

MAT 115 Final Exam

Dec. 13, 2021

Name: _____

C# : _____

Answer the questions in the spaces provided on the question sheets. Show an appropriate amount of work (including appropriate explanation) for each problem, so that graders can see not only your answer but also how you obtained it. Include units in your answer when possible. You may receive 0 points for a problem where you show no work.

Instructions:

1. Do not open this exam until told to do so.
2. No books or notes may be used on the exam.
3. Credit or partial credit will be given only when the appropriate explanation and/or algebra is shown.
4. Make sure your answer is clearly marked.
5. Read and follow directions carefully.
6. This exam has 11 questions, for a total of 100 points. There are 12 pages. Make sure you have them all.
7. You will have 120 minutes to complete the exam.
8. All cell phones and electronic devices (other than calculators) must be turned off during the exam.
9. Do not separate the pages of this exam. If they do become separated, write your name on every page and point this out to your instructor when you hand in the exam.
10. You may only use an *approved* calculator on the exam. No calculators with a CAS, QWERTY keyboards, or graphing utilities are allowed.
11. If you use graphs or tables to find an answer, be sure to include an explanation and sketch of the graph, and to write out the entries of the table that you use.

1. [10 points] Lyra deposits \$23,000 in a bank account. The account has an annual interest rate of 2.1% compounded **continuously**.

(a) How much money will be in the account after 3 years? *Write your interpretation in a complete sentence with units.*

(b) How many years will it take for her to double her savings? *Write your interpretation in a complete sentence with units.*

2. [10 points] Pablo is playing basketball and shoots a half-court shot at the buzzer. The height of the basketball after t seconds is given by the function $h(t) = -7t^2 + 14t + 6$
- (a) In a standard NBA court, the rim of the hoop is 10 feet in the air. If Pablo makes the shot, how long will it take the ball to reach the hoop? Round your answer to the nearest hundredth. *Write your interpretation in a complete sentence with units.*
- (b) Find when the basketball reaches its maximum height by completing the square. You must complete the square to receive credit. *Write your interpretation in a complete sentence with units.*

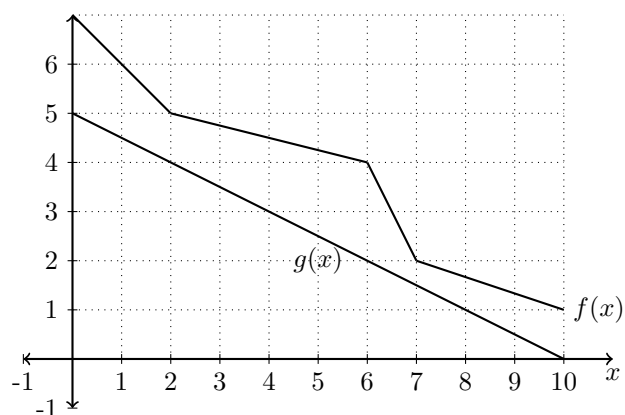
3. [12 points] This question is about lines.

(a) Find the equation for the horizontal line through $(2, 8)$.

(b) Find the equation for the line through the points $(1, 6)$ and $(3, -2)$.

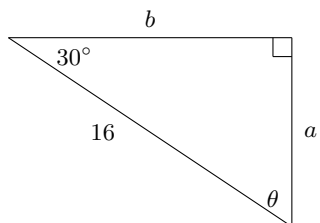
(c) Find the equation for the line parallel to the line in part (b) through the point $(0, 4)$.

4. [10 points] Use the graphs of $f(x)$ and $g(x)$, shown below, to answer parts (a)-(e).

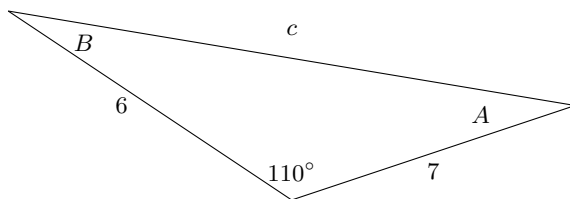


- (a) Evaluate $f(0)$.
- (b) Evaluate $f(g(6))$.
- (c) Evaluate $g(f(6))$.
- (d) Find x so that $f^{-1}(x) = 2$.
- (e) Evaluate $g^{-1}(8)$.

5. [10 points] (a) Using the triangle sketched below, find the length of side a and the measure in degrees of the angle θ . **Give your answers in exact form, do not round to a decimal.**



- (b) Using the triangle sketched below, find the side length c . **Round your answers to the nearest hundredth.**



6. [12 points] Suppose $f(x) = (x - 3)(2x + 1)^2$ and $g(x) = x^2 - 9$.

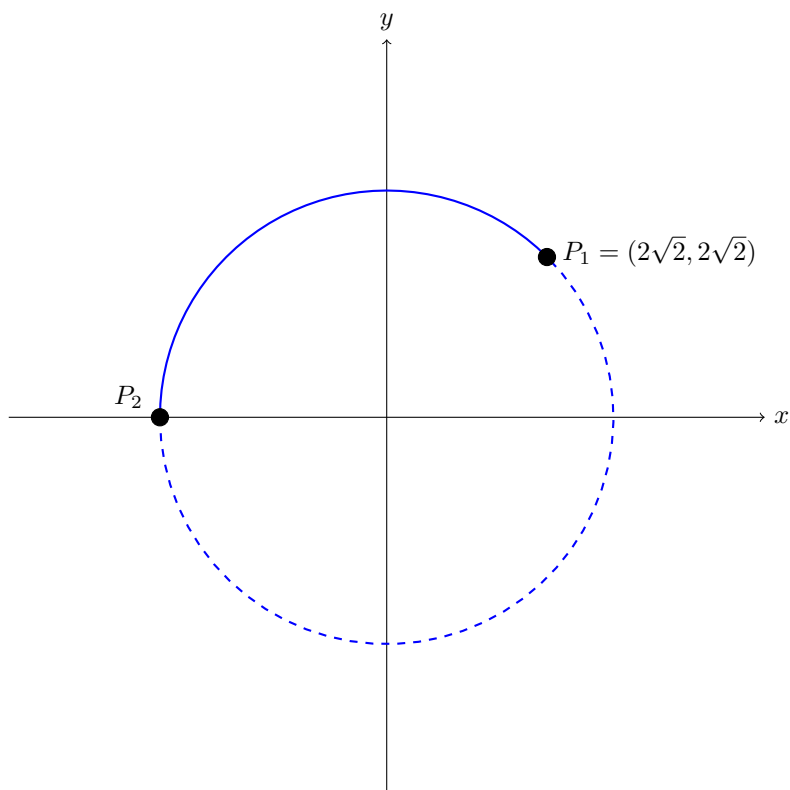
(a) What are the zeros of $f(x)$ and $g(x)$?

(b) What is the degree of $f(x)$?

(c) Let $H(x) = \frac{f(x)}{g(x)}$. Does the graph of $H(x)$ have any vertical asymptotes? If so, list all vertical asymptotes.

(d) Does the graph of $H(x)$ have any holes? If so, list the x -values of all holes.

7. [9 points] Consider the two points P_1 and P_2 whose Cartesian coordinates are given below.



- (a) What are the polar coordinates of P_1 ?
- (b) What are the polar coordinates of P_2 ?
- (c) Using your answers for parts (a) and (b), find the length of the solid (not dashed) arc between the points P_1 and P_2 .

8. [8 points] Suppose that $\cos(\theta) = \frac{5}{13}$ and θ is **in the fourth quadrant**. Use trigonometric identities to find the **exact** values of the following. (Note: There is a page with identities at the back of the exam.)
- (a) $\sin(\theta)$

(b) $\sin(2\theta)$

9. [5 points] Simplify the following as much as possible using fundamental trigonometric identities. **Show every step in order to receive credit.**

$$\frac{\cos \theta}{1 - \sin \theta} - \sec \theta \quad [\text{Hint: This can be reduced to an expression without fractions.}]$$

10. [5 points] Find the inverse of the function $f(x) = 4x - 1$.

11. [9 points] Consider the polynomial function $f(x) = x^3 + 2x^2 + 9x + 18$

- (a) What are the possible rational roots of f according to the rational root test? **You do not need to determine which values are roots.**

- (b) Given that $x = -2$ is a zero of f , find the remaining zeros of f . **You must show all work to receive credit.**

Additional Materials: This page contains a list of identities for you to reference.

Pythagorean Identities	$\sin^2(\theta) + \cos^2(\theta) = 1$ $1 + \tan^2(\theta) = \sec^2(\theta)$ $1 + \cot^2(\theta) = \csc^2(\theta)$
Double-Angle Formulae	$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$ $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ $\tan(2\theta) = \frac{2 \tan(\theta)}{1 - \tan^2(\theta)}$
Half-Angle Formulae	$\sin^2(\theta/2) = \frac{1}{2}(1 - \cos(\theta))$ $\cos^2(\theta/2) = \frac{1}{2}(1 + \cos(\theta))$
Angle-Sum Formulae	$\sin(A + B) = \sin(A) \cos(B) + \cos(A) \sin(B)$ $\cos(A + B) = \cos(A) \cos(B) - \sin(A) \sin(B)$
Angle-Difference Formulae	$\sin(A - B) = \sin(A) \cos(B) - \cos(A) \sin(B)$ $\cos(A - B) = \cos(A) \cos(B) + \sin(A) \sin(B)$
Law of Sines	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

This unit circle is not for a grade, but is intended to help you on the exam.

