THE OPTIONS GENIUS

Pricing expert and formula collector **Espen G Haug** relates a cautionary tale about pushing the numbers too far

September 1997 and a young buck by the name of Hoffneider has just graduated from Bridgeport University in Connecticut, USA. He now has a Bachelor degree in business and his goal is to become a superstar option trader as soon as possible. Well, basically he just wants to become a multi-millionaire without working too hard. He has been reading about how super-quants trading derivatives get millions and millions of dollars in bonuses, while making hundreds and hundreds of millions for some of the top-tier hedge funds. Unfortunately, Bridgeport University is not exactly well known (basically nobody has heard about it except his classmates). His grades are also just about average. The question is how will he compete for a job with all those Ivy League students from prestigious universities like Harvard, Wharton and Princeton, as well as all those rocket scientists from MIT? By simply sending his CV to a prestigious investment bank or hedge fund, there is no way they would contact him for an interview. However, there is a way around it. He had to make his weakness his strength. A week later, he sent his CV to what his finance professor had told him was the number one hedge fund: Short-Term Capital Management (STCM).

His CV stated: "I have no education at all, but I was born a genius. I can value any type of option in my head, only no-brainers need a computer. I am looking for an option-trading position where I can take big positions." Sincerely yours, Hoffneider.

USING YOUR HEAD

As expected, he was soon called in for an interview. Ten in the morning, he arrived at STCM in Greenwich, Connecticut. A very good-looking secretary (brunette with high convexity) brought him over to a huge office where a Mr Merioles and Dr Schowether where waiting for him. Merioles was known as a superstar trader and the founder of STCM. Schowether was head of quantitative research. After shaking hands, Merioles said: "Let's not waste time, let's see if you really are the option genius you claim to be."

Schowether: "What is the value of a three-month call on a futures contract, assuming the current futures price is 100, spot volatility of 20 per cent and, for simplicity, let's assume 0 per cent interest rate?" Hoffneider immediately answered: "4.0."

Merioles was typing the numbers into his computer and nodded positively to his partner. Schowether: "What if the option is Asian style, continuous monitoring of the average, same parameters?" Hoffneider immediately answered: "2.3."

Merioles selected Asian style on his computer screen. From his top-secret, state-of-the-art option system he got 2.3. Again Hoffneider was right.

Schowether: "Now assume lookback call, floating strike." Hoffneider immediately answered "7.7." Merioles checked the value and got 7.7.

Schowether: "What if the option is a simple chooser where you, after one month, have to decide if you want to keep a call or put, ceteris paribus?" Hoffneider: "6.3." Once again Merioles checked the value and got 6.3.

Schowether: "What if we have a spread option on asset one minus asset two. Asset one trades at 125, asset two at 100, strike price 25, volatility of both assets 20 per cent and correlation 0.4?"

Hoffneider now started counting out loud: "100, 99, 98, 97, 96."

Merioles: "Why are you counting?"

Hoffneider: "I am using a 100-step, three-dimensional binomial tree and am just rolling backwards doing backward induction. 95, 94, 93…3, 2, 1, the value must be 5.0."

Merioles touched the keyboard and waited a few seconds for the 100-step Rubinstein (1994) binomial pyramid to run through his 900Mhz Pentium processor. Out came exactly the same answer Hoffneider had just had calculated in his head. Merioles nodded his surprise to Schowether.

Schowether: "Let's go back to the standard call. So far we have naturally assumed geometric Brownian motion. What would the value be if we instead assume square root constant elasticity of variance process (SRCEV):

$dF = \sigma\sqrt{F}dZ$?"

Hoffneider: "5,000, 4,999, 4,998, …"

Merioles: "What are you counting now?"

Hoffneider: "I am running a Monte Carlo simulation using 5,000 simulations. Because I am using Antithetic variance reduction technique it should be accurate enough. 4,997, 4,996,…" About 45 minutes later: "...7, 6, 5, 4, 3, 2, 1, 0, the answer must be 0.4."

Merioles was now looking into his computer screen with big eyes, his state-of-the-art Monte Carlo simulation were giving exactly the same price. Merioles and Schowether was now walking over to one of the corners in the office,
whispering to each other for a minute or two before returning to Hoffneider.

Merioles: "Congratulations! You are hired as a senior option trader. You can start trading your own option book from tomorrow morning. We have plenty of capital and you will be able to take some big positions, but remember to have a leverage of minimum 25 to 1. Your fixed salary will be one Banana (one Banana = one million dollars), but you will naturally get much more in bonus."

The next morning, Hoffneider went to the headquarters of STCM. The secretary walked him over to his place on the trading desk. A phone and two computers, more than he needed. Hoffneider immediately started to sell deep-out-of-the money put options on the S&P 500 stock index. In January, he made more than 50 million dollars on his option trading. For this he got a nice bonus of four Bananas. Not too bad for somebody who had just got his Bachelor degree.

His strategy was a money machine, so, in 1998, he just continued with the same strategy, selling more and more deep-out-of-the-money puts on the S&P index. It went very well until July, when implied vols started to climb upwards. By the end of August, the S&P index had fallen about 20 per cent from its peak in July and implied vols had simultaneously gone from 15 per cent to 30 per cent. His position now showed a loss of several hundred million dollars. But this was nothing compared with the billions of dollars STCM had lost on other close to risk-free arbitrage strategies. So, Hoffneider among others lost his job. At least he had saved a few Bananas from his bonus, even after buying a few sports cars. He had always wanted to travel around the world. He is probably right now on a nice beach in Asia doing some surfing, or possibly back in another hedge fund or investment bank.

WORKING IT THROUGH

The interesting question is not how Hoffneider could blow up a few hundred million dollars. That only proves he is almost a real Wall Street genius (one should blow up at least a billion dollars to prove one is a real genius). The interesting question is how he could value all these options in his head. Let's start with the Black-76 formula:

$$\text{call} = e^{-rT}[FN(d_1) - XN(d_2)]$$

where

$$d_1 = \frac{\ln(F/X) + T\sigma^2/2}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T},$$

F =forward price, X =strike price, T =time to maturity, r=risk free rate, σ =volatilityand $N(x)$= cumulative Normal distribution function. When the option is at-the-money-forward (F=X), the option is pretty linear in the volatility. As shown by Brenner and Subrahmanyam (1988) we can then simplify the different parts of the Black-76 (Black-Scholes-Merton) formula quite a lot:

$$N(d_1) \approx 0.5 + 0.2\sigma\sqrt{T},$$

$$N(d_2) \approx 0.5 - 0.2\sigma\sqrt{T}.$$

Black-76 at-the-money-forward approximation:

$$\text{call} = \text{put} \approx e^{-rT} 0.4F\sigma\sqrt{T}.$$

If the interest rate is very low and/or the time to maturity is short, this can be further simplified to $\text{call} = \text{put} \approx 0.4\text{F}\sigma\sqrt{\text{T}}$ which is the formula Hoffneider used during his interview. Below we have extended this principle to find compact approximations for several popular exotic options. We will not go through all the boring math, but simply give you the results. All results assume that the option is at-the-money forward. For a simple chooser option (Rubinstein 1991[1]) we get

$$\text{call} = \text{put} \approx e^{-rT} 0.4F\sigma(\sqrt{T} - \sqrt{t})$$

where t is the time to the chooser period. The option holder at time $t(t < T)$ has the right to choose if she wants to keep a put or call, assuming the strike on the put and call initially equal the forward price. When it comes to Asian options it is well known that average volatility can be found from spot volatility by dividing the volatility by $\sqrt{3}$ This gives us the following approximation for an at-the-money forward Asian option.

$$\text{call} = \text{put} \approx e^{-rT} 0.23F\sigma\sqrt{T + 2t_1}.$$

where t_1 is the time to start of the average period $t_1 \geq 0$ and T is the time to maturity.

For a floating strike lookback call, which gives the option holder the right to buy the asset at the minimum - Goldman, Sosin, and Gatto, (1979) Haug, (1997) - we get

$$\text{call} \approx e^{-rT} 0.8F\sigma\sqrt{T} - 0.25\sigma^2 T.$$

For a put that gives the option holder the right to sell at the maximum

$$\text{put} \approx e^{-rT} 0.8F\sigma\sqrt{T} + 0.25\sigma^2 T.$$

An at-the-money forward spread option can be valued as

Table 1:

Volatility	Plain Vanilla CF[1]	Plain Vanilla Approx	Chooser CF[2]	Chooser Approx	Asian CF[3]	Asian Approx	SRCEV CF[4]	SRCEV Approx
10%	1.9453	1.9506	3.0684	3.0768	1.1233	1.1216	0.1945	0.1951
20%	3.8893	3.9012	6.1354	6.1536	2.2471	2.2432	0.3891	0.3901
30%	5.8309	5.8519	9.1995	9.2304	3.3718	3.3648	0.5836	0.5852
40%	7.7689	7.8025	12.2593	12.3072	4.4979	4.4864	0.7782	0.7802
60%	11.6291	11.7037	18.3600	18.4609	6.7561	6.7296	1.1673	1.1704

1: The Black-76 formula, 2: Simple Chooser Rubinstein (1991), 3: Turnbull-Wakeman (1991) extended for Asian futures options, 4: Hagan and Woodward (1999) approximation with β=0.5.

$$\text{call} = \text{put} \approx e^{-rT} 0.4 F_1 \sigma \sqrt{T}$$

when the correlation between the return of the two assets is $\rho = \frac{F_1}{2F_2}$, $\sigma_1 = \sigma_2$ and the at-the-money-forward is defined as $F_1 = F_2 + X$.

Next, let's assume the future/forward follows a general constant elasticity of variance model. See Cox and Ross, (1976) Beckers, (1978).

$$dF = \sigma F^{\beta} dz$$

where σ is the instantaneous volatility of the forward price, β is the elasticity parameter and dz is a Wiener process. Some well-known special cases of this model obtain for different values of β. β=0 gives a normally distributed asset, β=0.5 a square root constant elasticity of variance model (SRCEV), equivalent to the Cox, Ingersoll and Ross, (1985) model without the mean reverting part- β=1 gives the Black-Scholes-Merton model (log-normal price). To derive a simple expression for such a process can at first seem
impossible. However, thanks to Hagan and Woodward (1999), one can actually easily value options under such a process with a high level of accuracy. They show how one can use a Black-
equivalent volatility that corresponds to a constant elasticity of variance model

$$\hat{\sigma} = \frac{\sigma}{f^{1-\beta}} \left[1 + \frac{(1-\beta)(2+\beta)}{24} \left(\frac{F-X}{f} \right)^2 + \frac{(1-\beta)^2}{24} \frac{\sigma^2 T}{f^{2-2\beta}} + \ldots \right]$$

where ... represents additional 'negligible' terms and $f = \frac{1}{2}(F+X)$. When the option is at-the-money forward this equivalent Black-volatility collapses to (not necessary correlated to the collapse of STCM)

$$\hat{\sigma} = \frac{\sigma}{F^{1-\beta}} \left[1 + \frac{(1-\beta)^2}{24} \frac{\sigma^2 T}{F^{2-2\beta}} + \ldots \right].$$

The last term in this expression has a very small impact. We will, therefore, get a pretty good approximation by using only the first term $\hat{\sigma} \approx \frac{\sigma}{F^{1-\beta}}$. Feeding this equivalent volatility into the Brenner and Subrahmanyam (1988) at-the-money forward approximation we get

$$\text{call} = \text{put} \approx e^{-rT} 0.4 F^{\beta} \sigma \sqrt{T}$$

With this simple expression we can actually value at-the-money forward options with remarkably accuracy, assuming a generalized constant elasticity of variance diffusion process.

How accurate are these approximations? To get an idea, take a look at Table 1. Here we compare closed-form solutions (CF) with the at-the-money-forward approximations. We assume the forward price is 100, 0.25 years to maturity and 10 per cent interest rate (continuous compounding) for different volatilities.

It is evident that at-the-money forward approximations are remarkably accurate. Even if these simple approximations only hold for options that are close to at-the-money-forward, they can at least give us some intuition for the right price. More important these approximations should also be very useful when applying for a job in a prestigious hedge fund or investment bank. Just remember, to be considered a genius you need to leverage at least 25-1. Good luck!

REFERENCES

• Black, F. and Scholes, M: The Pricing of Options and Corporate Liabilities, *Journal of Political Economy*, 81, 637-654, 1973

• Black, F: The Pricing of Commodity Contracts, *Journal of Financial Economics*, 3, 167-179, 1976

• Brenner, M. and Subrahmanyam: A Simple Solution to Compute the Implied Standard Deviation, *Financial Analysts Journa*l, 80-83, 1988

• Cox, J.C. and Ross S.A: The Valuation of Options for Alternative Stochastic Processes, *Journal of Financial Economics*, 3, 145-166, 1976

• Haug, E. G: The Complete Guide To Option Pricing Formulas, McGraw-Hill, New York, 1997

• Cox, J.C., Ingersoll, J. and Ross S.A: A Theory of the Term Structure of Interest Rates, *Econometrica*, 53, 385-407, 1985

• Goldman, B. M., Sosin, H. B. and Gatto, M. A: Path Dependent Options: Buy at the Low, Sell at the High. *Journal of Finance*, 34(5), 1111-1127, 1979

• Hagan, P. S. and Woodward, D: Equivalent Black Volatilities, *Applied Mathematical Finance* 6, 147-157, 1999

• Turnebull, S.M. and Wakeman L. M: A Quick Algorithm for Pricing European Average Options, *Journal of Financial and Quantitative Analysis*, 26, 377-389, 1991

FOOTNOTE

1. For more references as well as implementation of this and many other exotic options see Haug (1997)

*** The views presented in this article do not necessarily reflect the views of my employer (an anonymous hedge fund in Greenwich,CT), and it definitely doesn't reflect my own view.Blondes with huge positive gamma can feel free to e-mail me at espenhaug@mac.com**