Due: 4/16

Mat 370: Written Assignment #9

Instructions: Solve each problem carefully on separate paper. To receive full credit, you must show all work and justify your answers. In addition, your work must be organized, legible, and include units and complete sentences where appropriate. Please staple your work if you use multiple pages.

1. Let G be a finite group. For each $x \in G$, define the function $\rho_x : G \to G$ by $\rho_x(g) = gx$. Show that ρ_x is a permutation for each $x \in G$.

2. Consider the following permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 7 & 1 & 5 & 3 & 2 & 8 & 4 & 9 \end{pmatrix}$$

   With proper justification, answer the following:

   (a) Write σ as product of disjoint cycles.

   (b) Find the order of σ

   (c) Write σ as a product of transpositions.

   (d) Is σ odd or even?

   (e) Is σ in the alternating group A_9? Why or why not?

3. Suppose you have a deck consisting of 13 cards in order. Find a way to shuffle the cards that you would need to repeat the same shuffle a total of 42 times to get the cards back in order.

4. (a) Prove that if τ is a cycle and that τ is even, then τ^2 is also a cycle. Hint: first consider the length of τ, then think about how it "moves" elements.

   (b) Show by example that if τ is a cycle and τ is odd that τ^2 may not be a cycle.

5. Prove the following: If τ is a transposition, then for any permutation σ, $\sigma\tau\sigma^{-1}$ is a transposition.

6. Explain why that if τ_1, τ_2, τ_3 are (not necessarily distinct) transpositions, then $\tau_1\tau_2\tau_3$ cannot be equal to the identity element ι.

7. Suppose that you are told that the permutation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 1 & 2 & & & 7 & 8 & 9 & 6 \end{pmatrix}$$

   is an even permutation, but the images of 4 and 5 have been lost. What must $\sigma(4)$ and $\sigma(5)$ be?

8. Find the elements of A_3.