Credit Default Swaps (CDS)

Alonso Peña, PhD, CQF
SDA Professor
SDA Bocconi School of Management

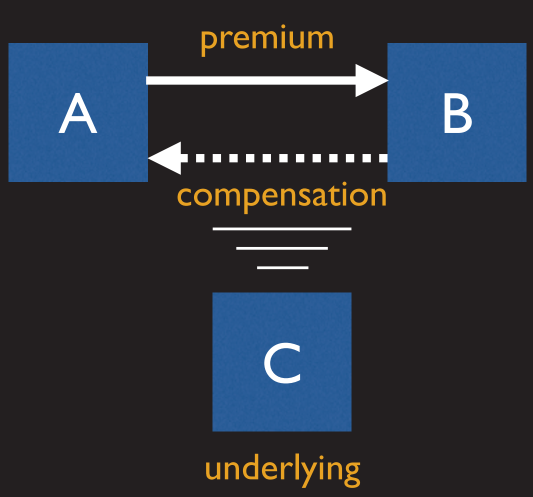
alonso.pena@sdabocconi.it

The CDS concept

Definition

credit default swap

A CDS is a financial contract between two counterparties A and B, in which one party pays to the other party a regular premium to buy credit protection against the possible default of an underlying C.



Definition

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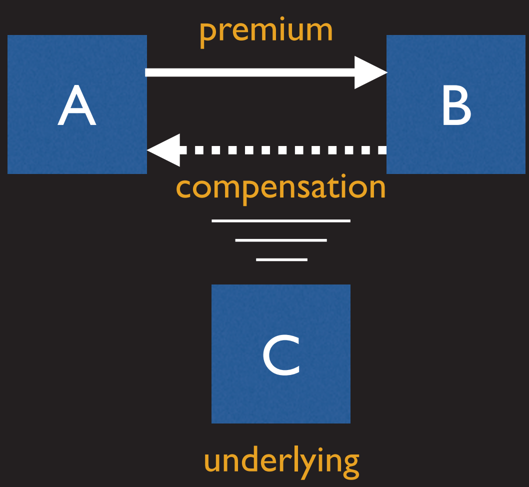
In structure, the CDS is similar to the plain vanilla IRS, as it can be considered as an exchange of cash flows between the parties.



Definition

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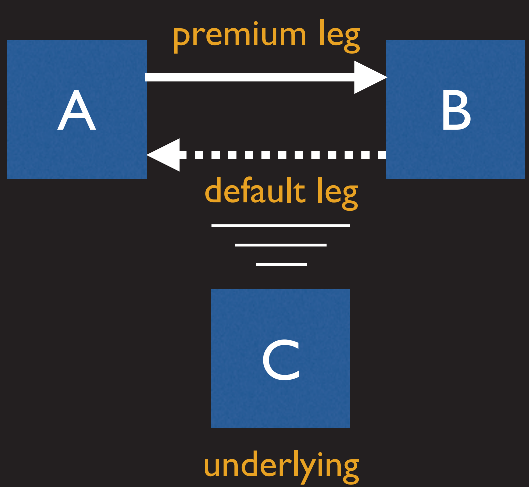
premium leg = the stream of cashflows that A pays B.

default leg = the protection payment paid by B to A in case of default of the underlying C.



Definition

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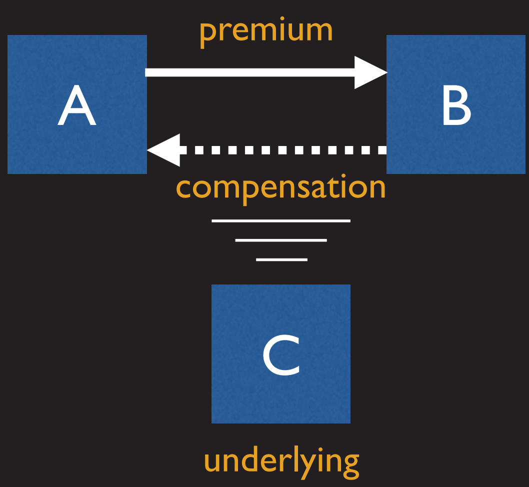
In a typical CDS with duration of five years (T=5), counterparty A pays B a series of premium payments at regular intervals (every 3M) upon an agreed notional (N).



Definition

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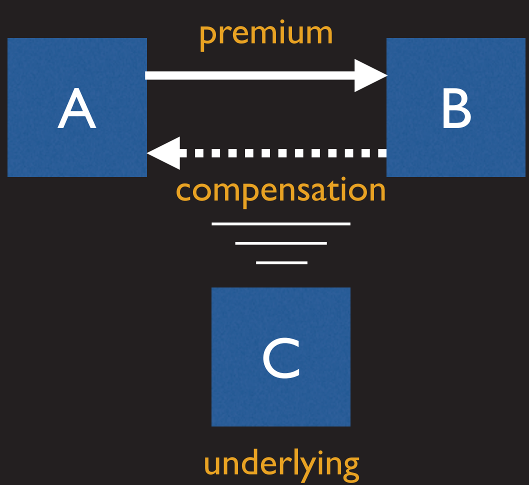
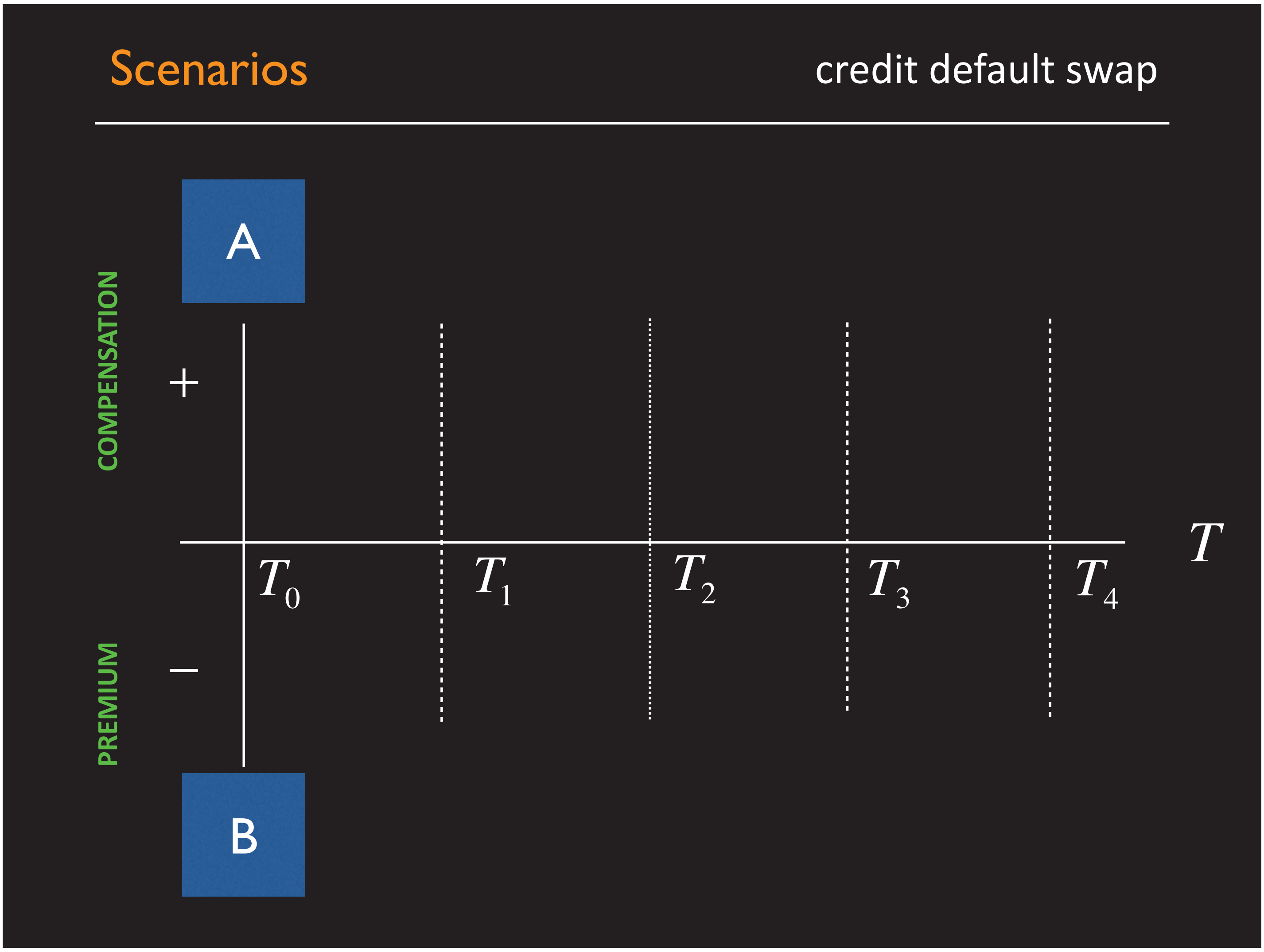
The payments from A to B will be made as long as underlying C doesn't default (i.e. survives).

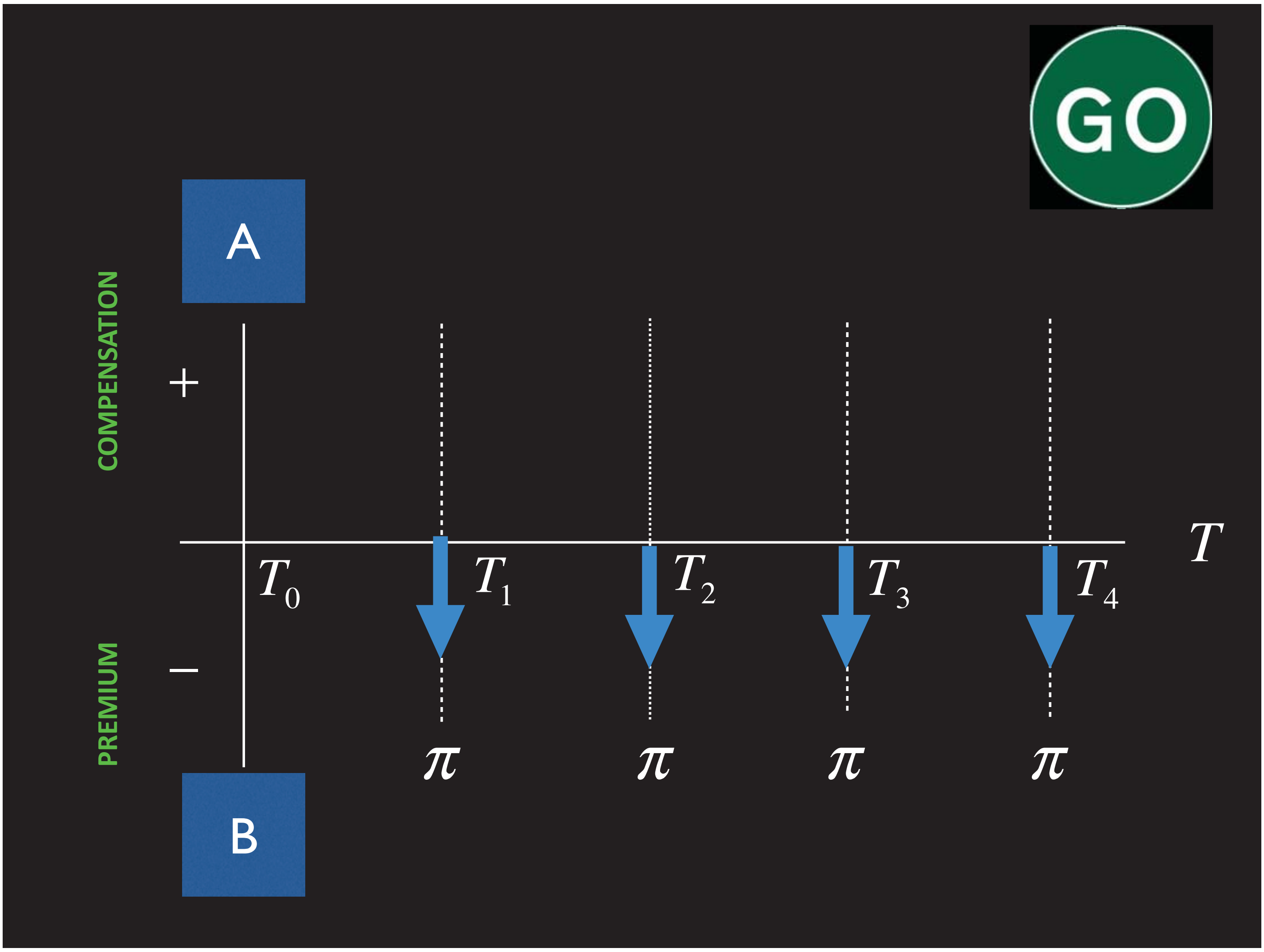
When default happens a single protection payment is made from B to A, and the contract ends.



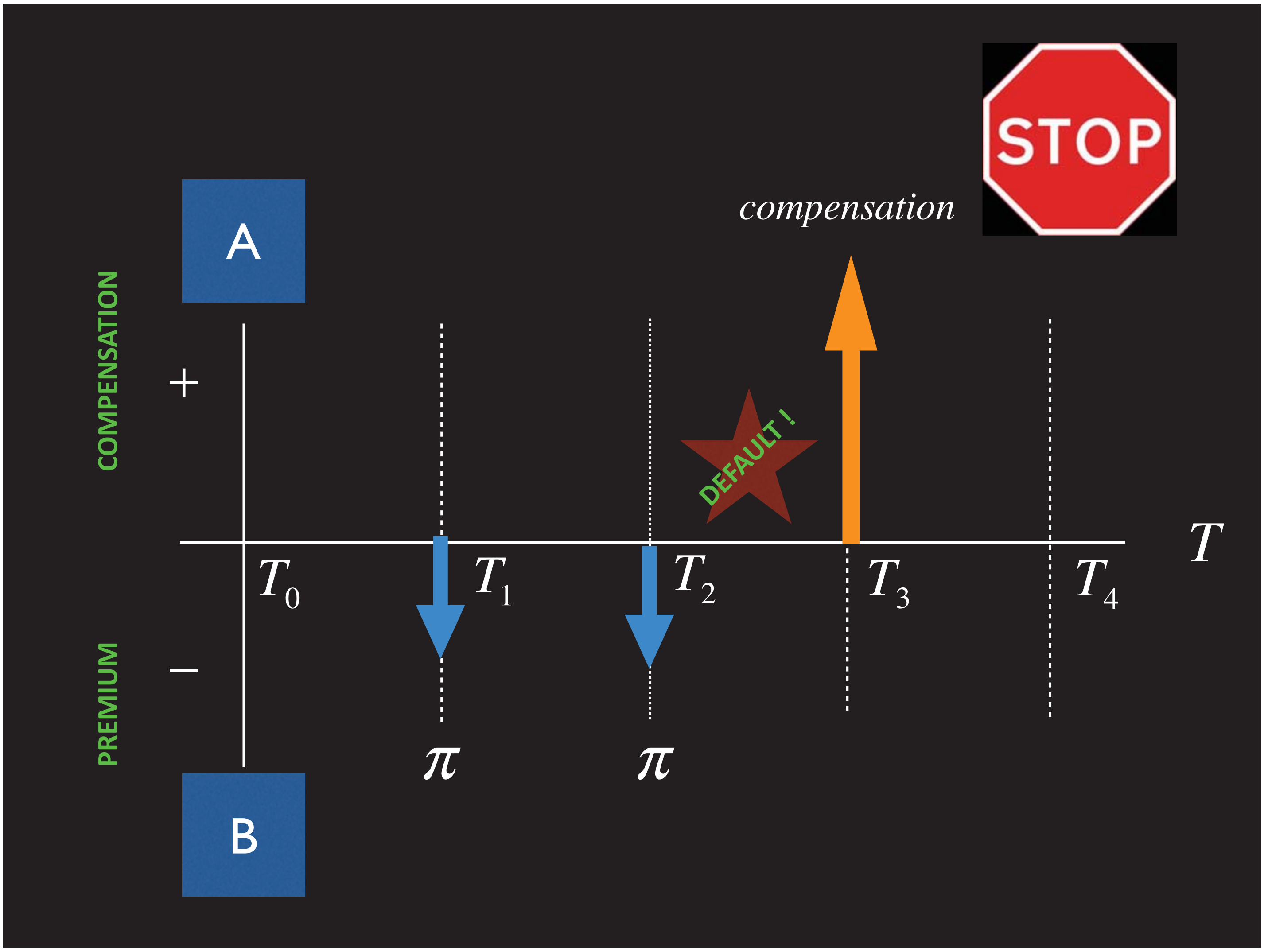
To default or not default, that is the question



scenario = no default



scenario = default



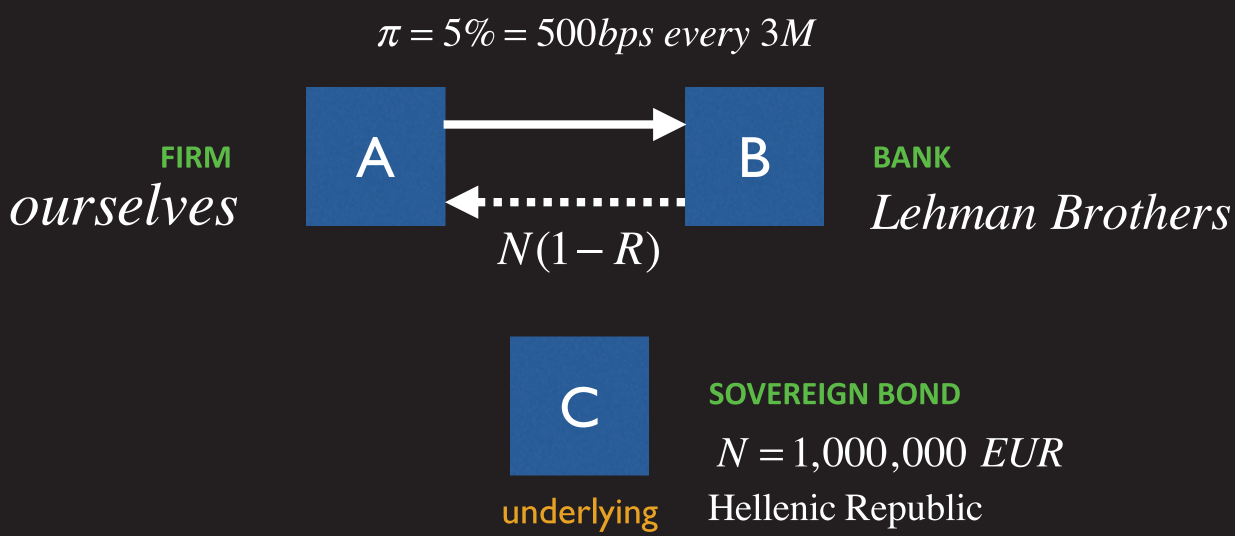
Pricing a CDS

π

Pricing

credit default swap

EXAMPLE: CDS contract duration is one year (T=1), quarterly payments (@3M), notional N= 1 million EUR.



Pricing

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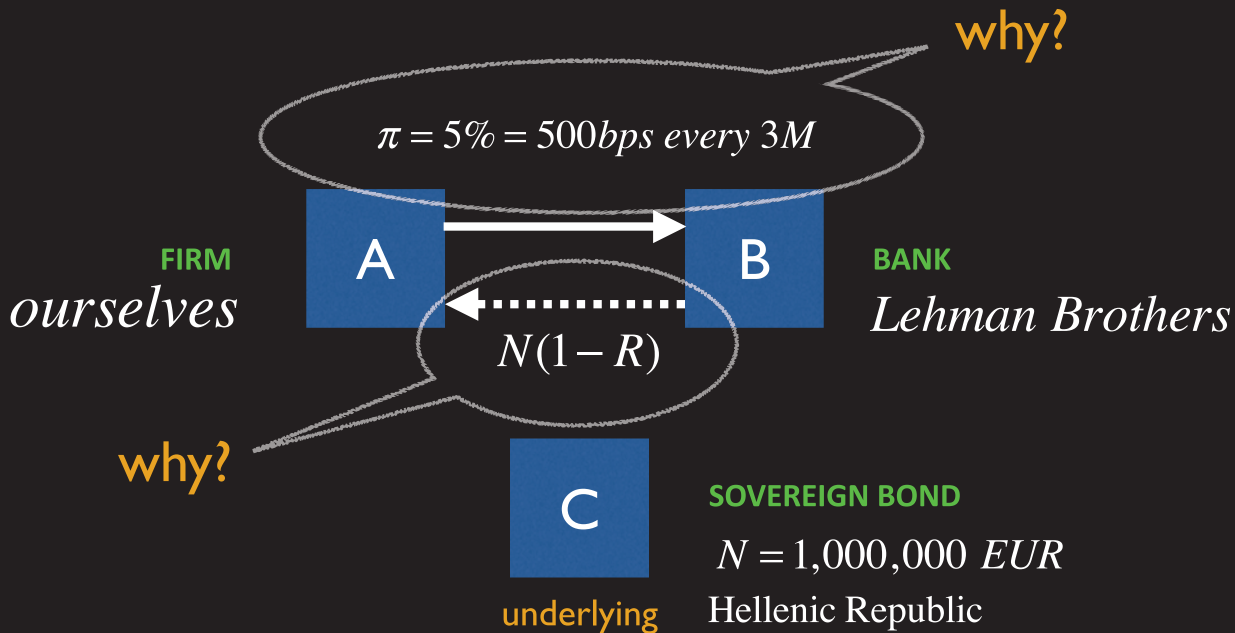

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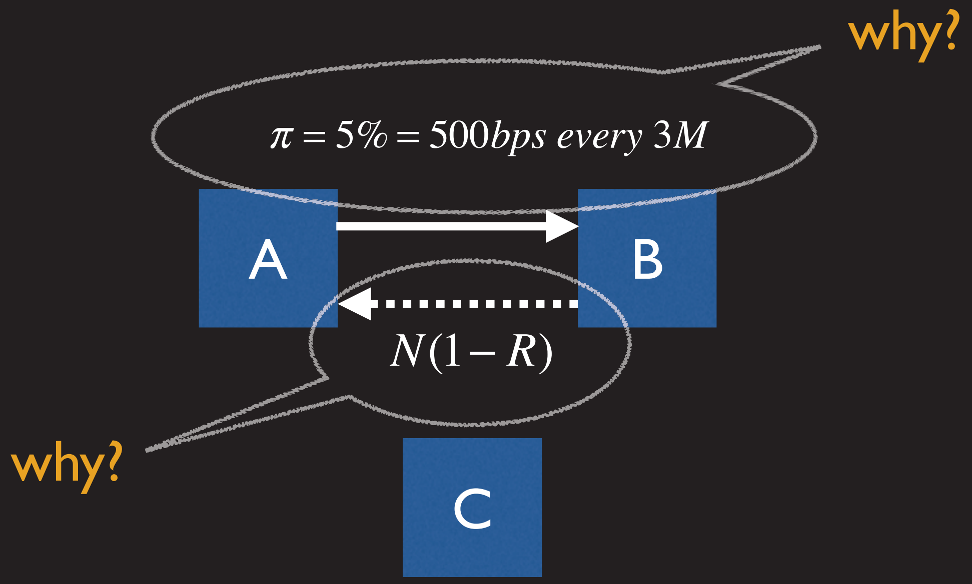

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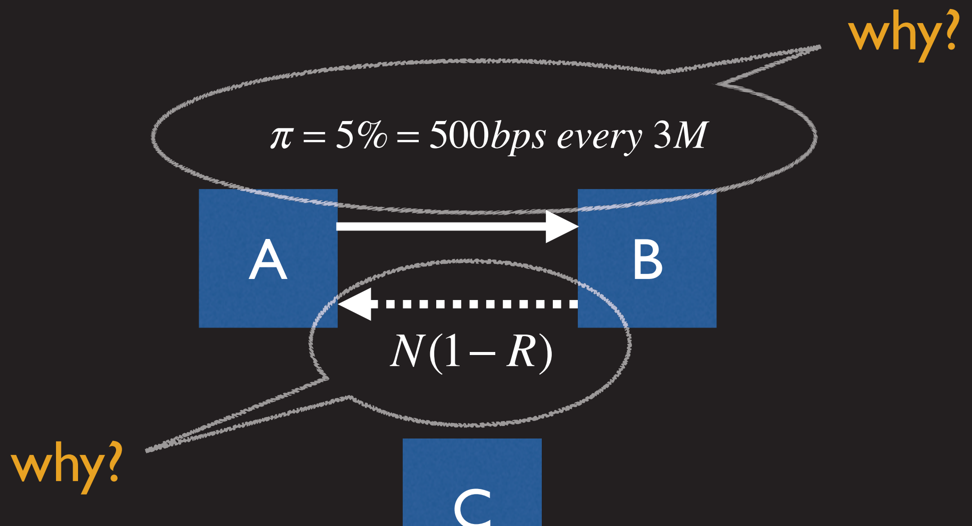
the premium is the price of the CDS,
should depend on the risk of default of C…



upon default there is usually
some recovery rate (R)…

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In summary to determine the premium (price) of a CDS contract we need:

1. maturity (T)
2. notional (N)
3. frequency payments (dt)
4. recovery rate (R)
5. ~~interest rates (r)~~
6. the "risk of default" of C

how?

Q: how do we model the risk of default of C ?

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A: The risk of default is modelled using the survival probability (P) and its complement the probability of default (PD).

$$P(t) = \exp(-\lambda \times t)$$

survival probability (function of time)

hazard rate

time

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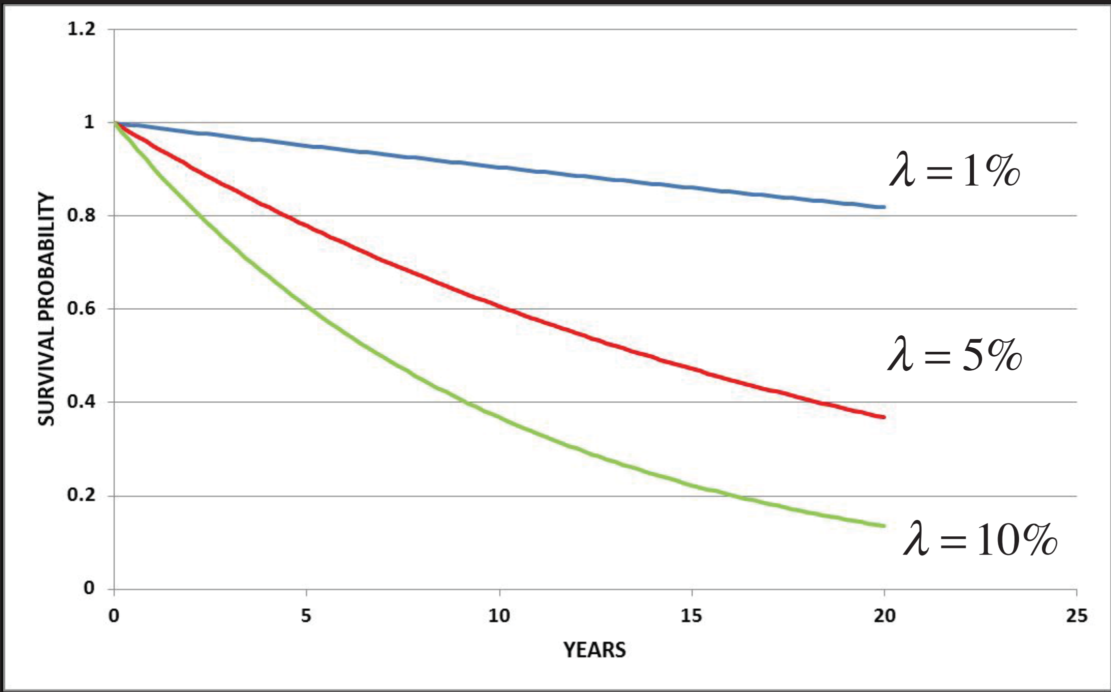

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We are going to use the survival probability to estimate the likelihood of the various cashflows in the CDS.

For the premium payments ->
P(t): survival

For the compensation (default) payment(s) ->
PD(t): probability of default

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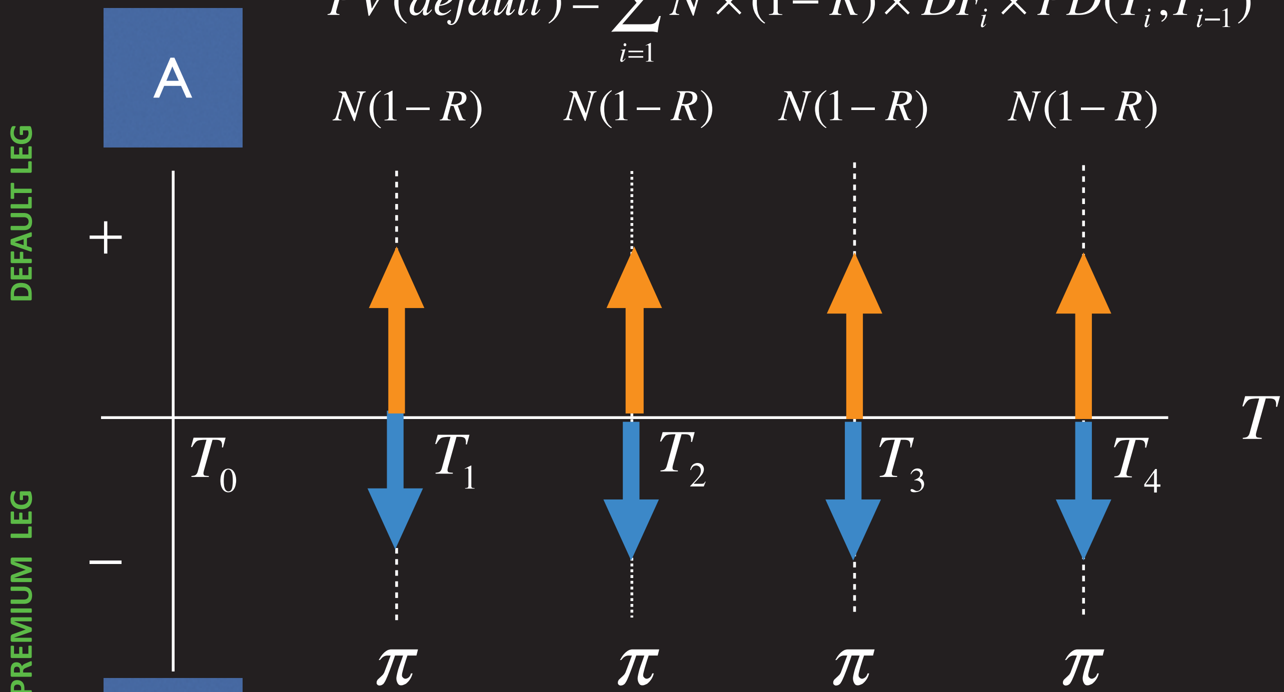
$$PV(default) = \sum_{i=1}^{4} N \times (1-R) \times DF_i \times PD(T_i, T_{i-1})$$



$$PV(premium) = \sum_{i=1}^{4} \pi \times N \times \Delta t \times DF_i \times P(T_i)$$

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From the point of view of counterpart A:

A

THIS IS RECEIVED... THUS POSITIVE...

$$PV(default) = \sum_{i=1}^{4} N \times (1-R) \times DF_i \times PD(T_i, T_{i-1})$$

THIS IS PAID... THUS NEGATIVE...

$$PV(premium) = \sum_{i=1}^{4} \pi \times N \times \Delta t \times DF_i \times P(T_i)$$

THE MARK TO MARKET IS THE SUM...

$$MTM = PV(default) - PV(premium)$$

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for fair pricing the MTM=0, thus

$$0 = PV(default) - PV(premium)$$

WE CAN EQUATE THE LEGS...

$$PV(premium) = PV(default)$$

$$\sum_{i=1}^{4} \pi \times N \times \Delta t \times DF_i \times P(T_i) = \sum_{i=1}^{4} N \times (1-R) \times DF_i \times PD(T_i, T_{i-1})$$

AND FINALLY ISOLATE THE PREMIUM...

$$\pi = \frac{\sum_{i=1}^{4} N \times (1-R) \times DF_i \times PD(T_i, T_{i-1})}{\sum_{i=1}^{4} N \times \Delta t \times DF_i \times P(T_i)}$$

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in terms of the survival probability only, the fair price (premium) of the CDS is:

$$\boxed{\pi = \frac{\sum_{i=1}^{4} (1-R) \times DF_i \times \left[P(T_{i-1}) - P(T_i)\right]}{\sum_{i=1}^{4} \Delta t \times DF_i \times P(T_i)}}$$