Instructions: Solve each problem carefully on separate paper. To receive full credit, you must show all work. In addition, your work must be organized, legible, and include units and complete sentences where appropriate. Please staple your work if you use multiple pages.

1. Use row operations to put the following matrix in reduced echelon form:

$$\left[\begin{array}{ccccc}
1 & 3 & 5 & 7 \\
3 & 5 & 7 & 9 \\
6 & 7 & 9 & 1
\end{array}\right]$$

2. Find the general solution of the following system of equations:

$$x_1 - 2x_2 - x_3 = 3$$

 $3x_1 - 6x_2 - 2x_3 = 2$

3. The following types of matrices are augmented matrices representing linear system. For each, answer the following: Is the system consistent? If so, does it have a unique solution? Explain your answers. (Here ■ represents a nonzero value, and * represents any possible value)

(a)
$$\begin{bmatrix} \blacksquare & * & * & * \\ 0 & \blacksquare & * & * \\ 0 & 0 & \blacksquare & * \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0 & \blacksquare & * & * & * \\ 0 & 0 & \blacksquare & * & * \\ 0 & 0 & 0 & 0 & \blacksquare \end{bmatrix}$$

4. Consider the following system of equations:

$$x_1 + hx_2 = 2$$

$$4x_1 + 8x_2 = k$$

Find values for h and k such that the system

- (a) has no solution
- (b) has a unique solution
- (c) has many solutions.