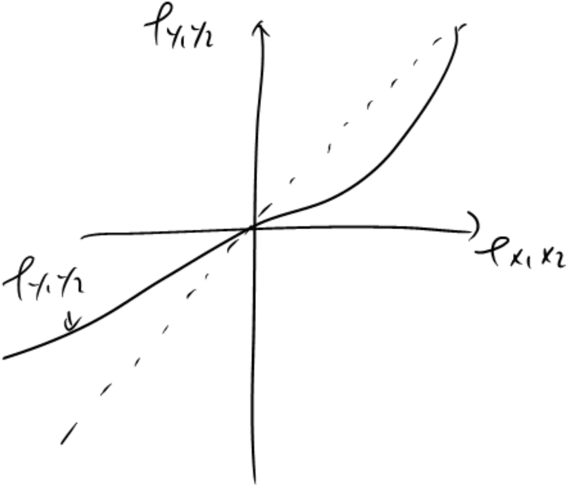
if $\sigma_1 = \sigma_2 = 1$

$$\rho_{Y_1 Y_2} = \frac{e^{\rho} - 1}{e - 1}$$



$$A_i = w_i Z + \cancel{\sqrt{1-w_i^2}\,\varepsilon_i}$$

$$A_j = w_j Z + \cancel{\sqrt{1-w_j^2}\,\varepsilon_j}$$

$$\rho_{ij} = \mathrm{Cov}(A_i, A_j) = w_i w_j \, \mathrm{Var}(Z)$$

$$= w_i w_j$$

$$w_i = w \quad \forall i$$

$$\rho_{ij} = w^2 = \rho$$

$$F(t \mid Z) = \Phi\left(\frac{d - \sqrt{\rho} Z}{\sqrt{1-\rho}} \right)$$

$$A_i = \boxed{wZ} + \sqrt{1-w^2}\,\varepsilon_i$$

$$A_j = \boxed{wZ} + \sqrt{1-w^2}\,\varepsilon_j$$

Conditional on Z

A_i & A_j are independant

because ε_i & ε_j are independent.

Conditional PD $F(t|Z)$ is same for all i

$$K \,|\, Z \sim \text{Binomial}(N, F(t|Z))$$

Numerical Integration

$$E\left\{ \Pr(K = k \mid z) \right]$$

$$= \int_{\mathbb{R}} \Pr(K = k \mid z) \, d\Phi(z)$$

$$= \sum_{k=1}^{n} \Pr(K = k \mid \underline{z = z_k}) \, w_k$$

$$= \sum_{k=1}^{n} \left\{ F(t \mid z_k)^{K} \left(1 - F(t \mid z_k)\right)^{n-K} \right] w_k .$$

$$F(t \mid z_k) = \Phi\left(\frac{d - \sqrt{\rho}\, z_k}{\sqrt{1-\rho}} \right).$$

Fraction of default in LHP

$$Y = \frac{K}{N}$$

$$E[Y | Z] = E\left(\frac{K}{N} \Big| Z\right) = \frac{1}{N} E(K|Z)$$

$$= \frac{1}{N} N \cdot F(t|Z)$$

$$= F(t|Z)$$

$$Var(Y|Z) = Var\left(\frac{K}{N} \Big| Z\right) = \frac{1}{N^2} Var(K|Z)$$

$$= \frac{1}{N^2} N p(1-p)$$

$$= \frac{p(1-p)}{N}$$

$$\lim_{N\to\infty} Var(Y) = \lim_{N\to\infty} \frac{p(1-p)}{N} \to 0.$$

$$\lim_{N\to\infty} Y \to F(t|z)$$

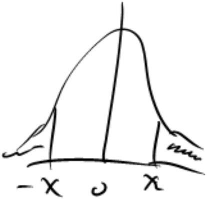
$$\lim_{N\to\infty} Y \to Y(z) = \Phi\left(\frac{d-\sqrt{\rho}\, z}{\sqrt{1-\rho}}\right).$$

Dis$^{\underline{n}}$ for Y

$$G(y) = \Pr(Y \le y) = \Pr\left(\Phi\left(\frac{d - \sqrt{\rho}\, z}{\sqrt{1-\rho}}\right) < y \right)$$

$$= \Pr\left(d - \sqrt{\rho}\, z < \sqrt{1-\rho}\, \Phi^{-1}(y) \right)$$

$$= \Pr\left(z > \frac{d - \sqrt{1-\rho}\, \Phi^{-1}(y)}{\sqrt{\rho}} \right)$$



$$= \Pr\left(z \le \frac{\sqrt{1-\rho}\, \Phi^{-1}(y) - d}{\sqrt{\rho}} \right)$$

$$= \Pr(z \le a) = \Phi(a)$$

Dis^n of $L(t)$

$$L(t) = Y(1-\theta)$$

$$\Pr(L \le l) = \Pr(Y(1-\theta) \le l)$$

$$= \Pr\left(Y \le \frac{l}{1-\theta}\right)$$

$$= G\left(\frac{l}{1-\theta}\right)$$

$$\rightarrow \quad = \Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{l}{1-\theta}\right) - d}{\sqrt{\rho}}\right)$$

$\frac{\partial}{\partial l}$.

Expected Value for $L(t; 0, \ell)$

$$E\{L(t;0,\ell)\} = E\{L(t)\,\mathbb{I}\{L(t)<\ell\}\} + \quad (1)$$

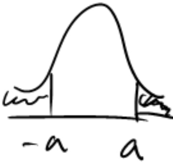
$$\ell\, E\{\mathbb{I}\{L(t)\geq \ell\}\} \quad (2)$$

$$(2) = \ell \Pr(L(t)\geq \ell) = \ell\{1-\Pr(L(t)<\ell)\}$$

$$= \ell\left(1-G\left(\frac{\ell}{1-\theta}\right)\right)$$

$$= \ell\left(1-\Phi\left(\underbrace{\frac{\sqrt{1-\rho}\,\Phi^{-1}\left(\frac{\ell}{1-\theta}\right)-d}{\sqrt{\rho}}}_{a}\right)\right)$$

$$= \ell(1-\Phi(a)) = \ell\,\Phi(-a)$$



$$E\Big[\underbrace{(1-\theta)F(t|z)}_{L(t)}\ \mathbb{I}\{(1-\theta)F(t|z) < \ell\}\Big]$$

$$= (1-\theta)\,E\Big(\Pr(A < d \mid z)\ \mathbb{I}\{F(t|z) < \tfrac{\ell}{1-\theta}\}\Big)$$

$$\mathbb{I}\Big\{\Phi\Big(\frac{d-\sqrt{\rho}z}{\sqrt{1-\rho}}\Big) < \frac{\ell}{1-\theta}\Big\}$$

$$\equiv \mathbb{I}\Big\{z \geq \frac{d-\sqrt{1-\rho}\,\Phi^{-1}\big(\frac{\ell}{1-\theta}\big)}{\sqrt{\rho}}\Big\}$$

$$\equiv \mathbb{I}\{z > -a\}$$

$$= (1-\theta)\,E\big(\Pr(A < d \mid z)\ \mathbb{I}(z > -a)\big)$$

$$= (1-\theta) \int_{-a}^{\infty} \Pr(A < d \mid z) \; d\Phi(z)$$

$$= (1-\theta) \int_{-a}^{\infty} \Pr(A < d \, , \, Z = z) \; dz$$

$$d\Phi(z) = f(z)\, dz$$

$$= (1-\theta) \Pr\left(A < d, \; Z > -a \right)$$

$$A = \sqrt{\rho}\, Z + \sqrt{1-\rho}\, \varepsilon$$

$$\tilde{Z} = -Z$$

$$A = -\sqrt{\rho}\, \tilde{Z} + \sqrt{1-\rho}\, \varepsilon$$

$$= (1-\theta) \Pr(A < d, \; \tilde{Z} < a)$$

$$A = -\sqrt{\rho}\,\tilde{z} + \sqrt{1-\rho}\,\varepsilon$$

$$\begin{pmatrix} A \\ \tilde{z} \end{pmatrix} \sim N\left(0, \begin{pmatrix} 1 & -\sqrt{\rho} \\ -\sqrt{\rho} & 1 \end{pmatrix} \right)$$

$$= (1-\theta) \Pr(A < d, \tilde{z} < a)$$

$$= (1-\theta)\, \Phi_2(d, a; -\sqrt{\rho})$$