Due: March 5

Mat 224: Homework #4

Instructions: Solve each problem carefully on separate paper. To receive full credit, you must show all work and justify your answers. In addition, your work must be organized, legible, and include units and complete sentences where appropriate. Please scan your work and submit it to Blackboard.

1. Write a formal proof for the following statements. You do not need to include a know-show table, but it may be helpful to make one first.

   (a) For each $n \in \mathbb{Z}$, if n is odd then n^3 is odd.

   (b) For each integer b, if 5 divides $b+2$, then 5 divides $b^2 - b - 6$.

2. The following two problems are about congruence modulo 9. Let a and b be integers and suppose that $a \equiv 7 \pmod 9$ and $b \equiv 5 \pmod 9$.

   (a) Prove that $a + b \equiv 3 \pmod 9$.

   (b) What should $a \cdot b$ be congruent to modulo 9? State your conjecture formally and then prove it.

3. Use a counterexample to show the following statements are false. Write your explanation formally.

   (a) For each natural number $n, (3 \cdot 2^n + 2 \cdot 3^n + 1)$ is a prime number.

   (b) For all real numbers x and y, $\sqrt{x^2 + y^2} > 2xy$.

4. For the following statements, determine if they are true or false. If true, prove the result. If false, find a counterexample and write a formal explanation.

   (a) If a, b and c are integers and $a \mid (bc)$ then $a \mid b$ or $a \mid c$.

   (b) For each integer n, if 3 divides $n+1$ then 3 divides $n^2 - 1$.