Parcial 1

1.

$x_j = x_0 + jh$

Usando el teorema de Taylor se tiene que

$$F(x+h) = F(x) + hF'(x) + \frac{h^2}{2}F''(x) + \cdots$$

$$\Rightarrow F'(x) = \frac{F(x+h) - F(x)}{h} - O(h) \quad \rightarrow \text{Progresiva}$$

$$\Rightarrow F'(x) \approx \frac{F(x_{j+1}) - F(x_j)}{h}$$

$$F(x-h) = F(x) - hF'(x) + \frac{h^2}{2}F''(x) + \cdots$$

$$\Rightarrow F'(x) = \frac{F(x) - F(x-h)}{h} + O(h) \quad \rightarrow \text{Regresiva}$$

$$\Rightarrow F'(x) \approx \frac{F(x_j) - F(x_{j-1})}{h}$$

central:

$$F(x+h) = F(x) + hF'(x) + \frac{h^2}{2}F''(x) + \frac{h^3}{3!}F'''(x) + \frac{h^4}{4!}F''''(x) + \cdots \quad (1)$$

$$F(x-h) = F(x) - hF'(x) + \frac{h^2}{2}F''(x) - \frac{h^3}{3!}F'''(x) + \frac{h^4}{4!}F''''(x) + \cdots \quad (2)$$

Entonces

$$(1) - (2) \Rightarrow F'(x) = \frac{F(x+h) - F(x-h)}{2h} \underbrace{- \frac{h^2}{3}F'''(x)}_{O(h^2)}$$

$$\Rightarrow F'(x_j) \approx \frac{F(x_{j+1}) - F(x_{j-1})}{2h}$$

2 Derivada

1 + 2

$$\Rightarrow f(x+h) + f(x-h) = 2F(x) + h^2 F''(x) + o(h^4)$$

$$\Rightarrow F''(x) = \frac{F(x+h) + F(x-h) - 2F(x)}{h^2} - \frac{o(h^4)}{h^2}$$

$$\Rightarrow F''(x) = \frac{F(x+h) - 2F(x) + F(x-h)}{h^2} + o(h^2)$$

$$\Rightarrow F''(x) \simeq \frac{F(x_{j+1}) - 2F(x_j) + F(x_{j-1})}{h^2}$$

Ahora para calcular la derivada de orden 4 se usan las siguientes definiciones

3 $F(x+2h) = F(x) + 2hF'(x) + \frac{4h^2}{2!}F''(x) + \frac{8h^3 F'''(x)}{3!} + \frac{16h^4 F''''(x)}{4!} + \cdots$

4 $F(x-2h) = F(x) - 2hF'(x) + \frac{4h^2}{2!}F''(x) - \frac{8h^3 F'''(x)}{3!} + \frac{16h^4 F''''(x)}{4!} + \cdots$

Entonces al tomar

$8(1 - 2) - (3 - 4) = 8\,1 - 8\,2 - 3 + 4$ se tiene que

$$= 8F(x+h) - 8F(x-h) - F(x+2h) + F(x-2h)$$

$$= 8\left(F(x) + hF'(x) + \frac{h^2 F''(x)}{2!} + \frac{h^3 F'''(x)}{3!} + \frac{h^4 F''''(x)}{4!} + o(h^5)\right)$$

$$-8\left(F(x) - hF'(x) + \frac{h^2 F''(x)}{2!} - \frac{h^3 F'''(x)}{3!} + \frac{h^4 F''''(x)}{4!} + o(h^5)\right)$$

$$-\left(F(x) + 2hF'(x) + \frac{4h^2}{2!}F''(x) + \frac{8h^3 F'''(x)}{3!} + \frac{16h^4 F''''(x)}{4!} + o(h^5)\right)$$

$$+\left(F(x) - 2hF'(x) + \frac{4h^2}{2!}F''(x) - \frac{8h^3 F'''(x)}{3!} + \frac{16h^4 F''''(x)}{4!} + o(h^5)\right)$$

④ + ③

$$\Rightarrow F(x+2h)+F(x-2h) = 2F(x) + \frac{8h^2}{2!}F''(x) + \frac{32h^4 F''''(x)}{4!} + 2O(h^5)$$

$$\Rightarrow F(x+2h)+F(x-2h)-2F(x)-\frac{8h^2}{2!}F''(x)-2O(h^5) = \frac{32h^4 F''''(x)}{4!}$$

$$\Rightarrow F''''(x) = \frac{4!}{32h^4}\left(F(x+2h)+F(x-2h)-2F(x)-\frac{8h^2}{2!}F''(x)-2O(h^5)\right)$$

$$F''''(x) = \frac{4!}{32h^4}\left(F(x+2h)+F(x-2h)-2F(x)-\frac{8h^2}{2!}\left(\frac{F(x+h)-2F(x)+F(x-h)}{h^2}+O(h^2)\right)-2O(h^5)\right.$$

$$F^{(4)}(x) = \frac{4!}{32h^4}\left(F(x+2h)+F(x-2h)-2F(x)-\frac{8h^2}{2!}\left(\frac{F(x+h)-2F(x)+F(x-h)}{h^2}+O(h^2)\right)-2O(h^5)\right.$$

$$= \frac{4!}{32h^4}F(x+2h)+\frac{4!}{32h^4}F(x-2h)-\frac{2\cdot 4!}{32h^4}F(x)-\frac{8\cdot 4!}{64h^2}\left(\frac{F(x+h)-2F(x)+F(x-h)}{h^2}+O(h^2)\right)-2O(h^5)$$

④ + ③

$$\Rightarrow F(x+2h)+F(x-2h) = 2F(x) + \frac{8h^2}{2!}F''(x) + \frac{32h^4 F''''(x)}{4!} + 2O(h^5)$$

$$\Rightarrow F(x+2h)+F(x-2h)-2F(x)-\frac{8h^2}{2!}F''(x) - 2O(h^5) = \frac{32h^4 F''''(x)}{4!}$$

$$\Rightarrow F''''(x) = \frac{4!}{32h^4}\left(F(x+2h)+F(x-2h)-2F(x)-\frac{8h^2}{2!}F''(x)-2O(h^5)\right)$$

$$F''''(x) = \frac{4!}{32h^4}\left(F(x+2h)+F(x-2h)-2F(x)-\frac{8h^2}{2!}\left(\frac{F(x+h)-2F(x)+F(x-h)}{h^2}+O(h^2)\right)-2O(h^5)\right)$$

$$F^{(4)}(x) = \frac{4!}{32h^4}\left(F(x+2h)+F(x-2h)-2F(x)-\frac{8h^2}{2!}\left(\frac{F(x+h)-2F(x)+F(x-h)}{h^2}+O(h^2)\right)-2O(h^5)\right)$$

$$= \frac{4!}{32h^4}F(x+2h)+\frac{4!}{32h^4}F(x-2h)-\frac{2\cdot 4!}{32h^4}F(x)-\frac{8\cdot 4!}{64h^2}\left(\frac{F(x+h)-2F(x)+F(x-h)}{h^2}+O(h^2)\right)-2O(h^5)$$

$$= \frac{3}{4h^4}F(x+2h)+\frac{3}{4h^4}F(x-2h)-\frac{3}{2h^4}F(x)-\frac{3}{h^2}\left(\frac{F(x+h)-2F(x)+F(x-h)}{h^2}+O(h^2)\right)-2O(h^5)$$

$$= \frac{3}{4h^4}F(x+2h)+\frac{3}{4h^4}F(x-2h)-\frac{3}{2h^4}F(x)-\frac{3F(x+h)+6F(x)-3F(x-h)}{h^4}+O(h^0)-2O(h^5)$$

$$= \frac{F(x+2h)-4F(x+h)+6F(x)-4F(x-h)+F(x-2h)}{h^4}$$

b. El orden de $O(h^k)$ es 2 es decir $O(h^2)$