Due: September 18 # Mat 370: Written Assignment #3

**Instructions:** Solve each problem carefully on separate paper. To receive full credit, you must show all work and justify your answers. In addition, your work must be organized, legible, and include units and complete sentences where appropriate. Please scan your work and submit it to Blackboard, with your name included in the file name, e.g. "Packuskas HW 3.pdf"

1. Find all solutions in $\mathbb{C}$ to the following equations:
   (a) $z^6 = 1$
   (b) $z^3 = -8$
2. Compute the following expressions using modular addition:
   (a) $\frac{7\pi}{4} +_{2\pi} \pi$
   (b) $10 +_{12} 9$
   (c) $20.5 +_{25} 19.3$
   (d) $3 +_9 6$
3. There is an isomorphism of the 7th roots of unity $U_7$ with $\mathbb{Z}_7$ in which $\zeta = e^{i(2\pi/7)} \leftrightarrow 4$. Find the elements in $\mathbb{Z}_7$ to which $\zeta^m$ must correspond for $m = 0, 2, 3, 4, 5$ and 6. [Hint: in such an isomorphism, if $a \leftrightarrow x$ and $b \leftrightarrow y$ for $a, b \in U_7$ and $x, y \in \mathbb{Z}_7$ then $a \cdot b \leftrightarrow x +_7 y$]
4. Determine, with proper justifcation, whether the following binary are commutative and whether they is associative.
   (a) $*$ defined on $\mathbb{Z}$ be letting $a * b = a - b$.
   (b) $\star$ defined on $\mathbb{Q}$ be letting $a \star b = ab + 1$.
   (c) $\bowtie$ defined on $\mathbb{Z}^+$ by setting $a \bowtie b = a^b$.
5. How many different binary operations can be described on a set $S$ with one element? Answer the same question if $S$ has exactly two, exactly three, and exactly $n$ elements. [Hint: One way to solve this is to think about the different possible tables corresponding to a binary operation on $S$]
6. For the following statements, either prove the statement or give a counterexample in the form of a multiplication table:
   (a) Every binary operation on a set consisting of a single element is both commutative and associative.
   (b) Every commutative binary operation on a set consisting of two elements is associative.