CDS
JP Morgan formulation

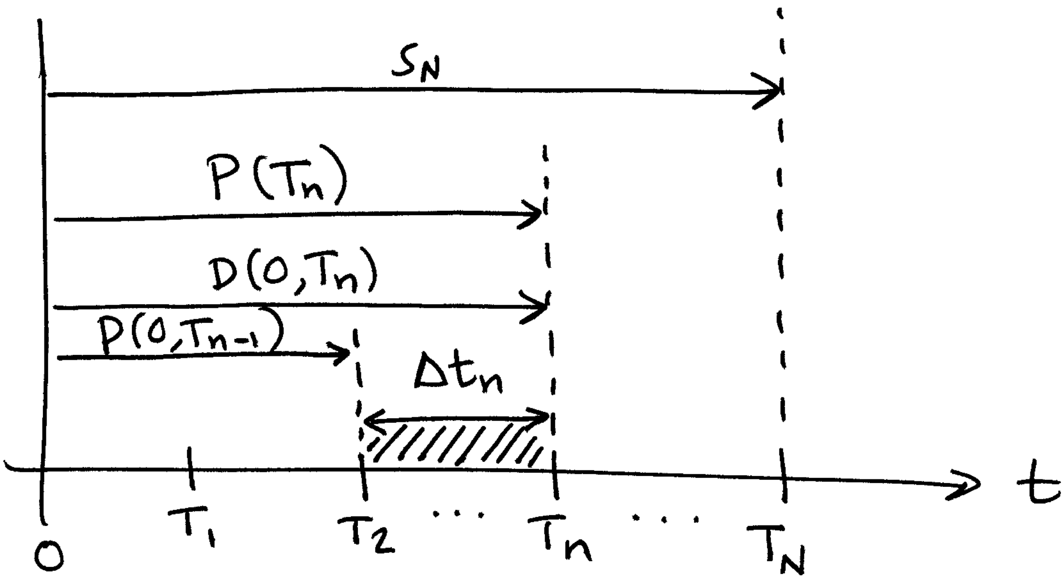
PREMIUM LEG

$$PL_N = S_N \sum_{n=1}^{N} D(0,T_n)\, P(T_n)\, \Delta t_n$$

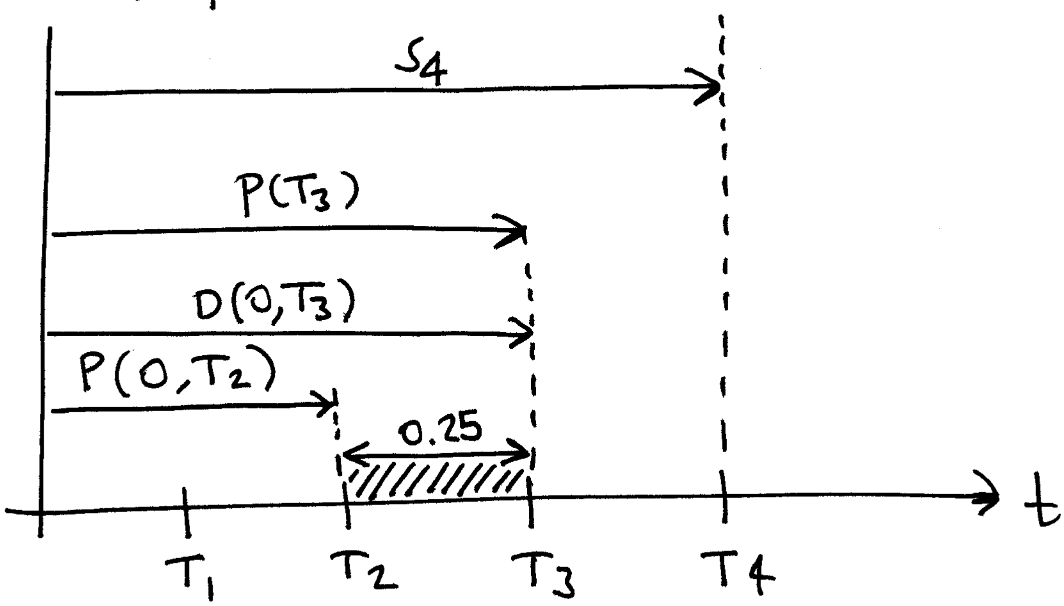
DEFAULT LEG

$$DL_N = (1-R) \sum_{n=1}^{N} D(0,T_n)\Big(P(T_{n-1}) - P(T_n)\Big)$$

based on time-grid:



for a quarterly CDS with maturity 1 year:
→ $N=4$



CDS Bootstrapping

We assume that we have a vector of CDS market spreads for increasing maturities $[S_1, S_2, \dots, S_N]$. We now determine their associated survival probabilities $[P(T_1), P(T_2), \dots, P(T_N)]$.

N = 1

$$PL_N = S_N \sum_{n=1}^{N} \left(D(0,T_n) P(T_n) \Delta t_n \right)$$

$$PL_1 = S_1 \left(D(0,T_1) P(T_1) \Delta t_1 \right)$$

$$DL_N = (1-R) \sum_{n=1}^{N} \left(D(0,T_n) \left(P(T_{n-1}) - P(T_n) \right) \right)$$

$$DL_1 = (1-R) D(0,T_1) \left(P(T_0) - P(T_1) \right)$$

$$PL_1 = DL_1$$

$$S_1 D(0,T_1) P(T_1) \Delta t_1 = \underbrace{(1-R)}_{L} D(0,T_1) \left[P(T_0) - P(T_1) \right]$$

$$S_1 D(0,T_1) P(T_1) \Delta t_1 = L\, D(0,T_1) P(T_0) - L\, D(0,T_1) P(T_1)$$

$$S_1 D(0,T_1) P(T_1) \Delta t_1 + L D(0,T_1) P(T_1) = L D(0,T_1) P(T_0)$$

$$P(T_1)\left[S_1 D(0,T_1)\Delta t_1 + L D(0,T_1)\right] = L D(0,T_1) P(T_0)$$

$$P(T_1)\cancel{D(0,T_1)}\left[S_1 \Delta t_1 + L\right] = L \cancel{D(0,T_1)} P(T_0)$$

with $P(T_0) = 1$

$$P(T_1) = \frac{L}{S_1 \Delta t_1 + L}$$

$\underline{\underline{N = 2}}$

$$PL_N = S_N \sum_{n=1}^{N} \left(D(0,T_n) P(T_n) \Delta t_n \right)$$

$$PL_2 = S_2 \left[D(0,T_1) P(T_1) \Delta t_1 + D(0,T_2) P(T_2) \Delta t_2 \right]$$

$$DL_N = (1-R) \sum_{n=1}^{N} D(0,T_n) \left(P(T_{n-1}) - P(T_n) \right)$$

$$DL_2 = (1-R) \left[D(0,T_1)\left(P(T_0) - P(T_1)\right) + D(0,T_2)\left(P(T_1) - P(T_2)\right) \right]$$

$$PL_2 = DL_2$$