iii) $\|A\|_{op,\infty} = \max_{1 \le i \le n} \|b_i\|_1 = \| b_i \|_1$

$$A = \begin{pmatrix} - & b_1 & - \\ & \vdots & \\ - & b_n & - \end{pmatrix}$$

" $\geq$ "

$$\left\| \begin{pmatrix} - & b_1 & - \\ & & \\ - & b_n & - \end{pmatrix} x \right\|_1 = \left\| \begin{pmatrix} b_1^T x \\ \vdots \\ b_n^T x \end{pmatrix} \right\|_\infty = \max \{ | b_i^T x | \} \geq | b_i^T x |$$

$$\begin{pmatrix} b_{11} & \cdots & b_{1n} \\ \vdots & & \vdots \\ \cdot & - - & - \end{pmatrix} \begin{pmatrix} \frac{|b_{11}|}{b_{11}} \\ \vdots \\ \frac{|b_{1n}|}{b_{1n}} \end{pmatrix} = \begin{pmatrix} \sum |b_{1i}| \\ \vdots \\ \vdots \end{pmatrix} .$$

" $\leq$ "

$$\|Ax\|_\infty = \max_{1\leq i\leq n} \left\{ \left| \sum_{j=1}^{n} b_{ij} x_j \right| \right\}$$

$$= \max_{1\leq i\leq n} \left\{ \sum_{j=1}^{n} |b_{ij}| |x_j| \right\}$$

$$\leq \max_{1\leq j\leq n} \{|x_j|\} \max_{1\leq i\leq n} \left\{ \sum_{j=1}^{n} |b_{ij}| \right\} = \|x\|_\infty \|b_1\|_1$$

$x \in \mathbb{C}^m, \quad A \in \mathbb{C}^{m \times n}$

1\. $\|x\|_\infty = \max|x_i| = ((\max|x_i|)^2)^{1/2} \leq (|x_1|^2 + \dots + |x_m|^2)^{1/2} = \|x\|_2$

2\. $\|x\|_2 = \left(\sum |x_i|^2\right)^{1/2} \leq \left(m(\max|x_i|)^2\right)^{1/2} = \sqrt{m}\,\|x\|_\infty$.

$\|Ay\|_\infty \leq \alpha \|y\|_\infty$

3\. $y \in \mathbb{C}^n$ 1.

$\|Ay\|_\infty \leq \|Ay\|_2 \leq \|A\|_2 \|y\|_2 \leq \underbrace{\|A\|_2 \cdot \sqrt{n}} \cdot \|y\|_\infty \Rightarrow \|A\|_\infty \leq \|A\|_2 \cdot \sqrt{n}$.

4\. $\|Ay\|_2 \leq \sqrt{m}\,\|Ay\|_\infty \leq \sqrt{m} \cdot \|A\|_\infty \|y\|_\infty \leq \underbrace{\sqrt{m}\,\|A\|_\infty} \|y\|_2 \Rightarrow \|A\|_2 \leq \sqrt{m}\,\|A\|_\infty$

$\|_{op,\infty} = \max\limits_{1\leq i\leq n}$

$$\begin{pmatrix} - b_1 - \\ \vdots \\ - b_n - \end{pmatrix}$$

<u>Conj</u>?

$\frac{1}{p}+\frac{1}{q}=1$

$$\|A\|_{op,p} = \|A^t\|_{op,q}$$

$$\begin{cases} \|A\|_{op,\infty} = \|A^t\|_{op,1} \\ \|A\|_{op,2} = \|A^t\|_{op,2} \end{cases}$$

$$A = \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix}$$

$A = U\Sigma V^*$ $\quad A^* = V\Sigma^* U^*$

$V\Sigma U^* V \Sigma U^*$

$V\Sigma\Sigma V^* = A^* A$

SVD para A (amano)

$$A^*A = \begin{bmatrix} 104 & -72 \\ -72 & 146 \end{bmatrix}$$

$\sigma_1 = 10\sqrt{2}$

$\sigma_2 = 5\sqrt{2}$

$$\begin{bmatrix} x-104 & 72 \\ 72 & x-146 \end{bmatrix} \quad (x-104)(x-146)-72^2=0$$

$(x-200)(x-50)=0$

$x=200 \quad x=50$

$$\begin{bmatrix} 204-200 & -72 \\ -72 & 146-200 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\hat{V}_1 = \frac{1}{\sqrt{(-1)^2+\left(\frac{4}{3}\right)^2}}\begin{pmatrix} -1 \\ \frac{4}{3} \end{pmatrix} = \begin{pmatrix} -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$$

$a = -1$

$b = \frac{4}{3}$

$$V_1 = \begin{pmatrix} -1 \\ \frac{4}{3} \end{pmatrix}$$

$\sqrt{2}$

$\sqrt{2}$

$$\begin{bmatrix} 204-50 & -72 \\ -72 & 146-50 \end{bmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$V_2 = \begin{pmatrix} 4/3 \\ 1 \end{pmatrix} \qquad \hat{V}_2 = \begin{pmatrix} \frac{4}{5} \\ \frac{3}{5} \end{pmatrix}$$

$$AV = U\Sigma$$

$$\boxed{AV\Sigma^{-1} = U}$$

$$(x - 146 - 72)^2 = 0$$

$$= \begin{pmatrix} -\frac{3}{5} \\ \frac{4}{5} \end{pmatrix}$$

$$\boxed{\begin{array}{l} u_1 = \frac{1}{\sqrt{200}} \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix} \begin{bmatrix} -\frac{3}{5} \\ \frac{4}{5} \end{bmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \\ u_2 = \frac{1}{\sqrt{50}} \begin{bmatrix} -2 & 11 \\ -10 & 5 \end{bmatrix} \begin{bmatrix} \frac{4}{5} \\ \frac{3}{5} \end{bmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \end{array}}$$

$$u_1 = \frac{A \vec{v}_i}{\sigma_i}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 10\sqrt{2} & 0 \\ 0 & 5\sqrt{2} \end{pmatrix} \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

$\|A\|_{Fb} = \sqrt{\operatorname{trace}(A^* A)} = \sqrt{104+146} = \sqrt{250}$. $\|A\|_{Fb} = \sqrt{\sum \sigma_i^2} = \|(\sigma_1, \dots, \sigma_k)\|_2$

$\|A\|_{op2} = \| \sum \binom{x}{y} \|_{op2} = \sqrt{(10\sqrt{2}\,1x1)^2 + (5\sqrt{2}\,1y1)^2} = \sqrt{200x^2 + 50x^2} = \sqrt{200} = \|(\sigma_1, \dots, \sigma_k)\|_\infty$

$\|(\sigma_1, \dots, \sigma_k)\|_1$

$A = U \Sigma V^*$

Norma Nuclear.

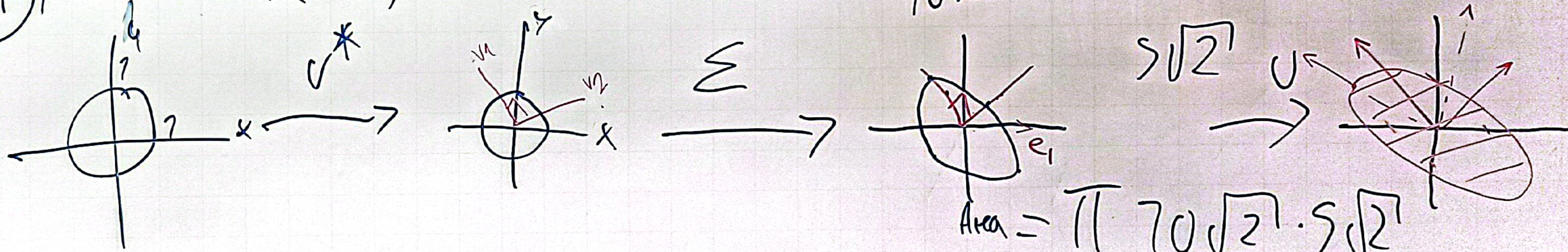
(3) $A^{-1} = V \Sigma^{-1} U^* = \begin{pmatrix} \frac{1}{20} & -\frac{11}{100} \\ \frac{1}{50} & -\frac{1}{50} \end{pmatrix}$

$\|(\sigma_1, \dots, \sigma_k)\|_p$

Ky-Fan norms.

Norma dual de la norma operador.

(4) $\mathbb{R}$ $x^2 + y^2 = 1$



Area $= \pi \, 10\sqrt{2} \cdot 5\sqrt{2}$

- Mínimo $n \in \mathbb{N}$ que no puedo representar en python (usando floats)

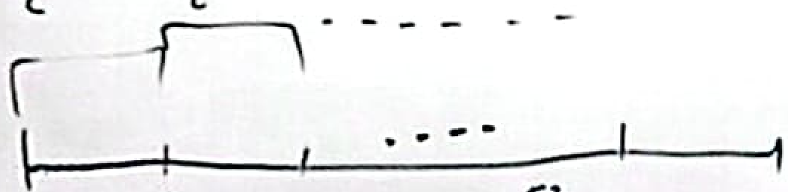
$2^{27} = \text{- - - -}$

str(----) 9

float( ) 8

$\underbrace{2^{53}+1}$

2^{53} 2^{52} - - - - - -



0 1

$0 \le m \le 2^{53}$

0 2^1 2^2 $\mathbb{N}$ 2^n 2^{n+1}

$2^{53-n} < 2^n = 2^{n+1} - 2^n$

2^n 2^{n+1}

2^n

$\neg\exists\, m \in \mathbb{N}$, $e \in \mathbb{N}$: $\left(\frac{m}{2^{53}}\right) 2^e$

$53 - n < n$ $n = 27$

$53 < 2n$

$26.5 < n$

$$\boxed{2^{53} + 1 = \frac{m}{2^{53}} 2^e}$$

$$\boxed{(2^{53})^2 + 2^{53} = m\, 2^e}$$

$$2^{53}+1 = \frac{m}{2^{53}} 2^{e}$$

$$\neg \exists\, e, m \in \mathbb{N}$$

$$0 \leq m \leq 2^{53}$$

$$2^{53}(2^{53}+1) = (m' 2^{d}) \cancel{2^{e}}$$

$$[d+e = 53]$$

$$m' \leq m \leq 2^{53}$$

$A = U \Sigma V^*$

(3)

$A^{-1} = V \Sigma$

$\underline{n \leq 2^{53}}$

$n = \frac{m}{2^{53}} 2^e$

$\mathbb{R}$

$x^2 +$

$= m' 2^d \cancel{2} \, e$

$\boxed{n 2^{53} = m 2^e}$

$n' 2^f 2^{53} = m 2^e$

r^*

$[d + e = 53]$

$n' \cancel{2}^{f+53} = m \cancel{2}^e$

$\underline{m'} \leq m \leq 2^{53}$

$\leq 2^{53}$

$e := f + 53$

$m = n'$

A matrix $\quad A = U\Sigma V^*$

$$B = \begin{bmatrix} 0 & A^* \\ A & 0 \end{bmatrix} \qquad \begin{pmatrix} V & 0 \\ 0 & U \end{pmatrix}\begin{pmatrix} 0 & \Sigma U^* \\ \Sigma V^* & 0 \end{pmatrix}$$

$$\| $$

$$\begin{bmatrix} 0 & V\Sigma U^* \\ U\Sigma V^* & 0 \end{bmatrix} \quad \begin{pmatrix} 0 & V\Sigma U^* \\ U\Sigma V^* & 0 \end{pmatrix}$$

$$\begin{pmatrix} V & 0 \\ 0 & U \end{pmatrix}\left[\begin{pmatrix} \Sigma & 0 \\ 0 & \Sigma \end{pmatrix}\begin{pmatrix} 0 & U^* \\ V^* & 0 \end{pmatrix}\right] = B$$

Float( )

$\lambda = 1 \cdot i$

$\underbrace{2^{53}+1}$

$\underline{B = P^* \Lambda P}$

$B^* B = P^* \Lambda \cancel{P P^*} \Lambda P$

$= P^* \Lambda^2 P$

$B = B^*$

$B^2 = B B^* =$