October 11, 2019 # Properties of Determinants Name: ____________________

Instructions: Carefully read each problem. Solve each problem with care and organize your work. Be sure to include units and write answers in complete sentences where appropriate.

1. Find the determinant of the following matrix using row reduction to echelon form.

$$\begin{bmatrix} 1 & 3 & 2 & 4 \\ 0 & 1 & 2 & -5 \\ 2 & 7 & 6 & -3 \\ -3 & -10 & -7 & 2 \end{bmatrix}$$

2. Combine the methods of row reduction and cofactor expansion to compute the determinant of the matrix below:

$$\begin{bmatrix} 1 & 5 & 4 & 1 \\ 0 & -2 & -4 & 0 \\ 3 & 5 & 4 & 1 \\ -6 & 5 & 5 & 0 \end{bmatrix}$$

3. Find the determinants below if

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$$

(a) $\begin{vmatrix} a & b & c \\ d & e & f \\ 5g & 5h & 5i \end{vmatrix}$

(b) $\begin{vmatrix} d & e & f \\ a & b & c \\ g & h & i \end{vmatrix}$

(c) $\begin{vmatrix} g & h & i \\ 6d & 6e & 6f \\ 2a & 2b & 2c \end{vmatrix}$

4. Use determinants to decide if the following three vectors are linearly dependent.

$$\begin{bmatrix} 4 \\ 6 \\ 2 \end{bmatrix}, \begin{bmatrix} -7 \\ 0 \\ 7 \end{bmatrix}, \begin{bmatrix} -3 \\ -5 \\ -2 \end{bmatrix}.$$

5. In general, for two square matrices of the same size, $\det(A+B) \neq \det(A)+\det(B)$. Find an examples of 2×2 matrices that demonstrate this fact.

6. Consider the matrix

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

(a) Compute $\det(B^5)$.

(b) Compute $\det(B^{-1})$.

7. Show that for two square matrices A and B, even though $AB \neq BA$ in general, it is always the case that $\det(AB) = \det(BA)$.

8. Let A and P be matrices with P invertible. Show that $\det(PAP^{-1}) = \det(A)$.

9. Find a formula for $\det(rA)$ where A is an $n \times n$ matrix.