

Instructions: Solve each problem carefully on separate paper. To receive full credit, you must show all work. In addition, your work must be organized, legible, and include units and complete sentences where appropriate. Please staple your work if you use multiple pages.

1. Describe the solutions of the following system of linear equations in parametric vector form. Then, give a geometric description of the solution set.

$$x_1 + 3x_2 - 5x_3 = 4$$

$$x_1 + 4x_2 - 8x_3 = 7$$

$$-3x_1 - 7x_2 + 9x_3 = -6$$

2. Construct a 3×3 nonzero matrix A such that the vector $\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$ is a solution of $A\mathbf{x} = \mathbf{0}$.

3. Is the following set of vectors linearly independent? Justify your answer.

$$\mathbf{v}_1 = \begin{bmatrix} -4 \\ 0 \\ 1 \\ 5 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -3 \\ -1 \\ 0 \\ 4 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 4 \\ 3 \\ 6 \end{bmatrix}.$$

4. How many pivot columns must a 5×7 matrix have if its columns span \mathbb{R}^5 ? Why?