Set theoretic notions have special interpretations in prob. theory

* Complement in Ω of event A, written A^c "NOT A"
* UNION $A \cup B$ is the event — "at least A or B occurs"
* Intersection $A \cap B$ "Both A and B occur"
* Inclusion $A \subseteq B$ "occurrence of A implies occurrence of B"

To a basic set of outcomes ω_i assign $\mathbb{R}$ numbers called probabilities

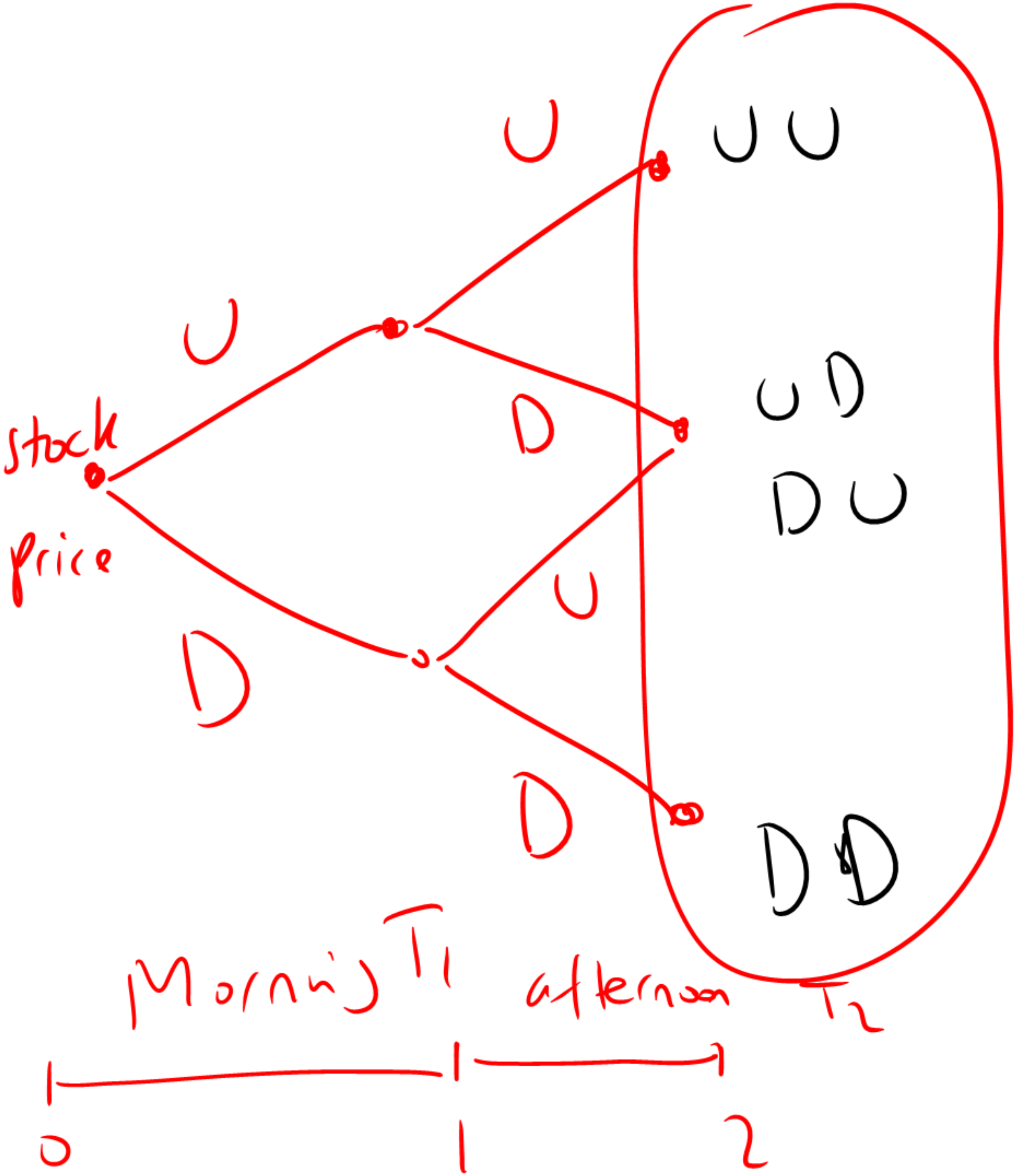
written $\mathbb{P}(\omega_i) = p_i$

Then for any event E_i

① $\mathbb{P}(E) = \sum_{\omega_i \in E} p_i$

② $0 \leq p_i \leq 1$

③ $\mathbb{P}(\Omega) = \sum_i p_i = 1$



Sample Space Ω

Examples of Trajectory:

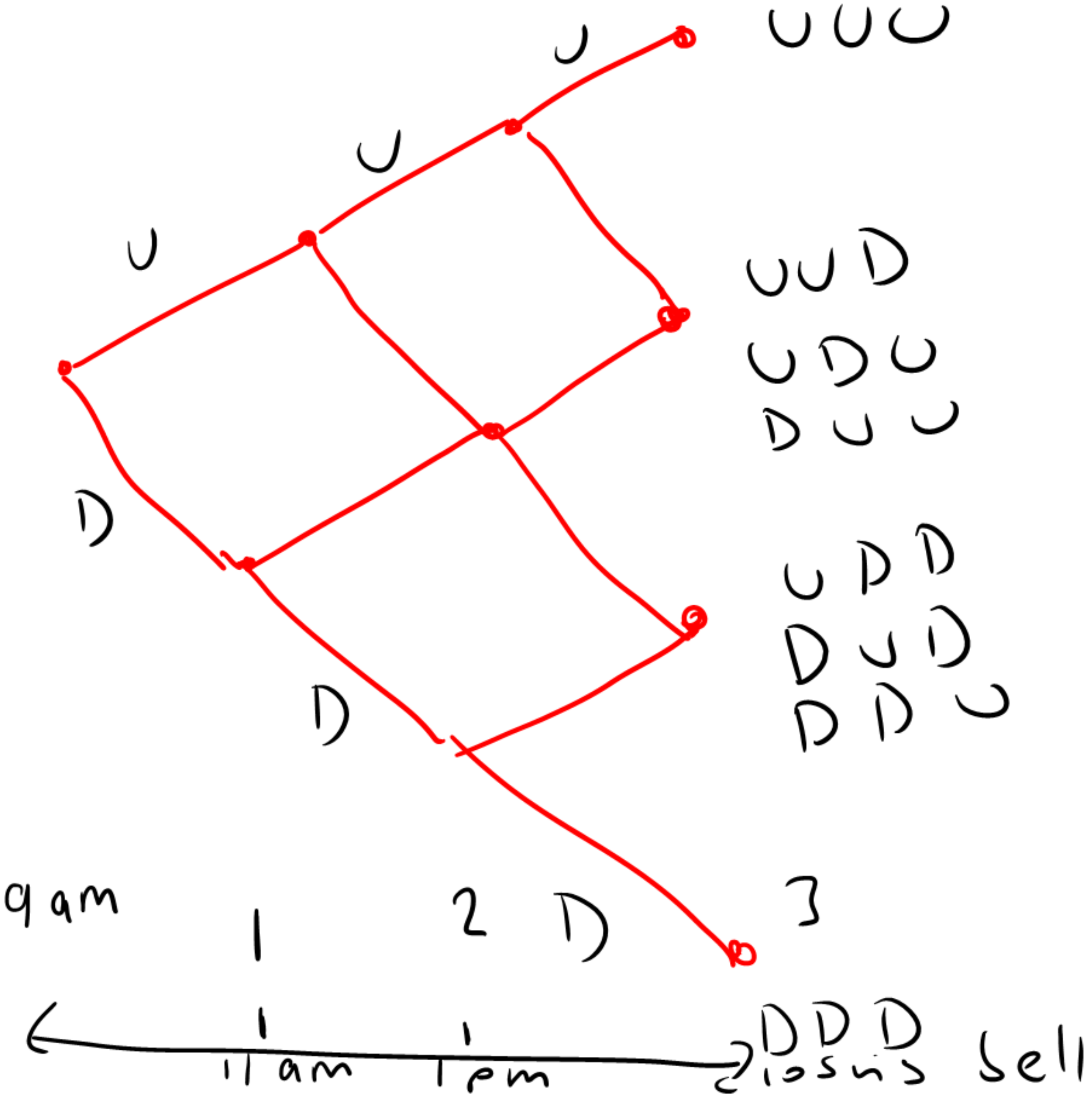
$\boxed{UU}$ ω_1

$\boxed{DD}$ ω_2

$\boxed{UD}$ ω_3

DU ω_4

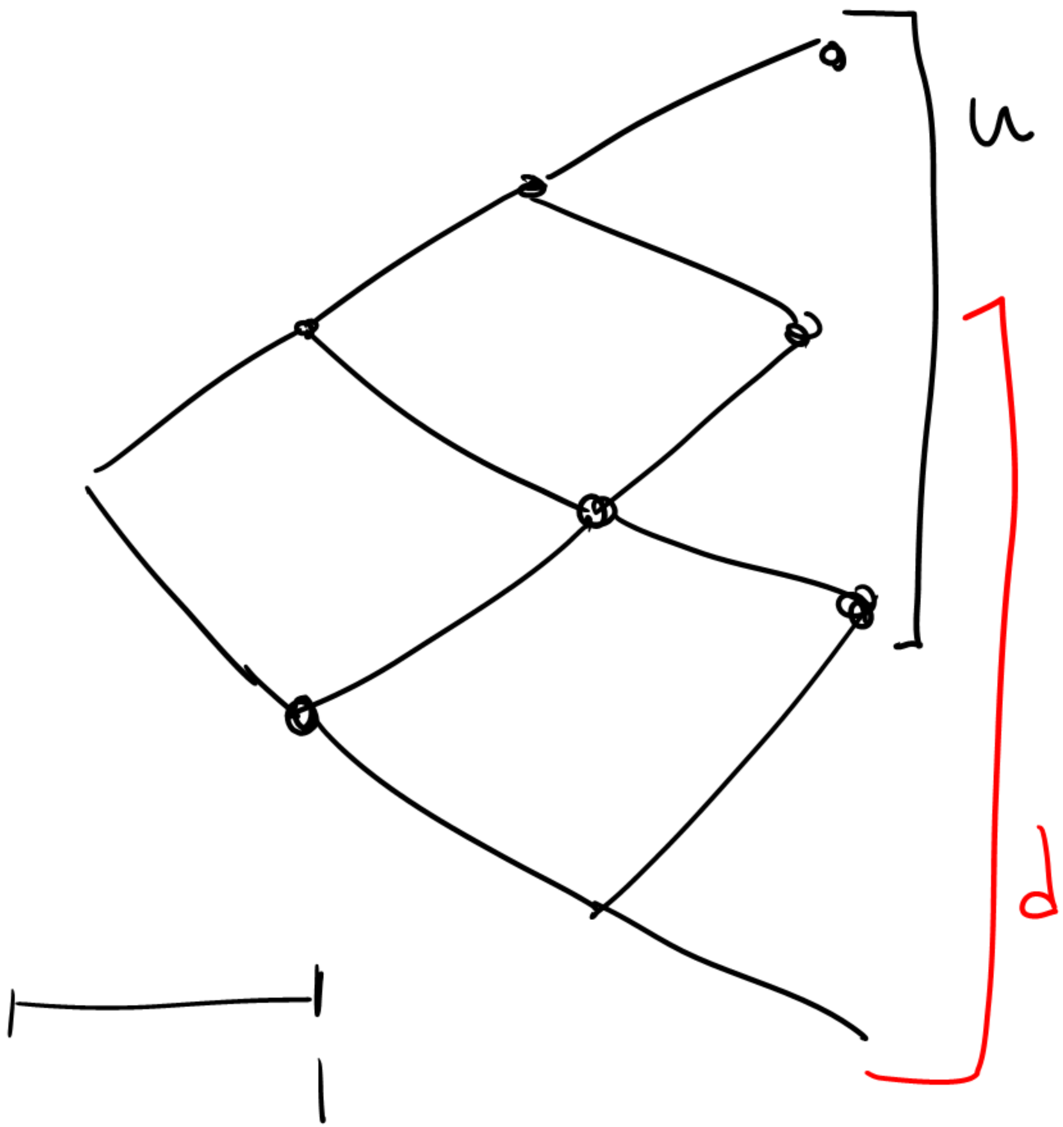
Seeing inside a filtration.



$$\Omega_3 = \{UUU, UUD, UDU, DUU, UDD, DUD, DDU, DDD\}$$

Period,	size of Ω
1	2
2	$2^2 = 4$
3	$2^3 = 8$
⋮	
10	$2^{10} = 1024$
⋮	
100	$2^{100} = 1.27 \times 10^{30}$

At time 1



$\mathbb{T} = \{0, 1, 2, \ldots, n\}$ discrete time set

Then

$\Omega = \Omega_n$ is set of all outcomes of

n coin tosses; each sample path

of length n; written

$$\omega = \omega_1 \omega_2 \ldots, \omega_n$$

e.g 3 time step binomial model $\longrightarrow$ $\omega = \omega_1 \omega_2 \omega_3$ length 3

$$P(x) = \mathbb{P}[X < x]$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}s^2} \, ds$$

$M_X(\theta)$ M.G.F

TSE $e^x = \sum_{0}^{\infty} \frac{x^n}{n!}$

$$M_X(\theta) = \mathbb{E}[e^{\theta X}] = \int_{-\infty}^{\infty} e^{\theta x} p(x)\,dx$$

$$= \int_{\mathbb{R}} \left(1 + \theta x + \frac{\theta^2 x^2}{2!} + \cdots\right) p(x)\,dx$$

$$= \int_{\mathbb{R}} p(x)\,dx + \theta \int_{\mathbb{R}} x\, p(x)\,dx + \frac{\theta^2}{2!} \int_{\mathbb{R}} x^2 p(x)\,dx + \cdots + \frac{\theta^n}{n!} \int_{\mathbb{R}} x^n p(x)\,dx$$

$$= 1 + \theta\, \mathbb{E}(X) + \frac{\theta^2}{2!} \mathbb{E}(X^2) + \cdots + \frac{\theta^n}{n!} \mathbb{E}(X^n) + \cdots$$

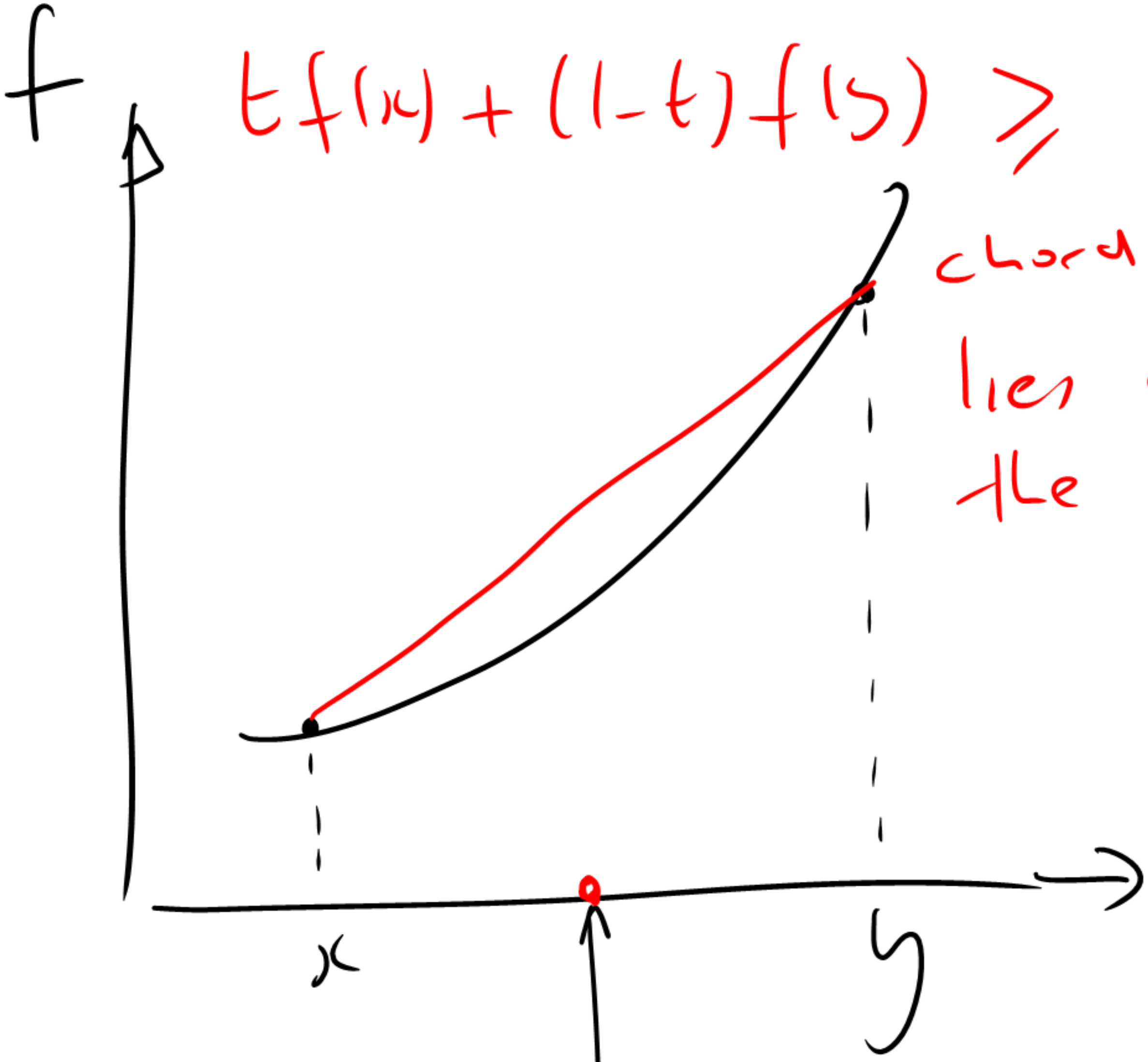
$$M_X(\theta) = \sum_{0}^{\infty} \frac{\theta^n}{n!} E[X^n]$$

for any k, the k^{th} moment written

$$m_k = \frac{d^k}{d\theta^k} M_X(\theta) \Big|_{\theta=0}$$

Convex Functions

$$t f(x) + (1-t) f(y) \geq f(tx + (1-t)y)$$



chord always lies above the curve

$f(x) = x^2$

variance, minimising

$f(x) = e^x$

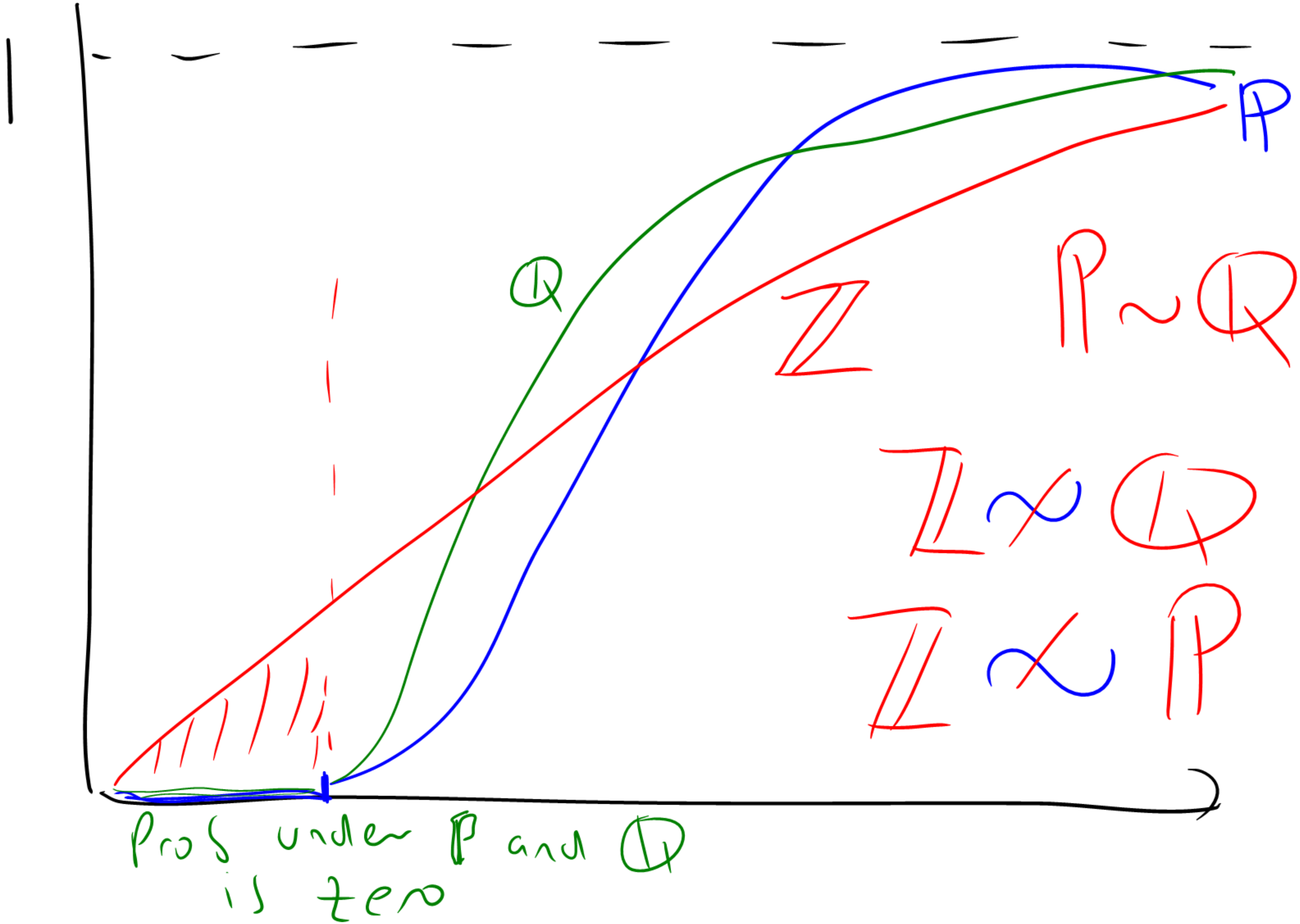
returns

$f(x) = |x|$

integrability

$tx + (1-t)y$ weighted average

$0 < t < 1$


1
$\mathbb{P}$
$\mathbb{Q}$
$\mathbb{Z}$
$\mathbb{P} \sim \mathbb{Q}$
$\mathbb{Z} \not\sim \mathbb{Q}$
$\mathbb{Z} \not\sim \mathbb{P}$
Prob under $\mathbb{P}$ and $\mathbb{Q}$
is zero