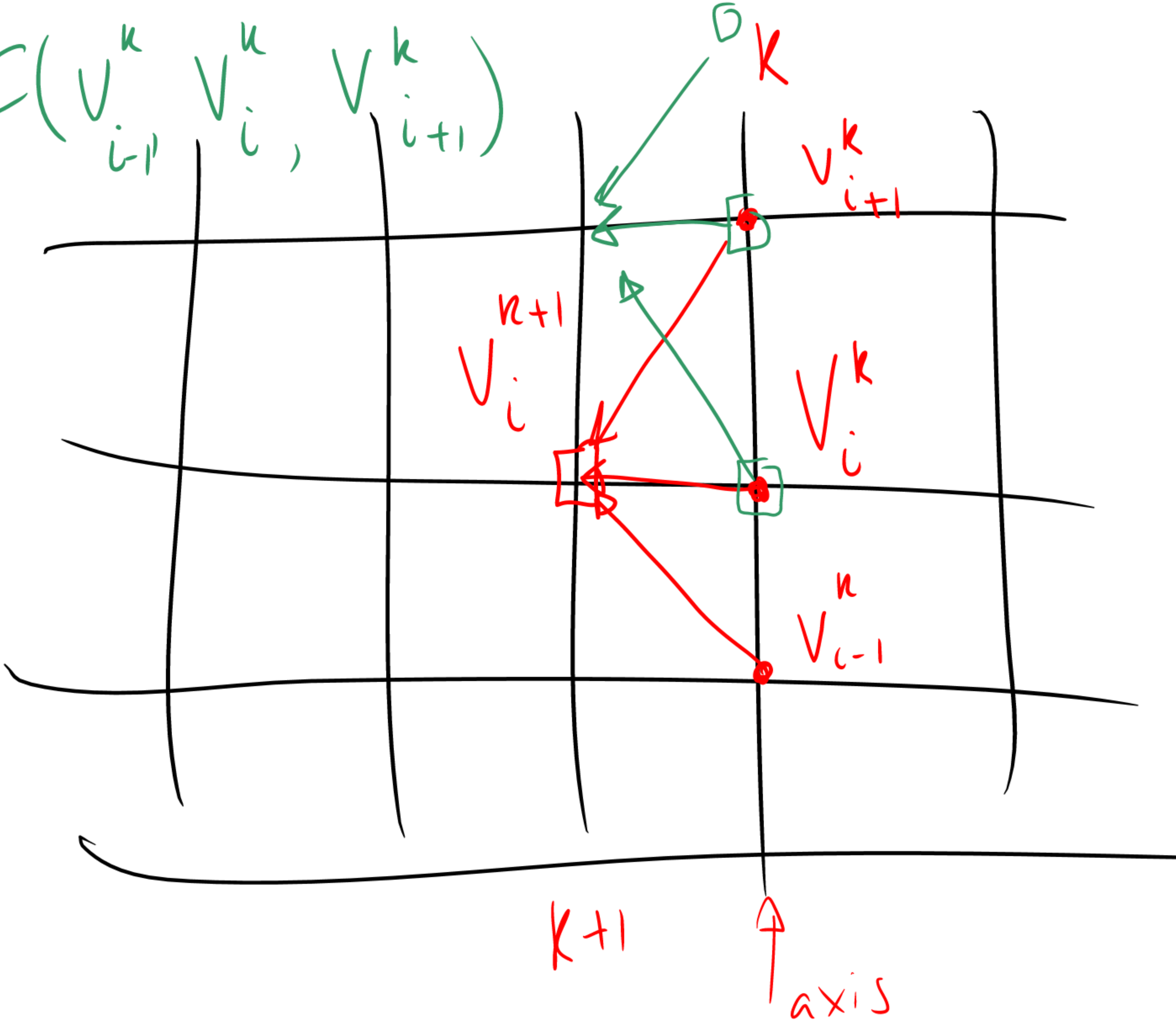
Explicit vs. Implicit

$y = f(x)$ $f(x, y) = 0$

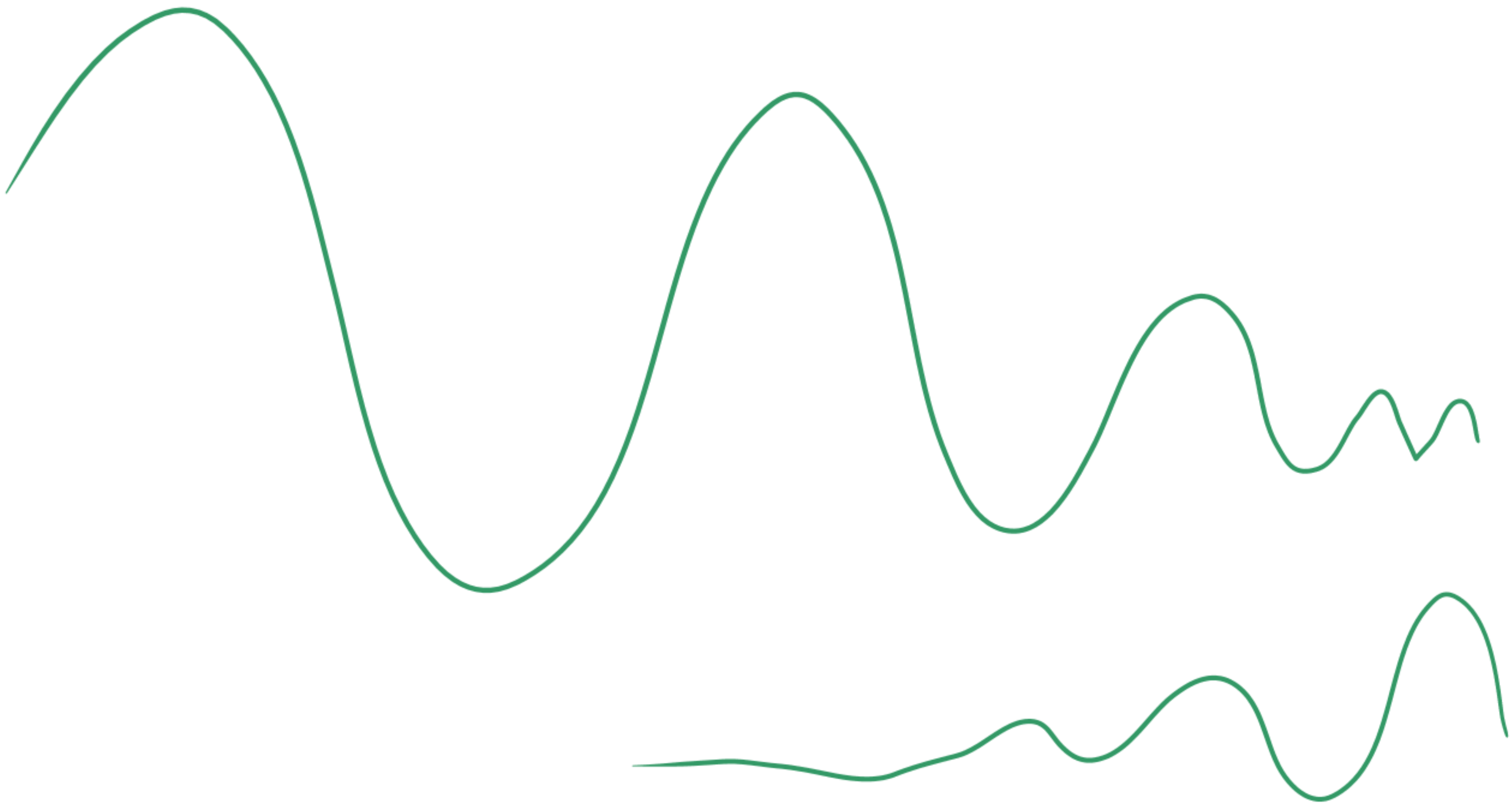
$$V_i^{k+1} = F(V_{i-1}^k, V_i^k, V_{i+1}^k)$$



Fourier stability

Stability Condition

$$\delta t \leq \frac{1}{\delta^2 I^2}$$



$$O(\delta t, \delta S^2)$$

δt – fixed

δS – fixed

Imp.

$$\frac{1}{2}\left(k + (k+1)\right)\delta t$$

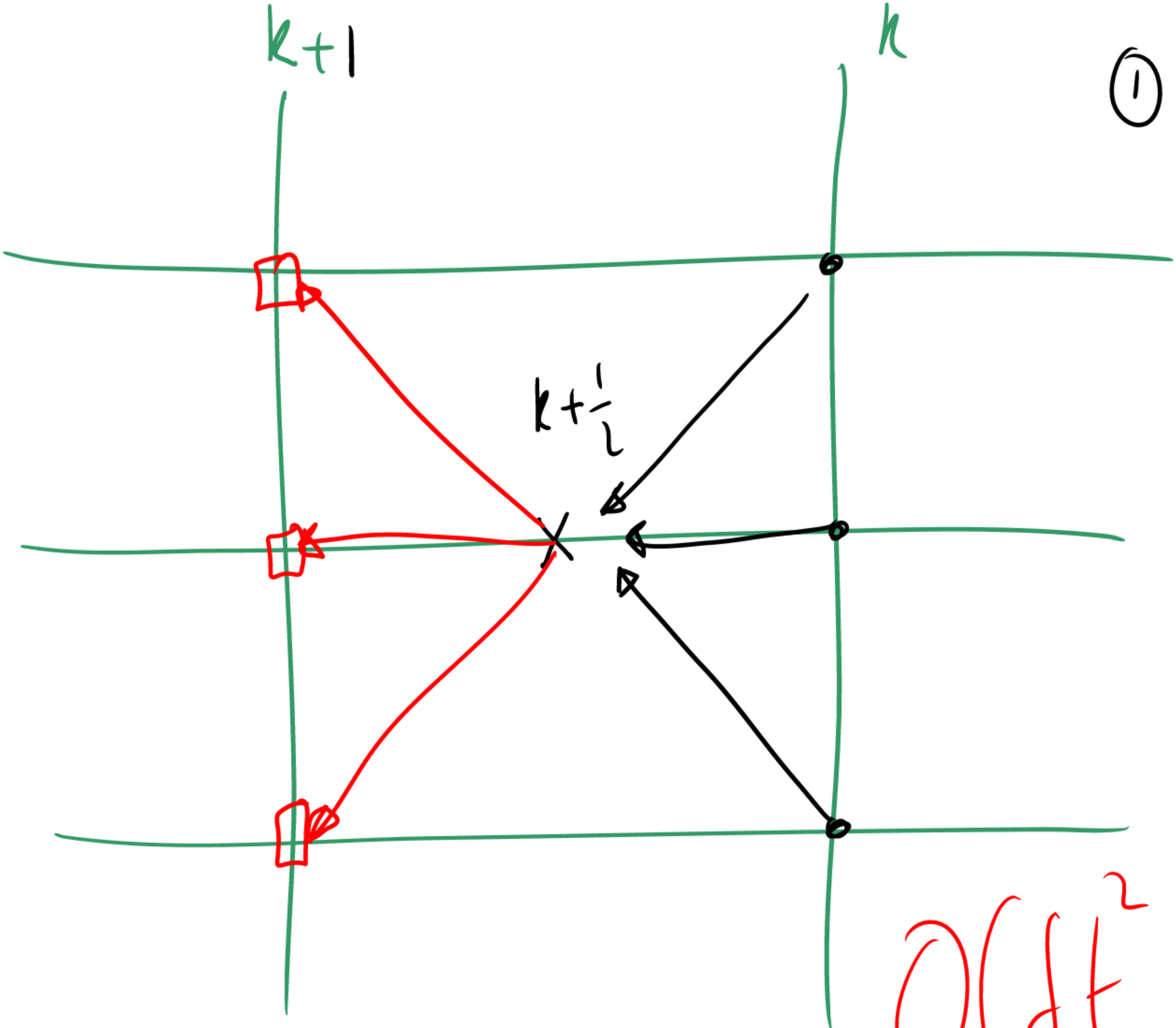
$\delta t = 1 \times 10^{-2}$

$\delta J = 1 \times 10^{-7}$

exp.

$$\left(k + \frac{1}{2}\right)\delta t$$

① Exp



$O(\delta t^2, \delta S^2)$

Numerical Analysis

Burden, Faires

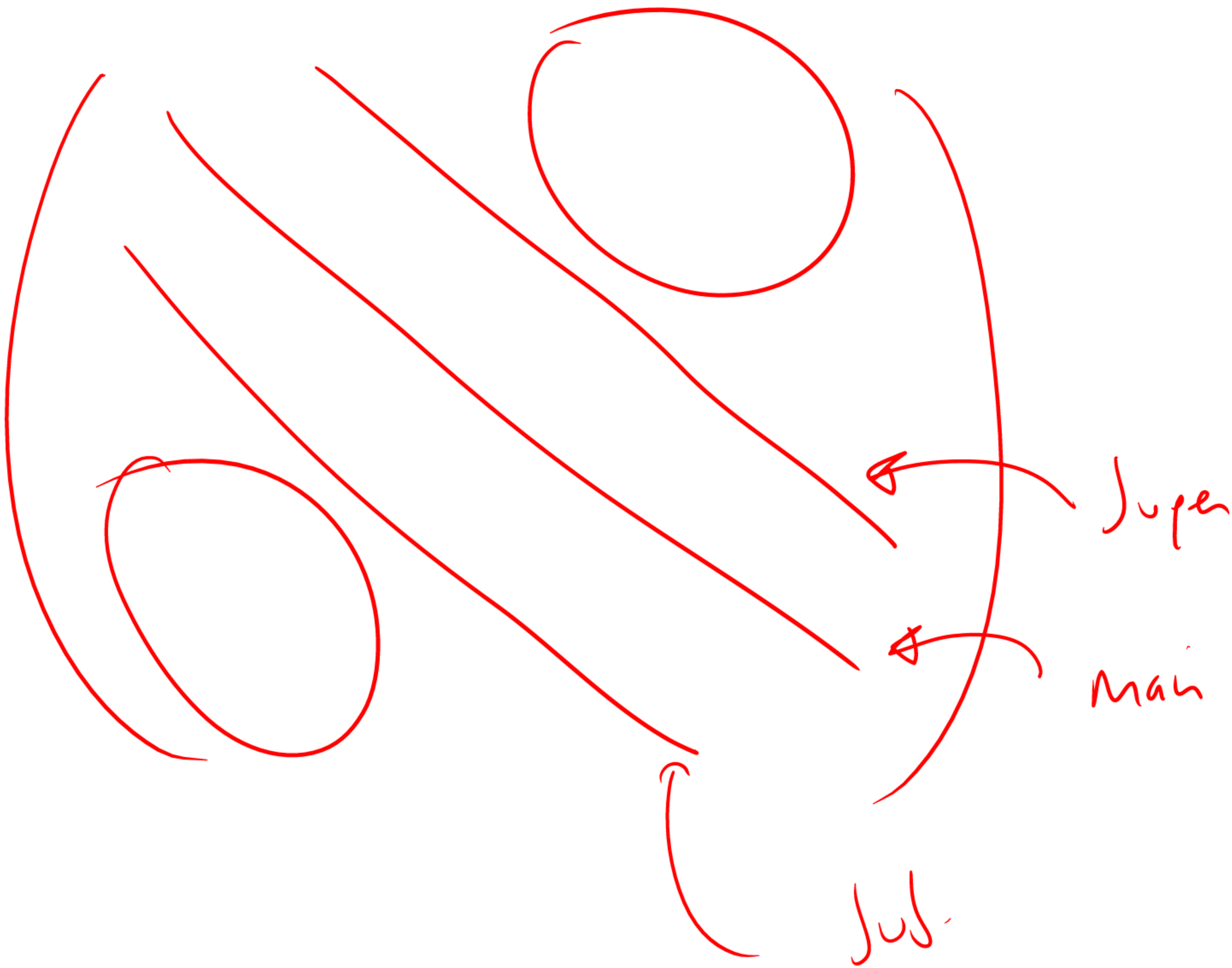
θ - Method $\quad 0 \leq \theta \leq 1$

$$\theta \times \text{Imp} + (1-\theta)\,\text{Exp}$$

$\theta = 0 \Longrightarrow$ Exp Scheme

$\theta = 1 \Longrightarrow$ Imp Scheme

$\theta = \frac{1}{2} \Longrightarrow$ C-N Scheme


Super
Main
Sub

$$\left(\begin{array}{c|ccc} & & I & \\ & 1 & 0 & 0 \\ A & 0 & 1 & 0 \\ & 0 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & \\ 0 & 1 & 0 & A^{-1} \\ 0 & 0 & 1 & \end{array}\right)$$

$$M \underline{v} = \underline{g}$$

$$L(U \underline{v}) = \underline{g}$$

$\underline{v}$ - unknown.

Put $U \underline{v} = \underline{w} \longrightarrow L \underline{w} = \underline{g}$

$\underline{w}$ unknown

Solve for $\underline{w}$

then put in $U \underline{v} = \underline{w} \longrightarrow \underline{v}$

$$M \underline{v} = \underline{g}$$

$$\underline{v} = T \underline{v} + \underline{c}$$

Convergence Criteria

$\underline{v} \in \mathbb{R}^N$ $\quad\quad |\underline{v}| = \sqrt{v_1^2 + \dots + v_N^2}$

$$\|\underline{x}\|_p = \left(\sum_{i=1}^{N} |x_i|^p \right)^{1/p}$$ Vector norm

$p = 2$ Euclidean norm $\quad$ l_2 norm

$p = \infty$ $\quad$ l_∞ norm

$$\|\underline{x}\|_\infty = \max_{1 \leq i \leq N} |x_i|$$

$$\frac{\| \underline{V}^{(k+1)} - \underline{V}^{(k)} \|_\infty}{\| \underline{V}^{(k)} \|_\infty} < \varepsilon$$

ε - level tolerance

$$A\underline{x} = \underline{b}$$

strictly diagonally dominant

coged sol^n using Jacobi | G-J

$$\begin{bmatrix} \alpha & \beta & \gamma \\ A & B & C \\ a & b & c \end{bmatrix}$$

$|\alpha| \geq |\beta| + |\gamma|$

$|B| > |A| + |C|$

$|c| > |a| + |b|$

$A\underline{x} = \underline{b}$

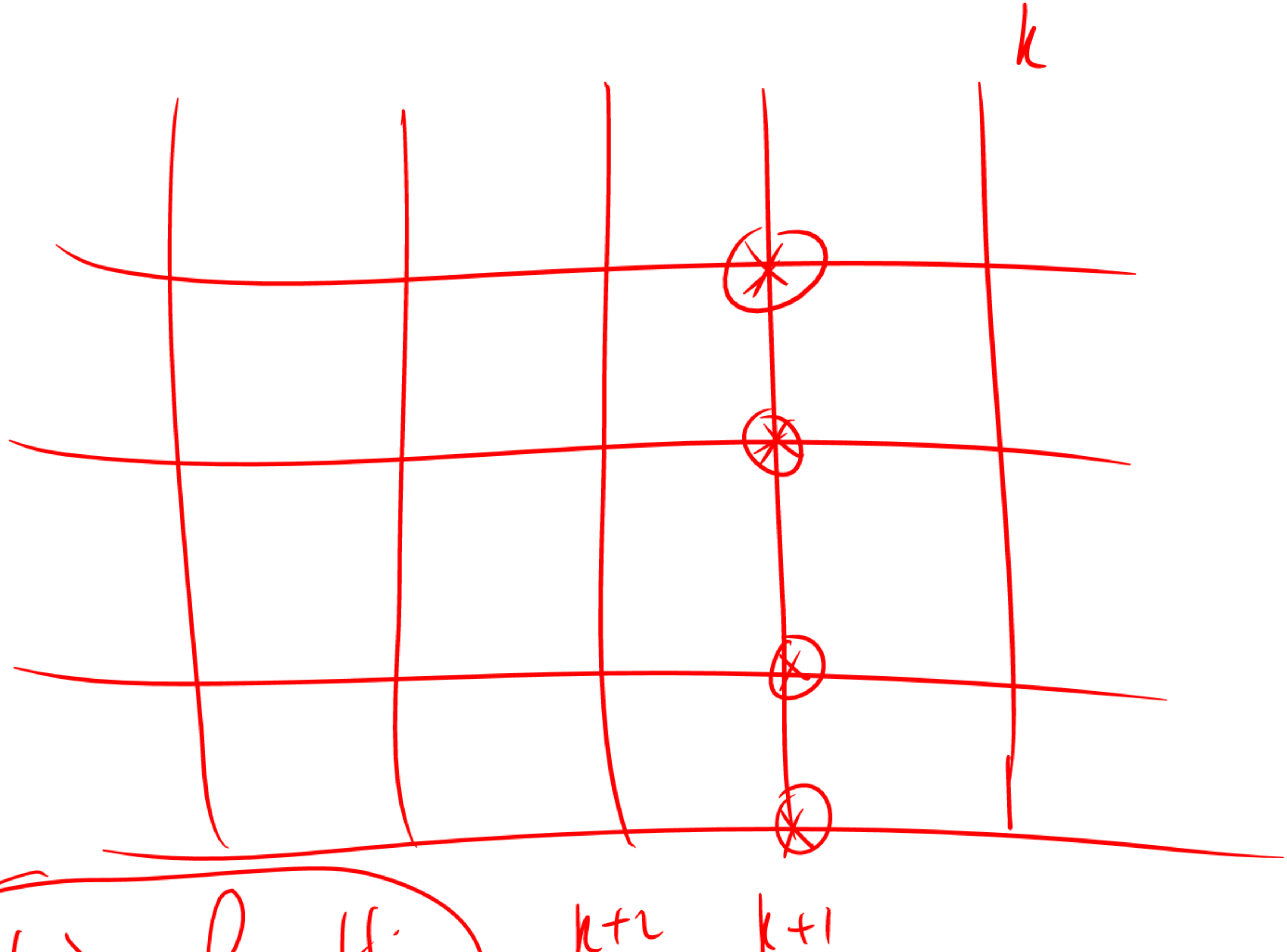
If $\underline{x}$ is exact then $A\underline{x} - \underline{b} = \underline{0}$

If $\underline{x}$ not exact then $A\underline{x} - \underline{b} = \underline{r}$

$$\lim_{n \to \infty} \underline{r} \longrightarrow 0$$

$$\boxed{\delta t \leq \frac{1}{\delta^2 \underbrace{N^2}}}$$

$$4\delta t \leq \frac{1}{\delta^2 N}$$



$V \geq$ Payoff

$$V(r, J, C, t)$$

$n\,dr$ $\quad$ $m\,dJ$ $\quad$ $i\,dC$ $\quad$ $k\,dt$

$$V^{k}_{nmi}$$

$$\frac{\partial}{\partial r}\left(\frac{\partial V}{\partial S}\right)$$

$$V(r, S, t) = V(j\delta r, i\delta S, k\delta t)$$

$$= V_{ij}^{k}$$

$$\frac{\partial}{\partial r}\left[\frac{V_{i+1,j}^{k} - V_{i-1,j}^{k}}{2\,\delta S}\right] = \frac{\dfrac{V_{i+1,j+1} - V_{i-1,j+1}}{2\,\delta S} - \dfrac{V_{i+1,j-1} - V_{i-1,j-1}}{2\,\delta S}}{2\,\delta r}$$