SCHOOL OF RANDOM DEPARTMENT NAMES

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1 Credit Curve Bootstrapping Methodology

We will first explain how credit curves are constructed (using a reduced-form model) before showing how the credule package can be used to build credit curves from the CDS quotes. We will look at 2 specific US Issuers as of 27 May 2014: Pfizer (Pfizer Inc - PFE) and Radioshack (RadioShack Corp - RSH).

Hazard rate and Survival Probability

The reduced-form model that we use here is based on the work of Jarrow and Turnbull (1995), who characterize a credit event as the first event of a Poisson counting process which occurs at some time t with a probability defined as : $\mathbb{P}(\tau)$. The probability of a default occurring within the time interval $(t, t+dt)$ conditional on surviving to time t, is proportional to some time dependent function $\lambda(t)$, known as the *hazard rate*, and the length of the time interval dt. We make the simplifying assumption that the hazard rate process is deterministic. By extension, this assumption also implies that the hazard rate is independent of interest rates and recovery rates. From this definition, we can calculate the continuous time survival probability to the time T conditional on surviving to the valuation time T by considering the limit $\rightarrow 0$. It can be shown that the survival probability is given by:

$$Q(t_V, T) = \exp\left(-\int_{t_V}^{T} \lambda(s)\,ds\right) \tag{1.1}$$

Finally, we assume that the hazard rate function is a step-wise constant function. In the below example, the hazard rate between time 0 and 1Y is $h_{0,1} = 1\%$. and the hazard rate between between 1Y and 3Y $h_{1,3} = 2.5\%$. Therefore, we have the 1Y survival probability $Q_{0,1} = exp(-h_{0,1} \times 1) = 99\%$ and the 3Y survival probability $Q_{1,3} = Q_{0,1} * exp(-h_{1,3} \times 2) = 91.9\%$

Furthermore, we can observe than the hazard rate does not have the same dynamic for both issuers. For Pfizer, the hazard rate curve is upward sloping (i.e hazard rate increase over time) whereas for Radioshack, the hazard rate curve is downward sloping. Downward sloping curve is commonly observed for stressed assets/speculative-grade firms (Radioshack rating is CCC as of 27 May 2014) and it translates the investors's expecation of a short term default. In other words, investors think that the issuer has room to improve with age (become less risky) or less potential to worsen considering that it is very risky today.

REFERENCES

- JP Morgan. Credit Derivatives: A Primer. JP Morgan Credit Derivatives and Quantitative Research (January 2005)
- D. Oḱane and S. Turnbull. Valuation of Credit Default Swaps. Lehman Brothers Quantitative Credit Research (Apr. 2003)
- Standard CDS Examples. Supporting document for the Implementation of the ISDA CDS Standard Model (Oct. 2012)

PROBABILITY OF SYSTEM FAILURE APPROACH: The hazard rate is generally accomplished through Weibull distribution analysis. The simulation article you provided also follows this method, but in a bootstrap fashion. However, you are dealing with actual CDS series. Let's start with two definitions: $H(t) = hazardrate$ and $S(t) = survivalrate$ where $S(t) = 1 - H(t)$.

Below are step-by-step procedures:

1. STEP 1: Rank you CDS data in ascending order, i.e. low to high. Call this data set $X(i) : (x1, x2, ..., xn)$ where $x_1 < x_2 < ... < x_n$. The subscript (i) in $X_(i)$ means raked data.
2. STEP 2: Find failure time. In this case, find your default time in CDS. This depends on the ample size you have. Select one of the following 2 methods:
   a) $F(t) = (i - 0.30)/(n + 0.40)$ if sample size $n < 100$
   b) $F(t) = i/(n+1)$ if sample size $n > 100$
3. STEP 3: Generate Weibull's QQ plot. From you observed series, you need to generate X and Y arrays in order to construct a linear regression equation. This is accomplished through the use of $F(t)$ from above. First, generate $X_w i$:
   a) $X_w i = \ln(\ln(1/1 - F(t))$ For all items of $F(ti)$. Now you have Weibull $X_w i : (x_w 1, X_w 2, \ldots, x_w n)$. Second, general the corresponding $Y_w i$:
   b) $Y_w i : \ln(x_w i)$ Now you have two arrays of $X_w i$ and $Y_w i$ which you are ready to run a linear regression to obtain Weibull linear equation: $Yw = a + bX$. See attached LAB Instruction sheet.
4. Calculate Weibull statistics and interpret result. If you want to do bootstrap, like that in your attached article, repeat the exercise and find the mean of the result.

   Conclude: $H(t)$ is the hazard rate, *i.e.* probability of failure.$S(t)$ is the survival rate or probability of success or survival.

tenor	hazardrate	survprob
1	1.00%	99.00%
3	2.50%	91.85%
5	3.00%	79.06%
7	4.00%	59.75%

Valuing the Premium Leg

The premium leg is the series of payments of the default swap spread made to maturity or to the time of the credit event, whichever occurs first. It also includes the payment of premium accrued from the previous premium payment date until the time of the credit event. Assume that there are $n = 1, \text{âĂę}, N$ contractual payment dates $t_1, \text{âĂę}, t_N$ where t_N is the maturity date of the default swap.

The present value of the premium leg is given by:

$$\text{PL PV}(t_V, t_N) = S(t_0, t_N) \sum_{n=1}^{N} \Delta(t_{n-1}, t_n, B) Z(t_V, t_n) \left[Q(t_V, t_n) + \frac{1_{PA}}{2} (Q(t_V, t_{n-1}) - Q(t_V, t_n)) \right] \tag{1.2}$$

- $N$is the total number of premiums. n=1,âĂę,N is the index for these premiums; premium dates are denoted t1, âĂę, $tN.tN$ is the maturity date of the swap
- $S(t_0, t_N)$ is the spread of a Credit Default Swap that matures on t_N
- $\Delta(t_{n\text{âĹŠ}1}, t_n, B)$ is the day count fraction between premium dates $t_{n\text{âĹŠ}1}$ and t_n in the appropriate basis convention denoted by B (ACT/360 in our model)
- $Q(t_V, t_n)$ is the arbitrage-free survival probability of the reference entity from valuation time t_V to premium time t_n
- $Z(t_V, t_n)$ is the risk-free discount factor from valuation date to the premium date n
- $1_{PA} = 1$ if premium accrued (PA) are included in the premium leg calculation, 0 otherwise.

Valuing the Default Leg

The default leg (or protection leg) is the contingent payment of (100% - R) on the face value of the protection made following the credit event. R is the expected recovery rate. In pricing the default leg, it is important to take into account the timing of the credit event because this can have a significant effect on the present value of the protection leg especially for longer maturity default swaps. Within the hazard rate approach we can solve this timing problem by conditioning on each small time interval $[s, s + ds]$ between time tV and time tN at which the credit event can occur. We calculate the expected present value of the recovery payment as:

$$\text{DL PV}(t_V, t_N) = (1 - R) \int_{t_V}^{t_N} Z(t_V, s) Q(t_V, s) \lambda(s) ds \tag{1.3}$$