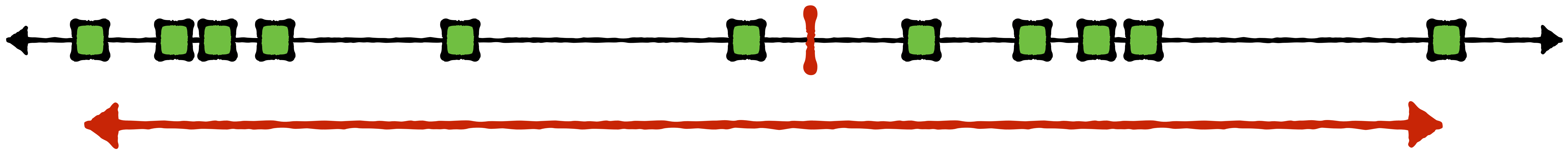
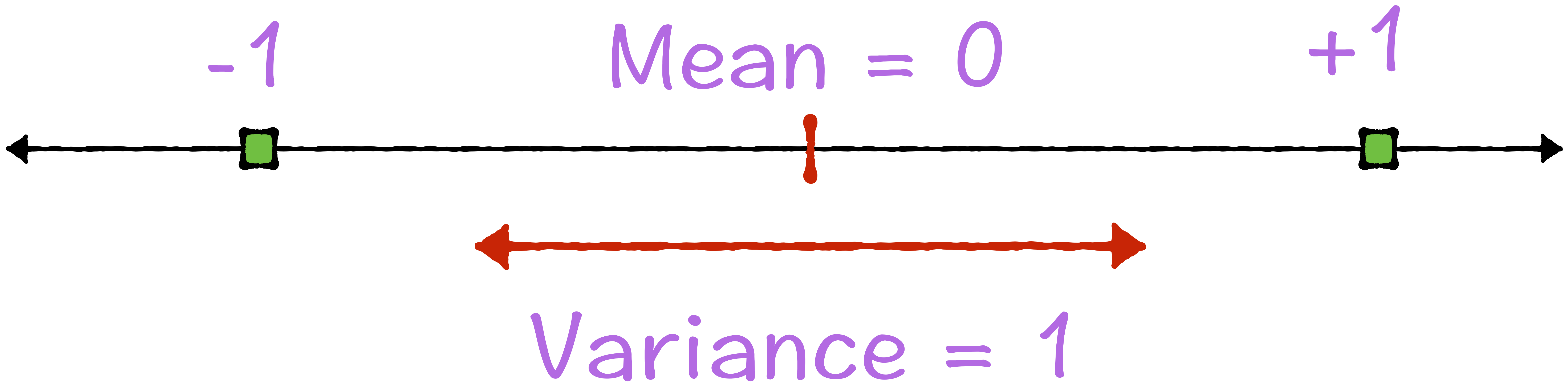
Mean vs Variance

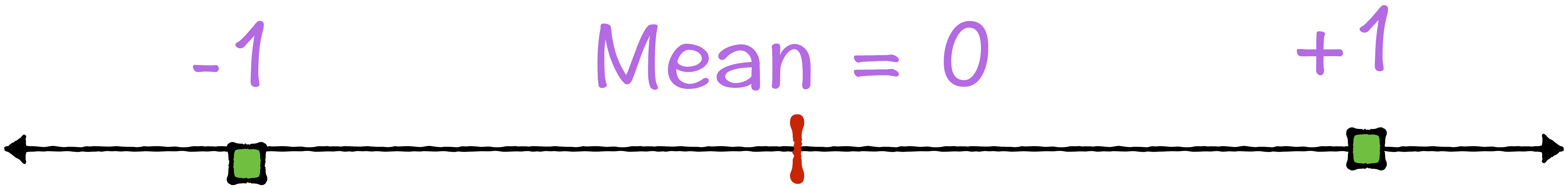


Variance measures risk

Small stakes game



Small stakes game

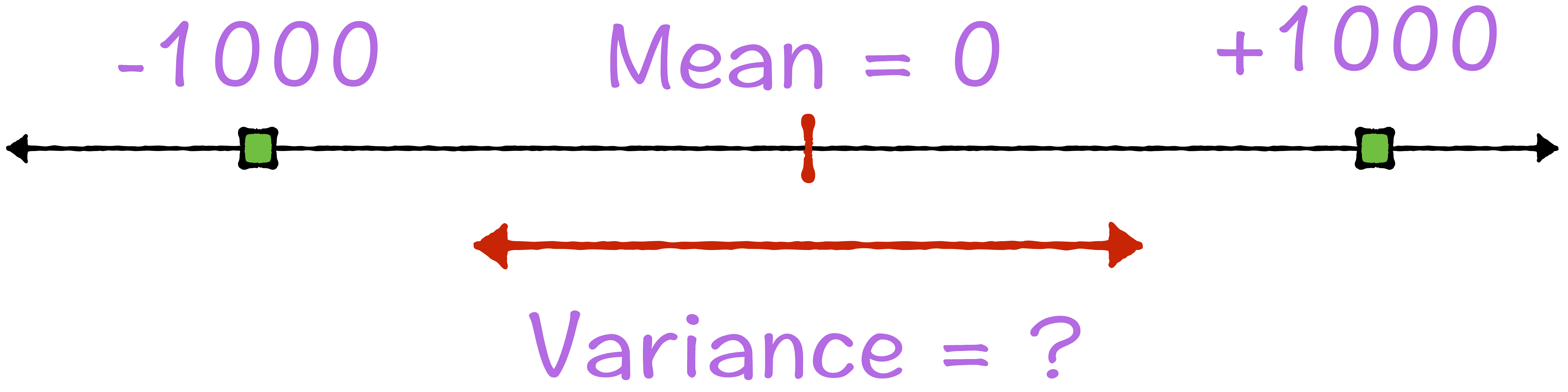


$$\text{Variance} = \frac{\sum (x_i - \bar{x})^2}{n}$$

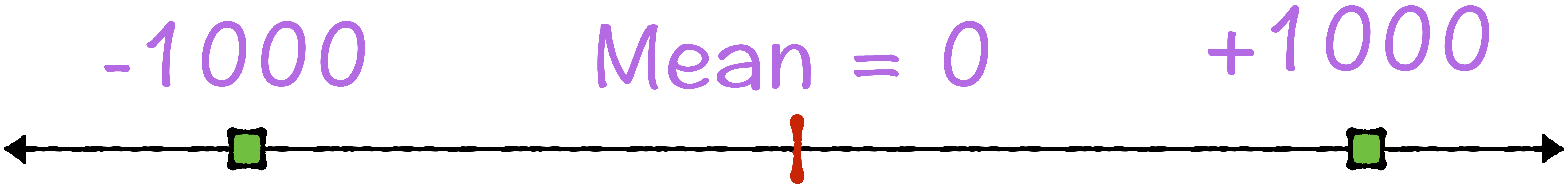
$$= \frac{(1-0)^2 + (-1-0)^2}{2}$$

$$= 1$$

Small stakes game



Small stakes game

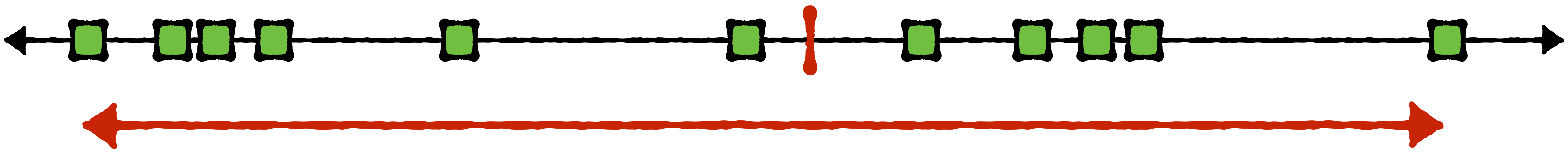


$$\text{Variance} = \frac{\sum (x_i - \bar{x})^2}{n}$$

$$= \frac{(1000-0)^2 + (-1000-0)^2}{2}$$

$$= 1000000$$

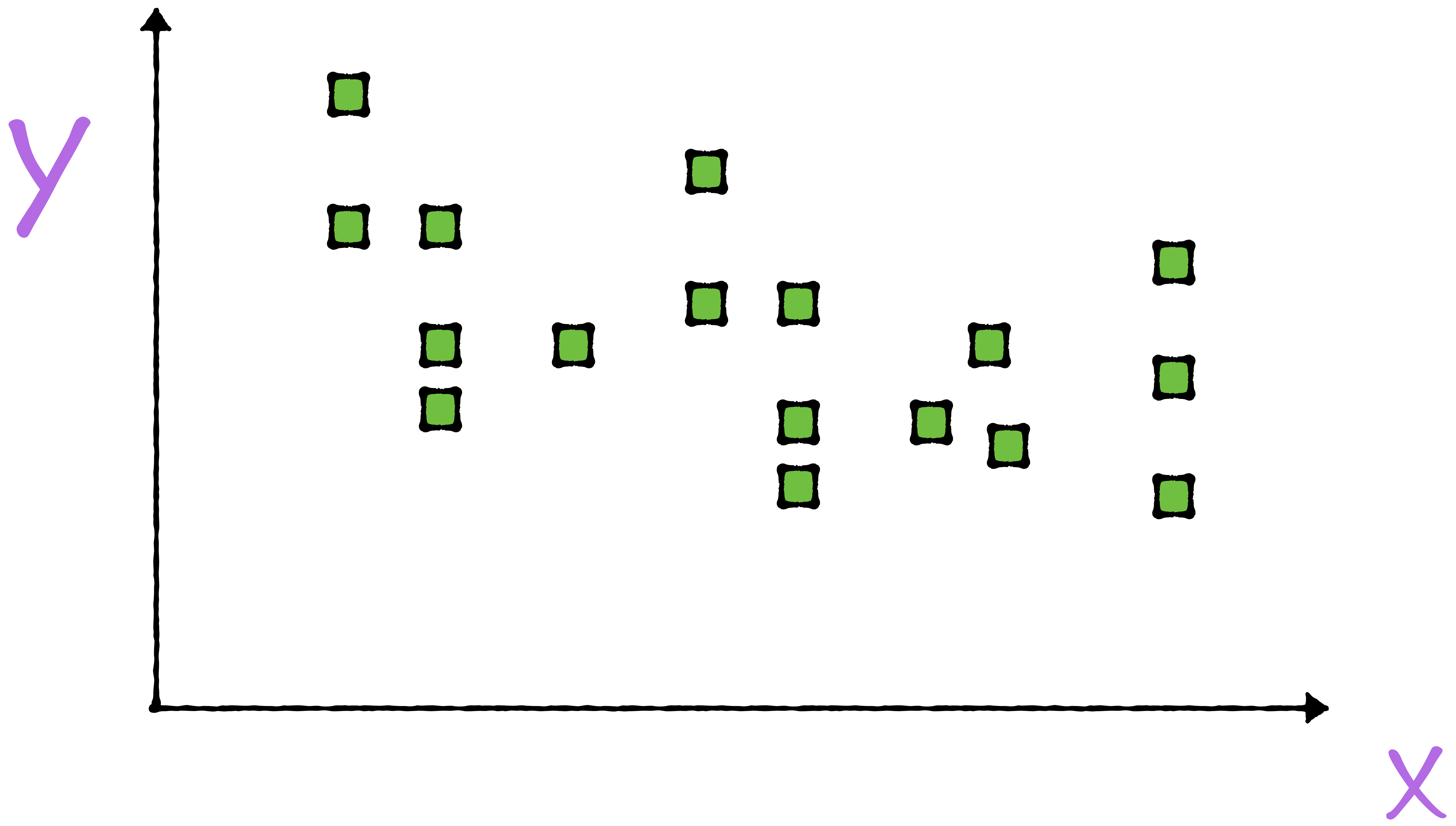
Mean vs Variance



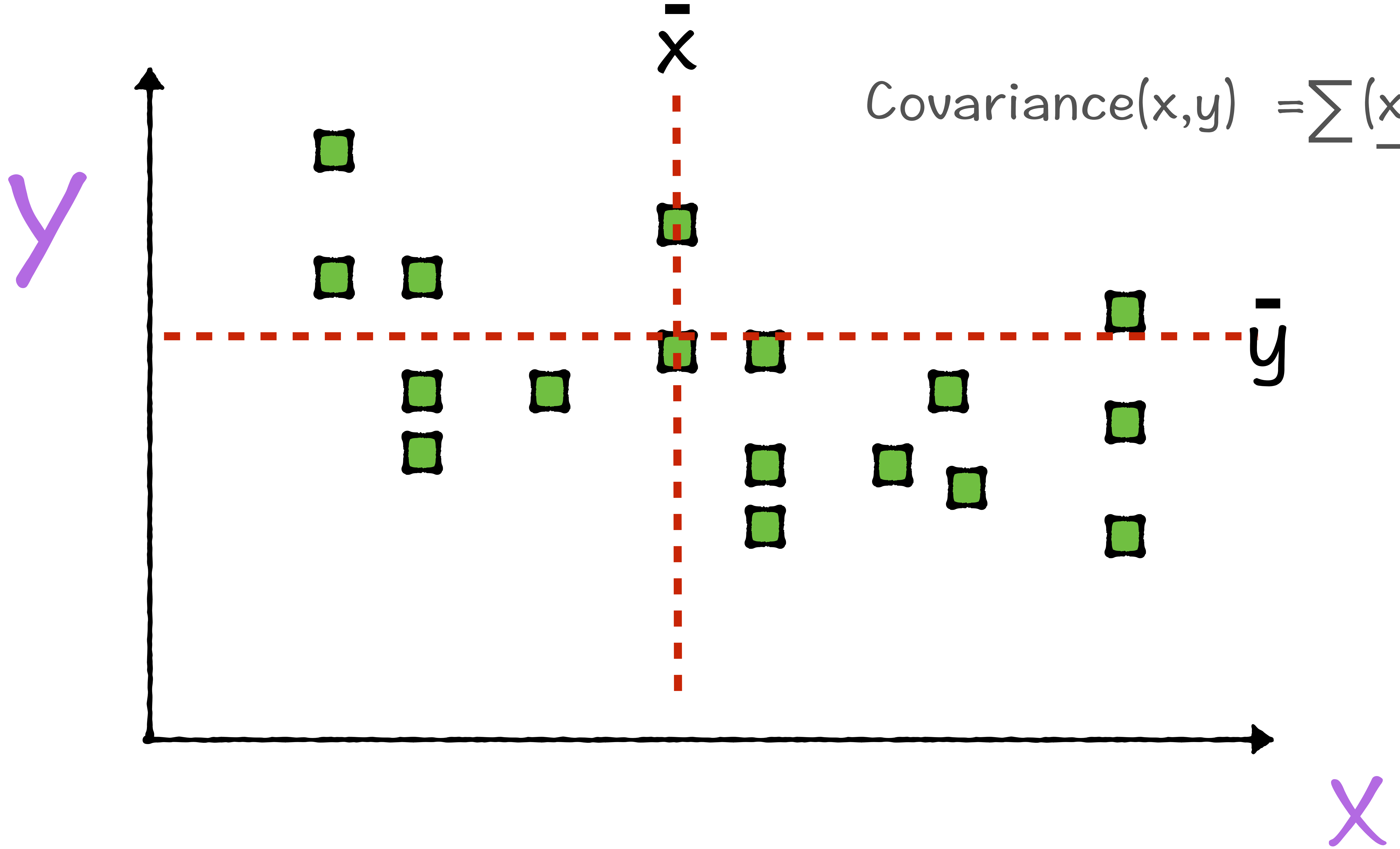
Variance measures risk

Variance grows faster than the mean

View data in relation to related data

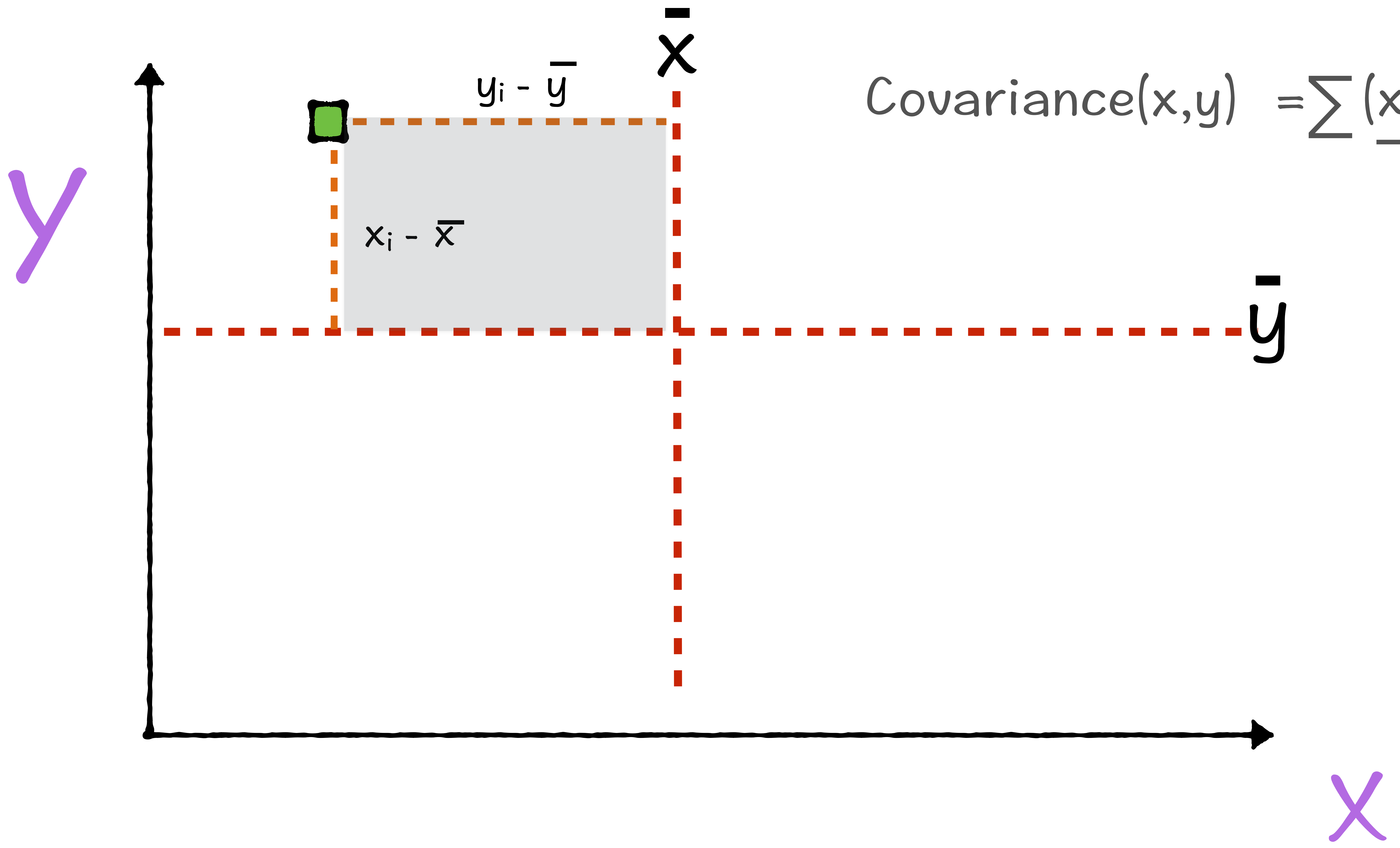


Covariance is variance in 2 dimensions



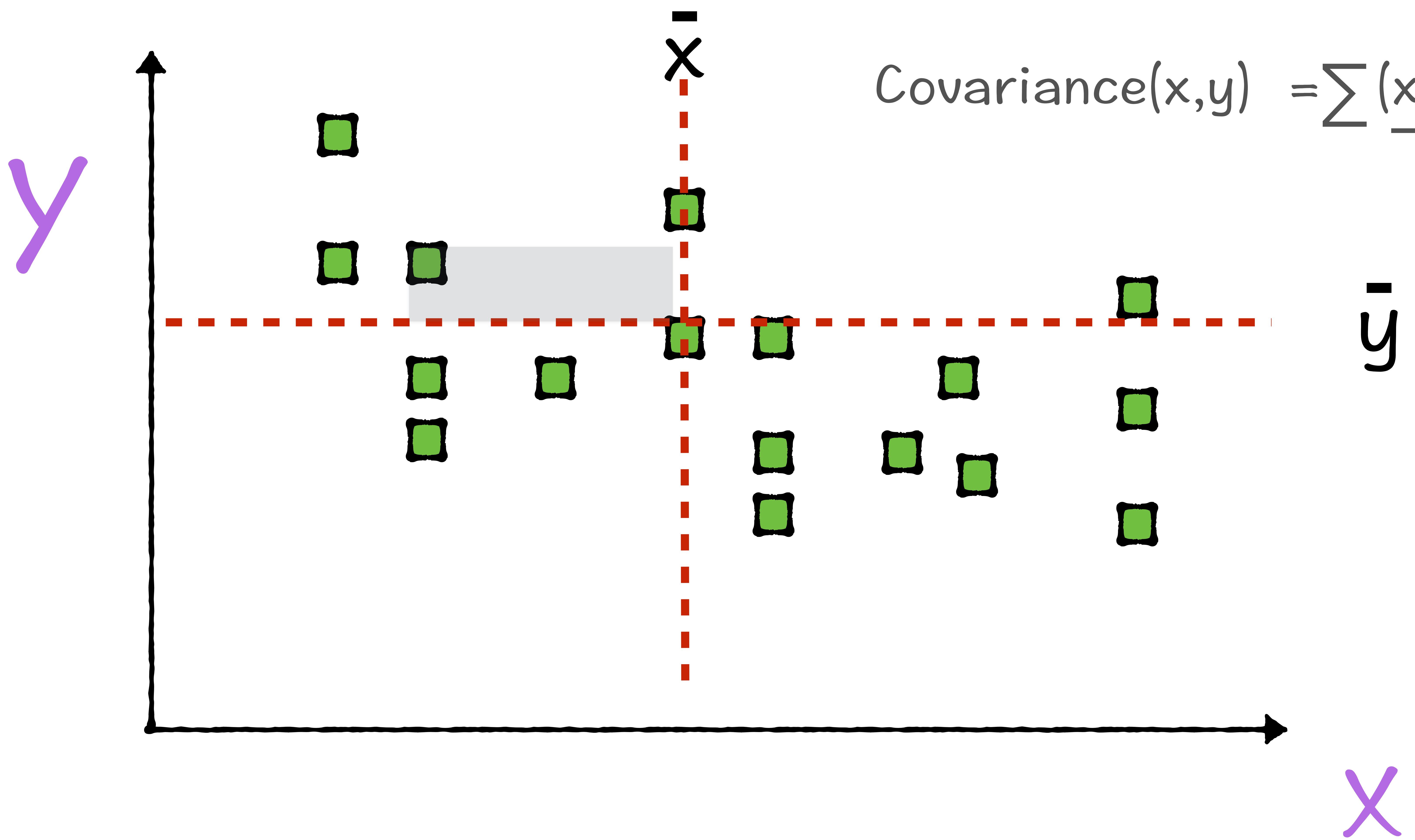
$$\text{Covariance}(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$

Covariance is variance in 2 dimensions



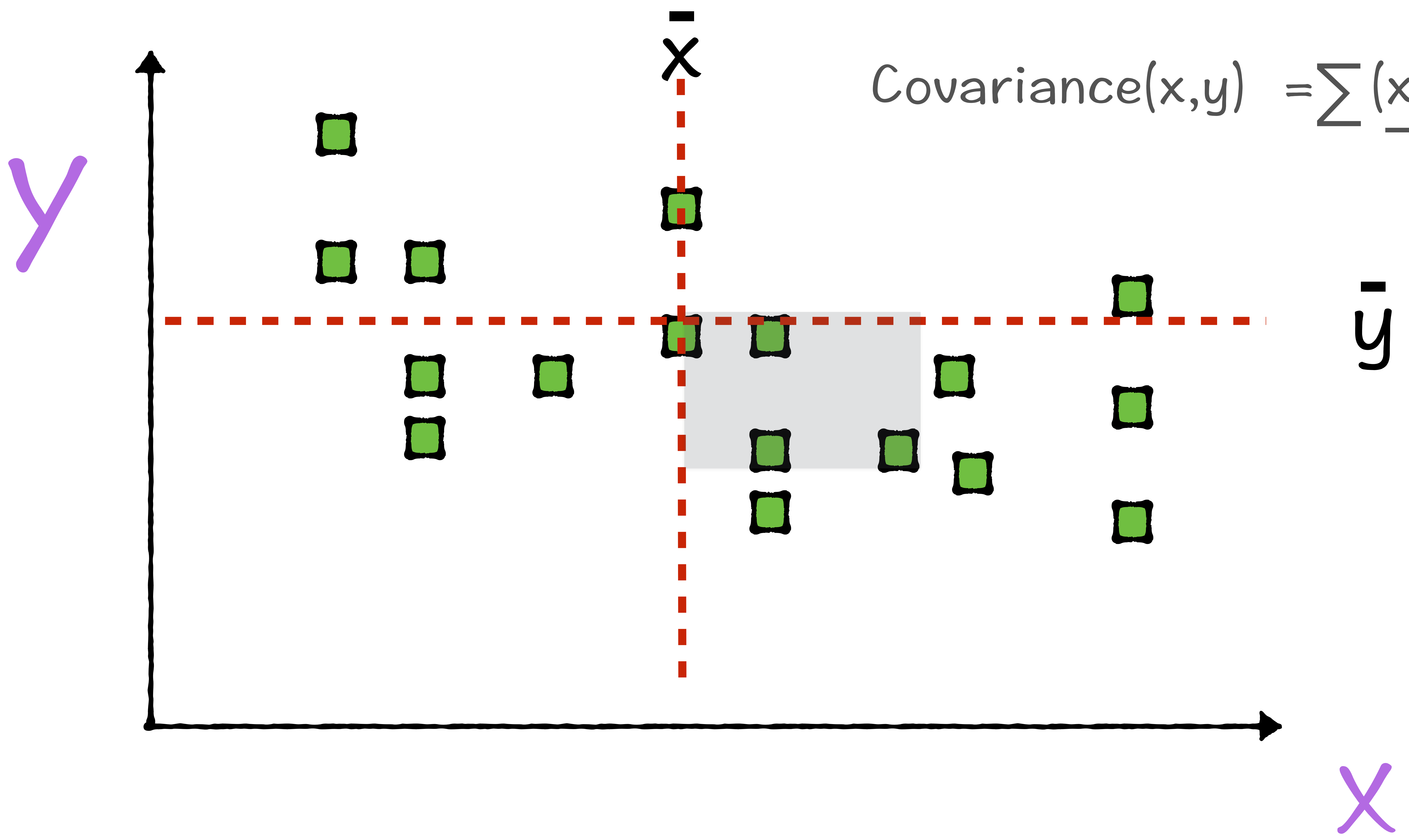
$$\text{Covariance}(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$

Covariance is variance in 2 dimensions

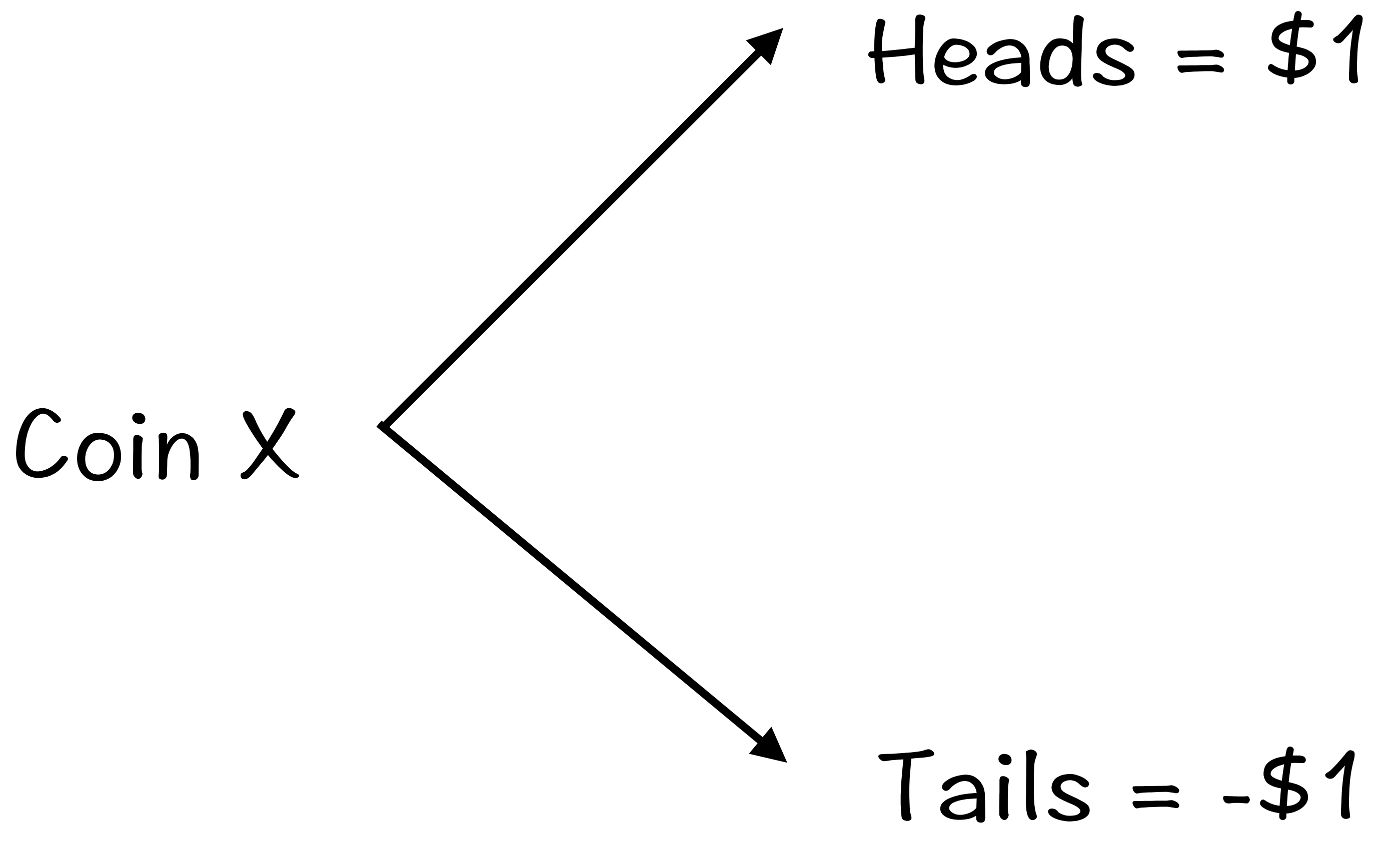


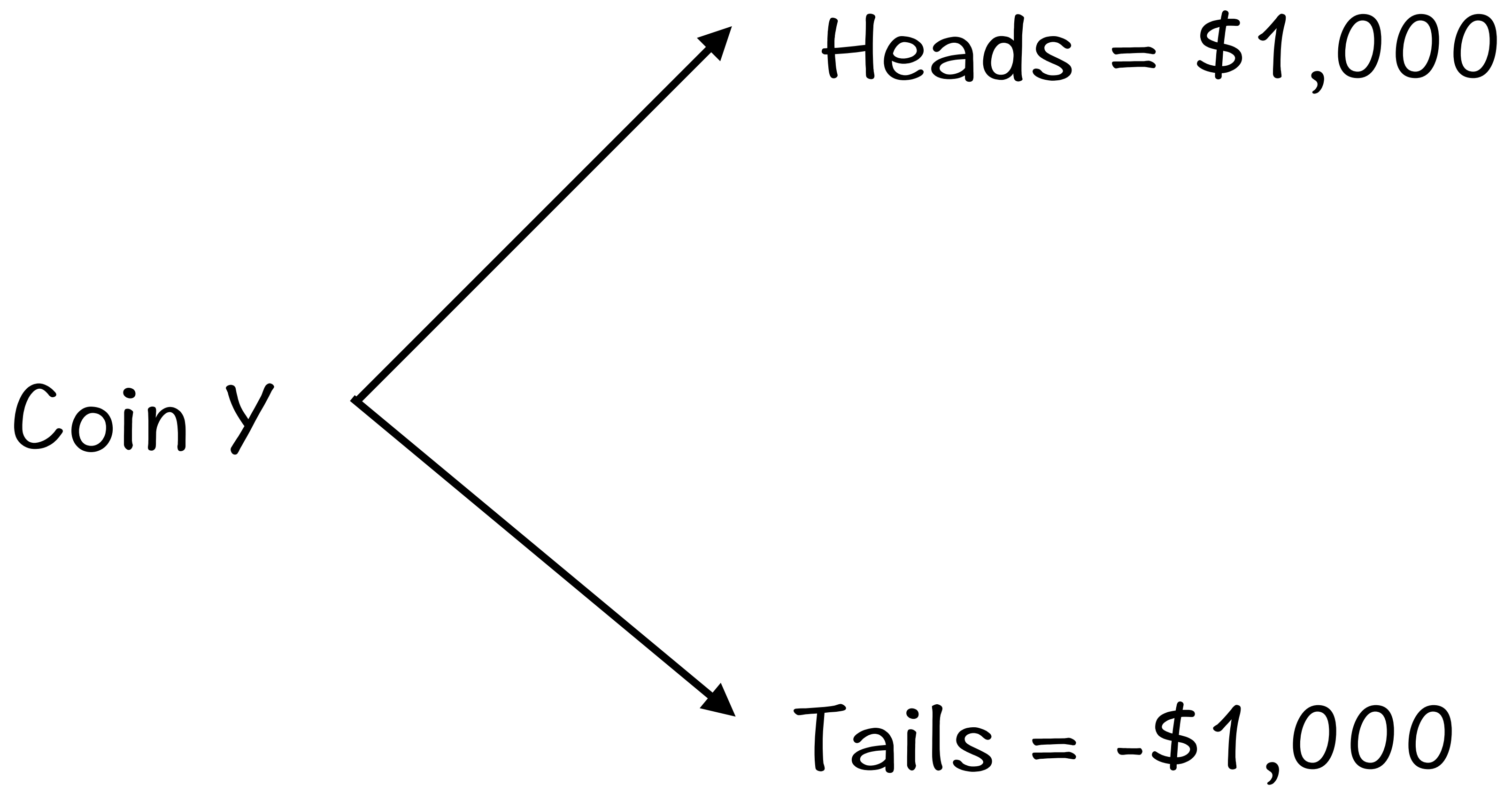
$$\text{Covariance}(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$

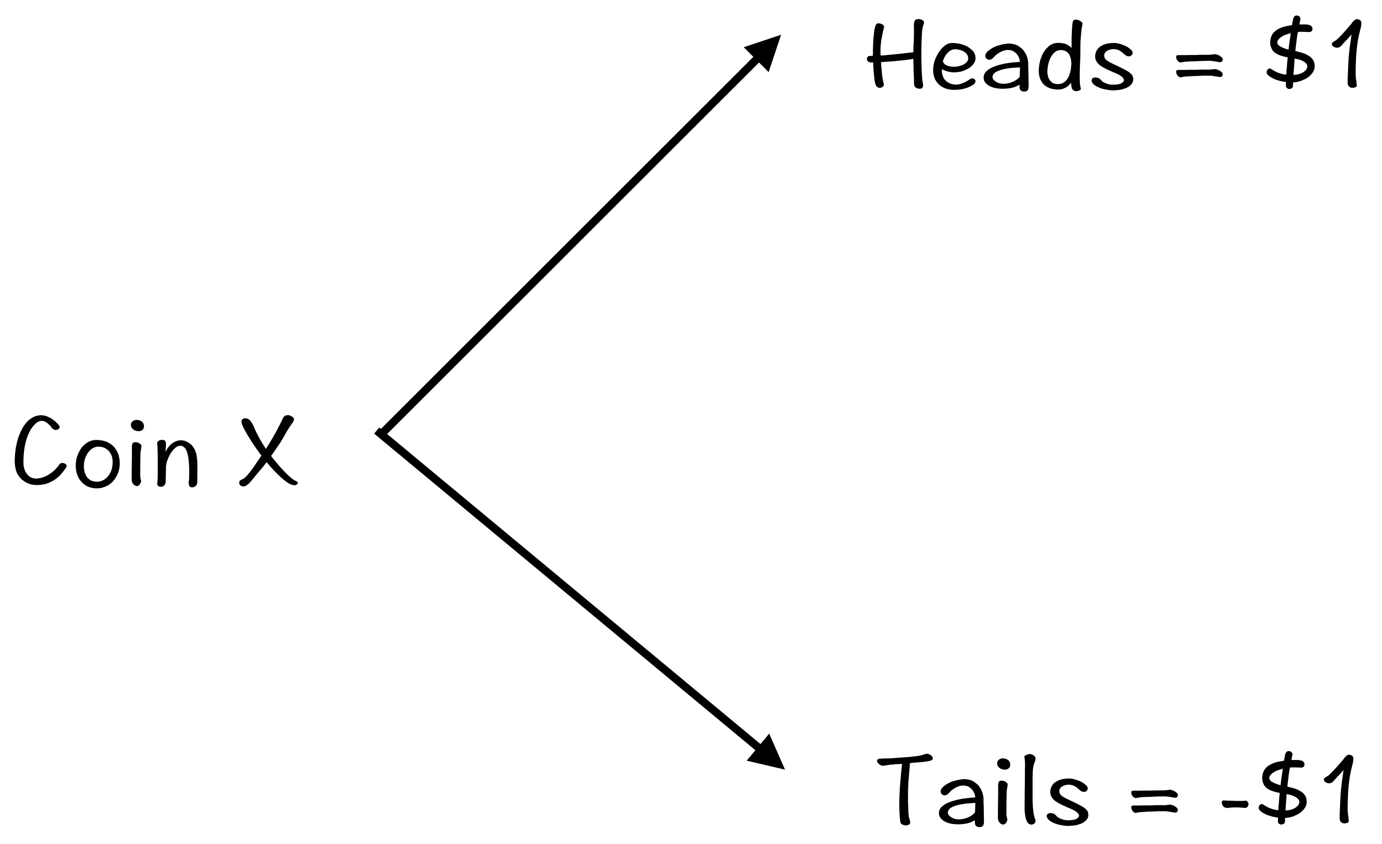
Covariance is variance in 2 dimensions

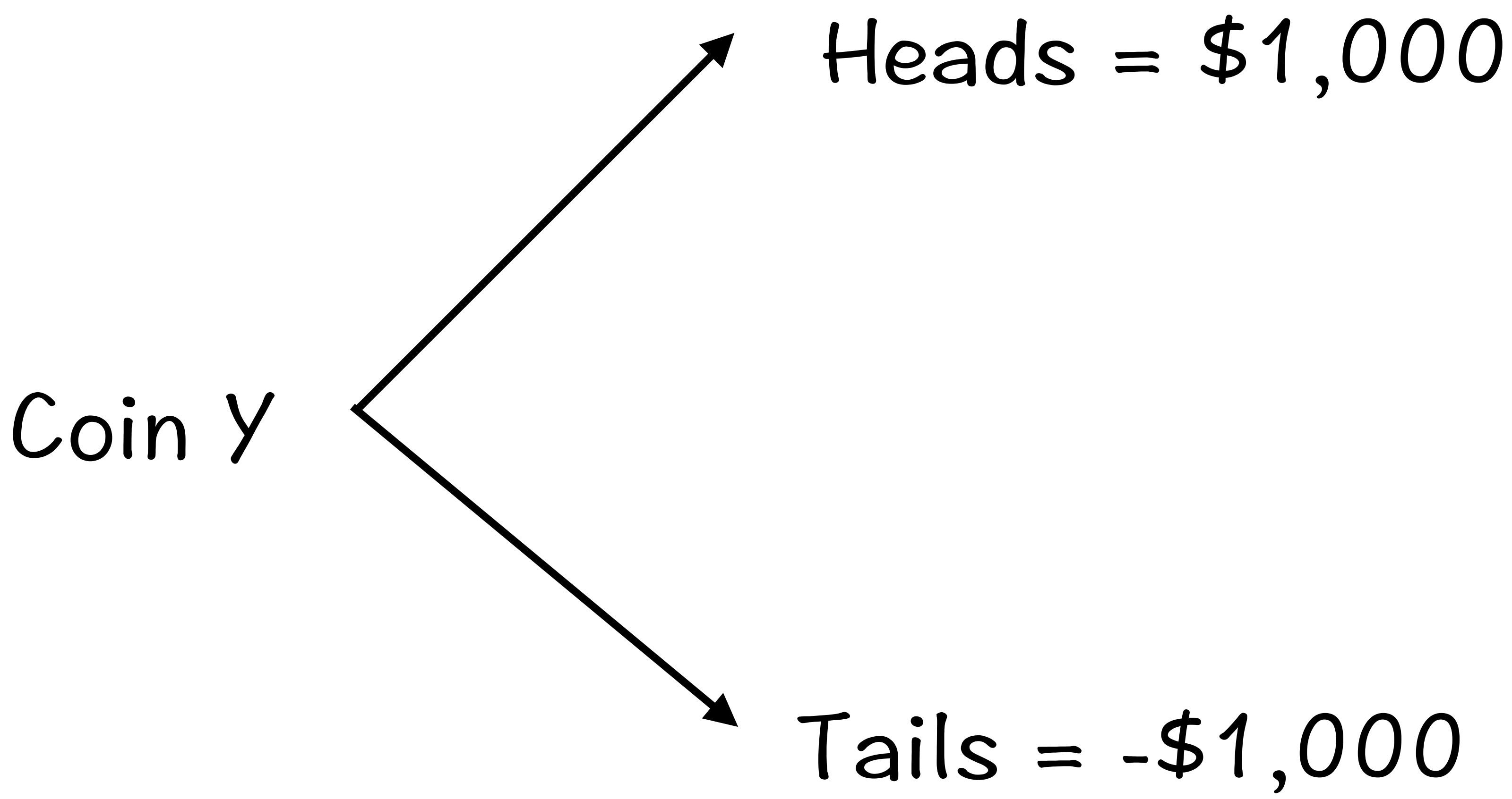


$$\text{Covariance}(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n}$$


Heads = $1
Coin X
Tails = -$1


Heads = $1,000
Coin Y
Tails = -$1,000





Coin X Result	Coin Y Result	Coin X Payoff	Coin Y Payoff
Heads	Heads	$1	$1,000
Heads	Tails	$1	-$1,000
Tails	Heads	-$1	$1,000
Tails	Heads	-$1	-$1,000

$x = 0$
$Var(x) = 1$

$y = 0$
$Var(y) = 1,000,000$

Coin X Result	Coin Y Result	Coin X Payoff	Coin Y Payoff
Heads	Heads	$1	$1,000
Heads	Tails	$1	-$1,000
Tails	Heads	-$1	$1,000
Tails	Heads	-$1	-$1,000

$x_i - \bar{x}$	$y_i - \bar{y}$	$(x_i - \bar{x})(y_i - \bar{y})$
$1	$1,000	1,000
$1	-$1,000	-1,000
-$1	$1,000	-1,000
-$1	-$1,000	1,000

$\bar{x} = 0$

$\mathrm{Var}(x) = 1$

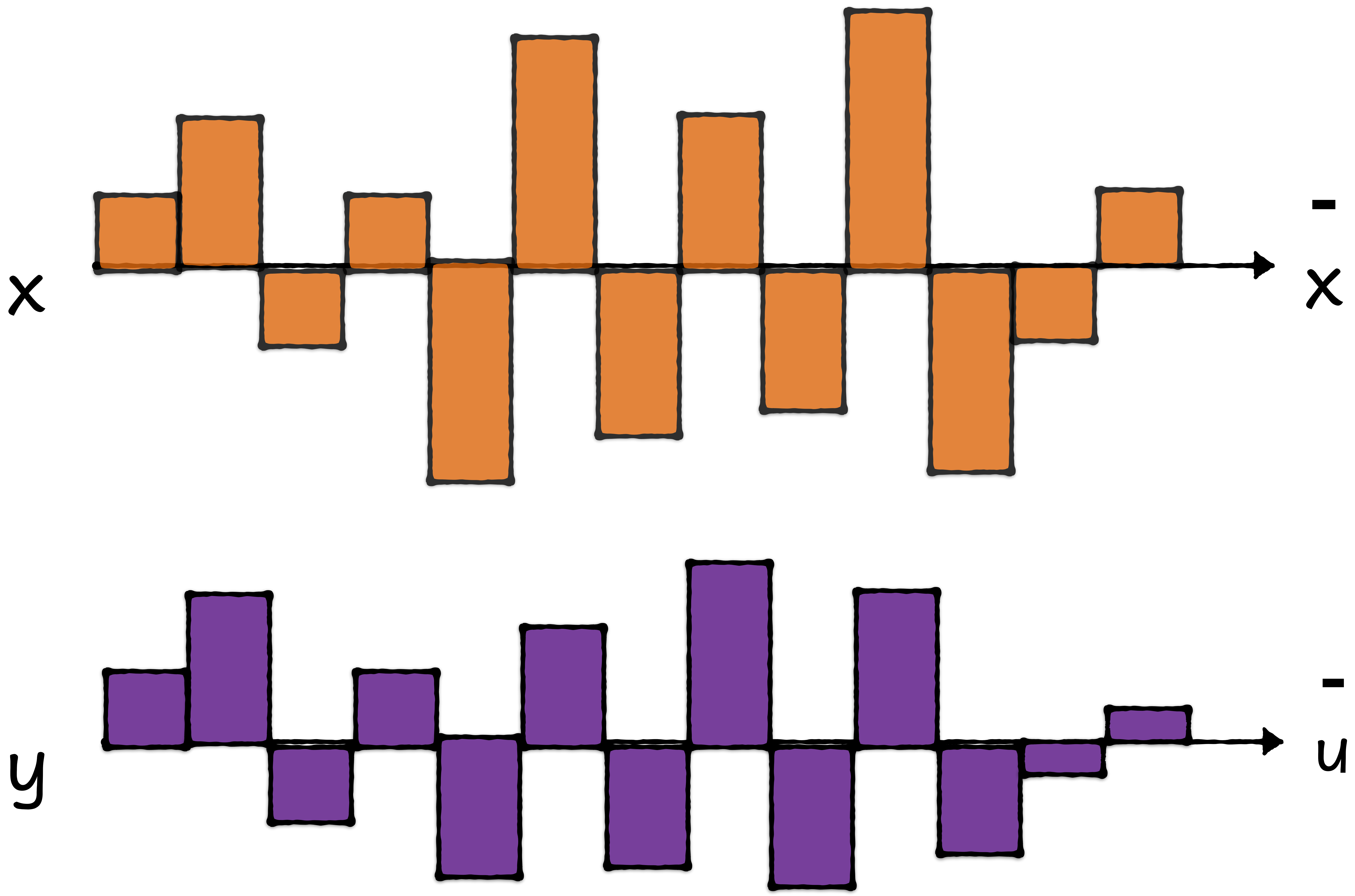
$\bar{y} = 0$

$\mathrm{Var}(y) = 1,000,000$

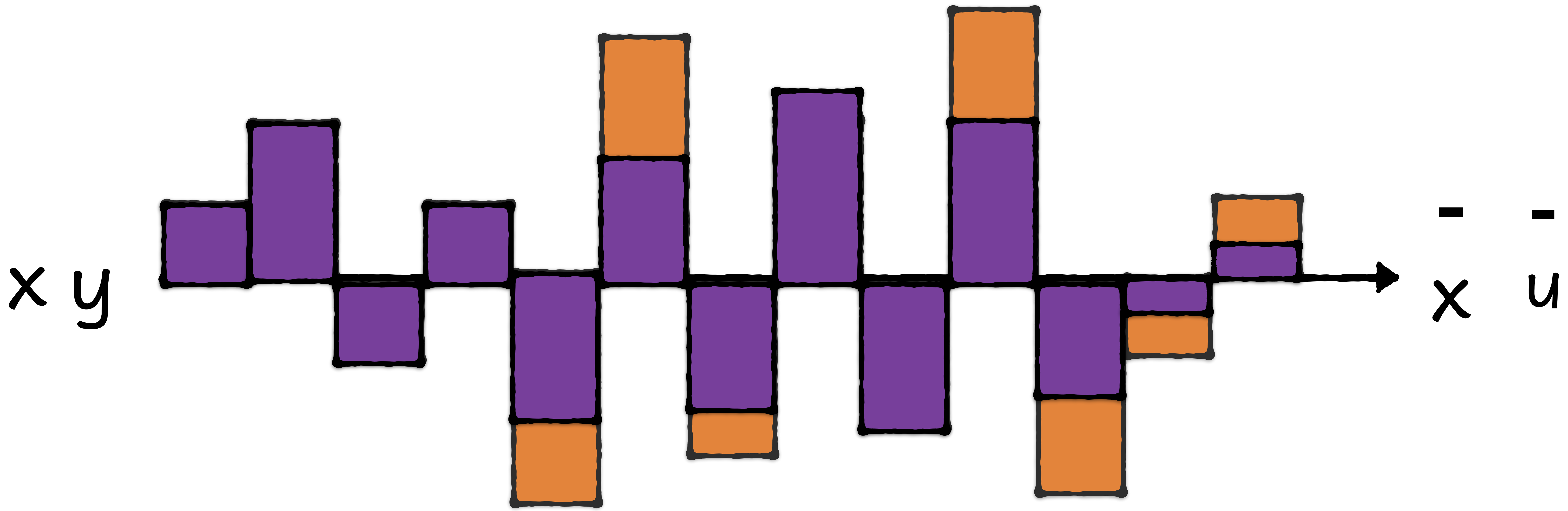
$$\text{Covariance}(x,y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{n} = 0$$

Covariance of independent variables is 0

Positive covariance

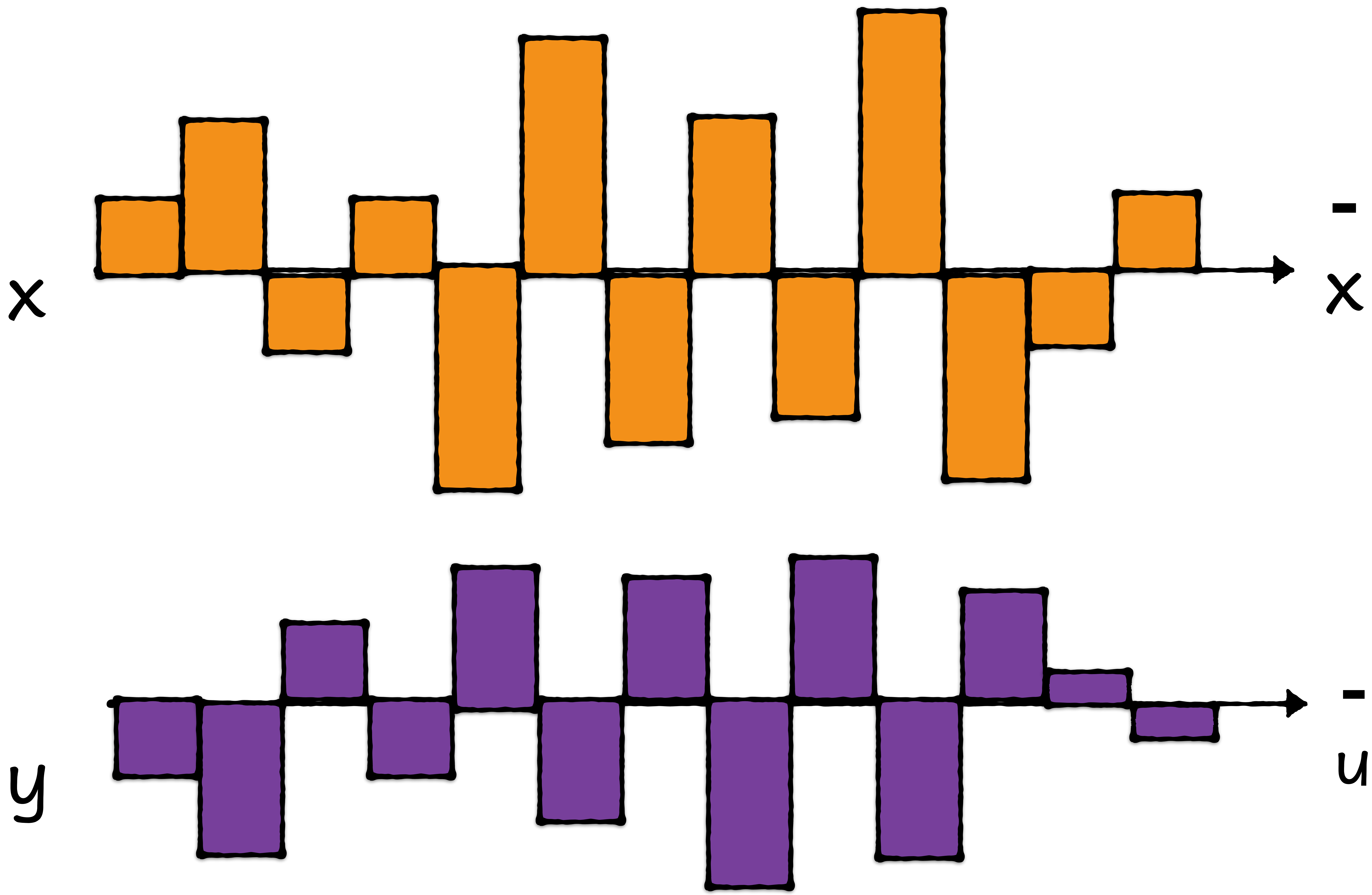


Positive covariance

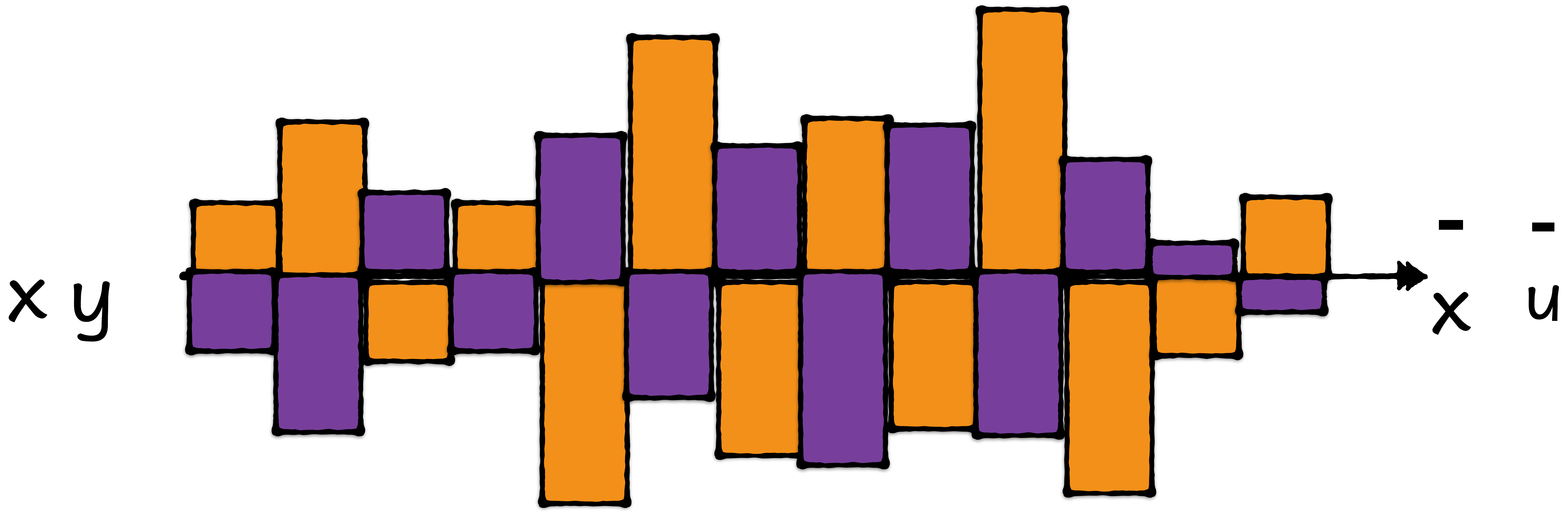


Deviations around the mean are in sync

Positive covariance

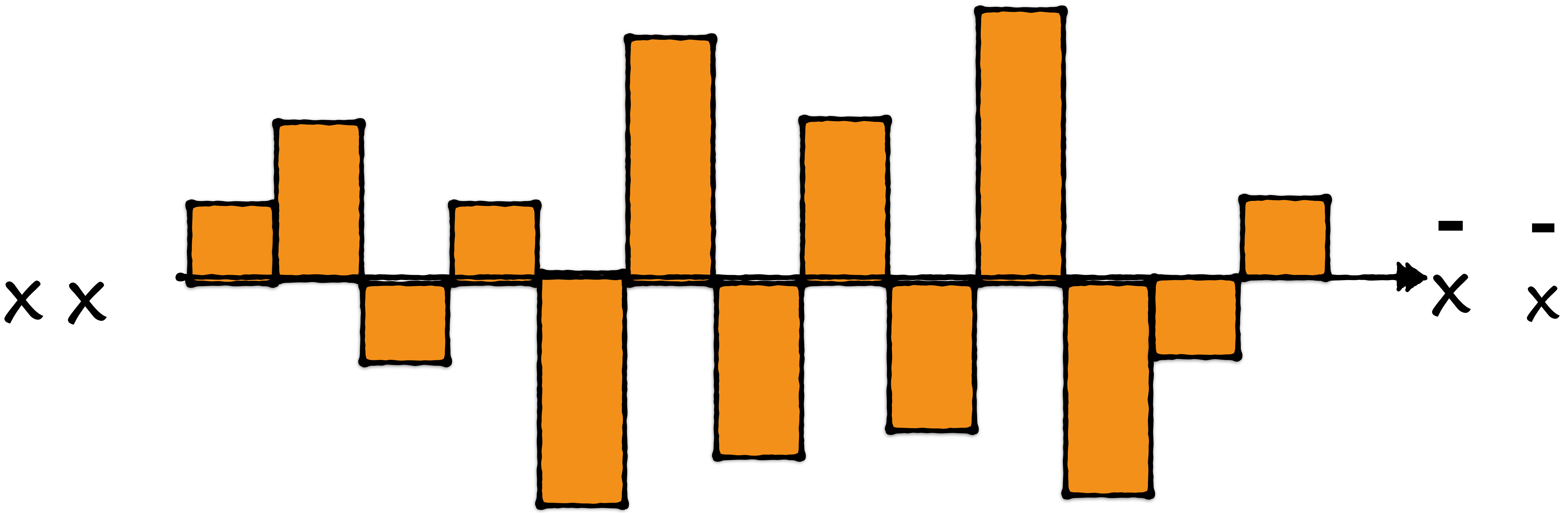


Positive covariance



Deviations around the mean are out of sync

Variance



Covariance of X with X

Random variables

An outcome which cannot be determined beforehand

Coin tosses

Dice rolls

Sporting events

Stock returns

Random variables

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ \cdots \\ E_n \end{bmatrix} \quad \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ \cdots \\ D_n \end{bmatrix} \quad \begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ \cdots \\ G_n \end{bmatrix} \quad \cdots \quad \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \cdots \\ A_n \end{bmatrix}$$

Exxon   Dow Jones   Google   Apple

Random variables

$$
\begin{bmatrix}
E_1 & D_1 & G_1 & & A_1 \\
E_2 & D_2 & G_2 & & A_2 \\
E_3 & D_3 & G_3 & \cdots & A_3 \\
\cdots & \cdots & \cdots & & \cdots \\
E_n & D_n & G_n & & A_n
\end{bmatrix}
$$

N rows

K columns

$$
\begin{array}{c}
\begin{bmatrix}
x_{11} & x_{12} & x_{13} & & x_{1k} \\
x_{21} & x_{22} & x_{23} & \cdots & x_{2k} \\
x_{31} & x_{32} & x_{33} & & x_{3k} \\
\cdots & \cdots & \cdots & & \cdots \\
x_{n1} & x_{n2} & x_{n3} & & x_{nk}
\end{bmatrix} \\
\begin{array}{ccccc}
X_1 & X_2 & X_3 & & X_k
\end{array}
\end{array}
$$

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

Each element X_i of this matrix is a vector with 1 column and n rows

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

Covariance matrix consists of pairwise covariances

Covariance matrix

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} Cov(X_1, X_1) & Cov(X_1, X_2) & \cdots & Cov(X_1, X_k) \\ Cov(X_2, X_1) & Cov(X_2, X_2) & \cdots & Cov(X_2, X_k) \\ Cov(X_k, X_1) & Cov(X_k, X_2) & \cdots & Cov(X_k, X_k) \end{bmatrix} \quad \text{K rows}$$

K columns

Covariance matrix

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} Cov(X_1, X_1) & Cov(X_1, X_2) & \cdots & Cov(X_1, X_k) \\ Cov(X_2, X_1) & Cov(X_2, X_2) & \cdots & Cov(X_2, X_k) \\ Cov(X_k, X_1) & Cov(X_k, X_2) & \cdots & Cov(X_k, X_k) \end{bmatrix}$$

Covariance of X1 with other random variables

Covariance matrix

$$\begin{bmatrix} X_1 & X_2 & X_3 & \dots & X_k \end{bmatrix}$$

$$\begin{bmatrix} Cov(X_1, X_1) & Cov(X_1, X_2) & \cdots & Cov(X_1, X_k) \\ Cov(X_2, X_1) & Cov(X_2, X_2) & \cdots & Cov(X_2, X_k) \\ Cov(X_k, X_1) & Cov(X_k, X_2) & \cdots & Cov(X_k, X_k) \end{bmatrix}$$

Diagonal elements are variances

Covariance matrix

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} & Cov(X_1, X_2) & \cdots & Cov(X_1, X_k) \\ Cov(X_2, X_1) & & \cdots & Cov(X_2, X_k) \\ Cov(X_k, X_1) & Cov(X_k, X_2) & \cdots & \end{bmatrix}$$

Symmetric matrix

Covariance matrix

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} Cov(X_1, X_1) & Cov(X_1, X_2) & \cdots & Cov(X_1, X_k) \\ Cov(X_2, X_1) & Cov(X_2, X_2) & \cdots & Cov(X_2, X_k) \\ Cov(X_k, X_1) & Cov(X_k, X_2) & \cdots & Cov(X_k, X_k) \end{bmatrix}$$

Covariance matrix

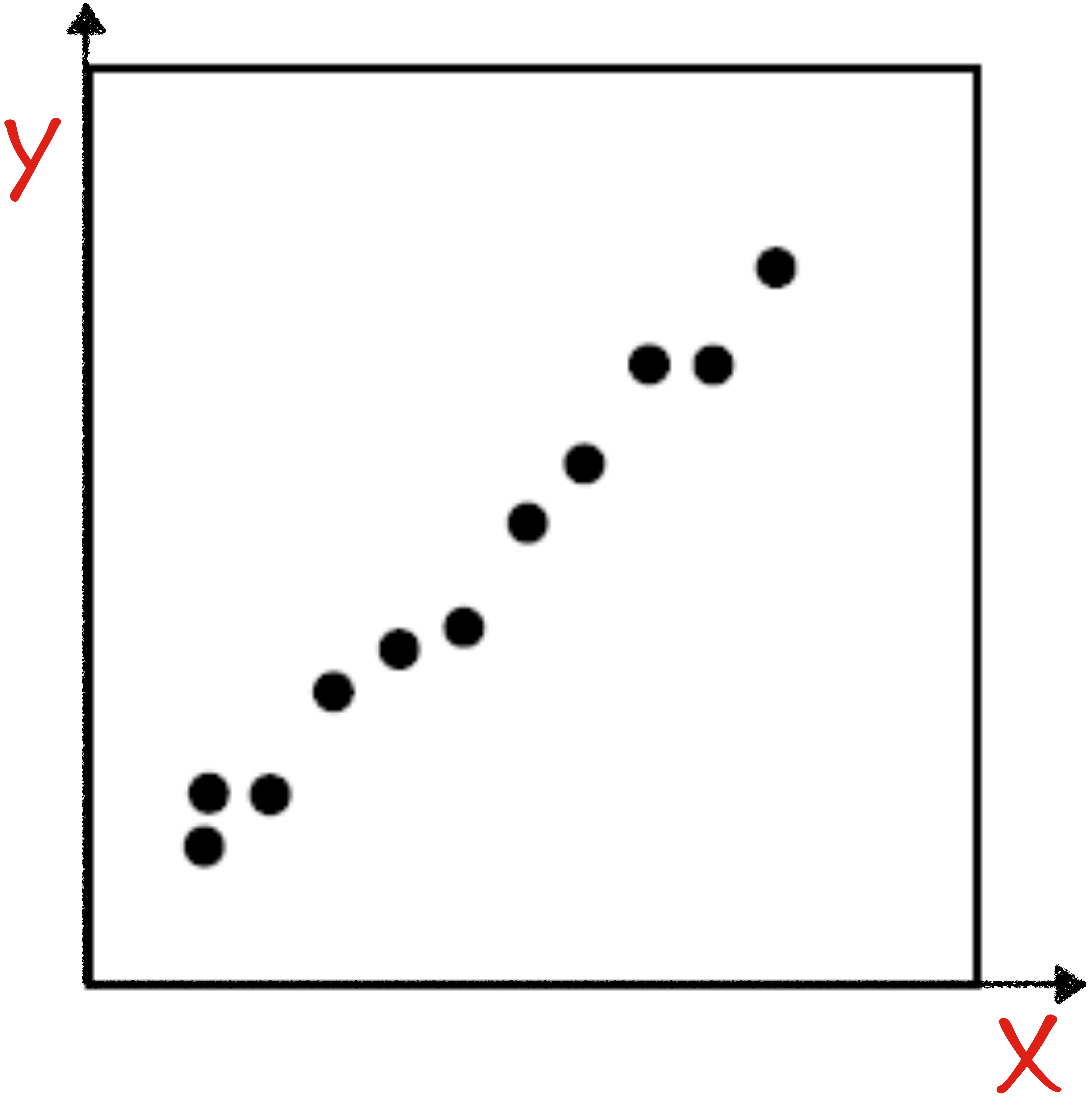
$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} \sigma^2_{x_1} & \sigma^2_{x_1 x_2} & \cdots & \sigma^2_{x_1 x_k} \\ \sigma^2_{x_2 x_1} & \sigma^2_{x_2} & \cdots & \sigma^2_{x_2 x_k} \\ \sigma^2_{x_k x_1} & \sigma^2_{x_k x_2} & \cdots & \sigma^2_{x_k} \end{bmatrix}$$

Correlation

When X increases , y increases linearly

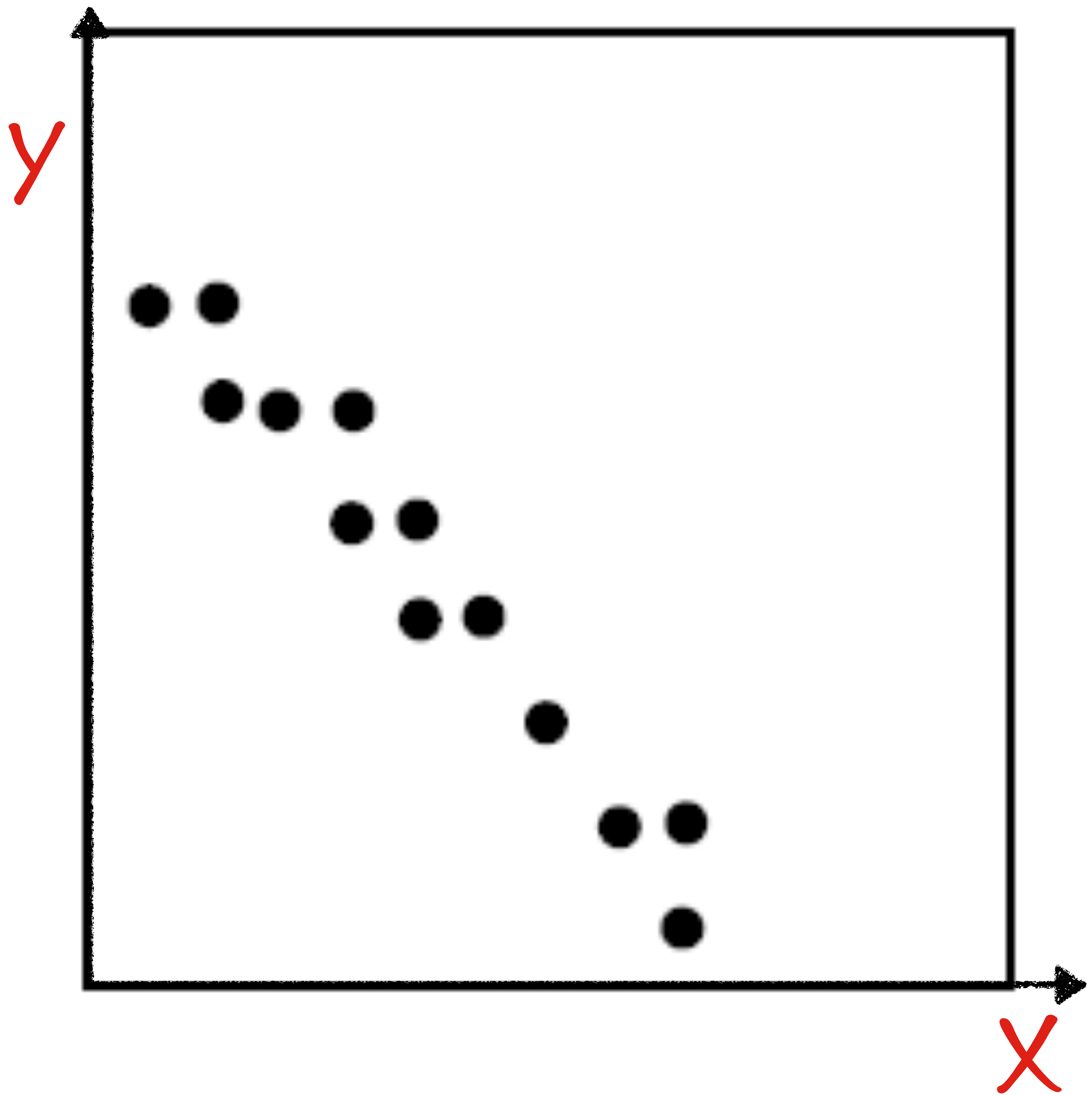
Correlation = +1



Correlation

When X increases , y decreases linearly

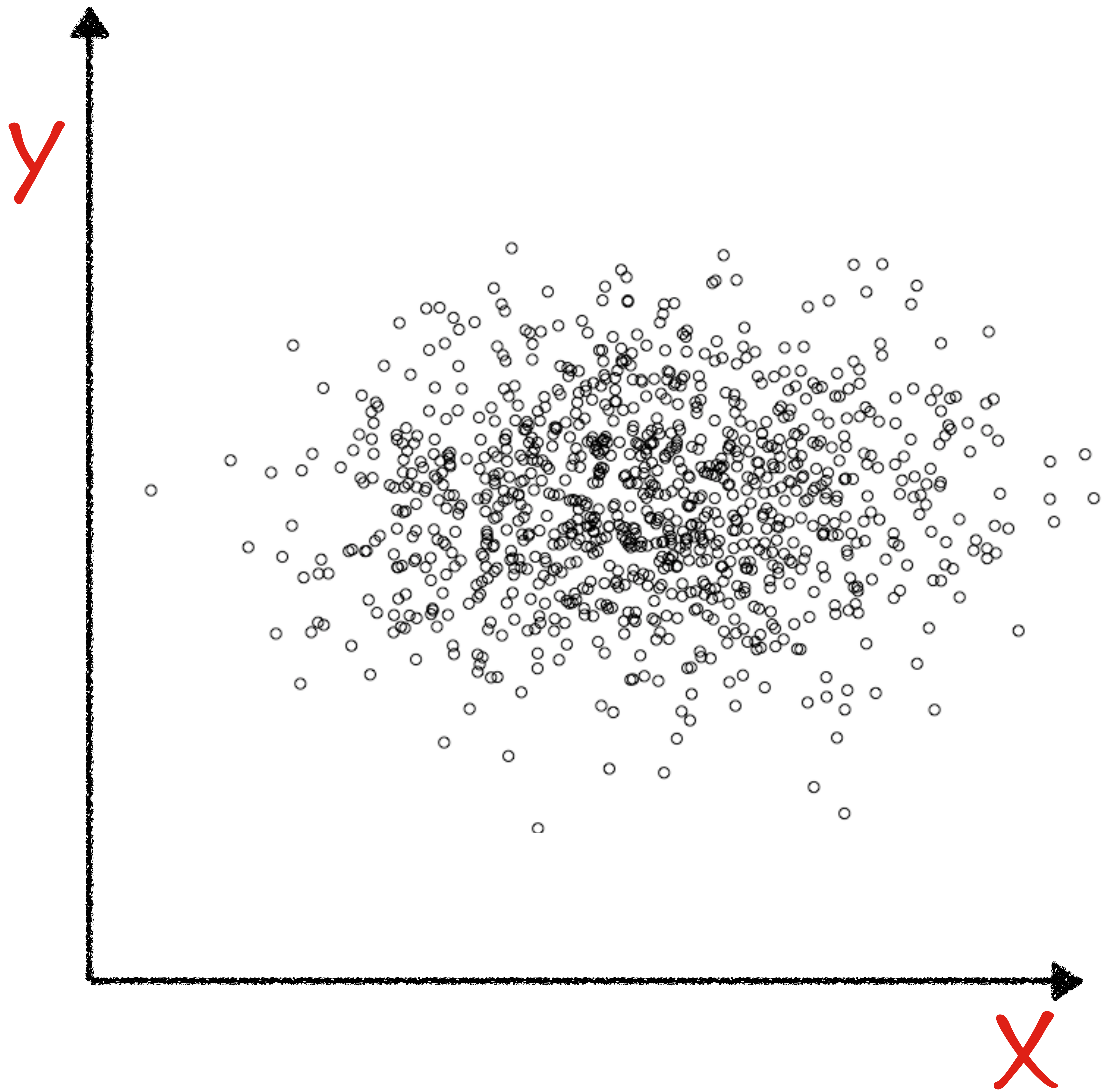
Correlation = -1



Correlation

Changes in X independent of changes in Y

Correlation = 0



Correlation

$$\rho_{xy} = \frac{\sigma^2_{xy}}{\sigma_x \sigma_y}$$

$$\text{Correlation }(x,y) = \frac{\text{Covariance }(x,y)}{\sqrt{\text{Variance }(x)}\sqrt{\text{Variance }(y)}}$$

Covariance matrix

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} \sigma^2_{x_1} & \sigma^2_{x_1 x_2} & \cdots & \sigma^2_{x_1 x_k} \\ \sigma^2_{x_2 x_1} & \sigma^2_{x_2} & \cdots & \sigma^2_{x_2 x_k} \\ \sigma^2_{x_k x_1} & \sigma^2_{x_k x_2} & \cdots & \sigma^2_{x_k} \end{bmatrix}$$

Correlation matrix

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} \rho_{x_1} & \rho_{x_1 x_2} & \cdots & \rho_{x_1 x_k} \\ \rho_{x_2 x_1} & \rho_{x_2} & \cdots & \rho_{x_2 x_k} \\ \rho_{x_k x_1} & \rho_{x_k x_2} & \cdots & \rho_{x_k} \end{bmatrix}$$

Correlation matrix

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} 1 & \rho_{x_1 x_2} & \cdots & \rho_{x_1 x_k} \\ \rho_{x_2 x_1} & 1 & \cdots & \rho_{x_2 x_k} \\ \rho_{x_k x_1} & \rho_{x_k x_2} & \cdots & 1 \end{bmatrix}$$

Covariance matrix of independent variables

$$\begin{bmatrix} X_1 & X_2 & X_3 & \cdots & X_k \end{bmatrix}$$

$$\begin{bmatrix} \sigma^2_{x_1} & 0 & \cdots & 0 \\ 0 & \sigma^2_{x_2} & \cdots & 0 \\ 0 & 0 & \cdots & \sigma^2_{x_k} \end{bmatrix}$$