Biomass fluxes [1] in SEAPODYM

Let $B_{a,t,x,y}$ be the biomass density (in mt/km^2) of tuna population at age $a = 1, ..., A+$, time t and grid cell (x, y). Let's consider two regions, r_1 and r_2. For a given age a, we denote

$$\beta_{r_1,r_2} = \sum_{i,j \in r_2} B_{t+\Delta t,x,y} \cdot \mathrm{A}_{xy}, \text{ when } B_{t,x,y} \; \forall x, y \in r_1 \text{ at time } t, \tag{1}$$

where A_{xy} is the grid cell surface area in km^2. Hence, we say β_{r_1,r_2} is the total regional biomass moving from region r_1 to region r_2 during time period $(t, t + \Delta t)$.

For a given age a and n regions we have $n \times n$ matrix with all elements corresponding to the same time $t + \Delta t$:

$$\mathbf{F}_{t,a} = \begin{pmatrix} \beta_{1,1} & \beta_{1,2} & \cdots & \beta_{1,n} \\ \beta_{2,1} & \beta_{2,2} & \cdots & \beta_{2,n} \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{n,1} & \beta_{n,2} & \cdots & \beta_{n,n} \end{pmatrix}, \tag{2}$$

where the $\sum_j \beta_{i,j}$ is the total biomass that was in region i before the movement occurred, i.e., at time t, and the $\sum_i \beta_{i,j}$ is the biomass in region j after the movement occurred, i.e., at time $t + \Delta t$. In other words,

$\sum_{j!=i} \beta_{i,j}$ – **outgoing biomass** from region i to other regions,

$\sum_{i!=j} \beta_{i,j}$ – **incoming biomass** to region j from other regions, and

β_{ii} – the **resident biomass** that stayed in the region i during the time period $(t, t + \Delta t)$.

As seen from above, the elements of matrix $\mathbf{F}_{t,a}$ have units of mass flow rate, i.e., mt/Δt. In current SEAPODYM implementation, Δt is set to 3-months period, hence β_{ij} units are mt/qtr.

A simulation with `seapodym_fluxes` application computes $n_a \times n_t$ of $n \times n$ matrices, written in n_a ASCII files and named `spname_FluxesRegion_age[a-1].txt`.

Converting F to movement probabilities for Multifan-CL

To convert the regional biomass fluxes to probabilities, we first convert the biomass to the number of individuals, which is necessary for the aggregation of several monthly age classes into a coarser Multifan-CL age structure. Simply using the mean weight-at-age $w(a)$ for each age class with mean age a, the total number of individuals moving from region i to j is $\nu_{a,i,j} = \frac{\beta_{a,i,j}}{w(a)}$. Then the probability $p_{i,j}$ to move from region i to region j is computed as

$$p_{i,j} = \frac{\nu_{i,j}}{\sum_j \nu_{i,j}}, \tag{3}$$

where the sum in the denominator is the total number of individuals in region i before the movement occurred (see above).

Finally, to match the Multifan-CL format, the movement matrices that are provided to the Multifan-CL model are the transpose of SEAPODYM-derived $\mathbf{P}_{t,a}$ matrices with elements (3) such as:

$$\mathbf{P}^{\mathrm{T}}{}_{t,a} = \begin{pmatrix} p_{1,1} & p_{2,1} & \cdots & p_{n,1} \\ p_{1,2} & p_{2,2} & \cdots & p_{n,2} \\ \vdots & \vdots & \vdots & \vdots \\ p_{1,n} & p_{2,n} & \cdots & p_{n,n} \end{pmatrix}, \tag{4}$$

so the sum over each column of $\mathbf{P}^{\mathrm{T}}{}_{t,a}$ is $\sum_j p_{i,j} = 1$.

Note that aggregation to Multifan-CL age classes as well as seasons is done before computing probabilities (3).

[1] Here in the sense of the mass flow rate