Integration Examples

$$\int \left(5x^3 - x^1 + \frac{1}{x}\right) dx$$

$$= 5 \cdot \frac{x^4}{4} - \frac{x^2}{2} + \ln|x| + C$$

- $\int 1\,dx = x + C$
- $\int a\,dx = ax + C$
- $\int x^n\,dx = ((x^{n+1})/(n+1)) + C\ ;\ n \neq 1$
- $\int \sin x\,dx = -\cos x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int \sec^2 x\,dx = \tan x + C$
- $\int (1/x)\,dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$
- $\int a^x\,dx = (a^x/\ln a) + C\ ;\ a > 0,\ a \neq 1$

$$\int \frac{x+2}{x} dx = \int \left(\frac{x}{x} + 2 \cdot \frac{1}{x}\right) dx$$

$$= \int \left(1 + 2 \cdot \frac{1}{x}\right) dx = x + 2 \cdot \ln|x| + C$$

Prove: $\int \frac{1}{ax+b} dx = \frac{1}{a} \cdot \ln|ax+b| + C$, $a, b \in \mathbb{R}$, $a \neq 0$

Proof.

$$\left(\frac{1}{a} \cdot \ln|ax+b| + C\right)' = \left(\frac{1}{a} \cdot \ln(ax+b) + C\right)'$$

$ax+b > 0$

$$= \frac{1}{a} \cdot \frac{1}{ax+b} \cdot (a) + 0$$

$$= \frac{1}{ax+b}$$

$$\Rightarrow \int \frac{1}{ax+b} dx = \frac{1}{a} \cdot \ln|ax+b| + C$$

$$\int \frac{x}{x+2}\,dx$$

Homework: Try to solve this problem.
Hint: Rewrite the numerator as x + 2 - 2.

Have an amazing day and see you next time !