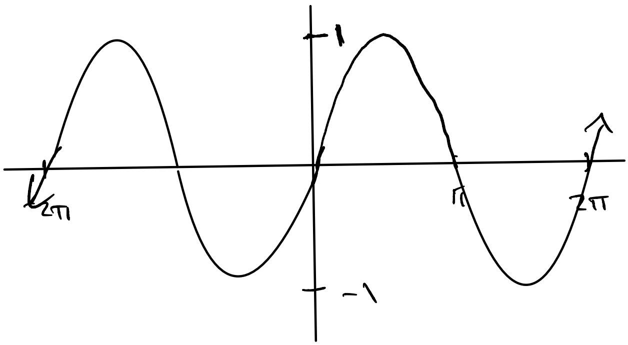
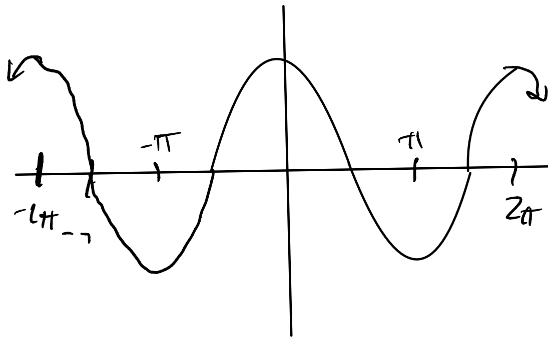
Announcements

1. My Math Lab #10 } Due Monday @ 8AM
2. Written Assignment #10 }
3. Exam 3: Next Friday 11/20



Sine: $y = \sin x$



$y = \cos x$

Goal for today: Graph functions of the form

$$y = A\sin(B(x-h)) + k \quad \text{or} \quad y = A\cos(B(x-h)) + k$$

Def: The graphs of sine, cosine and their transformations are called sinusoidal curves (or sine waves).

Amplitude – Loosely: How tall/deep are the waves?

Def: Let f be a periodic function and M be the maximum value of f and m be the minimum value of f then the amplitude of f is

$$|A| = \text{amplitude} = \frac{1}{2}(M - m)$$

(Note: amplitude is always positive)

E.g. $y = \sin x$

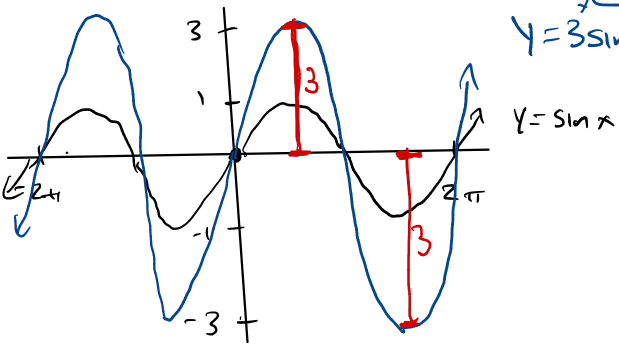
The maximum value is $M = 1$, min. value is $m = -1$, So the amplitude is $\frac{1}{2}(1-(-1)) = \frac{1}{2}\cdot 2 = 1$

E.g. $y = 3\sin x$

Vertical stretch by a factor of 3

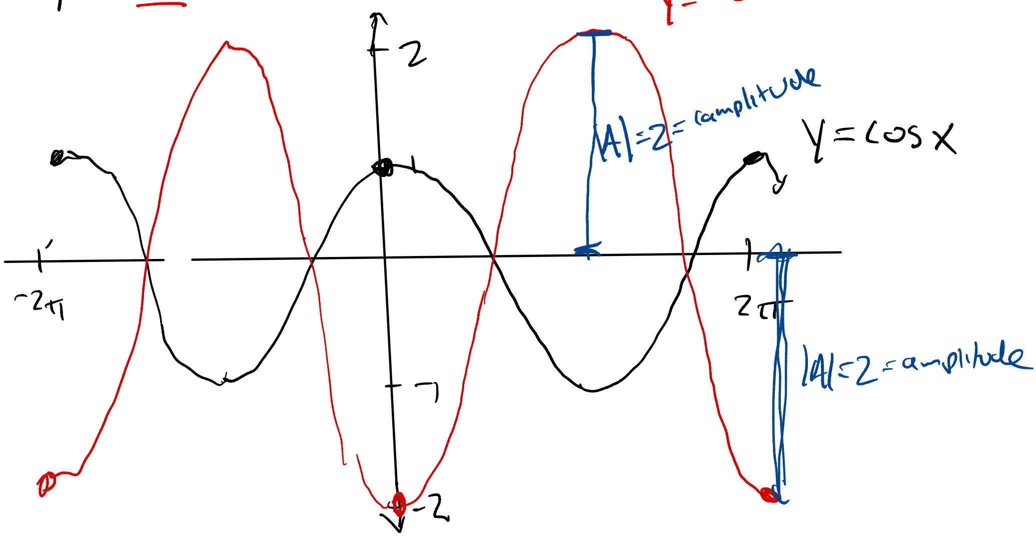
$\boxed{|A|=3}$

$y = 3\sin x \begin{cases} M = 3 \\ m = -3 \end{cases}$ amplitude $= \frac{1}{2}(3-(-3)) = \boxed{3}$



vertical stretch and reflect

E.g. $y = -2\cos x$



Here $A = -2 \Rightarrow |A| = 2$

Period under transformations

A Horizontal stretch/compression $\Longleftrightarrow$ increase/decrease the period.

Consider the function $y = \sin(Bx)$ or $y = \cos(Bx)$

The period of $y = \sin Bx$ should start when $Bx = 0$

and end when $|Bx| = 2\pi \Rightarrow |B|x = 2\pi$

$$\underbrace{0 \leq |B|x < 2\pi}_{\text{one full period}} \Rightarrow \underbrace{0 \leq x \leq \frac{2\pi}{|B|}}_{\text{one full period in transformed function}}$$

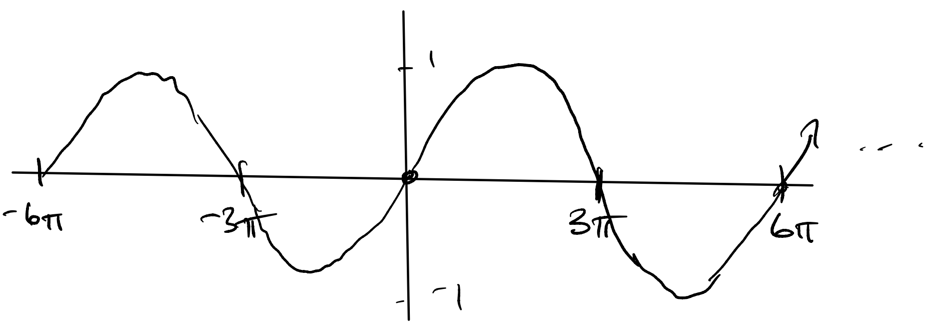
★ The period of $y = A\sin(B(x-h)) + k$ (or the cosine version)

is given by $\boxed{\text{period} = \frac{2\pi}{|B|}}$

E.g. $y = \sin\left(\frac{1}{3}x\right)$

Horizontal stretch, period should be longer than 2π

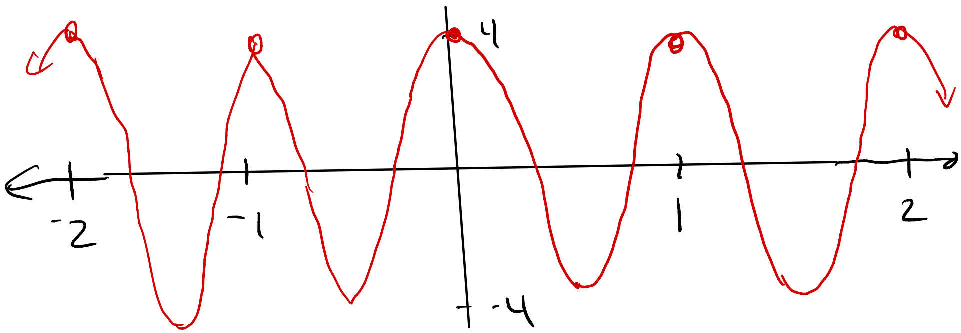
$$\text{period} = \frac{2\pi}{1/3} = \frac{2\pi}{1} \cdot \frac{3}{1} = 6\pi$$



E.g. $y = 4\cos(2\pi x)$

$A = 4 \Rightarrow$ amplitude is 4

$B = 2\pi \Rightarrow$ period is $\frac{2\pi}{2\pi} = 1$



<u>Midline</u>: The horizontal line in the middle of the highest point and lowest point is called the <u>midline</u>

The graph of $y = A\sin(B(x-h)) + k$ (or the cosine version)

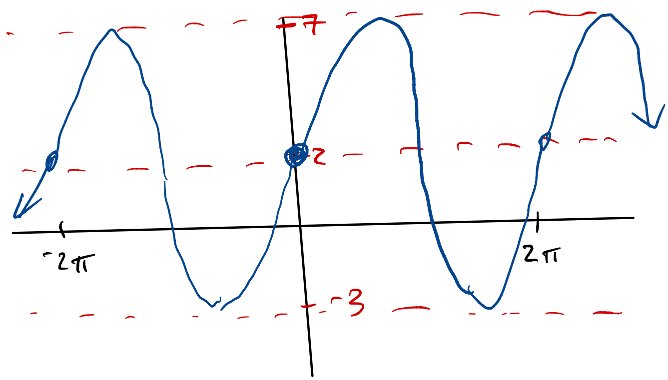
has midline $\boxed{y = k}$

<u>Ex</u>: $y = 5\sin(x) + 2$

$A = 5 \Rightarrow$ amplitude is 5

$B = 1$ period is 2π

$k = 2 \Rightarrow$ midline is $y = 2$



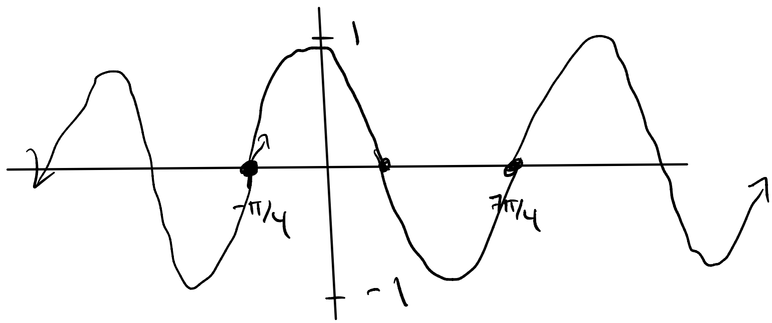
Phase Shift: A horizontal shift of a sinusoidal curve is called a phase shift.

$$y = A\sin(B(x-h)) + k$$ (or the cosine version)

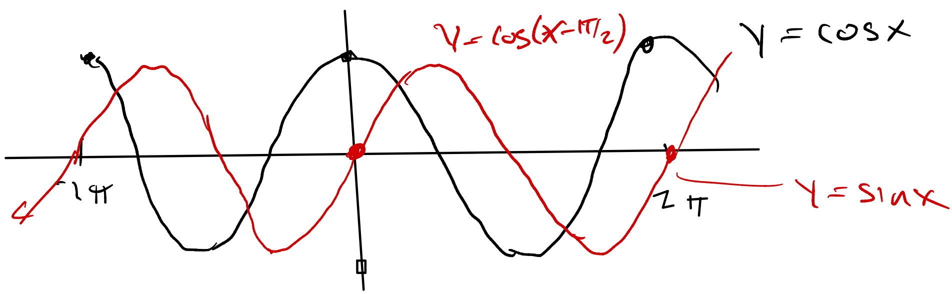
h is the phase shift

Ex. $y = \sin(x + \pi/4) = \sin(x - (-\pi/4))$

$h = -\pi/4$, shift left $\pi/4$ units



$y = \cos(x - \pi/2)$ ← shift $y = \cos x$ right by $\pi/2$



The graph of $y = \sin x$ is the same as $y = \cos(x - \pi/2)$

$y = \sin x$ is a shift of $y = \cos x$ and vice versa

★ Graphing $y = A\sin(B(x-h)) + k$ OR $y = A\cos(B(x-h)) + k$

① amplitude is $|A|$

② period is $2\pi/|B|$

③ midline is $y = k$

④ phase shift is h

If sine:

$A > 0$: Start at midline and go up

$A < 0$: Start at midline and go down

If cosine:

$A > 0$: Start at top

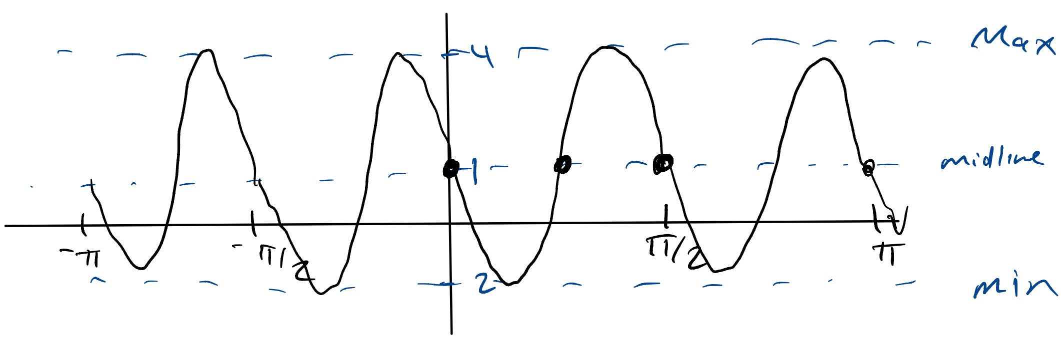
$A < 0$: Start at bottom

E.g. $y = -3\sin(4x) + 1$

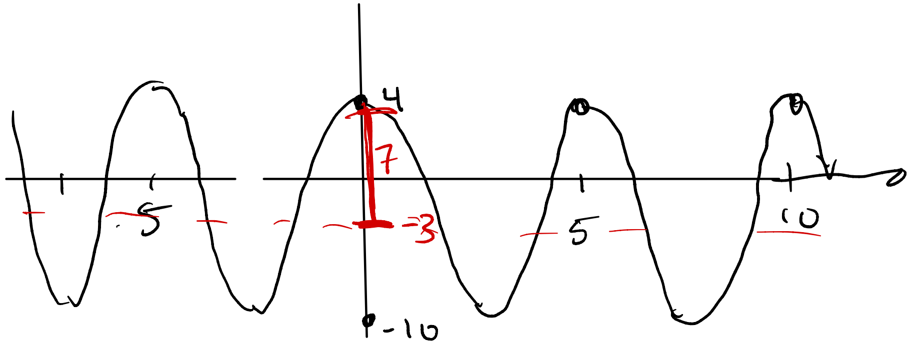
- $\boxed{\text{Amplitude is } 3}$
- period is $\frac{2\pi}{4} = \boxed{\pi/2 = \text{period}}$
- $h = 0$, no phase shift
- midline is $\boxed{y = 1}$

Sine: $A < 0$

Start at midline and go down.



Find a formula for the following graph:



- use $\boxed{\text{cosine}}$ because it starts at top
- Amplitude: $M=4$, $m=-10$ $\Rightarrow$ $|A|=\frac{1}{2}(4-(-10))$
  $= \frac{1}{2}(14) = 7$

  so $\boxed{A=7}$
- Period is $5 = \frac{2\pi}{|B|}$ $\Rightarrow$ $\boxed{B=\frac{2\pi}{5}}$
- midline is $y=-3$ $\Rightarrow$ $\boxed{k=-3}$
- no phase shift so $\boxed{h=0}$

$$y = A\cos(B(x-h))+k = 7\cos\left(\frac{2\pi}{5}x\right)-3$$