Due: Dec. 4

Mat 370: Practice Problems 3

Instructions: Solve each problem carefully on separate paper. To receive full credit, you must show all work and justify your answers. In addition, your work must be organized, legible, and include units and complete sentences where appropriate. Please staple your work if you use multiple pages.

1. Find the following:

   (a) All cosets of $6\mathbb{Z}$ in $\mathbb{Z}$

   (b) All cosets of $\langle 32 \rangle$ in $\mathbb{Z}_{48}$

   (c) All left and right cosets of $\langle \mu_2 \rangle$ in S_3.

2. Let U be a subgroup of V and let $x, y \in V$. Either prove the following statements or provide a counterexample:

   (a) If $xU = yU$ then $Ux^{-1} = Uy^{-1}$.

   (b) If $xU = yU$ then $x^2U = y^2U$. (Hint: think about D_4)

3. Let H be a subgroup of G such that $g^{-1}Hg = H$ for all $g \in G$ and $h \in H$. Show that every left coset gH is the same as the right coset Hg.

4. Show that if $|G| = n$ and $e \in G$ is the identity element, then $a^n = e$ for all $a \in G$.

5. Find the order of

   (a) $(3, 4, 5)$ in the group $\mathbb{Z}_6 \times \mathbb{Z}_{24} \times \mathbb{Z}_{25}$

   (b) $(\mu_3, 3)$ in the group $S_3 \times \mathbb{Z}_9$.

6. Find all abelian groups up to isomorphism of order 540.

7. Let P and Q be groups, and suppose $R = P \times Q$. Then you can view P and Q as subgroups of R by identifying P with $P \times \{e\}$ and Q with $\{e\} \times \{Q\}$, where e is the identity element. In this context, show the following:

   (a) Every element of R is a product of an element from P and an element from Q.

   (b) Viewed as elements of R, every element of P commutes with every element of Q.

   (c) As subgroups of R, the intersection of P and Q is the identity element of R