October 28, 2021 MAT 272: Exam #2 Name: _____

Instructions: Read the entire statement of each problem. Solve each problem carefully and organize your work. Be sure to include units and write your answers in complete sentences where appropriate. The exam is worth 100 points.

1. (16 Points) Consider the matrices below:

$$A = \begin{bmatrix} 5 & -1 \\ 2 & 0 \\ 8 & 4 \end{bmatrix} \quad B = \begin{bmatrix} \frac{1}{2} & 0 \\ -3 & 10 \\ 6 & 7 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 8 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

If possible, compute the following. If not possible, explain why.

(a) $2A^T - 4C$

(b) CB

(c) $B^T A$

(d) $4C - 8B$

2. (12 Points) Determine if the following matrices are invertible. Explain your reasoning. (You do not need to determine the inverses).

   (a) A 5×5 matrix A for which the equation $A\mathbf{x} = \mathbf{0}$ only has the trivial solution

   (b) $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 2 \\ 0 & 14 & 7 \end{bmatrix}$

   (c) The standard matrix of a one-to-one transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$.

3. (10 Points) Find the inverse of the matrix below:

4. $B = \begin{bmatrix} 2 & 0 & 6 \\ 2 & 1 & 0 \\ 3 & 0 & 3 \end{bmatrix}$

5. (16 Points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation which rotates vectors counterclockwise by $\pi/2$ radians then horizontally stretches them by a factor of 3.

   (a) Find the standard matrix for T.

   (b) Find the standard matrix for T^{-1}

   (c) Is T^{-1} one-to-one? Explain.

   (d) Give a geometric description of how T^{-1} transforms vectors.

6. (10 Points) Compute the determinant of the matrix below:

$$B = \begin{bmatrix} 3 & 1 & 4 \\ 6 & 1 & -2 \\ \frac{1}{2} & 0 & 5 \end{bmatrix}$$

7. (10 Points) Use inverse matrices to solve the following system of equations:

$$\begin{aligned} \frac{1}{2}x_1 + x_2 &= \frac{3}{2} \\ -\frac{1}{6}x_1 - \frac{2}{3}x_2 &= -\frac{5}{3} \end{aligned}$$

8. (10 Points) Let $T : \mathbb{R}^7 \to \mathbb{R}^6$ be a linear transformation.

   (a) If A is the standard matrix for T, how many rows and columns must A have?

   (b) How many pivot columns must A have in order for T to be an onto transformation? Explain your answer using complete sentences.

9. (10 Points) Compute the determinant of the matrix below:

$$A = \begin{bmatrix} 1 & 0 & 7 & 3 & 5 \\ 0 & 0 & -4 & 3 & 8 \\ 3 & -2 & -5 & 0 & -1 \\ -2 & 0 & -3 & -4 & 6 \\ 0 & 0 & 3 & 0 & 0 \end{bmatrix}$$

10. (16 Points) Determine whether the following statements are true or false. If true, provide a brief explanation why. If false, explain why or provide a counterexample.

(a) If the transpose of a matrix A is invertible, then A is invertible.

(b) Triangular matrices always have a nonzero determinant.

(c) If A, B and C are all square matrices of the same size and $AC = BC$, then $A = B$. .

(d) Invertible matrices always have a nonzero determinant.

(e) Every elementary matrix is invertible.