Certificate in Quantitative Finance
Final Project Brief

June 2016 Cohort

Please note that Workshops will be held to review the project topics.

This document outlines each available topic together with submission requirements for the project report. By-step instructions offer a structure not a limit to what you can implement. You are welcome to submit the project earlier to gain extra time for Final Exam preparation.

Exclusively for the use by current CQF delegates. No distribution.

Submission date is **Monday, 9 January 2017**

Submissions must match the Brief. There is no extension to the Final Project.

Projects without declaration or working code are incomplete and will be returned.

All projects are checked for originality. We reserve an option of *a viva voce* before project grade and therefore, qualification will be awarded.

1 Instructions

Projects are submitted in accordance with the current Brief. It is designed to give an opportunity for further study of numerical methods required to implement and validate a quantitative model. To complete the project, you must implement **one** of the listed topics **plus** CVA component.

1. **Pricing Fair Spread for a Credit Portfolio**
2. **Robust Portfolio Construction**
3. **Time Series Analysis and Backtesting**
4. **LIBOR and OIS Rates - Market Volatility**

All. This component is a mandatory addition for all topics as it balances exposure to the quant issues (interest rates, discounting) that would not be in focus otherwise.
CVA Calculation for an Interest Rate Swap

The Project Workshops, Brief, and Q&A are your primary guidance for the project. Please make sure you have (re-)watched relevant workshops and topical lectures with concentration. The Electives support the final project, explain concepts and some techniques. They give ideas on what to implement/write in Analysis and Discussion.

	Credit Spread	Robust Portfolio	Time Series	Rates Volatility
Algorithmic Trading			X	
Computational Methods				X
Risk Management	X			
Volatility Modeling				X
Portfolio Management		X		
Counterparty Credit Risk	X			
Behavioural Finance		X		
Data Analytics			X	
Python Applications		X	X	

Table 1: Relating Project Topics to CQF Electives.

1.1 Submission Requirements

- Submit working code together with a well-written report and originality declaration.
- Project report to have an exact title from the list above and content must correspond to it. Guided length is 25-40 pages, excluding code.
- Submissions to be uploaded to online portal **only**. Upload format: one written report (PDF), one zip archive with code and data files, and one scanned declaration (PDF).

Please upload only your final, completed version. For any queries on these requirements contact `Joanne.McCluskey@fitchlearning.com`.

1.2 Programming

- Programming environment must have appropriate strengths and facilities to implement the topic (pricing model) chosen. Common choices range from VBA to Python to C++, please exercise judgement as quants.
- Use of R/Matlab/Mathematica/Matlab is encouraged where time series or presentation involved. CQF-supplied Excel spreadsheets can be used as a starting point and to validate results but coding of numerical techniques/use of industry code libraries is expected.
- **'Scripted solution'** means the ready functionality from toolboxes and libraries is called, but the amount of own coding of numerical methods is minimal or non-existent. This particularly applies to Matlab/R as well as Excel spreadsheet functions (not robust).
- The aim of the project is to enable you to code numerical methods and develop model prototypes in a production environment. Excel spreadsheets-only or-scripted solutions are below the expected standard for completion of the project.
- To answer the question, "What should I code?" Delegates are expected to re-code numerical methods that are central to the model and exercise judgement in identifying them. Balanced use of libraries is allowed at the delegate's own discretion and subject to a description of limitations for ready functions/borrowed code (in the report).
- It is up to delegates to develop their own test cases, sensibility checks and validation. It is normal to observe irregularities when the model is implemented on real life data. If in doubt, reflect on the issue in the project report.
- The code must be thoroughly tested and well-documented: each function must be described, and comments must be used. Provide instructions on how to run the code.

1.3 Report

The main purpose of the report is to facilitate access to numerical methods' implementation (the code) and pricing results.

- The report must contain a sufficient description of the mathematical model, numerical methods and their properties. In-depth study is welcome but report must be relevant.
- Identify numerical methods recorded and include their code/algorithms in an appendix.
- Please give due attention and space for presentation and discussion of your pricing results. Present explicit sensitivity and/or risk analysis. Use charts, test cases and comparison to research results where available.
- Mathematical sections of the report can be prepared using LaTeX or Equation Editor (Word). For Mathematica and Python notebooks, make sure they are presentable.

CVA Calculation for an Interest Rate Swap

Summary

To recognise the importance of credit and counterparty risk adjustments to the derivatives business we introduce this mandatory component which must be implemented with **each topic**.

Calculate the credit valuation adjustment (taken by Counterparty A) to the price of an interest rate swap using credit spreads for Counterparty B. Plot MtM values (a good plot will show results from many simulations) and produce (a) a smoothed Expected Exposure profile. While EE is defined as $\max(\mathrm{MtM}_\tau, 0)$, you have to experiment with exposure distribution(s) at each tenor.[1] Produce smoothed Potential Future Exposure(s) using such indicators as (b) the median of positive exposure and (c) 97.5th percentile.

Provide a brief discussion of your observations, e.g., exposure over time, location of maximum exposure, impact of very small or negative rates. The advanced sensitivity analysis will illustrate the concept of the wrong-way risk.

Step-By-Step

The inputs for IRS valuation are (Forward) LIBORs and discounting factors. You also need default probabilities: bootstrap or make reasonable assumptions to supplement the data (e.g., flat credit spreads to 5Y tenor).

- Probability of default is bootstrapped from credit spreads for a reference name (any reasonable set values) in $6M$ increment. Linear interpolation over spreads and use of ready PD bootstrapping spreadsheet are acceptable, $RR = 40\%$. CVA LGD – own choice.
- Assume the swap is written on 6M LIBOR over 5Y. Notional $N = 1$ and payment frequency $\tau = 0.5$.

  To simulate the future values of L_{6M} at times $T_1, T_2, T_3, \dots$ take either (a) HJM MC spreadsheet, (b) one-factor or full calibrated LMM – see topic below or (c) ready/own one-factor calibrated model for $r(t)$, such as Hull & White.

  At each time T_{i+1} the SDE simulation produces a distribution of Forward LIBORs.
- Define MtM position as Floating Leg − Fixed Leg = $(L_{6M} - K)$ appropriately discounted. Depending on your inputs, choose fixed leg (rate) K to to have positive exposure.
- Discounting factors to be taken from the OIS curve, statically or with the use of LIBOR-OIS spread for choices (a) and (b).

[1] Using output of such models as HJM/LMM it is straightforward to come up with the distribution of Forward LIBORs at each swap fixing time T_{i+1} (tenor τ in HJM Model calibrated 'today').

Forward LIBOR from HJM and LMM

While based on historic volatility, the HJM offers a simple SDE, Gaussian simulation of interest rates (yield curve in continuous time) and affine calibration, even as the PCA factors are non-parametric.

HJM re-calibration would be on 2-3 years of recent data of the inst. forward rates from the BLC (LIBOR curve). Simple differences among daily rates are used.

Both models, HJM and LMM have SDEs that provide simulated forward curves – output of LMM SDE will look the same as HJM MC spreadsheet. The maths of obtaining LIBOR from a simulated forward curve is illustrated in the Yield Curve spreadsheet.

HJM and LMM have SDEs evolve forward curve – to match the rolling-forward measure, **discounting factors** must also be taken from the similarly evolved curves. However, (a) there are no caplets on OIS traded to calibrate and implement LMM model separately for discounting factors and (b) calibrating HJM to historical Forward OIS rates is a possibility but not market practice. In the absence of simulated Forward OIS, one can make assumptions about LIBOR-OIS spread and use it *to infer* the discounting curve from Forward LIBOR curve.

It is market practice to assume constant LIBOR-OIS spread but, as a challenge, you are welcome to explore the stochastic spread modelling between interest rates (funding basis spreads, tenor basis swaps).

Forward LIBOR CIR++ (Hull and White)

Calibration of one-factor interest rate models is covered in all common textbooks. While it is market practice to use these models, the preference for CQF project is towards full Forward LIBOR modelling either by HJM or LMM.

If using this path, one can obtain Forward LIBORs but not the evolution of the forward curve in continuous time.

Evolution of the spot process $r(t)$ with one-factor models can be improved by using market volatility and volatility fitting recipes from LMM model.

Data sources for various interest rates are also listed under LIBOR and OIS topic below.

Pricing Fair Spread for a Credit Portfolio

Summary

Price a fair spread, for a Basket CDS with at least 5 reference names, as an expectation over the joint distribution of default times. The distribution is unknown analytically and so, co-dependent uniform variables are sampled from a copula and then converted to default times using a marginal term structure of hazard rates (separately for each name). Copula is calibrated by estimating the appropriate default correlation (historical data of CDS differences is natural candidate but poses market noise issue). Initial results are histograms (uniformity checks) and scatter plots (co-dependence checks). Substantial result is sensitivity analysis by repricing.

A successful project will implement sampling from both, Gaussian and t copulae pricing all k-th to default instruments (1st to 5th) as well as elaborate sensitivity analysis. Spread convergence can require the low discrepancy sequences (e.g., Halton, Sobol) when sampling.

Data Requirements

Two **separate** datasets required, together with matching discounting curve data for each.

1. **A snapshot of credit curves** (on a particular day). Each debt issuer has a USD/EUR CDS curve – from which a term structure of hazard rates is bootstrapped and utilised to obtain exact default times, $u_i \to \tau_i$. Values can be visually stripped from financial media with reasonable assumptions, for instance, if you have 5Y CDS single value assume monotonically increasing values fro $1Y, 2Y, \ldots, 4Y$ for a concave down curve (positive slope).

2. **Historical credit spreads data** usually taken at the most liquid tenor $5Y$ for each of five reference names. The five columns of that sample data give the calculation input for 5×5 default correlation matrix. Default correlation can be obtained from different sources, in particular correlating equity returns is useful in the absence of historical credit spreads.

   Corporate credit spreads can be obtained from Bloomberg or Reuters terminals (via your firm or a colleague) – they are unlikely to be in open access. For sovereign credit spreads, time series of ready bootstrapped PD_{5Y} can be downloaded from DB Research: click on Excel icon to download file, *rename extension to .xls*, and check rows for missing values.
   `http://www.dbresearch.com/servlet/reweb2.ReWEB?rwnode=DBR_INTERNET_EN-PROD\$EM\&rwobj=CDS.calias\&rwsite=DBR_INTERNET_EN-PROD`

   Even if sovereign CDS_{5Y} and PD_{5Y} series are available with daily frequency, co-movement in daily changes might reflect market noise more than correlation of default events that are relatively rare to observe. Weekly changes give more appropriate input for estimation of such fundamental quantity as default correlation however that would entail using 2-3 years of historical data given that we need at least 100 data points to estimate correlation.

If access to historical credit spreads poses a problem remember, default correlation matrix can be estimated from historic equity returns or debt yields.

Step-by-Step Instructions

1. For each reference name, bootstrap implied default probabilities from quoted CDS and convert them to a term structure of hazard rates, $\tau \sim Exp(\hat{\lambda}_{1Y}, \ldots, \hat{\lambda}_{5Y})$.
2. Estimate default correlation matrices (near and rank) and d.f. parameter (ie, calibrate copulæ). You will need to implement pricing by Gaussian and t copulæseparately.
3. Using sampling form copula algorithm, repeat the following routine (simulation):
   (a) Generate a vector of correlated uniform random variable.
   (b) For each reference name, use its term structure of hazard rates to calculate exact time of default (or use semi-annual accrual).
   (c) Calculate the discounted values of premium and default legs for every instrument from *1st* to *5th*-to-default. Conduct MC separately or use one big simulated dataset.
4. Average premium and default legs across simulations separately. Calculate the fair spread.

Model Validation

- The fair spread for k*th*-to-default Basket CDS should be less than k-1 to default. Why?
- Project Report on this topic should have a section on **Risk and Sensitivity Analysis** of the fair spread *w.r.t.*
  1. default correlation among reference names: either stress-test by constant high/low correlation or $\pm$ percentage change in correlation from the actual estimated levels.
  2. credit quality of each individual name (change in credit spread, credit delta) as well as recovery rate.

  Make sure you discuss and compare sensitivities for all five instruments.
- Ensure that you explain historical sampling of default correlation matrix and copula fit (uniformity of pseudo-samples) – that is, Correlations Experiment and Distribution Fitting Experiment as will be described at the Project Workshop. Use histograms.

Copula, CDF and Tails for Market Risk

The recent practical tutorial on copula fitting and market risk is offered at the link below. Semi-parametric CDF fitting gives us percentile values, Extreme Value Theory comes as the tail exceedances correction for an Empirical CDF. Generalised Pareto Distribution applied to model the tails after percentile thresholds, while the CDF interior remains Gaussian kernel smoothed.

http://uk.mathworks.com/help/econ/examples/using-extreme-value-theory-and-copulas-to-evaluate-market-risk.html

http://uk.mathworks.com/help/stats/examples/nonparametric-estimates-of-cumulative-distribution-functions-and-their-inverses.html

Robust Portfolio Construction

Summary

Construct a portfolio of diverse assets using sufficient historic time series and estimate allocations for multiple levels of risk aversion together with the key risk measures (VaR, Expected Shortfall) and satisfaction indices (Sharpe Ratio). Utilise the key point of the Black-Litterman model, which is an introduction of analysts' recommendations into portfolio construction. You will have to identify sensible recommendations and formalise them as BL model views.

The effective advanced implementation will (a) improve on the sample covariance matrix estimator and (b) carefully diagnose and treat the optimisation problem, choosing from mean-variance, mean-TE, or mean-ES kinds of optimisation.[2] It is necessary to study the robustness of allocations (ie, what makes allocations to change) under different assumptions and constraints *ceteris paribus*. The study needs to be detailed and well-illustrated.

A successful project will have matrix form calculations and other numerical techniques coded (rather than spreadsheet calculations), robust optimisation results for different levels of risk aversion, and extensive graphic presentation. Compare your allocations to market benchmark weights and explore diversification. A naive mean-variance optimisation on sample mean and covariance is of little value.

Data Requirements (Portfolio Choice)

The objective is to come up with either **a.** a multi-asset diversified portfolio that can be seen as a global tactical allocation or **b.** specialised portfolio that focuses on an industry, emerging market(s), credit, etc. The first kind of portfolio can have under 8-10 diverse assets and sub-assets, while the specialised portfolio usually has more. If you follow portfolio choice from a reference please provide the source.

- Main asset classes are equity, fixed income, credit and volatility. Commodities, real estate and other assets are optional. ETFs can be used to represent the asset classes for which the data is hard to obtain (e.g., bonds, futures).
- Replication of broad equity indices is convenient.[3] However, the multi-asset approach comes from the need to account for most factors that drive performance – an equity index might have hundreds of names, for which performance is driven by only a few factors.
- You can consider investment into a factor(s) directly. The factor is a time series of returns of a long-short portfolio (eg, value factor, momentum factor, cointegrated combination).

[2] TE stands for Tracking Error and ES stands for Expected Shortfall mathematically known as CVaR.

[3] Because index weights can be converted into equilibrium weights. Without a ready market index/benchmark, the market cap approach can be used to obtain equilibrium weights.

The minimum historical time period is 2-3 years (for daily returns) though you might use the shorter periods of 1-3 months for variance estimation; that would require robust estimation and exclusion of stressed periods. Portfolios that are tradeable strategies themselves might require the higher frequency data (1-10min). A starting source for historical daily close prices of US equities and ETFs is Yahoo!Finance. Mean-variance optimisation is specified for *excess* simple returns (not compound). The technical challenges, such as the risk-free rate changing over time can be dealt using simplified assumptions.

Step-by-Step Instructions

Part I: The Black-Litterman Model

1. Construct the prior (reference distribution): equilibrium returns can come from a benchmark index, while covariance is estimated from historical data.

2. Define input views of both kinds, relative and absolute.

3. Estimate the posterior distribution of excess returns using the Black-Litterman formulae.

Part II: Robust Allocation

4. Choose at least one/two more optimisation designs in addition to variance minimisation. Experiment with sensible constraints in addition to views, for example, 'no short positions in bonds', 'no short positions at all'. Obtain allocations for three levels of risk aversion.

5. Prepare the sample covariance matrix and optionally, the matrix based on original correlations and GARCH-smoothed variances. Use both as input to optimisation and compare.[4]

6. Explore model behaviour (allocations resulting from the varied optimisation/covariance input), compare against the market benchmark weights, and check for common pitfalls such as 'corner solutions'. Formulate and answer questions such as: Are you over or underweight compared to a market? Can you think of the common themes and explanations? Are allocations artefacts of the model choice or make sense in terms of market factors?

A naive mean-variance optimisation on statistical mean and variance of returns is of little value without constraints, views and preferably robustness checks.

[4]Usually, GARCH is applied to forecast next day's volatility, which can be expressed in annualised terms. But, you are unlikely to rebalance a model and reallocate the portfolio every day, therefore, it makes sense to explore estimations over the longer horizons, e.g., estimate covariance using weekly/monthly returns.

Time Series Analysis and Backtesting

Summary

The aim to this topic is an estimation and analysis of tradeable relationships between two or more financial time series. Identifying and backtesting a robust cointegrated relationship means exposing a factor that drives both (or many) asset prices. The factor is traded by entering the long-short position given by cointegrating weights.

Through implementation you will have a hands-on introduction to Vector Autoregression (for returns) and Error Correction (for prices) models, which are the main variations of the multivariate regression. Instead of econometric forecasing, a range of techniques and considerations applied known as 'backtesting'. The techniques and quant recipes are specific to statistical arbitrage or systematic (algorithmic) strategy selected, for example, statistical arbitrage requires evaluating mean-reversion and optimality of trading of a spread.

A project that solely runs pre-programmed statistical tests on data is a preparation work, not the complete project. The project should have coding of necessary statistical tests from the first principles (explicit regression equations) by yourself. The least deliverables are **a.** implemented Engle-Granger procedure, **b.** statistical diagnosis and backtesting (split dataset in half or compute rolling estimates), and **c.** market factor backtesting via regressing returns from your strategy on market index returns or another factor. These are in addition to the underlying numerical methods on matrices and vector autoregression.[5]

Backtesting

The following notes offer choices to implement in aspects and questions of backtesting:

- All project designs (whether learning-level or in-depth) should include backtesting of a strategy. The strategy is realised by using cointegrating coefficients $\boldsymbol{\beta}_{Coint}$ as allocations $\boldsymbol{w}$. That creates a long-short portfolio that generates a mean-reverting spread (cointegrated residual).
- Does cumulative P&L behave as expected (for a cointegration trade)? Is P&L coming from a few or lot of trades/time period? What are the SR/Maximum Drawdown? Behaviour of risk measures (volatility/VaR)? Concentration in assets and attribution?
- Impact of transaction costs (plot an average P&L value vs. number of transactions).
- Optionally, introduce liquidity and algorithmic flow considerations (a model of order flow). How would you be entering and accumulating the position? What impact bid-ask spread and transaction costs will make?

[5] It is recommended that you use the environment with facilities for matrix and time series manipulation (R, Matlab) or code in Python/C++ with the use of quant libraries. The use of VBA will be cumbersome.

Step-by-Steps Designs

Design 1: 'Learning' and Cointegration in Pairs

An understanding-level design can use the ready specification tests, but matrix form regression estimation must be re-coded. The project can rely on the Engle-Granger procedure for cointegration testing among pairs.

1. Implement *concise matrix form* estimation for multivariate regression and conduct model specification tests for: (a) identifying optimal lag p and (b) stability check.

2. Optionally, test forecasting capability of regression with IRF and Granger Causality.

3. Implement Engle-Granger procedure and use it to identify a cointegrated pair. Estimate relationships both ways to select the appropriate lead variable.

4. ADF test for unit root must be coded and used.

Design 2: 'In-depth' and Multivariate Cointegration with Trends

The advanced implementation will re-code the essential tests (selection of lag p and unit root) and offer the multivariate estimation of cointegration. Analysis of relationships suitable for trading has to extend to exploring the quality of mean-reversion in cointegrated residual.

1. Implement *concise matrix form* estimation for multivariate regression and conduct your own coded tests for: (a) identifying optimal lag p and (b) stability check.

2. Apply Maximum Likelihood Estimation (Johansen Procedure) for multivariate cointegration on asset price data (in levels, not returns).

3. This is subject to (a) prior testing for unit roots by ADF test and (b) specification of deterministic trends for cointegrating equation. Use simple trends and check for overfitting.

4. Present clear analysis of results for *Maximum Eigenvalue* and *Trace* statistical tests, both are based on Likelihood Ratio test principle.

Before own implementation, consider R/Matlab functionality for Johansen Procedure and especially the tests for the number of cointegrated relationship in a multivariate system.

Strategy Backtest (both designs)

5. Use cointegration analysis to identify candidates for the long-short portfolios as given by cointegrating weights.[6]

6. Backtest a statistical arbitrage trade (or several) by investigating a mean-version in cointegrated residual $\boldsymbol{e_t} = \boldsymbol{\beta'_{Coint}} \boldsymbol{Y_t}$. Use the fitting to the OU process and give the speed of reversion, number of trades as well as other properties of P&L. Rely on backtesting considerations given above.

[6]Within an asset class, a robust cointegration relationship exposes *a factor*.

LIBOR and OIS Rates - Market Volatility

Summary

This advanced topic is for delegates with experience in interest rates who would like a challenge. The topic does require access to caplet and/or swaption data, an alternative will be to make up the data by simulating caplet prices from a calibrated HJM Model and converting them to implied volatility terms using the Black formula.

Discretised LMM SDE requires bootstrapped 3M caplet volatility data on input, to which volatility functions are fitted. Once the SDE is calibrated, simulations can be done for the purposes of pricing and risk evaluation. The outcomes are **a.** sensitivity analysis of caplet pricing *wrt* the market risks (ie, bumping the curve) and **b.** discussion of interest rate swaps and pricing of vanilla swaptions.

Above is classic LMM implementation. The market-driven alternative, formalised as Multi-curve LMM, starts with modelling the joint evolution of Forward OIS rates (available from BOE) and related LIBOR-OIS spreads for a given tenor. The twist of implementation is that the OIS rates and basis spreads as viewed factor and no market data of *volatilities* for OIS rates or bases is needed. Modelled explicitly, Forward LIBOROIS bases are guaranteed to be positive.

Data Sources

- Caplet or swaption data is usually maintained by trading desks and interdealer brokers. Data for certain markets is available from Thomson Reuters and Bloomberg. The simple input, a strip of market cap volatility prices (1M, 2M, 3M, 6M, 9M, 12M, 2Y, 3Y, etc.) can be taken from a textbook/research paper.
- LMM can also be calibrated to swaption volatility data (Option Maturity 1Y, 2Y, 3Y etc. vs. Swap Length 1Y, 2Y, 3Y, etc. that begins after maturity) – all at-the-money swaption volatilities (strike is equal to the rate on Forward Swap). That calibration is achieved via optimisation and called the Rebonato Method.
- For each tenor, you will need a discount factor and should make a decision on whether to use dual-curve discounting where DF is coming from the OIS curve. You will also need Forward LIBOR rates given deterministically by the forward curve *today*.
- Below are the links for Pound Sterling Bank Liability Curve (BLC) from LIBOR-linked instruments, OIS spot rates, Euro area curve (Government Liability) and FRB H.15 release (Treasuries and Interest Rate Swaps, each instrument gives its own spot curve):
  `http://www.bankofengland.co.uk/statistics/Pages/yieldcurve/archive.aspx`
  `http://www.bankofengland.co.uk/statistics/Documents/yieldcurve/ukois09_mdaily.xls`
  `http://www.ecb.europa.eu/stats/money/yc/html/index.en.html`
  `http://www.federalreserve.gov/releases/h15/data.htm`

Step-by-Step Instructions

Part I: Data

1. You will need market price data – cap price and discount factor (two columns). Caps are quoted in terms of implied volatility $\sigma^{cap}(t,T)$ with $T = 1Y, 2Y, 3Y, \ldots$.
   (a) For pre-simulated caplet data (ie, from the HJM model) the Black formula is conventional means of converting the caplet's cashflow to the implied volatility number.
2. The second set of data to which model fitting can be done is swaptions, for which the deliverable asset is an interest rate swap.

Part II: Calibration (Volatility Stripping)

3. Strip 3M caplet volatilities $\sigma^{cap}(T_i, T_{i+3M})$ or market traded caps traded in one-year increment using simplifying assumptions, e.g., flat volatility and pre-interpolated volatilities $\sigma^{cap}(t, T+3M)$ using definition of a cap:
   $$\text{cap}(t, 1Y) = \text{caplet}(t, T_{3M}, T_{6M}) + \text{caplet}(t, T_{6M}, T_{9M}) + \text{caplet}(t, T_{9M}, T_{12M}).$$
   To use Black formula you will need from *today's* curve, forward LIBOR f_i and strike K
   (a) The first step is to determine a strike for each caplet as a forward swap rate $S(t; T_{i-3M}, T_i)$.
   (b) The second step is to strip caplet volatilities.
4. Alternatively, volatilities can be calibrated from vanilla swaptions (European options on forward-starting swaps) where Rebonato method makes the Black formula suitable.
5. Fitting the *abcd* instantaneous volatility function is defined for each tenor as $\sigma_i(t)$. Coefficients *abcd* are estimated by optimisation that can be joint *wrt* caplet implied volatilities σ_i^{cap} and swaption implied volatilities v_i. Goal is to minimise the squared differences between two implied volatilities (for the same tenor enumerated i).
   (a) Use a parametric function for correlation structure ρ_{ij}.

Part II: Pricing and Sensitivity Analysis

Pricing of swaptions has already been done in the process of calibration (stripping) of caplet volatilities because forward swap rates $S(t; T_{i-3M}, T_i)$ were calculated. Pricing of path-dependent options, such as Bermudans that give exercise flexibility on some or all payment dates $T_i, T_{i+1}, \ldots, T_m$, would require the modified Least Squares Monte-Carlo simulation. Make sure that calibrated LMM model returns cap prices similar to the input data. Sensitivity Analysis means modifying the input forward curve (today) and/or discounting factors to evaluate the impact on derivatives pricing.

Another question is to explore how the LIBOR Market Model has to be modified with the introduction of OIS discounting. The change of numeraire to the discounting factor given by the OIS curve leads to an adjustment to the drift term of the LMM SDE.

Resources

Reading List: CVA

- CQF Lecture(s) on *Credit Value Adjustment.* For an alternative implementation example, `http://www.cvacentral.com/books/credit-value-adjustment/spreadsheets/`

Reading List: Rates Volatility

- Review *Methods for Constructing a Yield Curve* by Pat Hagan and Graeme West – best to start with version published in WILMOTT (May-June 2008).
- *The LIBOR Market Model in Practice* specialised textbook by Gatarek, et al. (2006) gives technical detail on cailbration from caplets and swaptions (Chapters 7 and 9 respectively) that will be useful to those working with LIBOR derivatives. (Please email the tutor.)

Reading List: Credit Portfolio

- The starting source on *sampling from copula* algorithm is *Monte Carlo Methods in Finance* textbook by Peter Jaekel (2002). Please see Chapter 5, particularly pages 46-50.
- Most likely, you will need to re-visit *CDO Lecture* material, particularly about slides 48-52 that illustrate Elliptical copula densities and discuss factorisation.
- Rank correlation coefficients are introduced *Correlation Sensitivity Lecture* and Jaekel (2002) as above. Project Q&A document gives the clarified formulae and explanations.
- Bootstrapping of survival probabilities is covered in *Credit Default Swaps Lecture.*

Reading List: Portfolio Construction

- CQF Lecture on *Fundamentals of Optimization and Application to Portfolio Selection*
- *A Step-by-step Guide to The Black-Litterman Model* (Incorporating user-specified confidence levels). Thomas Idzorek, 2002
- *The Black-Litterman Approach: Original Model and Extensions* Attilio Meucci, 2010. `http://ssrn.com/abstract=1117574`

Reading List: Time Series

- CQF Lectures on *Cointegration and Statistical Arbitrage* and *Statistical Methods for PD.*
- *Explaining Cointegration Analysis: Parts I and II*, by David Hendry and Katarina Juselius, *Energy Journal* 2000 and 2001.
- *Learning and Trusting Cointegration in Statistical Arbitrage* by Richard Diamond, *WILMOTT Magazine*, November 2013.
- *User Guide for Johansens Method* by Kyungho Jang and Masao Ogaki, 2001.