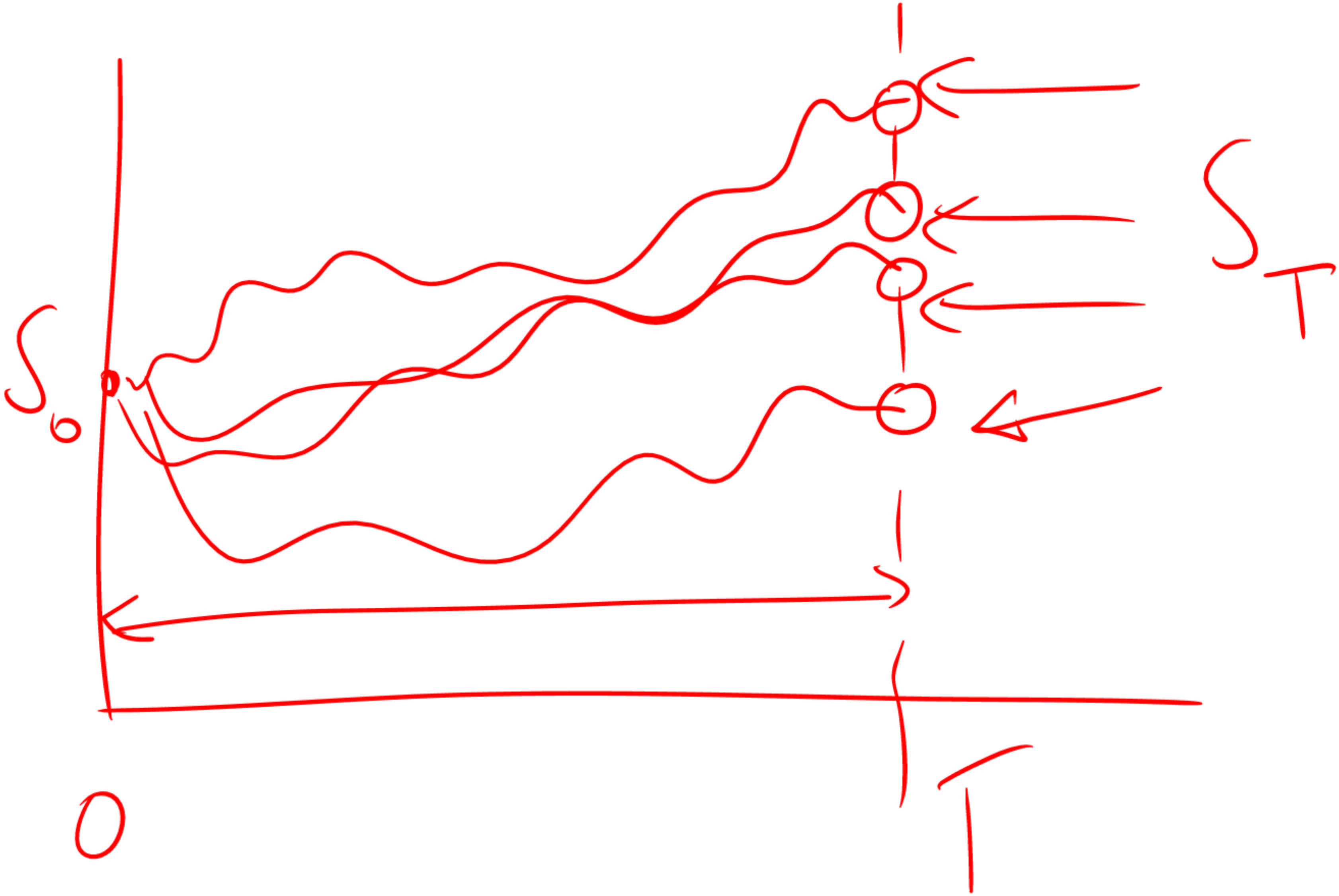

S_0
S_T
0
T

$$dt\left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2}\right) = r\left(V - S\frac{\partial V}{\partial S}\right)dt$$

1	2

$1 > 2$

$2 > 1$

$V \geq \text{Payoff}$

Perpetual American Put

E-S

$V \neq V(S,t)$ but $V = V(S)$

$$\frac{1}{2}\sigma^2 S^2 \frac{d^2V}{dS^2} + (r-D) S \frac{dV}{dS} - rV = 0$$

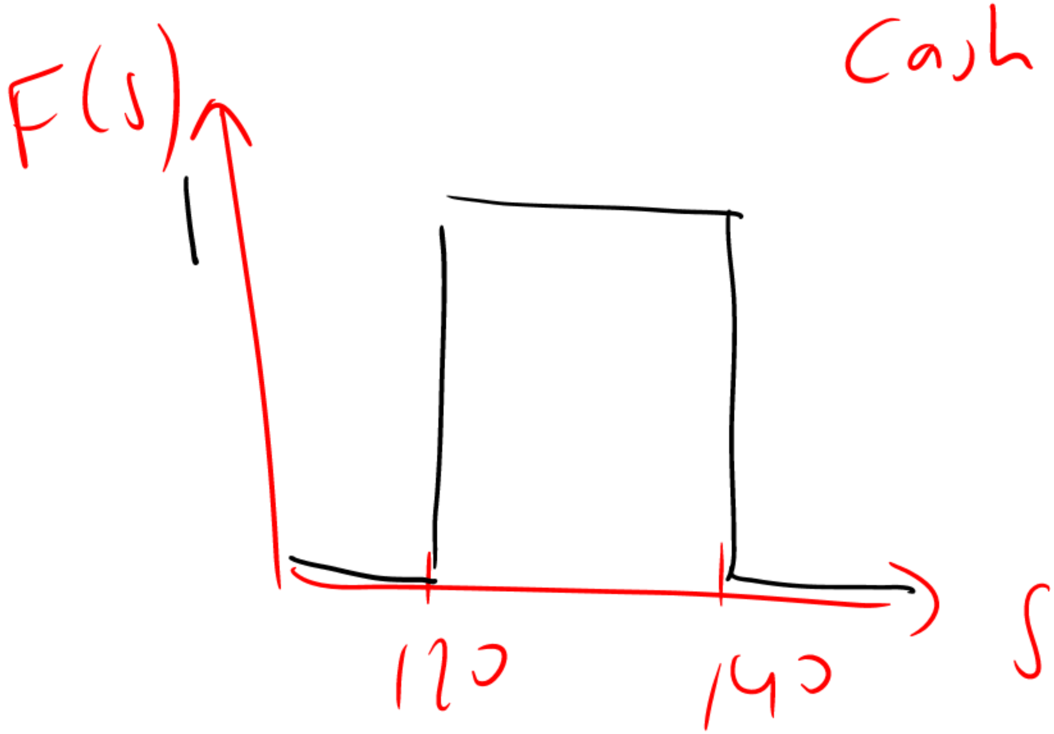
$\exists$ a sol^n of the form $V = S^\alpha$

S^* – exercise strategy.

$$\left.\begin{aligned} V(\infty) &= 0 \\ V(S^*) &= E - S^* \\ \frac{dV}{dS}\Big|_{S=S^*} &= -1 \end{aligned}\right\} \text{smooth pasting condition}$$

$$\mathcal{L} \equiv \frac{\partial}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2}{\partial S^2} + rS\frac{\partial}{\partial S} - r$$

$$\mathcal{L}[V] + \underbrace{F(S,t)}_{\text{cash flow}} = 0$$



B.S.E + cash flow

F - source | forcing term

Look at problem

Discrete	Cts
$A_a = \frac{1}{N} \sum_{i=1}^{N} s(t_i)$	$I_a = \frac{1}{T} \int_0^T s_t \, dt$
$A = \frac{1}{i} \sum_{k=1}^{i} s(t_k)$	$I = \frac{1}{t} \int_0^t s_\tau \, d\tau$
	$I = \frac{1}{t} \int_0^t f(s, \tau) \, d\tau$

$$A_g = \left(\prod_{i=1}^{N} J_i \right)^{1/N}$$

$$A_g = \exp\left[\frac{1}{T} \int_0^T \log J \, dt \right]$$

$$\log A_g = \frac{1}{N} \log \prod_{i=1}^{N} J_i$$

$$= \frac{1}{N} \sum_{i=1}^{N} \log J_i$$

$$A_g = \exp\left[\frac{1}{N} \sum_{i=1}^{N} \log J_i \right]$$

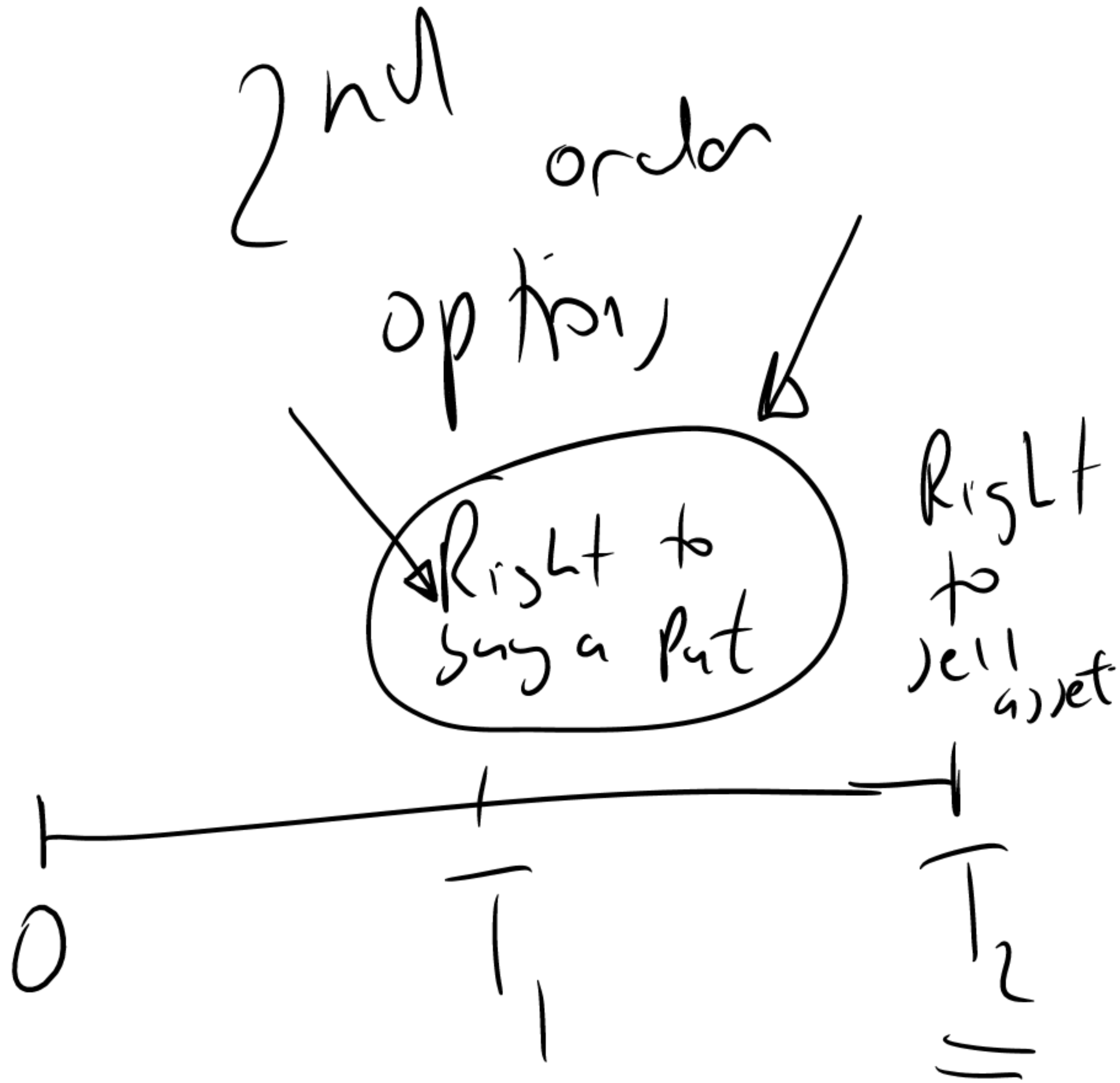
$\text{Call}\,(\text{Put})$

$\text{Put}\,(\text{Call})$

$\text{Put}\,(\text{Put})$

$\text{Call}\,(\text{Call})$

$C_1(P_2)$



$$\frac{\partial p}{\partial t} = c^2 \frac{\partial^2 p}{\partial y^2} \qquad \alpha = -\frac{1}{2}$$

$$\beta = 1/2$$

$$p(s,t) = t^{\alpha} f\left(\underbrace{\frac{s}{t^{\beta}}}_{\xi}\right)$$