# Chapter 6

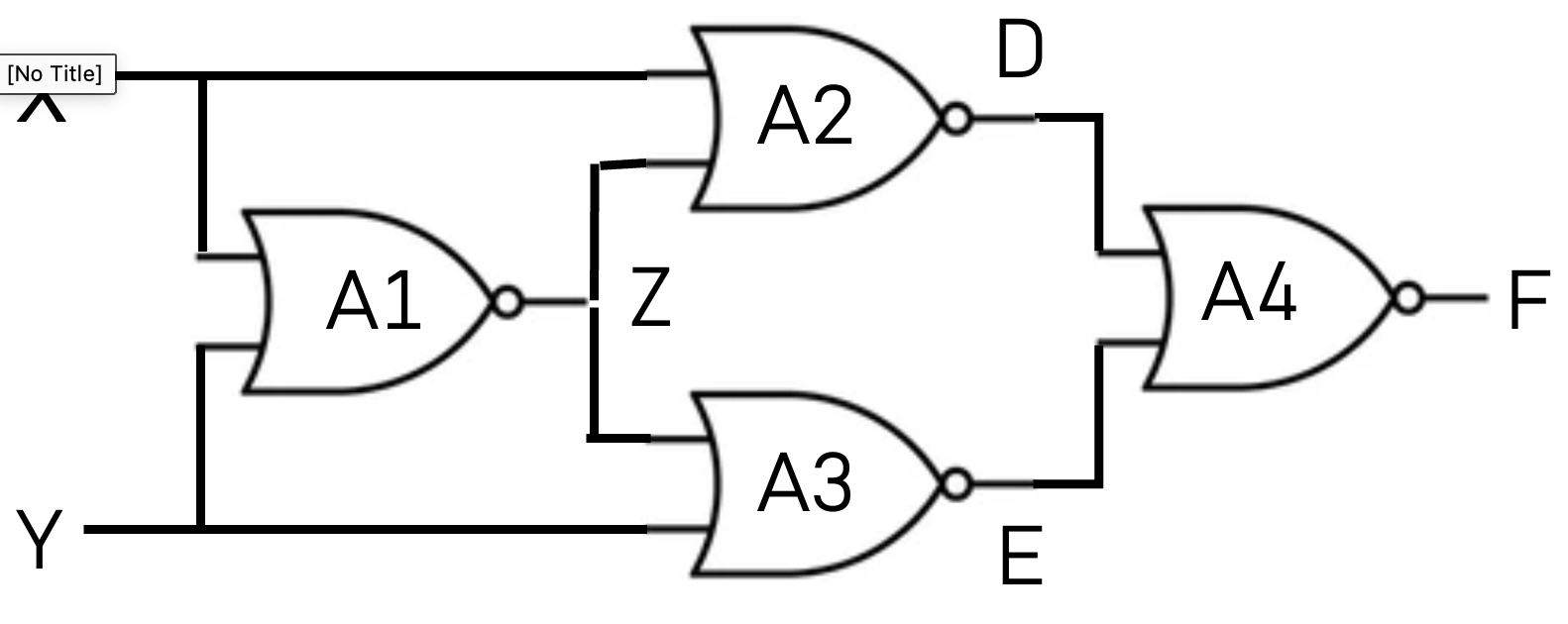
## Belief Propagation with Join Trees

First we import pgmpy libraries

from pgmpy.models import BayesianNetwork, MarkovNetwork  
from pgmpy.inference import BeliefPropagation  
from pgmpy.factors.discrete import TabularCPD  
from pgmpy.factors.discrete import DiscreteFactor  
from pgmpy.base import DAG  
from contextlib import redirect\_stdout  
from pgmpy.models import JunctionTree  
import warnings  
  
warnings.filterwarnings('ignore', category=DeprecationWarning)  
  
warnings.filterwarnings('ignore', category=np.DeprecationWarning) # It will ignore all numpy deprecation warnings

---------------------------------------------------------------------------  
  
NameError Traceback (most recent call last)  
  
Cell In[1], line 12  
 8 import warnings  
 10 warnings.filterwarnings('ignore', category=DeprecationWarning)  
---> 12 warnings.filterwarnings('ignore', category=np.DeprecationWarning) # It will ignore all numpy deprecation warnings  
  
  
NameError: name 'np' is not defined

## Circuit diagnosis Bayesian Network

 #### Create the BN model and specify its structure The following code creates the circuit diagnosis BN model. The model factors according to a DAG having 10 nodes and 12 edges.

The nodes are the following network variables.  
A1 A3 Y A2 X Z E D A4 F The following variables represent hypotheses about the working condition of the four NOR gates of the circuit.  
A1 A2 A3 A4 All network variables are binary taking values 0 and 1.

The directed edges of the DAG are as follows:  
A1->Z A3->E Y ->Z Y ->E A2->D X ->Z X ->D Z ->E Z ->D E ->F D ->F A4->F

def circuit():  
 circuit: BayesianNetwork = BayesianNetwork()  
 circuit.add\_nodes\_from(["X", "Y", "Z", "D", "E", "F", "A1", "A2", "A3", "A4"])  
 circuit.add\_edge("X", "Z")  
 circuit.add\_edge("Y", "Z")  
 circuit.add\_edge("A1", "Z")  
 circuit.add\_edge("X", "D")  
 circuit.add\_edge("Z", "D")  
 circuit.add\_edge("A2", "D")  
 circuit.add\_edge("Z", "E")  
 circuit.add\_edge("Y", "E")  
 circuit.add\_edge("A3", "E")  
 circuit.add\_edge("D", "F")  
 circuit.add\_edge("A4", "F")  
 circuit.add\_edge("E", "F")  
 return circuit

#### Create the Qualitative Component

The following code creates the graph object to represent the BN structure having the nodes and edges specified for the BN model.

def circuit\_dag():  
 model = circuit()  
 G = DAG()  
 nodes = model.nodes  
 edges = model.edges  
 G.add\_nodes\_from(nodes)  
 G.add\_edges\_from(edges)  
 return G  
  
print(f"creating the directed acyclic graph for the circuit diagnosis example")  
G = circuit\_dag()  
print(G)

creating the directed acyclic graph for the circuit diagnosis example

#### Create the Quantitative Component

The following code specifies the condition probability distributions for all the network variables and add the CPDs to the model. After quantifying the BN model for the circuit diagnosis example we check the model if it is specifying correctly. The function check\_model() should return True if the checking is successful.

model = circuit()  
cpd\_X = TabularCPD(variable="X", variable\_card=2, values=[[0.5], [0.5]])  
cpd\_Y = TabularCPD(variable="Y", variable\_card=2, values=[[0.5], [0.5]])  
cpd\_A1 = TabularCPD(variable="A1", variable\_card=2, values=[[0.95], [0.05]])  
cpd\_A2 = TabularCPD(variable="A2", variable\_card=2, values=[[0.95], [0.05]])  
cpd\_A3 = TabularCPD(variable="A3", variable\_card=2, values=[[0.95], [0.05]])  
cpd\_A4 = TabularCPD(variable="A4", variable\_card=2, values=[[0.95], [0.05]])  
cpd\_Z = TabularCPD(  
 variable="Z",  
 variable\_card=2,  
 values=[  
 [1e-9, 1 - 1e-9, 1 - 1e-9, 1 - 1e-9, 0.5, 0.5, 0.5, 0.5],  
 [1 - 1e-9, 1e-9, 1e-9, 1e-9, 0.5, 0.5, 0.5, 0.5],  
 ],  
 evidence=["A1", "X", "Y"],  
 evidence\_card=[2, 2, 2],  
)  
cpd\_D = TabularCPD(  
 variable="D",  
 variable\_card=2,  
 values=[  
 [1e-9, 1 - 1e-9, 1 - 1e-9, 1 - 1e-9, 0.5, 0.5, 0.5, 0.5],  
 [1 - 1e-9, 1e-9, 1e-9, 1e-9, 0.5, 0.5, 0.5, 0.5],  
 ],  
 evidence=["A2", "X", "Z"],  
 evidence\_card=[2, 2, 2],  
)  
cpd\_E = TabularCPD(  
 variable="E",  
 variable\_card=2,  
 values=[  
 [1e-9, 1 - 1e-9, 1 - 1e-9, 1 - 1e-9, 0.5, 0.5, 0.5, 0.5],  
 [1 - 1e-9, 1e-9, 1e-9, 1e-9, 0.5, 0.5, 0.5, 0.5],  
 ],  
 evidence=["A3", "Y", "Z"],  
 evidence\_card=[2, 2, 2],  
)  
cpd\_F = TabularCPD(  
 variable="F",  
 variable\_card=2,  
 values=[  
 [1e-9, 1 - 1e-9, 1 - 1e-9, 1 - 1e-9, 0.5, 0.5, 0.5, 0.5],  
 [1 - 1e-9, 1e-9, 1e-9, 1e-9, 0.5, 0.5, 0.5, 0.5],  
 ],  
 evidence=["A4", "D", "E"],  
 evidence\_card=[2, 2, 2],  
)  
  
model.add\_cpds(cpd\_X, cpd\_Y, cpd\_Z, cpd\_D, cpd\_E, cpd\_F, cpd\_A1, cpd\_A2, cpd\_A3, cpd\_A4, cpd\_F)  
  
print(model.check\_model())

True

markovmodel = model.to\_markov\_model()  
factors = markovmodel.get\_factors()  
for factor in factors:  
 print(factor)

+------+----------+  
| X | phi(X) |  
+======+==========+  
| X(0) | 0.5000 |  
+------+----------+  
| X(1) | 0.5000 |  
+------+----------+  
+------+----------+  
| Y | phi(Y) |  
+======+==========+  
| Y(0) | 0.5000 |  
+------+----------+  
| Y(1) | 0.5000 |  
+------+----------+  
+------+-------+------+------+-----------------+  
| Z | A1 | X | Y | phi(Z,A1,X,Y) |  
+======+=======+======+======+=================+  
| Z(0) | A1(0) | X(0) | Y(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(0) | X(0) | Y(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(0) | X(1) | Y(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(0) | X(1) | Y(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(1) | X(0) | Y(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(1) | X(0) | Y(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(1) | X(1) | Y(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(1) | X(1) | Y(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(0) | X(0) | Y(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(0) | X(0) | Y(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(0) | X(1) | Y(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(0) | X(1) | Y(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(1) | X(0) | Y(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(1) | X(0) | Y(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(1) | X(1) | Y(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(1) | X(1) | Y(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
+------+-------+------+------+-----------------+  
| D | A2 | X | Z | phi(D,A2,X,Z) |  
+======+=======+======+======+=================+  
| D(0) | A2(0) | X(0) | Z(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | X(0) | Z(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | X(1) | Z(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | X(1) | Z(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | X(0) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | X(0) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | X(1) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | X(1) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | X(0) | Z(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | X(0) | Z(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | X(1) | Z(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | X(1) | Z(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | X(0) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | X(0) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | X(1) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | X(1) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
+------+-------+------+------+-----------------+  
| E | A3 | Y | Z | phi(E,A3,Y,Z) |  
+======+=======+======+======+=================+  
| E(0) | A3(0) | Y(0) | Z(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(0) | Y(0) | Z(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(0) | Y(1) | Z(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(0) | Y(1) | Z(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(1) | Y(0) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(1) | Y(0) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(1) | Y(1) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(1) | Y(1) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(0) | Y(0) | Z(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(0) | Y(0) | Z(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(0) | Y(1) | Z(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(0) | Y(1) | Z(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(1) | Y(0) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(1) | Y(0) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(1) | Y(1) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(1) | Y(1) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
+------+-------+------+------+-----------------+  
| F | A4 | D | E | phi(F,A4,D,E) |  
+======+=======+======+======+=================+  
| F(0) | A4(0) | D(0) | E(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(0) | D(0) | E(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(0) | D(1) | E(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(0) | D(1) | E(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(1) | D(0) | E(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(1) | D(0) | E(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(1) | D(1) | E(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(1) | D(1) | E(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(0) | D(0) | E(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(0) | D(0) | E(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(0) | D(1) | E(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(0) | D(1) | E(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(1) | D(0) | E(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(1) | D(0) | E(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(1) | D(1) | E(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(1) | D(1) | E(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
+-------+-----------+  
| A1 | phi(A1) |  
+=======+===========+  
| A1(0) | 0.9500 |  
+-------+-----------+  
| A1(1) | 0.0500 |  
+-------+-----------+  
+-------+-----------+  
| A2 | phi(A2) |  
+=======+===========+  
| A2(0) | 0.9500 |  
+-------+-----------+  
| A2(1) | 0.0500 |  
+-------+-----------+  
+-------+-----------+  
| A3 | phi(A3) |  
+=======+===========+  
| A3(0) | 0.9500 |  
+-------+-----------+  
| A3(1) | 0.0500 |  
+-------+-----------+  
+-------+-----------+  
| A4 | phi(A4) |  
+=======+===========+  
| A4(0) | 0.9500 |  
+-------+-----------+  
| A4(1) | 0.0500 |  
+-------+-----------+

markovmodel.check\_model()

True

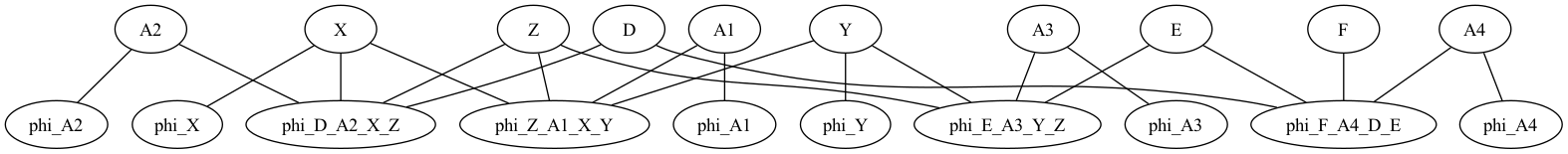
markovmodel.get\_partition\_function()

1.0000000000000002

factor\_graph = markovmodel.to\_factor\_graph()  
print(factor\_graph.nodes)  
print(factor\_graph.edges)

['X', 'Z', 'D', 'Y', 'A1', 'A2', 'E', 'A3', 'F', 'A4', 'phi\_X', 'phi\_Y', 'phi\_Z\_A1\_X\_Y', 'phi\_D\_A2\_X\_Z', 'phi\_E\_A3\_Y\_Z', 'phi\_F\_A4\_D\_E', 'phi\_A1', 'phi\_A2', 'phi\_A3', 'phi\_A4']  
[('X', 'phi\_X'), ('X', 'phi\_Z\_A1\_X\_Y'), ('X', 'phi\_D\_A2\_X\_Z'), ('Z', 'phi\_Z\_A1\_X\_Y'), ('Z', 'phi\_D\_A2\_X\_Z'), ('Z', 'phi\_E\_A3\_Y\_Z'), ('D', 'phi\_D\_A2\_X\_Z'), ('D', 'phi\_F\_A4\_D\_E'), ('Y', 'phi\_Y'), ('Y', 'phi\_Z\_A1\_X\_Y'), ('Y', 'phi\_E\_A3\_Y\_Z'), ('A1', 'phi\_Z\_A1\_X\_Y'), ('A1', 'phi\_A1'), ('A2', 'phi\_D\_A2\_X\_Z'), ('A2', 'phi\_A2'), ('E', 'phi\_E\_A3\_Y\_Z'), ('E', 'phi\_F\_A4\_D\_E'), ('A3', 'phi\_E\_A3\_Y\_Z'), ('A3', 'phi\_A3'), ('F', 'phi\_F\_A4\_D\_E'), ('A4', 'phi\_F\_A4\_D\_E'), ('A4', 'phi\_A4')]

from util.graphics import gviz  
gviz(factor\_graph.nodes, factor\_graph.edges, "factor\_graph")



Factor graph for the circuit Markov Network

#### Get the minimal d-separator set

The following returns the minimal d-separator set between the node ““E” and the node “X” in the directed acyclic graph G for the circuit model.

G.minimal\_dseparator("E", "X")

{'Y', 'Z'}

### Get all d-separations in the DAG

The following code gets all the d-separations in the directed acyclic graph for the BN circuit example and write them in a text file.

print(f"finding all d-separations in the circuit DAG")  
with open("circuit\_dag\_d-separations.txt", "w") as f:  
 with redirect\_stdout(f):  
 print(G.get\_independencies())  
f.close()  
print(  
 f"wrote all d-separations for the circuit DAG into file [circuit\_dag\_d-separations.txt]"  
)

finding all d-separations in the circuit DAG  
wrote all d-separations for the circuit DAG into file [circuit\_dag\_d-separations.txt]

### Get all conditional independencies

The following code gets all the conditional independencies in the circuit BN model and write them to a text file.

print(f"finding all independencies in the circuit BN")  
with open("circuit\_bn\_independencies.txt", "w") as f:  
 with redirect\_stdout(f):  
 print(model.get\_independencies())  
f.close()  
print(  
 f"wrote all independencies for the circuit BN model into file [circuit\_bn\_independencies.txt]"  
)

finding all independencies in the circuit BN  
wrote all independencies for the circuit BN model into file [circuit\_bn\_independencies.txt]

markovmodel.get\_local\_independencies()

(X ⟂ A4, E, A3, F | D, Z, A2, A1, Y)  
(Z ⟂ A4, F | D, A2, A1, A3, X, Y, E)  
(D ⟂ Y, A1, A3 | A4, Z, A2, X, E, F)  
(Y ⟂ D, A4, A2, F | Z, A1, A3, X, E)  
(A1 ⟂ D, A4, A2, A3, E, F | Y, Z, X)  
(A2 ⟂ A4, A1, A3, Y, E, F | D, Z, X)  
(E ⟂ A2, A1, X | D, A4, Z, A3, Y, F)  
(A3 ⟂ D, A4, A2, A1, X, F | Y, Z, E)  
(F ⟂ Z, A2, A1, A3, X, Y | D, A4, E)  
(A4 ⟂ Z, A2, A1, A3, X, Y | D, E, F)

import pgmpy  
import networkx as nx  
  
print(f"pgmpy version: {pgmpy.\_\_version\_\_}")  
print(f"networkx version: {nx.\_\_version\_\_}")

pgmpy version: 0.1.23  
networkx version: 2.5.1

import sys  
  
print(sys.executable)

/Users/yousrielfattah/anaconda3/bin/python

markovmodel = model.to\_markov\_model()  
JT = markovmodel.to\_junction\_tree()

print(JT.factors)

[<DiscreteFactor representing phi(D:2, A4:2, Z:2, X:2, A2:2, E:2, F:2) at 0x1b45102b0>, <DiscreteFactor representing phi(Z:2, A2:2, A1:2, A3:2, X:2, Y:2, E:2, F:2) at 0x1b4512d40>]

print(JT.nodes)

[('Z', 'A2', 'X', 'E', 'F', 'D', 'A4'), ('Z', 'A2', 'X', 'E', 'F', 'A1', 'Y', 'A3')]

G.is\_dconnected(['D', 'A4'], ['A1', 'Y', 'A3'], ['Z', 'A2', 'X', 'E', 'F'])

---------------------------------------------------------------------------  
  
TypeError Traceback (most recent call last)  
  
Cell In[36], line 1  
----> 1 G.is\_dconnected(['D', 'A4'], ['A1', 'Y', 'A3'], ['Z', 'A2', 'X', 'E', 'F'])  
  
  
File ~/anaconda3/lib/python3.10/site-packages/pgmpy/base/DAG.py:582, in DAG.is\_dconnected(self, start, end, observed)  
 556 def is\_dconnected(self, start, end, observed=None):  
 557 """  
 558 Returns True if there is an active trail (i.e. d-connection) between  
 559 `start` and `end` node given that `observed` is observed.  
 (...)  
 580 True  
 581 """  
--> 582 if end in self.active\_trail\_nodes(start, observed)[start]:  
 583 return True  
 584 else:  
  
  
TypeError: unhashable type: 'list'

circuit\_JT = JunctionTree()  
circuit\_JT.add\_nodes\_from([('Z', 'X', 'Y', 'A1'), ('Z', 'X', 'Y', 'E'), ('Z', 'A3', 'Y', 'E'), ('Z', 'X', 'D', 'E'), ('Z', 'X', 'A2', 'D'), ('A4', 'E', 'D', 'F')])  
circuit\_JT.add\_edges\_from([(('Z', 'X', 'Y', 'A1'), ('Z', 'X', 'Y', 'E')),(('Z', 'X', 'Y', 'E'), ('Z', 'A3', 'Y', 'E')),(('Z', 'X', 'Y', 'E'),('Z', 'X', 'D', 'E')),(('Z', 'X', 'D', 'E'),('Z', 'X', 'A2', 'D')),(('Z', 'X', 'D', 'E'),('A4', 'E', 'D', 'F'))])  
  
# circuit\_JT.check\_model()

phi\_A1 = cpd\_A1.to\_factor()  
phi\_A2 = cpd\_A2.to\_factor()  
phi\_A3 = cpd\_A3.to\_factor()  
phi\_A4 = cpd\_A4.to\_factor()  
phi\_X = cpd\_X.to\_factor()  
phi\_Y = cpd\_Y.to\_factor()  
phi\_Z = cpd\_Z.to\_factor()  
phi\_D = cpd\_D.to\_factor()  
phi\_E = cpd\_E.to\_factor()  
phi\_F = cpd\_F.to\_factor()  
  
# print all the variables before performing the product operation  
print(f"phi\_A1: {phi\_A1}")  
print(f"phi\_A2: {phi\_A2}")  
print(f"phi\_A3: {phi\_A3}")  
print(f"phi\_A4: {phi\_A4}")  
print(f"phi\_X: {phi\_X}")  
print(f"phi\_Y: {phi\_Y}")  
print(f"phi\_Z: {phi\_Z}")  
print(f"phi\_D: {phi\_D}")  
print(f"phi\_E: {phi\_E}")  
print(f"phi\_F: {phi\_F}")

phi\_A1: +-------+-----------+  
| A1 | phi(A1) |  
+=======+===========+  
| A1(0) | 0.9500 |  
+-------+-----------+  
| A1(1) | 0.0500 |  
+-------+-----------+  
phi\_A2: +-------+-----------+  
| A2 | phi(A2) |  
+=======+===========+  
| A2(0) | 0.9500 |  
+-------+-----------+  
| A2(1) | 0.0500 |  
+-------+-----------+  
phi\_A3: +-------+-----------+  
| A3 | phi(A3) |  
+=======+===========+  
| A3(0) | 0.9500 |  
+-------+-----------+  
| A3(1) | 0.0500 |  
+-------+-----------+  
phi\_A4: +-------+-----------+  
| A4 | phi(A4) |  
+=======+===========+  
| A4(0) | 0.9500 |  
+-------+-----------+  
| A4(1) | 0.0500 |  
+-------+-----------+  
phi\_X: +------+----------+  
| X | phi(X) |  
+======+==========+  
| X(0) | 0.5000 |  
+------+----------+  
| X(1) | 0.5000 |  
+------+----------+  
phi\_Y: +------+----------+  
| Y | phi(Y) |  
+======+==========+  
| Y(0) | 0.5000 |  
+------+----------+  
| Y(1) | 0.5000 |  
+------+----------+  
phi\_Z: +------+-------+------+------+-----------------+  
| Z | A1 | X | Y | phi(Z,A1,X,Y) |  
+======+=======+======+======+=================+  
| Z(0) | A1(0) | X(0) | Y(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(0) | X(0) | Y(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(0) | X(1) | Y(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(0) | X(1) | Y(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(1) | X(0) | Y(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(1) | X(0) | Y(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(1) | X(1) | Y(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(0) | A1(1) | X(1) | Y(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(0) | X(0) | Y(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(0) | X(0) | Y(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(0) | X(1) | Y(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(0) | X(1) | Y(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(1) | X(0) | Y(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(1) | X(0) | Y(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(1) | X(1) | Y(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| Z(1) | A1(1) | X(1) | Y(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
phi\_D: +------+-------+------+------+-----------------+  
| D | A2 | X | Z | phi(D,A2,X,Z) |  
+======+=======+======+======+=================+  
| D(0) | A2(0) | X(0) | Z(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | X(0) | Z(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | X(1) | Z(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | X(1) | Z(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | X(0) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | X(0) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | X(1) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | X(1) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | X(0) | Z(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | X(0) | Z(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | X(1) | Z(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | X(1) | Z(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | X(0) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | X(0) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | X(1) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | X(1) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
phi\_E: +------+-------+------+------+-----------------+  
| E | A3 | Y | Z | phi(E,A3,Y,Z) |  
+======+=======+======+======+=================+  
| E(0) | A3(0) | Y(0) | Z(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(0) | Y(0) | Z(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(0) | Y(1) | Z(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(0) | Y(1) | Z(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(1) | Y(0) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(1) | Y(0) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(1) | Y(1) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(0) | A3(1) | Y(1) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(0) | Y(0) | Z(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(0) | Y(0) | Z(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(0) | Y(1) | Z(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(0) | Y(1) | Z(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(1) | Y(0) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(1) | Y(0) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(1) | Y(1) | Z(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| E(1) | A3(1) | Y(1) | Z(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
phi\_F: +------+-------+------+------+-----------------+  
| F | A4 | D | E | phi(F,A4,D,E) |  
+======+=======+======+======+=================+  
| F(0) | A4(0) | D(0) | E(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(0) | D(0) | E(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(0) | D(1) | E(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(0) | D(1) | E(1) | 1.0000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(1) | D(0) | E(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(1) | D(0) | E(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(1) | D(1) | E(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(0) | A4(1) | D(1) | E(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(0) | D(0) | E(0) | 1.0000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(0) | D(0) | E(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(0) | D(1) | E(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(0) | D(1) | E(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(1) | D(0) | E(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(1) | D(0) | E(1) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(1) | D(1) | E(0) | 0.5000 |  
+------+-------+------+------+-----------------+  
| F(1) | A4(1) | D(1) | E(1) | 0.5000 |  
+------+-------+------+------+-----------------+

print("Before product operation: ", phi\_X)  
phi\_X.product(phi\_A1) # Performing the product operation  
print("After product operation: ", phi\_X)

Before product operation: +------+----------+  
| X | phi(X) |  
+======+==========+  
| X(0) | 0.5000 |  
+------+----------+  
| X(1) | 0.5000 |  
+------+----------+  
After product operation: +-------+------+-------------+  
| A1 | X | phi(A1,X) |  
+=======+======+=============+  
| A1(0) | X(0) | 0.4750 |  
+-------+------+-------------+  
| A1(0) | X(1) | 0.4750 |  
+-------+------+-------------+  
| A1(1) | X(0) | 0.0250 |  
+-------+------+-------------+  
| A1(1) | X(1) | 0.0250 |  
+-------+------+-------------+

print(phi\_X)

+-------+------+-------------+  
| A1 | X | phi(A1,X) |  
+=======+======+=============+  
| A1(0) | X(0) | 0.4750 |  
+-------+------+-------------+  
| A1(0) | X(1) | 0.4750 |  
+-------+------+-------------+  
| A1(1) | X(0) | 0.0250 |  
+-------+------+-------------+  
| A1(1) | X(1) | 0.0250 |  
+-------+------+-------------+

# [('Z', 'X', 'Y', 'A1'), ('Z', 'X', 'Y', 'E'), ('Z', 'A3', 'Y', 'E'), ('Z', 'X', 'D', 'E'), ('Z', 'X', 'A2', 'D'), ('A4', 'E', 'D', 'F')])  
import numpy as np  
# clique C1: ('Z', 'X', 'Y', 'A1')  
phi\_C1 = phi\_Z  
phi\_C1.product(phi\_X)  
phi\_C1.product(phi\_Y)  
phi\_C1.product(phi\_A1)  
print(phi\_C1)  
# clique C2: ('Z', 'X', 'Y', 'E')  
phi\_C2 = DiscreteFactor(['Z', 'X', 'Y', 'E'], [2, 2, 2, 2], np.ones(16) )  
print(phi\_C2)  
# clique C3: ('Z', 'A3', 'Y', 'E')  
phi\_C3 = phi\_E  
phi\_C3.product(phi\_A3)  
print(phi\_C3)  
# clique C4: ('Z', 'X', 'D', 'E')  
phi\_C4 = DiscreteFactor(('Z', 'X', 'D', 'E'), [2, 2, 2, 2], np.ones(16))  
print(phi\_C4)  
# clique C5: ('Z', 'X', 'A2', 'D')  
phi\_C5 = phi\_D  
phi\_C5.product(phi\_A2)  
print(phi\_C5)  
# clique C6: ('A4', 'E', 'D', 'F')  
phi\_C6 = phi\_F  
phi\_C6.product(phi\_A4)  
print(phi\_C6)

+------+-------+------+------+-----------------+  
| Y | A1 | Z | X | phi(Y,A1,Z,X) |  
+======+=======+======+======+=================+  
| Y(0) | A1(0) | Z(0) | X(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Y(0) | A1(0) | Z(0) | X(1) | 0.2256 |  
+------+-------+------+------+-----------------+  
| Y(0) | A1(0) | Z(1) | X(0) | 0.2256 |  
+------+-------+------+------+-----------------+  
| Y(0) | A1(0) | Z(1) | X(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Y(0) | A1(1) | Z(0) | X(0) | 0.0003 |  
+------+-------+------+------+-----------------+  
| Y(0) | A1(1) | Z(0) | X(1) | 0.0003 |  
+------+-------+------+------+-----------------+  
| Y(0) | A1(1) | Z(1) | X(0) | 0.0003 |  
+------+-------+------+------+-----------------+  
| Y(0) | A1(1) | Z(1) | X(1) | 0.0003 |  
+------+-------+------+------+-----------------+  
| Y(1) | A1(0) | Z(0) | X(0) | 0.2256 |  
+------+-------+------+------+-----------------+  
| Y(1) | A1(0) | Z(0) | X(1) | 0.2256 |  
+------+-------+------+------+-----------------+  
| Y(1) | A1(0) | Z(1) | X(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Y(1) | A1(0) | Z(1) | X(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| Y(1) | A1(1) | Z(0) | X(0) | 0.0003 |  
+------+-------+------+------+-----------------+  
| Y(1) | A1(1) | Z(0) | X(1) | 0.0003 |  
+------+-------+------+------+-----------------+  
| Y(1) | A1(1) | Z(1) | X(0) | 0.0003 |  
+------+-------+------+------+-----------------+  
| Y(1) | A1(1) | Z(1) | X(1) | 0.0003 |  
+------+-------+------+------+-----------------+  
+------+------+------+------+----------------+  
| Z | X | Y | E | phi(Z,X,Y,E) |  
+======+======+======+======+================+  
| Z(0) | X(0) | Y(0) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(0) | Y(0) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(0) | Y(1) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(0) | Y(1) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(1) | Y(0) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(1) | Y(0) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(1) | Y(1) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(1) | Y(1) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(0) | Y(0) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(0) | Y(0) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(0) | Y(1) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(0) | Y(1) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(1) | Y(0) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(1) | Y(0) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(1) | Y(1) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(1) | Y(1) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
+------+------+-------+------+-----------------+  
| Y | E | A3 | Z | phi(Y,E,A3,Z) |  
+======+======+=======+======+=================+  
| Y(0) | E(0) | A3(0) | Z(0) | 0.0000 |  
+------+------+-------+------+-----------------+  
| Y(0) | E(0) | A3(0) | Z(1) | 0.9500 |  
+------+------+-------+------+-----------------+  
| Y(0) | E(0) | A3(1) | Z(0) | 0.0250 |  
+------+------+-------+------+-----------------+  
| Y(0) | E(0) | A3(1) | Z(1) | 0.0250 |  
+------+------+-------+------+-----------------+  
| Y(0) | E(1) | A3(0) | Z(0) | 0.9500 |  
+------+------+-------+------+-----------------+  
| Y(0) | E(1) | A3(0) | Z(1) | 0.0000 |  
+------+------+-------+------+-----------------+  
| Y(0) | E(1) | A3(1) | Z(0) | 0.0250 |  
+------+------+-------+------+-----------------+  
| Y(0) | E(1) | A3(1) | Z(1) | 0.0250 |  
+------+------+-------+------+-----------------+  
| Y(1) | E(0) | A3(0) | Z(0) | 0.9500 |  
+------+------+-------+------+-----------------+  
| Y(1) | E(0) | A3(0) | Z(1) | 0.9500 |  
+------+------+-------+------+-----------------+  
| Y(1) | E(0) | A3(1) | Z(0) | 0.0250 |  
+------+------+-------+------+-----------------+  
| Y(1) | E(0) | A3(1) | Z(1) | 0.0250 |  
+------+------+-------+------+-----------------+  
| Y(1) | E(1) | A3(0) | Z(0) | 0.0000 |  
+------+------+-------+------+-----------------+  
| Y(1) | E(1) | A3(0) | Z(1) | 0.0000 |  
+------+------+-------+------+-----------------+  
| Y(1) | E(1) | A3(1) | Z(0) | 0.0250 |  
+------+------+-------+------+-----------------+  
| Y(1) | E(1) | A3(1) | Z(1) | 0.0250 |  
+------+------+-------+------+-----------------+  
+------+------+------+------+----------------+  
| Z | X | D | E | phi(Z,X,D,E) |  
+======+======+======+======+================+  
| Z(0) | X(0) | D(0) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(0) | D(0) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(0) | D(1) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(0) | D(1) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(1) | D(0) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(1) | D(0) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(1) | D(1) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(0) | X(1) | D(1) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(0) | D(0) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(0) | D(0) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(0) | D(1) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(0) | D(1) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(1) | D(0) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(1) | D(0) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(1) | D(1) | E(0) | 1.0000 |  
+------+------+------+------+----------------+  
| Z(1) | X(1) | D(1) | E(1) | 1.0000 |  
+------+------+------+------+----------------+  
+------+-------+------+------+-----------------+  
| D | A2 | Z | X | phi(D,A2,Z,X) |  
+======+=======+======+======+=================+  
| D(0) | A2(0) | Z(0) | X(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | Z(0) | X(1) | 0.9500 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | Z(1) | X(0) | 0.9500 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(0) | Z(1) | X(1) | 0.9500 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | Z(0) | X(0) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | Z(0) | X(1) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | Z(1) | X(0) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(0) | A2(1) | Z(1) | X(1) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | Z(0) | X(0) | 0.9500 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | Z(0) | X(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | Z(1) | X(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(0) | Z(1) | X(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | Z(0) | X(0) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | Z(0) | X(1) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | Z(1) | X(0) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(1) | A2(1) | Z(1) | X(1) | 0.0250 |  
+------+-------+------+------+-----------------+  
+------+-------+------+------+-----------------+  
| D | A4 | E | F | phi(D,A4,E,F) |  
+======+=======+======+======+=================+  
| D(0) | A4(0) | E(0) | F(0) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A4(0) | E(0) | F(1) | 0.9500 |  
+------+-------+------+------+-----------------+  
| D(0) | A4(0) | E(1) | F(0) | 0.9500 |  
+------+-------+------+------+-----------------+  
| D(0) | A4(0) | E(1) | F(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(0) | A4(1) | E(0) | F(0) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(0) | A4(1) | E(0) | F(1) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(0) | A4(1) | E(1) | F(0) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(0) | A4(1) | E(1) | F(1) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(1) | A4(0) | E(0) | F(0) | 0.9500 |  
+------+-------+------+------+-----------------+  
| D(1) | A4(0) | E(0) | F(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A4(0) | E(1) | F(0) | 0.9500 |  
+------+-------+------+------+-----------------+  
| D(1) | A4(0) | E(1) | F(1) | 0.0000 |  
+------+-------+------+------+-----------------+  
| D(1) | A4(1) | E(0) | F(0) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(1) | A4(1) | E(0) | F(1) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(1) | A4(1) | E(1) | F(0) | 0.0250 |  
+------+-------+------+------+-----------------+  
| D(1) | A4(1) | E(1) | F(1) | 0.0250 |  
+------+-------+------+------+-----------------+

circuit\_JT.add\_factors(phi\_C1, phi\_C2, phi\_C3, phi\_C4, phi\_C5, phi\_C6)

circuit\_JT.check\_model()

True

from pgmpy.inference.ExactInference import BeliefPropagation  
  
belief\_propagation = BeliefPropagation(circuit\_JT)  
res1 = belief\_propagation.query(variables=['A1', 'A2', 'A3', 'A4'],  
 evidence={'X': 1, 'Y': 1, 'F': 0})  
print(res1)

+-------+-------+-------+-------+--------------------+  
| A1 | A2 | A3 | A4 | phi(A1,A2,A3,A4) |  
+=======+=======+=======+=======+====================+  
| A1(0) | A2(0) | A3(0) | A4(0) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(0) | A3(0) | A4(1) | 0.3129 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(0) | A3(1) | A4(0) | 0.3129 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(0) | A3(1) | A4(1) | 0.0165 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(1) | A3(0) | A4(0) | 0.3129 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(1) | A3(0) | A4(1) | 0.0165 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(1) | A3(1) | A4(0) | 0.0247 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(1) | A3(1) | A4(1) | 0.0009 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(0) | A3(0) | A4(0) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(0) | A3(0) | A4(1) | 0.0009 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(0) | A3(1) | A4(0) | 0.0009 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(0) | A3(1) | A4(1) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(1) | A3(0) | A4(0) | 0.0009 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(1) | A3(0) | A4(1) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(1) | A3(1) | A4(0) | 0.0001 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(1) | A3(1) | A4(1) | 0.0000 |  
+-------+-------+-------+-------+--------------------+

res2 = belief\_propagation.query(variables=['A1', 'A2', 'A3', 'A4'],  
 evidence={'X': 1, 'Y': 1, 'F': 0, 'D': 1})  
print(res2)

+-------+-------+-------+-------+--------------------+  
| A1 | A2 | A3 | A4 | phi(A1,A2,A3,A4) |  
+=======+=======+=======+=======+====================+  
| A1(0) | A2(0) | A3(0) | A4(0) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(0) | A3(0) | A4(1) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(0) | A3(1) | A4(0) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(0) | A3(1) | A4(1) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(1) | A3(0) | A4(0) | 0.9231 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(1) | A3(0) | A4(1) | 0.0243 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(1) | A3(1) | A4(0) | 0.0486 |  
+-------+-------+-------+-------+--------------------+  
| A1(0) | A2(1) | A3(1) | A4(1) | 0.0013 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(0) | A3(0) | A4(0) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(0) | A3(0) | A4(1) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(0) | A3(1) | A4(0) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(0) | A3(1) | A4(1) | 0.0000 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(1) | A3(0) | A4(0) | 0.0026 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(1) | A3(0) | A4(1) | 0.0001 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(1) | A3(1) | A4(0) | 0.0001 |  
+-------+-------+-------+-------+--------------------+  
| A1(1) | A2(1) | A3(1) | A4(1) | 0.0000 |  
+-------+-------+-------+-------+--------------------+

res3 = belief\_propagation.map\_query(variables=['A1', 'A2', 'A3', 'A4','Z', 'D','E'],  
 evidence={'X': 1, 'Y': 1, 'F': 0})  
print(res3)

{'A1': 0, 'A2': 0, 'A3': 0, 'A4': 1, 'Z': 0, 'D': 0, 'E': 0}

res4 = belief\_propagation.map\_query(variables=['A1', 'A2', 'A3', 'A4', 'Z', 'E'],  
 evidence={'X': 1, 'Y': 1, 'F': 0, 'D': 1})  
print(res4)

{'A1': 0, 'A2': 1, 'A3': 0, 'A4': 0, 'Z': 0, 'E': 0}