September 27, 2019 MAT 272: Exam #1 Name: ____________________

Instructions: Read the entire statement of each problem. Solve each problem carefully and organize your work. Be sure to include units and write your answers in complete sentences where appropriate. The exam is worth 100 points.

1. (20 Points) Solve the following system of linear equations, and write the solution set in parametric vector form.

$$\begin{aligned} x_1 + x_2 + x_3 &= 4 \\ x_2 - x_3 &= 3 \\ 2x_1 - 2x_2 + 6x_3 &= -4 \end{aligned}$$

2. (15 Points) For each of the following, determine whether the set of vectors is linearly independent. Briefly explain why or why not.

(a) $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}.$

(b) $\mathbf{u}_1 = \begin{bmatrix} 3 \\ -2 \\ 5 \\ 6 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} -9 \\ 6 \\ 15 \\ -18 \end{bmatrix},$

(c) The columns of the $n \times n$ identity matrix.

3. (15 Points) For the matrices below, compute the indicated operations.

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & -3 & -7 \\ 2 & 4 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 \\ 0 & -1 \\ 3 & -2 \end{bmatrix}$$

(a) $2A + B$

(b) AC

(c) CB

4. (10 Points) Find the standard matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that first reflects over the horizontal x_1 axis followed by a horizontal stretch by a factor of 4.

5. (10 Points) Find the inverse of the following matrix:

$$A = \begin{bmatrix} 0 & 8 & 0 \\ 4 & 0 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

6. (a) (5 Points) How many rows and columns must a matrix A have if it is the standard matrix for a transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$? Briefly explain your answer.

(b) (5 Points) If the mapping above is one-to-one, how many pivot columns must it have? Briefly explain.

7. (20 Points) Determine whether the following statements are true or false, and provide a brief explanation why.

   (a) If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are vectors in $\mathbb{R}^2$ then one of the vectors is a linear combination of the other two.

   (b) Elementary matrices have the same number of rows and columns.

   (c) The determinant of $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ is $wz - yx$.

   (d) A linear transformation $T : \mathbb{R}^a \to \mathbb{R}^b$ is onto if its range is $\mathbb{R}^b$.

   (e) If A, B, and C are all $n \times n$ matrices, then $AB = AC$ implies $B = C$.