

Instructions: Read the entire statement of each problem. Solve each problem carefully and organize your work. Be sure to write your answers in complete sentences where appropriate. The exam is worth 100 points.

1. (4 Points) State the word-for-word definition of what it means for an integer n to be odd.

2. (15 Points) Write a know-show table for the following proposition. **Your know-show table must be neat, give yourself adequate room to write!**

If n is an odd integer, then $-2n^2 + 5n + 4$ is an odd integer.

3. (15 Points) The following problems are related. Answer them below. **Your truth tables must be readable, give yourself plenty of room and use the back of the page if you need to.**
- (a) Write a truth-table for the statement $\neg(P \vee Q)$
 - (b) Write a truth-table for the statement $P \rightarrow \neg Q$
 - (c) Which of the previous statements is equivalent to $\neg P \wedge \neg Q$? **Explain your answer.**

4. (16 Points) Answer the questions below for the following statements:

P is “It is getting late”

Q is “I am not tired”

R is “I will go to bed”

(a) Translate the sentence “If am tired and it is getting late then I will go to bed.” into symbols.

(b) Translate the symbolic statement “ $(R \wedge P) \rightarrow \neg Q$ ” into a readable English sentence.

(c) Translate the symbolic statement “ $\neg R \leftrightarrow (Q \vee \neg P)$ ” into a readable English sentence.

(d) If Q is false and P and R are true, which of the statements above are true? Justify your answer.

5. (15 Points) Consider the statement “If x is a positive real number, then $x^2 - 2x > 0$. ”

(a) Write the converse of this statement.

(b) Write the contrapositive of this statement.

(c) The value of $x = -1$ demonstrates which of the three statements above to be false? **Explain your answer.**

6. (15 Points) The following questions all regard sets and set-builder notation.

(a) Write the set consisting of “natural numbers whose square is less than 50” using both the roster method and set-builder notation.

(b) Write the set consisting of “integers which are multiples of 10” using both the roster method and set builder notation.

(c) Is the following statement true or false? Explain your answer.

$$-5 \in \{4n - 1 \mid n \in \mathbb{Z}\}$$

7. (20 Points) Consider the following statement:

$$(\forall x \in \mathbb{R})(\exists y \in \mathbb{R})(xy = 1)$$

- (a) Write the statement out in a readable English sentence.

- (b) Write the negation of the statement in symbols without using the negation symbol.

- (c) Write the negation of the statement in a readable english sentence.

- (d) Which is true, the original statement or its negation? **Explain your answer.**