 5-year time to maturity, and the current interest rate is 3 percent.

Example 2: Recapitalization

Now, the Chief Financial Officer of XYZ Ltd. is considering recapitalizing the balance sheet; in other words, he wants to eliminate some of the debt. Specifically he considers issuing 20 million USD of equity so that the firm can repurchase 20 million USD of debt. However, the CFO is not sure how that would impact the company's credit spread if XYZ Ltd. lowered its debt to 20 million USD and raised its equity to 80 million USD. Put more accurately, what will be the firm's marginal credit spread, or the credit spread on the next dollar of issuance, after the recapitalization? Lets solve this problem with the Merton model.

Example 2: Recapitalization

As in the previous example, our goal is to calculate the credit spread. However, this time we assume that the firm is a technology company whose assets are not traded. That means we cannot observe the company's asset volatility directly. (In the previous example, we simplified the situation by saying that the assets were a portfolio of traded securities.) Because asset volatility is required in the formulas we will use, we have to assess it somehow for this particular company. One method is to imply it from what we can observe.

Example 2: Recapitalization

Let's now compute the implied asset volatility of XYZ Ltd, the technology company we introduced earlier. We are going to calibrate the Merton model such that it prices the debt to its known market price, which is 40 million USD. We plug known parameters for face value, interest rate, and time to maturity into equation the equation for calculating debt. In this equation, the only unknown is the volatility σ:

$$D_0 = D \exp(-rT) - ...$$

$$[D \exp(-rT) N(-d_2) - V_0 N(-d_1)]$$

Example 2: Recapitalization

We plug in all the other numbers to obtain:

$$40 = (50)\exp\left(-(0.03)(5)\right) - ...$$

$$[(50)\exp\left(-(0.03)(5)\right) N(-d_2) - (100)N(-d_1)]$$

where the formulas for d_1, and d_2, are

$$d_1 = \frac{1}{\sigma_V\sqrt{5}}\left[\log\left(\frac{100}{50}\right) + \left((0.03) + \frac{1}{2}\sigma_V^2\right)(5)\right]$$

$$d_2 = \frac{1}{\sigma_V\sqrt{5}}\left[\log\left(\frac{100}{50}\right) + \left((0.03) + \frac{1}{2}\sigma_V^2\right)(5)\right] - \sigma_V\sqrt{5}$$

Example 2: Recapitalization

Using a spreadsheet program, we can solve for σ_A which ends up being 0.334. The asset volatility value we needed is, in other words, 33.4 percent.

Now that the firm has a value for asset volatility, it can set out to calculate the credit spread after recapitalization. We return to equation 19, which asks us for the values of d_1 and d_2. We calculate these to be

$$d_1 = \frac{1}{(0.334)\sqrt{5}}\left[\log\left(\frac{100}{30}\right) + \left((0.03) + \frac{1}{2}(0.334)^2\right)(5)\right] = 2.186$$

$$d_2 = d_1 - (0.334)\sqrt{5} = 1.439$$

Example 2: Recapitalization

Bringing all these together we arrive to the following debt calculation:

$$D_0 = D\exp(-rT) - [D\exp(-rT)\,N(-d_2) - V_0 N(-d_1)]$$

$$D_0 = (20)\exp(-(0.03)(5)) - ...$$

$$[(20)\exp(-(0.03)(5))\,N(-1.439) - (100)N(-2.186)]$$

In other words, the debt value after recapitalization is 25.32 million USD. We now need to convert this debt value into a credit spread, just as we did in the previous example.

Example 2: Recapitalization

We start by figuring out the yield to maturity, y. The face value after recapitalization is 30 million USD, because the firm just paid off 20 million USD of the 50 million USD debt. We just calculated today's market value to be 25.32 million USD, and the time to maturity is 5 years. This gives us the following equation:

$$25.32 = 30 \exp\left(-y(5)\right)$$

Solving for y, yield to maturity, gives us $y = 3.39\%$.

Subtracting the corresponding Treasury rate gives us the credit spread. Our assumed risk-free rate is 3 percent, which then gives us a credit spread of 3.39% - 3% = 0.39%. The credit spread for this particular debt, in a Black -Scholes economy, is therefore 0.39 percent or 39 basis points.

Example 2: Recapitalization

Compare this result with the original yield to maturity before capitalization can be computed (using the same approach we just used) to be 4.46 percent, meaning a 146 basis points spread. The recapitalization would therefore diminish the company's credit spread by 105 basis points.

Example 3: Merton versus Black and Cox

Remember that the Black and Cox model relaxes two of Merton's assumptions by allowing early default timing and by using a threshold as a signal of default instead of the debt value. Let's now use a numerical example to show how these two extensions affect an actual credit spread. To do so, we will return to the ABC Corporation example we used earlier to calculate a spread using the Merton model. We'll use the same data, repeated below in the table, to calculate a credit spread using the Black and Cox model.

Example 3: Merton versus Black and Cox

Asset Value	100 million USD
Principal Value	70 million USD
Risk-free rate	5%
Volatility	20%
Time to maturity	4 years

Example 3: Merton versus Black and Cox

For this exercise, we will assume that the default barrier, K, has been set at 60 million USD. Note how this default barrier is lower than the principal value of the debt.

This follows a standard approach that calculates the default barrier as recovery rate times principal debt to the exponential value of $-rT$, which always leads to a default barrier that is lower than the principal debt value.

Example 3: Merton versus Black and Cox

Finding the Default Probability We start by finding the default probability. We plug the given values into our equation to obtain

$$h_1 = \frac{1}{\sigma_V\sqrt{T}}\left[\log \quad \frac{K}{\exp(rT)V_0}\right) + \left(+\frac{1}{2}\sigma_V^2 T\right)\Bigg]$$

$$h_1 = \frac{1}{(0.2)\sqrt{4}}\left[\log \quad \frac{60}{\exp\left((0.05)(4)\right)(100)}\right) + \left(+\frac{1}{2}(0.2)^2(4)\right)\Bigg]$$

$$h_1 = 1.577$$

Example 3: Merton versus Black and Cox

We then use a normal distribution to find the values of $N(h_1)$ and $N(h_2)$

$$N(h_1) = N(-1.577) = 0.0574$$

$$N(h_2) = N(-1.977) = 0.0240$$

Example 3: Merton versus Black and Cox

Plugging these two values into the previous equation gives us the default probability

$$P = N(h_1) + \exp\left[2 \left.\quad r - \frac{\sigma_V^2}{2}\right) \log \left.\quad \frac{K}{V_0}\right) \frac{1}{\sigma_V^2}\right] N(h_2)$$

$$P = (0.0574) + \exp\left[2 \left.\quad (0.05) - \frac{(0.2)^2}{2}\right) \log\left(\frac{60}{100}\right) \frac{1}{(0.2)^2}\right] (0.0240)$$

$$P = 0.0686$$

We obtain that the default probability of ABC Corp. is 6.86 percent.

Example 3: Merton versus Black and Cox

Arriving at the Credit Spread We then plug the default probability into the equation which describes the price of a zero-coupon risky bond that pays off 1 USD at maturity. This gives us

$$\exp(-rT)(1 - PD) = \exp(-(0.05)(4))(1 - 0.0686) = 0.7626$$

The market value of ABC Corp's debt is then calculated as 70 million USD $\times$ 0.7626 = 53.38 million USD.

Example 3: Merton versus Black and Cox

We have now all the values to make a comparison with the Merton model. We have a face value of 70 million USD, a newly computed market debt value of 56.38 million USD (compared to the 56.21 million USD that the Merton model) and a time to maturity of 4 years. To calculate the credit spread, we need to find a value for the yield to maturity, y. Using the values we do know, we can solve for y using the equation

$$53.238 = 70 \exp\left(-4y\right)$$

Example 3: Merton versus Black and Cox

Solving for y gives us $y = 0.0678 = 6.78\%$.

Because our aim is to find the credit spread, we need to subtract the risk-free interest rate from the yield to maturity. Consequently, the credit spread is calculated by

$$6.78\% - 5\% = 1.78\%$$

Under the Black and Cox model, the credit spread for the risky debt of ABC Corp. is 1.78 percent or 178 basis points.

Example 3: Merton versus Black and Cox

For the same risky debt, recall that the Merton model gave us a credit spread of 49 basis points. In other words, the Black and Cox model's result is 129 basis points higher. It should have been expected, though, that Black and Cox would deliver a higher credit spread than Merton. The early default arrival, which the Black and Cox model allows for, accelerates default probability, and as we know from our sensitivity analysis of the Merton model, the higher the default probability, the higher the credit spread. This in part explains the higher credit spread. The other part of the explanation lies in the default barrier function. However, Black and Cox allows for early default, which also speeds up the default probability. Again, a higher default probability results in a higher credit spread.

Example 3: Merton versus Black and Cox

In comparing this special case of the Black and Cox model to the basic Merton model, we can make the intuitive guess that the credit spread resulting from the Black and Cox model will always be higher than that coming out of the Merton model. In the Black and Cox model, not only can the firm default at the debt's expiration if the asset value is below the principal value of the debt, but it can also default at any point in the life of the debt. The probability of default is therefore always greater than in the Merton model, meaning investors should ask for a higher credit spread as compensation for taking on this extra risk.

Part 5

Structural Models: Advanced

5.1 Black and Cox (1976): default re-defined

5.2 Black and Cox (1976): curved barrier

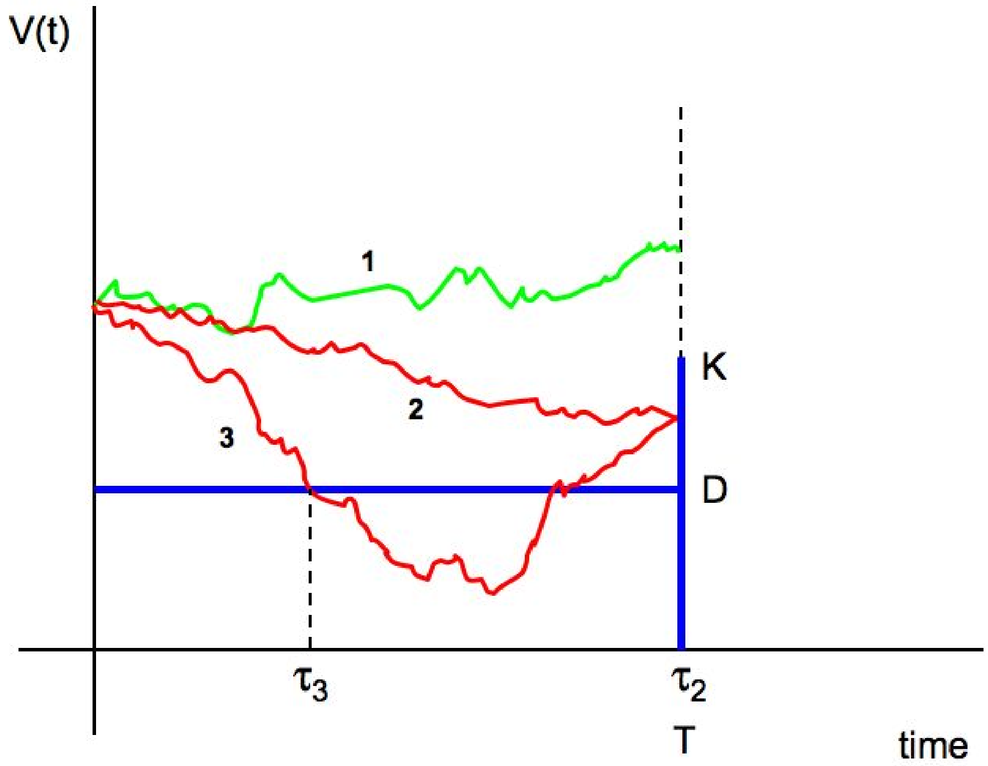
5.3 Other Models

Black-Cox (1976): default re-defined

We redefine default as firm value falling below the barrier $K < D$ at any time before maturity *or* firm value falling below face value D at maturity. Formally, the default time is now given by

$$\tau = \min\left\{\tau^1, \tau^2\right\}$$

In other words, the default time is defined as the minimum of the Black & Cox default time and Merton's default time. This definition of default is consistent with the payoff to equity and bonds. Even if the firm value does not fall below the barrier, if assets are below the bond's face value at maturity the firm defaults, see figure. We obtain for the corresponding default probabilities



Default Re-defined. Three possible paths for the evolution of the firm. Path 1 does not default as its always above the lower barrier K and above D (the face value of debt) at maturity. Path 2 defaults as its below D at maturity ($\tau_2 = T$). Path 3 defaults the moment it touches the lower barrier (τ_3).

Black-Cox (1976): default re-defined

$$
\begin{aligned}
p(T) &= 1 - P\left[\min\left\{\tau^1, \tau^2\right\} > T\right] \\
p(T) &= 1 - P\left[\tau^1 > T, \tau^2 > T\right]
\end{aligned}
$$

Using the joint distribution of an arithmetic Brownian and its running minimum, we obtain

$$
p(T) = N\left(\frac{\log\left(\frac{D}{V_0}\right) - mT}{\sigma\sqrt{T}}\right) + \quad \frac{K}{V_0}\Bigg)^{\frac{2m}{\sigma^2}} N\left(\frac{\log\left(\frac{K^2}{DV_0}\right) + mT}{\sigma\sqrt{T}}\right)
$$

where $m = \mu - \frac{1}{2}\sigma^2$

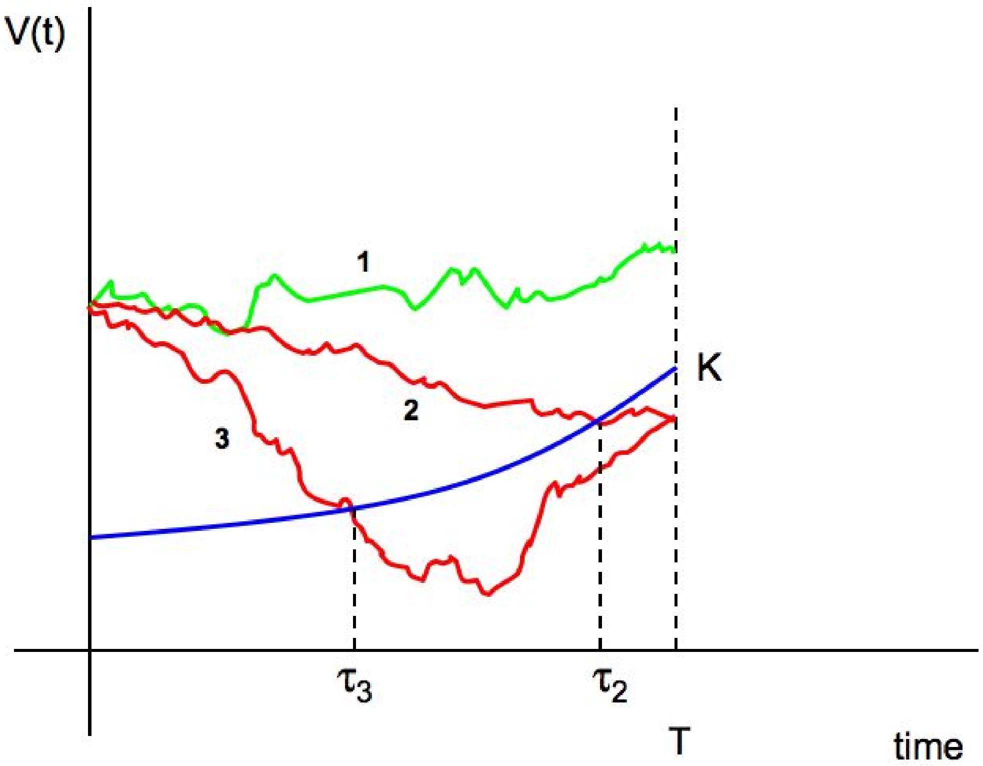
Black-Cox (1976): Time-varying barrier

The second way to avoid the inconsistency discussed above is to introduce a time-varying default barrier $K(t) \leq D$ for all $t \leq T$. For some constant $\alpha > 0$, consider the deterministic function

$$K(t) = De^{-\alpha(T-t)}$$

which can be thought of as the face value of the debt, discounted back to time t at a continuously compounding rate α. The firm defaults at

$$\tau = \text{inf}\,\{t > 0 : V_T < K(t)\}$$



Time-Varying Barrier. Three possible paths for the evolution of the firm. Path 1 does not default as its always above the curved barrier $K(t)$. Path 2 and Path 3 default the moment they touch the lower barrier, τ_2 and τ_3, respectively.

Other Models

Stochastic lower barrier: Hsu, Saa-Requejo and Santa-Clara (2004) suggest that V_t and D do not matter directly to the valuation of default risky bonds but only through their ratio, which is a measure of the solvency of the firm. They model the default threshold as a stochastic process, which together with the stochastic process assumed for the firm's asset value, allow them to obtain the stochastic process.

Other Models

Optimial lower barrier The default threshold can also be chosen endogenously by the stockholders to maximize the value of the equity. See for example Mello and Parsons (1992), Nielsen et al. (1993), Leland (1994), Anderson and Sundaresan (1996), Leland and Toft (1996), Mella-Barral and Perraudin (1997), and Francois and Morellec (2004).

Other Models

Stochastic interest rates Nielsen et al. (1993) and Longstaff and Schwartz (1995) consider a Vasicek process for the interest rate, correlated with the firms' asset value:

$$dV_t = (c - d)V_t dt + \sigma_V V_t dW_t$$

$$dr_t = (a - br_t)dt + \sigma_{V\,t} d\tilde{W}_t$$

$$dW_t d\tilde{W}_t = \rho dt$$

where dW_t and d_t are correlated Brownian motions.

Other Models

Earnings Wilmott et al. (1998) assume that a firms earnings follow the mean-reverting process

$$dE = \theta \left(\bar{E} - E\right) E dt + \sigma E dW$$

where E is earnings, θ a parameter for the speed of mean-reversion, $\bar{E}$ the mean reversion level and dW a Wiener process.

Other Models

Other specifications for the stochastic process of the short rate have been considered. For example Kim, Ramaswamy and Sundaresan (1993) suggest a CIR process

$$dr_t = (a - br_t)dt + \sigma_r \sqrt{r_t} d\tilde{W}_t$$

and Briys and de Varenne (1997) a generalized Vasicek process

$$dr_t = (a(t) - b(t)r_t)dt + \sigma_r(t) d\tilde{W}_t$$

Workshop

Implementing Structural Models with Monte Carlo Simulation: Merton

% Merton MATLAB SCRIPT, A. Pena, Oct 2011

```
clear all; close all;

% STEP 1: Initialize input data
V0 = 100;
mu = 0.05;
sigma=0.40;
D = 90;
T = 1;
M=365; % total num timesteps, daily
nsim = 1000; % total num mc simulations
default_count=0;
dt=T/M;

% MAIN Monte Carlo simulations
for i = 1:nsim
VOld=V0;
for j=1:M % time integration

% STEP 2: Generate draw from N(0,1), std normal distribution
phi = randn;
```

```
% STEP 3: Integrate SDE for one timestep with EULER
VNew = VOld + mu*VOld*dt + sigma*VOld*sqrt(dt)*phi;
V(i,j)=VNew;
VOld=VNew;
end % time integration
VT(i) = VNew;

% STEP 4: Check if barrier touched (i.e. default) at maturity
if VT(i)< D
default_count=default_count+1;
end

end % simulations
```

```
% STEP 5: Plot results and figures
default_count/nsim % print percent defaults

figure(1)
plot(1:M,V(1:100,:),'b-','LineWidth',1,'Color','blue')
xlabel('TIME: t');
ylabel('FIRM VALUE: V(t)');
axis([0 M 0 300])
grid
saveas(gcf,['paths.jpg'])

figure(2)
hist(VT,30)
xlabel('V(T)'); ylabel('count');
grid
saveas(gcf,['histogram.jpg'])

figure(3)
x = [default_count nsim-default_count];
pie3(x)
title('percent defaults [red] and no default [green] ');
colormap(prism)
saveas(gcf,['piechart.jpg'])
```

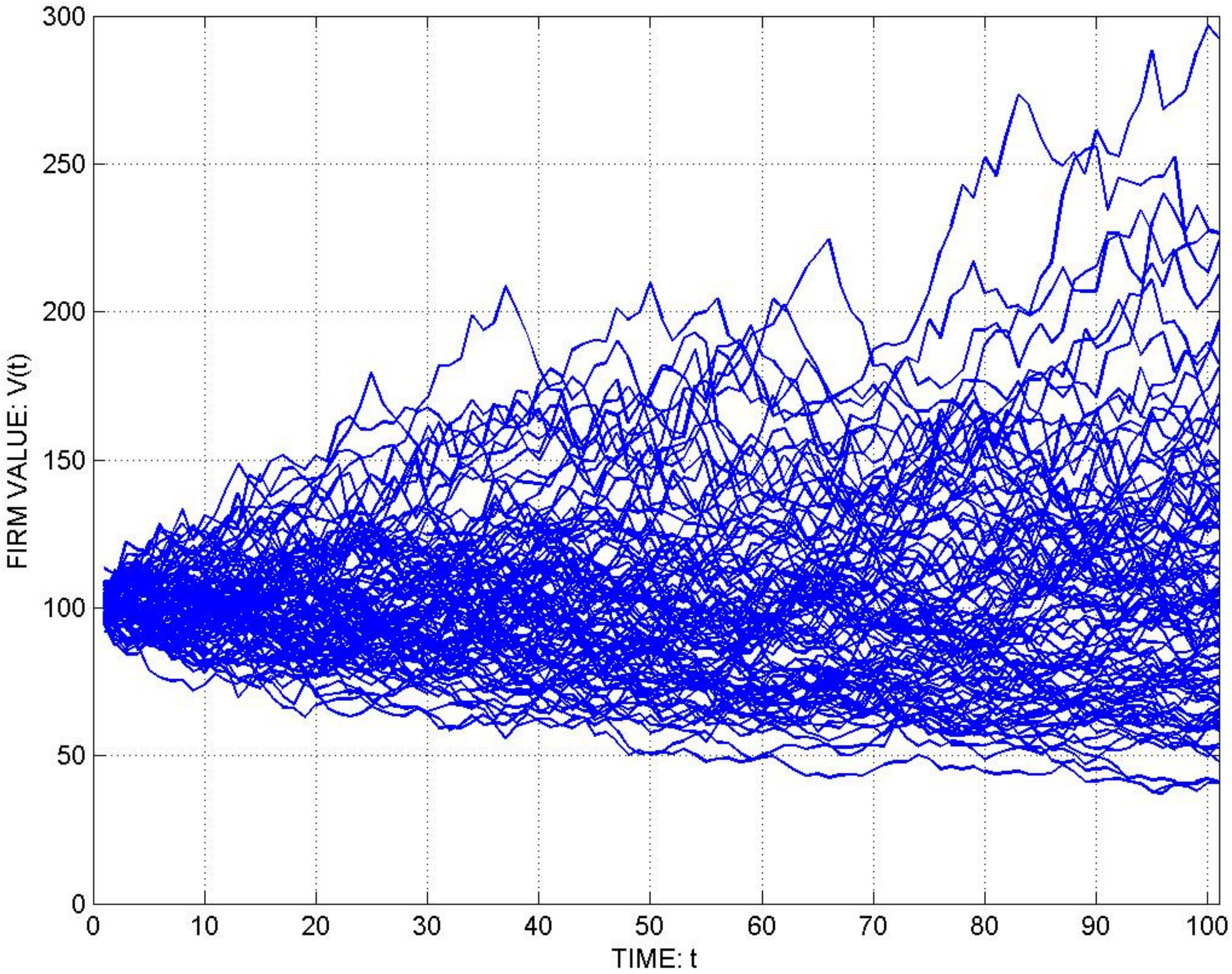


Figure 1 Merton: first 100 Monte Carlo simulations

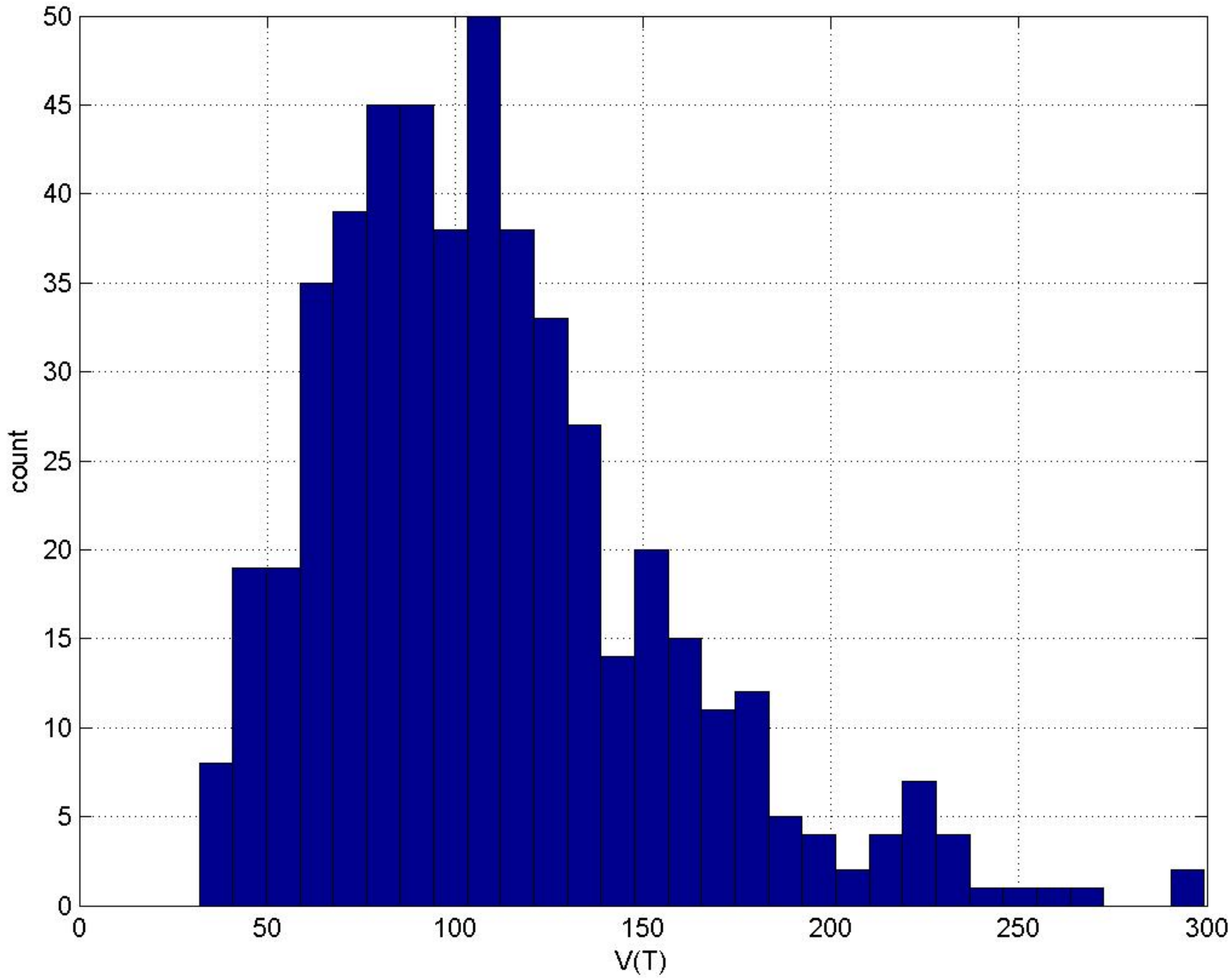


Figure 2 Merton: histogram of firm value at maturity $V(T)$

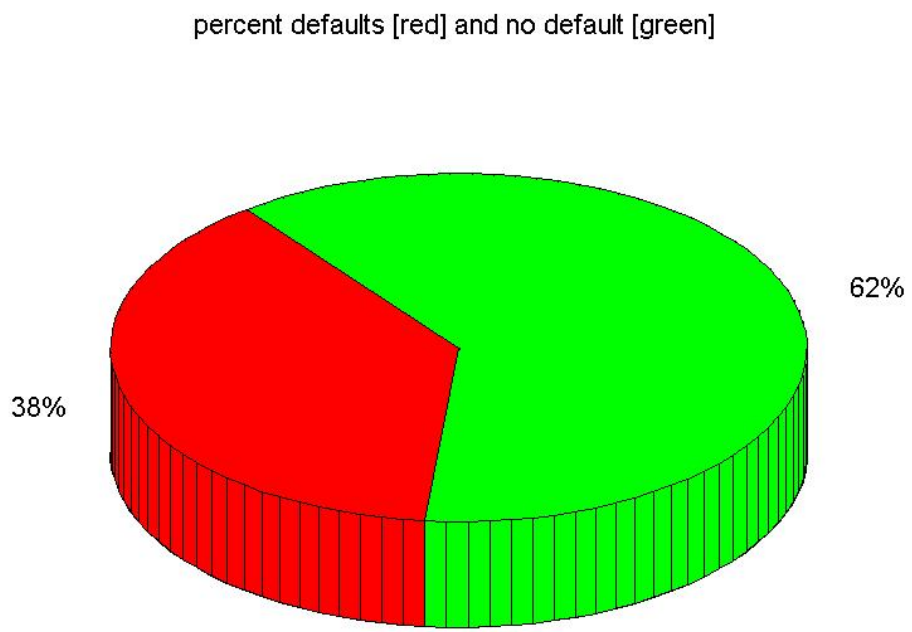


Figure 3 Merton: firms in default (red) and no default (green)

References

[1] Anderson, R, Sundaresan, S., and Tychon, P., 1996, "Strategic Analysis of Contingent Claims," European Economic Review 40, 871-881. [2] Bielecki, T. R, and Rutkowski, M., 2002, "Credit Risk: Modeling, Valuation and Hedging," Springer Finance. [3] Black, F., and J. C. Cox, 1976 "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions," Journal of Finance 31, 351-367. [4] Black, F., and Scholes, M., 1973, "The Pricing of Options and Corporate Liabilities," Journal of Political Economy 81, 637-654. [5] Briys, E., and de Varenne, F., 1997, "Valuing Risky Fixed Rate Debt: An Extension," Journal of Financial and Quantitative Analysis 31, 239-248. [6] Bruche, M., 2005, "Estimating structural bond pricing models via simulated maximum likelihood," London School of Economics, Financial Markets Group Discussion Paper 534. [7] Chacko G, Sjoman A, Motohashi H, Dessain V, Credit Derivatives: Understanding Credit Risk and Credit Instruments, Wharton School Publishing, 2006. [8] Delianedis, G., and Geske, R., 2001, "The Components of Corporate Credit Spreads: Default, Recovery, Tax, Jumps, Liquidity, and Market Factors," Working Paper, UCLA. [9] Duan, J. C., 1994, "Maximum Likelihood Estimation Using Price Data of the Derivative Contract," Mathematical Finance 4, 155-157. [10] Duan, J. C., Gauthier, G., Simonato, J. G., and Zaanoun, S., 2003, "Estimating Merton's Model by Maximum Likelihood with Survivorship Consideration." [11] Duffie, D., and Lando, D., 2001, "Term Structure of Credit Spreads with Incomplete Accounting Information," Econometrica 69, 633-664. [12] Francois, P., and Morellec, E., 2004, "Capital Structure and Asset Prices: Some Effects of Bankruptcy Procedures," Journal of Business 77, 387-411. [13] Geske, R,

1977, "The Valuation of Corporate Liabilities as Compound Options," Journal of Financial and Quantitative Analysis 12, 541-552. [14] Geske, R, 1979, "The Valuation of Compound Options," Journal of Financial Economics 7, 63-81. [15] Giesecke, K, 2005, "Default and Information," Working Paper, Cornell University. [16] Giesecke, K, and Goldberg, L. R, 2004, "Sequential defaults and incomplete information," Journal of Risk 7, 1-26. [17] Hilberink, B., and Rogers, L. C. G., 2002, "Optimal Capital Structure and Endogenous Default," Finance and Stochastics 6, 237-263. [18] Hsu, J., Saa-Requejo, J., and Santa-Clara, P., 2004, "Bond Pricing with Default Risk," UCLA Working Paper. [19] Jarrow, R. A., and Protter, P., 2004, "Structural versus reduced form models: a new information based perspective," Journal of Investment Management 2, 1-10. [20] Jones, P., Mason, S., and Rosenfeld, E., 1984, "Contingent Claim Analysis of Corporate Capital Structures: An Empirical Investigation," Journal of Finance 39, 611-625. [21] Kim, 1. J., Ramaswamy, K., and Sundaresan, S. M., 1993, "Does default risk in coupons affect the valuation of corporate bonds?: A contingent claims model," Financial Management 22, 117-131. [22] Leland, H. E., 1994, "Risky Debt, Bond Covenants and Optimal Capital Structure," Journal of Finance 49, 1213-1252. [23] Leland, H. E., and Toft, K. B., 1996, "Optimal Capital Structure, Endogenous Bankruptcy and the Term Structure of Credit Spreads," Journal of Finance 50, 789-819. [24] Longstaff, F. A., and Schwartz, E. S., 1995, "A Simple Approach to Valuing Risky Fixed and Floating Rate Debt," Journal of Finance 50, 789-819. [25] Mella-Barral, P., and Perraudin, W., 1997, "Strategic Debt Service," Journal of Finance 52, 531-566. [26] Mello, A., and Parsons, J., 1992, "Measuring the agency cost of debt," Journal of Finance 47, 1887-1904. [27] [illegible]4, "On the Pricing of Corporate

Debt: the Risk Structure of Interest Rates," Journal of Finance 29, 449-470. [28] Nielsen, L. T., Saa-Requejc, J., and Santa-Clara, P., 1993, "Default Risk and Interest Rate Risk: The Term Structure of Default Spreads," Working Paper, INSEAD. [29] Ronn, E. 1., and Verma, A. K., 1986, "Pricing Risk-Adjusted Deposit Insurance: An Option Based Model," Journal of Finance 41, 871-895. [30] Wilmott P, Whalley E, Schonbucher PJ, Mayor N and Epstein D, The Valuation of a Firm Advertising Optimally. Quarterly Review of Economics and Finance, Vol. 38 No. 2, Summer 1998. Available at SSRN: http://ssrn.com/abstract=52501 [31] Wilmott P, Paul Wilmott on Quantitative Finance, John Wiley & Sons, 2nd Edition edition, 2006. [32] Zhou, C., 1997, "A Jump-Diffusion Approach to Modelling Credit Risk and Valuing Defaultable Securities," Federal Reserve Board, Washington. [33] Zhou, C., 2001a, "The Term Structure of Credit Spreads with Jump Risk," Journal of Banking and Finance 25, 2015-2040.