8

PID Controllers and Modified PID Controllers

8–1 INTRODUCTION

In previous chapters, we occasionally discussed the basic PID controllers. For example, we presented electronic, hydraulic, and pneumatic PID controllers. We also designed control systems where PID controllers were involved.

It is interesting to note that more than half of the industrial controllers in use today are PID controllers or modified PID controllers.

Because most PID controllers are adjusted on-site, many different types of tuning rules have been proposed in the literature. Using these tuning rules, delicate and fine tuning of PID controllers can be made on-site. Also, automatic tuning methods have been developed and some of the PID controllers may possess on-line automatic tuning capabilities. Modified forms of PID control, such as I-PD control and multi-degrees-of-freedom PID control, are currently in use in industry. Many practical methods for bumpless switching (from manual operation to automatic operation) and gain scheduling are commercially available.

The usefulness of PID controls lies in their general applicability to most control systems. In particular, when the mathematical model of the plant is not known and therefore analytical design methods cannot be used, PID controls prove to be most useful. In the field of process control systems, it is well known that the basic and modified PID control schemes have proved their usefulness in providing satisfactory control, although in many given situations they may not provide optimal control.

In this chapter we first present the design of a PID controlled system using Ziegler and Nichols tuning rules. We next discuss a design of PID controller with the conventional frequency-response approach, followed by the computational optimization approach to design PID controllers. Then we introduce modified PID controls such as PI-D control and I-PD control. Then we introduce multi-degrees-of-freedom control systems, which can satisfy conflicting requirements that single-degree-of-freedom control systems cannot. (For the definition of multi-degrees-of-freedom control systems, see Section 8–6.)

In practical cases, there may be one requirement on the response to disturbance input and another requirement on the response to reference input. Often these two requirements conflict with each other and cannot be satisfied in the single-degree-of-freedom case. By increasing the degrees of freedom, we are able to satisfy both. In this chapter we present two-degrees-of-freedom control systems in detail.

The computational optimization approach presented in this chapter to design control systems (such as to search optimal sets of parameter values to satisfy given transient response specifications) can be used to design both single-degree-of-freedom control systems and multi-degrees-of-freedom control systems, provided a fairly precice mathematical model of the plant is known.

Outline of the Chapter. Section 8–1 has presented introductory material for the chapter. Section 8–2 deals with a design of a PID controller with Ziegler–Nichols Rules. Section 8–3 treats a design of a PID controller with the frequency-response approach. Section 8–4 presents a computational optimization approach to obtain optimal parameter values of PID controllers. Section 8–5 discusses multi-degrees-of-freedom control systems including modified PID control systems.

8–2 ZIEGLER–NICHOLS RULES FOR TUNING PID CONTROLLERS

PID Control of Plants. Figure 8–1 shows a PID control of a plant. If a mathematical model of the plant can be derived, then it is possible to apply various design techniques for determining parameters of the controller that will meet the transient and steady-state specifications of the closed-loop system. However, if the plant is so complicated that its mathematical model cannot be easily obtained, then an analytical or computational approach to the design of a PID controller is not possible. Then we must resort to experimental approaches to the tuning of PID controllers.

The process of selecting the controller parameters to meet given performance specifications is known as controller tuning. Ziegler and Nichols suggested rules for tuning PID controllers (meaning to set values K_p, T_i, and T_d) based on experimental step responses or based on the value of K_p that results in marginal stability when only proportional control action is used. Ziegler–Nichols rules, which are briefly presented in the following, are useful when mathematical models of plants are not known. (These rules can, of course, be applied to the design of systems with known mathematical

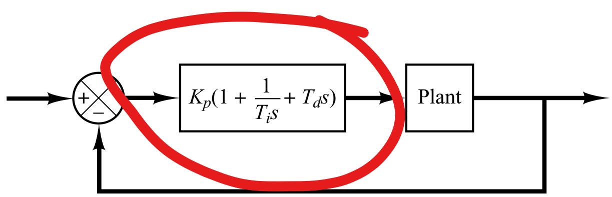


Figure 8–1 PID control of a plant.

models.) Such rules suggest a set of values of K_p, T_i, and T_d that will give a stable operation of the system. However, the resulting system may exhibit a large maximum overshoot in the step response, which is unacceptable. In such a case we need series of fine tunings until an acceptable result is obtained. In fact, the Ziegler–Nichols tuning rules give an educated guess for the parameter values and provide a starting point for fine tuning, rather than giving the final settings for K_p, T_i, and T_d in a single shot.

Ziegler–Nichols Rules for Tuning PID Controllers. Ziegler and Nichols proposed rules for determining values of the proportional gain K_p, integral time T_i, and derivative time T_d based on the transient response characteristics of a given plant. Such determination of the parameters of PID controllers or tuning of PID controllers can be made by engineers on-site by experiments on the plant. (Numerous tuning rules for PID controllers have been proposed since the Ziegler–Nichols proposal. They are available in the literature and from the manufacturers of such controllers.)

There are two methods called Ziegler–Nichols tuning rules: the first method and the second method. We shall give a brief presentation of these two methods.

First Method. In the first method, we obtain experimentally the response of the plant to a unit-step input, as shown in Figure 8–2. If the plant involves neither integrator(s) nor dominant complex-conjugate poles, then such a unit-step response curve may look S-shaped, as shown in Figure 8–3. This method applies if the response to a step input exhibits an S-shaped curve. Such step-response curves may be generated experimentally or from a dynamic simulation of the plant.

The S-shaped curve may be characterized by two constants, delay time L and time constant T. The delay time and time constant are determined by drawing a tangent line at the inflection point of the S-shaped curve and determining the intersections of the tangent line with the time axis and line $c(t) = K$, as shown in Figure 8–3. The transfer

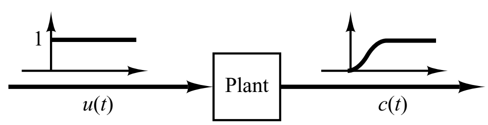


Figure 8–2 Unit-step response of a plant.

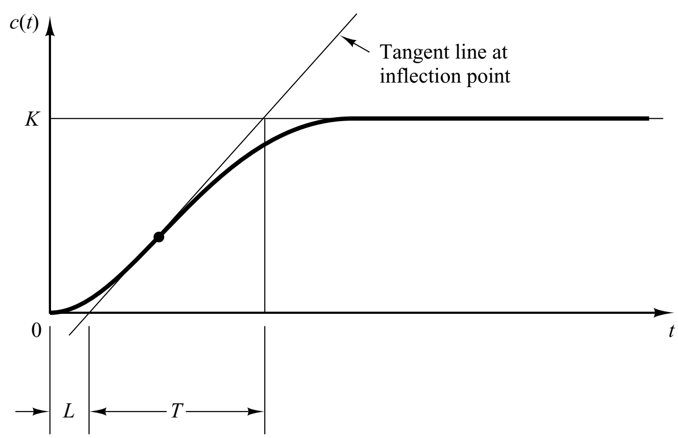


Figure 8–3 S-shaped response curve.

Table 8–1 Ziegler–Nichols Tuning Rule Based on Step Response of Plant (First Method)

Type of Controller	K_p	T_i	T_d
P	$\frac{T}{L}$	∞	0
PI	$0.9\frac{T}{L}$	$\frac{L}{0.3}$	0
PID	$1.2\frac{T}{L}$	$2L$	$0.5L$

function $C(s)/U(s)$ may then be approximated by a first-order system with a transport lag as follows:

$$\frac{C(s)}{U(s)} = \frac{Ke^{-Ls}}{Ts + 1}$$

Ziegler and Nichols suggested to set the values of K_p, T_i, and T_d according to the formula shown in Table 8–1.

Notice that the PID controller tuned by the first method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 1.2\frac{T}{L}\left(1 + \frac{1}{2Ls} + 0.5Ls\right) \\ &= 0.6T\frac{\left(s + \frac{1}{L}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -1/L$.

Second Method. In the second method, we first set $T_i = \infty$ and $T_d = 0$. Using the proportional control action only (see Figure 8–4), increase K_p from 0 to a critical value K_{cr} at which the output first exhibits sustained oscillations. (If the output does not exhibit sustained oscillations for whatever value K_p may take, then this method does not apply.) Thus, the critical gain K_{cr} and the corresponding period P_{cr} are experimentally

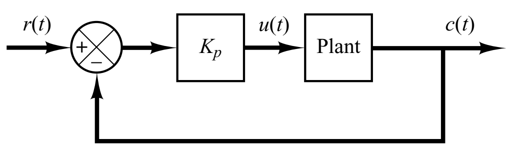


Figure 8–4 Closed-loop system with a proportional controller.

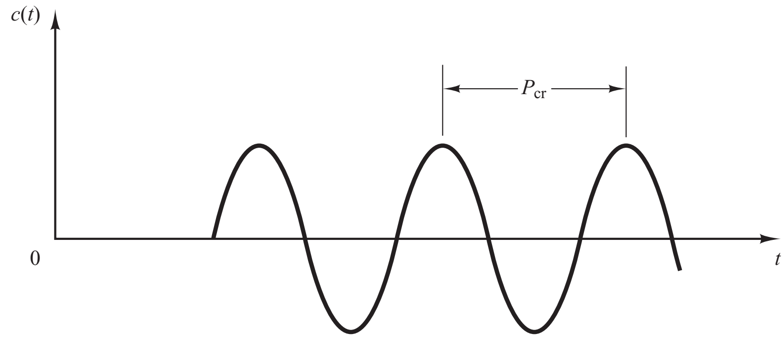


Figure 8–5 Sustained oscillation with period P_{cr}. (P_{cr} is measured in sec.)

determined (see Figure 8–5). Ziegler and Nichols suggested that we set the values of the parameters K_p, T_i, and T_d according to the formula shown in Table 8–2.

Table 8–2 Ziegler–Nichols Tuning Rule Based on Critical Gain K_{cr} and Critical Period P_{cr} (Second Method)

Type of Controller	K_p	T_i	T_d
P	$0.5K_{cr}$	∞	0
PI	$0.45K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6K_{cr}$	$0.5P_{cr}$	$0.125P_{cr}$

Notice that the PID controller tuned by the second method of Ziegler–Nichols rules gives

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 0.6K_{cr}\left(1 + \frac{1}{0.5P_{cr}s} + 0.125P_{cr}s\right) \\ &= 0.075K_{cr}P_{cr}\frac{\left(s + \dfrac{4}{P_{cr}}\right)^2}{s} \end{aligned}$$

Thus, the PID controller has a pole at the origin and double zeros at $s = -4/P_{cr}$.

Note that if the system has a known mathematical model (such as the transfer function), then we can use the root-locus method to find the critical gain K_{cr} and the frequency of the sustained oscillations ω_{cr}, where $2\pi/\omega_{cr} = P_{cr}$. These values can be found from the crossing points of the root-locus branches with the $j\omega$ axis. (Obviously, if the root-locus branches do not cross the $j\omega$ axis, this method does not apply.)

Comments. Ziegler–Nichols tuning rules (and other tuning rules presented in the literature) have been widely used to tune PID controllers in process control systems where the plant dynamics are not precisely known. Over many years, such tuning rules proved to be very useful. Ziegler–Nichols tuning rules can, of course, be applied to plants whose dynamics are known. (If the plant dynamics are known, many analytical and graphical approaches to the design of PID controllers are available, in addition to Ziegler–Nichols tuning rules.)

EXAMPLE 8–1 Consider the control system shown in Figure 8–6 in which a PID controller is used to control the system. The PID controller has the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

Although many analytical methods are available for the design of a PID controller for the present system, let us apply a Ziegler–Nichols tuning rule for the determination of the values of parameters K_p, T_i, and T_d. Then obtain a unit-step response curve and check to see if the designed system exhibits approximately 25% maximum overshoot. If the maximum overshoot is excessive (40% or more), make a fine tuning and reduce the amount of the maximum overshoot to approximately 25% or less.

Since the plant has an integrator, we use the second method of Ziegler–Nichols tuning rules. By setting $T_i = \infty$ and $T_d = 0$, we obtain the closed-loop transfer function as follows:

$$\frac{C(s)}{R(s)} = \frac{K_p}{s(s + 1)(s + 5) + K_p}$$

The value of K_p that makes the system marginally stable so that sustained oscillation occurs can be obtained by use of Routh's stability criterion. Since the characteristic equation for the closed-loop system is

$$s^3 + 6s^2 + 5s + K_p = 0$$

the Routh array becomes as follows:

$$\begin{array}{lcc} s^3 & 1 & 5 \\ s^2 & 6 & K_p \\ s^1 & \dfrac{30 - K_p}{6} & \\ s^0 & K_p & \end{array}$$

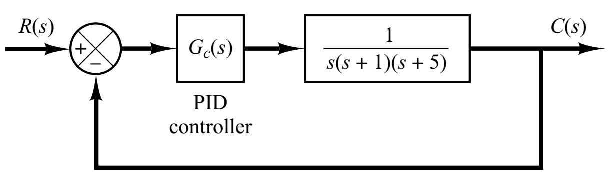


Figure 8–6 PID-controlled system.

Examining the coefficients of the first column of the Routh table, we find that sustained oscillation will occur if $K_p = 30$. Thus, the critical gain K_{cr} is

$$K_{cr} = 30$$

With gain K_p set equal to K_{cr} $(= 30)$, the characteristic equation becomes

$$s^3 + 6s^2 + 5s + 30 = 0$$

To find the frequency of the sustained oscillation, we substitute $s = j\omega$ into this characteristic equation as follows:

$$(j\omega)^3 + 6(j\omega)^2 + 5(j\omega) + 30 = 0$$

or

$$6(5 - \omega^2) + j\omega(5 - \omega^2) = 0$$

from which we find the frequency of the sustained oscillation to be $\omega^2 = 5$ or $\omega = \sqrt{5}$. Hence, the period of sustained oscillation is

$$P_{cr} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{5}} = 2.8099$$

Referring to Table 8–2, we determine K_p, T_i, and T_d as follows:

$$K_p = 0.6K_{cr} = 18$$
$$T_i = 0.5P_{cr} = 1.405$$
$$T_d = 0.125P_{cr} = 0.35124$$

The transfer function of the PID controller is thus

$$\begin{aligned} G_c(s) &= K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \\ &= 18\left(1 + \frac{1}{1.405s} + 0.35124s\right) \\ &= \frac{6.3223(s + 1.4235)^2}{s} \end{aligned}$$

The PID controller has a pole at the origin and double zero at $s = -1.4235$. A block diagram of the control system with the designed PID controller is shown in Figure 8–7.

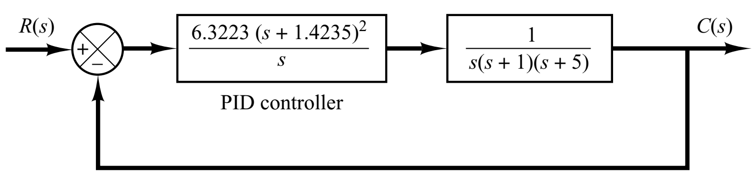


Figure 8–7 Block diagram of the system with PID controller designed by use of the Ziegler–Nichols tuning rule (second method).

Next, let us examine the unit-step response of the system. The closed-loop transfer function $C(s)/R(s)$ is given by

$$\frac{C(s)}{R(s)} = \frac{6.3223s^2 + 18s + 12.811}{s^4 + 6s^3 + 11.3223s^2 + 18s + 12.811}$$

The unit-step response of this system can be obtained easily with MATLAB. See MATLAB Program 8–1. The resulting unit-step response curve is shown in Figure 8–8. The maximum overshoot in the unit-step response is approximately 62%. The amount of maximum overshoot is excessive. It can be reduced by fine tuning the controller parameters. Such fine tuning can be made on the computer. We find that by keeping $K_p = 18$ and by moving the double zero of the PID controller to $s = -0.65$—that is, using the PID controller

$$G_c(s) = 18\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 13.846\frac{(s + 0.65)^2}{s} \tag{8–1}$$

the maximum overshoot in the unit-step response can be reduced to approximately 18% (see Figure 8–9). If the proportional gain K_p is increased to 39.42, without changing the location of the double zero ($s = -0.65$), that is, using the PID controller

$$G_c(s) = 39.42\left(1 + \frac{1}{3.077s} + 0.7692s\right) = 30.322\frac{(s + 0.65)^2}{s} \tag{8–2}$$

MATLAB Program 8–1

```
% ---------- Unit-step response ----------

num = [6.3223  18  12.811];
den = [1  6  11.3223  18  12.811];
step(num,den)
grid
title('Unit-Step Response')
```

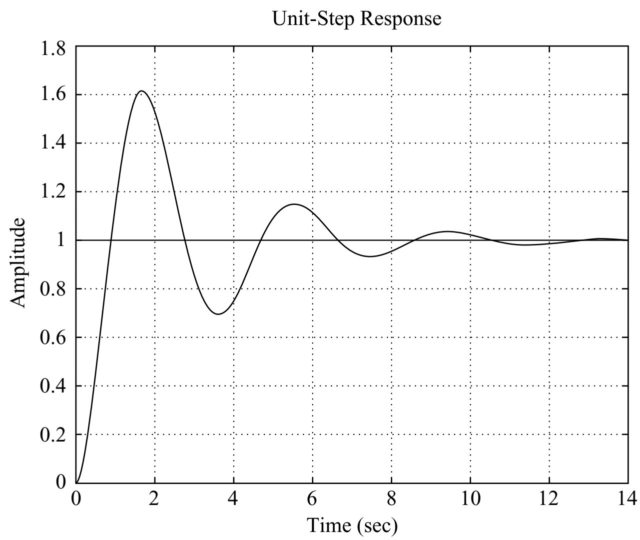


Figure 8–8
Unit-step response curve of PID-controlled system designed by use of the Ziegler–Nichols tuning rule (second method).

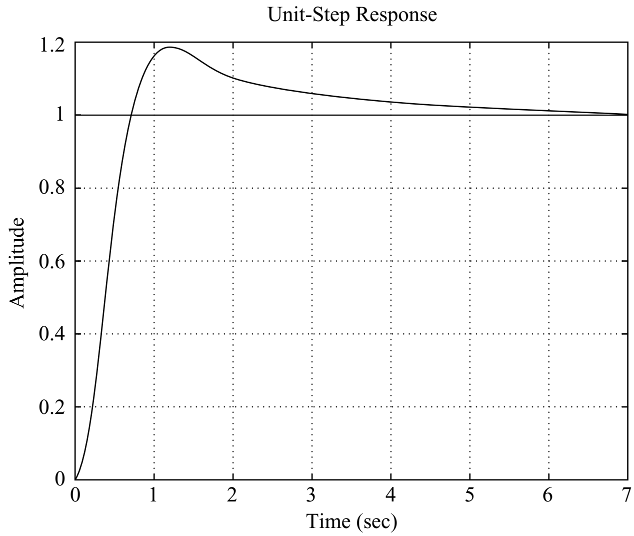


Figure 8–9
Unit-step response of the system shown in Figure 8–6 with PID controller having parameters $K_p = 18$, $T_i = 3.077$, and $T_d = 0.7692$.

then the speed of response is increased, but the maximum overshoot is also increased to approximately 28%, as shown in Figure 8–10. Since the maximum overshoot in this case is fairly close to 25% and the response is faster than the system with $G_c(s)$ given by Equation (8–1), we may consider $G_c(s)$ as given by Equation (8–2) as acceptable. Then the tuned values of K_p, T_i, and T_d become

$$K_p = 39.42, \qquad T_i = 3.077, \qquad T_d = 0.7692$$

It is interesting to observe that these values respectively are approximately twice the values suggested by the second method of the Ziegler–Nichols tuning rule. The important thing to note here is that the Ziegler–Nichols tuning rule has provided a starting point for fine tuning.

It is instructive to note that, for the case where the double zero is located at $s = -1.4235$, increasing the value of K_p increases the speed of response, but as far as the percentage maximum overshoot is concerned, varying gain K_p has very little effect. The reason for this may be seen from

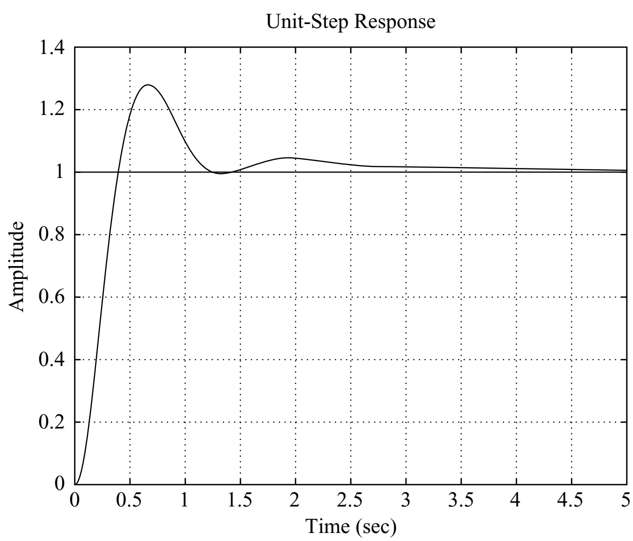


Figure 8–10
Unit-step response of the system shown in Figure 8–6 with PID controller having parameters $K_p = 39.42$, $T_i = 3.077$, and $T_d = 0.7692$.

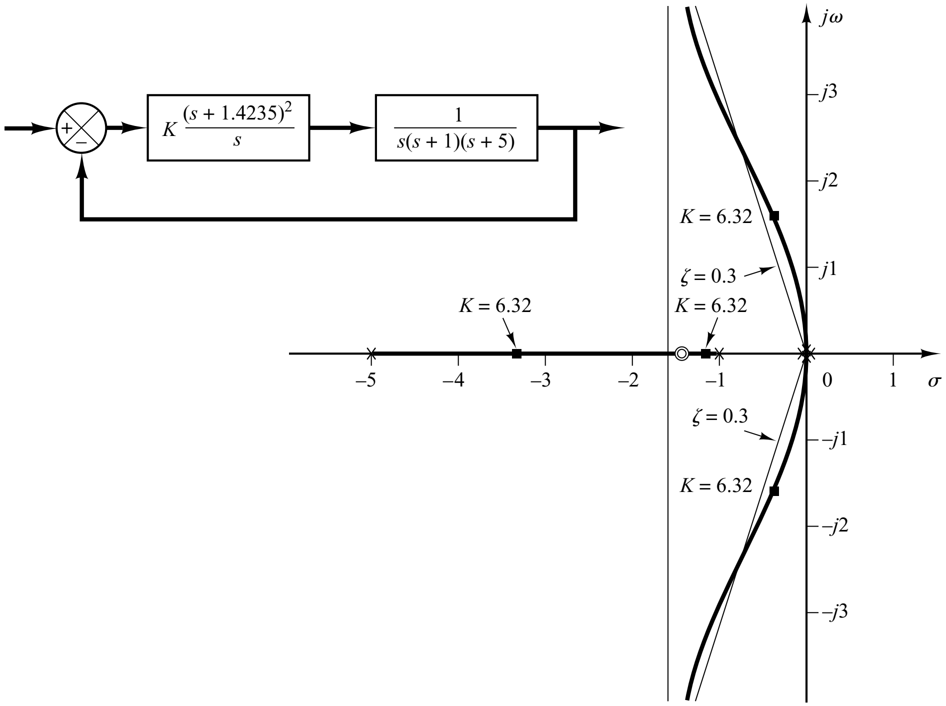


Figure 8–11 Root-locus diagram of system when PID controller has double zero at $s = -1.4235$.

the root-locus analysis. Figure 8–11 shows the root-locus diagram for the system designed by use of the second method of Ziegler–Nichols tuning rules. Since the dominant branches of root loci are along the $\zeta = 0.3$ lines for a considerable range of K, varying the value of K (from 6 to 30) will not change the damping ratio of the dominant closed-loop poles very much. However, varying the location of the double zero has a significant effect on the maximum overshoot, because the damping ratio of the dominant closed-loop poles can be changed significantly. This can also be seen from the root-locus analysis. Figure 8–12 shows the root-locus diagram for the system where the PID controller has the double zero at $s = -0.65$. Notice the change of the root-locus configuration. This change in the configuration makes it possible to change the damping ratio of the dominant closed-loop poles.

In Figure 8–12, notice that, in the case where the system has gain $K = 30.322$, the closed-loop poles at $s = -2.35 \pm j4.82$ act as dominant poles. Two additional closed-loop poles are very near the double zero at $s = -0.65$, with the result that these closed-loop poles and the double zero almost cancel each other. The dominant pair of closed-loop poles indeed determines the nature of the response. On the other hand, when the system has $K = 13.846$, the closed-loop poles at $s = -2.35 \pm j2.62$ are not quite dominant because the two other closed-loop poles near the double zero at $s = -0.65$ have considerable effect on the response. The maximum overshoot in the step response in this case (18%) is much larger than the case where the system is of second order and having only dominant closed-loop poles. (In the latter case the maximum overshoot in the step response would be approximately 6%.)

It is possible to make a third, a fourth, and still further trials to obtain a better response. But this will take a lot of computations and time. If more trials are desired, it is desirable to use the computational approach presented in Section 10–3. Problem **A–8–12** solves this problem with the computational approach with MATLAB. It finds sets of parameter values that will yield the maximum overshoot of 10% or less and the settling time of 3 sec or less. A solution to the present problem obtained in Problem **A–8–12** is that for the PID controller defined by

$$G_c(s) = K\,\frac{(s + a)^2}{s}$$

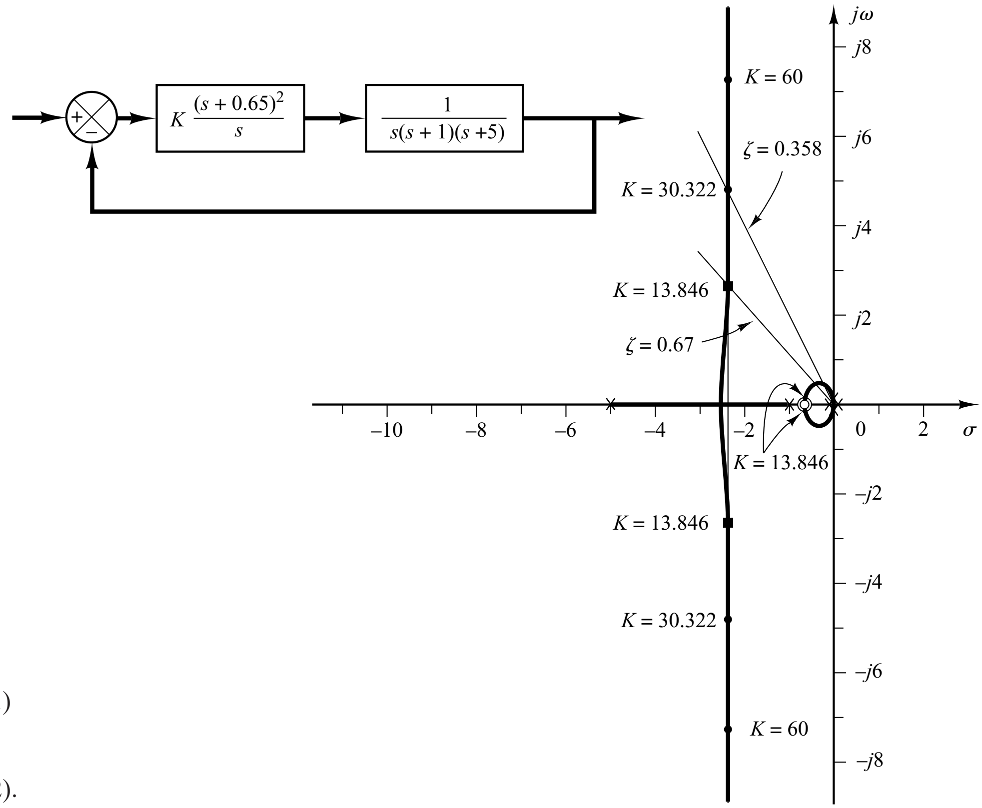


Figure 8–12
Root-locus diagram of system when PID controller has double zero at $s = -0.65$. $K = 13.846$ corresponds to $G_c(s)$ given by Equation (8–1) and $K = 30.322$ corresponds to $G_c(s)$ given by Equation (8–2).

the values of K and a are

$$K = 29, \qquad a = 0.25$$

with the maximum overshoot equal to 9.52% and settling time equal to 1.78 sec. Another possible solution obtained there is that

$$K = 27, \qquad a = 0.2$$

with the 5.5% maximum overshoot and 2.89 sec of settling time. See Problem **A–8–12** for details.

8–3 DESIGN OF PID CONTROLLERS WITH FREQUENCY-RESPONSE APPROACH

In this section we present a design of a PID controller based on the frequency-response approach.

Consider the system shown in Figure 8–13. Using a frequency-response approach, design a PID controller such that the static velocity error constant is 4 $\sec^{-1}$, phase margin is 50° or more, and gain margin is 10 dB or more. Obtain the unit-step and unit-ramp response curves of the PID controlled system with MATLAB.

Let us choose the PID controller to be

$$G_c(s) = \frac{K(as + 1)(bs + 1)}{s}$$

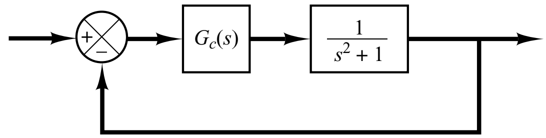


Figure 8–13 Control system.

Since the static velocity error constant K_v is specified as 4 sec^{-1}, we have

$$K_v = \lim_{s\to 0} sG_c(s)\frac{1}{s^2+1} = \lim_{s\to 0} s\frac{K(as+1)(bs+1)}{s}\frac{1}{s^2+1}$$
$$= K = 4$$

Thus

$$G_c(s) = \frac{4(as+1)(bs+1)}{s}$$

Next, we plot a Bode diagram of

$$G(s) = \frac{4}{s(s^2+1)}$$

MATLAB Program 8–2 produces a Bode diagram of $G(s)$. The resulting Bode diagram is shown in Figure 8–14.

MATLAB Program 8–2

```
num = [4];
den = [1  0.00000000001  1  0];
w = logspace(-1,1,200);
bode(num,den,w)
title('Bode Diagram of 4/[s(s^2+1)]')
```

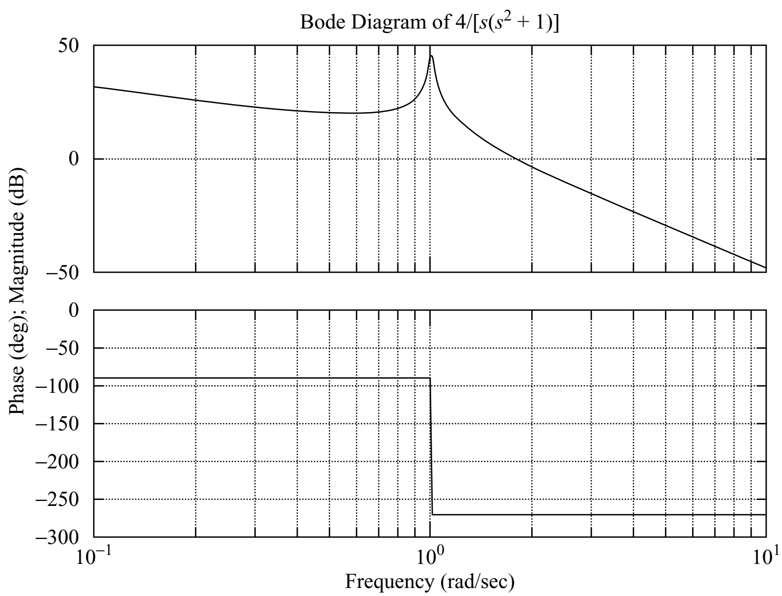


Figure 8–14 Bode diagram of $4/[s(s^2+1)]$.

We need the phase margin of at least 50° and gain margin of 10 dB or more. From the Bode diagram of Figure 8–14, we notice that the gain crossover frequency is approximately $\omega = 1.8$ rad/sec. Let us assume the gain crossover frequency of the compensated system to be somewhere between $\omega = 1$ and $\omega = 10$ rad/sec. Noting that

$$G_c(s) = \frac{4(as + 1)(bs + 1)}{s}$$

we choose $a = 5$. Then, $(as + 1)$ will contribute up to 90° phase lead in the high-frequency region. MATLAB Program 8–3 produces the Bode diagram of

$$\frac{4(5s + 1)}{s(s^2 + 1)}$$

The resulting Bode diagram is shown in Figure 8–15.

MATLAB Program 8–3

```
num = [20  4];
den = [1 0.00000000001  1  0];
w = logspace(-2,1,101);
bode(num,den,w)
title('Bode Diagram of G(s) = 4(5s+1)/[s(s^2+1)]')
```

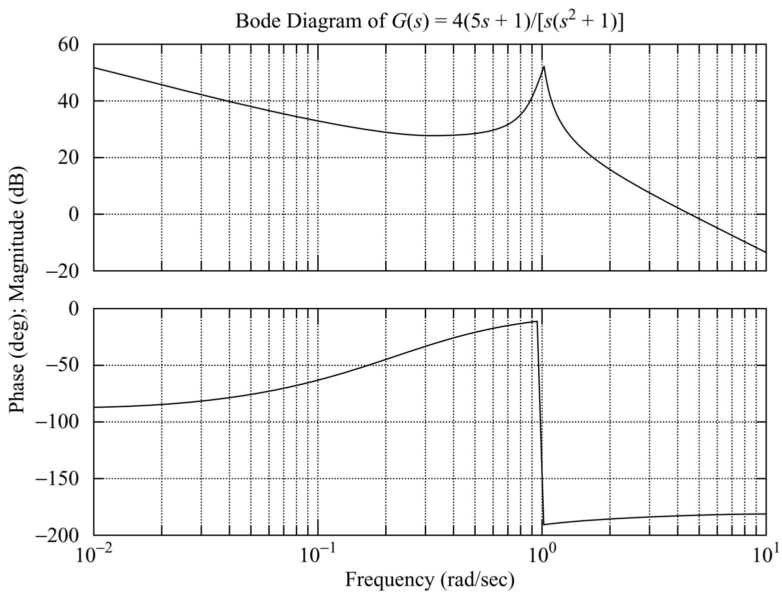


Figure 8–15 Bode diagram of $G(s) = 4(5s + 1)/[s(s^2 + 1)]$.

Based on the Bode diagram of Figure 8–15, we choose the value of b. The term $(bs + 1)$ needs to give the phase margin of at least 50°. By simple MATLAB trials, we find $b = 0.25$ to give the phase margin of at least 50° and gain margin of $+\infty$ dB. Therefore, by choosing $b = 0.25$, we have

$$G_c(s) = \frac{4(5s + 1)(0.25s + 1)}{s}$$

and the open-loop transfer function of the designed system becomes

$$\text{Open-loop transfer function} = \frac{4(5s + 1)(0.25s + 1)}{s}\frac{1}{s^2 + 1}$$

$$= \frac{5s^2 + 21s + 4}{s^3 + s}$$

MATLAB Program 8–4 produces the Bode diagram of the open-loop transfer function. The resulting Bode diagram is shown in Figure 8–16. From it we see that the static velocity error constant is 4 $\sec^{-1}$, the phase margin is 55°, and the gain margin is $+\infty$ dB.

MATLAB Program 8–4

```
num = [5  21  4];
den = [1  0  1  0];
w = logspace(-2,2,100);
bode(num,den,w)
title('Bode Diagram of 4(5s+1)(0.25s+1)/[s(s^2+1)]')
```

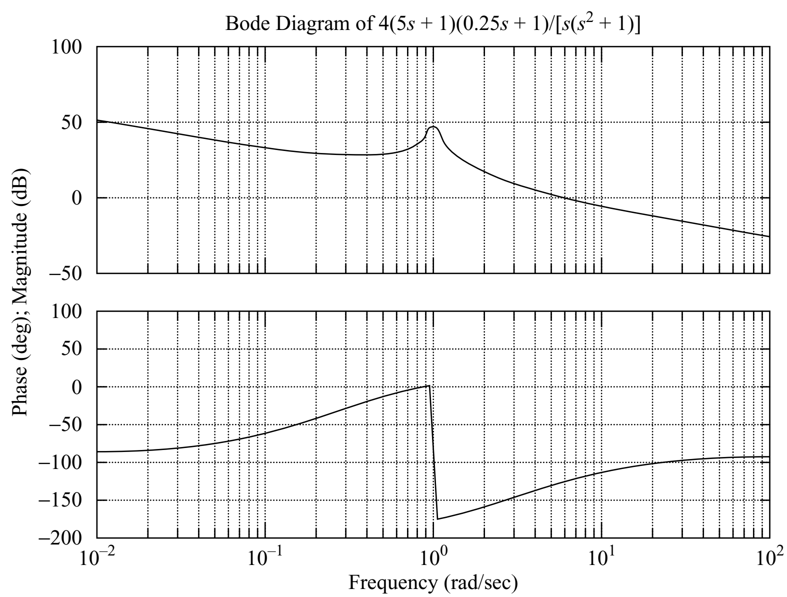


Figure 8–16
Bode diagram of $4(5s + 1)(0.25s + 1)/[s(s^2 + 1)]$.

Therefore, the designed system satisfies all the requirements. Thus, the designed system is acceptable. (Note that there exist infinitely many systems that satisfy all the requirements. The present system is just one of them.)

Next, we shall obtain the unit-step response and the unit-ramp response of the designed system. The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{5s^2 + 21s + 4}{s^3 + 5s^2 + 22s + 4}$$

Note that the closed-loop zeros are located at

$$s = -4, \qquad s = -0.2$$

The closed-loop poles are located at

$$s = -2.4052 + j3.9119$$
$$s = -2.4052 - j3.9119$$
$$s = -0.1897$$

Notice that the complex-conjugate closed-loop poles have the damping ratio of 0.5237. MATLAB Program 8–5 produces the unit-step response and the unit-ramp response.

MATLAB Program 8–5

```
%***** Unit-step response *****

num = [5  21  4];
den = [1  5  22  4];
t = 0:0.01:14;
c = step(num,den,t);
plot(t,c)
grid
title('Unit-Step Response of Compensated System')
xlabel('t (sec)')
ylabel('Output c(t)')

%***** Unit-ramp response *****

num1 = [5  21  4];
den1 = [1  5  22  4  0];
t = 0:0.02:20;
c = step(num1,den1,t);
plot(t,c,'-',t,t,'--')
title('Unit-Ramp Response of Compensated System')
xlabel('t (sec)')
ylabel('Unit-Ramp Input and Output c(t)')
text(10.8,8,'Compensated System')
```

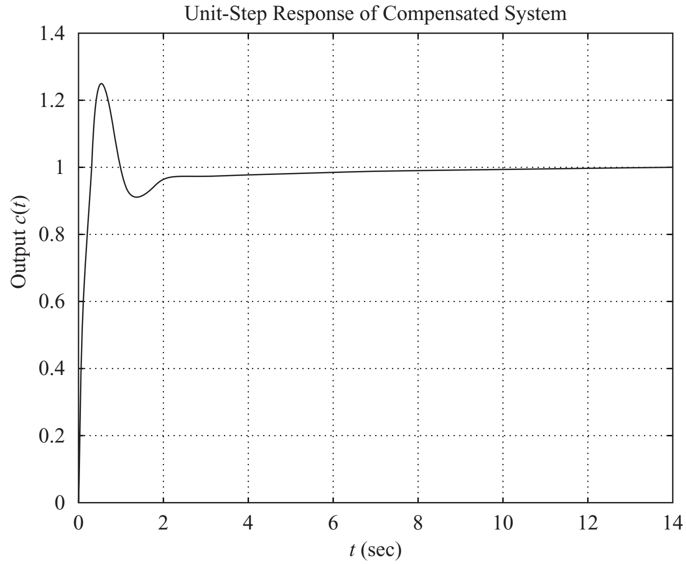


Figure 8–17
Unit-step response curve.

The resulting unit-step response curve is shown in Figure 8–17 and the unit-ramp response curve in Figure 8–18. Notice that the closed-loop pole at $s = -0.1897$ and the zero at $s = -0.2$ produce a long tail of small amplitude in the unit-step response.

For an additional example of design of a PID controller based on the frequency-response approach, see Problem **A–8–7**.

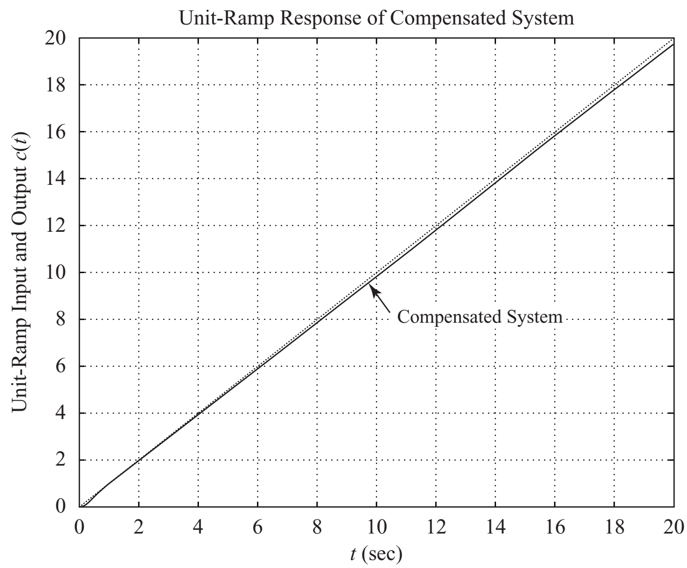


Figure 8–18
Unit-ramp input and the output curve.

8–4 DESIGN OF PID CONTROLLERS WITH COMPUTATIONAL OPTIMIZATION APPROACH

In this section we shall explore how to obtain an optimal set (or optimal sets) of parameter values of PID controllers to satisfy the transient response specifications by use of MATLAB. We shall present two examples to illustrate the approach in this section.

EXAMPLE 8–2 Consider the PID-controlled system shown in Figure 8–19. The PID controller is given by

$$G_c(s) = K\frac{(s+a)^2}{s}$$

It is desired to find a combination of K and a such that the closed-loop system will have 10% (or less) maximum overshoot in the unit-step response. (We will not include any other condition in this problem. But other conditions can easily be included, such as that the settling time be less than a specified value. See, for example, Example 8–3.)

There may be more than one set of parameters that satisfy the specifications. In this example, we shall obtain all sets of parameters that satisfy the given specifications.

To solve this problem with MATLAB, we first specify the region to search for appropriate K and a. We then write a MATLAB program that, in the unit-step response, will find a combination of K and a which will satisfy the criterion that the maximum overshoot is 10% or less.

Note that the gain K should not be too large, so as to avoid the possibility that the system require an unnecessarily large power unit.

Assume that the region to search for K and a is

$$2 \le K \le 3 \qquad \text{and} \qquad 0.5 \le a \le 1.5$$

If a solution does not exist in this region, then we need to expand it. In some problems, however, there is no solution, no matter what the search region might be.

In the computational approach, we need to determine the step size for each of K and a. In the actual design process, we need to choose step sizes small enough. However, in this example, to avoid an overly large number of computations, we choose the step sizes to be reasonable—say, 0.2 for both K and a.

To solve this problem it is possible to write many different MATLAB programs. We present here one such program, MATLAB Program 8–6. In this program, notice that we use two "for" loops. We start the program with the outer loop to vary the "K" values. Then we vary the "a" values in the inner loop. We proceed by writing the MATLAB program such that the nested loops in the program begin with the lowest values of "K" and "a" and step toward the highest. Note that, depending on the system and the ranges of search for "K" and "a" and the step sizes chosen, it may take from several seconds to a few minutes for MATLAB to compute the desired sets of the values.

In this program the statement

```
solution(k,:) = [K(i)  a(j)  m]
```

will produce a table of K, a, m values. (In the present system there are 15 sets of K and a that will exhibit $m < 1.10$—that is, the maximum overshoot is less than 10%.)

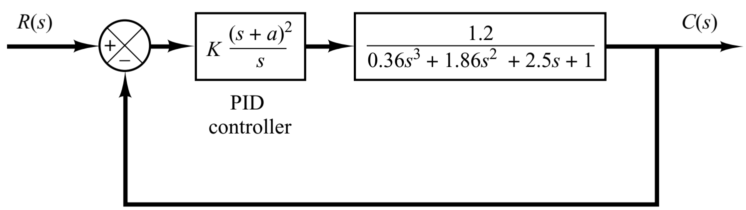


Figure 8–19 PID-controlled system.

To sort out the solution sets in the order of the magnitude of the maximum overshoot (starting from the smallest value of m and ending at the largest value of m in the table), we use the command

sortsolution = sortrows(solution,3)

MATLAB Program 8–6

```
%'K' and 'a' values to test

K = [2.0  2.2  2.4  2.6  2.8  3.0];
a = [0.5  0.7  0.9  1.1  1.3  1.5];

% Evaluate closed-loop unit-step response at each 'K' and 'a' combination
% that will yield the maximum overshoot less than 10%

t = 0:0.01:5;
g = tf([1.2],[0.36  1.86  2.5  1]);
k = 0;
for i = 1:6;
   for j = 1:6;
      gc = tf(K(i)*[1  2*a(j)  a(j)^2], [1  0]);  % controller
         G = gc*g/(1 + gc*g);  % closed-loop transfer function
         y = step(G,t);
         m = max(y);
         if m < 1.10
         k = k+1;
         solution(k,:) = [K(i)  a(j)  m];
         end
      end
   end
solution  % Print solution table

solution =

   2.0000  0.5000  0.9002
   2.0000  0.7000  0.9807
   2.0000  0.9000  1.0614
   2.2000  0.5000  0.9114
   2.2000  0.7000  0.9837
   2.2000  0.9000  1.0772
   2.4000  0.5000  0.9207
   2.4000  0.7000  0.9859
   2.4000  0.9000  1.0923
   2.6000  0.5000  0.9283
   2.6000  0.7000  0.9877
   2.8000  0.5000  0.9348
   2.8000  0.7000  1.0024
   3.0000  0.5000  0.9402
   3.0000  0.7000  1.0177
sortsolution = sortrows(solution,3)  % Print solution table sorted by
                                     % column 3
```

(*continues on next page*)

```
sortsolution =

   2.0000  0.5000  0.9002
   2.2000  0.5000  0.9114
   2.4000  0.5000  0.9207
   2.6000  0.5000  0.9283
   2.8000  0.5000  0.9348
   3.0000  0.5000  0.9402
   2.0000  0.7000  0.9807
   2.2000  0.7000  0.9837
   2.4000  0.7000  0.9859
   2.6000  0.7000  0.9877
   2.8000  0.7000  1.0024
   3.0000  0.7000  1.0177
   2.0000  0.9000  1.0614
   2.2000  0.9000  1.0772
   2.4000  0.9000  1.0923

% Plot the response with the largest overshoot that is less than 10%

K = sortsolution(k,1)

K =

   2.4000

a = sortsolution(k,2)

a =

   0.9000

gc = tf(K*[1  2*a  a^2], [1  0]);
G = gc*g/(1 + gc*g);
step(G,t)
grid  % See Figure 8–20

% If you wish to plot the response with the smallest overshoot that is
% greater than 0%, then enter the following values of 'K' and 'a'

K = sortsolution(11,1)

K =

   2.8000

a = sortsolution(11,2)

a =

   0.7000

gc = tf(K*[1  2*a  a^2], [1  0]);
G = gc*g/(1 + gc*g);
step(G,t)
grid  % See Figure 8–21
```

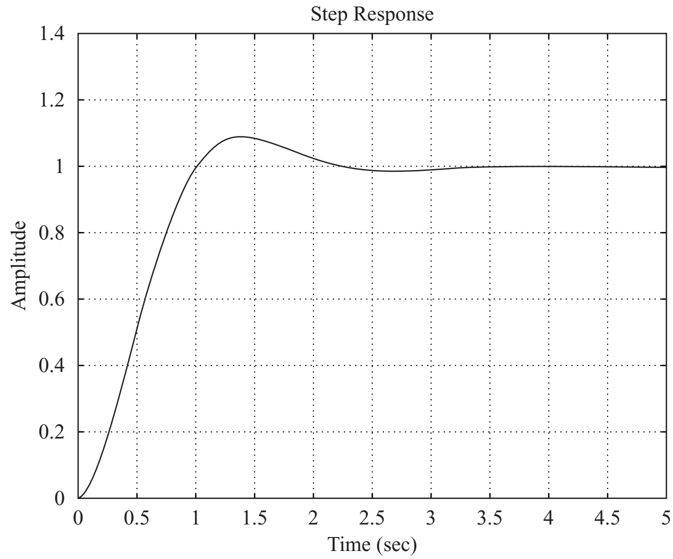


Figure 8–20 Unit-step response of the system with $K = 2.4$ and $a = 0.9$. (The maximum overshoot is 9.23%.)

To plot the unit-step response curve of the last set of the K and a values in the sorted table, we enter the commands

```
K = sortsolution (k,1)
a = sortsolution (k,2)
```

and use the step command. (The resulting unit-step response curve is shown in Figure 8–20.) To plot the unit-step response curve with the smallest overshoot that is greater than 0% found in the sorted table, enter the commands

```
K = sortsolution (11,1)
a = sortsolution (11,2)
```

and use the step command. (The resulting unit-step response curve is shown in Figure 8–21.)

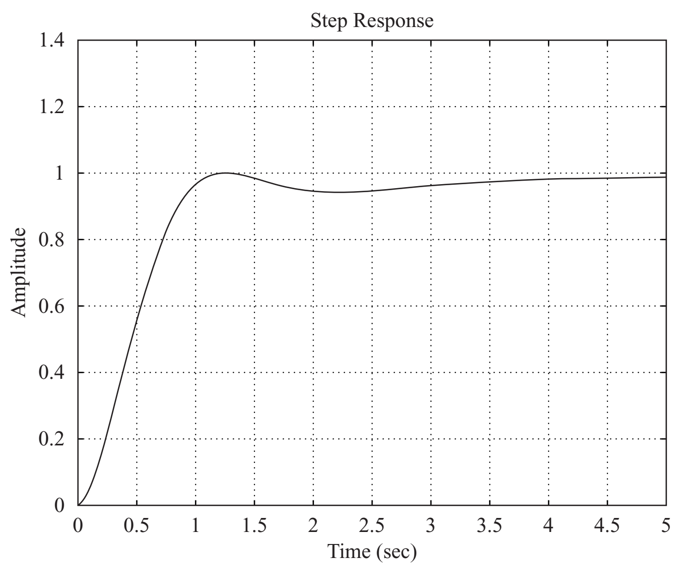


Figure 8–21 Unit-step response of the system with $K = 2.8$ and $a = 0.7$. (The maximum overshoot is 0.24%.)

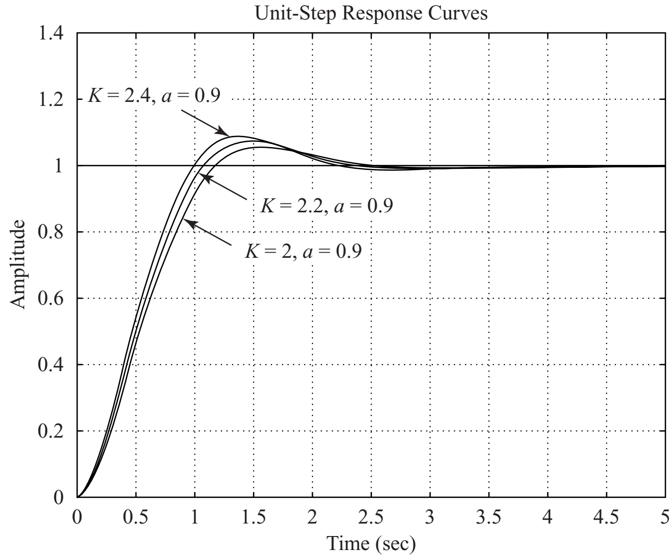


Figure 8–22 Unit-step response curves of system with $K = 2, a = 0.9$; $K = 2.2, a = 0.9$; and $K = 2.4$, $a = 0.9$.

To plot the unit-step response curve of the system with any set shown in the sorted table, we specify the K and a values by entering an appropriate sortsolution command.

Note that for a specification that the maximum overshoot be between 10% and 5%, there would be three sets of solutions:

$$K = 2.0000, \qquad a = 0.9000, \qquad m = 1.0614$$

$$K = 2.2000, \qquad a = 0.9000, \qquad m = 1.0772$$

$$K = 2.4000, \qquad a = 0.9000, \qquad m = 1.0923$$

Unit-step response curves for these three cases are shown in Figure 8–22. Notice that the system with a larger gain K has a smaller rise time and larger maximum overshoot. Which one of these three systems is best depends on the system's objective.

EXAMPLE 8–3 Consider the system shown in Figure 8–23. We want to find all combinations of K and a values such that the closed-loop system has a maximum overshoot of less than 15%, but more than 10%, in the unit-step response. In addition, the settling time should be less than 3 sec. In this problem, assume that the search region is

$$3 \le K \le 5 \quad \text{and} \quad 0.1 \le a \le 3$$

Determine the best choice of the parameters K and a.

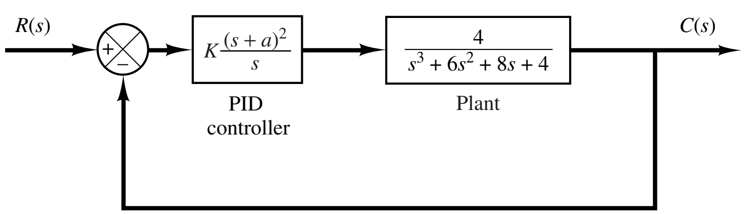


Figure 8–23 PID-controlled system with a simplified PID controller.

In this problem, we choose the step sizes to be reasonable, — say 0.2 for K and 0.1 for a. MATLAB Program 8–7 gives the solution to this problem. From the sortsolution table, it looks like the first row is a good choice. Figure 8–24 shows the unit step response curve for $K = 3.2$ and $a = 0.9$. Since this choice requires a smaller K value than most other choices, we may decide that the first row is the best choice.

MATLAB Program 8–7

```
t = 0:0.01:8;
k = 0;
for K = 3:0.2:5;
   for a = 0.1:0.1:3;
      num = [4*K  8*K*a  4*K*a^2];
      den = [1  6  8+4*K  4+8*K*a  4*K*a^2];
         y = step(num,den,t);
         s = 801;while y(s)>0.98 & y(s)<1.02; s = s – 1;end;
      ts = (s–1)*0.01; % ts = settling time;
      m = max(y);
      if m<1.15 & m>1.10; if ts<3.00;
         k = k+1;
         solution(k,:) = [K  a  m  ts];
        end
      end
    end
   end
   solution

solution =
      3.0000    1.0000    1.1469    2.7700
      3.2000    0.9000    1.1065    2.8300
      3.4000    0.9000    1.1181    2.7000
      3.6000    0.9000    1.1291    2.5800
      3.8000    0.9000    1.1396    2.4700
      4.0000    0.9000    1.1497    2.3800
      4.2000    0.8000    1.1107    2.8300
      4.4000    0.8000    1.1208    2.5900
      4.6000    0.8000    1.1304    2.4300
      4.8000    0.8000    1.1396    2.3100
      5.0000    0.8000    1.1485    2.2100
sortsolution = sortrows(solution,3)

sortsolution =

   3.2000    0.9000    1.1065    2.8300
   4.2000    0.8000    1.1107    2.8300
   3.4000    0.9000    1.1181    2.7000
   4.4000    0.8000    1.1208    2.5900
   3.6000    0.9000    1.1291    2.5800
   4.6000    0.8000    1.1304    2.4300
   4.8000    0.8000    1.1396    2.3100
   3.8000    0.9000    1.1396    2.4700
```

(*continues on next page*)

```
    3.0000    1.0000    1.1469    2.7700
    5.0000    0.8000    1.1485    2.2100
    4.0000    0.9000    1.1497    2.3800

    % Plot the response curve with the smallest overshoot shown in
sortsolution table.

    K = sortsolution(1,1), a = sortsolution(1,2)

K =

    3.2000

a =

    0.9000

    num = [4*K    8*K*a    4*K*a^2];
    den = [1    6    8+4*K    4+8*K*a    4*K*a^2];
    num

num =

    12.8000    23.0400    10.3680

    den

den =

    1.0000    6.0000    20.8000    27.0400    10.3680

    y = step(num,den,t);
    plot(t,y) % See Figure 8–24.
    grid
    title('Unit-Step Response')
    xlabel('t sec')
    ylabel('Output y(t)')
```

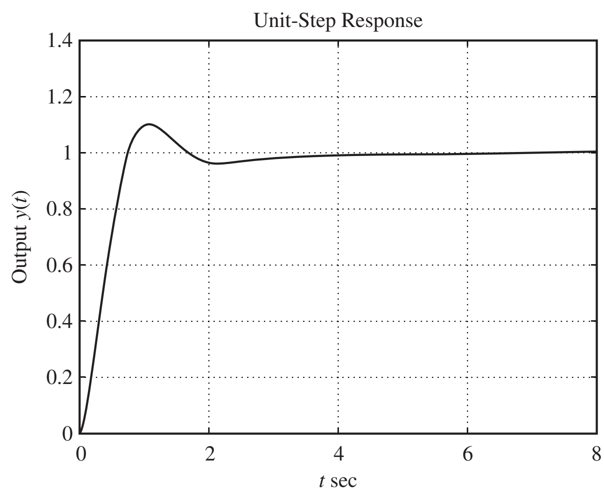


Figure 8–24 Unit-step response curve of the system with $K = 3.2$ and $a = 0.9$.

Consider the basic PID control system shown in Figure 8–25(a), where the system is subjected to disturbances and noises. Figure 8–25(b) is a modified block diagram of the same system. In the basic PID control system such as the one shown in Figure 8–25(b), if the reference input is a step function, then, because of the presence of the derivative term in the control action, the manipulated variable $u(t)$ will involve an impulse function (delta function). In an actual PID controller, instead of the pure derivative term $T_d s$, we employ

$$\frac{T_d s}{1 + \gamma T_d s}$$

where the value of γ is somewhere around 0.1. Therefore, when the reference input is a step function, the manipulated variable $u(t)$ will not involve an impulse function, but will involve a sharp pulse function. Such a phenomenon is called *set-point kick*.

PI-D Control. To avoid the set-point kick phenomenon, we may wish to operate the derivative action only in the feedback path so that differentiation occurs only on the feedback signal and not on the reference signal. The control scheme arranged in this way is called the PI-D control. Figure 8–26 shows a PI-D-controlled system.

From Figure 8–26, it can be seen that the manipulated signal $U(s)$ is given by

$$U(s) = K_p\left(1 + \frac{1}{T_i s}\right)R(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)B(s)$$

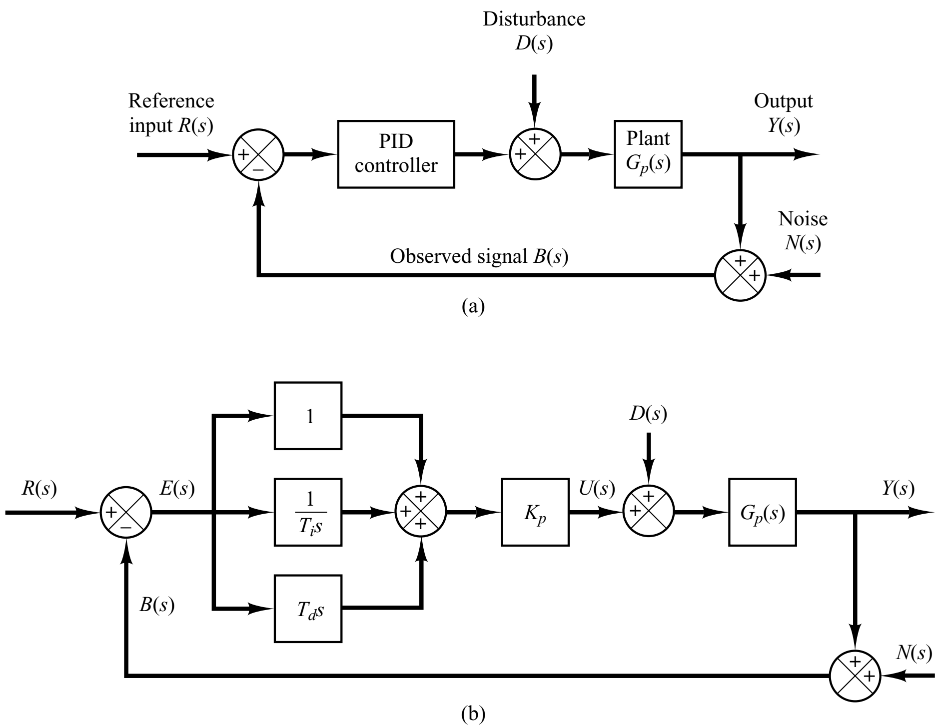


Figure 8–25
(a) PID-controlled system;
(b) equivalent block diagram.

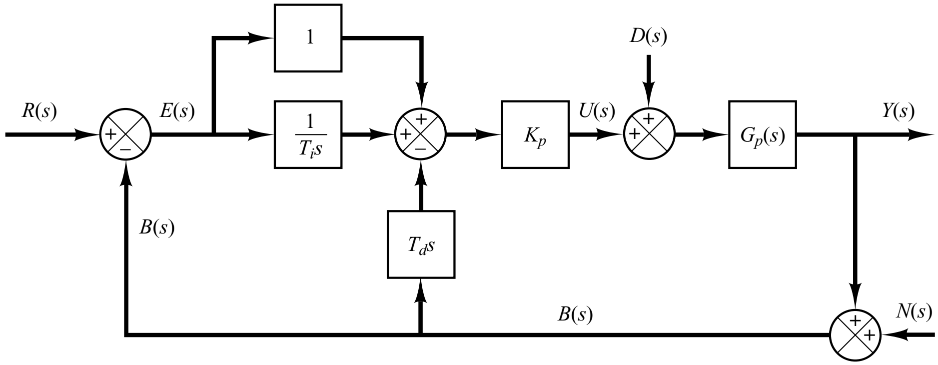


Figure 8–26
PI-D-controlled system.

Notice that in the absence of the disturbances and noises, the closed-loop transfer function of the basic PID control system [shown in Figure 8–25(b)] and the PI-D control system (shown in Figure 8–26) are given, respectively, by

$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s} + T_d s\right)\frac{K_p G_p(s)}{1 + \left(1 + \dfrac{1}{T_i s} + T_d s\right)K_p G_p(s)}$$

and

$$\frac{Y(s)}{R(s)} = \left(1 + \frac{1}{T_i s}\right)\frac{K_p G_p(s)}{1 + \left(1 + \dfrac{1}{T_i s} + T_d s\right)K_p G_p(s)}$$

It is important to point out that in the absence of the reference input and noises, the closed-loop transfer function between the disturbance $D(s)$ and the output $Y(s)$ in either case is the same and is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s)\left(1 + \dfrac{1}{T_i s} + T_d s\right)}$$

I-PD Control. Consider the case where the reference input is a step function. Both PID control and PI-D control involve a step function in the manipulated signal. Such a step change in the manipulated signal may not be desirable in many occasions. Therefore, it may be advantageous to move the proportional action and derivative action to the feedback path so that these actions affect the feedback signal only. Figure 8–27 shows such a control scheme. It is called the I-PD control. The manipulated signal is given by

$$U(s) = K_p \frac{1}{T_i s} R(s) - K_p\left(1 + \frac{1}{T_i s} + T_d s\right)B(s)$$

Notice that the reference input $R(s)$ appears only in the integral control part. Thus, in I-PD control, it is imperative to have the integral control action for proper operation of the control system.

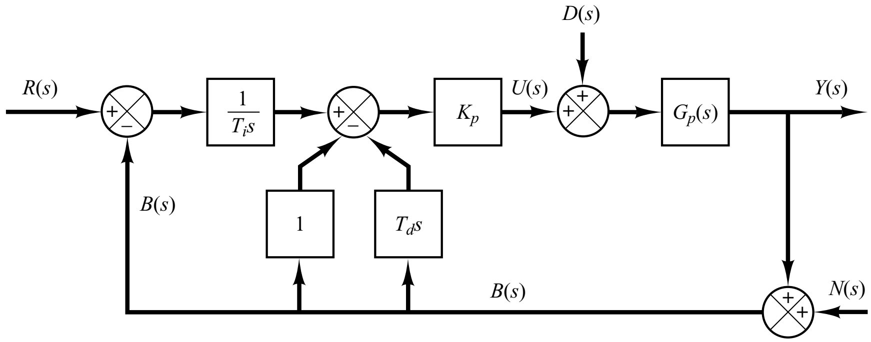


Figure 8–27
I-PD-controlled system.

The closed-loop transfer function $Y(s)/R(s)$ in the absence of the disturbance input and noise input is given by

$$\frac{Y(s)}{R(s)} = \left(\frac{1}{T_i s}\right) \frac{K_p G_p(s)}{1 + K_p G_p(s)\left(1 + \dfrac{1}{T_i s} + T_d s\right)}$$

It is noted that in the absence of the reference input and noise signals, the closed-loop transfer function between the disturbance input and the output is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p(s)}{1 + K_p G_p(s)\left(1 + \dfrac{1}{T_i s} + T_d s\right)}$$

This expression is the same as that for PID control or PI-D control.

Two-Degrees-of-Freedom PID Control. We have shown that PI-D control is obtained by moving the derivative control action to the feedback path, and I-PD control is obtained by moving the proportional control and derivative control actions to the feedback path. Instead of moving the entire derivative control action or proportional control action to the feedback path, it is possible to move only portions of these control actions to the feedback path, retaining the remaining portions in the feedforward path. In the literature, PI-PD control has been proposed. The characteristics of this control scheme lie between PID control and I-PD control. Similarly, PID-PD control can be considered. In these control schemes, we have a controller in the feedforward path and another controller in the feedback path. Such control schemes lead us to a more general two-degrees-of-freedom control scheme. We shall discuss details of such a two-degrees-of-freedom control scheme in subsequent sections of this chapter.

8–6 TWO-DEGREES-OF-FREEDOM CONTROL

Consider the system shown in Figure 8–28, where the system is subjected to the disturbance input $D(s)$ and noise input $N(s)$, in addition to the reference input $R(s)$. $G_c(s)$ is the transfer function of the controller and $G_p(s)$ is the transfer function of the plant. We assume that $G_p(s)$ is fixed and unalterable.

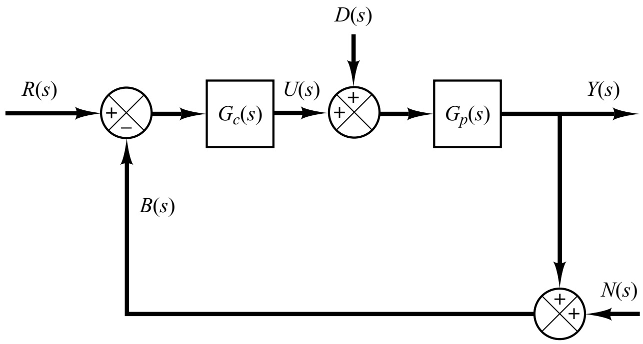


Figure 8–28 One-degree-of-freedom control system.

For this system, three closed-loop transfer functions $Y(s)/R(s) = G_{yr}$, $Y(s)/D(s) = G_{yd}$, and $Y(s)/N(s) = G_{yn}$ may be derived. They are

$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_c G_p}{1 + G_c G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_c G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_c G_p}{1 + G_c G_p}$$

[In deriving $Y(s)/R(s)$, we assumed $D(s) = 0$ and $N(s) = 0$. Similar comments apply to the derivations of $Y(s)/D(s)$ and $Y(s)/N(s)$.] The degrees of freedom of the control system refers to how many of these closed-loop transfer functions are independent. In the present case, we have

$$G_{yr} = \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

Among the three closed-loop transfer functions G_{yr}, G_{yn}, and G_{yd}, if one of them is given, the remaining two are fixed. This means that the system shown in Figure 8–28 is a one-degree-of-freedom control system.

Next consider the system shown in Figure 8–29, where $G_p(s)$ is the transfer function of the plant. For this system, closed-loop transfer functions G_{yr}, G_{yn}, and G_{yd} are given, respectively, by

$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1} G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{(G_{c1} + G_{c2})G_p}{1 + (G_{c1} + G_{c2})G_p}$$

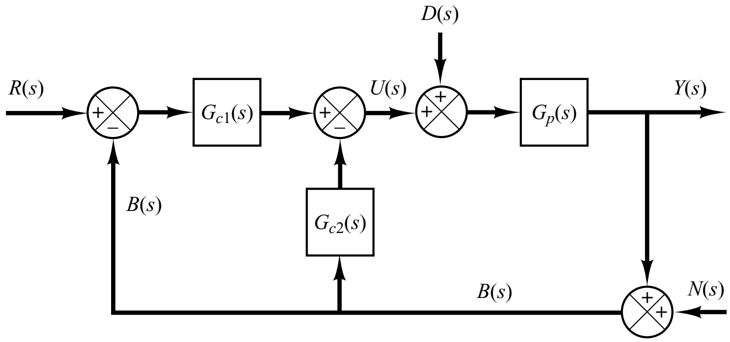


Figure 8–29 Two-degrees-of-freedom control system.

Hence, we have

$$G_{yr} = G_{c1} G_{yd}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

In this case, if G_{yd} is given, then G_{yn} is fixed, but G_{yr} is not fixed, because G_{c1} is independent of G_{yd}. Thus, two closed-loop transfer functions among three closed-loop transfer functions G_{yr}, G_{yd}, and G_{yn} are independent. Hence, this system is a two-degrees-of-freedom control system.

Similarly, the system shown in Figure 8–30 is also a two-degrees-of-freedom control system, because for this system

$$G_{yr} = \frac{Y(s)}{R(s)} = \frac{G_{c1} G_p}{1 + G_{c1} G_p} + \frac{G_{c2} G_p}{1 + G_{c1} G_p}$$

$$G_{yd} = \frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_{c1} G_p}$$

$$G_{yn} = \frac{Y(s)}{N(s)} = -\frac{G_{c1} G_p}{1 + G_{c1} G_p}$$

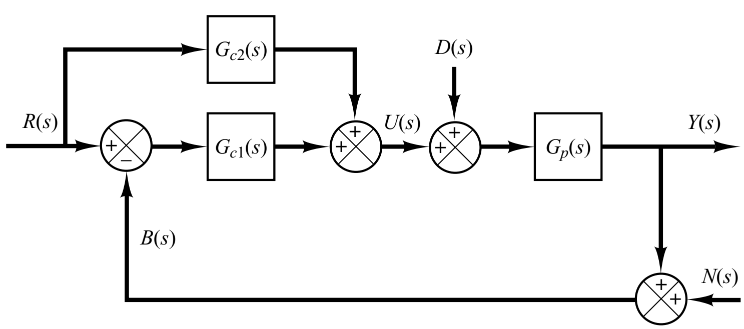


Figure 8–30 Two-degrees-of-freedom control system.

Hence,

$$G_{yr} = G_{c2}G_{yd} + \frac{G_p - G_{yd}}{G_p}$$

$$G_{yn} = \frac{G_{yd} - G_p}{G_p}$$

Clearly, if G_{yd} is given, then G_{yn} is fixed, but G_{yr} is not fixed, because G_{c2} is independent of G_{yd}.

It will be seen in Section 8–7 that, in such a two-degrees-of-freedom control system, both the closed-loop characteristics and the feedback characteristics can be adjusted independently to improve the system response performance.

8–7 ZERO-PLACEMENT APPROACH TO IMPROVE RESPONSE CHARACTERISTICS

We shall show here that by use of the zero-placement approach presented later in this section, we can achieve the following:

> The responses to the ramp reference input and acceleration reference input exhibit no steady-state errors.

In high-performance control systems it is always desired that the system output follow the changing input with minimum error. For step, ramp, and acceleration inputs, it is desired that the system output exhibit no steady-state error.

In what follows, we shall demonstrate how to design control systems that will exhibit no steady-state errors in following ramp and acceleration inputs and at the same time force the response to the step disturbance input to approach zero quickly.

Consider the two-degrees-of-freedom control system shown in Figure 8–31. Assume that the plant transfer function $G_p(s)$ is a minimum-phase transfer function and is given by

$$G_p(s) = K\frac{A(s)}{B(s)}$$

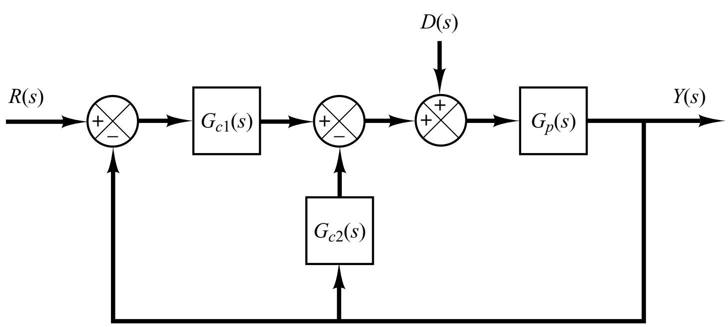


Figure 8–31 Two-degrees-of-freedom control system.

where

$$A(s) = (s + z_1)(s + z_2) \cdots (s + z_m)$$
$$B(s) = s^N (s + p_{N+1})(s + p_{N+2}) \cdots (s + p_n)$$

where N may be 0, 1, 2 and $n \geq m$. Assume also that G_{c1} is a PID controller followed by a filter $1/A(s)$, or

$$G_{c1}(s) = \frac{\alpha_1 s + \beta_1 + \gamma_1 s^2}{s} \frac{1}{A(s)}$$

and G_{c2} is a PID, PI, PD, I, D, or P controller followed by a filter $1/A(s)$. That is

$$G_{c2}(s) = \frac{\alpha_2 s + \beta_2 + \gamma_2 s^2}{s} \frac{1}{A(s)}$$

where some of α_2, β_2, and γ_2 may be zero. Then it is possible to write $G_{c1} + G_{c2}$ as

$$G_{c1} + G_{c2} = \frac{\alpha s + \beta + \gamma s^2}{s} \frac{1}{A(s)} \tag{8–3}$$

where α, β, and γ are constants. Then

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p} = \frac{K\dfrac{A(s)}{B(s)}}{1 + \dfrac{\alpha s + \beta + \gamma s^2}{s} \dfrac{K}{B(s)}}$$
$$= \frac{sKA(s)}{sB(s) + (\alpha s + \beta + \gamma s^2)K}$$

Because of the presence of s in the numerator, the response $y(t)$ to a step disturbance input approaches zero as t approaches infinity, as shown below. Since

$$Y(s) = \frac{sKA(s)}{sB(s) + (\alpha s + \beta + \gamma s^2)K} D(s)$$

if the disturbance input is a step function of magnitude d, or

$$D(s) = \frac{d}{s}$$

and assuming the system is stable, then

$$y(\infty) = \lim_{s \to 0} s \left[\frac{sKA(s)}{sB(s) + (\alpha s + \beta + \gamma s^2)K} \right] \frac{d}{s}$$
$$= \lim_{s \to 0} \frac{sKA(0)d}{sB(0) + \beta K}$$
$$= 0$$

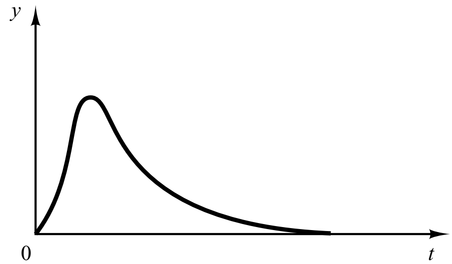


Figure 8–32 Typical response curve to a step disturbance input.

The response $y(t)$ to a step disturbance input will have the general form shown in Figure 8–32.

Note that $Y(s)/R(s)$ and $Y(s)/D(s)$ are given by

$$\frac{Y(s)}{R(s)} = \frac{G_{c1}G_p}{1 + (G_{c1} + G_{c2})G_p}, \qquad \frac{Y(s)}{D(s)} = \frac{G_p}{1 + (G_{c1} + G_{c2})G_p}$$

Notice that the denominators of $Y(s)/R(s)$ and $Y(s)/D(s)$ are the same. Before we choose the poles of $Y(s)/R(s)$, we need to place the zeros of $Y(s)/R(s)$.

Zero Placement. Consider the system

$$\frac{Y(s)}{R(s)} = \frac{p(s)}{s^{n+1} + a_n s^n + a_{n-1}s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

If we choose $p(s)$ as

$$p(s) = a_2 s^2 + a_1 s + a_0 = a_2(s + s_1)(s + s_2)$$

that is, choose the zeros $s = -s_1$ and $s = -s_2$ such that, together with a_2, the numerator polynomial $p(s)$ is equal to the sum of the last three terms of the denominator polynomial—then the system will exhibit no steady-state errors in response to the step input, ramp input, and acceleration input.

Requirement Placed on System Response Characteristics. Suppose that it is desired that the maximum overshoot in the response to the unit-step reference input be between arbitrarily selected upper and lower limits—for example,

$$2\% < \text{maximum overshoot} < 10\%$$

where we choose the lower limit to be slightly above zero to avoid having overdamped systems. The smaller the upper limit, the harder it is to determine the coefficient a's. In some cases, no combination of the a's may exist to satisfy the specification, so we must allow a higher upper limit for the maximum overshoot. We use MATLAB to search at least one set of the a's to satisfy the specification. As a practical computational matter, instead of searching for the a's, we try to obtain acceptable closed-loop poles by searching a reasonable region in the left-half s plane for each closed-loop pole. Once we determine all closed-loop poles, then all coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ will be determined.

Determination of G_{c2}. Now that the coefficients of the transfer function $Y(s)/R(s)$ are all known and $Y(s)/R(s)$ is given by

$$\frac{Y(s)}{R(s)} = \frac{a_2 s^2 + a_1 s + a_0}{s^{n+1} + a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0} \tag{8–4}$$

we have

$$\begin{aligned}\frac{Y(s)}{R(s)} &= G_{c1} \frac{Y(s)}{D(s)} \\ &= \frac{G_{c1} s K A(s)}{sB(s) + \left(\alpha s + \beta + \gamma s^2\right) K} \\ &= \frac{G_{c1} s K A(s)}{s^{n+1} + a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}\end{aligned}$$

Since G_{c1} is a PID controller and is given by

$$G_{c1} = \frac{\alpha_1 s + \beta_1 + \gamma_1 s^2}{s} \frac{1}{A(s)}$$

$Y(s)/R(s)$ can be written as

$$\frac{Y(s)}{R(s)} = \frac{K\left(\alpha_1 s + \beta_1 + \gamma_1 s^2\right)}{s^{n+1} + a_n s^n + a_{n-1} s^{n-1} + \cdots + a_2 s^2 + a_1 s + a_0}$$

Therefore, we choose

$$K\gamma_1 = a_2, \qquad K\alpha_1 = a_1, \qquad K\beta_1 = a_0$$

so that

$$G_{c1} = \frac{a_1 s + a_0 + a_2 s^2}{Ks} \frac{1}{A(s)} \tag{8–5}$$

The response of this system to the unit-step reference input can be made to exhibit the maximum overshoot between the chosen upper and lower limits, such as

$$2\% < \text{maximum overshoot} < 10\%$$

The response of the system to the ramp reference input or acceleration reference input can be made to exhibit no steady-state error. The characteristic of the system of Equation (8–4) is that it generally exhibits a short settling time. If we wish to further shorten the settling time, then we need to allow a larger maximum overshoot—for example,

$$2\% < \text{maximum overshoot} < 20\%$$

The controller G_{c2} can now be determined from Equations (8–3) and (8–5). Since

$$G_{c1} + G_{c2} = \frac{\alpha s + \beta + \gamma s^2}{s} \frac{1}{A(s)}$$

we have

$$G_{c2} = \left[\frac{\alpha s + \beta + \gamma s^2}{s} - \frac{a_1 s + a_0 + a_2 s^2}{Ks} \right] \frac{1}{A(s)}$$

$$= \frac{(K\alpha - a_1)s + (K\beta - a_0) + (K\gamma - a_2)s^2}{Ks} \frac{1}{A(s)} \tag{8–6}$$

The two controllers G_{c1} and G_{c2} can be determined from Equations (8–5) and (8–6).

EXAMPLE 8–4 Consider the two-degrees-of-freedom control system shown in Figure 8–33. The plant transfer function $G_p(s)$ is given by

$$G_p(s) = \frac{10}{s(s+1)}$$

Design controllers $G_{c1}(s)$ and $G_{c2}(s)$ such that the maximum overshoot in the response to the unit-step reference input be less than 19%, but more than 2%, and the settling time be less than 1 sec. It is desired that the steady-state errors in following the ramp reference input and acceleration reference input be zero. The response to the unit-step disturbance input should have a small amplitude and settle to zero quickly.

To design suitable controllers $G_{c1}(s)$ and $G_{c2}(s)$, first note that

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_p(G_{c1} + G_{c2})}$$

To simplify the notation, let us define

$$G_c = G_{c1} + G_{c2}$$

Then

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_p G_c} = \frac{\dfrac{10}{s(s+1)}}{1 + \dfrac{10}{s(s+1)} G_c}$$

$$= \frac{10}{s(s+1) + 10G_c}$$

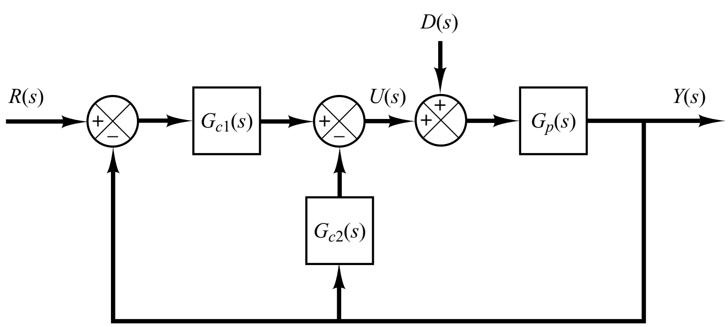


Figure 8–33 Two-degrees-of-freedom control system.

Second, note that

$$\frac{Y(s)}{R(s)} = \frac{G_p G_{c1}}{1 + G_p G_c} = \frac{10G_{c1}}{s(s+1) + 10G_c}$$

Notice that the characteristic equation for $Y(s)/D(s)$ and the one for $Y(s)/R(s)$ are identical.

We may be tempted to choose a zero of $G_c(s)$ at $s = -1$ to cancel a pole at $s = -1$ of the plant $G_p(s)$. However, the canceled pole $s = -1$ becomes a closed-loop pole of the entire system, as seen below. If we define $G_c(s)$ as a PID controller such that

$$G_c(s) = \frac{K(s+1)(s+\beta)}{s} \tag{8–7}$$

then

$$\frac{Y(s)}{D(s)} = \frac{10}{s(s+1) + \dfrac{10K(s+1)(s+\beta)}{s}}$$

$$= \frac{10s}{(s+1)\left[s^2 + 10K(s+\beta)\right]}$$

The closed-loop pole at $s = -1$ is a slow-response pole, and if this closed-loop pole is included in the system, the settling time will not be less than 1 sec. Therefore, we should not choose $G_c(s)$ as given by Equation (8–7).

The design of controllers $G_{c1}(s)$ and $G_{c2}(s)$ consists of two steps.

Design Step 1: We design $G_c(s)$ to satisfy the requirements on the response to the step-disturbance input $D(s)$. In this design stage, we assume that the reference input is zero.

Suppose that we assume that $G_c(s)$ is a PID controller of the form

$$G_c(s) = \frac{K(s+\alpha)(s+\beta)}{s}$$

Then the closed-loop transfer function $Y(s)/D(s)$ becomes

$$\frac{Y(s)}{D(s)} = \frac{10}{s(s+1) + 10G_c}$$

$$= \frac{10}{s(s+1) + \dfrac{10K(s+\alpha)(s+\beta)}{s}}$$

$$= \frac{10s}{s^2(s+1) + 10K(s+\alpha)(s+\beta)}$$

Note that the presence of "s" in the numerator of $Y(s)/D(s)$ assures that the steady-state response to the step disturbance input is zero.

Let us assume that the desired dominant closed-loop poles are complex conjugates and are given by

$$s = -a \pm jb$$

and the remaining closed-loop pole is real and is located at

$$s = -c$$

Note that in this problem there are three requirements. The first requirement is that the response to the step disturbance input damp out quickly. The second requirement is that the maximum overshoot in the response to the unit-step reference input be between 19% and 2% and the settling time be less than 1 sec. The third requirement is that the steady-state errors in the responses to both the ramp and acceleration reference inputs be zero.

A set (or sets) of reasonable values of a, b, and c must be searched using a computational approach. To satisfy the first requirement, we choose the search region for a, b, and c to be

$$2 \leq a \leq 6, \qquad 2 \leq b \leq 6, \qquad 6 \leq c \leq 12$$

This region is shown in Figure 8–34. If the dominant closed-loop poles $s = -a \pm jb$ are located anywhere in the shaded region, the response to a step disturbance input will damp out quickly. (The first requirement will be met.)

Notice that the denominator of $Y(s)/D(s)$ can be written as

$$\begin{aligned} & s^2(s + 1) + 10K(s + \alpha)(s + \beta) \\ &= s^3 + (1 + 10K)s^2 + 10K(\alpha + \beta)s + 10K\alpha\beta \\ &= (s + a + jb)(s + a - jb)(s + c) \\ &= s^3 + (2a + c)s^2 + \left(a^2 + b^2 + 2ac\right)s + \left(a^2 + b^2\right)c \end{aligned}$$

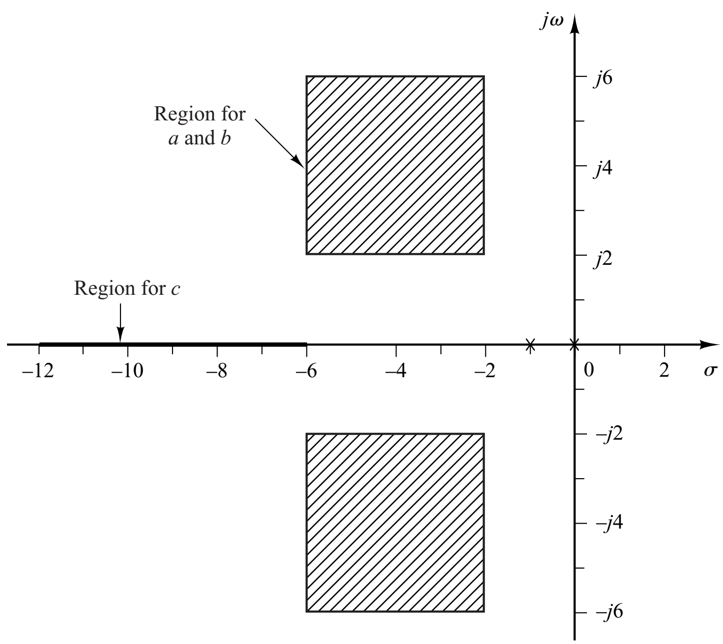


Figure 8–34
Search regions for a, b, and c.

Since the denominators of $Y(s)/D(s)$ and $Y(s)/R(s)$ are the same, the denominator of $Y(s)/D(s)$ determines also the response characteristics for the reference input. To satisfy the third requirement, we refer to the zero-placement method and choose the closed-loop transfer function $Y(s)/R(s)$ to be of the following form:

$$\frac{Y(s)}{R(s)} = \frac{(2a + c)s^2 + (a^2 + b^2 + 2ac)s + (a^2 + b^2)c}{s^3 + (2a + c)s^2 + (a^2 + b^2 + 2ac)s + (a^2 + b^2)c}$$

in which case the third requirement is automatically satisfied.

Our problem then becomes a search of a set or sets of desired closed-loop poles in terms of a, b, and c in the specified region, such that the system will satisfy the requirement on the response to the unit-step reference input that the maximum overshoot be between 19% and 2% and the settling time be less than 1 sec. (If an acceptable set cannot be found in the search region, we need to widen the region.)

In the computational search, we need to assume a reasonable step size. In this problem, we assume it to be 0.2.

MATLAB Program 8–8 produces a table of sets of acceptable values of a, b, and c. Using this program, we find that the requirement on the response to the unit-step reference input is met by any of the 23 sets shown in the table in MATLAB Program 8–8. Note that the last row in the table corresponds to the last search point. This point does not satisfy the requirement and thus it should simply be ignored. (In the program written, the last search point produces the last row in the table whether or not it satisfies the requirement.)

MATLAB Program 8–8

```
t = 0:0.01:4;
k = 0;
for i = 1:21;
   a(i) = 6.2-i*0.2;
   for j = 1:21;
      b(j) = 6.2-j*0.2;
      for h = 1:31;
         c(h) = 12.2-h*0.2;
      num = [0  2*a(i)+c(h)  a(i)^2+b(j)^2+2*a(i)*c(h)  (a(i)^2+b(j)^2)*c(h)];
      den = [1  2*a(i)+c(h)  a(i)^2+b(j)^2+2*a(i)*c(h)  (a(i)^2+b(j)^2)*c(h)];
            y = step(num,den,t);
            m = max(y);
            s = 401; while y(s) > 0.98 & y(s) < 1.02;
            s = s-1; end;
            ts = (s-1)*0.01;
         if m < 1.19 & m > 1.02 & ts < 1.0;
         k = k+1;
         table(k,:) = [a(i)  b(j)  c(h)  m  ts];
         end
      end
   end
 end
```

(*continues on next page*)

```
table(k,:) = [a(i)  b(j)  c(h)  m  ts]

table =

    4.2000  2.0000  12.0000  1.1896  0.8500
    4.0000  2.0000  12.0000  1.1881  0.8700
    4.0000  2.0000  11.8000  1.1890  0.8900
    4.0000  2.0000  11.6000  1.1899  0.9000
    3.8000  2.2000  12.0000  1.1883  0.9300
    3.8000  2.2000  11.8000  1.1894  0.9400
    3.8000  2.0000  12.0000  1.1861  0.8900
    3.8000  2.0000  11.8000  1.1872  0.9100
    3.8000  2.0000  11.6000  1.1882  0.9300
    3.8000  2.0000  11.4000  1.1892  0.9400
    3.6000  2.4000  12.0000  1.1893  0.9900
    3.6000  2.2000  12.0000  1.1867  0.9600
    3.6000  2.2000  11.8000  1.1876  0.9800
    3.6000  2.2000  11.6000  1.1886  0.9900
    3.6000  2.0000  12.0000  1.1842  0.9200
    3.6000  2.0000  11.8000  1.1852  0.9400
    3.6000  2.0000  11.6000  1.1861  0.9500
    3.6000  2.0000  11.4000  1.1872  0.9700
    3.6000  2.0000  11.2000  1.1883  0.9800
    3.4000  2.0000  12.0000  1.1820  0.9400
    3.4000  2.0000  11.8000  1.1831  0.9600
    3.4000  2.0000  11.6000  1.1842  0.9800
    3.2000  2.0000  12.0000  1.1797  0.9600
    2.0000  2.0000   6.0000  1.2163  1.8900
```

As noted above, 23 sets of variables a, b, and c satisfy the requirement. Unit-step response curves of the system with any of the 23 sets are about the same. The unit-step response curve with

$$a = 4.2, \qquad b = 2, \qquad c = 12$$

is shown in Figure 8–35(a). The maximum overshoot is 18.96% and the settling time is 0.85 sec. Using these values of a, b, and c, the desired closed-loop poles are located at

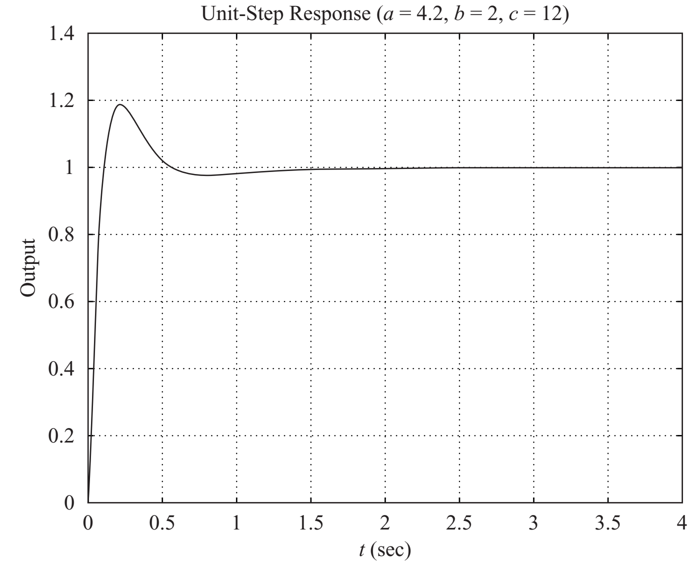
$$s = -4.2 \pm j2, \qquad s = -12$$

Using these closed-loop poles, the denominator of $Y(s)/D(s)$ becomes

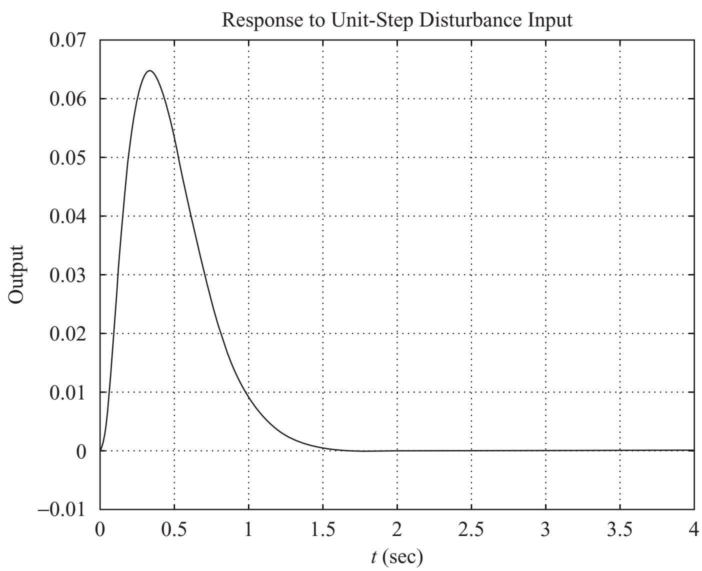
$$s^2(s + 1) + 10K(s + \alpha)(s + \beta) = (s + 4.2 + j2)(s + 4.2 - j2)(s + 12)$$

or

$$s^3 + (1 + 10K)s^2 + 10K(\alpha + \beta)s + 10K\alpha\beta = s^3 + 20.4s^2 + 122.44s + 259.68$$



(a)



(b)

Figure 8–35
(a) Response to unit-step reference input ($a = 4.2, b = 2, c = 12$); (b) response to unit-step disturbance input ($a = 4.2, b = 2, c = 12$).

By equating the coefficients of equal powers of s on both sides of this last equation, we obtain

$$1 + 10K = 20.4$$

$$10K(\alpha + \beta) = 122.44$$

$$10K\alpha\beta = 259.68$$

Hence

$$K = 1.94, \qquad \alpha + \beta = \frac{122.44}{19.4}, \qquad \alpha\beta = \frac{259.68}{19.4}$$

Then $G_c(s)$ can be written as

$$\begin{aligned} G_c(s) &= K\frac{(s+\alpha)(s+\beta)}{s} \\ &= \frac{K\left[s^2 + (\alpha+\beta)s + \alpha\beta\right]}{s} \\ &= \frac{1.94s^2 + 12.244s + 25.968}{s} \end{aligned}$$

The closed-loop transfer function $Y(s)/D(s)$ becomes

$$\begin{aligned} \frac{Y(s)}{D(s)} &= \frac{10}{s(s+1) + 10G_c} \\ &= \frac{10}{s(s+1) + 10\,\dfrac{1.94s^2 + 12.244s + 25.968}{s}} \\ &= \frac{10s}{s^3 + 20.4s^2 + 122.44s + 259.68} \end{aligned}$$

Using this expression, the response $y(t)$ to a unit-step disturbance input can be obtained as shown in Figure 8–35(b).

Figure 8–36(a) shows the response of the system to the unit-step reference input when a, b, and c are chosen as

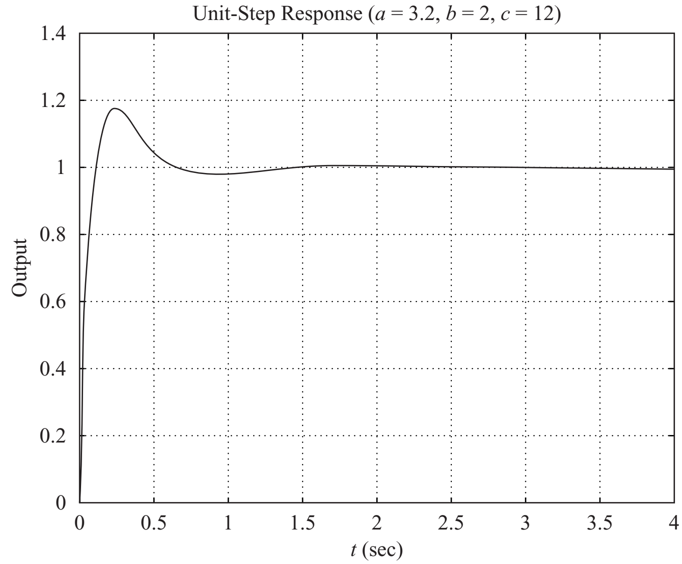
$$a = 3.2, \qquad b = 2, \qquad c = 12$$

Figure 8–36(b) shows the response of this system when it is subjected to a unit-step disturbance input. Comparing Figures 8–35(a) and Figure 8–36(a), we find that they are about the same. However, comparing Figures 8–35(b) and 8–36(b), we find the former to be a little bit better than the latter. Comparing the responses of systems with each set in the table, we conclude the first set of values ($a = 4.2$, $b = 2$, $c = 12$) to be one of the best. Therefore, as the solution to this problem, we choose

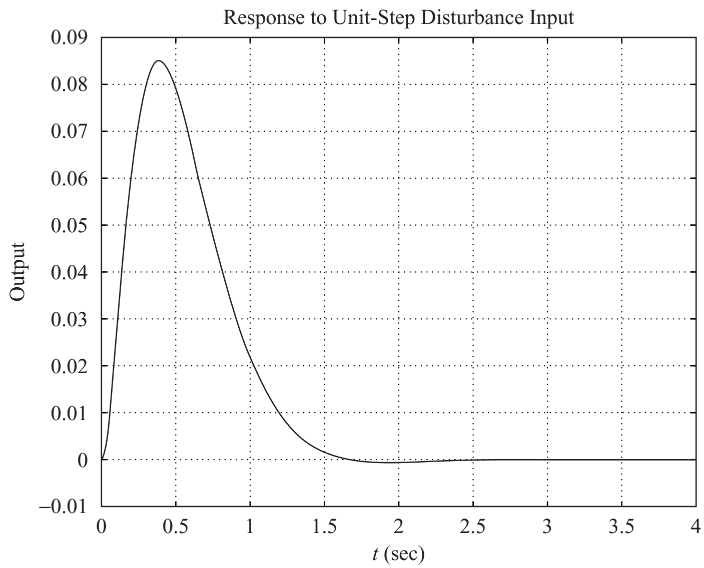
$$a = 4.2, \qquad b = 2, \qquad c = 12$$

Design Step 2: Next, we determine G_{c1}. Since $Y(s)/R(s)$ can be given by

$$\begin{aligned} \frac{Y(s)}{R(s)} &= \frac{G_pG_{c1}}{1 + G_pG_c} \\ &= \frac{\dfrac{10}{s(s+1)}G_{c1}}{1 + \dfrac{10}{s(s+1)}\,\dfrac{1.94s^2 + 12.244s + 25.968}{s}} \\ &= \frac{10sG_{c1}}{s^3 + 20.4s^2 + 122.44s + 259.68} \end{aligned}$$



(a)



(b)

Figure 8–36
(a) Response to unit-step reference input $(a = 3.2, b = 2, c = 12)$; (b) response to unit-step disturbance input $(a = 3.2, b = 2, c = 12)$.

our problem becomes that of designing $G_{c1}(s)$ to satisfy the requirements on the responses to the step, ramp, and acceleration inputs.

Since the numerator involves "s", $G_{c1}(s)$ must include an integrator to cancel this "s". [Although we want "s" in the numerator of the closed-loop transfer function $Y(s)/D(s)$ to obtain zero steady-state error to the step disturbance input, we do not want to have "s" in the numerator of the closed-loop transfer function $Y(s)/R(s)$.] To eliminate the offset in the response to the step reference input and eliminate the steady-state errors in following the ramp reference input and acceleration reference input, the numerator of $Y(s)/R(s)$ must be equal to the last three terms of the denominator, as mentioned earlier. That is,

$$10sG_{c1}(s) = 20.4s^2 + 122.44s + 259.68$$

or

$$G_{c1}(s) = 2.04s + 12.244 + \frac{25.968}{s}$$

Thus, $G_{c1}(s)$ is a PID controller. Since $G_c(s)$ is given as

$$G_c(s) = G_{c1}(s) + G_{c2}(s) = \frac{1.94s^2 + 12.244s + 25.968}{s}$$

we obtain

$$\begin{aligned} G_{c2}(s) &= G_c(s) - G_{c1}(s) \\ &= \left(1.94s + 12.244 + \frac{25.968}{s}\right) - \left(2.04s + 12.244 + \frac{25.968}{s}\right) \\ &= -0.1s \end{aligned}$$

Thus, $G_{c2}(s)$ is a derivative controller. A block diagram of the designed system is shown in Figure 8–37.

The closed-loop transfer function $Y(s)/R(s)$ now becomes

$$\frac{Y(s)}{R(s)} = \frac{20.4s^2 + 122.44s + 259.68}{s^3 + 20.4s^2 + 122.44s + 259.68}$$

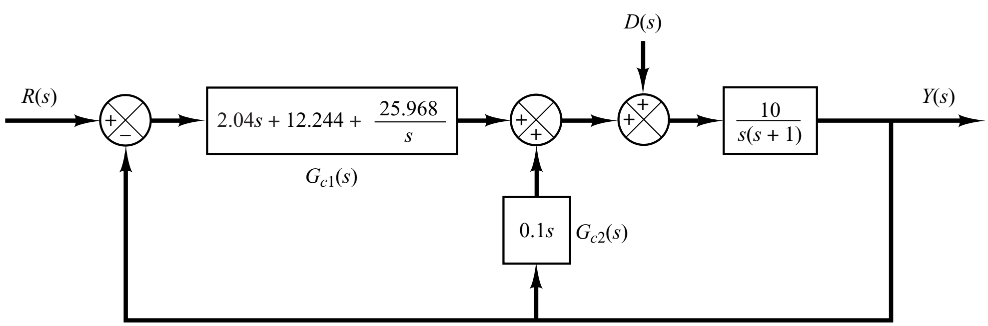


Figure 8–37 Block diagram of the designed system.

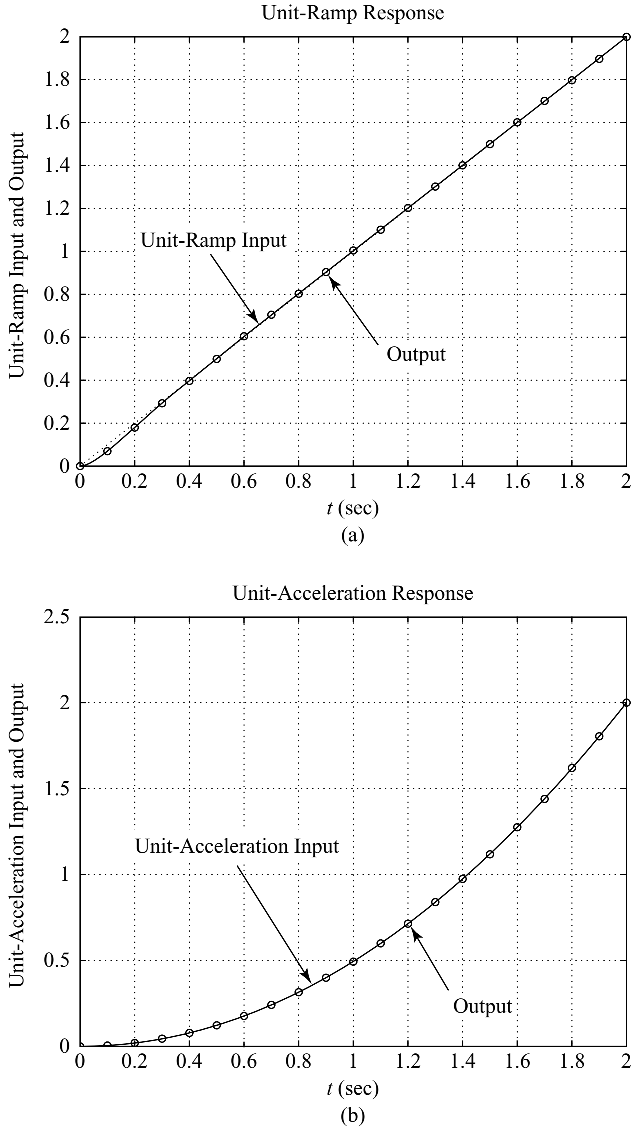


Figure 8–38
(a) Response to unit-ramp reference input; (b) response to unit-acceleration reference input.

The response to the unit-ramp reference input and that to the unit-acceleration reference input are shown in Figures 8–38(a) and (b), respectively. The steady-state errors in following the ramp input and acceleration input are zero. Thus, all the requirements of the problem are satisfied. Hence, the designed controllers $G_{c1}(s)$ and $G_{c2}(s)$ are acceptable.

EXAMPLE 8–5 Consider the control system shown in Figure 8–39. This is a two-degrees-of-freedom system. In the design problem considered here, we assume that the noise input $N(s)$ is zero. Assume that the plant transfer function $G_p(s)$ is given by

$$G_p(s) = \frac{5}{(s+1)(s+5)}$$

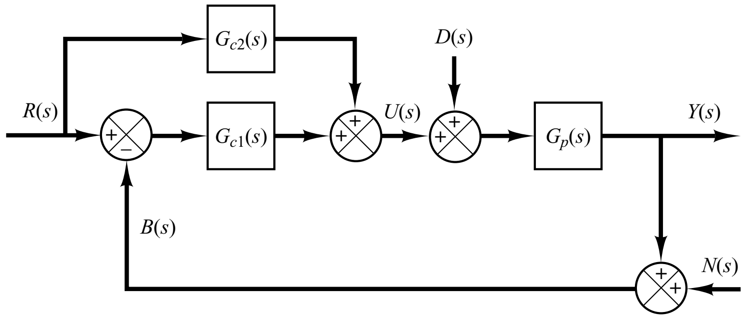


Figure 8–39 Two-degrees-of-freedom control system.

Assume also that the controller $G_{c1}(s)$ is of PID type. That is,

$$G_{c1}(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

The controller $G_{c2}(s)$ is of P or PD type. [If $G_{c2}(s)$ involves integral control action, then this will introduce a ramp component in the input signal, which is not desirable. Therefore, $G_{c2}(s)$ should not include the integral control action.] Thus, we assume that

$$G_{c2}(s) = \hat{K}_p(1 + \hat{T}_d s)$$

where $\hat{T}_d$ may be zero.

Let us design controllers $G_{c1}(s)$ and $G_{c2}(s)$ such that the responses to the step-disturbance input and the step-reference input are of "desirable characteristics" in the sense that

1. The response to the step-disturbance input will have a small peak and eventually approach zero. (That is, there will be no steady-state error.)
2. The response to the step reference input will exhibit less than 25% overshoot with a settling time less than 2 sec. The steady-state errors to the ramp reference input and acceleration reference input should be zero.

The design of this two-degrees-of-freedom control system may be carried out by following the steps **1** and **2** below.

1. Determine $G_{c1}(s)$ so that the response to the step-disturbance input is of desirable characteristics.
2. Design $G_{c2}(s)$ so that the responses to the reference inputs are of desirable characteristics without changing the response to the step disturbance considered in step **1**.

Design of $G_{c1}(s)$: First, note that we assumed the noise input $N(s)$ to be zero. To obtain the response to the step-disturbance input, we assume that the reference input is zero. Then the block diagram which relates $Y(s)$ and $D(s)$ can be drawn as shown in Figure 8–40. The transfer function $Y(s)/D(s)$ is given by

$$\frac{Y(s)}{D(s)} = \frac{G_p}{1 + G_{c1}G_p}$$

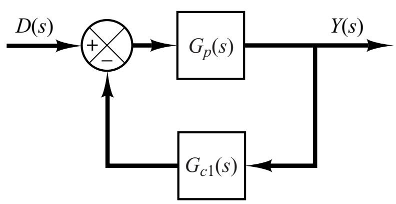


Figure 8–40 Control system.

where

$$G_{c1}(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

This controller involves one pole at the origin and two zeros. If we assume that the two zeros are located at the same place (a double zero), then $G_{c1}(s)$ can be written as

$$G_{c1}(s) = K\frac{(s + a)^2}{s}$$

Then the characteristic equation for the system becomes

$$1 + G_{c1}(s)G_p(s) = 1 + \frac{K(s + a)^2}{s}\frac{5}{(s + 1)(s + 5)} = 0$$

or

$$s(s + 1)(s + 5) + 5K(s + a)^2 = 0$$

which can be rewritten as

$$s^3 + (6 + 5K)s^2 + (5 + 10Ka)s + 5Ka^2 = 0 \tag{8–8}$$

If we place the double zero between $s = -3$ and $s = -6$, then the root-locus plot of $G_{c1}(s)G_p(s)$ may look like the one shown in Figure 8–41. The speed of response should be fast, but not faster than necessary, because faster response generally implies larger or more expensive components. Therefore, we may choose the dominant closed-loop poles at

$$s = -3 \pm j2$$

(Note that this choice is not unique. There are infinitely many possible closed-loop poles that we may choose from.)

Since the system is of third order, there are three closed-loop poles. The third one is located on the negative real axis to the left of point $s = -5$.

Let us substitute $s = -3 + j2$ into Equation (8–8).

$$(-3 + j2)^3 + (6 + 5K)(-3 + j2)^2 + (5 + 10Ka)(-3 + j2) + 5Ka^2 = 0$$

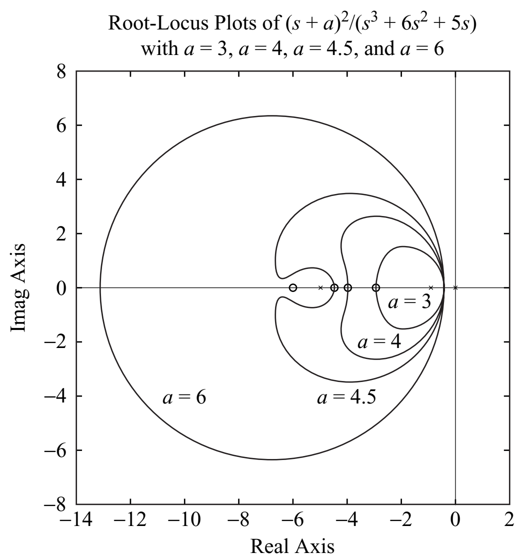


Figure 8–41 Root-locus plots of $5K(s + a)^2/[s(s + 1)(s + 5)]$ when $a = 3$, $a = 4$, $a = 4.5$, and $a = 6$.

which can be simplified to

$$24 + 25K - 30Ka + 5Ka^2 + j(-16 - 60K + 20Ka) = 0$$

By equating the real part and imaginary part to zero, respectively, we obtain

$$24 + 25K - 30Ka + 5Ka^2 = 0 \tag{8–9}$$

$$-16 - 60K + 20Ka = 0 \tag{8–10}$$

From Equation (8–10), we have

$$K = \frac{4}{5a - 15} \tag{8–11}$$

Substituting Equation (8–11) into Equation (8–9), we get

$$a^2 = 13$$

or $a = 3.6056$ or -3.6056. Notice that the values of K become

$$K = 1.3210 \qquad \text{for } a = 3.6056$$

$$K = -0.1211 \qquad \text{for } a = -3.6056$$

Since $G_{c1}(s)$ is in the feedforward path, the gain K should be positive. Hence, we choose

$$K = 1.3210, \qquad a = 3.6056$$

Then $G_{c1}(s)$ can be given by

$$\begin{aligned} G_{c1}(s) &= K\frac{(s + a)^2}{s} \\ &= 1.3210\frac{(s + 3.6056)^2}{s} \\ &= \frac{1.3210s^2 + 9.5260s + 17.1735}{s} \end{aligned}$$

To determine K_p, T_i, and T_d, we proceed as follows:

$$\begin{aligned} G_{c1}(s) &= \frac{1.3210(s^2 + 7.2112s + 13)}{s} \\ &= 9.5260\left(1 + \frac{1}{0.5547s} + 0.1387s\right) \end{aligned} \tag{8–12}$$

Thus,

$$K_p = 9.5260, \qquad T_i = 0.5547, \qquad T_d = 0.1387$$

To check the response to a unit-step disturbance input, we obtain the closed-loop transfer function $Y(s)/D(s)$.

$$\begin{aligned} \frac{Y(s)}{D(s)} &= \frac{G_p}{1 + G_{c1}G_p} \\ &= \frac{5s}{s(s + 1)(s + 5) + 5K(s + a)^2} \\ &= \frac{5s}{s^3 + 12.605s^2 + 52.63s + 85.8673} \end{aligned}$$

The response to the unit-step disturbance input is shown in Figure 8–42. The response curve seems good and acceptable. Note that the closed-loop poles are located at $s = -3 \pm j2$ and $s = -6.6051$. The complex-conjugate closed-loop poles act as dominant closed-loop poles.

Design of $G_{c2}(s)$: We now design $G_{c2}(s)$ to obtain the desired responses to the reference inputs. The closed-loop transfer function $Y(s)/R(s)$ can be given by

$$\frac{Y(s)}{R(s)} = \frac{(G_{c1} + G_{c2})G_p}{1 + G_{c1}G_p}$$

$$= \frac{\left[\dfrac{1.321s^2 + 9.526s + 17.1735}{s} + \hat{K}_p(1 + \hat{T}_d s)\right]\dfrac{5}{(s+1)(s+5)}}{1 + \dfrac{1.321s^2 + 9.526s + 17.1735}{s}\,\dfrac{5}{(s+1)(s+5)}}$$

$$= \frac{(6.6051 + 5\hat{K}_p\hat{T}_d)s^2 + (47.63 + 5\hat{K}_p)s + 85.8673}{s^3 + 12.6051s^2 + 52.63s + 85.8673}$$

Zero placement. We place two zeros together with the dc gain constant such that the numerator is the same as the sum of the last three terms of the denominator. That is,

$$(6.6051 + 5\hat{K}_p\hat{T}_d)s^2 + (47.63 + 5\hat{K}_p)s + 85.8673 = 12.6051s^2 + 52.63s + 85.8673$$

By equating the coefficients of s^2 terms and s terms on both sides of this last equation,

$$6.6051 + 5\hat{K}_p\hat{T}_d = 12.6051$$

$$47.63 + 5\hat{K}_p = 52.63$$

from which we get

$$\hat{K}_p = 1, \qquad \hat{T}_d = 1.2$$

Therefore,

$$G_{c2}(s) = 1 + 1.2s \tag{8–13}$$

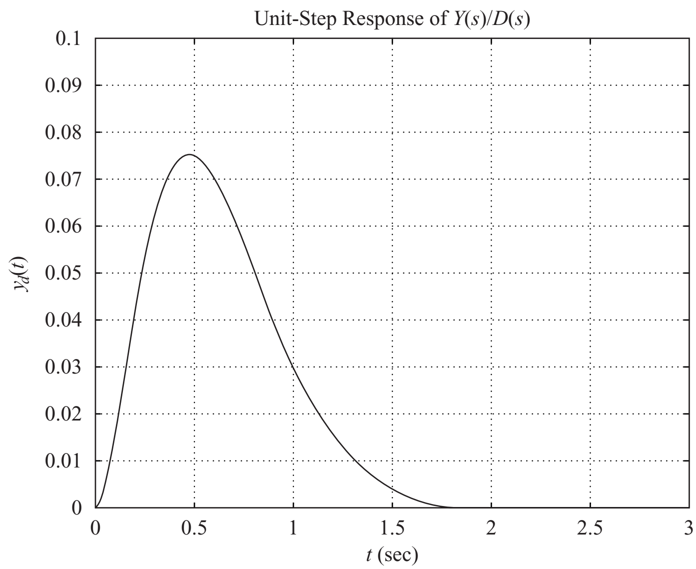
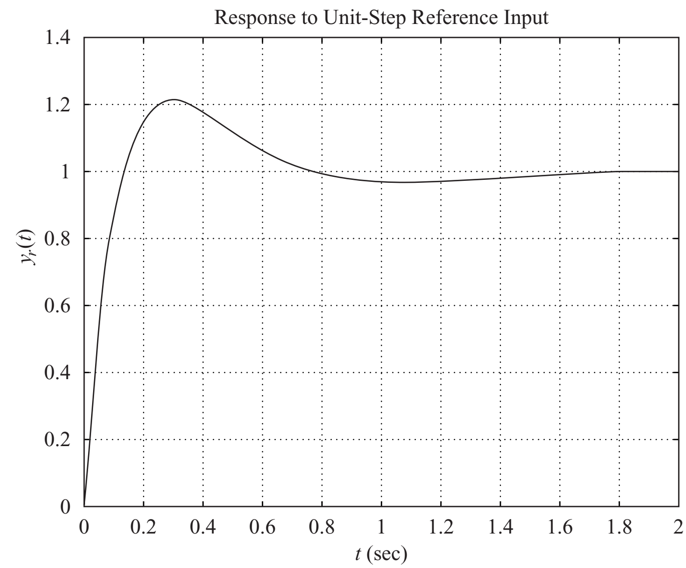


Figure 8–42 Response to unit-step disturbance input.

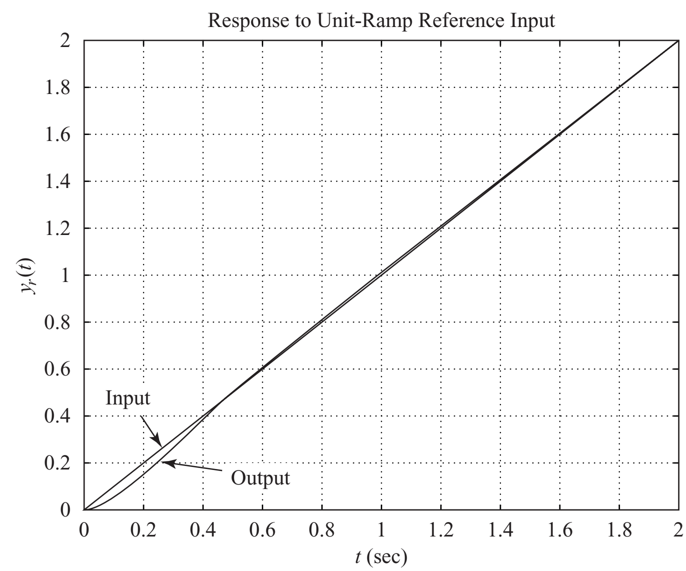
With this controller $G_{c2}(s)$, the closed-loop transfer function $Y(s)/R(s)$ becomes

$$\frac{Y(s)}{R(s)} = \frac{12.6051s^2 + 52.63s + 85.8673}{s^3 + 12.6051s^2 + 52.63s + 85.8673}$$

The response to the unit-step reference input becomes as shown in Figure 8–43(a).

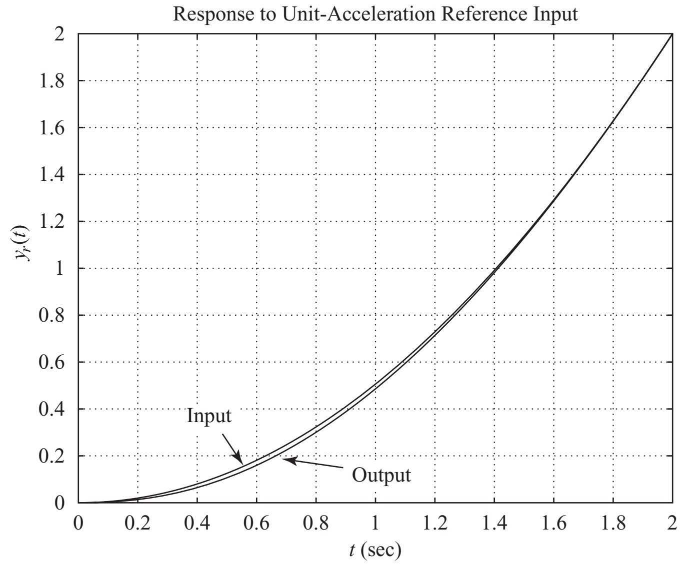


(a)



(b)

Figure 8–43
(a) Response to unit-step reference input; (b) response to unit-ramp reference input; (c) response to unit-acceleration reference input.



(c)

Figure 8–43 *(continued)*

The response exhibits the maximum overshoot of 21% and the settling time is approximately 1.6 sec. Figures 8–43(b) and (c) show the ramp response and acceleration response. The steady-state errors in both responses are zero. The response to the step disturbance was satisfactory. Thus, the designed controllers $G_{c1}(s)$ and $G_{c2}(s)$ given by Equations (8–12) and (8–13), respectively, are satisfactory.

If the response characteristics to the unit-step reference input are not satisfactory, we need to change the location of the dominant closed-loop poles and repeat the design process. The dominant closed-loop poles should lie in a certain region in the left-half s plane (such as $2 \le a \le 6$, $2 \le b \le 6, 6 \le c \le 12$). If the computational search is desired, write a computer program (similar to MATLAB Program 8–8) and execute the search process. Then a desired set or sets of values of a, b, and c may be found such that the system response to the unit-step reference input satisfies all requirements on maximum overshoot and settling time.

EXAMPLE PROBLEMS AND SOLUTIONS

A–8–1. Describe briefly the dynamic characteristics of the PI controller, PD controller, and PID controller.

Solution. The PI controller is characterized by the transfer function

$$G_c(s) = K_p\left(1 + \frac{1}{T_i s}\right)$$

The PI controller is a lag compensator. It possesses a zero at $s = -1/T_i$ and a pole at $s = 0$. Thus, the characteristic of the PI controller is infinite gain at zero frequency. This improves the steady-state characteristics. However, inclusion of the PI control action in the system increases the