Computational Methods - Problem Sheet

1. Write down $p_4(x)$, the fourth degree Lagrange polynomial, defining in separate expressions $l_i(x)$, $i = 0, 1, 2, 3, 4$. By considering each expression for $l_i(x)$, show that
$$p_4(x_2) = y_2.$$

2. Write down in full an expression for
$$S = \sum_{i=0}^{2} l_i(\alpha).$$
By forming a common denominator and multiplying out the expression show that $S = 1$.

3. Estimate $\sin(\pi/4)$ by fitting a Lagrange polynomial of degree three to the data points $(x, \sin x)$ at $x = 0, \pi/6, \pi/3, \pi/2$. Check your calculation by taking an appropriate sum. Obtain the actual error in your estimate.

4. Given the data
$$\ln(1) = 0, \ \ln(1.1) = 0.09531, \ \ln(1.3) = 0.26236$$
use a Lagrange interpolating quadratic to estimate $\ln(1.2)$. What is the actual error in the approximation?

5. Use the Composite Trapezoidal rule to evaluate the following integrals:

   (i)
$$I = \int_1^2 (x + 1/x)\, dx$$
   (ii)
$$I = \int_0^1 \frac{dx}{1 + x^2}$$
   (iii)
$$I = \int_0^{\pi/2} \sin x dx$$
   For each integral calculate the actual error.

6. Repeat question 5 by using the Composite Simpson's Rule (for the same number of strips). Calculate the actual error. What do you notice?

7. The curve $y = 2\ln(2e - x)$ intersects the line $y = x$ at a single point where $x = \alpha$. Show that α lies between 1 and 3. Solve the recurrence relation
$$x_{n+1} = 2\ln(2e - x_n)$$
with $x_1 = 1$ to obtain x_2 and x_3 to three decimal places.