MAT 115, Midterm 2α

October 23, 2020

Name: _____   C# : _____

> Answer the questions in the spaces provided on the question sheets. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but also how you obtained it. Include units in your answer when possible. You may receive 0 points for a problem where you show no work.

Instructions:

1. Do not open this exam until told to do so.

2. <u>No</u> books or notes may be used on the exam.

3. Credit or partial credit will be given <u>only</u> when the appropriate explanation and/or <u>algebra</u> is shown.

4. Make sure your answer is clearly marked.

5. Read and follow directions carefully.

6. This exam has 9 questions, for a total of 100 points. There are 7 pages. Make sure you have them all.

7. You will have <u>50 minutes</u> to complete the exam.

8. All cell phones and electronic devices (other than calculators) must be turned off during the exam.

9. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.

10. You may only use an *approved* calculator on the exam. No calculators with a CAS, QWERTY keyboards, or graphing utilities are allowed.

11. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

1. [12 points] Consider the composition of functions

$$f(g(x)) = \left(\frac{2^{x-1} - 3}{6}\right)^2$$

(a) If $g(x) = x - 1$, what should $f(x)$ be?

(b) If $f(x) = \left(\frac{x}{6}\right)^2$, what should $g(x)$ be?

(c) If $g(x) = 2^{x-1}$ what should $f(x)$ be?

2. [16 points] The price for a chartered bus to New York City from Cortland goes down as more people buy tickets. When n people purchase tickets, the price per ticket is given by the function

$$P(t) = \frac{420}{15 + t}$$

(a) What is the price per ticket when 15 people are riding the bus? *Write your interpretation in a complete sentence with units.*

(b) How many people will it take for tickets to be 10 dollars? *Write your interpretation in a complete sentence with units.*

(c) Find a formula for $P^{-1}(t)$.

(d) Calculate and interpret $P^{-1}(21)$. *Write your interpretation in a complete sentence with units.*

3. [15 points] Jack plays basketball, and takes his training very seriously. He notices that the points $S(t)$ he scores in a game is a function of the number of hours t he practices the week before, and is given by

$$S(t) = -0.25t^2 + 7t \quad (0 \le t \le 28)$$

(a) Evaluate and interpret $S(2)$. *Write your interpretation in a complete sentence with units.*

(b) How long should Jack practice to maximize his score? What is the maximum points Jack can hope to score in a game? *Write your interpretation in a complete sentence with units.*

(c) Write $S(t)$ in vertex form.

4. [10 points] For the following polynomials, state their degree and end behavior (written in the form "as $x \to \infty$...").

  (a) $f(x) = 5x - 12x^2 - 30x^6 + 100$

  (b) $g(x) = \frac{1}{2}x^{10}(x+1)^3(x-4)^2$.

5. [8 points] Find the vertical and horizontal asymptotes of the following rational function:

$$f(x) = \frac{x^2 + 5x + 6}{x^3 - x^2 - 12x}$$

6. [12 points] Consider the polynomial below:

$$p(x) = x^3 - 3x^2 + 25x - 75$$

(a) What are the possible rational zeros of $p(x)$?

(b) Given that $x = 5i$ is a zero of $p(x)$, what are the other zeros? Show all work.

7. [8 points] Find the formula for the exponential function that passes through the points $(5, 16)$ and $(-2, \frac{1}{8})$.

8. [10 points] Suppose Carly has \$13,000 that she will invest for a period of 6 years at an interest rate r. Interest will be compounded monthly.

   (a) How much money will be in the account if the interest rate is 3%? *Write your interpretation in a complete sentence with units.*

   (b) What should the interest rate be if Carly hopes to have \$18,000 at the end of the investment? *Write your interpretation in a complete sentence with units.*

9. [9 points] Come up with a possible formula for the graph below: