Behavioural Finance For Quants

In this session..

1. I am a quant, so why do I need to care about behavioural stuff?

2. How individuals make their decisions: behavioural biases and psychological pitfalls in decision-making?

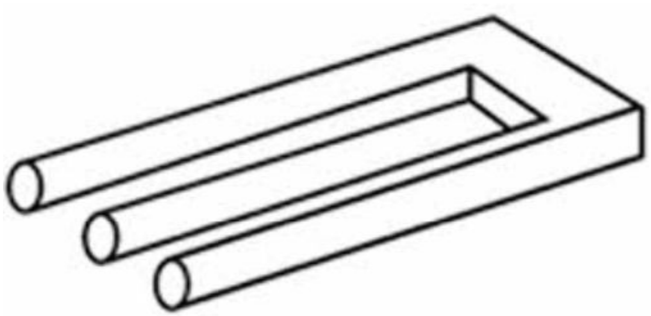
3. How mathematics can help you and your team?

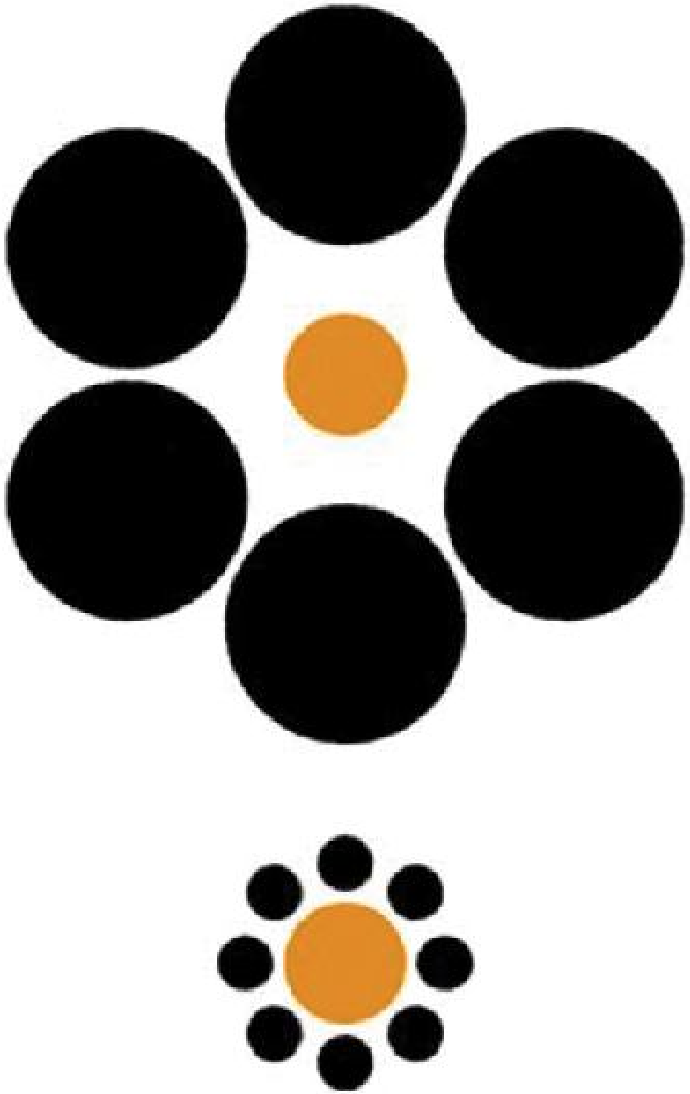
4. An excursion into game theory.

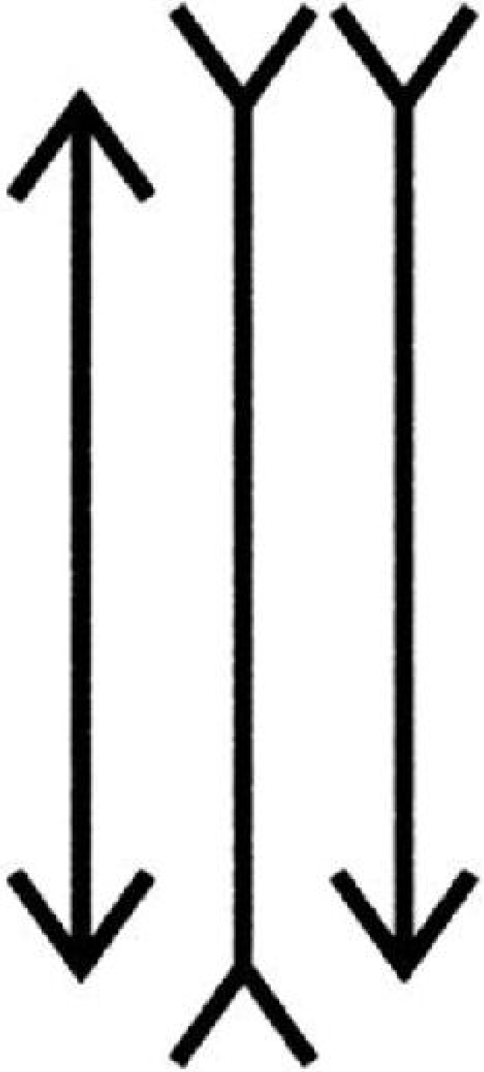
I am a quant, so why do I need to care about behavioural stuff?

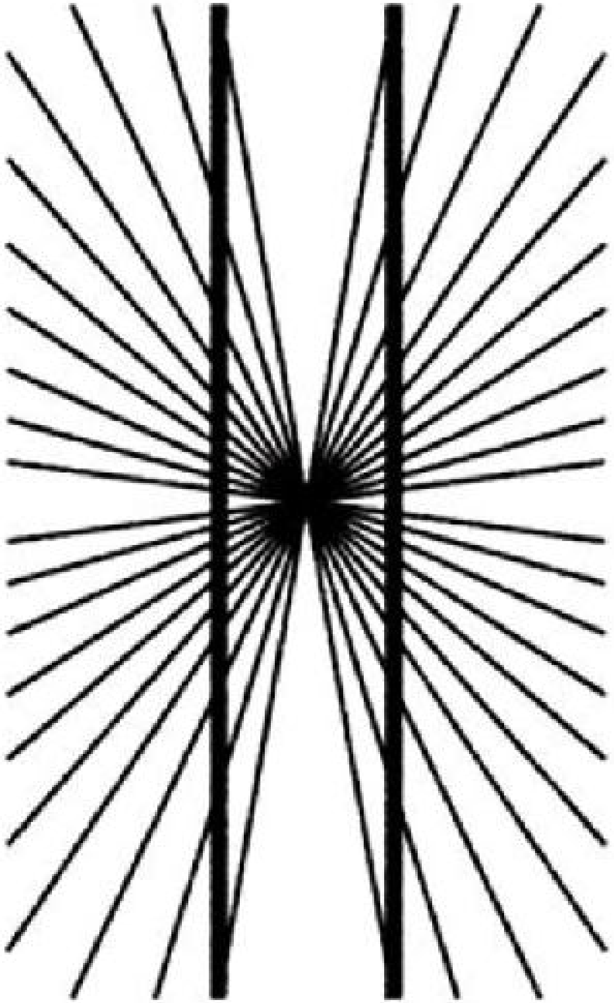
Our Mind Can Be Tricked

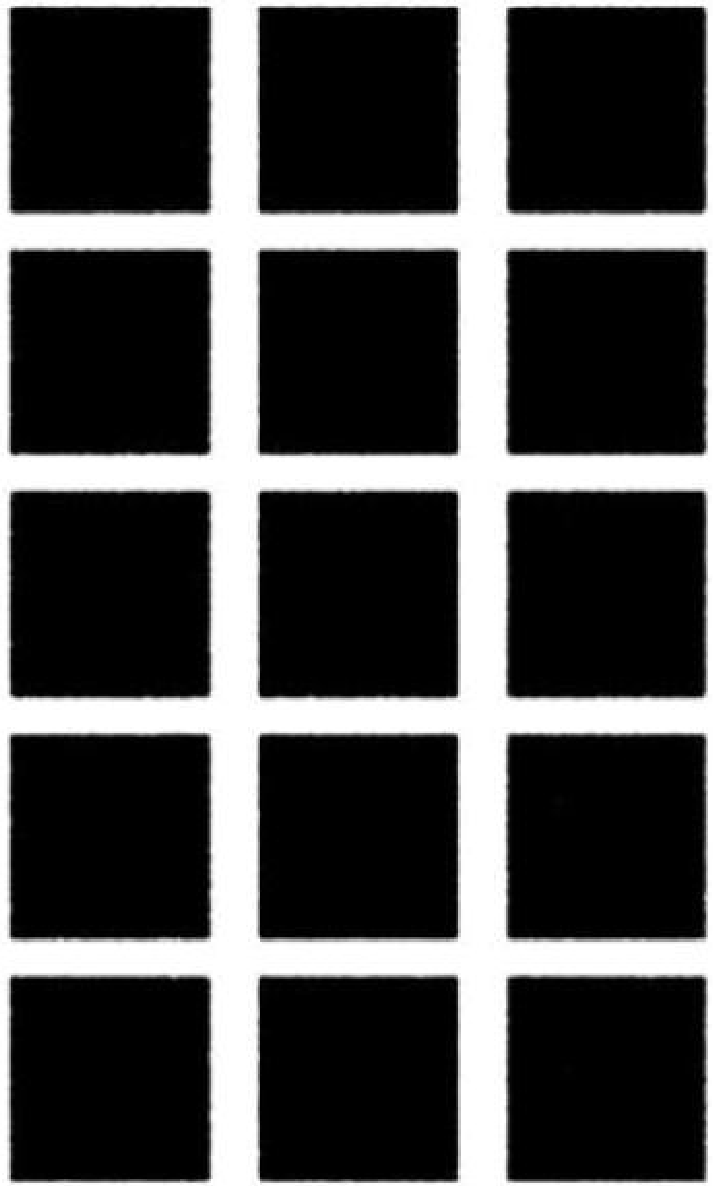
- Our mind is very good at filling blanks and completing patterns.
- Sometimes, it is a bit too good at it, and this makes it easy to be tricked!
- Even more so when we have incomplete information or face uncertainty;







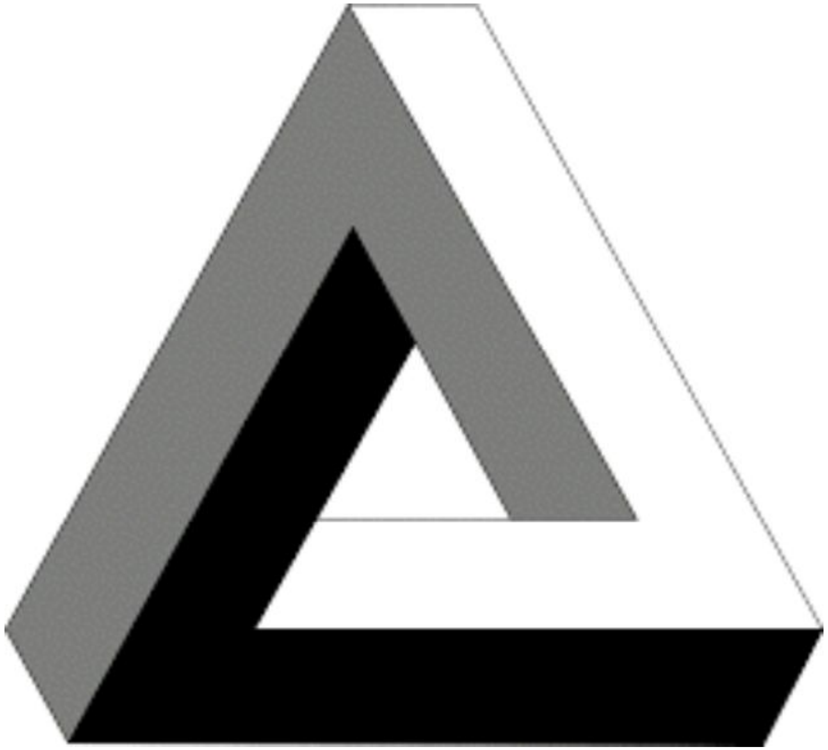




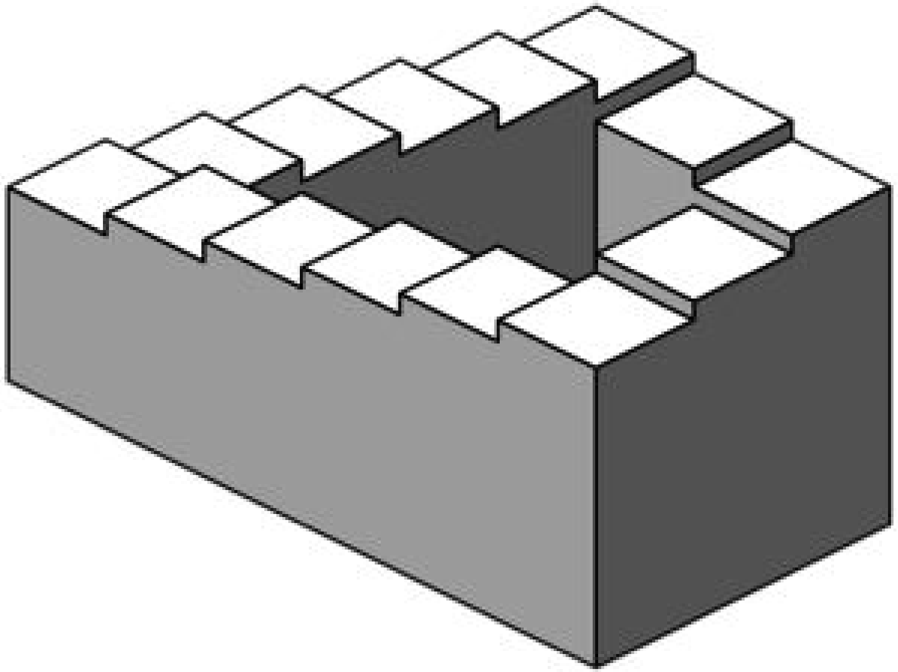
M.C. Escher

M.C. Escher's Waterfall (1961)

The Penrose Triangle



Variation On The Penrose Triangle: The Penrose Rectangle!



The Evolution of Mathematical Proofs

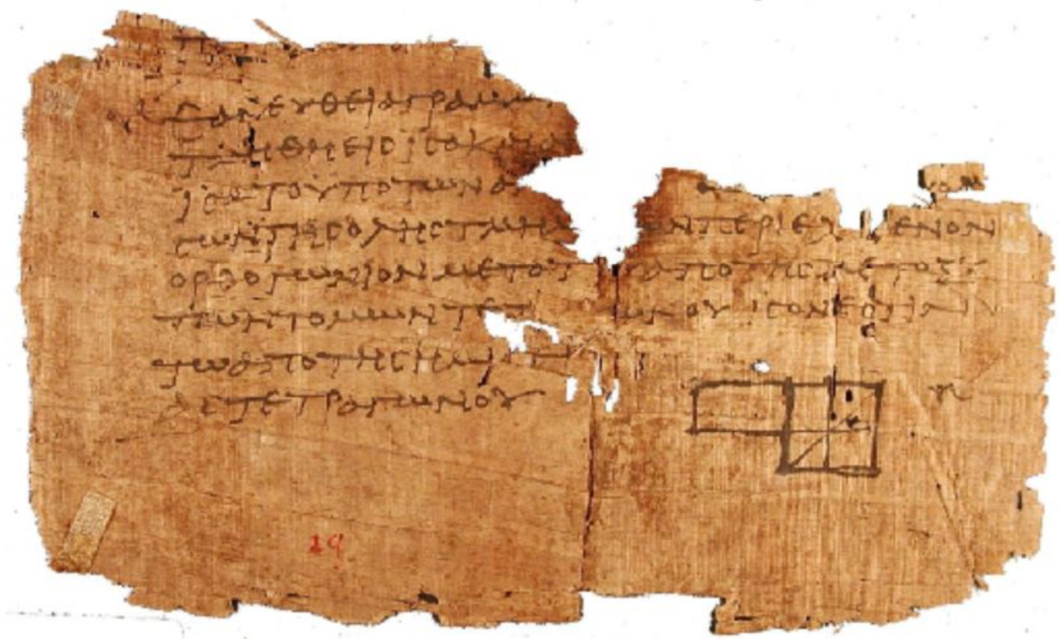
- Mathematics (if not mathematicians!) is keenly aware of the risk of letting our observations and intuition deceive us.

- This is one of the reasons why the idea of what constitues a mathematical proof has evolved considerably since the ancient Greeks placed the ability to prove results rigorously at the center of mathematics.

Ancient Greeks

- In Ancient Greece, it was commonly accepted to use geometric figures as part of the main argument in your proof.

- This practice has a definite disadvantage: get the figure wrong and you will get the argument wrong!



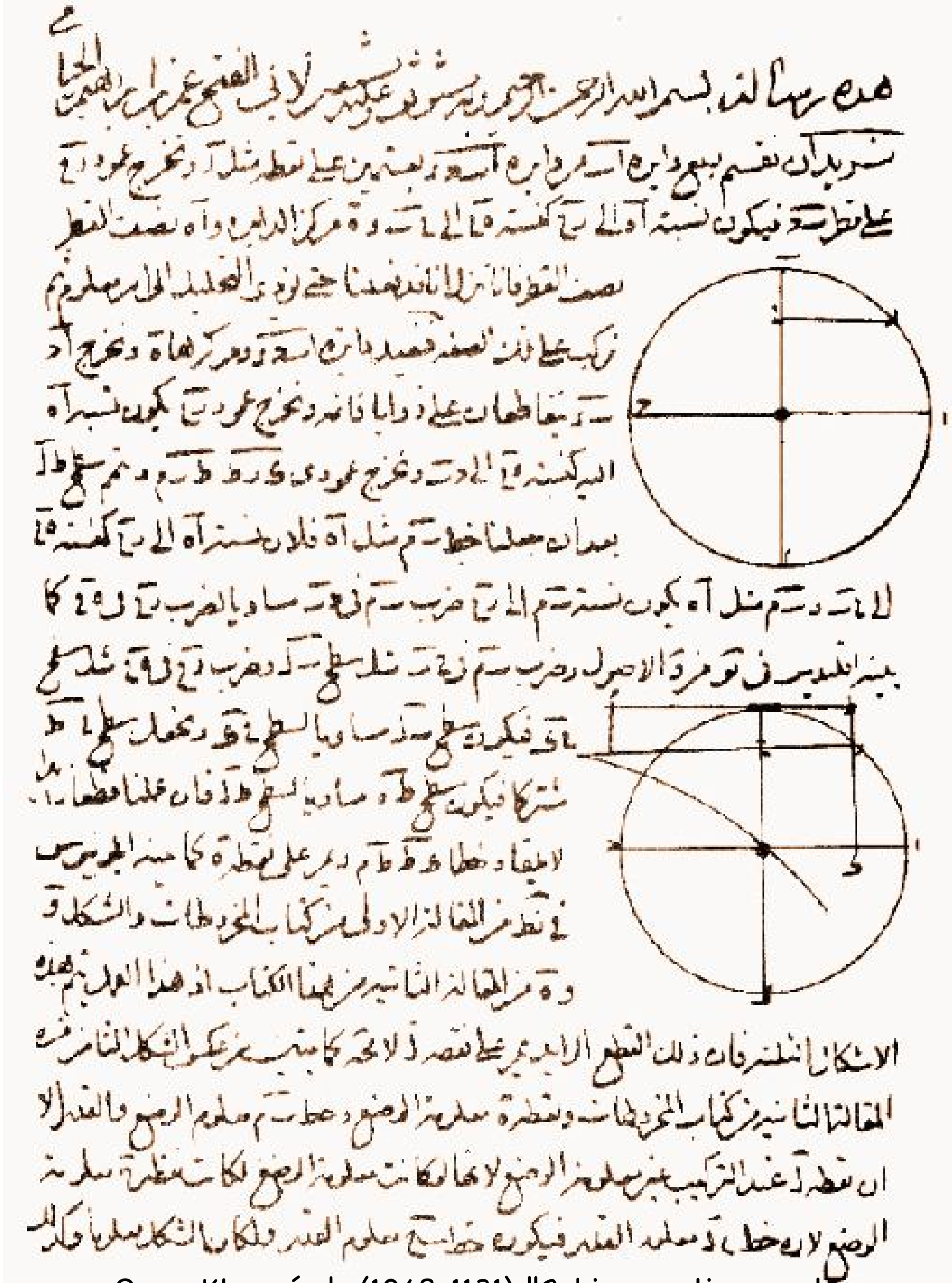
One of the oldest surviving fragments of Euclid's Elements (15-125 AD). In modern terms, this fragment proves the algebraic relation:

$$ab + (a-b)2/4 = (a+b)2/4$$

Medieval Age

- The practice of using geometric figures in proofs continued into the Medieval Age.

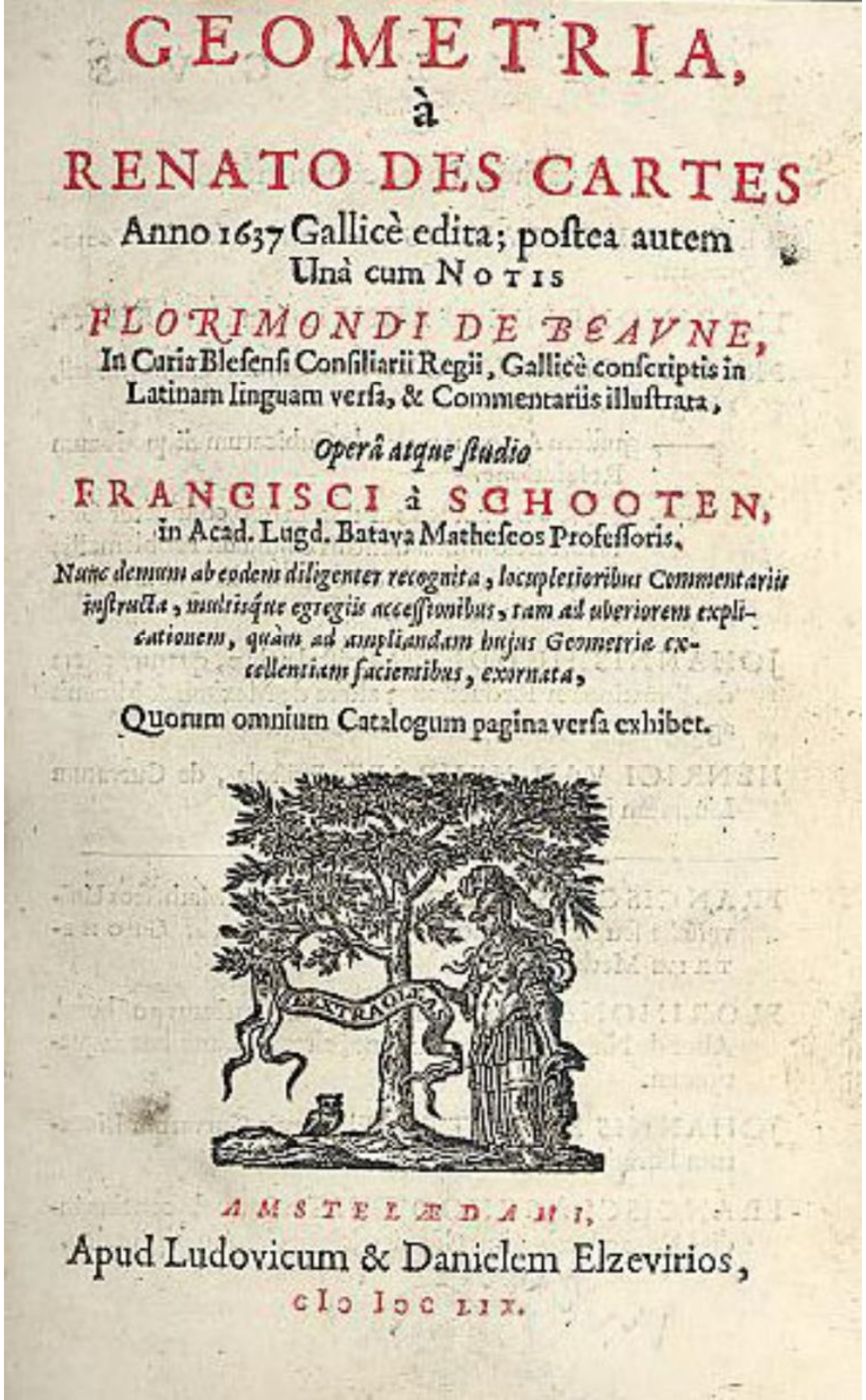
- But gradually, figures slipped from the main argument to being used as an illustrations of the main argument.



Omar Khayyám's (1048-1131) "Cubic equations and intersections of conic sections"

Descartes and Analytic Geometry

- The quandary was finally settled René Descartes’ use of analytical geometry.
- With analytical geometry, you can represent any geometric figure using algebra.
- No need for geometric figures in your proof ever again!
- No chance to get it wrong ever again… or almost…


GEOMETRIA,
à
RENATO DES CARTES
Anno 1637 Gallicè edita; postea autem
Unà cum NOTIS
FLORIMONDI DE BEAVNE,
In Curia Blesensi Consiliarii Regii, Gallicè conscriptis in
Latinam linguam versa, & Commentariis illustrata,
Operâ atque studio
FRANCISCI à SCHOOTEN,
in Acad. Lugd. Batava Matheseos Professoris.
Nunc demum ab eodem diligenter recognita, locupletioribus Commentariis instructa, multisque egregiis accessionibus, tam ad uberiorem explicationem, quàm ad ampliandam hujus Geometriæ excellentiam facientibus, exornata,
Quorum omnium Catalogum pagina versa exhibet.
AMSTELÆDAMI,
Apud Ludovicum & Danielem Elzevirios,
CIↃ IↃC LIX.

Not Just Our Eyes…

- Our eyes are not the only sense that can trick our mind into making the wrong judgement decision.
- Our mind is particularly apt at tricking itself…

A Word Game

- I am going to show you a few words

A Word Game

- I am going to show you a few words

Cloud

Muffin

Dinner

Pasta

Sunshine

Croissant

Salad

Car

Entrée

Towel

Meal

Desert

- Now, I will give you 3 letters of a 4 letter word and ask you to find the word.

- Now, I will give you 3 letters of a 4 letter word and ask you to find the word.

S O _ P

Other Possible “S O _ P” Words…

- soap,
- soop,
- soup,
- sowp.

From Mathematics to Statistics

- Predominantly, we live in a world of incomplete information.
- This problem is typical of finance.
- We need to draw inference from a finite sample of data.
    - This situation leaves us open to being misled by the data and drawing the wrong inference;
    - But there is more, because our brains might misapprehend the data or fail to incorporate adequately new information.
- One more reason to understand human behaviour!

How individuals decide: behavioural biases and psychological pitfalls in decision-making

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Putting psychology back into the decision process

Finance is About Decisions

- We all make decisions, every day from what to wear and what to eat to how to invest for retirement.
- Unsurprisingly, decisions are a critical part of management and finance.
- The basic idea behind **decision theory** is to build a coherent set of tools that will help us make decisions.

In this section,

- The difference between behavioural finance and standard finance
- Thinking fast and slow: System 1 vs System 2
- Behavioural biases: overconfidence, excessive optimism, confirmation bias, illusion of control
- Heuristic processes: representativeness, availability, anchoring and adjustment, affect, extrapolation bias
- Framing effects: narrow frame, opaque frame, transparent frame, mental accounting, aversion to a sure loss
- Group processes: herd behaviour, groupthink
- Loss aversion vs. risk aversion:
  - Loss aversion and prospect theory
  - SP/A theory

Conventional Financial Theory

- Conventional financial theory, assumes that the world and its participants are, for the most part, rational **"utility of wealth maximisers"**.

Behavioural Observation

- However, there are many instances where **emotions** and **psychology** influence our decisions, causing us to behave in unpredictable or irrational ways.

The Reality in Finance: Emotions Lead To Mistakes

US Stock Market Performance 1994–2010 (S&P 500 Index)

Source: Barclays, SSgA and MSCI as of 30 November 2012
Index returns are unmanaged and do not reflect the deduction of any fees or expenses.
Index returns reflect all items of income, gain and loss and the reinvestment of dividends and other income.
Past performance is not a guarantee of future results. Performance returns for periods of less than one year are not annualised.

EMPRES-2485

STATE STREET GLOBAL ADVISORS

Behavioural Finance Provides Insights Into…

- Private Banking
- Product Development
- Asset Management
- Risk Management
- Marketing

Behavioural Finance At Work

Behavioural Finance At Work

Allianz Global Investors

Center for Behavioral Finance

Fidelity WORLDWIDE INVESTMENT

BARCLAYS

UBS

Vanguard

STATE STREET GLOBAL ADVISORS

BLACKROCK

MORNINGSTAR

J.P.Morgan

United-ICAP
the technical advisory service

Bloomberg

The Difference Between Behavioral Finance and Standard Finance…

"The difference between us is that you assume people are as smart as you are, while I assume people are as dumb as I am."

Remark from Richard Thaler, a founding father of behavioral finance, to Robert Barro at a National Bureau of Economic Research (NBER) Conference

Richard Thaler (born 1945), behavioural economist at the University of Chicago

Behavioural Finance

- Behavioural finance is a relatively new field that is seeking:

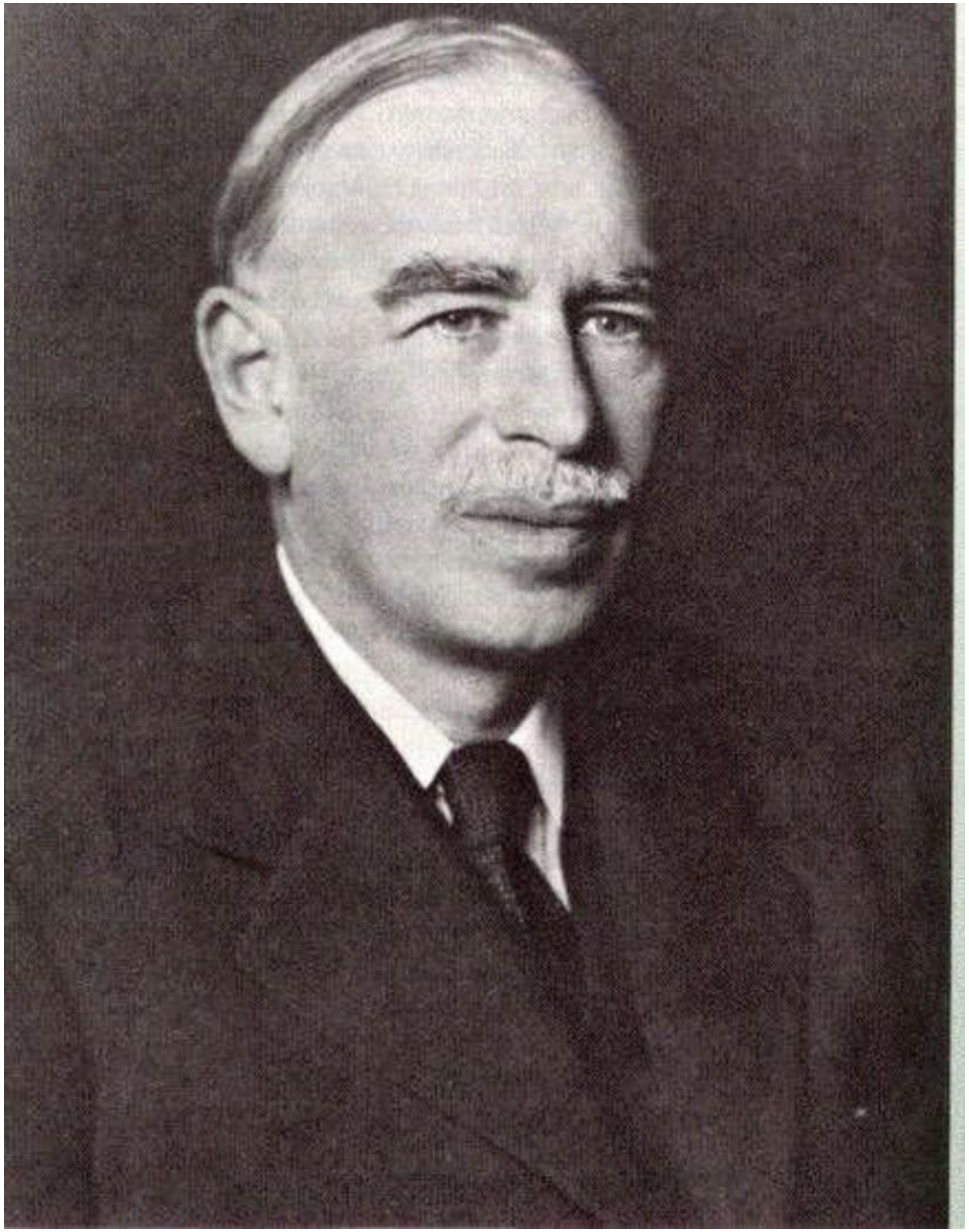
  - to combine behavioural and cognitive psychological theory with conventional economics and finance, and

  - to provide explanations for why people make irrational financial decisions.

John Maynard Keynes

- **Animal spirits** is the term John Maynard Keynes used in his 1936 book *The General Theory of Employment, Interest and Money* to describe emotions which influence human behavior.

- "Human nature desires quick results, there is a peculiar zest in making money quickly, and remoter gains are discounted by the average man at a very high rate."



5 June 1883 – 21 April 1946

Behavioural Finance Is Not New!

- The first published work in behavioural finance was by economists: Shefrin and Statman (1984) in the Journal of Finance article "The Disposition to Sell Winners too Early and Ride Losers too Long: Theory and Evidence".

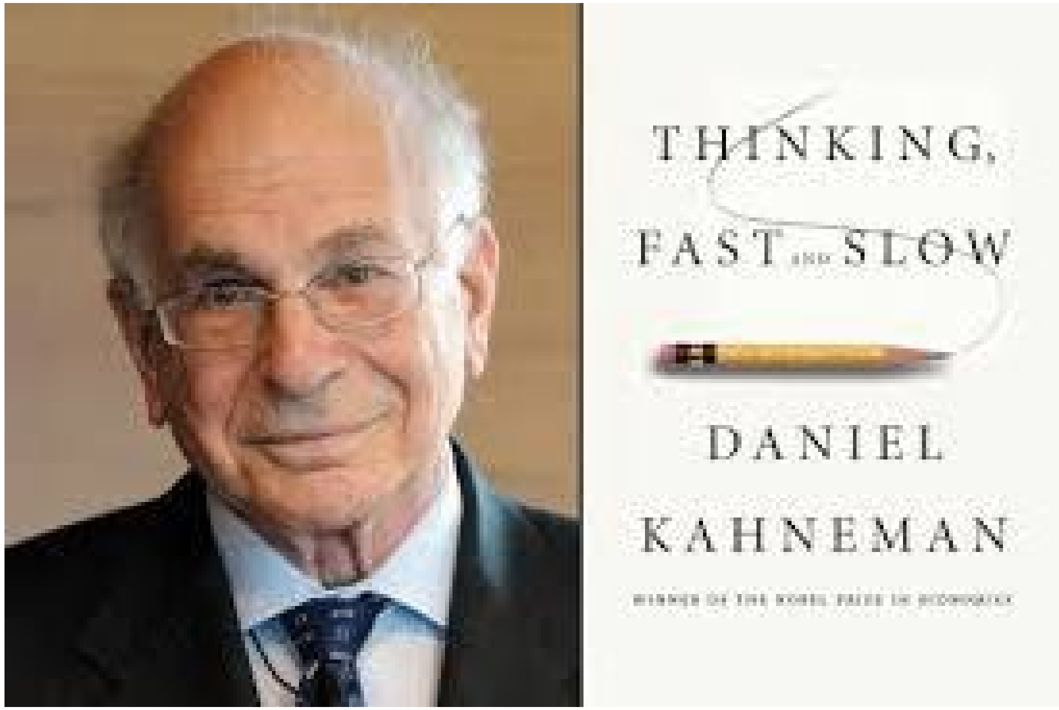
Hersh Shefrin

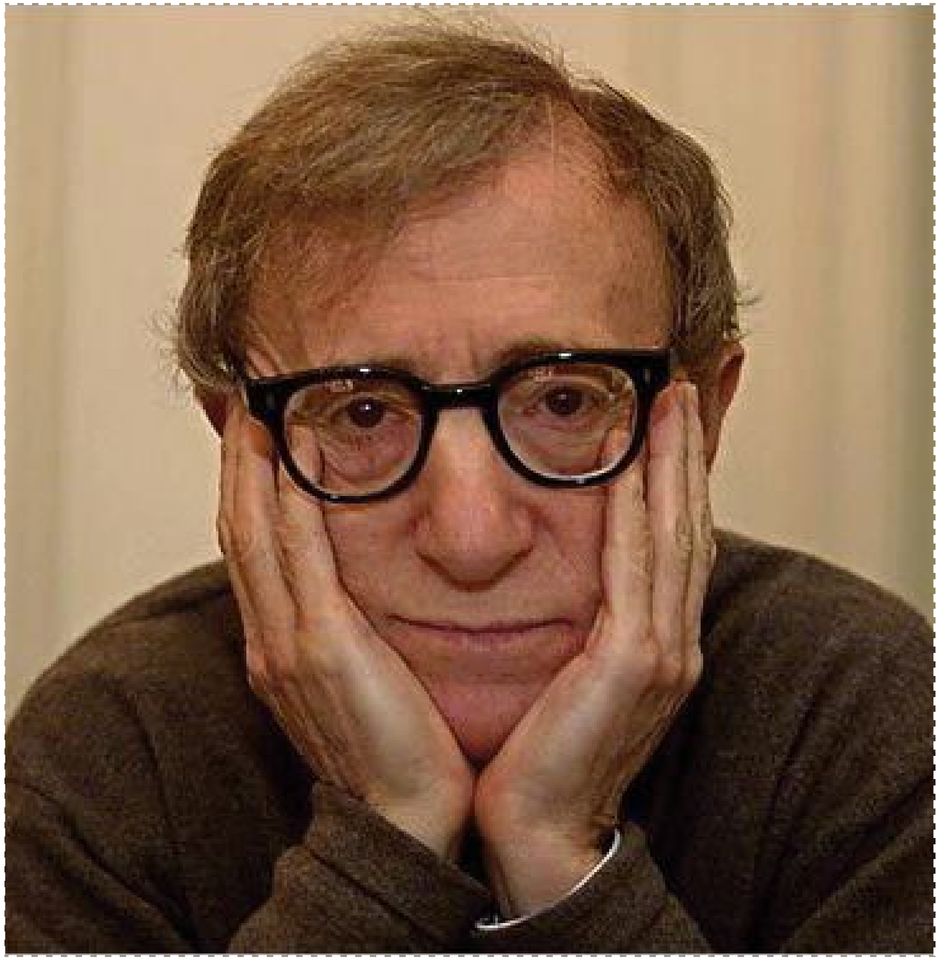
Meir Statman

How Do We Decide: A Tale of Two Systems

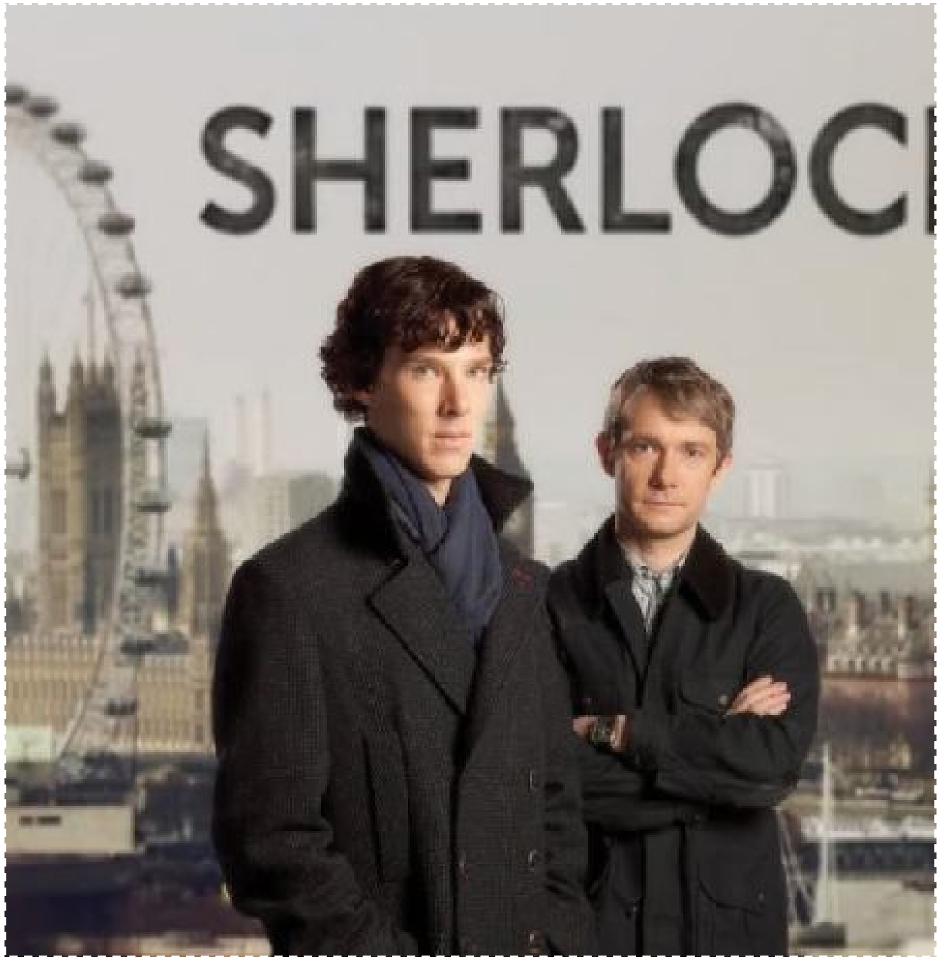
- **System 1** is fast; it's intuitive, associative, metaphorical, automatic, impressionistic, and it can't be switched off.

- **System 2** is slow, deliberate, effortful. Its operations require attention.





Not rational?



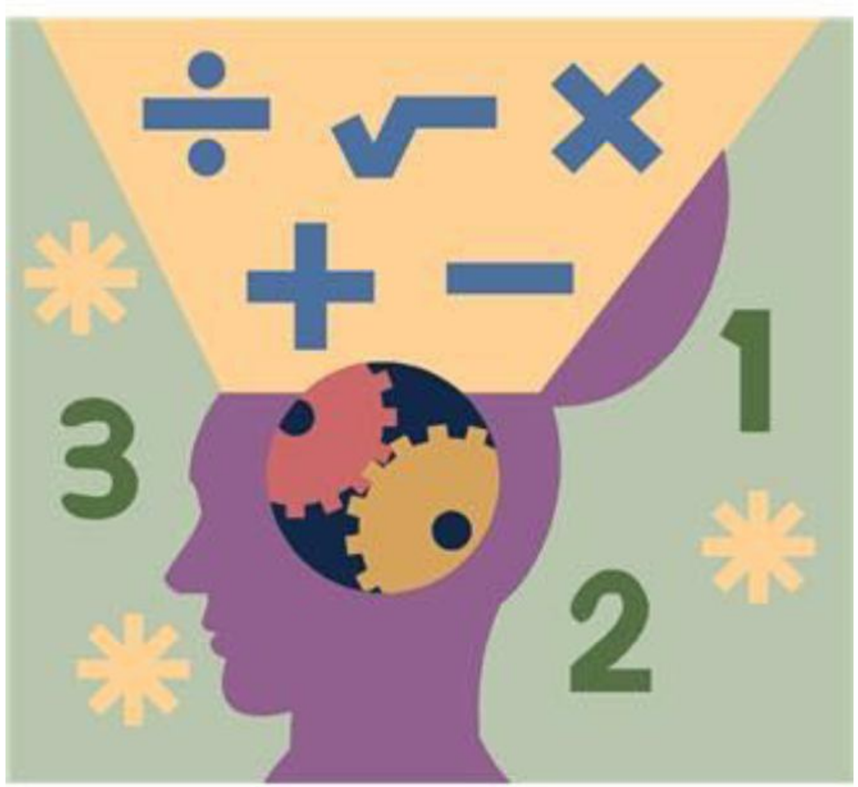
Too rational?

Woody vs. Sherlock: both ends of the spectrum?

How Does The Brain Work?

Reflexive

Reflective



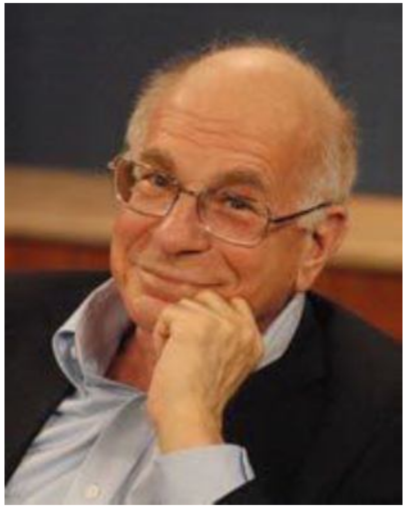
Impact On Our Decision Making

- The coexistence of two information-processing systems complicates our decision making:
  - The brain is hardwired to like short-term gratification. → **make quick and easy decisions.**
  - We tend to dislike social-exclusion behaviour. → **adopt to herd-like decisions.**
- As a result, these innate tendencies often cause less-than-rational decisions (despite our hopes/beliefs that we are logical decision makers).

[Montier, 2007]

Important Contributors

- 3 of the biggest names associated with the field:
  - Daniel Kahneman and Amos Tversky
  - Richard Thaler



Standard vs Behavioural Finance

Standard Finance

- Investors are rational.
- Markets are efficient.
- Apply mean-variance portfolio theory in portfolio design.
- Expected returns are a function of risk and risk alone, following capital asset pricing theory (Capital Asset Pricing Model)

Expected Utility Theory

The Efficient Market Hypothesis

Behavioural Finance

- Investors are normal, not rational.
- Markets are not efficient
- Apply Behavioural Portfolio Theory (BPT) in portfolio design.
- Expected returns follow Behavioural asset pricing theory (Behavioural Asset Pricing Model).

Prospect Theory

Question

- Imagine a bingo cage containing 36 balls, numbered from 1 to 36.
- Consider two risky alternatives whose outcomes depend on the number on a single ball drawn from the bingo cage.
- A. If the number drawn is 29 or less, you win $20; otherwise you win $0.
- B. If the number drawn is 30 or more, you win $90; otherwise you win $0.

1. What is the maximum amount you would be willing to pay for the opportunity to face alternative A?

2. What is the maximum amount you would be willing to pay for the opportunity to face alternative B?

3. If you could only choose one of these alternatives, which one would you choose, A or B?

Concept Preview: Answers

- Expected payoff
  - Alternative B: $17.50=(7/36)*90
  - Alternative A: $16.10=(29/36)*20
- Many people, typically 40%, assign a higher willingness to pay (WTP) to the **riskier alternative B** than the safer alternative A.

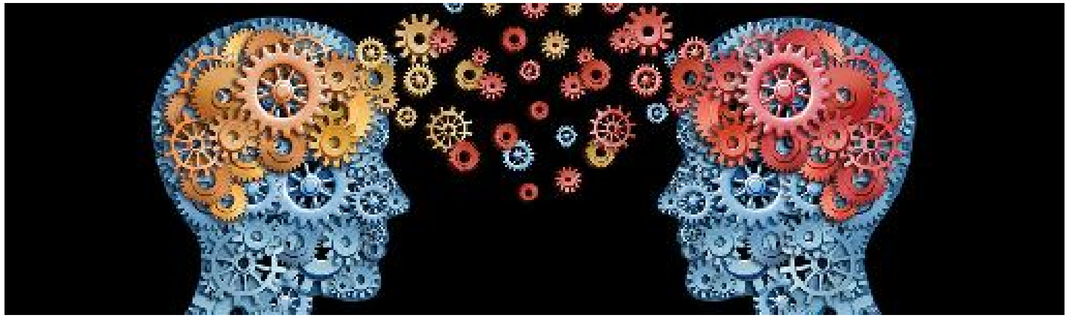
Study: Affect Heuristic

- Psychological studies have used this question to investigate
- how people make decisions, and
  - assign value when the outcomes are risky.
- The major conclusion from these studies is
  - that people rely heavily on the affect heuristic, and
  - that doing so leads them to attach a lower value to the alternative they choose than the alternative they reject.

10 Psychological Phenomena in Finance

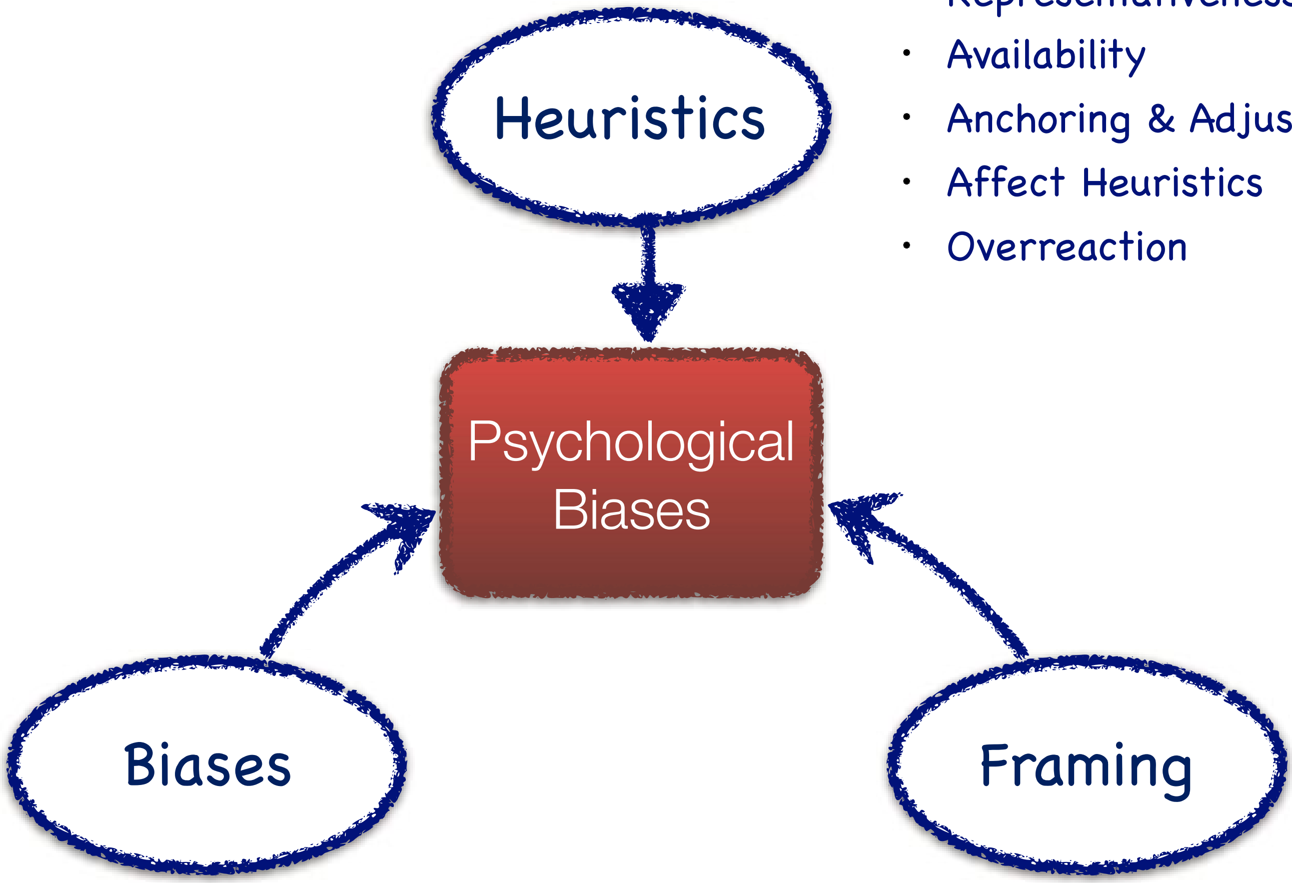
- These phenomena cause us to make faulty decisions, individually and in a group.

- They are divided into:
  - **Heuristics**,
  - **Biases**, and
  - **Framing effects**



Psychological Phenomena

- Representativeness
- Availability
- Anchoring & Adjustment
- Affect Heuristics
- Overreaction



- Excessive Optimism
- Overconfidence
- Confirmation Bias
- Illusion of Control

- Loss Aversion
- Aversion to a Sure Loss

Chapter 1 - Heuristics

- “When individuals are faced with a complex judgment involving a statistical probability, frequency or incomplete information, many subjects usually utilize a limited number of heuristics that reduce the decision to a simpler task”.

[Kahneman, Slovic, and Tversky, 1982]

Defining Heuristics

Heuristics

Defining Heuristics

Heuristics (noun): A heuristic process or method.

Heuristics (adjective):

- Enabling a person to discover or learn something for themselves.
- Proceeding to a solution by trial and error or by rules that are only loosely defined.

Trial and Error: The process of experimenting with various methods of doing something until one finds the most successful.

[Oxford English Dictionary]

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[Ricciardi & Simon 2001]

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[Ricciardi 2004]

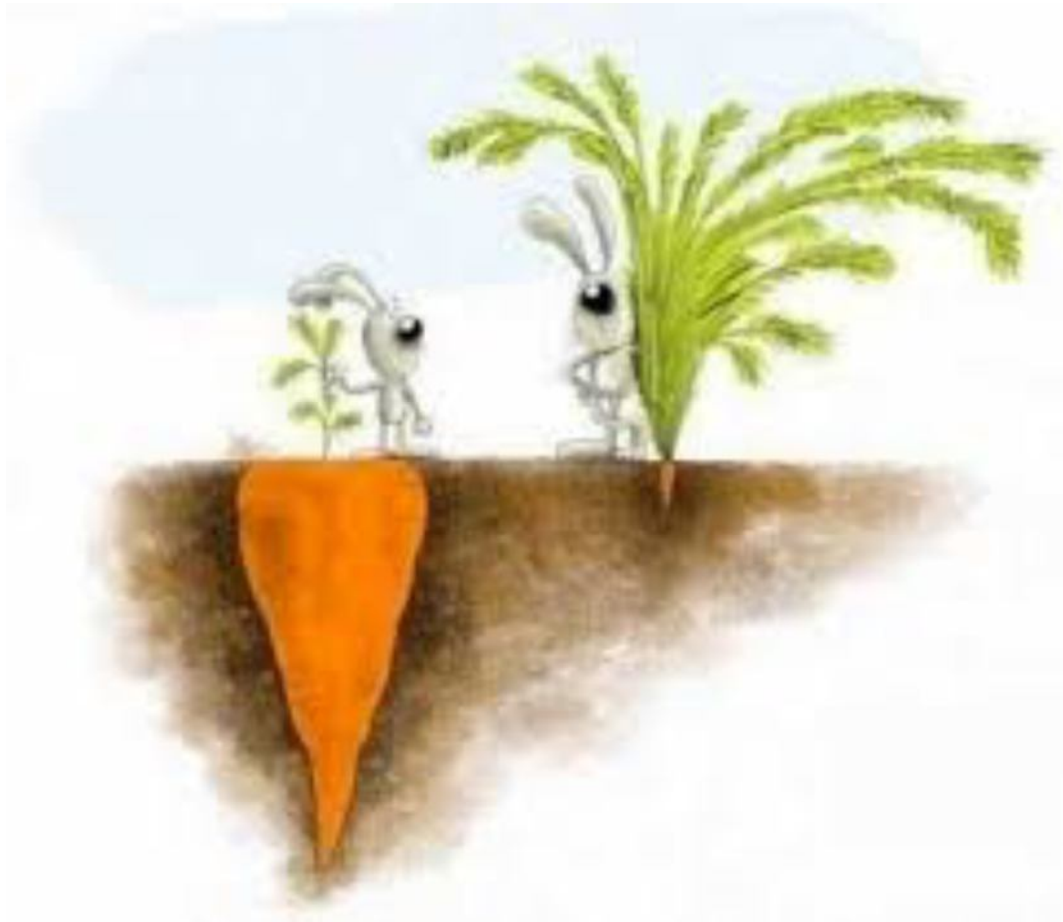
Heuristics give us something close... but not exact!!!

4 Heuristics

- We will examine 4 heuristics:

1. **Representativeness**
2. **Availability**
3. **Anchoring & Adjustment**
4. **Affect Heuristics**

Heuristics #1: Representativeness



Decision Problem: Representativeness

- One measure of successful admission decision to universities is that students who are admitted perform well scholastically. Imagine a situation where an admission officer is attempting to predict the grade point average (GPA) of some prospective students based upon their high school GPA levels. Consider the following data:

- For the past three years, the mean high school GPA of students who entered as freshmen and graduated was 3.44 (standard deviation was 0.36). The mean college GPA of those same students was 3.08 (standard deviation 0.40).

- Suppose you are given the task of predicting the graduating GPA for three undergraduate students, based solely on their high school GPA scores. The three high school GPA scores are 2.20, 3.00, and 3.80. What are your predications for the college GPAs of these students upon graduation?

[Shefrin 2002]

How Did You Answer?

How Do Most People Answer?

- Shefrin administered this question to large groups → very consistent mean responses:

- Actual GPAs are closer to the mean than predicted GPAs

[Shefrin 2002]

How Do Most People Answer?

- Shefrin administered this question to large groups → very consistent mean responses:

High School GPA	Predicted College GPA	Actual College GPA
2.20	2.03	2.70
3.00	2.77	2.93
3.80	3.46	3.30

- Actual GPAs are closer to the mean than predicted GPAs

[Shefrin 2002]

How Do Most People Answer?

- At both the low end and high end, the predictions are **far from the mean** of 3.08!
  - illustrates that people do not appreciate regression to the mean.
- Keep in mind that the question pertains to those who graduated from college.
  - **Survivorship bias**

[Shefrin 2002]

Concept: Representativeness

- **Representativeness** is the tendency to rely on stereotypes to make a judgment. [Shefrin 2010]

Applying the Definition

- Based on this principle,
  - The simplest is to predict that college GPA will be the same as high school GPA.
- But in fact, people base their predictions on **how representative a student appears to be**.
  - A student with a high GPA in high school is seen as representative of a good student.
  - Especially hard on students with low high school GPAs.
- This fails to recognise **regression to the mean**.
- Therefore, they are predisposed to making errors when they predict the future GPA of particular individuals.

When We Rely on It,

- Representativeness can lead us to:
  - Overweight recent evidence (**recency bias**),
  - Draw inferences which are too strong in the face of small samples ("**law of small numbers**"),
  - Overpredict reversals when we know the process governing transitions ("**gambler's fallacy**"), and
  - Overpredict continuation when we infer the process from recent observations ("**hot hand fallacy**").

[Shefrin 2010]

From An Investment Perspective

- Montier points out:
  - People like a good story rather than hard facts, and if numbers are involved, people are often quite bored.
- For example,
  - Investors view dividends as boring, small numbers. They lack the “story” that growth stocks offer.
  - Montier notes that more than half the historic return of U.S. stocks has come from their dividends.

[Montier, 2007]

“Good” Firm Vs “Good” Investment

1. **Ethical / socially responsible**: Investors might feel that all firms with management promoting environmental awareness are “good” firms (and then go on to infer that their stocks are a good investments).

2. **Good earnings announcements**: Investors interpret all good earnings announcements as predictors of good future performance, without determining whether the performance will continue for the individual firm making the announcement.

“Good” Firm Vs “Good” Investment

3. **Investors tend to form opinions on stocks based on very limited information.**

    - Like with people, they may view a stock as “good” because the company’s chairman is the chairman of another successful company, when in fact the chairman may be the only similarity and therefore, categorising these two companies together may be a big mistake.

“Good” Firm Vs “Good” Investment

4. Investors might be especially bold in predicting that the future return of a particular stock will be very favorable because its past long-term performance has been very favorable.

    - This is because they form the judgment that favorable past performance is representative of good stocks.

    - However, representativeness leads such predictions to be overly bold, because of insufficient attention to factors that induce regression to the mean.

[Shefrin 2010]

Impact on Investment Decision Making

- Representativeness can take many forms.
  - Anytime an investor (or anyone else for that matter) bases expectations for the future on some past or current characteristic or measure, the individual is applying an **"if-then" heuristic**.
  - That is, **"IF this has happened, THEN that will happen."**

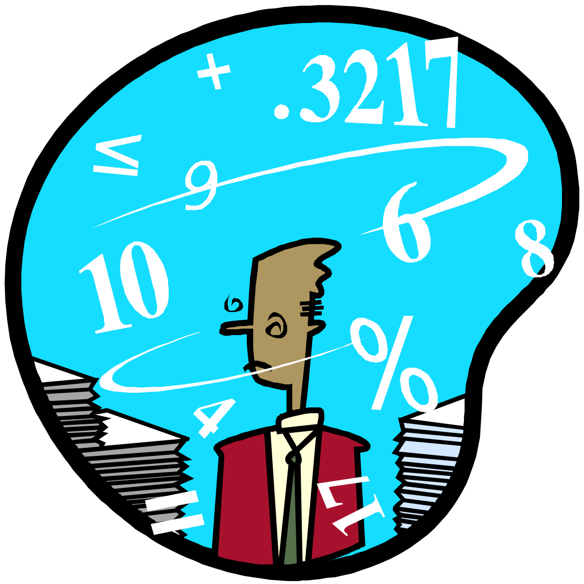
Decision Problem: Biases Associated with Representativeness

- If five tosses of a fair coin all turn out to be heads, what is the probability that the sixth toss will be a tail?

How Did You Answer?

How Do Most People Answer?

- **"A tail is due"**
- After a run of five heads, people tend to predict tails on the sixth toss, because of the representativeness heuristic.



Heuristics #2: Availability

Decision Problem: Availability Bias

- Which is the more frequent cause of death in the United States, homicide or stroke?

How Did You Answer?

- Homicide:

- Stroke:

How Do Most People Answer?

- The majority rely on recall.
- By seeing how many events of each type come readily to mind.
- If people more readily recall instances of homicide than of stroke → answer "homicide."

How Do Most People Answer?

- Why? & How?
  - In this case, most people rely on the media for their information about homicides and strokes.
  - Suppose that the media tends to report one case of death more than the other, because one is newsworthy and the other is not.
  - Then people who rely on an availability heuristic may recall instances related to one type of death more readily than the other.

Concept: Availability Heuristic

- **Availability** is the degree to which information is readily available.

- **Availability heuristic** is the tendency to form judgments based on information which is readily available, but to underweight information that is not readily available. [Shefrin 2010]

Concept: Availability Heuristic

- According to the availability heuristics,
  - People tend to heavily weigh their decisions toward more recent information, making any new opinion biased toward that latest news.
- Availability bias can result in us paying more attention to companies covered heavily by the media.

Flying or Driving?

Availability Bias: Shark Attack?

- By Franklin Templeton Investments

- https://www.youtube.com/watch?v=od33LBtXtO0results_video

[Franklin Templeton Investments]

Heuristics #3: Anchoring and Adjustment

Decision Problem: Anchoring

- You will see a number showing up. Please put a check mark.
- Please indicate whether the percentage of U.N. membership accounted for by African countries is higher or lower than the number shown.
- Then, give your actual estimate.

[Shefrin 2002]

How Did You Answer?

How Do Most People Answer?

- The random number had an anchoring effect on the subjects' responses,

→ pulling their estimates closer to the number they were just shown,

→ even though the number had absolutely no correlation at all to the question.

[Shefrin 2002]

How Do Most People Answer?

- In a 1974 paper entitled "Judgment Under Uncertainty: Heuristics And Biases", Kahneman and Tversky conducted a study in which a wheel containing the numbers 1 though 100 was spun.

- Tversky and Kahneman found that the seemingly random anchoring value of the number on which the wheel landed had a pronounced effect on the answer that the subjects gave.

- For example,

  - When the wheel landed on 10, the average estimate given by the subjects was 25%,

  - Whereas when the wheel landed on 60, the average estimate was 45%.

Decision Problem: Anchoring

- You will see a number showing up. Please put a check mark.

- How would you answer these two questions?

    - Is the population of Turkey greater than the number shown in the previous question?

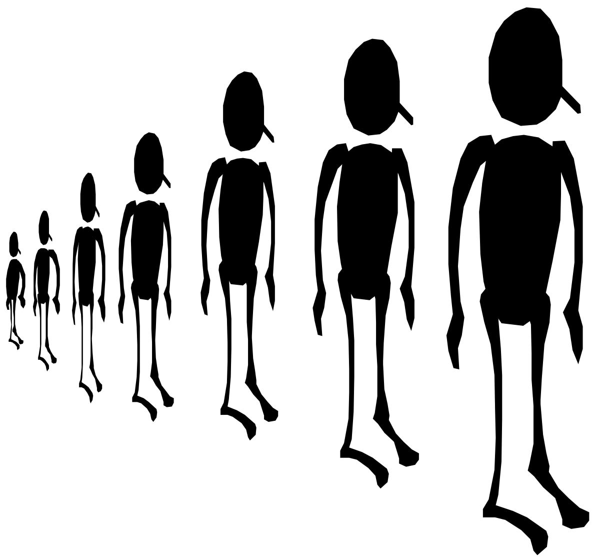
    - What's your best estimate of Turkey's population in millions?

[Ricciardi 2004]

How Did You Answer?

How Do Most People Answer?

- Most people who reply to the 2nd question are influenced by the population of 35 or 100 million figure that was revealed in the 1st question even though it has no factual foundation.



[Ricciardi 2004]

Why?

- These findings reveal that when people make judgments, **their minds give inappropriate significance (or overweighs) the importance of the initial information it obtains.**

- Thus, first impressions, rough calculations, or statistical figures anchor subsequent thinking and choices. [Hammond, Keeney, and Raiffa 1998]

[Ricciardi 2004]

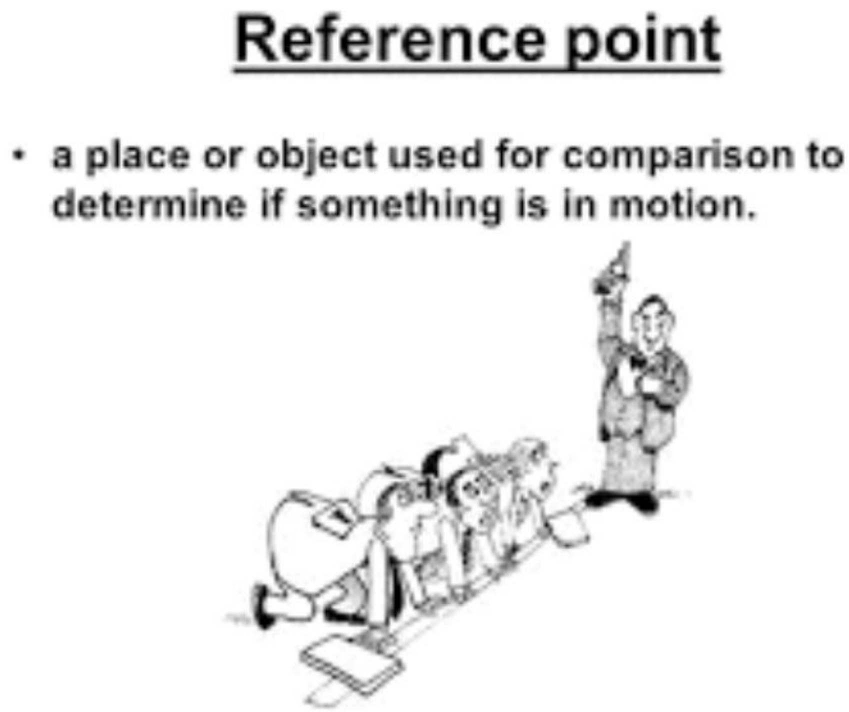
Concept: Anchoring

- **Anchoring** demonstrates the tendency to attach or "anchor" our thoughts to a **reference point** - even though it may have no logical relevance to the decision at hand.

- A **reference point** might be a purchase price used to define gains and losses, or a level of aspiration. [Shefrin 2009]

Reference Point–Induced Risk Seeking

- **Reference point-induced risk seeking** is the tendency of people to behave in a risk-seeking fashion to avoid an outcome that lies below the reference point. [Shefrin 2009]



Anchoring on What?

- Anchoring on an **aspiration**:
  - Being the industry leader.
- Anchoring on **historical perceptions**:
  - Good company vs. good investment
- Anchoring on **initial data/ perceptions**:
  - Such as purchase price, historical price, opening bid.
- Anchoring on **meaningless numbers**:
  - Tversky and Kahneman’ 1974 “spin the wheel” study.

Warren Buffet vs. Anchoring

- "When I bought something at X and it went up to X and 1/8th, I sometimes stopped buying, perhaps hoping it would come back down. We've missed billions when I've gotten anchored.

- It cost us about $10 billion [by not buying enough Wal-Mart].

- I set out to buy 100 million shares, pre-split, at $23. We bought a little and it moved up a bit and I thought it might come back a bit –who knows?

- That thumb-sucking, the reluctance to pay a little more, cost us a lot."

Decision Problem: Conservatism

- Imagine 100 book bags, each of which contains 1,000 poker chips.
    - Forty-five bags contain 700 black chips and 300 red chips.
    - The other 55 bags contain 300 black chips and 700 red chips. You cannot see inside either bag.
- One of the bags is selected at random by means of a coin toss.

[Shefrin 2002]

Decision Problem: Conservatism

- Consider the following two questions about the book bag.

  a) What probability would you assign to the event that the selected bag contains predominantly black chips?

  b) Now imagine that 12 chips are drawn, with replacement, from the selected bag.

  These twelve draws produce 8 blacks and 4 reds.

  Would you use the new information about the drawing of chips to revise your probability that the selected bag contains predominantly black chips?

  If so, what new probability would you assign?

[Shefrin 2002]

How Did You Answer?

- a)
- b)

How Do Most People Answer?

- The first question: the most frequent answer given is 45%.
- The second question: the most frequent answer given is 45% or 67%
  - A lot more difficult than the first.
  - 45%: the number of bags containing predominantly black chips.
  - 67%: the fraction of black chips drawn with replacement.

[Shefrin 2002]

Why 45% or 67% for Question B?

- Possible reasoning of the answers:
  - Those who respond with 45% essentially do not know how to incorporate the new information. → **they stick to their initial beliefs.**
  - People who answer 67% focus on the fact that 8/12 = 2/3 of the chips drawn with replacement are black. → **they ignore their prior information.**

[Shefrin 2002]

What Is the Correct Answer for Question B?

- The correct answer to the second questions is 96.04%.

- Most people respond too conservatively to the new information in this problem.

- They are perhaps anchored on to 45% and do not adjust sufficiently to the new information.

[Shefrin 2002]

So, How Did We Get To 96.04%?

Using Bayes' Rule!

Bayes' Formula

- **Bayes' formula** is closely related to the multiplication rule and the total probability rule.

- Bayes' formula is routinely used to update probabilities when new information becomes available:

$$\begin{aligned}&\text{Updated probability}\\ = \;&\frac{\text{Probability of a new information for a given event}}{\text{Unconditional probability of new information}}\\ &\times \text{Prior probability of the event}\end{aligned}$$

Bayes' Formula

- Mathematically,

$$P(E|I) = \frac{P(I|E)}{P(I)} \times P(E)$$

- A bit of jargon:
    - $P(E)$ is called the **prior** probability. It is the uninformed probability of event E.
    - $P(E|I)$ is called the **posterior** probability and represents the probability of event E once we have incorporated information I.
    - $P(E|I) = \mathcal{L}(E|I)$ is the **likelihood** of information I given observation E.
    - $P(I)$ is the **marginal likelihood** or **evidence**. This probability is often regarded as a **normalisation constant**.

Thomas Bayes (1701-1751)

Symmetry

- Bayes' formula is a direct application of the **multiplication rule**:

$$P(I) \times P(E|I) = P(E) \times P(I|E)$$

- The multiplication rule reveals an interesting **symmetry** in Bayes' formula:

$$P(E|I) = \frac{P(I|E)}{P(I)} \times P(E) \qquad P(I|E) = \frac{P(E|I)}{P(E)} \times P(I)$$

  - Bayes' formula does not imply any causality between E and I. Both events/parameters are treated equally!
  - We could start with I to get E, or alternatively start from E to get I.

- Up until the 1930s, this symmetry meant that Baye's formula was considered with suspicion by most statisticians and probabilists.
  - The classical view of statistics assumes that
    - the data (or measurements) contain error, while
    - the parameters of the underlying distribution are viewed as constant.
  - **Inverse problems**, in which
    - the data (or observations) are assumed to be accurate, but
    - the parameters of the distribution are viewed as random

    were frowned upon as "unnatural".

- The attitude has changed radically since the 1950s:
  - recognition that parameters need to be estimated and that their estimator is noisy!
  - rise of personal probabilities and decision theory;
  - development of dynamic parameter estimation, and statistical learning and information theory;
  - Successful applications to
    - *aeronautics*: filtering techniques to help direct the first missions to the moon;
    - *telecommunications*: digital signal processing, speech recognition;
    - *data science and computing*: machine learning!

Bayes' Formula is so elegant that it has been made it to…

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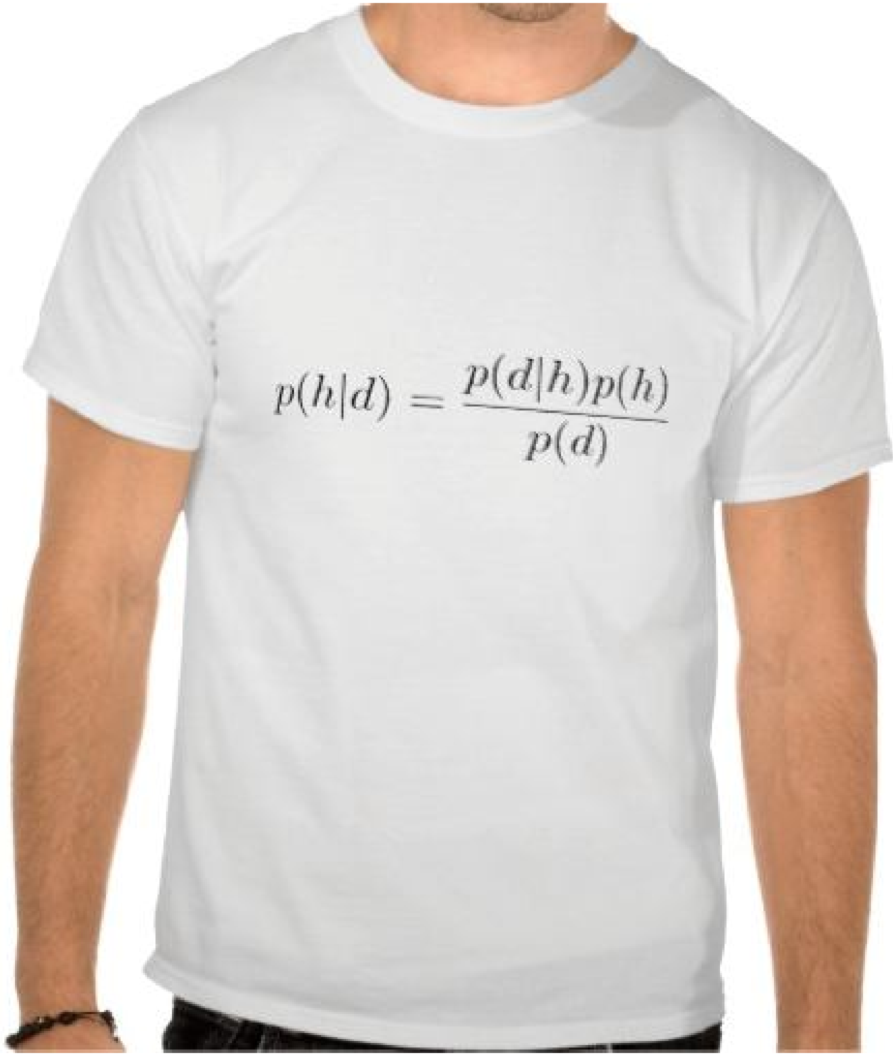
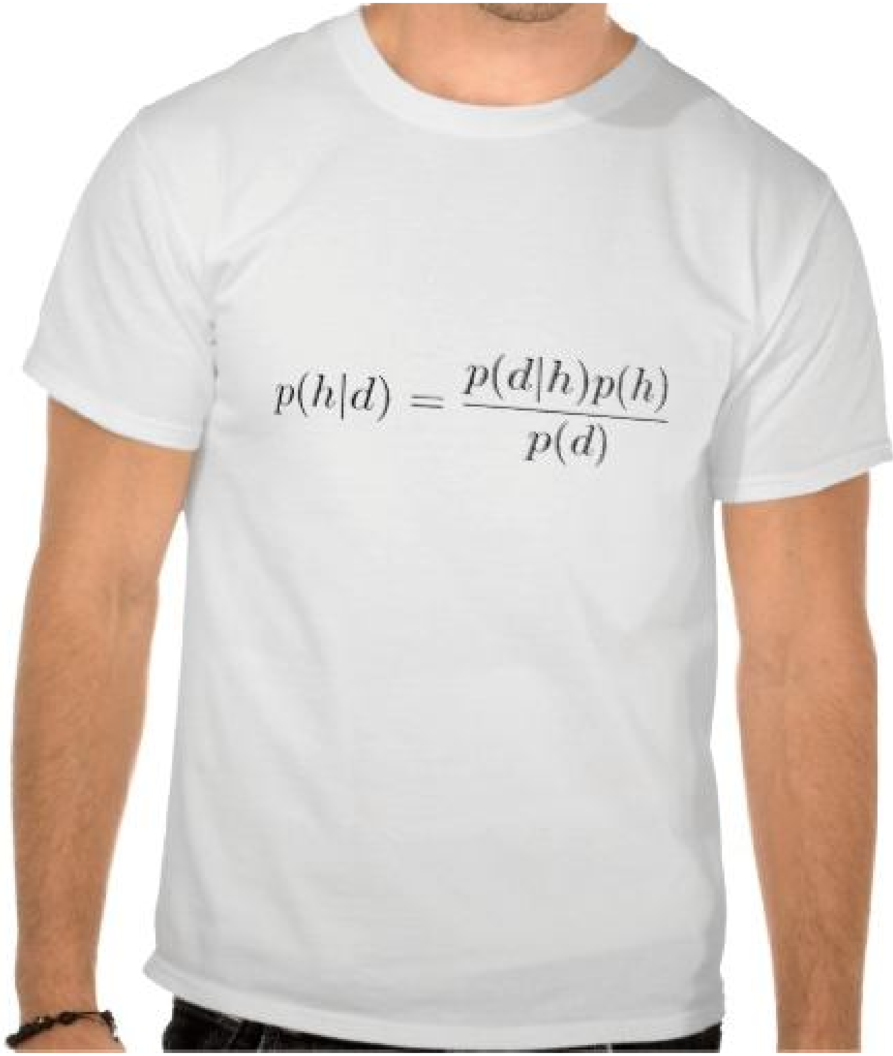
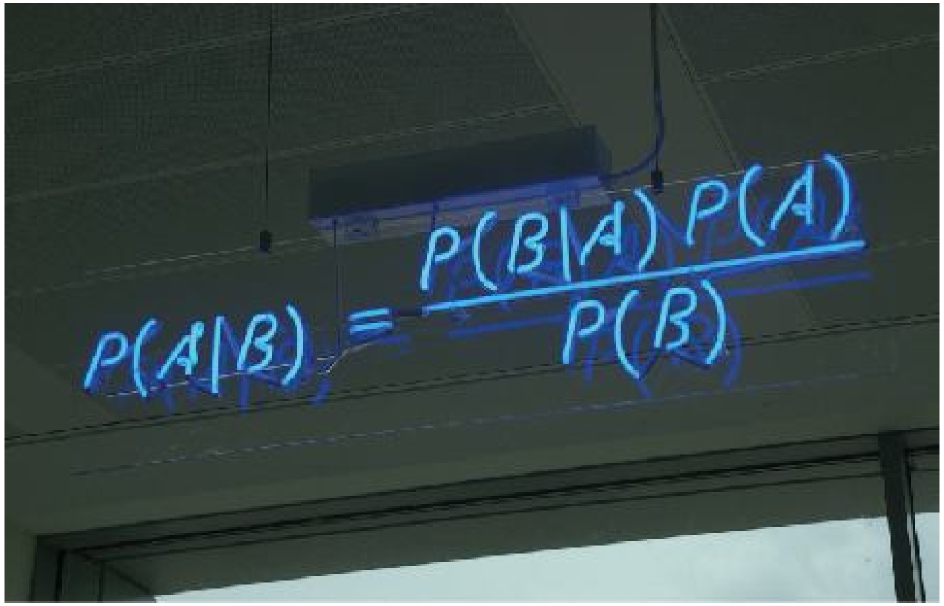


T-shirts…

Bayes' Formula is so elegant that it has been made it to…



T-shirts…



… and even neon signs!

L. Jimmie Savage

- Mathematician and statistician.
- Worked closely with John Von Neumann during World War II.
- Friend of both Paul Samuelson and Milton Friedman.
- Popularised the use of Bayes’ formula through his book “Foundations of Statistics” published in 1954.
- Among other things, Savage:
    - Worked on subjective probabilities,
    - Extended the von Neuman-Morgenstern definition of classical utility into subjective utility,
    - Formalised the concept of “ellicitability”,
    - Rediscovered Bachelier’s thesis on option pricing during one of his stays in Paris.

L. Jimmie Savage (1917–1971)

Solving the Problem

- Let’s start with a picture…

Solving the Problem

- Let's start with a picture…

Group 1: 45 bags

Solving the Problem

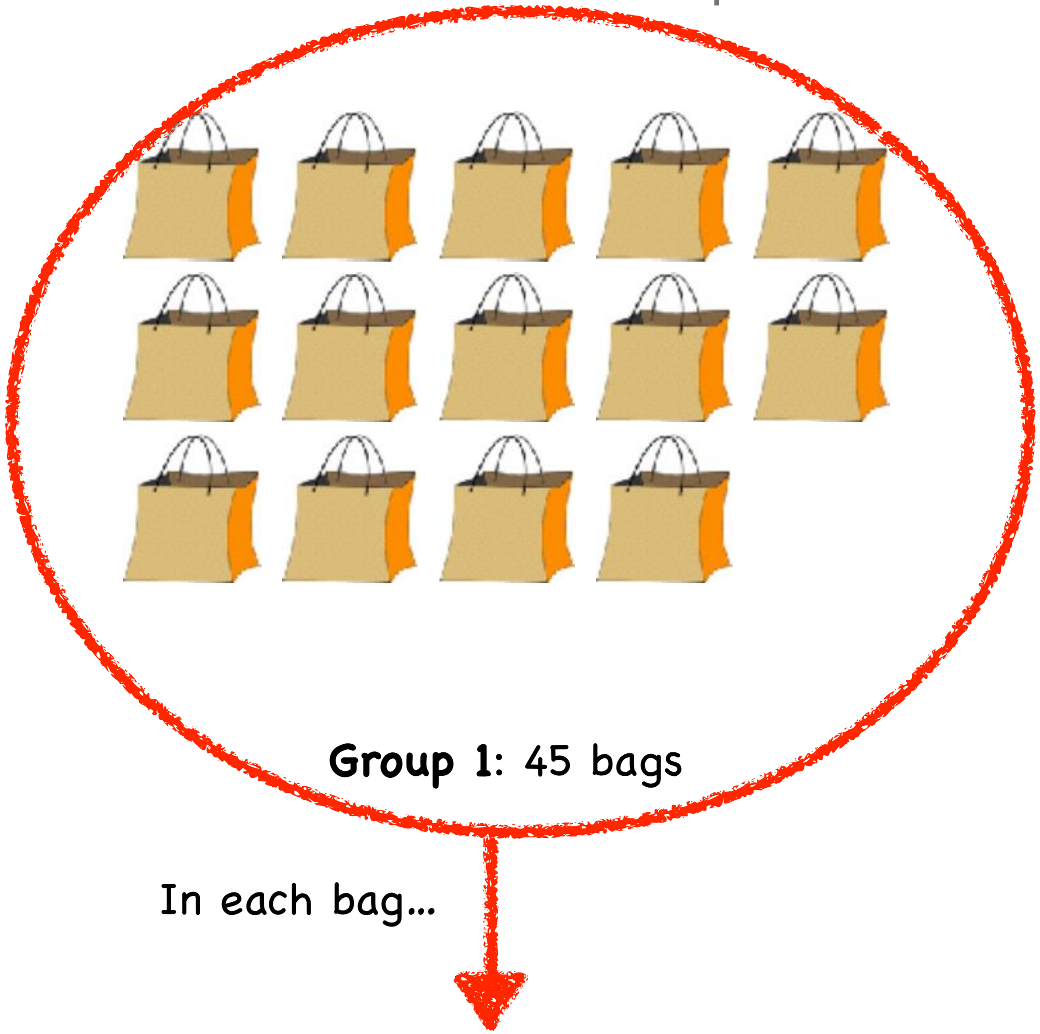
- Let's start with a picture…

Group 1: 45 bags

In each bag…

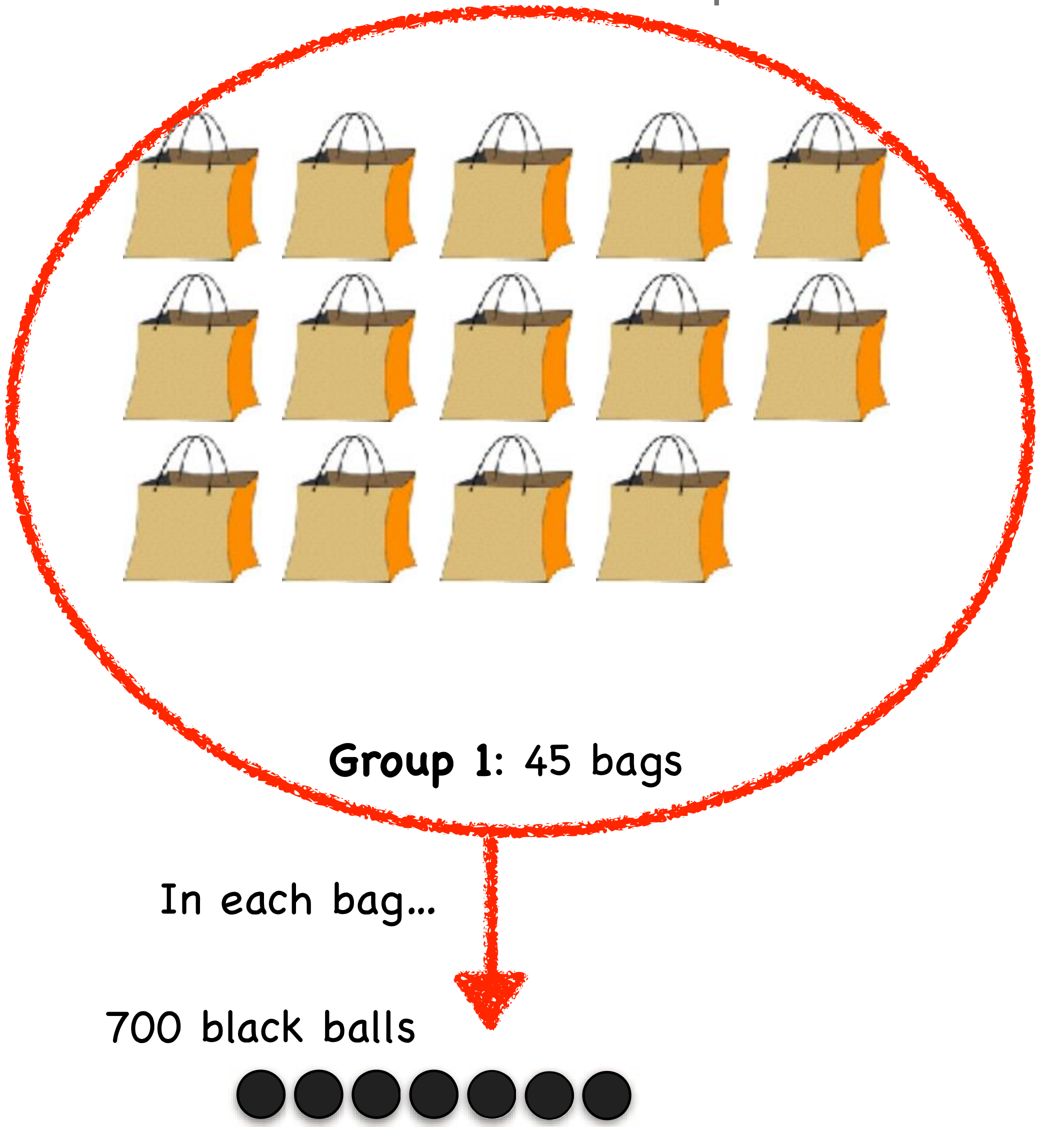
Solving the Problem

- Let's start with a picture…



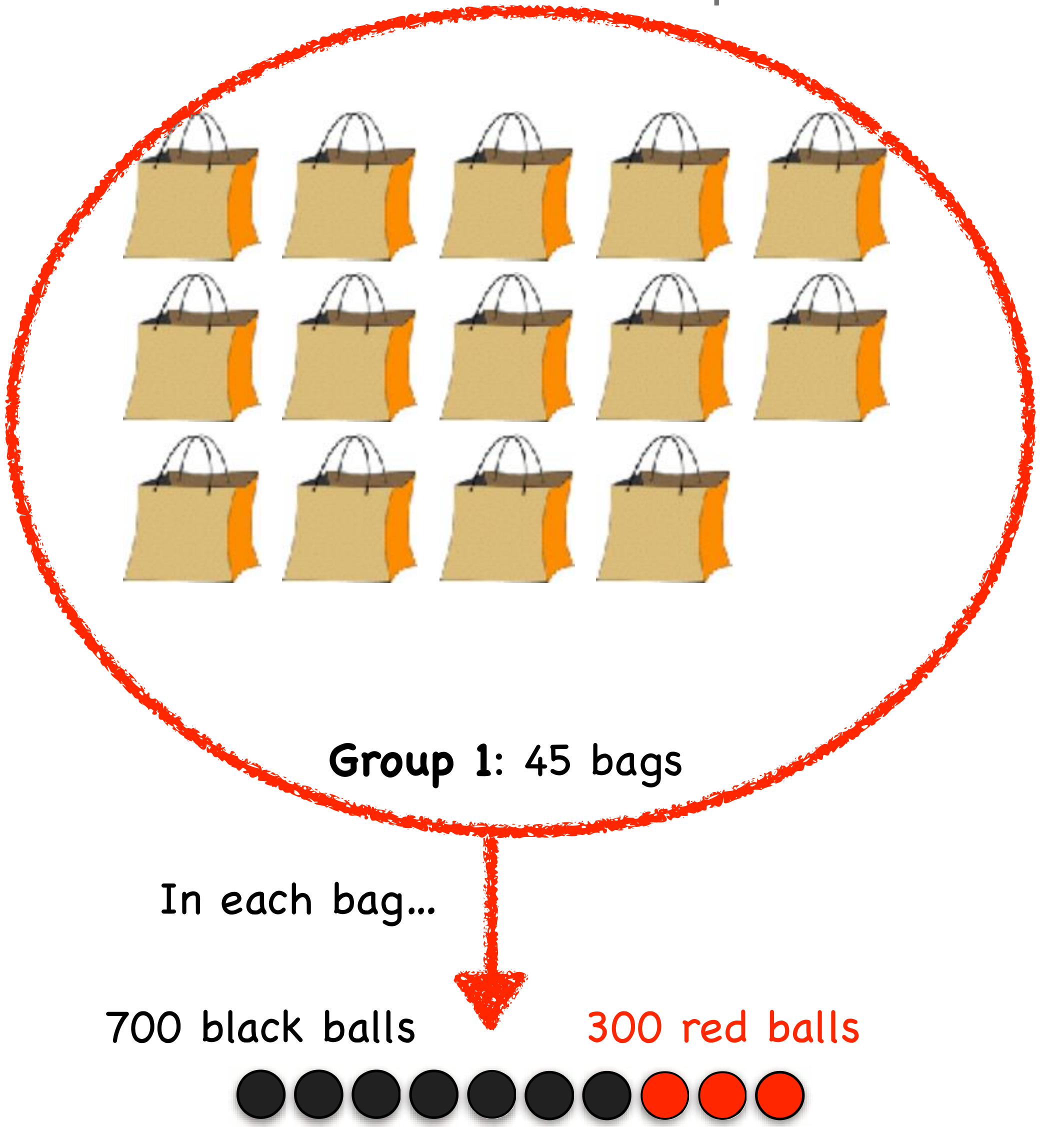
Solving the Problem

- Let’s start with a picture…



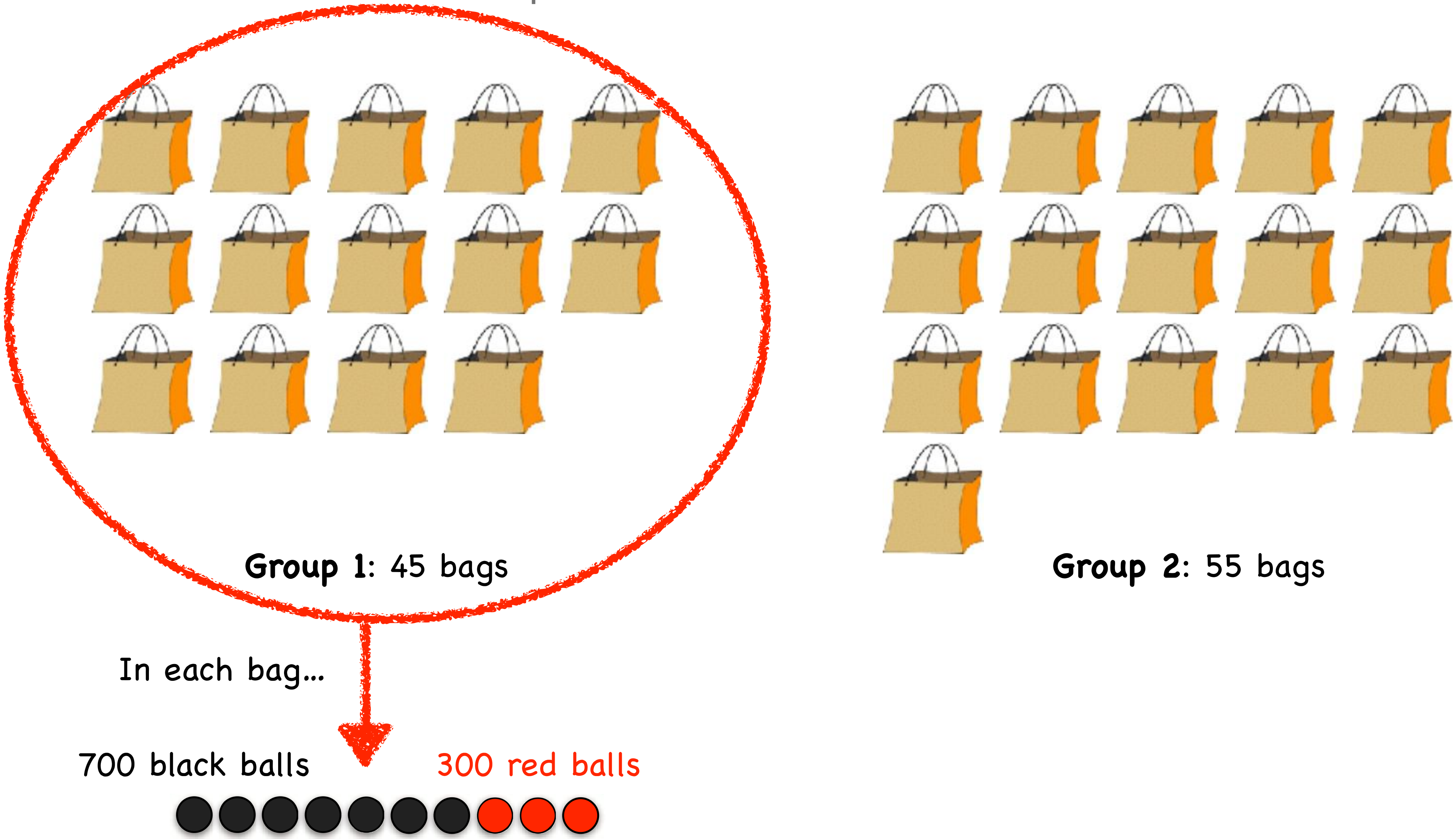
Solving the Problem

- Let’s start with a picture…



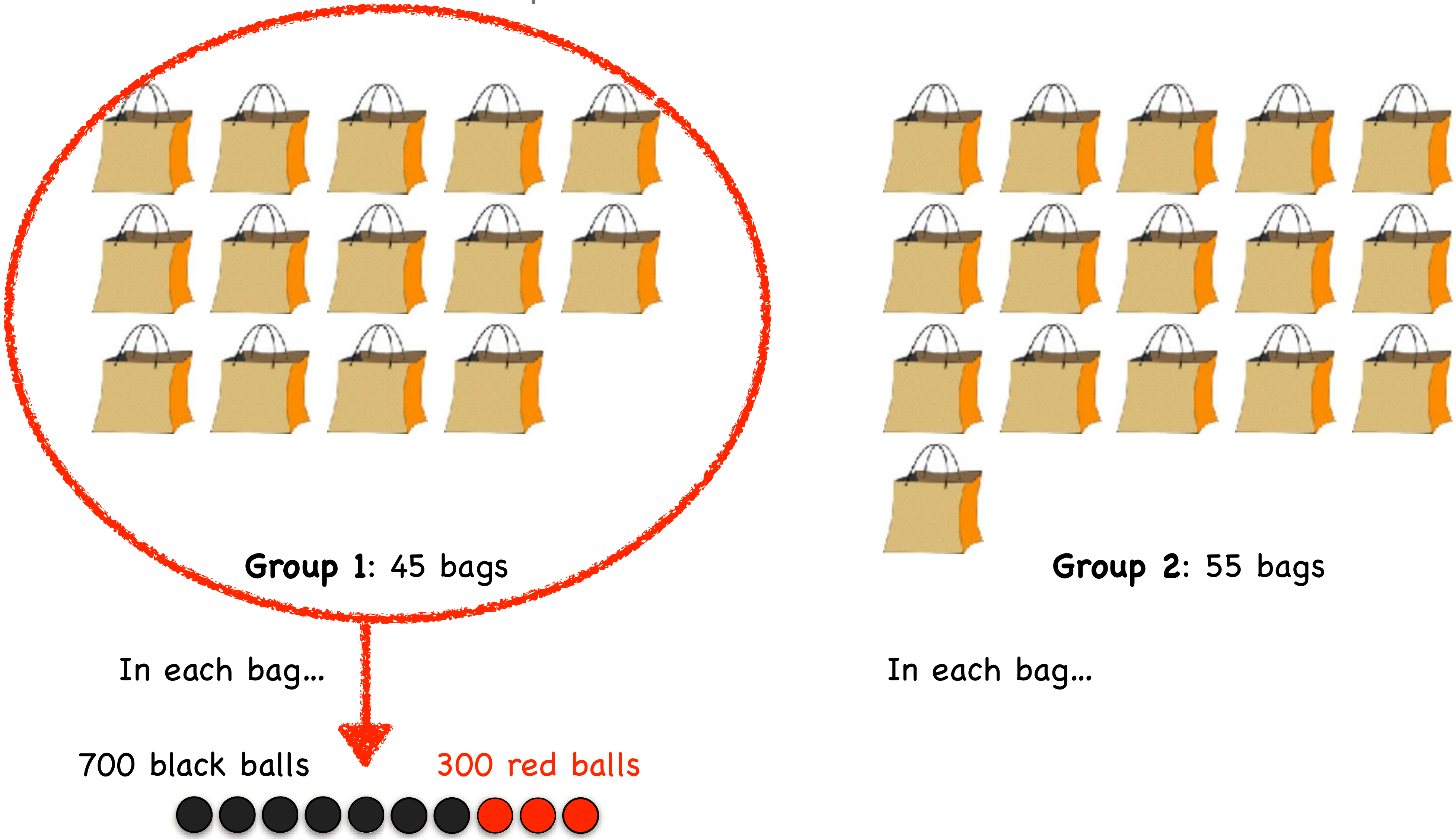
Solving the Problem

- Let's start with a picture…



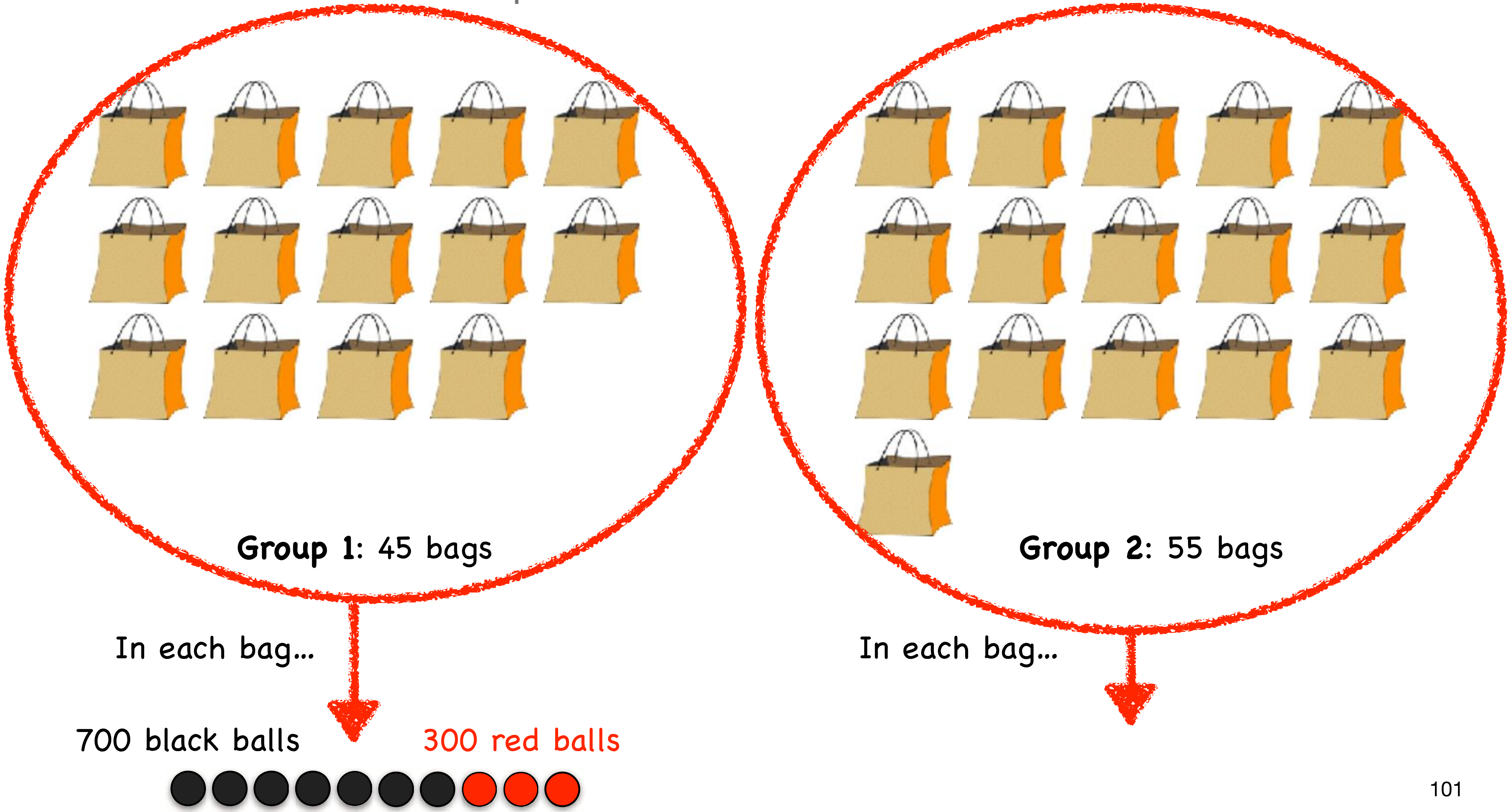
Solving the Problem

- Let's start with a picture…



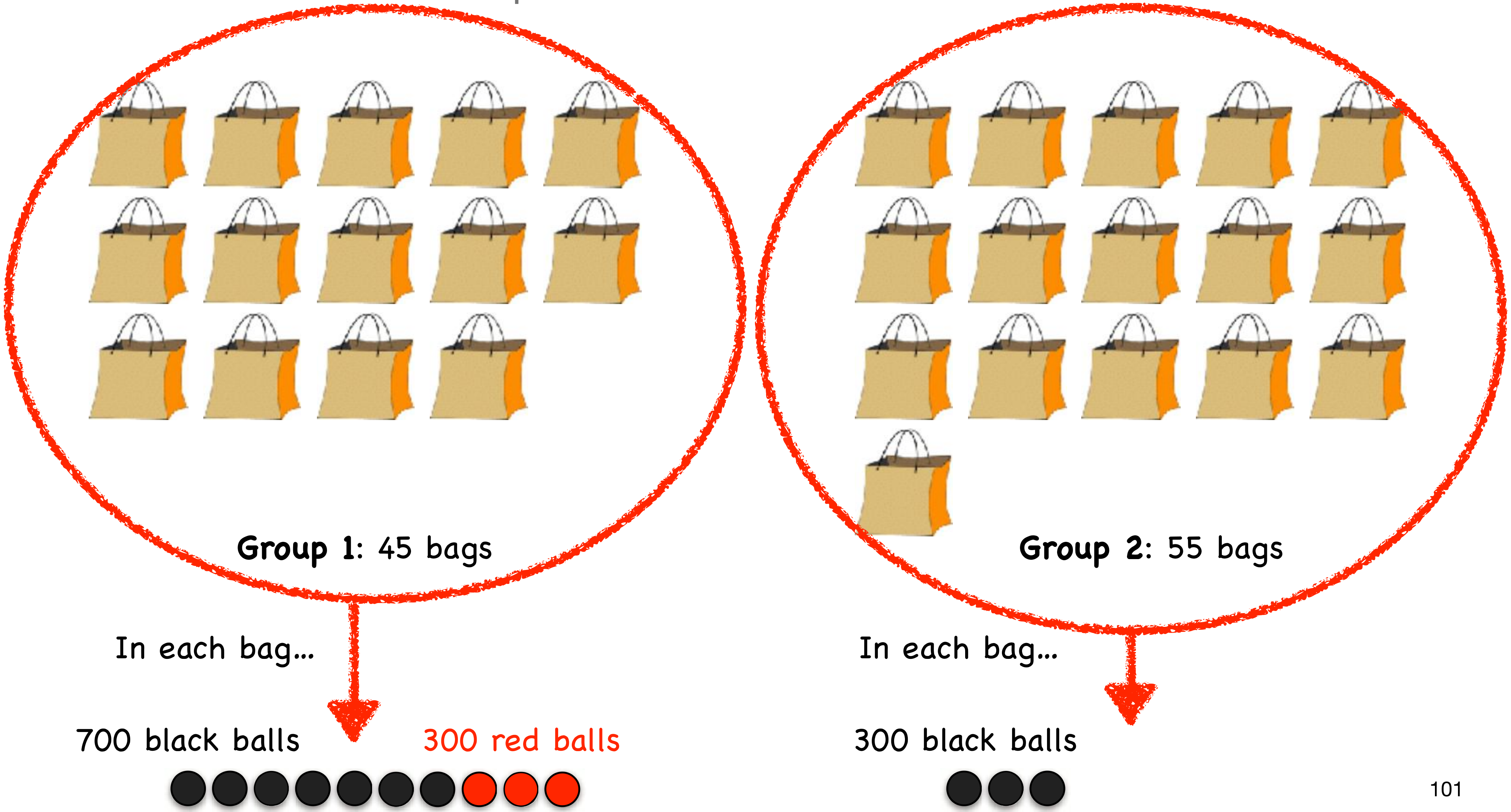
Solving the Problem

- Let's start with a picture…



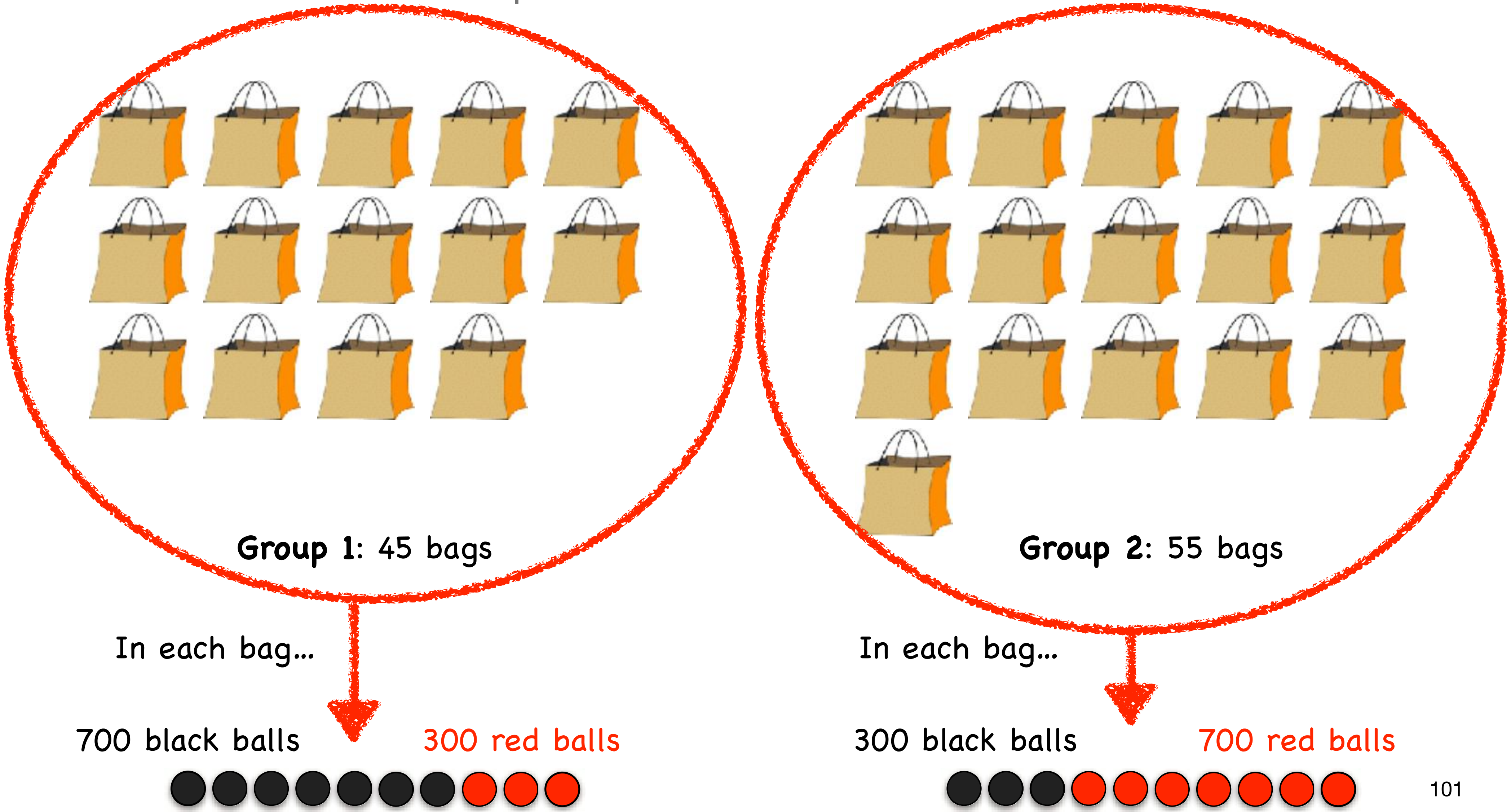
Solving the Problem

- Let’s start with a picture…



Solving the Problem

- Let's start with a picture…



Rewriting the Problem…

- In the first group of bags, denoted as $\mathbf{G_1}$:
  - Proportion of $\mathbf{G_1}$ bags in the total population of bags = 45/100 = 0.45
  - Proportion of ***black*** chips = 700/1000 = 0.7
  - Proportion of ***red*** chips = 300/1000 = 0.3
- In the second group of bags, denoted as $\mathbf{G_2}$:
  - Proportion of $\mathbf{G_2}$ bags in the total population of bags = 55/100 = 0.55
  - Proportion of ***black*** chips = 300/1000 = 0.3
  - Proportion of ***red*** chips = 700/1000 = 0.7

… And Now In Terms of Probabilities:

- In the first group of bags, denoted as $\mathbf{G_1}$:
  - Probability of drawing a $\mathbf{G_1}$ bag at random among the bag population:

$$P(G_1) = \frac{45}{100} = 0.45$$

  - Probability of drawing a ***black*** chip from a $\mathbf{G_1}$ bag:

$$P(B|G_1) = \frac{70}{100} = 0.70$$

  - Probability of drawing a ***red*** chip from a $\mathbf{G_1}$ bag:

$$P(R|G_1) = \frac{30}{100} = 0.30$$

- In the first group of bags, denoted as $\mathbf{G_2}$:
  - Probability of drawing a $\mathbf{G_2}$ bag at random among the bag population:

$$P(G_2) = \frac{55}{100} = 0.55$$

  - Probability of drawing a ***black*** chip from a $\mathbf{G_2}$ bag:

$$P(B|G_2) = \frac{30}{100} = 0.30$$

  - Probability of drawing a ***red*** chip from a $\mathbf{G_2}$ bag:

$$P(R|G_2) = \frac{70}{100} = 0.70$$

… And Now In Terms of Probabilities:

- In the first group of bags, denoted as $\mathbf{G_1}$:
  - Probability of drawing a $\mathbf{G_1}$ bag at random among the bag population:

$$P(G_1) = \frac{45}{100} = 0.45$$

  - Probability of drawing a ***black*** chip from a $\mathbf{G_1}$ bag:

$$P(B|G_1) = \frac{70}{100} = 0.70$$

  - Probability of drawing a ***red*** chip from a $\mathbf{G_1}$ bag:

$$P(R|G_1) = \frac{30}{100} = 0.30$$

- In the first group of bags, denoted as $\mathbf{G_2}$:
  - Probability of drawing a $\mathbf{G_2}$ bag at random among the bag population:

$$P(G_2) = \frac{55}{100} = 0.55$$

  - Probability of drawing a ***black*** chip from a $\mathbf{G_2}$ bag:

$$P(B|G_2) = \frac{30}{100} = 0.30$$

  - Probability of drawing a ***red*** chip from a $\mathbf{G_2}$ bag:

$$P(R|G_2) = \frac{70}{100} = 0.70$$

Note 1: as expected, these probabilities are just the "proportions" we calculated on the previous slide.

… And Now In Terms of Probabilities:

- In the first group of bags, denoted as **G_1**:
  - Probability of drawing a **G_1** bag at random among the bag population:
$$P(G_1) = \frac{45}{100} = 0.45$$
  - Probability of drawing a ***black*** chip from a **G_1** bag:
$$P(B|G_1) = \frac{70}{100} = 0.70$$
  - Probability of drawing a ***red*** chip from a **G_1** bag:
$$P(R|G_1) = \frac{30}{100} = 0.30$$

- In the first group of bags, denoted as **G_2**:
  - Probability of drawing a **G_2** bag at random among the bag population:
$$P(G_2) = \frac{55}{100} = 0.55$$
  - Probability of drawing a ***black*** chip from a **G_2** bag:
$$P(B|G_2) = \frac{30}{100} = 0.30$$
  - Probability of drawing a ***red*** chip from a **G_2** bag:
$$P(R|G_2) = \frac{70}{100} = 0.70$$

Note 1: as expected, these probabilities are just the "proportions" we calculated on the previous slide.

Note 2: $P(G_1)$ and $P(G_2)$ are unconditional probabilities. All the other probabilities are conditional.

Solving Question a)

- Group 1 bags contain predominantly ***black*** chips while Group 2 bags contain predominantly ***red*** chips.

- The probability that the bag contains predominantly ***black*** chips is therefore the unconditional probability of picking a group 1 bag:

$$P(G_1) = \frac{45}{100} = 0.45$$

- Question a) is easy enough, now let's go to question b)…

An Now For Question b)…

- We observe the following sequence *X* of 12 draws (with replacement): 8 ***black*** and 4 ***red***.
  - We are drawing with replacement.
- Question b) asks us to compute the conditional probability that we are drawing for a **G1** bag given that we have observed the sequence *X*:

$$P(G_1|X)$$

Enter Bayes' Formula

- **Bayes' formula** tells us that

$$P(G_1|X) = \frac{P(G_1) \times P(X|G_1)}{P(X)}$$

- We now the prior probability $P(G_1)$: we just computed it for question a)!
- Now we just need to compute:
  - $P(X|G_1)$, the likelihood of G_1 given the observation sequence X;
  - $P(X)$, the marginal likelihood, that is the (unconditional) probability of observing sequence X.

Likelihood of G_1 Given the Observation Sequence X, $P(X|G_1)$

- The likelihood is the likelihood of G_1 given the observation sequence X is simply the conditional probability $P(X|G_1)$ of observing the sequence given that we have picked a bag from the first group:

$$P(X|G_1) = C_{12}^{8} \times 0.7^{8} \times 0.3^{4} = 0.2311$$

Marginal Likelihood P(*X*)

- We can compute the marginal likelihood *P*(*X*) using the **Total Probability formula**:

$$
\begin{aligned}
P(X) &= P(XG_1) + P(XG_2) \\
&= P(G_1) \times P(X|G_1) + P(G_2) \times P(X|G_2)
\end{aligned}
$$

- Here,

$$P(X) = 0.45 \times 0.2311 + 0.55 \times 0.0078 = 0.1083$$

And The Answer Is…

- We go back to Bayes' formula to conclude that

$$\begin{aligned} P(G_1|X) &= \frac{P(G_1) \times P(X|G_1)}{P(X)} \\ &= \frac{0.45 \times 0.2311}{0.1083} \\ &= 0.9604 \end{aligned}$$

as claimed!

- Now, what would be the probability that X is actually drawn from a Group 2 bag with mostly red chips?

- Once again we use Bayes' formula to get

$$
\begin{aligned}
P(G_2|X) &= \frac{P(G_2) \times P(X|G_2)}{P(X)} \\
&= \frac{0.55 \times 0.2311}{0.1083} \\
&= 0.0396
\end{aligned}
$$

where

$$P(X|G_2) = C_{12}^{8} \times 0.3^{8} \times 0.7^{4} = 0.0078$$

Concept: Conservatism

- **Conservatism** refers to the inability to fully incorporate (adjust) the impact of new information on projections.

- **Conservatism** is the tendency to underreact by overweighting base-rate information relative to new (or singular) information.

  - This phenomenon is sometimes called **underreaction**. [Shefrin 2009]

Ex: Analysts' Earnings Revisions

- The bag is like a company that in the future may operate in the black or in the red.
  - **Black** chips stand for good future earnings,
  - **Red** for poor future earnings.

[Shefrin 2002]

Ex: Analysts' Earnings Revisions

- In the **first question**, analysts start out with information that leads them to form their initial beliefs.
  - In this case, beliefs concerns the probability that the bag contains predominantly black chips. ➔ 45%
  - The bag of chips is like a company that **appears more likely to generate poor future earnings (Red 55%)** than good future earnings (Black 45%).

[Shefrin 2002]

Ex: Analysts' Earnings Revisions

- The **second question** is akin to a positive earnings announcements.

- So the question now is how to react to a positive earnings announcement made by a company that has not been performing all that well.

  - 45%: since the "earnings announcement" is favourable, they underreact.

  - 67%: ignore their prior information, in accordance with the representativeness heuristic. **Do they over-react, under-react, or get it just right?**

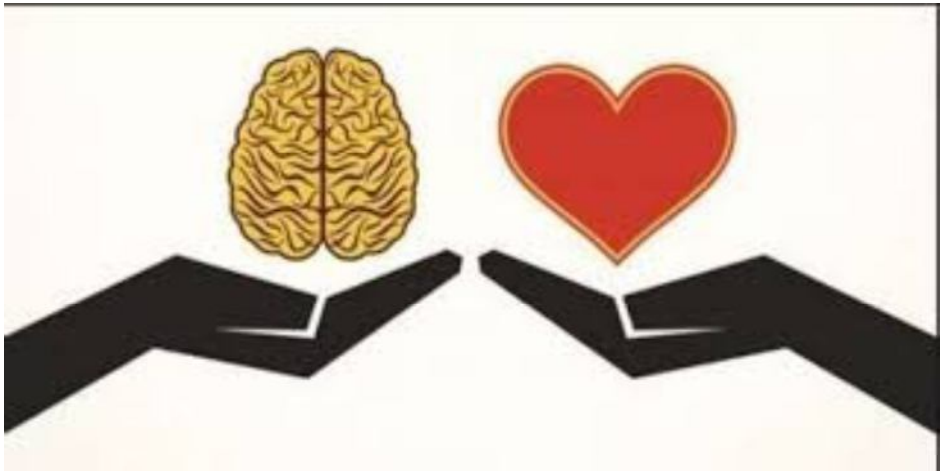
[Shefrin 2002]

Ex: Analysts' Earnings Revisions

- So how security analysts react to earnings announcements in the real world?
  - They do not revise their earnings estimates enough to reflect the new information.
  - Consequently, positive earnings surprises tend to be followed by more positive earnings surprises, and negative surprises by more negative surprises.

[Shefrin 2002]

Heuristics #4:
Affect Heuristics



Concept: Affect Heuristic

- **Affect**: an emotional feeling.
- The **affect heuristic** refers to the making of judgments on the basis of positive or negative feelings rather than underlying fundamentals.
- Reliance on the affect heuristic is often described as using "**gut feel**" or **intuition**.

[Shefrin 2009]

Research on Affect Heuristics

- Hirshleifer and Shumway (2001) examined the relation between morning sunshine at a country's leading stock exchange and market index stock returns that day at 26 stock exchanges internationally from 1982-1997.

Research on Affect Heuristics

- They found:
  - Sunshine is strongly significantly correlated with daily stock returns.
  - After controlling for sunshine, rain and snow are unrelated to returns.
  - There were positive net-of-transaction costs profits to be made from substantial use of weather-based strategies, but the magnitude of the gains was fairly modest.
- These findings are difficult to reconcile with fully rational price-setting.

[Shefrin 2009]

Heuristics #5:
Overreaction

The Winner-Loser Effect

- In 1985, Werner De Bondt and Richard Thaler released a study called "**Does the Market Overreact?**"
  - In this study, they examined returns on the New York Stock Exchange for a three-year period.
  - They separated the best 35 performing stocks into a "winners portfolio" and the worst 35 performing stocks were then added to a "losers portfolio".
  - Then tracked each portfolio's performance against a representative market index for three years.

[Shefrin 2002]

The Winner-Loser Effect

- Surprisingly, it was found that
  - The losers portfolio consistently beat the market index, while the winners portfolio consistently underperformed.
  - In total, the cumulative difference between the two portfolios was almost 25% during the three-year time span.
  - In other words, it appears that the original "winners" would became "losers", and vice versa.

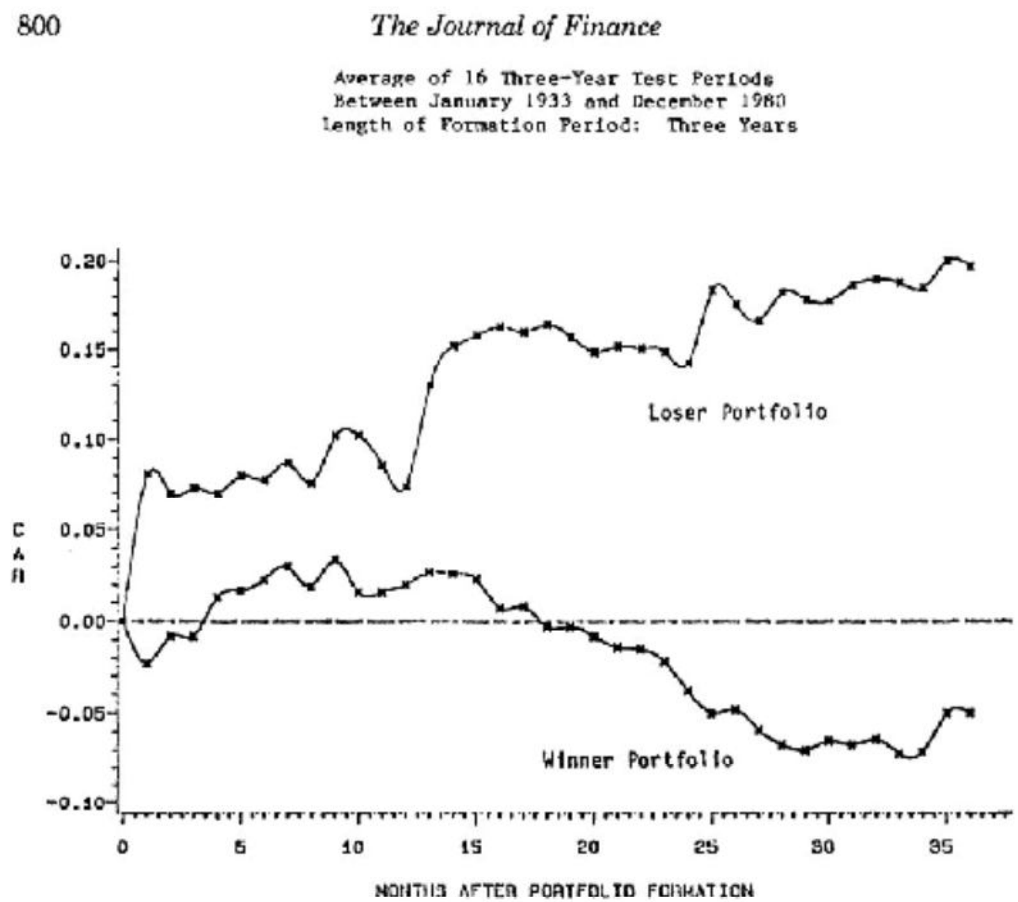


Figure 1. Cumulative Average Residuals for Winner and Loser Portfolios of 35 Stocks (1–36 months into the test period)

[Shefrin 2002]

So What Happened?

- In both the winners and losers portfolios, investors essentially **overreacted**.
  - In the case of loser stocks, investors overreacted to bad news, driving the stocks' share prices down disproportionately.
  - After some time, investors realised that their pessimism was not entirely justified, and these losers began rebounding as investors came to the conclusion that the stock was underpriced.
  - The exact opposite is true with the winners portfolio: investors eventually realised that their exuberance wasn't totally justified.

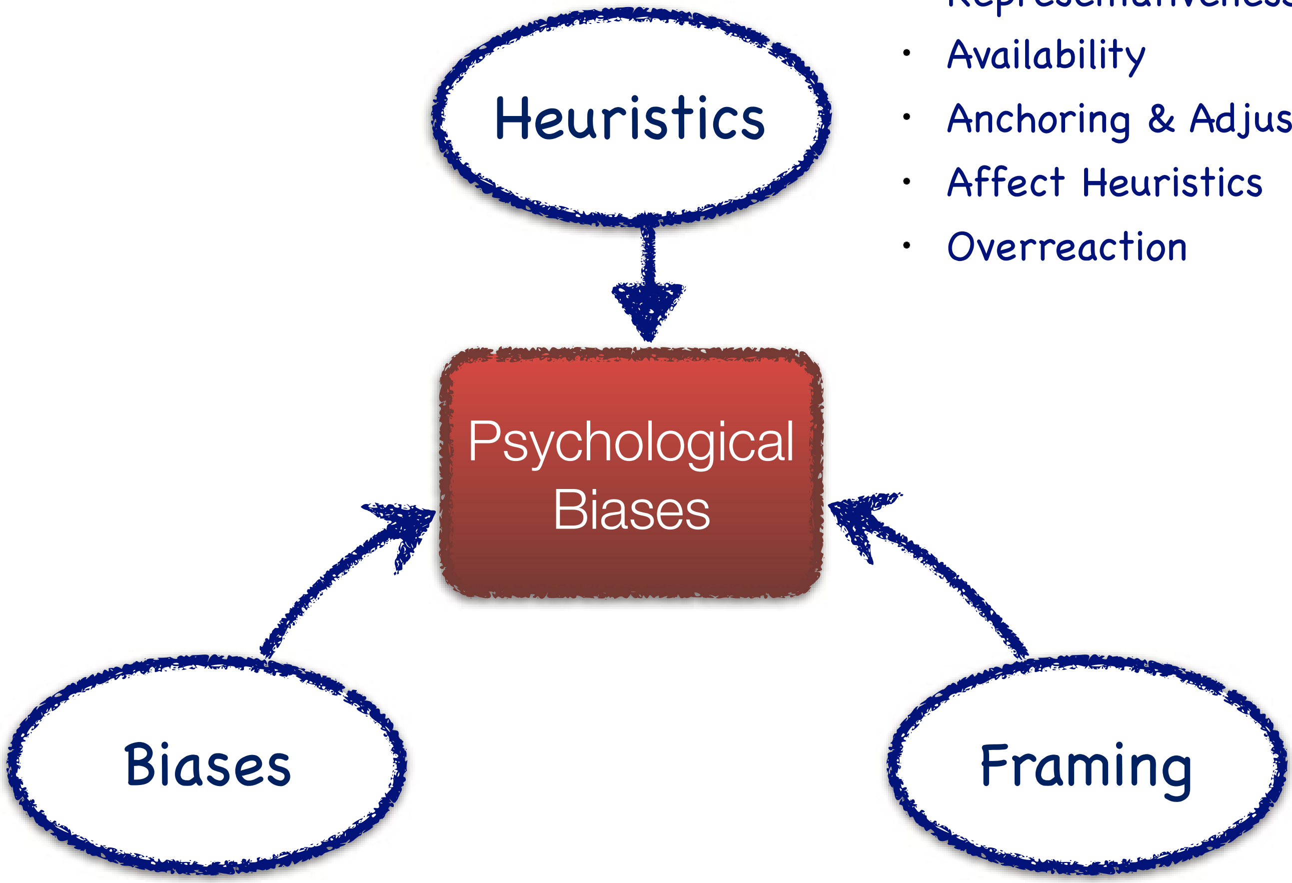
[Shefrin 2002]

Earnings Forecasts By Security Analysts

- De Bondt (1992) also shows that
  - The long-term earnings forecasts made by security analysts tend to be biased in the direction of recent success.
  - Specifically, analysts overreact in that they are much more optimistic about recent winners than they are about recent losers.
- Similar to the GPA question.
  - Investors treat past losers like high school students with low GPAs, and past winners like as high school students with high GPAs.

[Shefrin 2002]

Psychological Phenomena



Heuristics:

- Representativeness
- Availability
- Anchoring & Adjustment
- Affect Heuristics
- Overreaction

Biases:

- Excessive Optimism
- Overconfidence
- Confirmation Bias
- Illusion of Control

Framing:

- Loss Aversion
- Aversion to a Sure Loss

Chapter 2 - Biases

Biases

- **Bias**: a predisposition toward error. [Shefrin 2009]

- **Bias**: a distortion in cognition and/or decision making due to non-rational and/or emotional factors. [United-ICAP, 2011]

- We examine 4 biases:
  - Overconfidence
  - Excessive Optimism
  - Confirmation Bias
  - Illusion of Control

Bias #1: Overconfidence

Concept: Overconfidence

- **Overconfidence** is a bias that pertains to how well people understand their own abilities and the limits of their knowledge.

Overconfident Managers

- In general, managers display overconfidence when it comes to difficult tasks and their own abilities.
  - People who are overconfident about their abilities think they are better than they actually are.
  - People who are overconfident about their **level of knowledge** think they know more than they actually know.

Confident vs Overconfident

- Overconfidence does not necessarily mean that these people are ignorant or incompetent.
    - It just means that in their own eyes that are smarter and better than is actually the case.

- Keep in mind that there's a fine line between confidence and overconfidence.
    - Confidence implies **realistically trusting** in one's abilities.
    - While overconfidence usually implies an **overly optimistic** assessment of one's knowledge or control over a situation.

Research Findings

- In general,
    - 19% of people think they belong to the richest 1% of U.S. households.
    - 68% of lawyers in civil cases believe that their side will prevail.
    - Doctors consistently overestimate their ability to detect certain diseases.
- For students,
    - 80% of students think they will finish in the top half of their class.
    - Tilson relates that 86% of his Harvard Business School classmates say they are better looking than their classmates.

[Tilson 2004]

Gender Difference

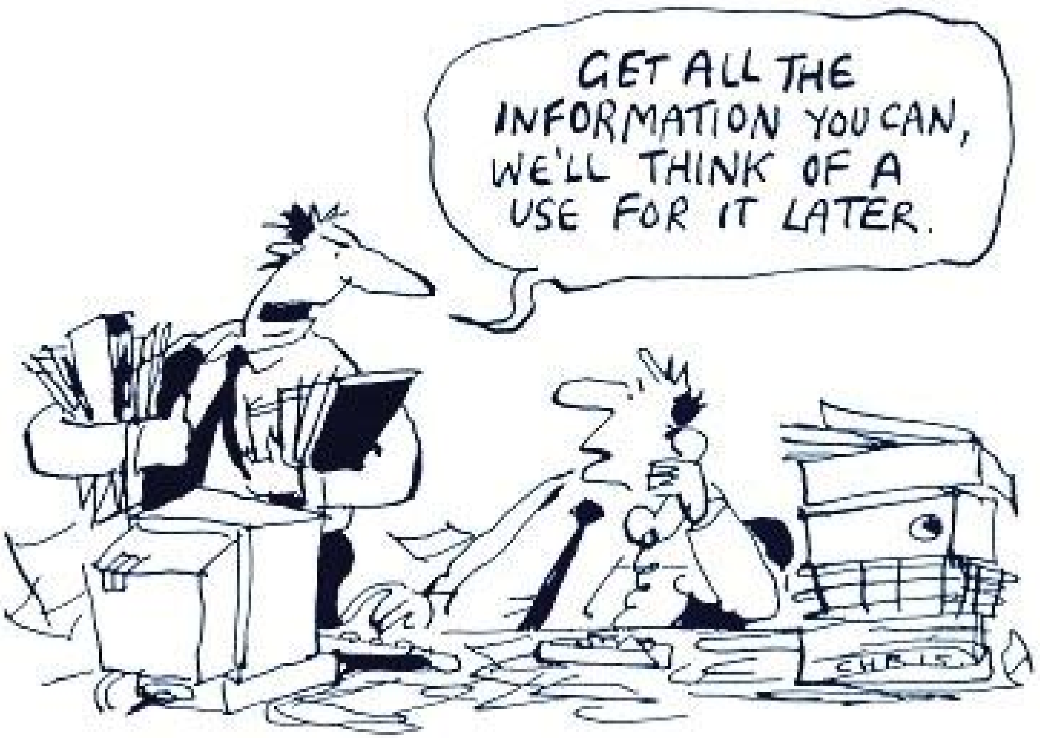
- Men exhibit this trait more than women.
  - Studies have shown that greater hubris in men leads to excessive turnover and underperformance in investing.

[Montier, 2007]

Quantity of Information

- Behavioural experiments have shown that the more (and often irrelevant) information we accumulate, the more confident we become.

- Data hoarding may lead to overconfidence!



[Montier, 2007]

Surprises

- Overconfidence can lead to surprises.

- Because investors continually underestimate the range of possible returns, there is a higher than normal probability of a return outside the confidence interval (i.e., a surprise).

- When asked to make a prediction at the 98% confidence level, people are right only 60-70% of the time. [Tilson 2004]

- For example,

  - When ask investors to predict a confidence interval around the expected return on a stock.

  - The investors will consistently make the interval too narrow (i.e., they will set the range of possible returns too narrow).

  - That is, they tend to systematically underestimate the risk (standard deviation) of the returns on the stock.

Bias #2: Excessive Optimism

Decision Problem: Excessive Optimism

- How good a driver are you?

  (1) above average?

  (2) average? Or

  (3) below average?

- Here “average” is actually defined as the median.

[Shefrin 2002]

How Did You Answer?

How Do Most People Answer?

- 82% of people say they are in the top 30% of safe drivers. [Tilson 2004]

→ A statistical impossibility.

Better Than Average Effect

- By definition, the median lies exactly in the middle, with the population equally divided on either side.
  - The point here is that very few people rate themselves below average.
  - Instead, almost everyone rates themselves as either above average or average.

Concept: Excessive Optimism

- **Excessive optimism**: people overestimate how frequently they will experience favourable outcomes and underestimate how frequently they will experience unfavourable outcomes. [Shefrin 2009]

- Excessive optimism leads people to look at the world through rose-colored glasses.

Investment Experts

- Mutual fund managers, analysts, and business executives at a conference were asked to write down how much money they would have at retirement and how much the average person in the room would have.
  - The average figures were $5 million and $2.6 million, respectively.
- The professor who asked the question said that, regardless of the audience, the ratio is always approximately 2:1

[Tilson 2004]

Excessive Optimism & Overconfident

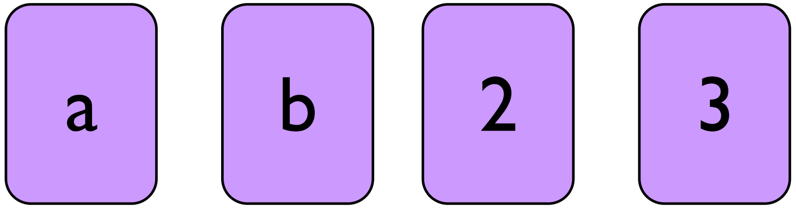
- **Overconfidence leads people to be too sure of their opinions, a tendency that frequently results in their underestimating risk.**
- Overconfidence and excessive optimism often go hand in hand, but are not the same.
  - They are really quite different psychological shortcomings in a decision maker.
  - Managers can be pessimistic, yet overconfidently so.
  - The point is that overconfident managers are overly convinced that their views are correct.

Bias #3: Confirmation Bias

Decision Problem: Confirmation Bias

- Imagine that you are presented with 4 cards flat on a table in front of you. There is a letter appearing on one side of the card and a number on the other side of the card.

- You see the following on the 4 cards: a, b, 2, and 3.



- Suppose you are asked to test the following hypothesis about these 4 cards: "Any card having a vowel on one side has an even number on the other side."

- Imagine that you are asked to select those cards, and only those cards, that will determine whether the hypothesis is true. That is, please select the minimum number of cards that will enable you to determine whether or not the hypothesis is true.

- Of the 4 cards, which would you turn over to verify the hypothesis?

[Shefrin 2009]

How Did You Answer?

How Do Most People Answer?

- In this card task, most people turn over the card with the $\boxed{a}$ and some turn over the card with the $\boxed{2}$ as well.
  - Typically less than 1/3rd choose to turn over just the $\boxed{3}$ and the $\boxed{a}$.
  - Yet, turning over only the $\boxed{a}$ and $\boxed{3}$ turns out to be the correct answer.
  - This is because the efficient way of testing the validity of the hypothesis is to turn over only the cards that might falsify the hypothesis.

[Shefrin 2009]

Spotlight on Falsification

- In finance, we all know about **black swans**.

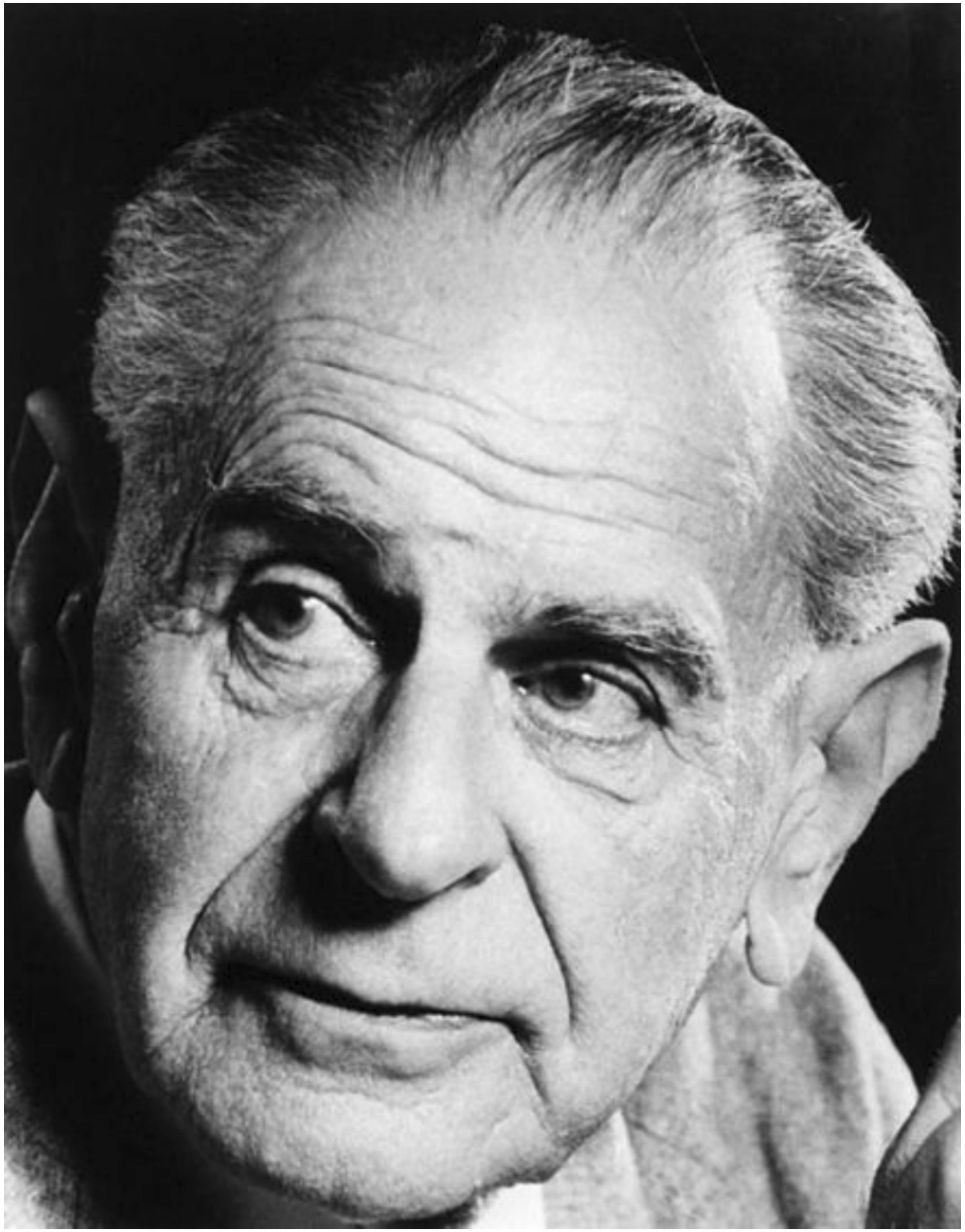
- The basic idea is the following: assume that you live in a world were in your experience all the swans (actually the sample of data you have collected so far) are white.

- Now, if we extrapolate that this sample is representative of the entire population of swans, we will conclude that all the swans in the world are white.

- … but it only takes one **black swan** for our theory to collapse.

Falsifiability

- This is the key idea behind **falsifiability**.
- Falsifiability essentially tells you that a finite sample of data, no matter how large, can never prove any statement.
- It can only be used to disprove, or **falsify**, the statement.
- This is a crucial idea for **statistical inference**.



Sir Karl Raimund Popper (1902 –1994)

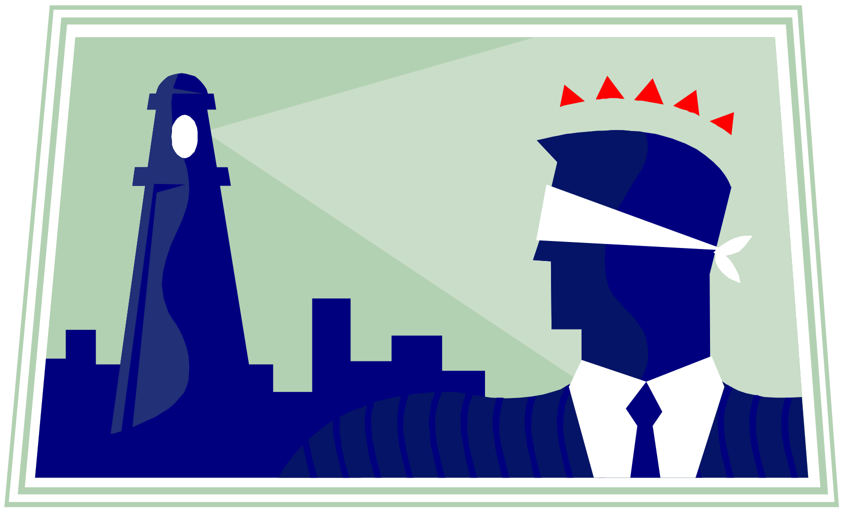
Concept: Confirmation Bias

- **Confirmation bias** leads people to overweight information that confirms their prior views and to underweight information that disconfirms those views. [Shefrin 2009]

- Without realising it, we emphasise information which reinforces our view whilst tending to downplay, avoid or even ignore contradictory information.
  - Hot stocks
  - First impression
  - Just bought a car?

Managers: Turning A Blind Eye

- Managers often only hear what want to hear.
- They spend too much time searching for reasons to support why their views are right and too little time searching for reasons that might lead them to conclude that their views are wrong.



Confirmation Bias

Have a think about it… we all do it, almost every day, in one way or another.

Bias #4:
Illusion of Control

Decision Problem: Illusion of Control

- Imagine that you agree to participate in a baseball pool.
- The pool works as follows:
    - Lying in front of you are two identical pile of 227 baseball cards, with the face of each card displaying the picture of a different baseball player.
- Two ways to play:
    1. The organiser of the pool picks a card and hands to you, or
    2. You look through the pile, select one card, and show it to him.
- Then, the organiser looks through the second (duplicate) pile, finds the twin of the card, and deposits the twin into a brown cardboard carton.

[Shefrin 2009]

Decision Problem: Illusion of Control (Cont'd)

- In order to participate, you pay $1 to a pool organiser for each card you select.

- Because you were the first participant approached, when the pool organiser approaches the next participant, she will do so with two identical pool containing 226 cards, not 227 cards.

- After all the cards have been sold, the organiser then draws exactly one card from the brown cardboard carton.

- The owner of the winning card receives a $50 prize.

[Shefrin 2009]

Decision Problem: Illusion of Control (Cont'd)

- How do you want to play?

  1. The organiser of the pool picks a card and hands to you, or

  2. You look through the pile, select one card, and show it to him.

[Shefrin 2009]

How Did You Answer?

Decision Problem: Illusion of Control (Cont'd)

- Suppose that all the cards have been sold, but the drawing has yet to take place.
- The pool organiser approaches you to say that someone who really wanted to participate can't, because all the cards have been sold.
- He asks you how much you would be willing to accept in exchange for the card you drew. What is the minimum amount you would ask to give up your card?

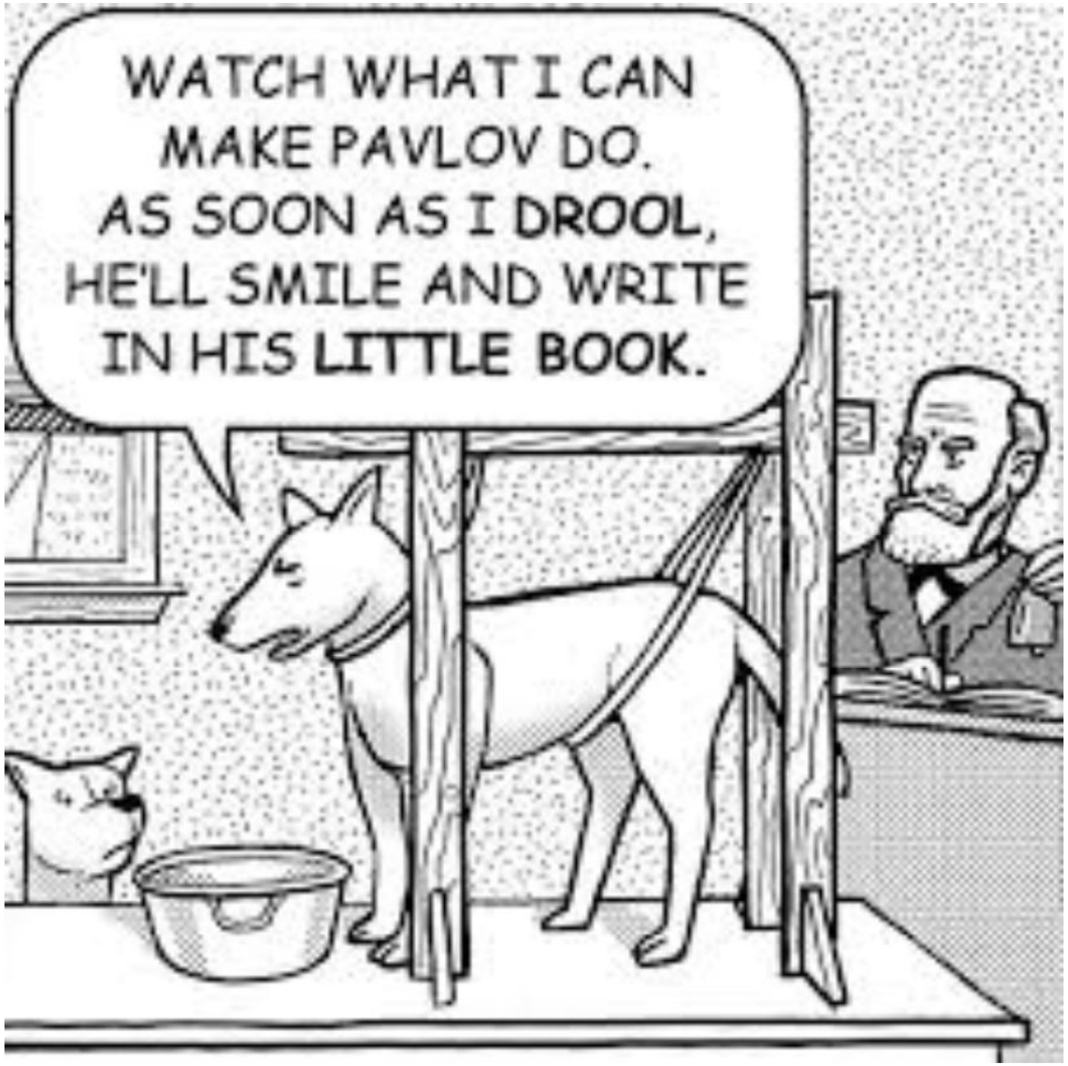
[Shefrin 2009]

How Did You Answer?

- Range:

How Do Most People Answer?

- Among participants who selected their own cards,
  - the typical response was $8.67.
- In the alternative version, where the organiser selected the card,
  - the typical response was $1.96.



[Shefrin 2009]

3 Questions

1. Is it important that people be able to choose their own cards instead of being handed their cards?
2. Does it affect their valuations?
3. Is a pool participant more likely to win the $50 just because he or she selected the card instead of being handed the card?

Answers

- Of course NOT;
  - the winner of the pool is determined entirely by a chance drawing.
- The odds of winning $50 from a single card in this pool are 1 in 227.
  - The expected payoff is $0.22.
- And participants have been asked to pay $1 per card no matter who selects the card.

So What Accounts for the Difference?

- Psychologists have concluded that the illusion of control leads people to place a higher value on their cards when they select the cards than when the organiser selects their cards.

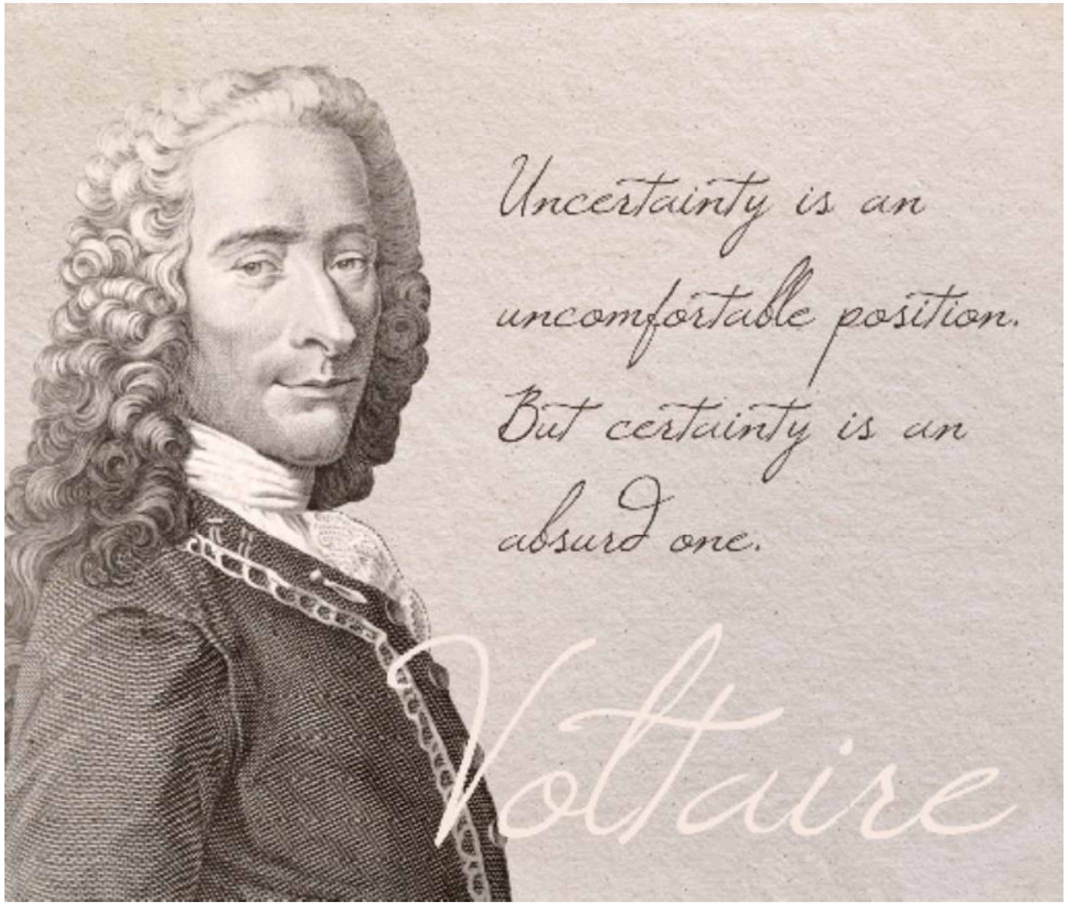
  - That is, people seem to act as if they can control the odds of winning the pool by selecting the card themselves rather than letting somebody else do it.

- However the odds are the same, no matter who selects the card.

- Being able to control the odds is an illusion.

Concept: Illusion Of Control

The **illusion of control** is overestimating the role of skill relative to luck in the determination of outcomes. [Shefrin 2010]



Impact on Managers

- When a manager makes a decision, the outcome typically depends on a combination of luck and skill.
- In general, managers have an exaggerated view of how much control they exert over outcomes.
- Psychological studies have found that an increase in perceived control leads to an increase in excessive optimism.

Psychological Phenomena

Heuristics

- Representativeness
- Availability
- Anchoring & Adjustment
- Affect Heuristics
- Overreaction

Psychological Biases

Biases

- Excessive Optimism
- Overconfidence
- Confirmation Bias
- Illusion of Control

Framing

- Loss Aversion
- Aversion to a Sure Loss

Chapter 3 – Framing Effect

Defining Framing

- **Framing**:
  - Refers to the form used to describe a decision problem.
  - People can give different answers depending on the same, but differently framed, questions. [Montier 2007]
- A decision is either **frame dependent** or **frame independent**.

Frame Independent vs Dependent

- If people acted with frame independence,
  - People would make purely economic decisions, and
  - The form within which information is received and the individual's current circumstances would have no effect on their decision making.
  - People would base each decision purely on its expected merits.
- If people act with frame dependence,
  - they will behave (make decisions and take actions) depends on the way that their decision problems are framed:
    - within which information is received (i.e., the media), or
    - the individual's circumstances at the time (i.e., emotional state).

Behavioural Finance → Frame Dependent

- **In behavioural finance**, frame dependence is the main issue when people make decisions.

- Frame dependence reflects a mix of **cognitive** and **emotional** elements.
  - The **cognitive issues** pertain to the way that information is mentally organised, especially the coding of outcomes into gains and losses.
  - The **emotional aspects** deal with the way people feel as they register the information.

Decision Problem: Concurrent Decisions

Imagine that you face the following pair of concurrent decisions. Think of making your choices in the morning, with the outcome to the first decision being determined in the afternoon, and the outcome of the second decision being determined in the evening.

Imagine that the current time is morning. First examine both decisions, and then indicate the option you prefer.

First decision. Choose:

a. sure gain of $2,400, or

b. 25% chance to gain $10,000 and a 75% chance to gain nothing.

Second decision. Choose:

c. sure loss of $7,500, or

d. 75% chance to lose $10,000 and a 25% chance to lose nothing.

[Shefrin 2002]

How Did You Answer?

- First question:

- Second question:

How Do Most People Answer?

- Most people
  - Choose A (risk-free) over B (risky), but
  - Choose D (risky) over C (risk-free).
- End up facing
  - a 25% chance of winning $2,400, and
  - a 75% chance of losing $7,500.

[Shefrin 2002]

How Do Most People Answer?

- The way people respond to this problem tells us a lot about their approach to making decision.

  - Choose A over B in the first decision → **risk-averse choice.**

  - Choose D in the second decision → **they want to change to get even.**

- The point is that **when only losses are involved, people may seek risk rather than be averse to risk**.

[Shefrin 2002]

However, They Could Do Better!

- They could choose B & C offer:
  - a 25% change of winning $2,500, and
  - a 75% change of losing $7,500.
- But most people don’t see through the opaque / interconnected frame.
  - The two decision problems together constitute a concurrent “package”.
  - But most people do not see the package.
  - They separate the choices into mental accounts.

[Shefrin 2002]

Framing: Outbreak

- Assume that the outbreak of a new disease is expected to kill 600 people.
  - Assume also that two alternative programs have been proposed to tackle the disease.
- In the first set of options,
  - **Program A** will result in 200 people being saved,
  - **Program B** will result in a one third probability that 600 people will be saved, but two thirds probability that no one will be saved.

Framing: Outbreak

- Roughly 75% of people elect Program A as the preferable one.

Framing: Outbreak

- Now for a second set of options.
  - **Program C** will result in the death of 400 people,
  - **Program D** will result in a one third probability that no one will die, but two thirds probability that 600 people will die.

Framing: Outbreak

- Roughly 75% of people choose Program D as the preferable one.

Framing: Outbreak

- In fact,
  - **Program A** is the same as **Program C**, but framed differently, and
  - **Program B** is the same as **Program D**, but framed differently.

Framing: Outbreak

- So why?
  - when information is framed as a gain, people generally go with the guaranteed option.
  - when information is framed as a loss, people generally go with the risky option.
- The first set of options used the word '**save**' (framed as a gain) in **A**, whereas the second set of options used the word '**die**' (framed as a loss) in **C**.
- And conversely for **B** and **D**.

How Exactly Does Framing Relate to Investing?

- We see this exact phenomenon in the stock markets:
  - When making money on a trade (a gain), people often take the guaranteed option by taking profits and locking in that ‘guaranteed’ gain.
  - However, when losing money on a trade (a loss), most people choose to take the risky option by running losses and holding on to the stock.

However,

- Any market expert will tell you that good investors do the exact opposite: **they cut their losses and run their profits!**

Framing Effect: Loss Aversion & Aversion to Sure Loss

How do people respond to the prospect of a loss?

Decision Problem: Loss Aversion

- Suppose you had an opportunity to take a 25 percent chance of winning $7,500, but a 75 percent chance of losing $2,500.

- Would you be willing to take this chance? Yes or no?

[Shefrin 2002]

How Did You Answer?

How Do Most People Answer?

- Most answer **NO**.

- Since the risky alternative described unappealing, they judge that the potential gain of $7,500 does not justify a three in four chance (75%) of losing $2,500.

[Shefrin 2002]

Decision Problem: Loss Aversion

Suppose you face a choice between:

1. accepting a sure loss of $7,500, or

2. taking a chance where there is a 75% chance you will lose $10,000 and a 25% chance you will lose nothing.

Would you choose to take the guaranteed loss or take a chance?

[Shefrin 2002]

How Did You Answer?

How Do Most People Answer?

- Most people opt for the latter (2) **- take a chance!**

[Shefrin 2002]

Why?

- The expected loss in both choices is $7,500.

- Why is that?
  - Because they hate to lose! and
  - The uncertain choice holds out the hope they won't have to lose.

[Shefrin 2002]

Concept: Loss Aversion

- Kahnmen and Tversky call this phenomenon - Individual’s reluctance to accept a loss - **loss aversion**.
    - The idea of loss aversion was first introduced by Kahneman and Tversky (1979).
    - "The central assumption of the theory is that losses and disadvantages have greater impact on preferences than gains and advantages.” [Tversky and Kahneman 1991]

Concept: Loss Aversion

- People tend to feel losses much more acutely than they feel gains of comparable magnitude.
  - Kahneman and Tversky find that a loss has about **two and a half times (2½)** the impact of a gain of the same magnitude.

- *"It is not so much that people hate uncertainty–but rather, they hate losing."* [Tversky 1991]

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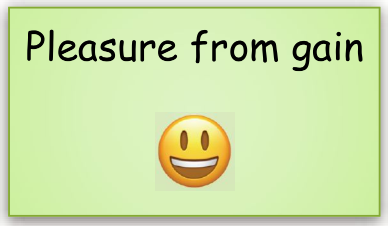

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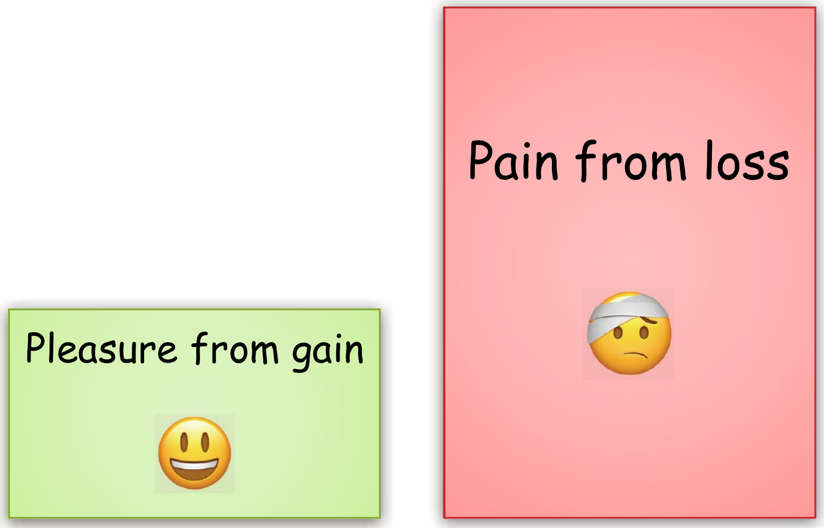

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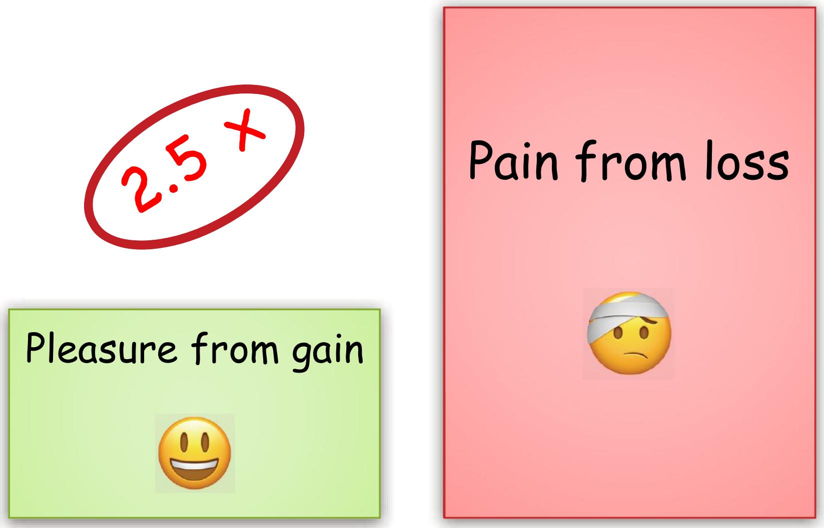

Explanation

- From an **evolutionary psychology** perspective, this makes a great deal of sense when you think about it.
  - If you only have $1,000, making $1,000 would give you $2,000, but losing $1,000 would wipe you out entirely.
  - Given the human need for money to procure food and shelter, it is far more motivating to avoid losing that $1,000 than it is to make that $1,000.
- Therefore, psychologists have concluded that instead of accepting sure losses, people are prone to accept risky, actuarially unfair prospects.

Highly Important in Finance

- The idea that **investors are not risk-averse but loss-averse** is one of the main tenets of behavioural finance. [Nevins 2004]
    - Thus, **the assumption that investors are always risk-averse, which underpins modern portfolio theory, is incorrect.**

Decision Problem:

Loss Aversion → Risk-Seeking Behaviour

- Imagine that you face the following choice.
- You can accept a guaranteed loss of $750 or accept a risk.
- The risk involves a 50-50 chance of losing $525 or losing $975. Would you accept the sure loss or take the risk?

[Shefrin 2002]

How Did You Answer?

How Do Most People Answer?

- In the situation just described, the majority of people accept the sure loss.

- To be sure, the risky alternative is <u>actuarially fair</u>, relative to the sure loss.

- However, the risky alternative does not offer the possibility of <u>breaking even</u>.

[Shefrin 2002]

Get-Even → Risk-Seeking Behaviour

- The possibility of breaking even serves as a compelling reason for people to choose risk-seeking behaviour.

- **Get-even**: investors will not sell anything at a loss, and hope to get even before they get out.

Get-Even → Risk-Seeking Behaviour

- *For example*, a portfolio manager may have experienced recent losses.

- Knowing that he must report at the end of the quarter and being reluctant to report losses, he might start taking progressively riskier positions in hopes of at least breaking even.

Monkey Business Experiment

- In an article: “Too much monkey business: From how monkeys react to losses, to how footballers take on penalty shoot-outs can help influence your investing decisions.”

Monkey Business Experiment → Loss Aversion

- In an experiment that illustrates loss aversion, monkeys were taught how to trade tokens for food.

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One
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Monkey Business Experiment → Loss Aversion

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One person would give the monkeys one slice of apple and then 50 per cent of the time a second slice as a bonus.

Another person would give them two slices of apple but 50 per cent of the time would then take one of the slices away.

Monkey Business Experiment → Loss Aversion

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One person would give the monkeys one slice of apple and then 50 per cent of the time a second slice as a bonus.

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The net result was the same.

Monkey Business Experiment → Loss Aversion

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One person would give the monkeys one slice of apple and then 50 per cent of the time a second slice as a bonus.

Another person would give them two slices of apple but 50 per cent of the time would then take one of the slices away.

The net result was the same.

But the monkeys didn't see it that way!

Monkey Business Experiment

- The monkey traded
  - 70 per cent of the time with the person who gave them another slice, and
  - only 30 per cent of the time with the person who took one away.
- Rationally it didn't make any difference, but a painful loss was associated with one of the people.

Managers' Faulty Financial Decisions

- Managers are inclined to make faulty decisions about investment policy and financing because they are
  - Unduly sensitive about potential future losses, and
  - Find it difficult to accept losses that have already occurred.

Concept Preview

- Imagine two people, Ginger and Jane, who have each just received parking tickets that will cost them $50 apiece. As it happens, both Ginger and Jane have won office lotteries for $75. Ginger won her office lottery the month before she received her parking ticket. Jan won her office lottery the same day she received her parking ticket. Both Ginger and Jane feel the pain of their parking tickets.

- Who do you think feels the pain more intensely? You may answer Ginger, Jane, no difference, or no opinion.

- Suppose that instead of receiving a parking ticket that will cost her $50, Ginger learns that she missed out on winning $50 in another office lottery because she was absent from work that day. Which do you believe would cause Ginger more pain, receiving the $50 parking ticket or having missed winning $50 in an office lottery?

Hersh Shefrin, 2009, Behavioral Corporate Finance: Decisions that Create Value, Page 85

We Learn 2 Lessons

1. Most people feel that Ginger experiences the pain from receiving her parking ticket more than Jane feels the pain from receiving hers.

    - Presumably, Jane can mentally offset her $50 loss with the $75 lottery win and view herself as $25 ahead at the end of the day.

    - However, that mental operation is much more difficult to do for Ginger, who is less likely to associate the $75 gain with the $50 loss because the gain occurred a month earlier.

    - Therefore, Ginger is likely to view herself as being down $50 at the end of the day because of the parking ticket, instead of up $25.

Concept Preview: We Learn 2 Lessons

2. Ginger feels the $50 loss from the parking ticket more than she feels having lost out on winning $50 in the office lottery.

   - Generally, people experience a cost that is out of pocket more intensely than an opportunity cost.

Decision Problem: Concurrent Decisions

Imagine that you face the following pair of concurrent decisions. Think of making your choices in the morning, with the outcome to the first decision being determined in the afternoon, and the outcome of the second decision being determined in the evening.

Imagine that the current time is morning. First examine both decisions, and then indicate the option you prefer.

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Second decision. Choose:

c. sure loss of $7,500, or

d. 75% chance to lose $10,000 and a 25% chance to lose nothing.

[Shefrin 2002]

Answer?

- How did you answer this question?
  - First question:
  - Second question:

[Shefrin 2002]

How Do Most People Answer?

- At first, the conclusion of this experiment appear extremely similar two our previous exemple:

  - Choose A over B in the first decision → risk-averse choice.

  - Choose D in the second decision → risk-seeking behaviour for a chance to **get even**.

- On the surface, the point is still that when only losses are involved, people may seek risk rather than be averse to risk.

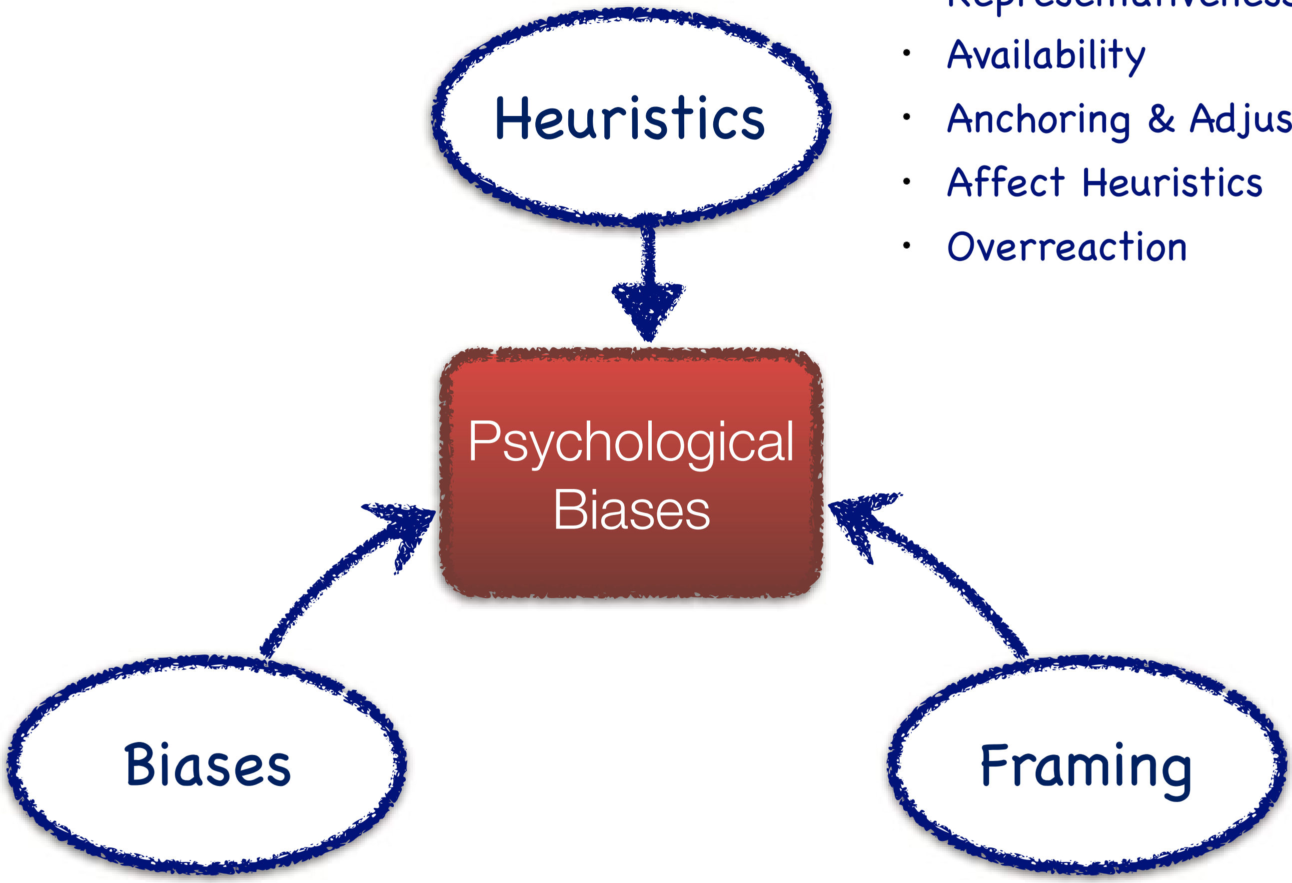
[Shefrin 2002]

How Do Most People Answer?

- However, people could do better.
  - They could choose B & C: offer a 25% change of winning $2,500 and a 75% change of losing $7,500.
  - But most people don’t see through the **opaque frame**.
- This is an example of **opaque framing**.
  - The two decision problems together constitute a concurrent “package”.
  - But most people do not see the package.
  - They separate the choices into **mental accounts**.
- On the other hand, **transparent frames** make concurrent packages easier to perceive, leading to better decisions.

[Shefrin 2002]

Psychological Phenomena



Heuristics:
- Representativeness
- Availability
- Anchoring & Adjustment
- Affect Heuristics
- Overreaction

Biases:
- Excessive Optimism
- Overconfidence
- Confirmation Bias
- Illusion of Control

Framing:
- Loss Aversion
- Aversion to a Sure Loss

Group Processes

Group Process

- Group process is critical to the effectiveness of institutional and corporate financial decisions:
    - Given most institutional investment decisions are made by **committees**.
    - Given the major decision made by managers take place in **managerial meetings**.

- Traditional textbooks in both corporate and investment finance do not include much material on group process.

[Montier, 2007]

Effective Group → Process Gain

- **Effective groups** exploit potential synergies from bringing together people with different skills, perspectives and values.

- Effective groups are said to experience **process gains**.

    - In theory, the key to process gains is the constructive use of individual differences among group members.

Process Loss

- Despite the potential for process gain, many groups are unable to exploit potential synergies and instead experience **process loss**.

- The **source of this loss is typically psychological**, in that **group psychology** often leads people to make different decisions when they operate as part of a group than when they act as individuals.

Germany 7 – Brazil 1

Group / Herd Behaviour

- **Herd behaviour**, is the tendency for individuals to mimic the actions (rational or irrational) of a larger group.

- Individually, most people would not necessarily make the same choice.

Stampede! the Dotcom bubble was a classic example of herding behaviour by investors

[Montier, 2007]

Herd Behaviour

- By Franklin Templeton Investments

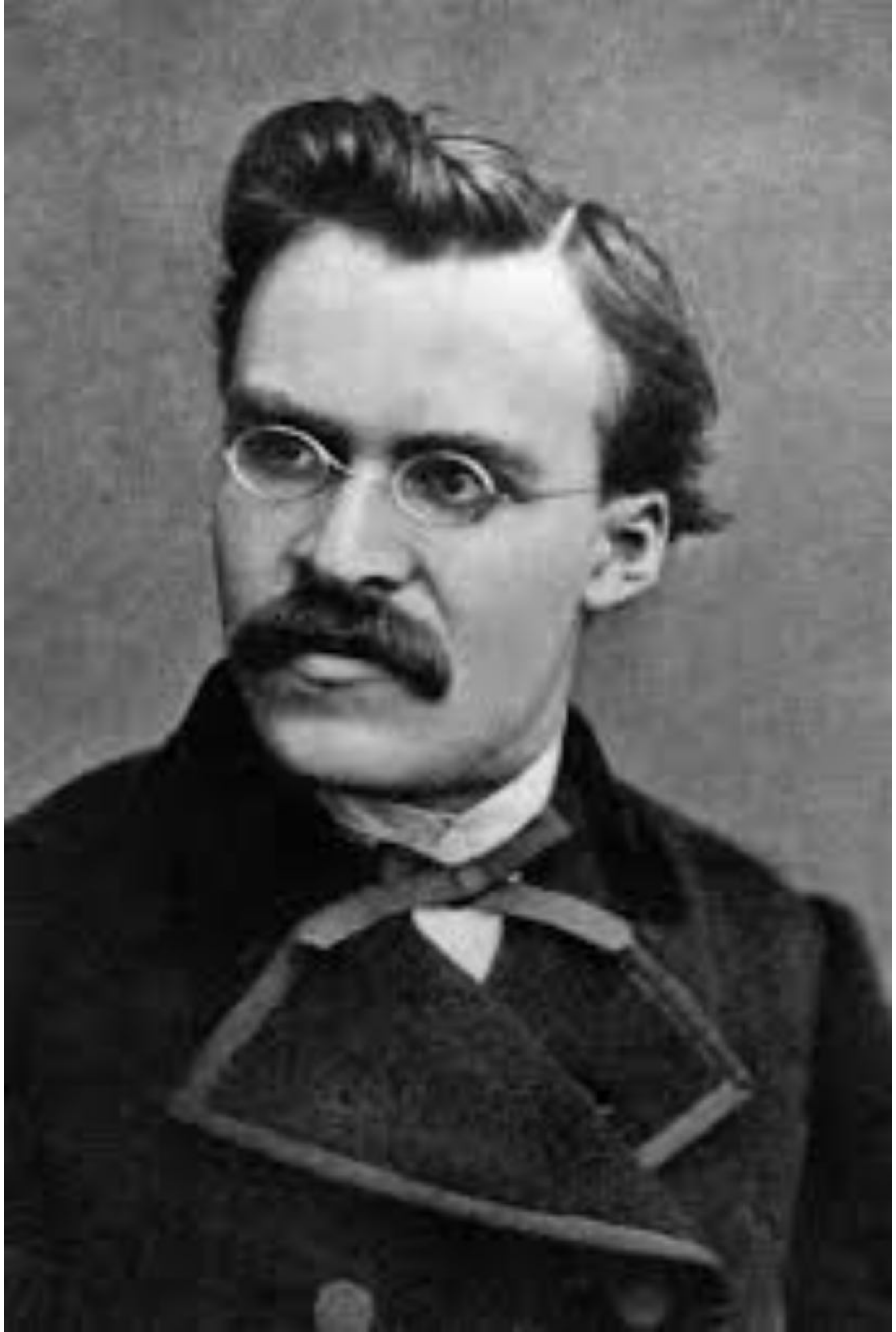
- https://www.youtube.com/watch?v=0R8H9ID8wm8results_video

[Franklin Templeton Investments]

Herding/Group Behaviour

- Quote from Friedrich Nietzsche (German philosopher, poet, composer and classical philologist):

  "Madness is a rare thing in individuals, but in groups … it is the rule."




JUST A NORMAL DAY AT THE NATION'S MOST IMPORTANT FINANCIAL INSTITUTION...
I'VE GOT A STOCK HERE THAT COULD REALLY EXCEL
REALLY EXCEL?
EXCEL?
SELL?
SELL?
SELL
SE
SELL!
SELL!
KAL '89 BALTIMORE SUN
SELL!
SELL!
ELL!
SELL!
THIS IS MADNESS! I CAN'T TAKE ANYMORE
GOOD BYE!
GOOD BYE?
BYE?
Buy?
BUY!
Buy!
BUY!!!
Buy!
Bu
BUY!
I'VE GOT A STOCK HERE..

Why Herd Behaviour?

1. **The social pressure of conformity.**

- Most people are very sociable and have a natural desire to be accepted by a group, rather than be branded as an outcast.
- Therefore, following the group is an ideal way of becoming a member.

Social Conformity, Why?

- Neurologists have found that real pain and social pain are felt in the same part of the brain.
  - Contrarian strategies are the investment equivalent of seeking out social pain.

Group pressure and pressure to conform are recurring themes in "The Big Short."

Why Herd Behaviour?

2. **The common rationale that it's unlikely that such a large group could be wrong.**

- After all, even if you are convinced that a particular idea or course or action is irrational or incorrect, you might still follow the herd, **believing they know something that you don't.**

- This is especially prevalent in situations in which an individual has very little experience.

Three Important Features

- Behavioural studies have identified 3 important features about group behaviour:

1. **Accuracy**
2. **Polarisation**
3. **Unwarranted acceptance**

1. Accuracy

- Groups tend to outperform individuals in particular types of tasks known as intellectual tasks.
    - An **intellectual task** is a problem with a “correct” answer that once identified, group members would readily acknowledge as being correct.
    - In this respect, an intellectual task is sometimes called a **eureka problem.**
- Although groups tend to outperform individuals on intellectual tasks, the same is not true for **judgmental tasks that are more subjective in nature**.

2. Polarisation

- Groups often become polarised in respect to **risk tolerance**.

  - *For example*, if at the individual level the members of a group are slightly risk seeking, group discussion typically **amplifies** the degree of risk-seeking behavior.

3. Unwarranted Acceptance

- Group discussion leads the members of a group to accept a decision readily.
- However, such acceptance is often unwarranted, producing a phenomenon akin to **collective overconfidence** known as the **illusion of effectiveness** (i.e. unwarranted confidence in the decision).

General Reasons for Group Errors

- 3 main factors that underlie group inaccuracy and unwarranted acceptance.

    1. groupthink
    2. poor information sharing
    3. inadequate motivation

1. Group Think

- **Group think** is the drive for achieving group consensus overrides the realistic appraisal of alternative courses of action.
  - A collective form of confirmation bias.
  - May lead "**false consensus effect**".

Collective Confirmation Bias

- **Conformity effect**, is the tendency of people's judgments and decisions to be skewed toward supporting other members of one's group.
    - a key bias studied in social psychology.

False Consensus Effect

- **False consensus effect** means people come to believe that others share their beliefs to a greater degree than is actually the case. [Ross et al. 1977].

"Then we are agreed nine to one that we will say our previous vote was unanimous!"

Emergence of Group Think

- The following **conditions are especially conducive to the emergence of groupthink**:
  - The group dynamics feature amiability and esprit de corps.
  - A powerful, opinionated leader leads the group.
  - Group members operate under stress.
  - Group members are strongly influenced by a desire for social conformity.
  - There is no explicit decision-making procedure.

2. Poor Information Sharing

- **Poor information** sharing: group members fail to share enough information with other group members.
  - People are often ineffective when it comes to sharing information within groups.
- One behavioural study analysed information sharing about job candidates.
  - Relevant information was distributed across group members to see if they would find a way to share the relevant information.
  - The key finding in the study is that people refrain from sharing relevant information with others in their groups, even when the members of the group share a common goals.

3. Inadequate Motivation

- Inadequate motivation leads to **free-rider agency conflicts** known as **social loafing** (i.e. group members reduce their contributions, instead relying on others to exert the requisite effect).
  - In this regard, some embers of the group might choose to reduce their level of effort, relying on the efforts of others to generate group benefit.
  - In other works, individuals who work in groups might not work as hard as individuals who work alone.
  - Setting incentives to deal appropriately with social loafing is difficult when the link between effort and outcome is weak and when responsibility within the group is diffused.

Herd Behaviour → Biases

- **Fundamental attribution error** is when people, in the course of determining the behaviour of others, overestimate the importance of disposition (nature) and underestimate the role of external circumstances.

Group Pitfalls

- Psychologists have found that:
  - Groups amplify rather than alleviate decision-making biases.
  - Groups tend to reduce the variance of options.
  - They falsely lead members to increased confidence.
  - They are not especially good at uncovering new information.
  - People seek group credibility by telling the group what it wants to hear.

[Montier, 2007]

To Conclude on Group Processes,

- **In theory**, group process adds synergistic value to the efforts of the individual group participants.

- **In practice**, three factors lead this synergy to be less than maximal, and sometimes negative.

1. Although synergy is positive for tasks that are intellectual, it is typically negative for tasks that are judgmental.

2. Group process often leads to polarisation in respect to risk attitude.

3. Group discussion typically leads its members to feel more effective than warranted, a form of group overconfidence known as the illusion of effectiveness.

Winner's Curse

Anomalies: The Winner's Curse

- The **winner's curse**: a tendency for the winning bid in an auction setting to exceed the intrinsic value of the item purchased.

- In the famous LBO of RJR Nabisco by KKR in 1988, the curse is twofold:

    - not only has the winning bidder overpaid for RJR Nabisco,

    - but it also increase the cost of financing and prevent future deals.

Prospect Theory

Introduction

- In this section:
    - Expected utility theory and its alternatives
    - Prospect theory
    - Value function
    - Prospect theory vs. expected utility theory

Expected Utility theory

Expected Utility theory

- Standard Finance:
    - Standard decision theory suggests that the net effect of the gains and losses involved with each choice are combined to present an overall evaluation of whether a choice is desirable.
    - Economists tend to use "**utility**" to describe enjoyment and contend that we prefer instances that maximise our **expected utility**.

Alternative To Expected Utility Theory

Alternative To Expected Utility Theory

- Behavioural Finance:
  - Research has found that we don't actually process information in such a rational way.
  - Kahneman and Tversky (1979) proposed a critique of expected utility theory as a descriptive model of decision making under risk and develop an alternative model, which they call prospect theory.
  - **Prospect theory** deals with the idea that people value gains and losses differently and do not always behave rationally.

Decision Problem 1

- You have $1,000 and you must pick one of the following choices:
- **Choice A**:
  - You have a 50% chance of <u>gaining</u> $1,000, and a 50% chance of gaining $0.
- **Choice B**:
  - You have a 100% chance of <u>gaining</u> $500.

[Shefrin 2002]

Decision Problem 2

- You have $2,000 and you must pick one of the following choices:
- **Choice A**:
  - You have a 50% chance of losing $1,000, and 50% of losing $0.
- **Choice B**:
  - You have a 100% chance of losing $500.

[Shefrin 2002]

Your Answer

How Most People Answer

- The implication is that people:
  - are willing to settle for a reasonable level of gains (even if they have a reasonable chance of earning more), but
  - are willing to engage in risk-seeking behaviours where they can limit their losses.
- In other words, losses are weighted more heavily than an equivalent amount of gains.
- It is this line of thinking that created the asymmetric value function (see details later!).

How Most People Answer

- If the subjects had answered logically, they would have picked either "A" or "B" in both situations.
- People choosing "B" would be more risk adverse than those choosing "A“.
- Thus, if a person were given two equal choices, one expressed in terms of possible gains and the other in possible losses, people would choose the former - even when they achieve the same economic end result. [Nevins 2004]

Concept: Prospect Theory

- **Prospect theory**: a descriptive framework for the way people make choices in the face of risk and uncertainty.

- Landmark work by Daniel Kahneman and Amos Tversky (1979) - "Prospect Theory: An Analysis of Decision under Risk".

- The most cited paper ever to appear in Econometrica,

- Prospect theory holds that there are persistent biases motivated by psychological factors that influence people's choices under conditions of uncertainty.

Concept: Prospect Theory

- Prospect theory considers:
  - Preferences as a function of "**decision weights**," and it assumes that these weights do not always match with probabilities.
  - Specifically, prospect theory suggests that decision weights tend to over-weigh small probabilities and under-weigh moderate and high probabilities.

Two Stages of Decision Making

- Concretely, according to the Prospect theory, the investor goes through 2 stages of decision making:

- The '**editing stage**' frames all choices in terms of **potential gains and/or losses relative to a fixed reference point**.

- The '**evaluation stage**' in which the decision maker employs **an S-shaped valuation function** (meaning a utility function on the domain of gains and a related function in the domain of losses) which is concave in the gains region and convex on the loss region.

Decision Weights

- Decision weights are generally lower than the corresponding probabilities, except in the range of low probabilities.

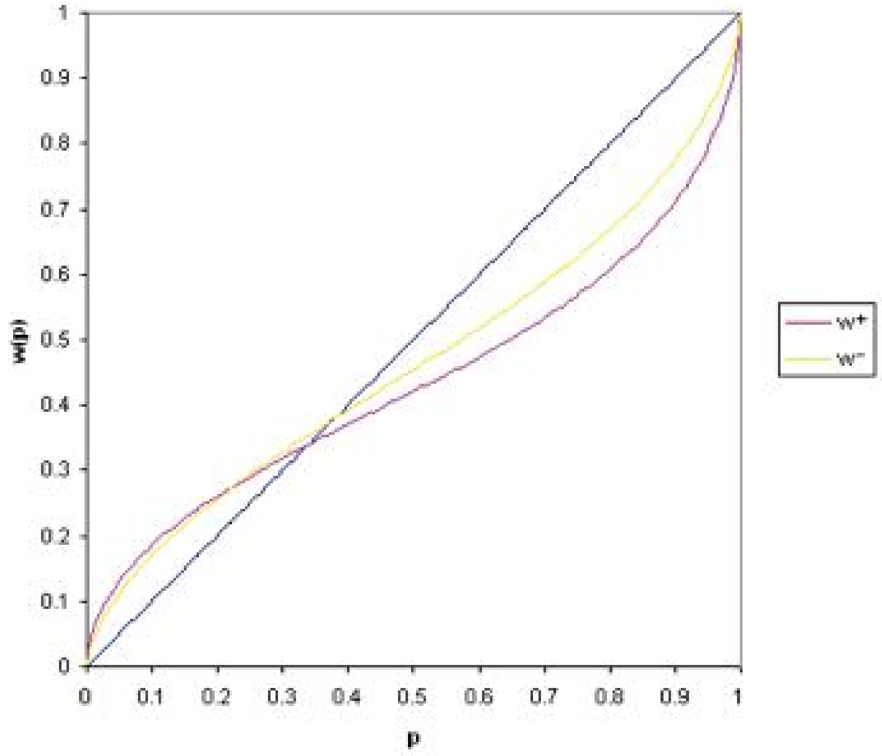


Figure 2: Weighting functions for gains (w+) and losses (w-) based on median estimates of parameters

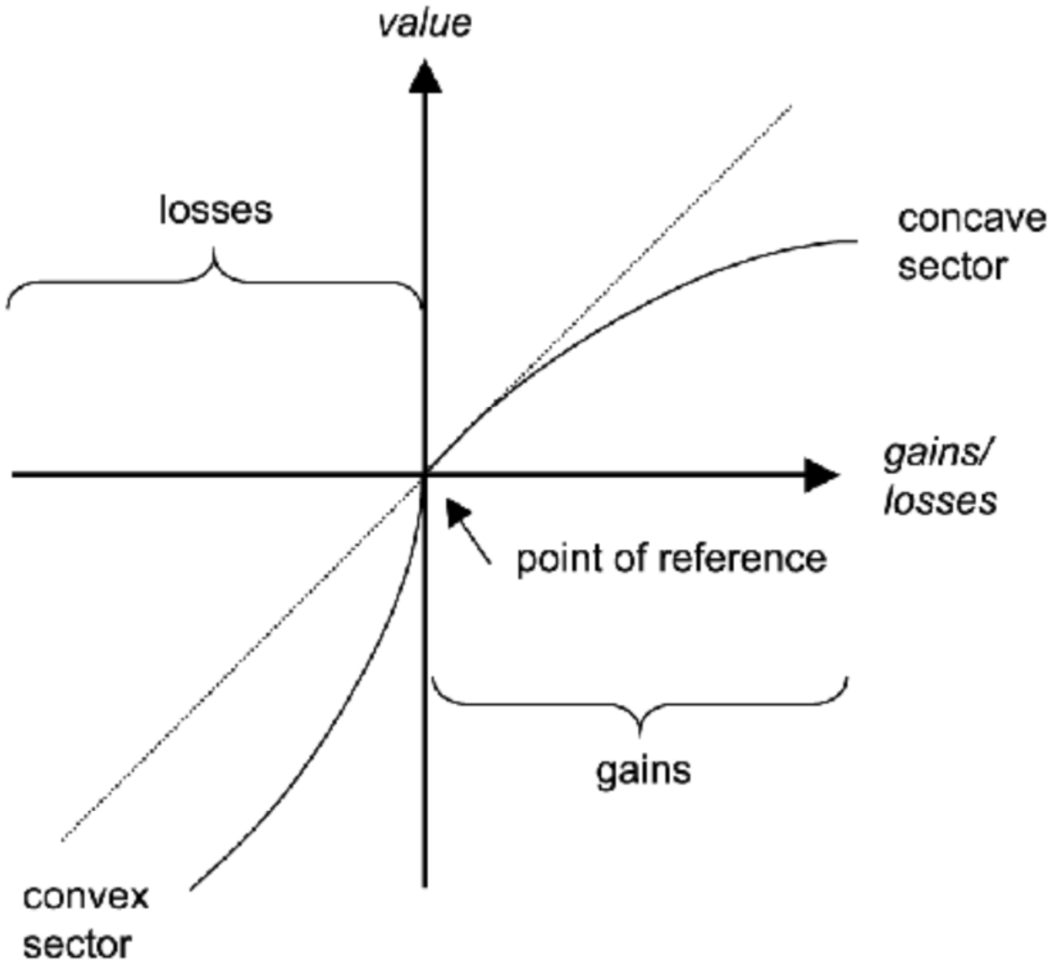
Value Function

- Kahneman and Tversky introduced a model of investor preferences called a **value function**.
    - The value function is defined on deviations from a reference point and is normally **concave for gains** (implying risk aversion), commonly **convex for losses** (risk seeking) and is generally steeper for losses than for gains (loss aversion).
- It is key to note that although not everyone would have a value function that looks exactly like this, this is the general trend.

Value Function

- The most evident feature is how a loss creates a greater feeling of pain compared to the joy created by an equivalent gain.

- For example, the absolute joy felt in finding $50 is a lot less than the absolute pain caused by losing $50.



An Example

- Consider an investor who purchased a stock for \$50 one month ago and the stock now is selling at \$40

- There are 2 outcomes to this situation:

  - Sell the stock now and realise a loss of \$10, or

  - Hold the stock for one more period, with a 50-50 odds between losing an additional \$10 or "breaking even".

An Example

- Since the choice between these two is associated with the convex portion of the S-shaped value function, prospect theory implies that B will be selected over A.
- This seems to apply even if the odds of breaking even were something less than 50-50.

Expected Utility vs Prospect Theory

- Consider the following gamble or prospect:
  - Payoff x with probability p;
  - Payoff y with probability q.
- Note that:
  - x or y could be positive (winnings) or negative (loss)
  - We only require at this stage that $p + q \leq 1$;

[Thaler 2000]

Expected Utility vs Prospect Theory

- Under utility theory, the expected utility an individual with initial wealth w derives from the gamble is:

$$E[U] = pU(w+x) + qU(w+x)$$

- This is true, only when $p + q = 1$.

Expected Utility vs Prospect Theory

- Under prospect theory, we use a value function v
- For “regular prospects,” i.e. $p + q < 1$ or $x \geq 0 \geq y$ or $x \leq 0 \leq y$

$$V(x, p; y, q) = \pi(p)v(x) + \pi(q)v(x)$$

- If $p + q = 1$ and either $x > y > 0$ or $x < y < 0$

$$V(x, p; y, q) = v(y) + \pi(p)[v(x) - v(y)]$$

- Where, $\pi(p)$ is the decision weight assigned to a probability p, same for $\pi(q)$.

Expected Utility vs Prospect Theory

- There are **two fundamental reasons** why prospect theory (which calculates value) is not consistent with expected utility theory.

  1. Whilst expected utility is necessarily linear with the probabilities, value is not.

  2. Whereas utility is dependent on final wealth, value is defined in terms of gains and losses (deviations from current wealth).

To Wrap Up: Expected Utility vs Prospect Theory

- **Expected utility theory**:
  - A **prescriptive** approach: concerns how decisions under uncertainty should be made.
  - Assuming investors are risk-averse.
  - Utility is assigned to final assets.
  - Investors want more utility than less.
  - Utility is linear with the probability.

Gains and Losses in Time…

- According to prospect theory, losses have more emotional impact than an equivalent amount of gains, but are **path-dependent**.

- For example,

  - In a traditional way of thinking, the amount of utility gained from receiving $50 should be equal to a situation in which you gained $100 and then lost $50. In both situations, the end result is a net gain of $50.

  - However, despite the fact that you still end up with a $50 gain in either case, most people view a single gain of $50 more favourably than gaining $100 and then losing $50.

Example: Extra Money… or Extra Taxes?

- Prospect theory can be used to explain a number of illogical financial behaviours.

- For example,

  - There are people who do not wish to put their money in the bank to earn interest or who refuse to work overtime because they don't want to pay more taxes.

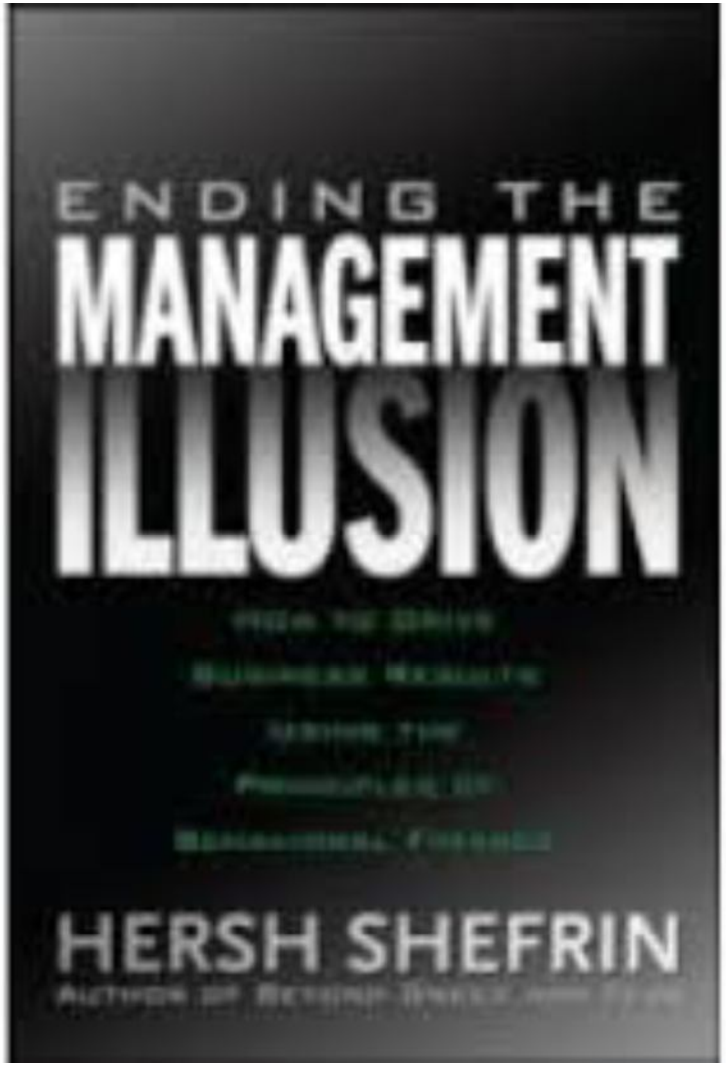
  - Although these people would benefit financially from the additional after-tax income, prospect theory suggests that the benefit (or utility gained) from the extra money is not enough to overcome the feelings of loss incurred by paying taxes.

SP/A Theory

- Lola Lopes (1987) proposed the **SP/A theory**, a model of decision-making revolving around three emotions:
    - The need to **assuage fear** by providing **Security**;
    - The need to **offer hope**, by providing an upside **Potential**;
    - The **need to succeed**, by achieving a predefined **Aspiration** level or goal.
- SP/A stands for “security, potential, and aspiration.”
- People rank alternatives by comparing the way in which these alternatives provide security, upside potential and the chance of success.
- People differ in terms of their emotional ned for security and potential, and in the degree to which they set aspiration levels that reflect their ambitions.

Ending the Management Illusion

How to Debias in a Corporate Environment



Two Key Ideas (Shefrin 2008, 2009)

- Common threads link the psychological pitfalls that affect financial judgments and decisions.
  - A relatively small set of psychological pitfalls were especially germane to the creation of the crisis.
  - Key mistakes made were specific phenomena lying at the heart of behavioural finance.
- How can we debias?
  1. Recognise the pitfalls;
  2. Understand their impact on the management process.

Component 1: Psychological Pitfalls

Reminder: A Short List of the Main Psychological Biases in Finance

- Here is a reminder of the main psychological biases:
  - Reference point–induced risk seeking,
  - Narrow framing,
  - Opaque framing,
  - Excessive optimism,
  - Overconfidence,
  - Extrapolation bias,
  - Confirmation bias,
  - Conservatism,
  - Affect heuristic,”
  - “Groupthink,”
  - Hindsight bias,
  - Categorisation bias.

This list is NOT exhaustive!

Impact of Psychological Pitfalls

- Psychological pitfalls have three impacts that analysts should be aware of (Shefrin 2008a):

    1. Impact on pricing of assets, particularly the securities of firms followed by analysts.

    2. Impact on decisions by corporate managers that are germane to companies' operational risks.

    3. Impact on the judgments of analysts themselves.

Component 2: Business Processes

Business Processes

- Sherfrin (2008b) identifies four key business processes:

  1. **Planning**:
     - development of strategy,
     - preparation of pro forma financial statements.
  2. **Standards**:
     - Design and implementation of goals and performance metrics.
     - From a risk management perspective, standards include targets and goals that relate to accounting controls and include position limits and other risk-control mechanisms

Business Processes

3. **Information sharing**:

   - Information sharing results from the nature of organisational design. Is information sharing unidirectional (top-down vs bottom-up), segmented (silos), opaque or transparent?

4. **Incentives**:

   - Compensation system - in theory, compensation provides managers with incentives to maximise the value of their firms;

   - Incentive compensation frameworks (beyond the base salary) often rely on a combination of

     - bonus plan that relates to the short term, and;

     - equity-based compensation that relates to the long term;

   - Key aspect, but not the only aspect of corporate governance.

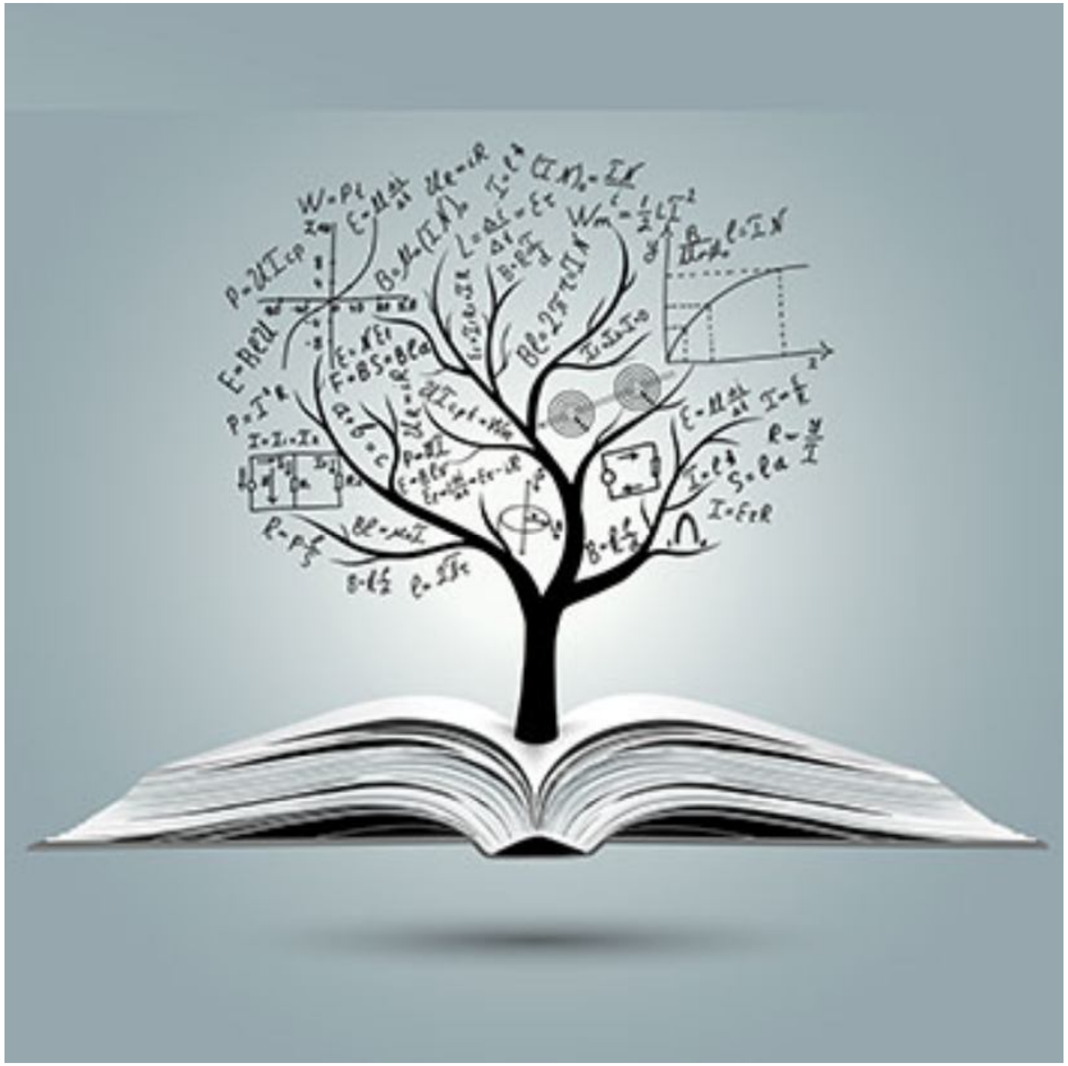
Bringing It Together: Ending the Management Illusion

Interaction Between Psychological Pitfalls and Business Processes (Shefrin 2008,2009)

Process	Reference Point-Induced Risk Seeking	Narrow Framing	Opaque Framing	Excessive Optimism	Overconfidence
Planning					
Standards					
Information sharing					
Incentives					

How Mathematics Can Help You and Your Team

How to use math to debias!



Tackling Behavioural Challenges in Quantitative Finance: A few Simple Rules

1. Beware of implicit assumptions!
2. Linear locally, but nonlinear globally
3. The data illusion
4. Understand regression to the mean
5. Statistics cannot prove anything, but they are extremely useful to raise question and understand a problem.
6. When in doubt, experiment and randomise!
7. Think like there is no box!
8. Do not be afraid to say “I do not know”
9. Most of what we think we know is wrong…
10. … And humans are pretty bad at updating what they know about the world.
11. Not everybody around you was trained in mathematics!

Beware of implicit assumptions!

During WW2, the US army asked: where should we put armour on a plane?

You cannot put armour <u>everywhere</u>!

You cannot put armour everywhere!

... Or you will end up with a tank!

So, we should put more armour where the plane is most likely to be shot.

A statistics question: where do planes get shot?

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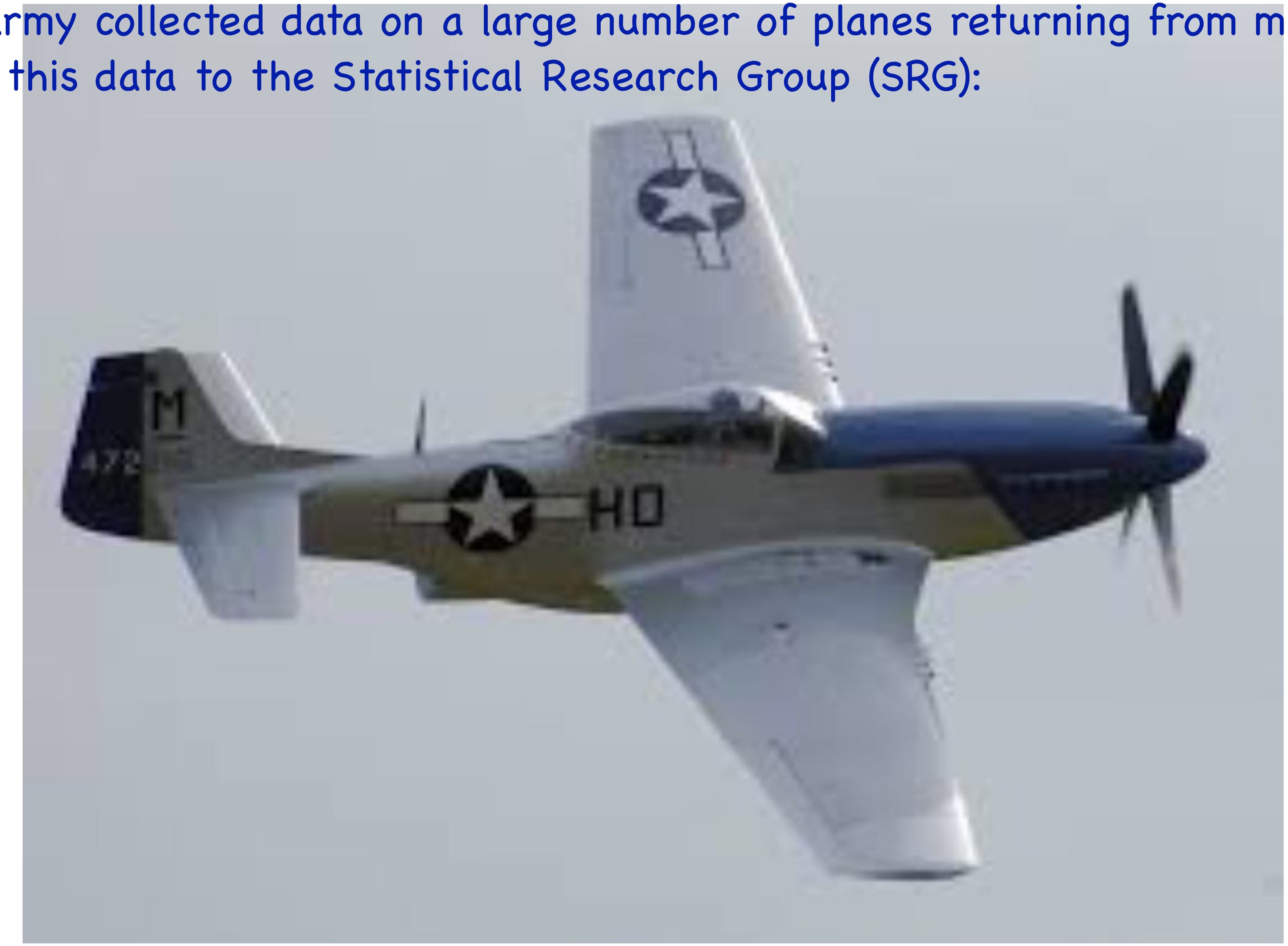
The US army collected data on a large number of planes returning from missions, and sent this data to the Statistical Research Group (SRG):



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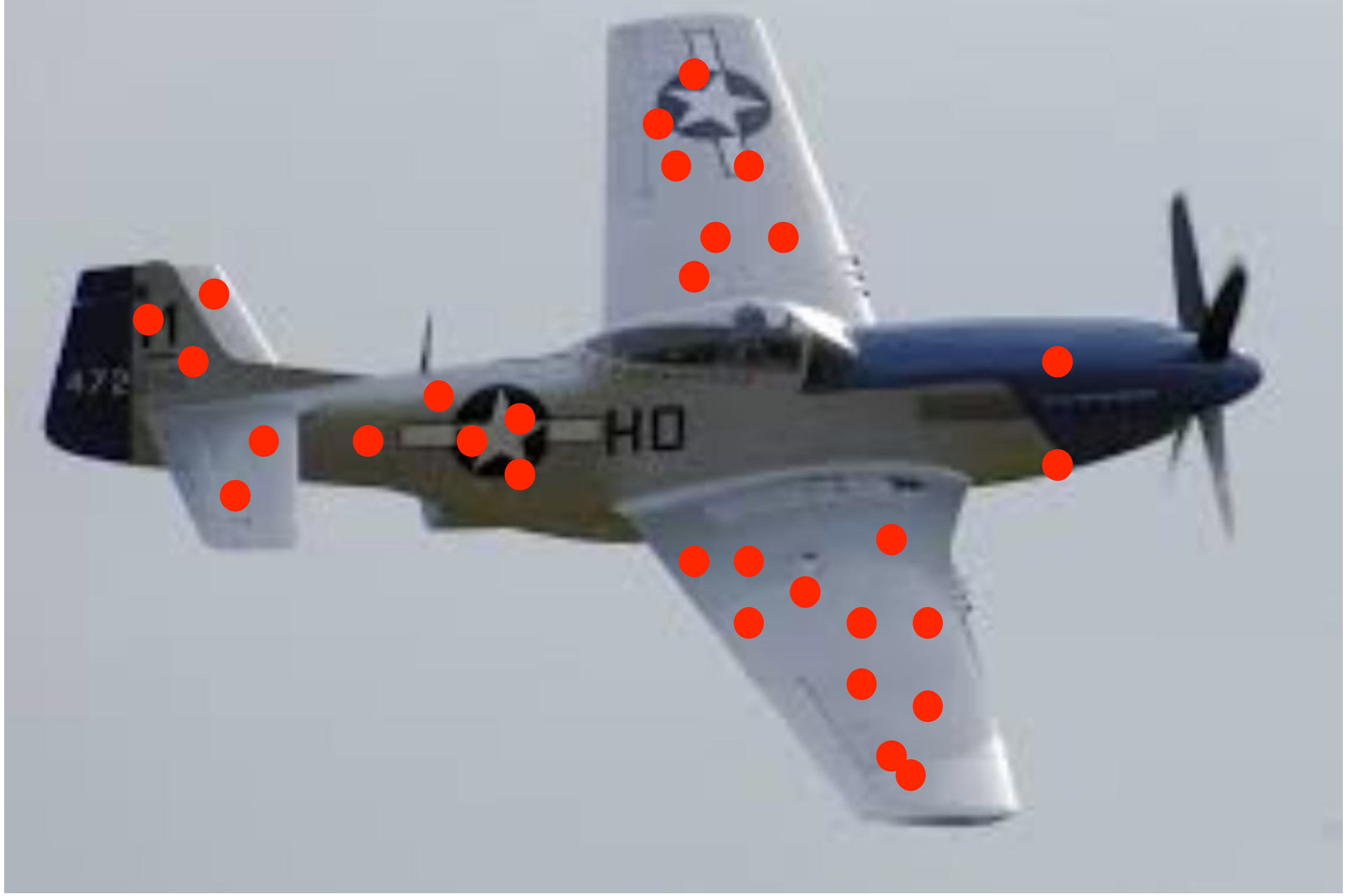

Abraham Wald asked: Why don't we have the same concentration of bullet holes on the engine?

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... so it would not be coming back home...

... and make it to the database!

Abraham Wald asked: Why don't we have the same concentration of bullet holes on the engine?

Because in most cases, a plane would not survive a direct hit to its engine!

… so it would not be coming back home…

… and make it to the database!

This is an example of survivorship bias: the sample we have is biased because it includes only surviving aircrafts and no downed aircraft!

Conclusion

Conclusion

You should put more armour where the bullets are NOT: on the engine!

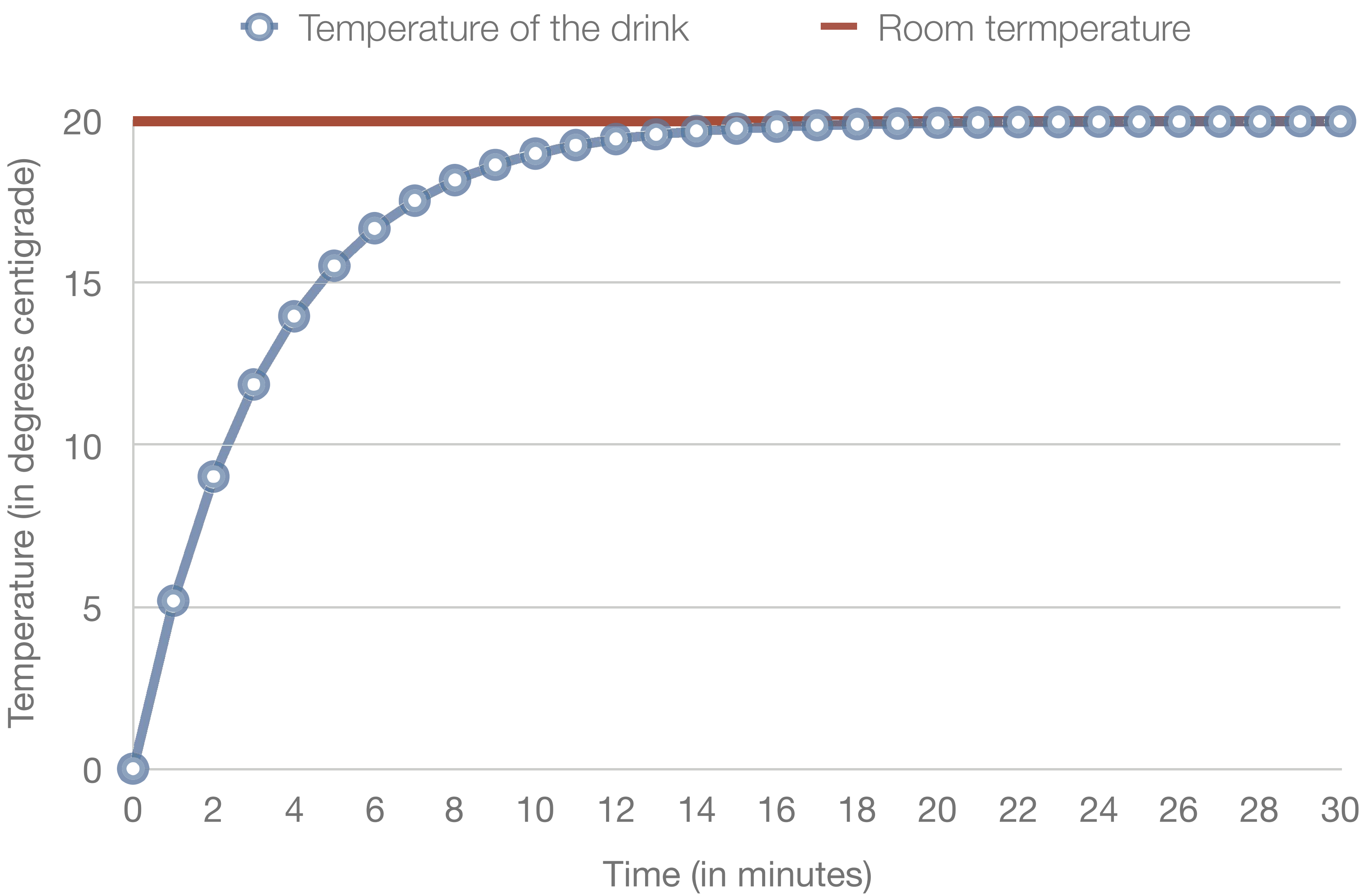
Mathematician Abraham Wald reached (and proved mathematically) this remarkable conclusion.

Linear locally, but nonlinear globally

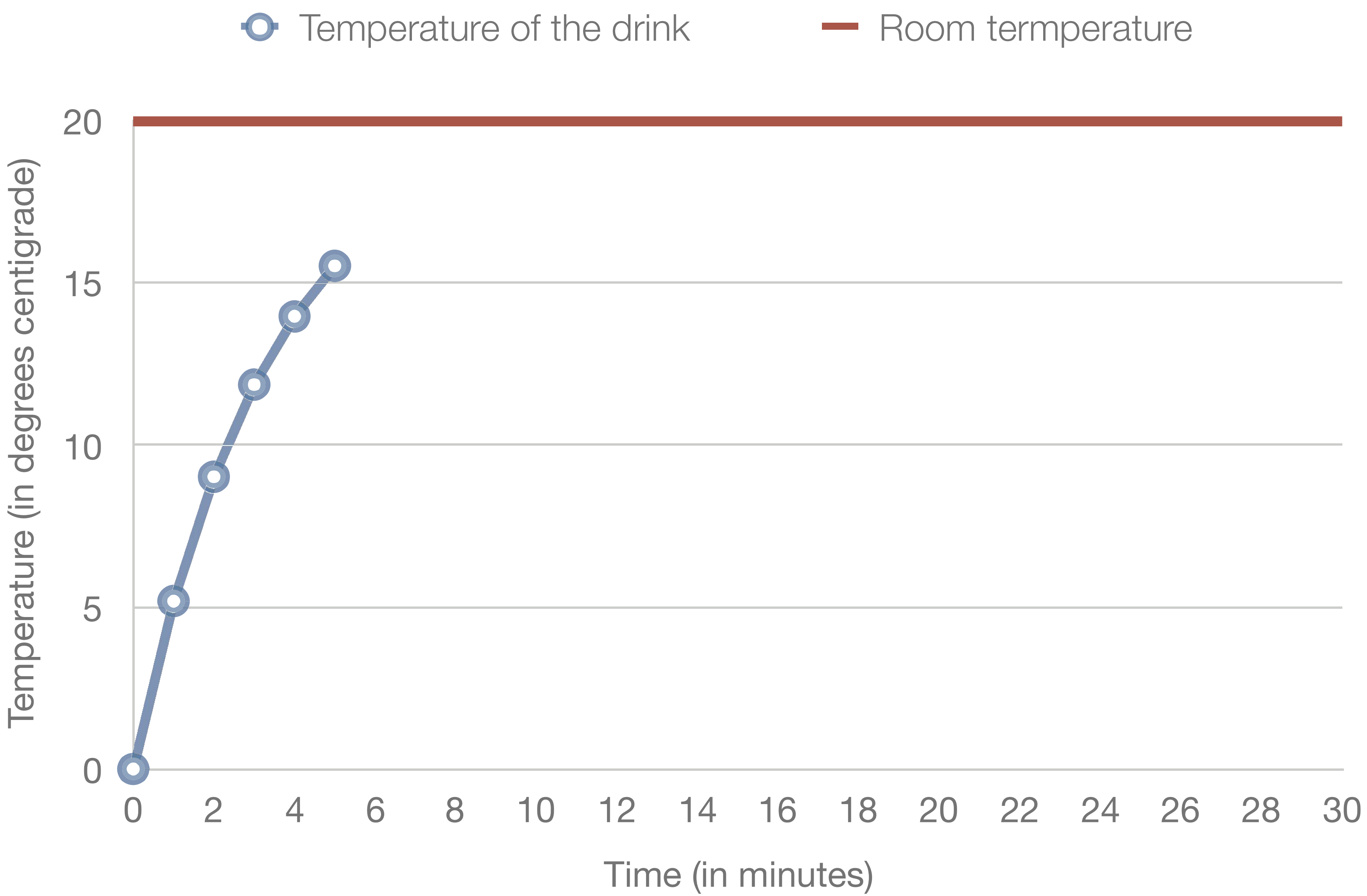
It is a hot day out there…

… you take an ice-cold soda can out of the cooler…

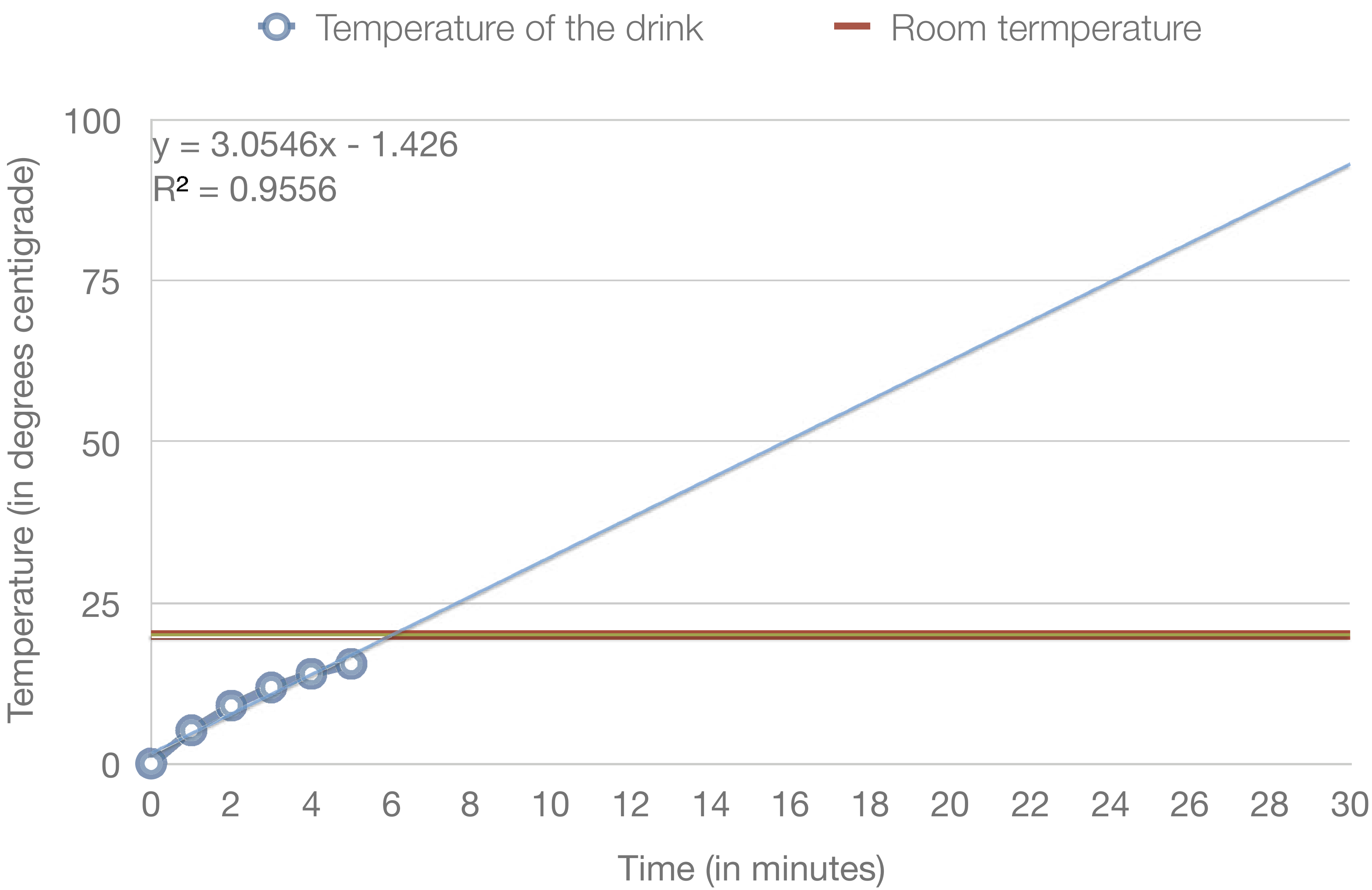
The drink’s temperature increases in time, but not in straight line.



Now, what happens if we try to forecast the change in temperature based on the first five minutes?



Let's try a linear regression!



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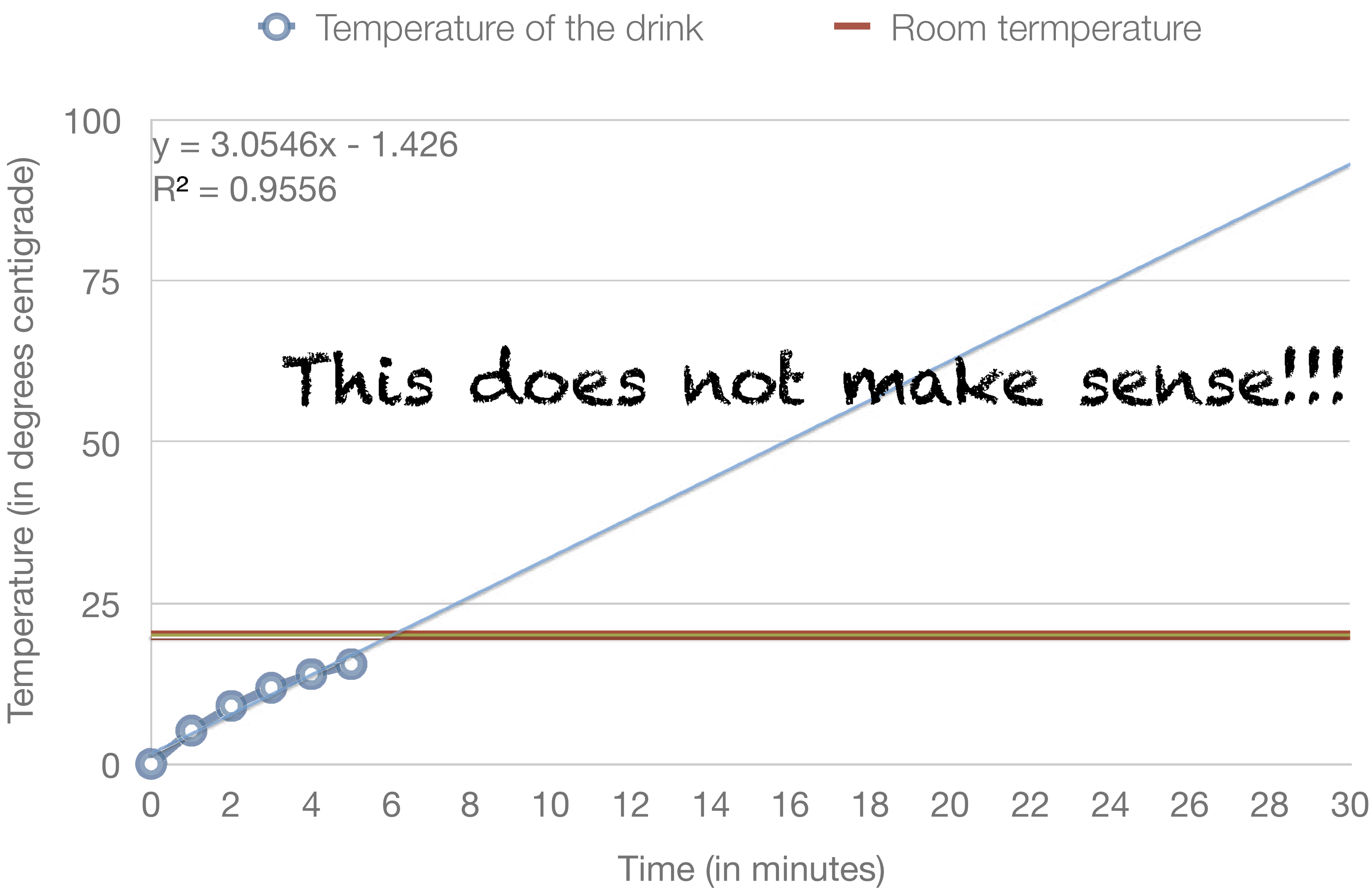
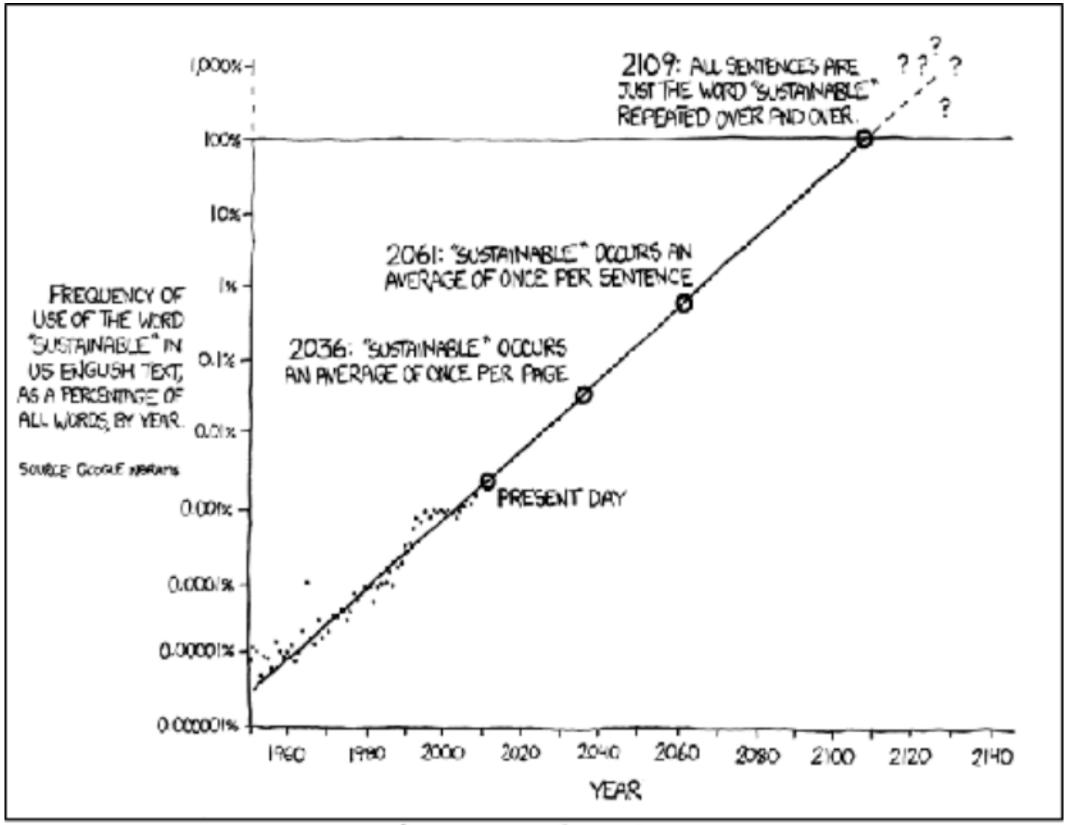


Conclusion

- Most real-world phenomena:
- are nonlinear **globally**;
- can be safely approximated by a straight line, but only **locally**!
    - This is the idea behind first order Taylor expansions… and basic linear regressions!



THE WORD "SUSTAINABLE" IS UNSUSTAINABLE.

The Data Illusion

The Data Illusion

- Recall that behavioural experiments have shown that the more (and often irrelevant) information we accumulate, the more confident we become.

- But quantity $\not\Rightarrow$ quality!

- This is very important in the era of big data!

- You do not need more data, you only need more **relevant** data!

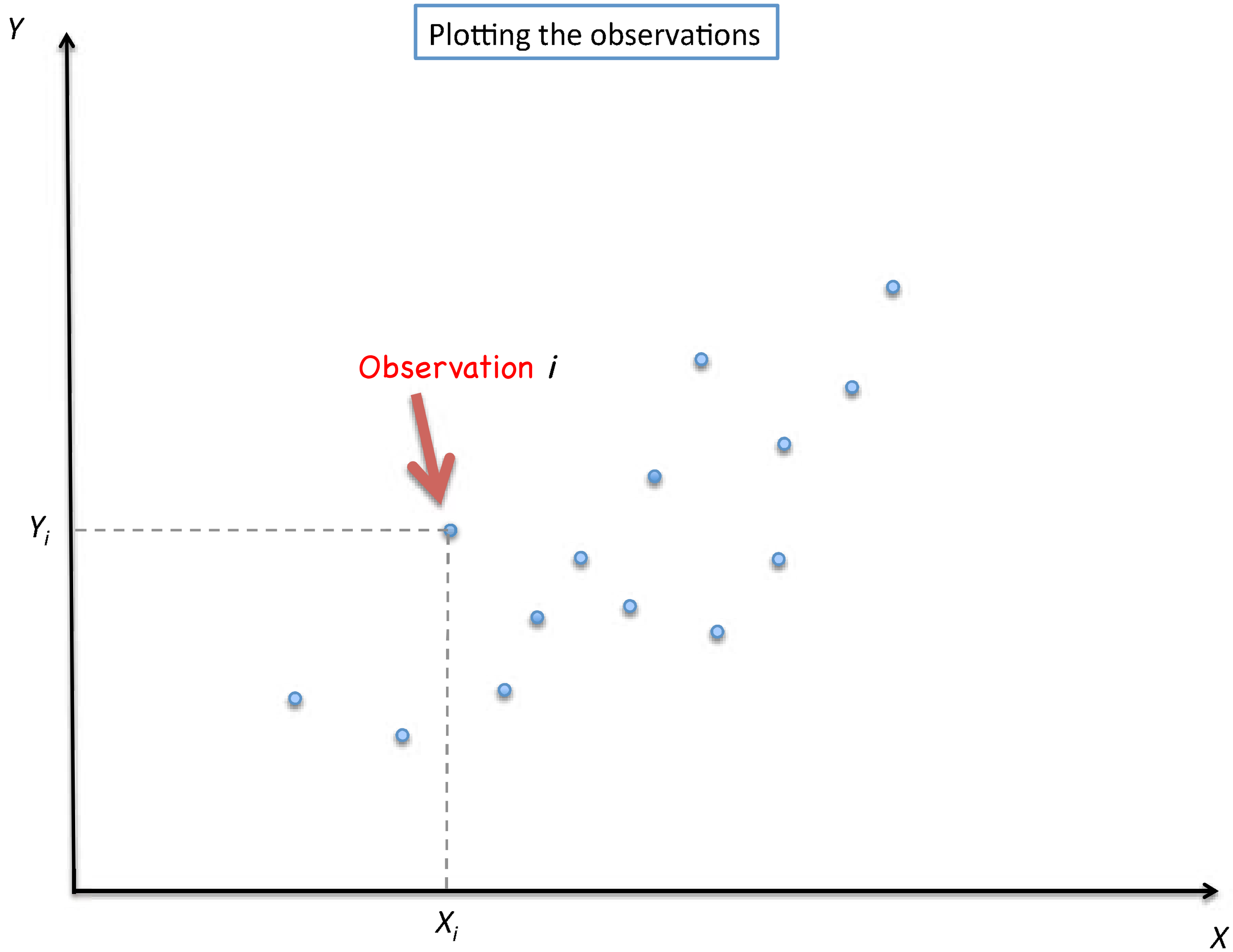
Understand Regression to the Mean

Understand Regression to the Mean

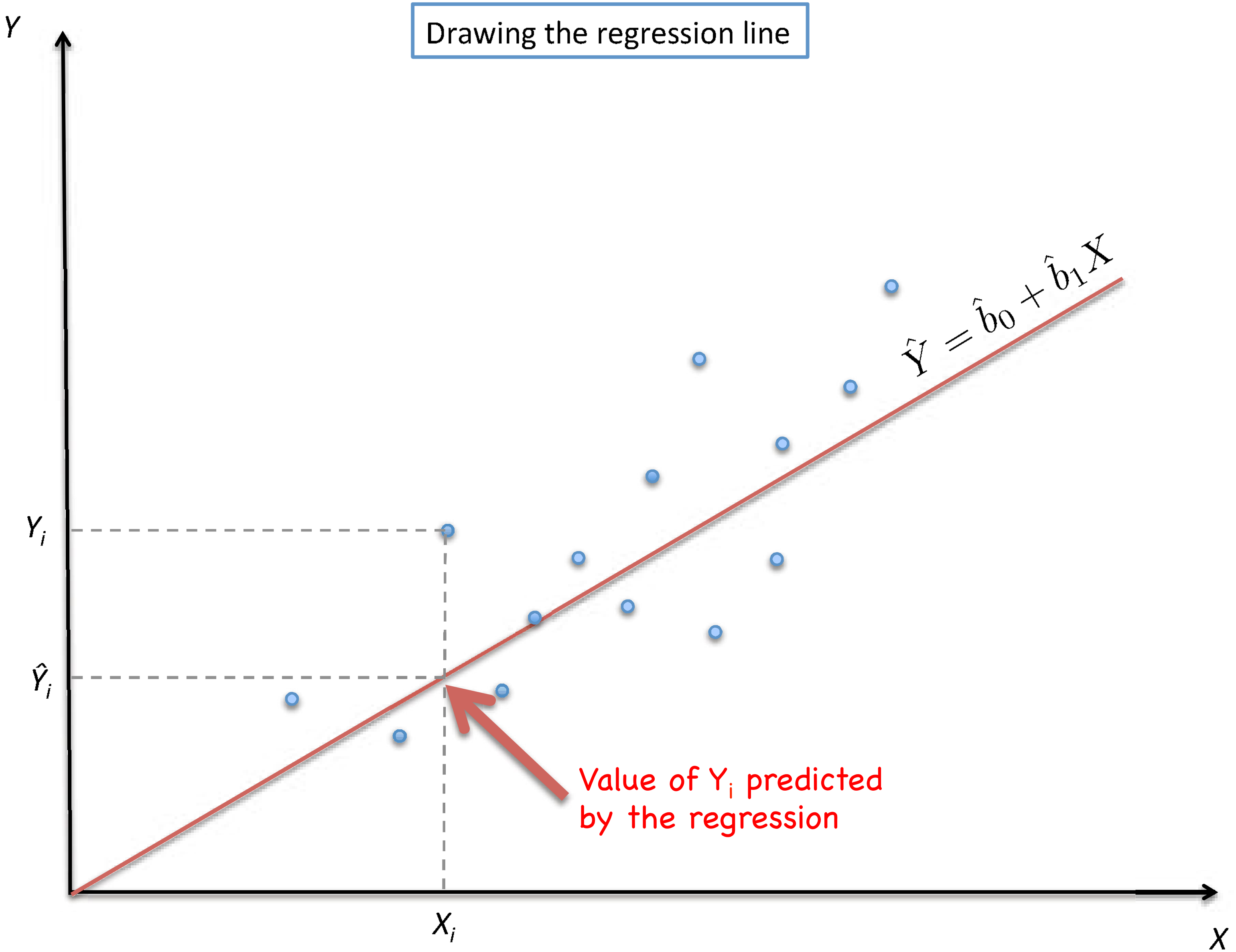
- Regression to the mean is a powerful concept, but it is often misunderstood.
- This misunderstanding leads to “if-then” heuristic error such as:
  - hot hand fallacy;
  - gambler’s fallacy.

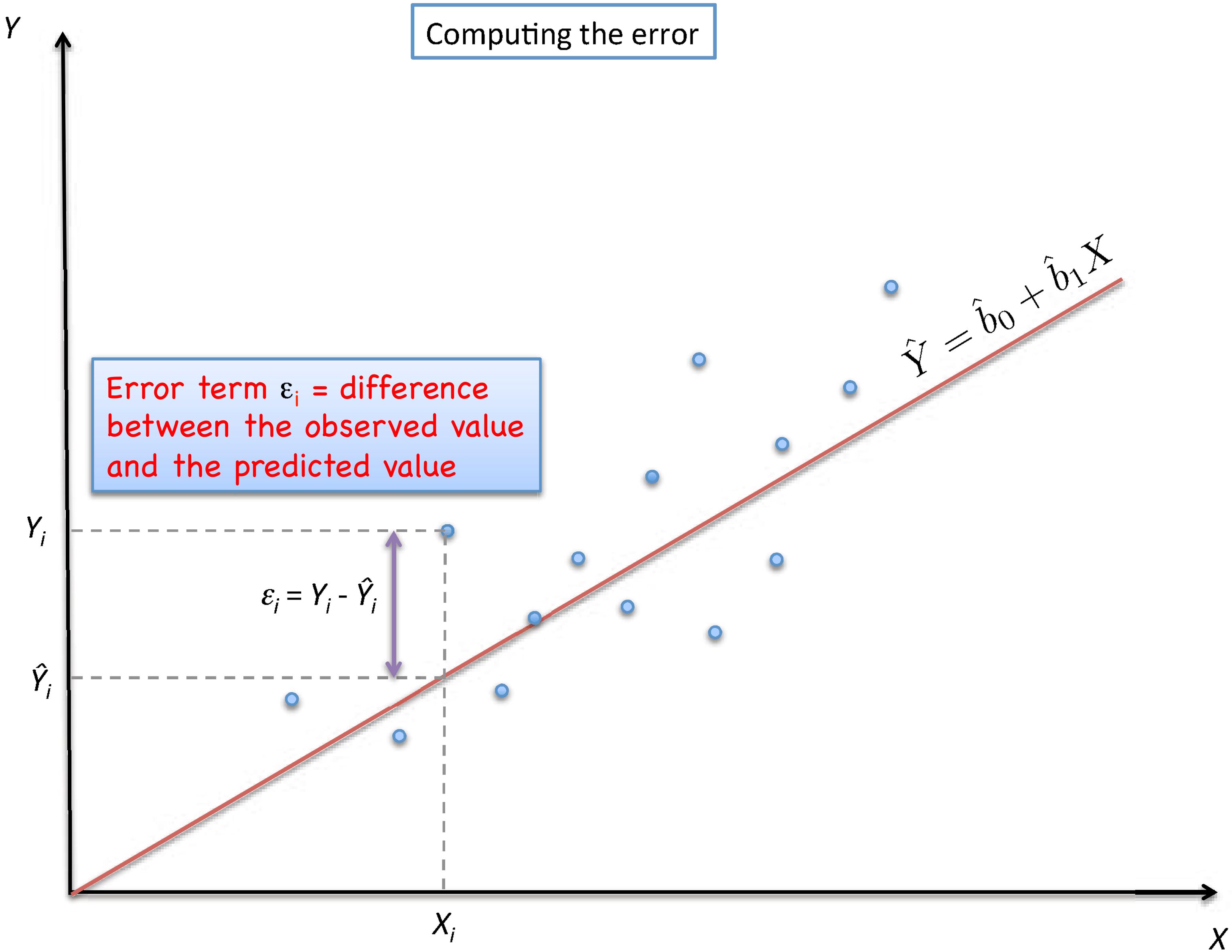
- In reality, regression to the mean states that:
    - Every observation or measurement contains some error;
    - This error is random and unpredictable;
    - However, the error will cancel **on average** over many measurements.

Linear Regression As An Example



Drawing the regression line




Computing the error
Y
X
Error term ε_i = difference between the observed value and the predicted value
$\hat{Y} = \hat{b}_0 + \hat{b}_1 X$
Y_i
$\hat{Y}_i$
$\varepsilon_i = Y_i - \hat{Y}_i$
X_i

- Now, we tend to pay more attention to extreme observations, for example an asset manager wth one stellar year;
    - These extreme observations are part signal, part randomness;
    - Regression to the mean hints that the error term of the next observation (next year's return) is likely to be lower… but does not guarantee it!
    - This is one of the key reasons why a manager who performed extremely well over a period of time, might perform comparatively poorly over the next.
- This phenomenon has little to do with the manager's skill and has probably more to do with random events.

Statistics cannot prove anything, but they are most useful to question hypotheses

Statistics cannot prove anything…

- Statisticians have long realised that you only need to one datapoint to prove a hypothesis wrong…

- … and this datapoint might not eleven be in the sample that you are looking at.

- This is another illustration of the **falsification** idea we encountered earlier.

… but they are extremely useful to raise question and understand a problem.

- Start from the data you have.
- Then use the data to question what you think you know about the problem.
- Once you understand the problem clearly, look for a solution.
- Finally, test the solution on your data:
    - this does not guarantee success…
    - … but it prevents costly failure.
- Keep updating what you know!

When in doubt, experiment and randomise!

Studies In Crop Variations

- As with many aspects of statistics, Ronald Fisher seems to have got there first.
- Karl Pearson's intense dislike for young Fisher, meant that Fisher had a difficult time getting published or finding a job.
- Eventually, Fisher gained employment at a small agricultural station:

  Rothamsted Experimental Station in Harpenden, Hertfordshire, England.
- There, Fisher was to undertake some of his most important work, including his celebrated series of articles "*Studies in Crop Variations*" I, II and III.
- Fisher also set the foundations for sampling and experimental design.

Ronald Aylmer Fisher (1890-1962)

Think like there is NO box

Do not be afraid of saying “I do not know!”

Most of what we think we know about the world is wrong…

Watch Hans and Ola Rosling’ s “How Not to Be Ignorant About the World”!

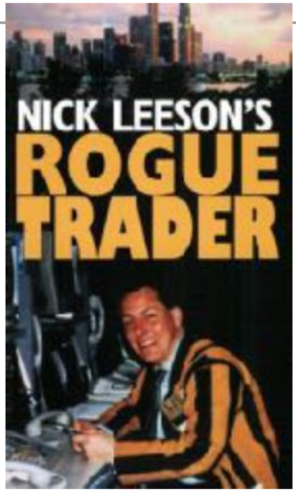
… And humans are pretty bad at updating what they know about the world.

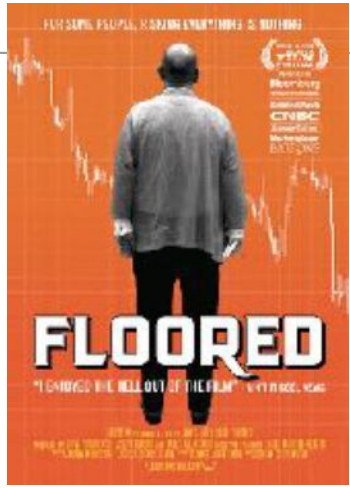
Remember conservatism a.k.a anchoring-and-adjustment and Baye’s rule!

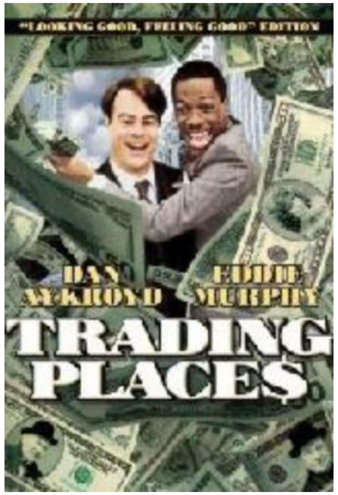
Not Everybody Trained In Mathematics

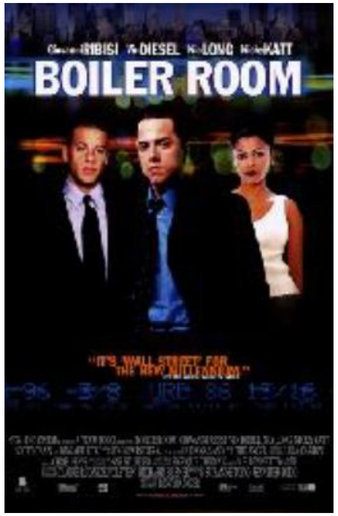
- Remember, not everybody in your organisation has had a mathematical training.
- You will need to be patient in your explanations!
- Remember Abraham Wald!
  - The U.S. military command believed that the extra armour should go where the bullets holes were…
  - But Wald managed to convince them that they should put extra armour where the bullets were **not**!
  - And he did just that… using mathematics.

Behavioural Finance… *at the movies*





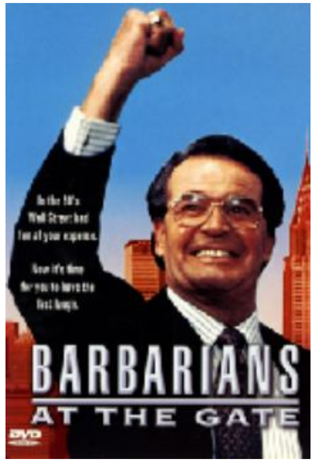




Happy watching!

TV Documentaries and Movies

- The Ascent of Money
- The Money Masters (BBC)
- PBS NOVA: Mind over Money
- Goldman Sachs CNBC Documentary:
  - Trading Techniques of an Investment Bank
- Million Dollar Traders (3 Episodes)
- ENRON the Smartest Guys in the Room (Movie)
- Barbarians at the Gates (Movie)
- Billions (TV Series)



An Excursion Into Game Theory

Decision Theory vs. Game Theory

- The whole class goes to a restaurant to celebrate the end of the CQF.
  - If each of you pays for your own meal separately $\Longrightarrow$ **decision problem**.
  - If you all agree to split the bill $\Longrightarrow$ **game**!

Game 1

- Your two 8-year old nephews, John and Paul are coming to visit you today. You have baked a cake for them.
- They are constantly squabbling over which of them is your favorite.
- How do you propose to cut the cake to satisfy both John and Paul?

The Importance of Considering Interactions…

- So far, all the theories (standard and *behavioural*) that we have introduced in this course
    - Centre on individual decisions.
    - Limit social effects to “crowd effects” in which the crowd is considered as a single, homogenous, entity.
- However, the human world is made from and through interactions.
- Game theory is the first attempt at analysing and understanding these interactions
    - From a strategic perspective;
    - Under the assumption of **`rationality`**.

Game Theory

- Game theory started in the 1920s as the recreational study of board game and card games by mathematicians.
  - Emile Borel and John Von Neumann made significant early contributions.
- Game Theory became mainstream when Von Neumann teamed up with Morgenstern in an effort to rewrite economic theory by taking into consideration the interactions between agents.
  - This led to considerable developments first in economics and later in corporate finance…
- In the early 1950s, John Nash presented an *elegant* definition of the solution of a game in a one–page article and a 27-page (!!!) PhD thesis.

Game Theory

- In the 1960s, Cold War political scientists used game theory as a rational formalization of the Nuclear standoff.
- In total, 8 game theorists have received a Nobel prize[1].
- Game theory has also gained a place in the popular culture through novels, movies or TV series.

[1] And a number of other Nobel prize winners have worked on or used game theory in their research.

Meet John and John: Von Neumannn (1903-1957) and Nash (1928-2015)

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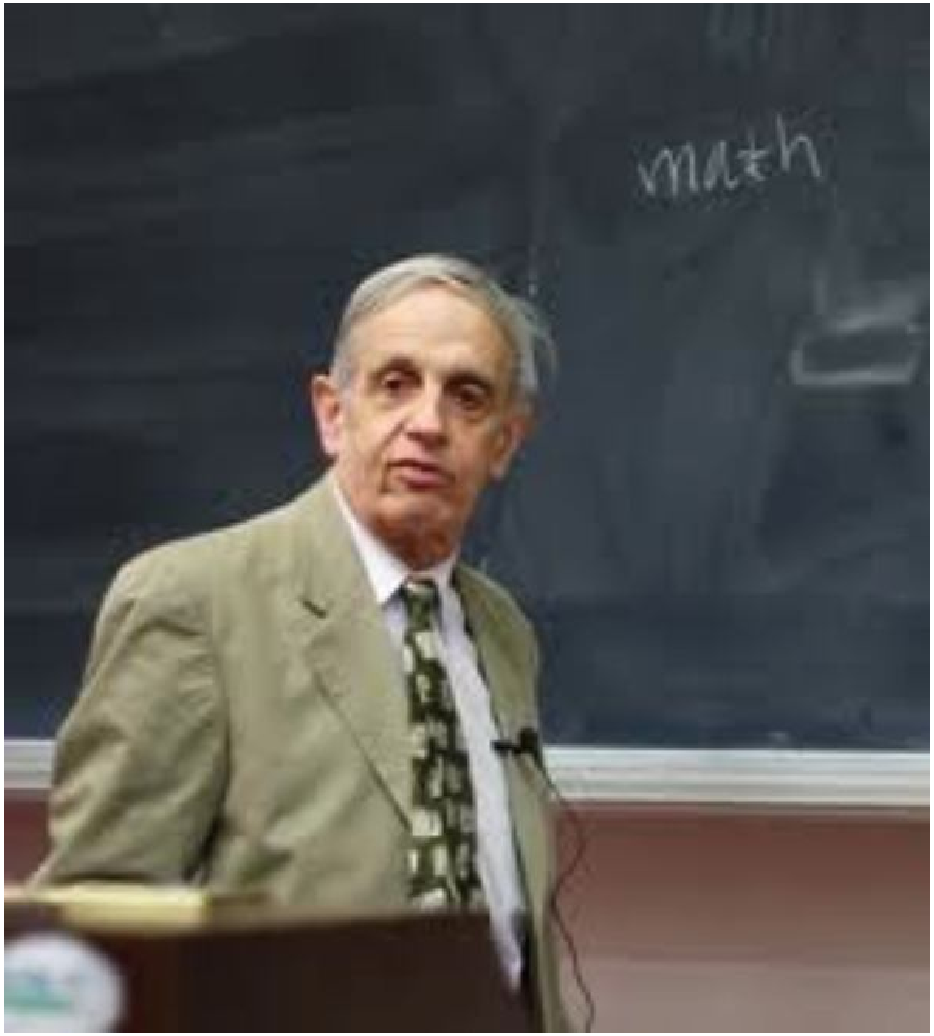

Robert Duncan Luce (1925-2012)

- Intellectual giant and pioneer of mathematical psychology.
- PhD in Mathematics from MIT.
- Was awarded the title of Benjamin Franklin Professor of Psychology at the University of Pennsylvania in 1968.
- Received the 2003 National Medal of Science in behavioral and social science for his contributions to the field of mathematical psychology.
- Wrote an influential book on game theory (with Howard Raifa) and tested empirically the predictions of game theory against the actual behaviour of individuals.
- Worked with and influenced the work of Amos Tversky.

Game 2

- You play a game with a partner with whom you cannot communicate.
- Your and your partner's objective is to maximise your individual gains.
- You can pick either the colour **black** or the colour **red**:
    - If you pick **red**, and your partner picks **red**, you gain 10 and your partner gains 5;
    - If you pick **red**, and your partner picks **black**, you gain 3 and your partner gains 4;
    - If you pick **black**, and your partner picks **red**, you gain 7 and your partner gains 4;
    - If you pick **black**, and your partner picks **black**, you gain 7 and your partner gains 3;
- What colour do you pick?

A First Representing of Your Choices

- In a game, the decision of your partner (or opponent) will affect your decision.
- Visualising your choices is crucial.
- The table below represents your choices as a list.

Your colour	Partner’s colour	Your gains	Partner’s gains
R	R	10	5
R	B	3	4
B	R	7	4
B	R	7	3

Main Elements of a Game

- There are four main elements to a game:

  1. Players;
  2. Strategies;
  3. Type of Game;
  4. Payoffs.

Player

- **Players**:
  - The players are the decision makers.
  - Their objective is to maximise their payoffs.
  - Games can have from 2 to any arbitrary number of players. In this case we talk about **n-player games**.

Strategy

- **Strategy**[1]:
  - Informally, a strategy is a **plan of action**.
  - Formally, a strategy is a rule for choosing an action at every point that a decision has to be made.
  - The strategy set can contain:
    - A finite number of actions (*ex:* "***Black***" or "***Red***");
    - An infinite number of actions (*ex:* select a number in the interval [0,1]).

[1] The term "strategy" evolved from the Greek ***strategos*** (στρατεγοσ) meaning military commander.

Type of Game

- To classify games, we need to look at three main dimensions :

    1. Sequential vs. simultaneous games;

    2. Cooperative vs. non-cooperative games;

    3. Static vs. dynamic (or evolutionary) games.

Sequential vs. Simultaneous Games

- In most (but far from all) games, players act **simultaneously**:
  - **Imperfect information**: at the time players make their decision, they do not know what the decisions of the other players are;
  - This assumptions eliminates any advantage stemming from the order of play and focuses on the rational decision.
- Some games allow players to act **sequentially**, meaning that the strategy of the first player is known by the time the second player makes his move
  - At times it is better to move first to capture a new market ("the early bird gets the worm");
  - Other times it is better to move last in order to get some additional information ("the early bird gets shot")

Cooperative vs. Non-Cooperative Games

- In **non-cooperative** games, players cannot communicate with each other in order to design a coordinated strategy.
  - Most games are non-cooperative;
- In **cooperative** games, players are allowed to communicate to design a joint strategy.

Static vs. Dynamic and Evolutionary Games

- Static games are played only once.
- Dynamic games are played sequentially:
  - Dynamic games may occur in finite or infinite time.
  - **Differential games**[1]: games played in continuous time.
  - **Evolutionary games**[2]: dynamic games used in biology to understand the evolution of a population according to their biological traits or behaviour.

[1] Rufus Isaacs (1965) provided the first actual treatment of the subject.

[2] The pioneering work in the field is due to John Maynard Smith (1982), although the argument appeared informally in earlier works on natural selection (includinG Darwin).

Game 2

- **Game 2**:
  - Has two players: “you” and “your partner”;
  - The set of strategies for each player is “***Black***” or “***Red***”;
  - The game type is:
    - **Simultaneous**: neither you nor your partner knows how the other will play;
    - **Non-cooperative**: you cannot communicate with your partner;
    - **Static**: we only play this game once.

Payoffs and Game Representations

- The payoffs represent the amounts won or lost by each player subject to the different strategies:
  - The payoffs can be monetary amounts, gain or loss of utility, market share... etc;
  - The payoff is the most important part of the game ➔ it gives its structure to the game.
- There are two useful ways to visually represent static games:
  - Strategic form;
  - Tree.

The Game in Strategic Form

- The game can be represented in its **strategic form** as a payoff matrix;

- This representation is mostly used for non-cooperative games.

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Game 2 in strategic form

You \ Partner	Black	Red
Black	7,3	7,4
Red	3,4	10,5

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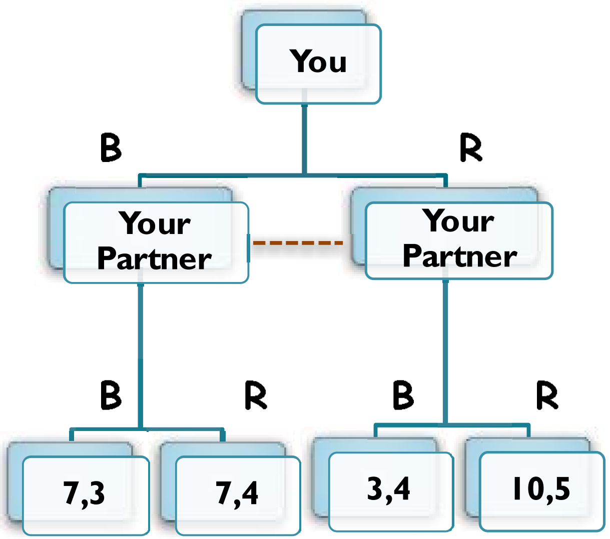
The Game as a Decision Tree

- In a **tree form**:
  - Each node represents a decision that must be taken.
  - Each line represents a possible decision or action.
  - The doted line at the level of "your partner" indicates that the two players act simultaneously.



Empirical Result

- When faced with **Game 2**,
  - About 70% of respondents pick **red**;
  - About 30% of respondents pick **black**.
- So, how do we solve the game?

Solving the Game

- To solve the game, we need to:
  - Define what we mean by "**solution**;"
  - Design an appropriate methodology to find the solution.

Nash Equilibrium

- The solution to a two-player game is a pair of strategies adopted by each of the two players.
- The standard definition of a solution is as a **Nash equilibrium.**
- A **Nash equilbrium** is a solution to the game such that no player can improve his/her payoff by changing strategy **unilaterally**.
- The definition of a Nash equilibrium is appealing:
  - It is intuitive;
  - It implicitly refers to the idea of minimising regret.

Existence of Nash Equilibria

- Every game that has a finite strategic form, i.e.
  - Finite number of players;
  - Finite number of (pure) strategies for each player;

has **at least one** **Nash equilibrium**.

Solving the Game by Elimination of Dominated Strategies

- The **elimination of dominated strategy** is a particularly simple way of looking for Nash equilbria.

- The basic idea behind the **elimination of dominated strategy** is that

  - A **Nash equilibrium** is a strategy that offers the **best response** to every strategy of the other player.

  - We can then go through all of the strategies available to each player and eliminate sequentially all the strategies that are not best response strategies.

  - Whatever strategy is left should be a **Nash equilibrium**.

Dominated and Inadmissible Actions

- The first thing to do is to try to simplify your decision problem by removing all the dominated (inadmissible) actions.
- An action is **dominated** (or **inadmissible**) if we can find another action that has
    - a <u>strictly greater payoff in at least one state</u> of nature, and
    - <u>no worse payoff</u> in any other states.
- Dominated actions can safely be excluded from the analysis because we can always find a better alternative among the other choices.

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  - Etc… etc… etc…
- This chain of assumptions on rationality is called **Common Knowledge of Rationality** (CKR).
  - This assumption guarantees that each player is rational and can put himself/herself in another player's situation.

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  - Hence, as far as player 2 is concerned, we can eliminate ***Black*** and focus solely on ***Red***.
- Now we can look at the problem as ***Player 1***:
  - Since Player 2 will pick ***Red***, the best response for Player I is to choose ***Red*** as well.

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- Note:

  - Starting from ***Player 2***'s perspective proved convenient as we were able to identify and eliminate a strictly dominated strategy almost immediately;

  - However, we could have started from the perspective of ***Player I*** and then move to ***Player 2***. We would still have found the same Nash equilibrium ➔ the order in which we perform the elimination is <u>irrelevant</u>.

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  - ***Black*** provides the highest degree of certainty regardless of the strategy adopted by Player 2.
  - ***Black*** is a pragmatic strategy if Player 1 thinks Player 2 is irrational, does not understand the game, has been given a different information about the game or is just plain malevolent…
  - … but ***Black*** is suboptimal if both players are rational and play the game according to the same rules.

Solving the Game by Elimination of Dominated Strategies

- Most games cannot be solved by elimination of dominated strategies.
  - However, when a game can be solved this way, the elimination of dominated strategies will automatically yield the Nash equilibrium or equilibria.
- In a general situation, we would either:
  - Solve a few equations;
  - Look at the payoff matrix entry by entry to determine where the Nash equilibria are.

Pure Strategies

- So far, we have considered **pure strategies**, that is strategies that commit us to a definite course of action….

Question 3

- This is the world cup final, and you are set to shoot the last penalty kick. If you score, you and your team win the world cup. If you fail to score, you and your team lose.
- Do you shoot:
  - Left of the keeper?
  - Right of the keeper?

Game Theory in Sports

- Top athletes, and in particular football players, ice hockey players and tennis players routinely engage in games against an opponent: another tennis player or a goalkeeper.

- How does a football player or ice hockey player “decide” where to shoot?

- How does a goalkeeper decide whether to go left or right to intercept the shot?

Game Theory in Sports

- Most professional athletes engaged in games naturally randomise their strategy:
  - Over their career, football players will shoot their penalties left/right of the keeper at about 50%-50% probability[1].
  - Over a season, top tennis players will serve left/right of their opponent at about 54%-46% probability[2].
- This observation leads us to extend our definition of strategies to include these randomised strategies, which are called **mixed strategies** in game theory.

[1] Left-handed and right-handed have slightly different probabilities.

[2] The departure from 50%-50% is meant to maximise the probability of winning the point.

Mixed Strategies

- Formally, a **mixed strategy** is a probability distribution over all pure strategies.

- Penalty shootout:

    - The two pure strategies are "**shoot left**" and "**shoot right**."

    - The mixed strategy adopted by most players is (50%,50%), meaning **shoot left** 50% of the time and **shoot right** the remaining 50% of the time.

- **Nash equilibria** can occur among either pure strategies or mixed strategies.

Two Interpretations of Mixed Strategies

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- **Naïve interpretation**: the player chooses randomly one of the possible actions according to the probabilities of the mixed strategy.

- **In a population**: the probabilities of the mixed strategy represent the proportions of the population selecting a given action.

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- Example - in tennis, the 54%-46% can be interpreted as:

    - On each of his serve, a given tennis player will pick a side randomly with a 54% chance of serving left and a 46% chance of serving right.

    - Out of all the population of tennis players, 54% will serve on the left and 46% will serve on the right.

Zero-Sum Games

- In a zero-sum game, one player's gains are another player's losses.
    - One player wins and the other(s) lose(s).
    - The net payoff of the game is 0 under every possible strategy.
- Zero-sum games were the first games to be studied:
- The motivation for the study of zero-sum games comes from card games and board games.
- Well specified (generic) zero-sum games have a unique solution.
    - One can obtain the optimal strategy by solving a minimax or maximin optimisation problem.
    - The optimal solution corresponds to the **unique Nash equilibrium**.

Illustration: Rock-Scissors-Paper

- Two children, Adam and Beth are playing the "Rock-Scissors-Paper" game.
- The children simultaneously make the shape of one of the items with their hands:
  - Rock (R) beats Scissors (S);
  - Scissors (S) beats Paper (P);
  - Paper (P) beats Rock (R);
- If both players choose the same item, then the game is a draw.

Rock-Scissors-Paper in Strategic Form

- The “Rock-Scissors-Paper” game is a zero-sum game;
- It has a unique **Nash equilibrium**: the mixed strategy (1/3,1/3,1/3);

Adam / Beth	R	S	P
R	0,0	1,-1	-1,1
S	-1,1	0,0	1,-1
P	1,-1	-1,1	0,0

Game 4: The Prisoner's Dilemma

- The police caught you and your partner. They do not have enough evidence to convict either of you and absolutely need one (or both) of you to testify.
- The detective in charge of the investigation puts each of you in a separate interrogation room to prevent any communication.
- You have two choices: you can either "rat out" (denounce your partner) or "keep quiet" (refuse to talk):
  - If both you and your partner keep quiet, you will both be sentenced to 2 years in jail.
  - If you "rat out" but your partner keeps quiet, he gets 10 years in jail and you walk free.
  - If your partner "rat out" but you keep quiet, you get 10 years in jail and your partner walks free.
  - If you both "rat out," you will both be jailed for 6 years.
- Do you "rat out" or "keep quiet"?

The Prisoner's Dilemma in Strategic Form

- The Prisoner's dilemma can be solved by elimination of dominated strategies.
- The **Nash equilibrium** for the prisoner's dilemma is (*Rat out*, *Rat out*).
- This solution is **socially inefficient**: by following their self interest, both prisoners end up worse off.

You \ Partner	Quiet	Rat out
Quiet	-2, -2	-10, 0
Rat out	0, -10	-6, -6

Pareto Optimality

- A solution is said to be **Pareto[1] Optimal** if no player's payoff can be increased without decreasing the payoff to another player.

- Said otherwise, an outcome of a game is Pareto optimal if no other outcome makes every player as well off and at least one player better off.

- Pareto optimal solutions are termed **socially efficient**, or just **efficient**.

- The **Pareto optimal** solutions for the prisoner's dilemma is for both prisoners to keep quiet.

[1] Named after Italian economist Wilfredo Pareto (1848-1923).

Socially Suboptimal Choices

- The prisoner's dilemma is a starting point for discussion of the notion of **social contract**.
- *Example*: there are two certainties in life...
  - Individually, everyone is better off by minimising the amount of taxes paid;
  - But if no-one pays taxes, then there is no money to pay for community services (healthcare, education, roads, garbage collection, etc...) and everybody ends up worse off...
- This is also the origin of the **free rider problem**.

Another Example: Game Theory at the Opera[1]

- In Puccini's opera *Tosca*, Tosca's lover has been condemned to death.
- The police chief, Scarpia, offers to fake the execution if Tosca will sleep with him.
- The bargain is struck.
- However,
  - In order to keep her honor, Tosca stabs and kills Scarpia;
  - Unknown to Tosca, Scarpia has also reneged on the deal and Tosca's lover has been set for execution.

[1] Example from James Webb's book *Game Theory*.

The Tosca Game

- Tosca (presumably) prefers to keep her honour so,
  - a > c;
  - b > d.
- Scarpia (presumably) prefers to do his duty so,
  - $\beta > \alpha$;
  - $\delta > \gamma$;
- … resulting in the outcome presented in the Libreto[1].

Scarpia \ Tosca	Fake execution	Real execution
Kill	a, α	b, β
Sleep	c, γ	d, δ

[1] Also, do you know many operas that end on a happy note?

Dynamic Games

- Can we solve the prisoner's dilemma so that the dominant strategy coincides with the Pareto optimal strategy?
- We could introduce loyalty, threats and retaliation:
  - This would affect the payoff structure…
  - … and would change the prisoner's dilemma into an entirely different game.
- We could also repeat the prisoner's dilemma by considering that the two prisoners will repeat the same game over and over again ➔ would this change anything?

Tit for Tat

- The answer is yes!
- Consider the following "**Tit for Tat**" strategy[1]:
    - Stay quiet in the first round;
    - After that, do whatever the other player did in the previous round.
- The "**Tit for Tat**" has the following characteristics:
    - It is more effective than "Always Quiet" and less aggressive than a "Always rat out" strategy;
    - It is not a game winning strategy…
    - … but overall it is a more efficient strategy than any other because it rewards cooperation and punishes defection.
    - It can be a Nash equilibrium in some circumstances.

[1] The two pioneers are Robert Axelrod and Anatol Rapoport.

Back to "One-Time" Games

- Some games do not extend easily to dynamic or evolutionary games and can only be played once.
  - Generally, "one-time" games have a large negative payoff;
  - The main example is the **game of chicken**.

Game 5: Chicken

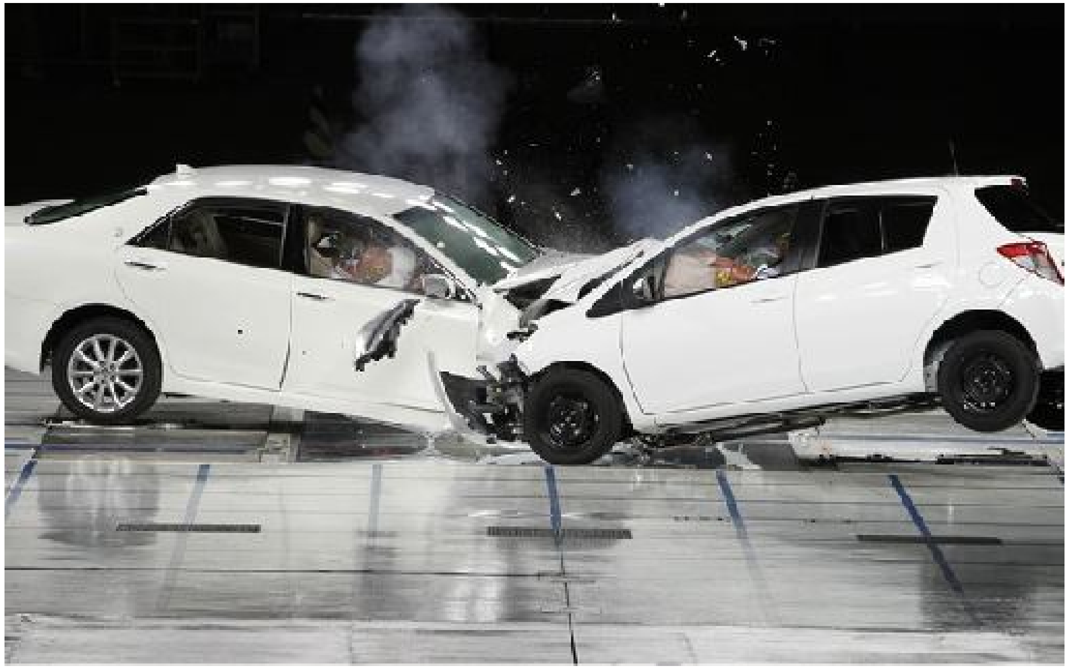
- We are travelling back to the 1960s. Andy has challenged Bud to a *game of chicken*: they will both race their car toward each other at full speed. The first one who yields is a *chicken*!

- Payoffs:

  - Both chicken out: let's just forget about it! ➔ payoff of 0 for both;

  - One chickens out and the other stand his ground: the chicken loses his pride and the one who stood his ground earns some bragging rights ➔ payoff of -2 for the chicken and of +1 for the one who stood his ground.

  - Both stand their ground: car crash! ➔ payoff of -8 for both.



Game 5: Chicken

- The game of chicken has three Nash equilbria:
  - Two **pure** strategies: (Chicken, Stand) and (Stand, Chicken)
  - One **mixed** strategy: (6/7,1/7)
- The game of chicken is an example of an **anti coordination** game:
  - In an anti coordination game, the best strategy is to play the opposite action of your opponent's.

You \ Partner	Chicken	Stand
Chicken	0,0	-2,1
Stand	1,-2	-8,-8

“Rigging” the Game of Chicken

- The game of chicken is often used as an analogy for geopolitical games.
- How do you rig the game in order to ensure your own survival? By making **threats of retaliation**!
- The threat of retaliation must:
  - Be credible;
  - Have dire consequences;
  - Be public
- Examples of threats:
  - Disabling the stirring wheel of a car;
  - Netherlands threatening to open the dams in the event of an invasion.
  - Joint threat: Mutually Assured Destruction (MAD);

Commitment

- Deterrents and threats are examples of **commitments**.
- Commitments can be
  - **Negative**: threats, deterrents;
  - **Positive**: promises.
- *Example of commitment*:
  - Alliance treaties;
  - Scorched earth (USSR during operation Barbarossa);
  - Burning bridges (or ships) to avoid a retreat.
- Commitments can be used to rig the game… as long as they are **credible**.

Behavioural Games And The Limits of Rationality

- We do not make purely rational decisions and cannot expect that our opponent or partner will be fully rational.
- Enter behavioural games.
- To further illustrate the limits of rationality and introduce behavioural games, let's go back to the game-deciding penalty shootout in the world cup.

Behavioural Games And The Limits of Rationality

- The optimal strategy for the shooter is neither to shoot left nor right, but to aim in the centre!
    - In all likelihood, the keeper will jump either left or right;
    - This leaves the centre open regardless of the keeper's choice!
    - Shooting in the centre maximises the probability of scoring.
- So, why do so few players aim at the centre?
    - Behavioural factors come into play!

Just imagine you shoot in the center, that the keeper stays where he is and stops the ball…

Concluding Thoughts

- For the past 60 years (at least), game theory has been a place of exchange for mathematicians, economists, social scientists, psychologists, computer scientists and political scientists.
  - Mathematicians have led theoretical developments;
  - Economists and political scientists have led applications;
  - Social scientists and psychologists have tested the predictions of game theory against the actual behaviour of individuals.
- More often than not, the predictions of game theoretical models have been validated by actual experimentations.

Concluding Thoughts

- Our fascination for game theory is fourfold:
  - Game theory takes into consideration human interactions;
  - It tries to remove emotions in a context that is naturally charged with emotions;
  - It spans the full range of strategies from full cooperation to complete conflict;
  - Game theory is also just plain fun…
- However, by and large game theoretic models are often regarded as either simplistic toy models or models that are too difficult to implement.
  - A possible psychological reason is that game theory makes a heavy use of the reflective part of the brain by forcing us to think about somebody else's actions and decisions ➔cognitive overdrive.

Concluding Thoughts

- There is much (much) more to game theory.
  - More advanced topics include:
  - Dynamic games;
  - Auction theory;
  - The limits of rationality (concept of “trembling hand perfection”);
  - Information asymmetry and communication (the classic examples are the agency problem and the dividend announcement problem in corporate finance);
  - Behavioural games;
  - Games on networks.

Game Theory… *at the movies*

- Game theory features in many movies, including:
  - Dr Strangelove
  - The Good, The Bad and the Ugly
  - Princess Bride
  - A Beautiful Mind
  - The Hunt for Red October
  - The Sum of All Fears
  - Ronin

Happy watching!

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Happy watching!

Wrap-Up

- In this lecture, we have seen…
  - The difference between behavioural finance and standard finance
  - Thinking fast and slow: System 1 vs System 2
  - Behavioural biases: overconfidence, excessive optimism, confirmation bias, illusion of control
  - Heuristic processes: representativeness, availability, anchoring and adjustment, affect, extrapolation bias
  - Framing effects: narrow frame, opaque frame, transparent frame, mental accounting, aversion to a sure loss
  - Group processes: herd behaviour, groupthink
  - Loss aversion vs. risk aversion:
  - Loss aversion and prospect theory
  - SP/A theory
  - A primer on game theory