# Diagnostics for a tagging-based **mo**vement **mo**del (**momo**)

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# Outline

- Why should I care about momo?
- What is momo?
- Does it work?
- Estimating residuals for momo
- Conclusion

# Why should I care about momo?

- New state-space movement model implemented in TMB and now RTMB
- Allows to estimate animal movement based on tagging data (conventional and/or archival tags)
- Allows to estimate natural and fishing mortality and biomass based on tagging data and spatial effort and catch data
- Incorporates environmental information into stock assessments
- Was used to estimate relative and absolute biomass indices for stock assessment of skipjack tuna in the Eastern Pacific Ocean

# What is momo?

- Model that describes movement based on the advection-diffusion equation
- Uses habitat preference functions to estimate advection and diffusion rates as smooth functions of any field (e.g. environmental fields)
- Allow differentiation between active advection (taxis) informed by gradient of fields and passive advection (advection) informed by rate fields
- Reconstruct tag history either by matrix exponential approach or by Kalman filter

# Advection-diffusion equation and habitat preference functions

- Advection-diffusion equation

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial N}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial N}{\partial y}\right) - \frac{\partial}{\partial x}(uN) - \frac{\partial}{\partial y}(vN)$$

- Habitat preference function

$$h(g,t) = \sum_{i=1}^{n} f_i(\xi_i(g,t), k_i, \alpha_i)$$

where $f_i$ is a smooth function of the $i$th environmental or geographical field $\xi_i(g,t)$ with knot vector $k_i$ and parameter vector $\alpha_i$

# Matrix exponential approach (à la Thorson et al. 2021)

- Instantaneous diffusion rate: $\boldsymbol{D}^*$

$$d^*(g_2, g_1, t) = \begin{cases} e^{h(g_1,t)} & \text{if } g_1 \text{ and } g_2 \text{ are adjacent} \\ -\sum_{g' \neq g_1} \{d^*(g', g_1, t)\} & \text{if } g_1 = g_2 \\ 0 & \text{otherwise} \end{cases}$$

- Instantaneous taxis rate: $\boldsymbol{Z}^*$

$$z^*(g_2, g_1, t) = \begin{cases} h(g_2, t) - h(g_1, t) & \text{if } g_1 \text{ and } g_2 \text{ are adjacent} \\ -\sum_{g' \neq g_1} \{h(g_2, t) - h(g_1, t)\} & \text{if } g_1 = g_2 \\ 0 & \text{otherwise} \end{cases}$$

- Instantaneous advection rate: $\boldsymbol{V}^*$

$$v^*(g_2, g_1, t) = \sum_{i=1}^{n} \gamma_i w^*(g_2, g_1, t, i)$$

- Movement: $\boldsymbol{M}$

$$\boldsymbol{M}(t) = e^{(\boldsymbol{A}^* + \boldsymbol{D}^*)\Delta t} = e^{(\boldsymbol{Z}^* + \boldsymbol{V}^* + \boldsymbol{D}^*)\Delta t}$$

# Kalman filter

- Data
  - Archival tag:

$$o_{t_r} = (x_{t_r}, y_{t_r}), o_{t_r+\Delta t} = (x_{t_r+\Delta t}, y_{t_r+\Delta t}), ..., o_{t_c} = (x_{t_c}, y_{t_c})$$

  - Conventional tag:

$$o_{t_r} = (x_{t_r}, y_{t_r}), o_{t_c} = (x_{t_c}, y_{t_c})$$

  where $t_r$ is the time of release and $t_c$ is the time of capture.

- Model

$$o_t \sim N(\psi_t, \Sigma_o)$$

$$\psi_{t+\Delta t} \sim N(\psi_t + \boldsymbol{A}^*(\psi_t, t),\ 2\boldsymbol{D}^*(\psi_t, t)\Delta t I_{2x2})$$

where $\psi_t$ is the predicted position at time $t$, $\boldsymbol{\Sigma}_o$ is the observation noise covariance matrix for observation $o_t$, and $\boldsymbol{I}_{2x2}$ is the identity matrix.

- As $\psi$ depends on parameter values, differentiable representation of fields are required $\rightarrow$ local interpolation

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# Survival and probability of capture

Likelihood contribution of an (archival) tag $i$ recaptured in position $pos_m = (x_m, y_m)$ at time $t_m$:

$$\boldsymbol{L}_i(M, \lambda | E) = \frac{F_{pos_m, t_m, f}}{M + \sum_{f=1}^{F} F_{pos_m, t_m, f}} (1 - e^{-(M + \sum_{f=1}^{F} F_{pos_m, t_m, f}) \Delta t_m}) \prod_{s=1}^{m-1} e^{-(M + \sum_{f=1}^{F} F_{pos_s, t_s, f}) \Delta t_s}$$

Likelihood contribution of a non-recaptured tag $i$:

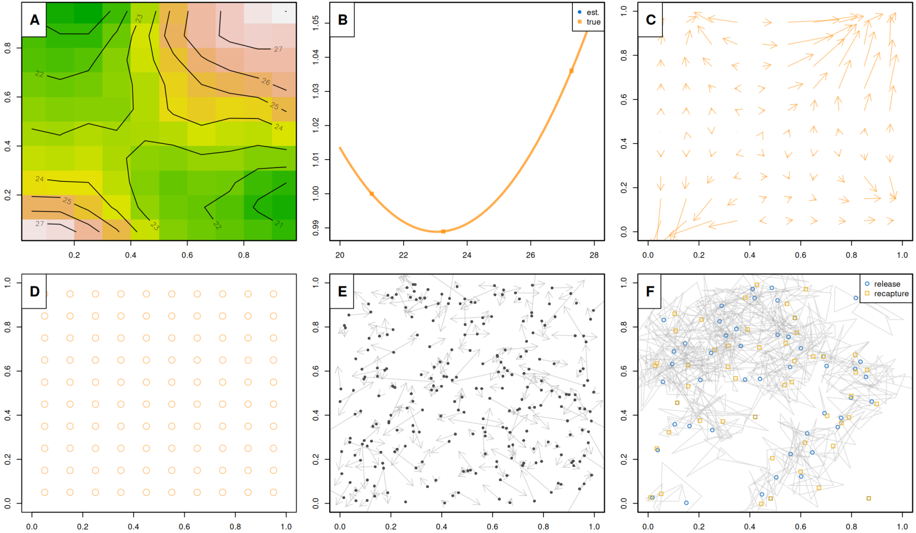
$$\begin{aligned} \boldsymbol{L}_i(M, \lambda | E) = & \frac{M}{M + \sum_{f=1}^{F} F_{pos_1, t_1, f}} (1 - e^{-(M + \sum_{f=1}^{F} F_{pos_1, t_1, f}) \Delta t_1}) \\ & + \sum_{a=1}^{A} \frac{M}{M + \sum_{f=1}^{F} F_{pos_a, t_a, f}} (1 - e^{-(M + \sum_{f=1}^{F} F_{pos_a, t_a, f}) \Delta t_a}) \prod_{s=1}^{a-1} e^{-(M + \sum_{f=1}^{F} F_{pos_s, t_s, f}) \Delta t_s} \\ & + \prod_{a=1}^{A} e^{-(M + \sum_{f=1}^{F} F_{pos_a, t_a, f}) \Delta t_a} \end{aligned}$$

where $M$ is the constant instantaneous natural mortality rate and $F_f$ is the instantaneous fishing mortality proportional to the locally interpolated effort of fleet $f$.

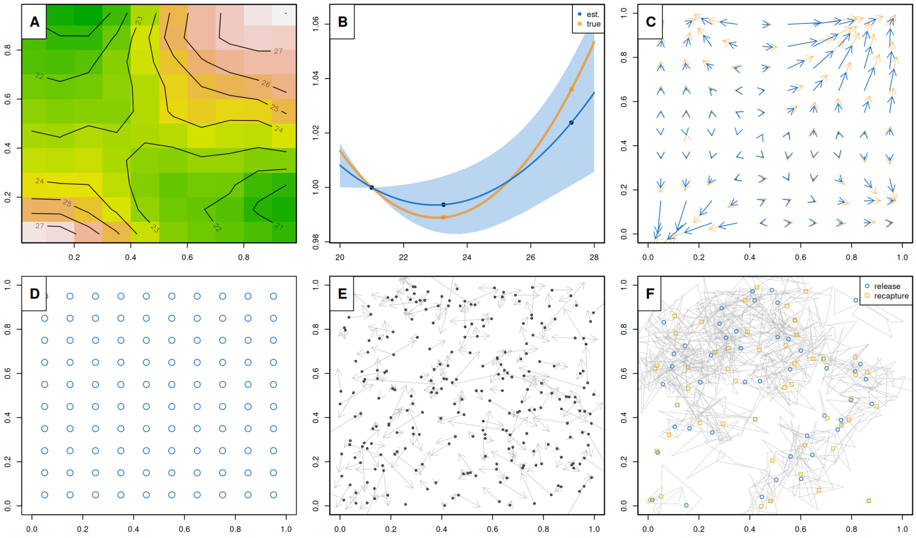
# Does it work?

- Simulation-estimation study
- Generate random environmental fields
- Draw random parameters defining habitat preference function for taxis and constant diffusion
- Compare c-tags vs. a-tags vs. both combined
- Varying number of tags (c-tags/a-tags): 25/5, 100/25, 500/100
- 500 replicates
- Apply KF and EXPM to same data sets
- Calculate relative errors for parameters ($\frac{\hat{\theta}-\theta}{\theta}$)

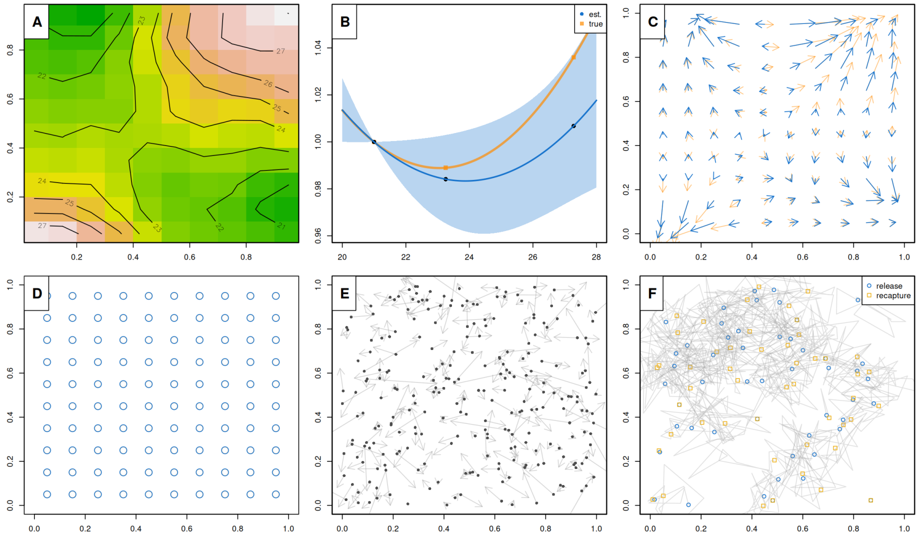
# Simulated data example I



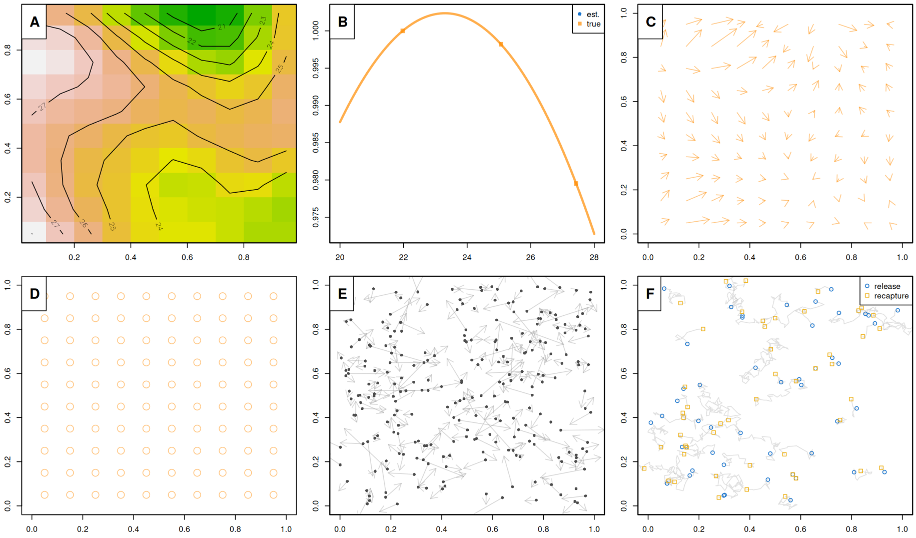
# Simulated data example I (KF)



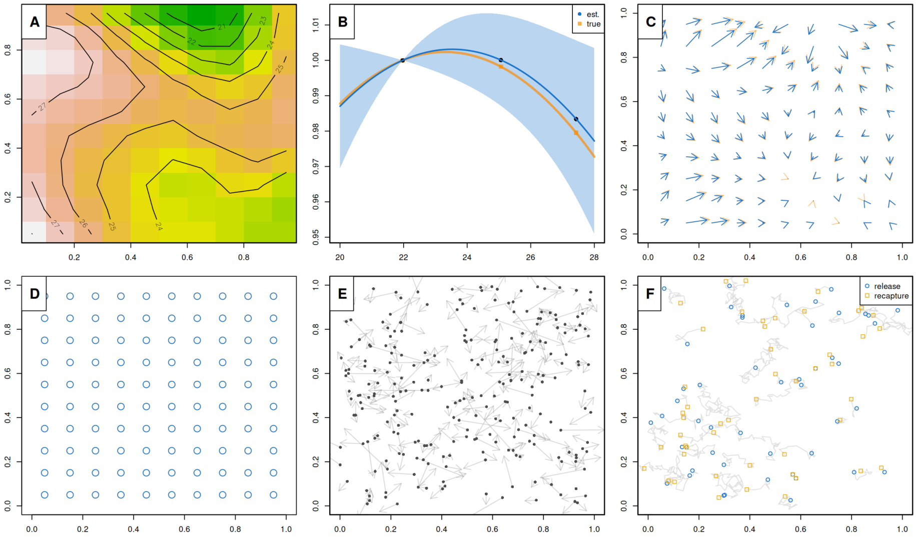
# Simulated data example I (EXPM)



# Simulated data example II



# Simulated data example II (KF)



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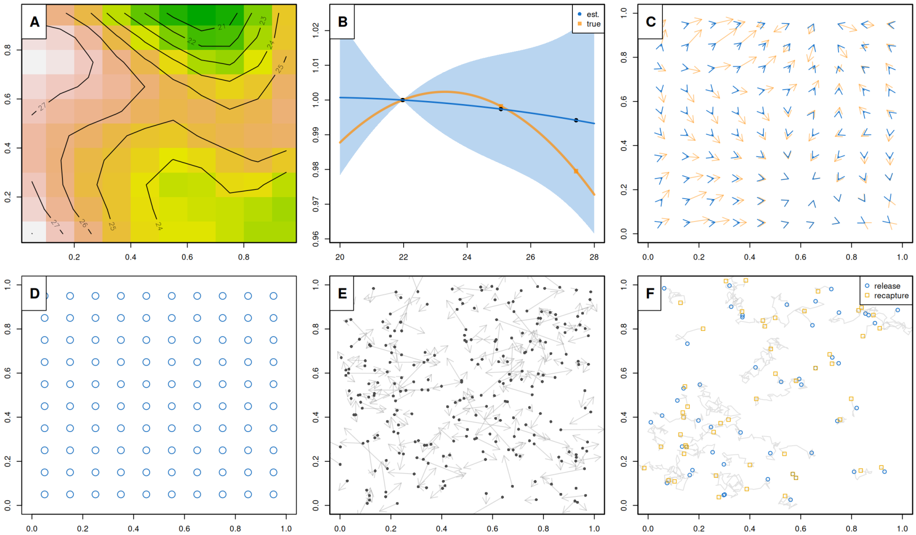

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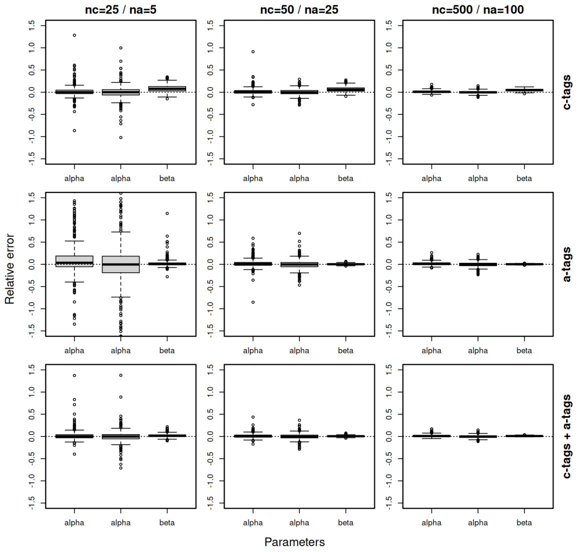

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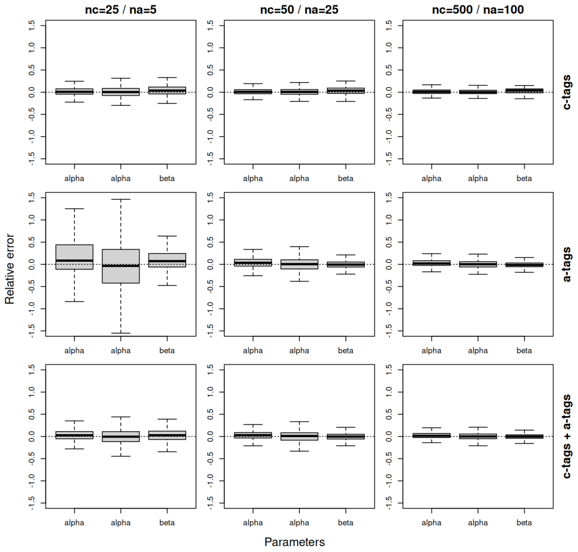

# Recapture position residuals

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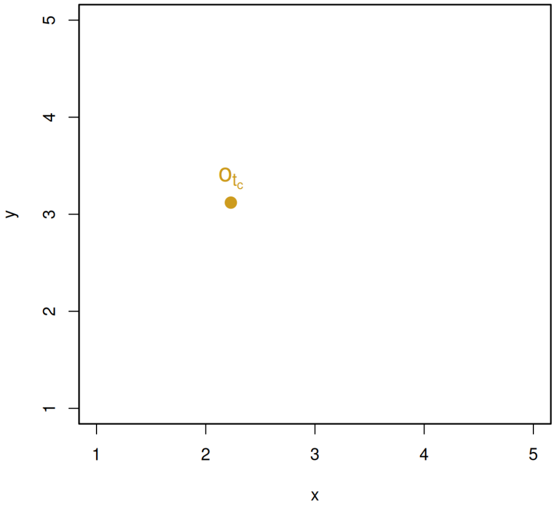
# Recapture position residuals (KF)



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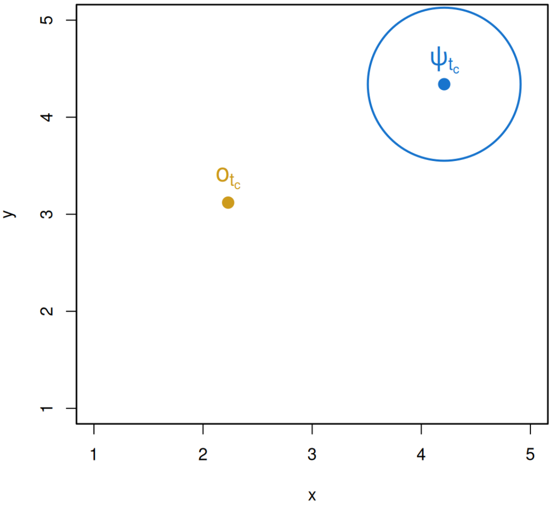

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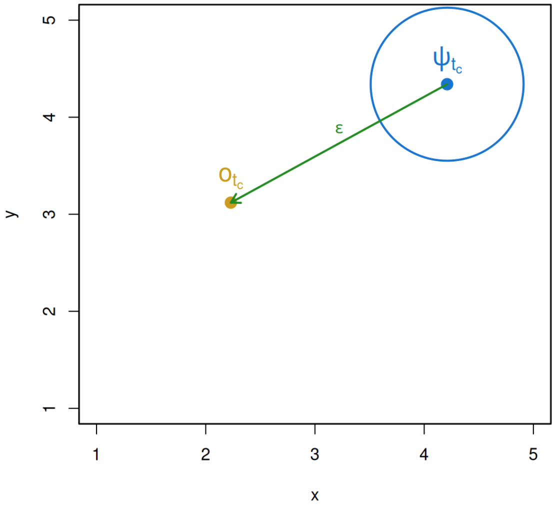

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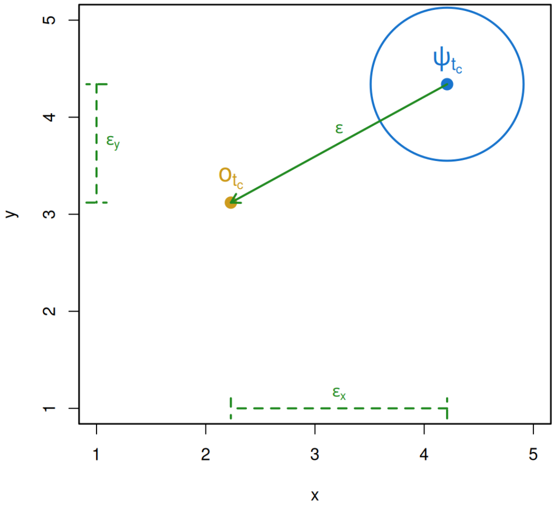

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$$\epsilon^{\mathrm{kf}} = (\epsilon_x, \epsilon_y) = \left( \frac{x_{t_c} - \hat{x}_{t_c}}{\sqrt{2\boldsymbol{D}^*(\psi_{t_c}, t_c)\Delta t \boldsymbol{I}_{2x2}}}, \frac{y_{t_c} - \hat{y}_{t_c}}{\sqrt{2\boldsymbol{D}^*(\psi_{t_c}, t_c)\Delta t \boldsymbol{I}_{2x2}}} \right)$$

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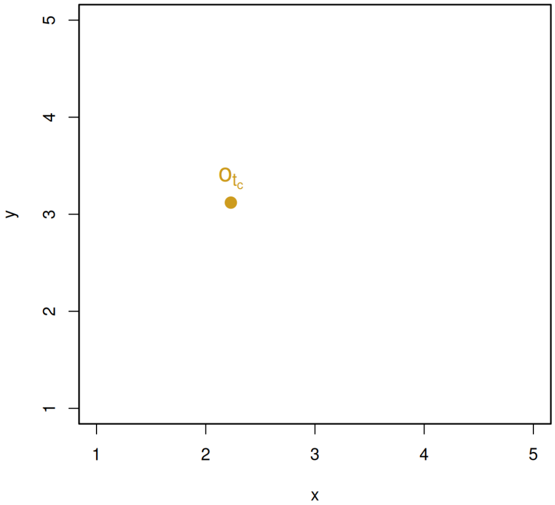
- Matrix exponential:

$$\epsilon^{\text{expm}} = (\epsilon_x, \epsilon_y)$$

# Recapture position residuals for $x$



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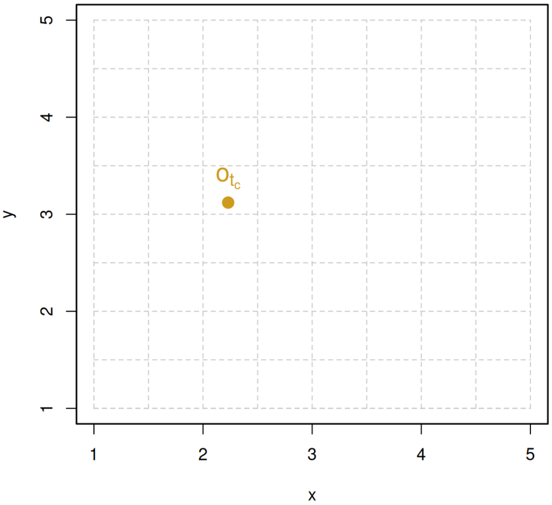

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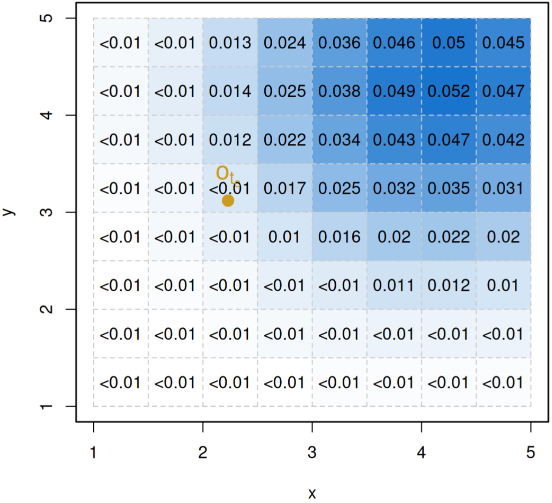

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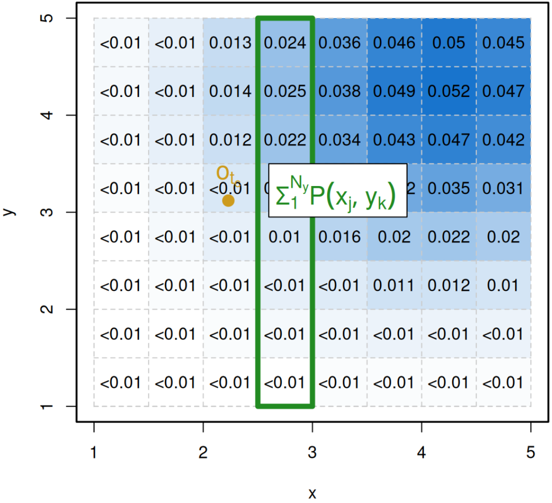

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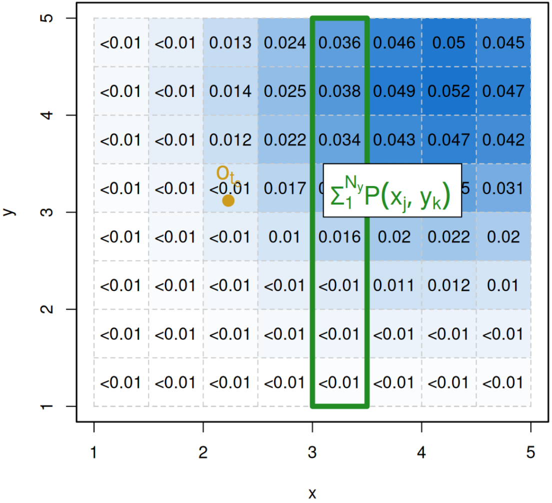

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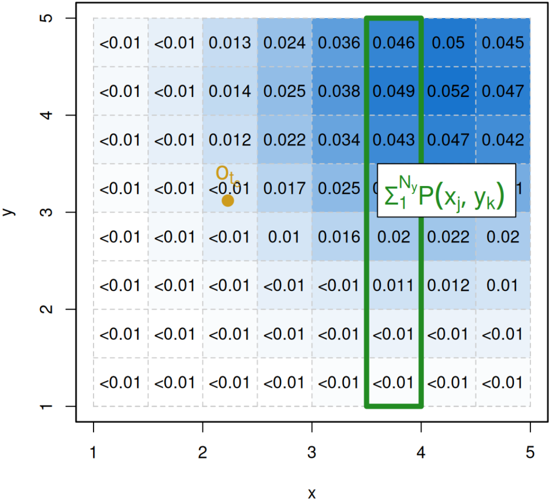

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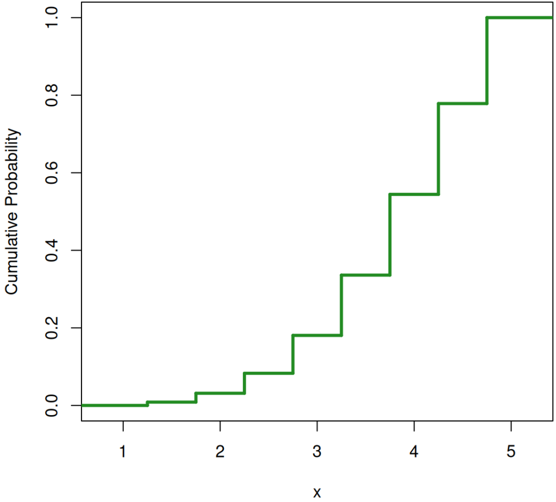

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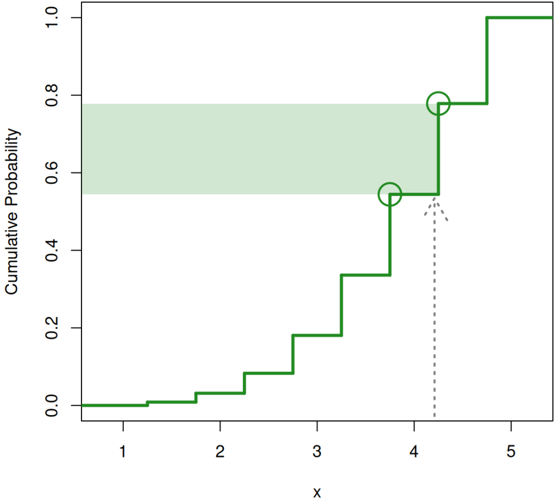

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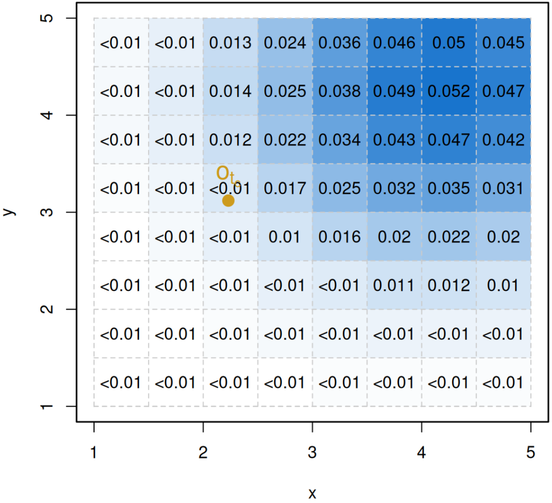
- Matrix exponential:

$$\epsilon^{\text{expm}} = (\epsilon_x, \epsilon_y)$$

$$\epsilon_x^{\text{expm}} = \Phi^{-1}\left(\text{Uniform}\left(\sum_{j=1}^{i-1}\sum_{k=1}^{N_y} P(x_j, y_k), \sum_{j=1}^{i}\sum_{k=1}^{N_y} P(x_j, y_k)\right)\right)$$

# Recapture position residuals for $y$



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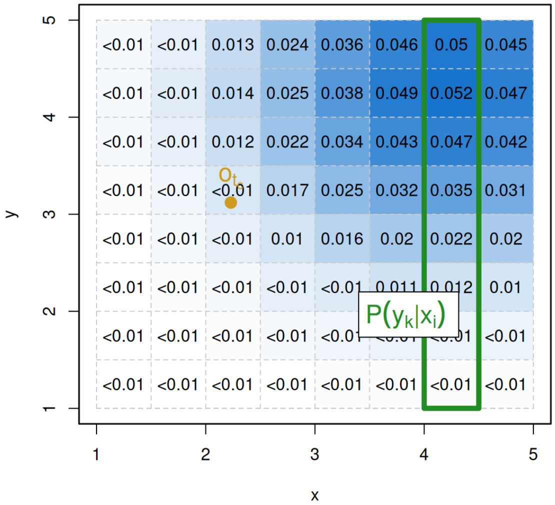

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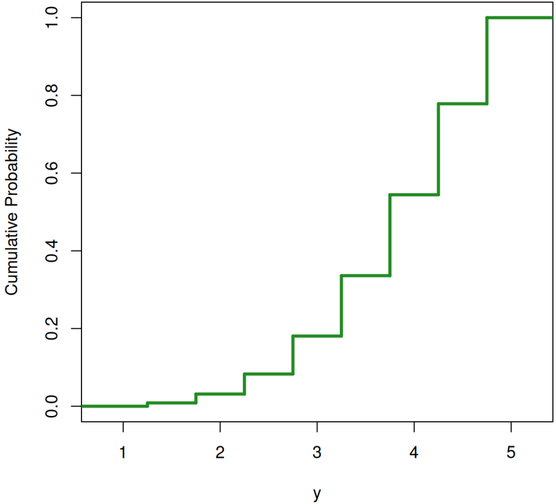

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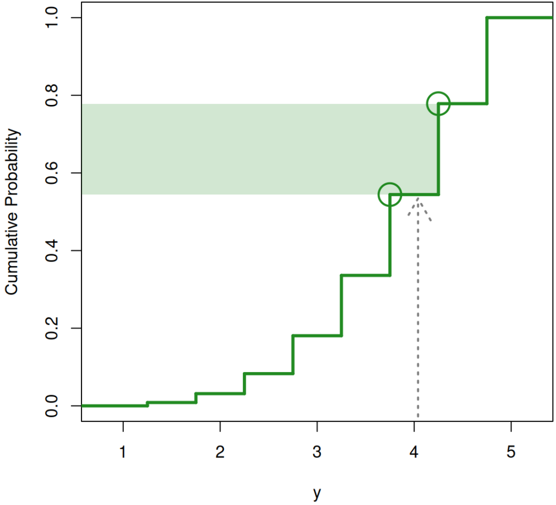

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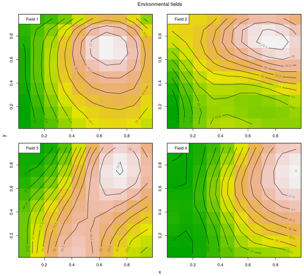
$$\epsilon_y^{\text{expm}} = \Phi^{-1}\left(\text{Uniform}\left(\sum_{k=1}^{j-1} P(y_k|x_i), \sum_{k=1}^{j} P(y_k|x_i)\right)\right)$$

where $\Phi^{-1}$ is the inverse normal distribution function, $P(x_j, y_k)$ is the probability of a tag being located in the $j^{th}$ row ($x$-coordinate) and $k^{th}$ column ($y$-coordinate) and $P(y_k|x_i)$ is the probability of a tag begin located in the $k^{th}$ column ($y$-coordinate) given that is located in the $i^{th}$ row ($x$-coordinate).

# Correct model (KF)



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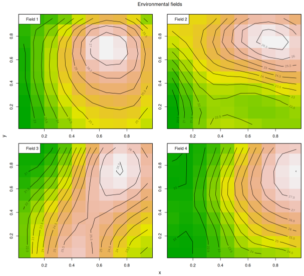


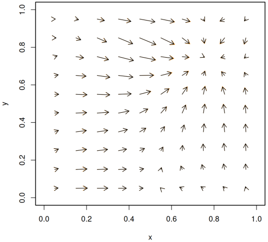

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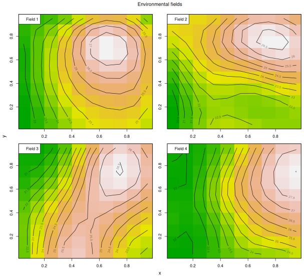


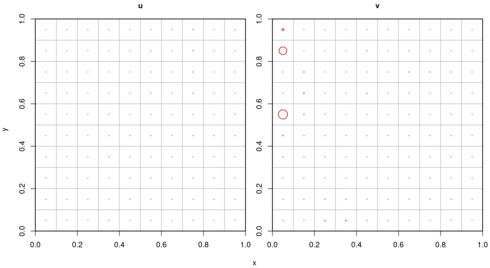

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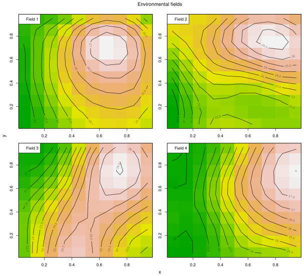
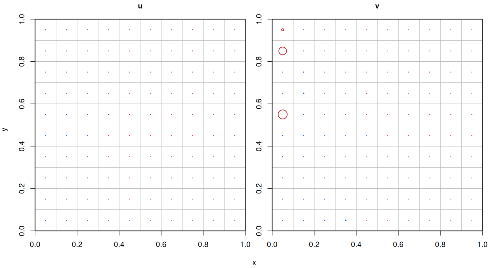
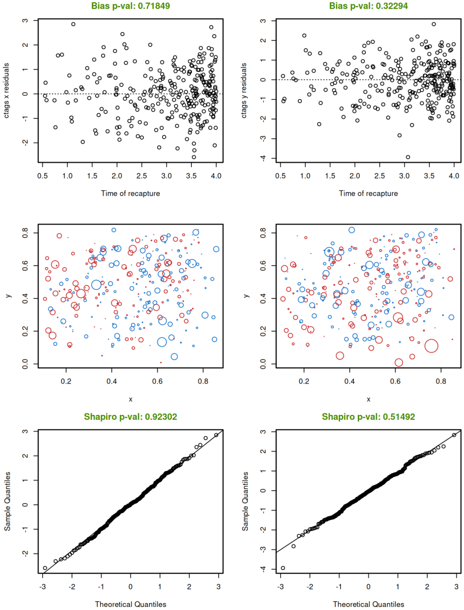
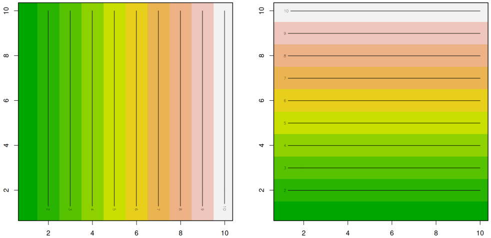


# Using x&y-coordinates fields (KF)



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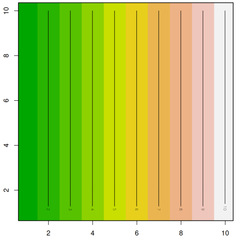


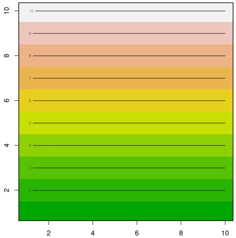


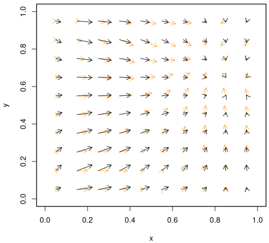

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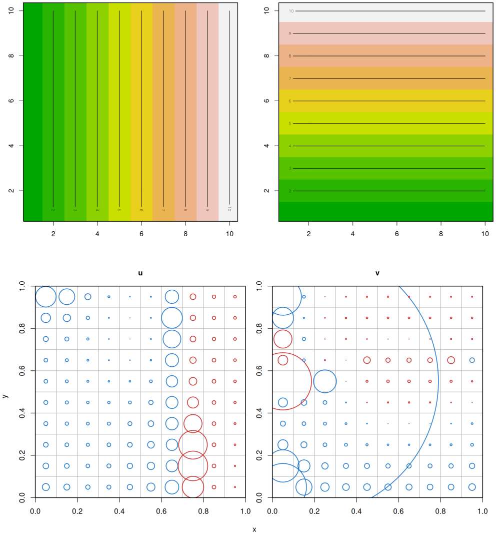

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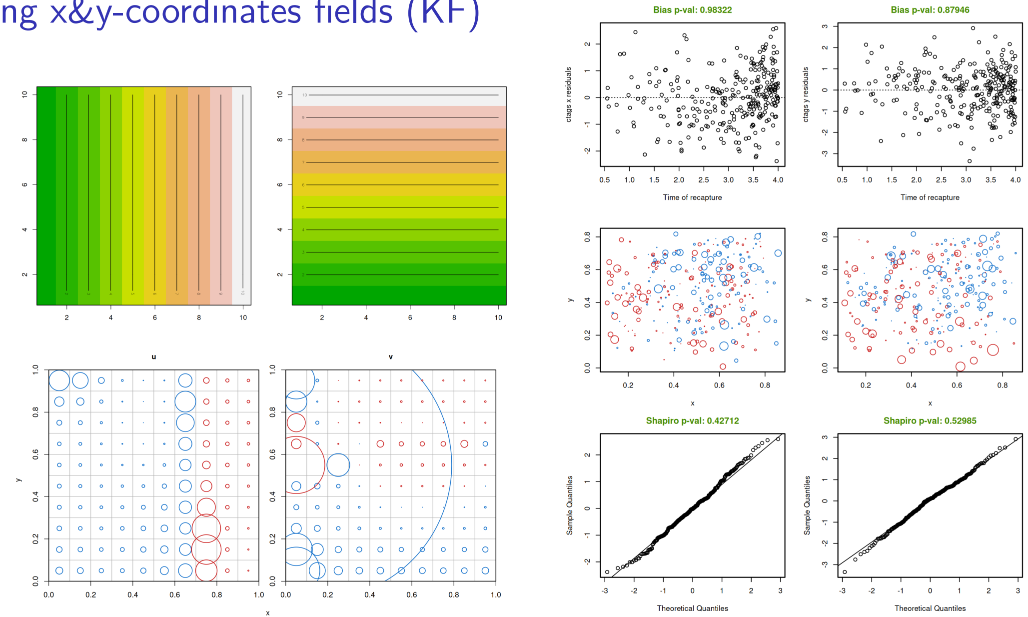

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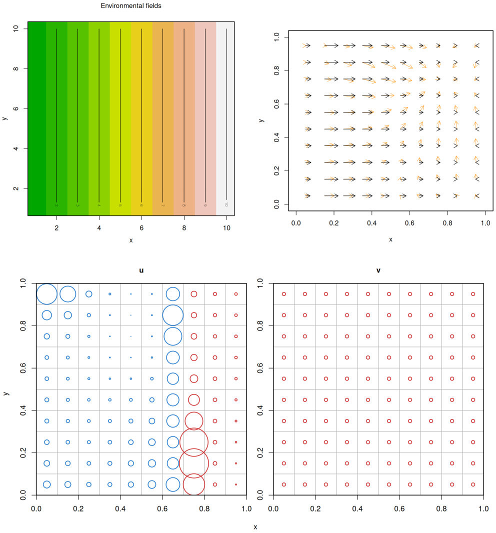
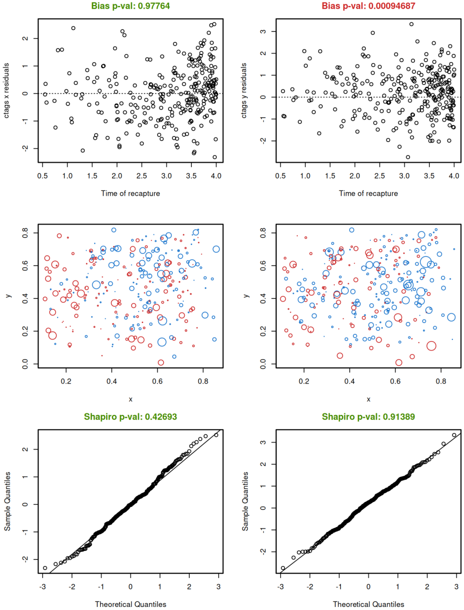
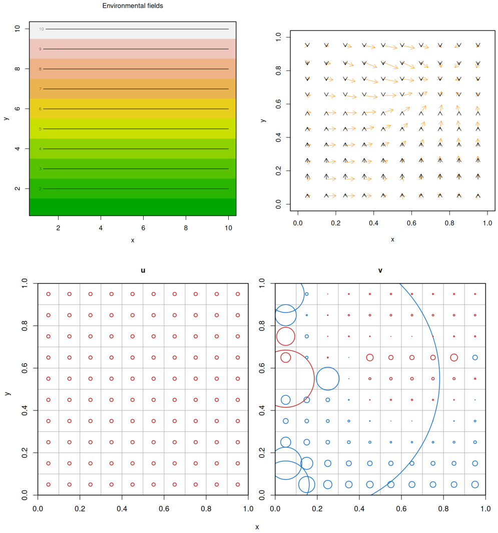
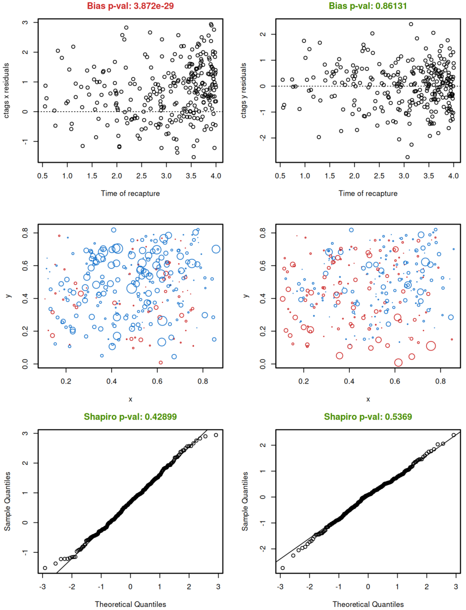


# Using y-coordinates fields (KF)



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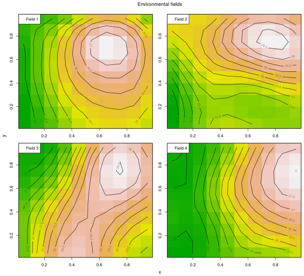
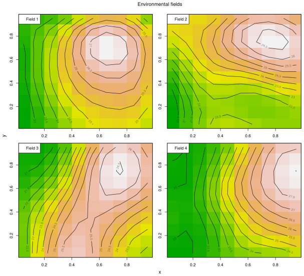
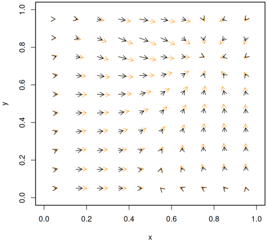


# Correct model (EXPM)



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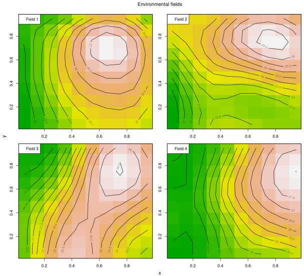


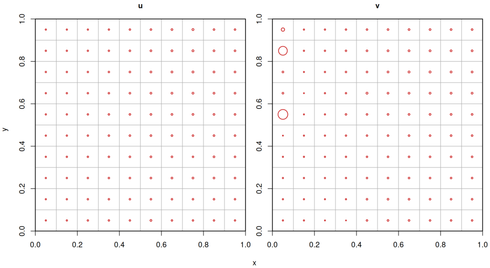

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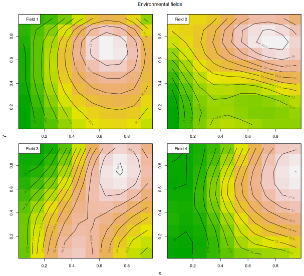


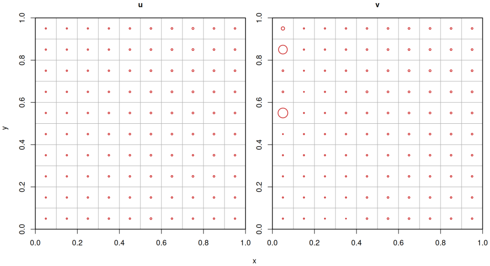


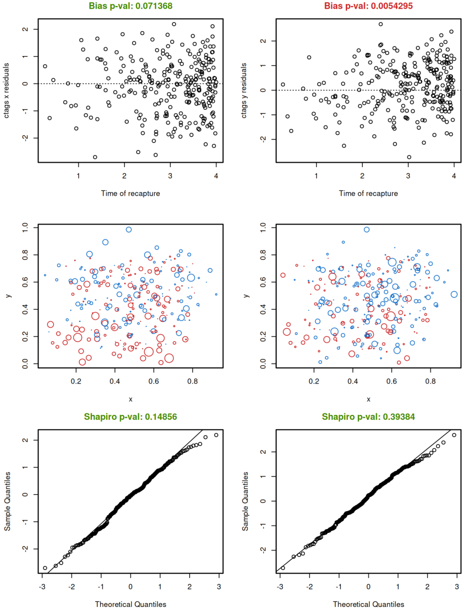

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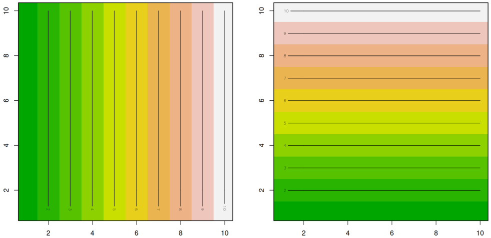

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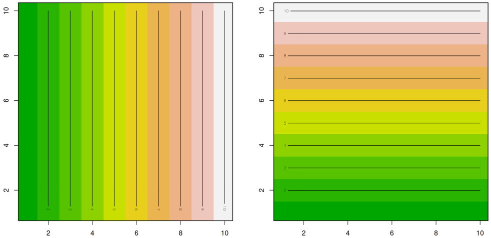

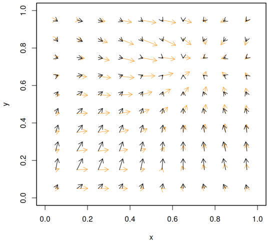

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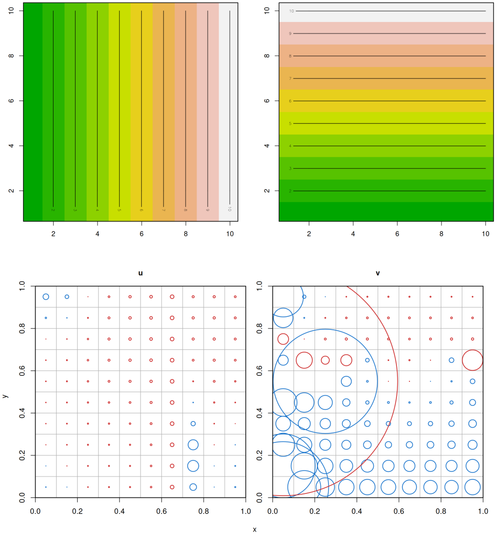

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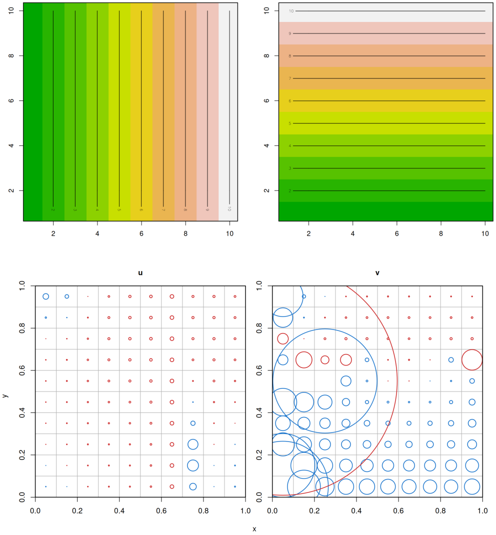


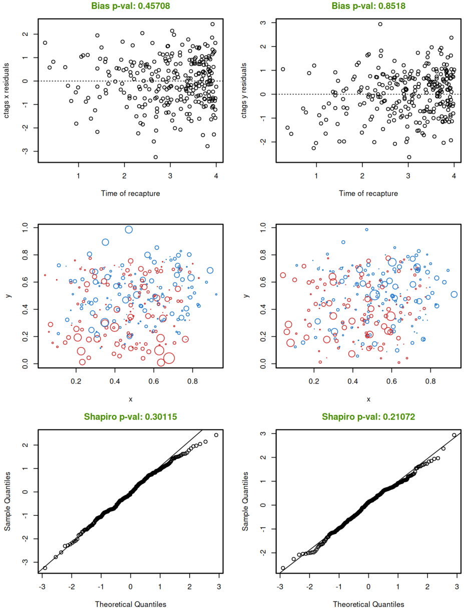

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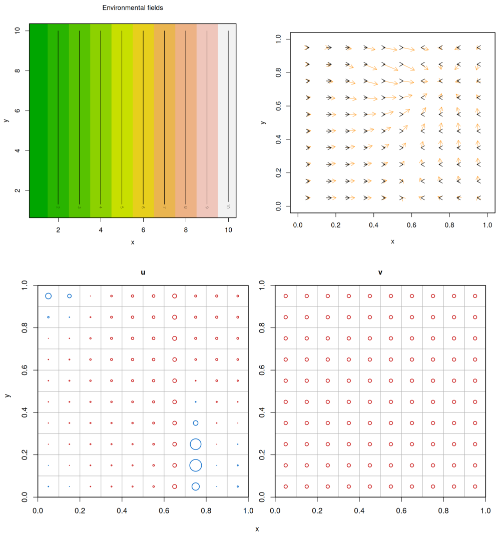


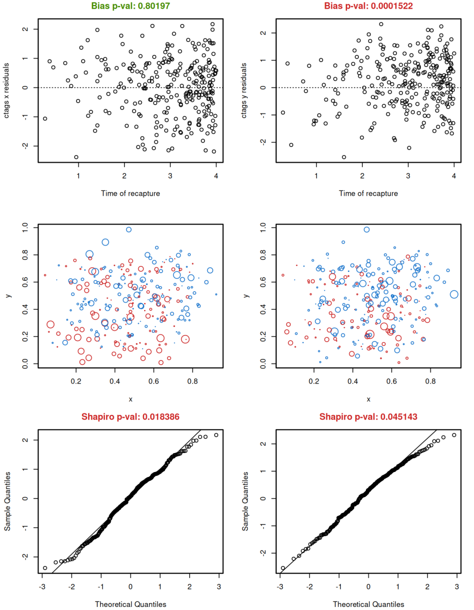

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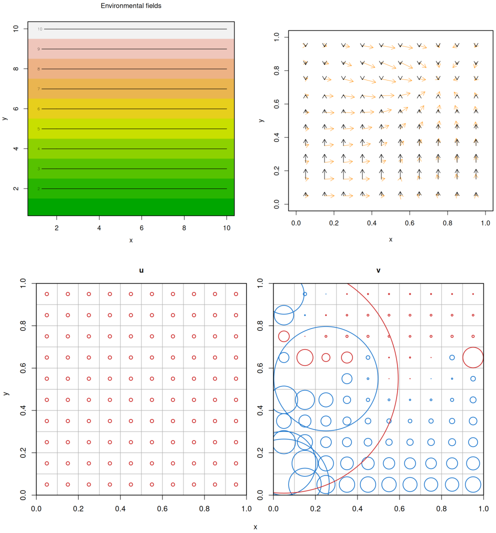


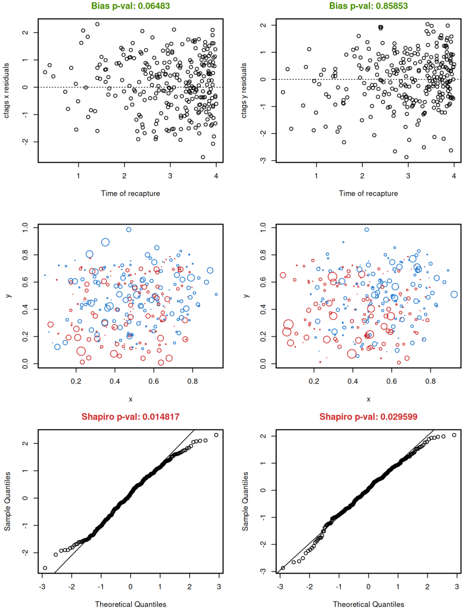

# Conclusion

- Work in progress - development & testing ongoing
- Possible to speed up expm?
- Implement flexible rate and gradient flexible fields
- Thanks to IATTC for funding this work

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