

Simple Practice Problems & Solution for Electric and Hybrid Vehicle Component sizing

Vehicle Specifications:

Gross Vehicle Weight = 1500 Kg

Tyre Specifications = 195/55 R 16

Transmission Gear Ratio =

Final Drive Ratio =

Rolling Resistance Coefficient = 0.013

Air density = 1.184 Kg/m^3 @ 25 deg Celsius

Vehicle Dimensions = L | B | H = 4445 mm*1770 mm*1550 mm

Frontal Area = $1.77\text{m} \times 1.55\text{m} = 2.7435 \text{ m}^2$

Drag Coefficient = 0.28

Basic Unit Conversions

$1\text{N} = 1 \text{ Kg} \cdot \text{m/s}^2$

$1 \text{ Watt} = 1 \text{ N} \cdot \text{m/s} = 1 \text{ Kg} \cdot \text{m}^2/\text{s}^3$

1. What is the wheel power in kW required for acceleration of an electric vehicle from 0 to 100 Km/h in 11.5 seconds?

Now for a vehicle accelerating from rest, the following resistance will be applicable

Rolling Resistance + Acceleration Resistance + Drag resistance

Formulae Applied:

Total Wheel Force Required = Rolling Resistance + Acceleration Resistance + Drag resistance

Acceleration Resistance Force = mass(kg) * acceleration(m/s²)

Rolling Resistance $F_r = m * g * \mu_r = 1500 * 9.8 * 0.013 = 191 \text{ N}$

Drag resistance Force $F_d = \frac{1}{2} (\text{air density} * \text{Frontal Area} * \text{Drag Coefficient} * ((\text{Vehicle speed} - \text{Wind speed})^2))$

Solution:

Final speed = 100 Km/h = 27.77 m/s

Wheel Power Required = Force (N) * Velocity (m/s)

Acceleration Resistance Force = mass(kg) * acceleration(m/s²)

$$= \text{mass} * ((\text{Final Velocity (m/s)} - \text{Initial Velocity(m/s)}) / \text{Acceleration Time(s)})$$

$$= 1500 * ((27.77 - 0) / 11.5)$$

$$= 3622.17 \text{ Kg} * \text{m/s}^2 = 3622 \text{ N}$$

Drag resistance Force $F_d =$

$$\frac{1}{2} (\text{air density} * \text{Frontal Area} * \text{Drag Coefficient} * ((\text{Vehicle speed} - \text{Wind speed})^2))$$

$$= 0.5 * 1.184 \text{ Kg/m}^3 * 2.7435 \text{m}^2 * 0.28 * (20.83)^2$$

$$= 197.3 \text{ N}$$

Total Force Required = Rolling Resistance + Acceleration Resistance + Drag resistance

$$= 191 + 3622 + 197.3 = 4010 \text{ N}$$

Distance Travelled for Acceleration from 0-100 Km/h in 11.5 seconds = $s = ut + \frac{1}{2} at^2$

$$= 0 + (0.5 * 2.414 * 11.5 * 11.5) = 159 \text{ m}$$

Required Wheel Power = Force (N) * Distance travelled (m) = $4010 \text{ N} * 159 \text{ m} = 637590 \text{ Nm} = 637590 \text{ J}$

Now 637590 J energy has been spent for 11.5 seconds

Then Power delivered = $55442 \text{ J/s} = 55442 \text{ Watt} = 55 \text{ kW}$

Required wheel Power for accelerating from 0 to 100 Km/h in 11.5 seconds = 55 kW

2. What is the wheel power in kW required in an electric vehicle for negotiating a gradient of 20 % when cruising at a speed of 40 Kmph for 30 seconds?

Now for a vehicle negotiating a gradient of 20 % when cruising at a speed of 40 Kmph for 30 seconds, the following resistance will be applicable

Total Resistance = Rolling Resistance + Drag resistance + Gradient Resistance

Formulae Applied:

Wheel Power Required = Force (N) * Velocity (m/s)

Force Required for Gradient climbing = $m * g * \sin \Theta$

Solution:

Rolling Resistance $F_r = m * g * \mu_r = 1500 * 9.8 * 0.013 = 191 \text{ N}$

Drag resistance Force $F_d = 0.5 * 1.184 \text{ Kg/m}^3 * 2.7435 \text{ m}^2 * 0.28 * (11.11)^2 = 56.13 \text{ N}$

Gradient resistance Force = $m * g * \sin \Theta$

Convert the gradient from percentage to degree

Slope in Degrees = $[\tan^{-1} (\text{Slope in \%} / 100)]$ – If you have percentage value

$= \tan^{-1}(20/100) = \tan^{-1}(0.2) = 11.3 \text{ Deg}$

$F = 1500 \text{ kg} * 9.8 \text{ m/s}^2 * \sin (11.30)$

$= 1500 * 9.8 * 0.1959$

$= 2879.73 \text{ kg} * \text{m/s}^2 = 2880 \text{ N}$

Total resistance = $191 + 56 + 2880 = 3127 \text{ N}$

Total Power Required to negotiate the Gradient @ 40kmph (11.11 m/s)

$= \text{Force (N)} * \text{Velocity (m/s)} = 3127 \text{ N} * 11.11 \text{ m/s}$

$= 34740.97 \text{ Nm/s} = 34741 \text{ Watt} = 34.741 \text{ kW}$

3. What is the wheel power required in kW to maintain a constant cruising speed of 150 Kmph in an electric vehicle?

A vehicle running at a top speed need to overcome the rolling resistance and aerodynamic resistance.

$$\text{Rolling resistance Force} = F_r = m * g * \mu_r * \cos \alpha$$

$$= 1500 \text{ kg} * 9.8 \text{ m/s}^2 * 0.013 * \cos 0^\circ$$

$$= 191.1 \text{ kg m/s}^2 = 191 \text{ N}$$

$$\text{Power Required to overcome rolling resistance} = 191 \text{ N} * 41.66 \text{ m/s}$$

$$= 7957.06 \text{ Watt} = 8 \text{ kW}$$

$$\text{Aerodynamic Resistance Force } F_w = \frac{1}{2} \rho A_f C_D (V - V_w)^2$$

$$= \frac{1}{2} (\text{air density} * \text{Frontal Area} * \text{Drag Coefficient} * ((\text{Vehicle speed} - \text{Wind speed})^2))$$

$$= 0.5 * 1.184 \text{ Kg/m}^3 * 2.7435 \text{ m}^2 * 0.28 * (41.66 - 0)^2$$

$$= 789.26 \text{ kg} * \text{m/s}^2 = 790 \text{ N}$$

$$\text{Power Required to overcome aerodynamic resistance} = 790 \text{ N} * 41.66 \text{ m/s}$$

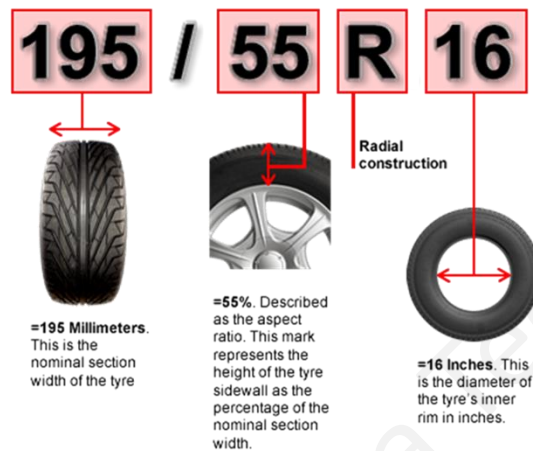
$$= 32953.06 \text{ Watt} = 32.94 \text{ kW}$$

$$\text{Total Power Required to travel at 150 Kmph}$$

$$= 8 \text{ kW} + 32.94 \text{ kW} = 40.94 \text{ kW}$$

4. What is the wheel radius in metres of an electric passenger car with the following tyre and wheel rim specifications 195/55 R 16

Wheel Radius Calculation:



Wheel specification → 195/55 R16 means

Wheel radius = Wheel Diameter / 2 = [(2*Sidewall height in m) + Rim Diameter in m]/2

= Side wall height = 55% of section width = $0.55 \times 0.195\text{m} = 0.10725\text{ m}$

Rim Diameter = 16 inches = 0.4064 m

Wheel radius = [(2*0.10725 in m) + 0.4064 in m]/2 = 0.31045 m

5. What is the electric motor speed in rpm required for achieving a top speed of 180Kmph in an electric vehicle?

Let us first calculate the Wheel speed in rpm

Vehicle speed = 180 Kmph = 50 m/s

To convert m/s to rpm = Vehicle speed (m/s) * (60/ (2*pi*Wheel radius))

Wheel speed in rpm = Vehicle speed (m/s) * (60/ (2*pi*Wheel radius))

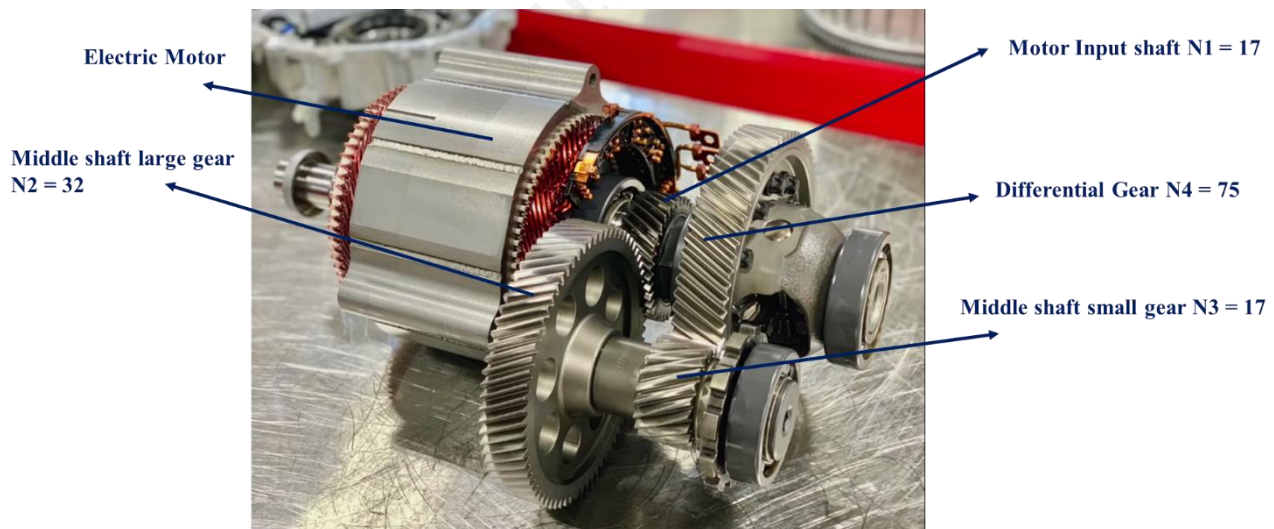
= 50 m/s * (60/ (2*3.14*0.31045m))

= 50 * 30.78

= 1500 rpm

Now the wheel needs to rotate at 1500 rpm to operate at 180 Kmph

Now consider the following drivetrain assembly.



Total Gear Ratio = $i_g + i_o = N2/N1 + N4/N3 = (32/17) + (75/17) = 1.8823 + 4.41 = 6.29$

Motor speed = (Wheel speed * i_g * i_o)

= 1500*1.88*4.41 = 12436 rpm

For the vehicle to operate @ 180 Kmph, the motor needs to operate @ 12436 rpm

So to reach the target top speed, the motor selected should have a capability to operate at a maximum speed of 12436 rpm

6. What is the continuous power for an electric vehicle required for the following drive cycle?

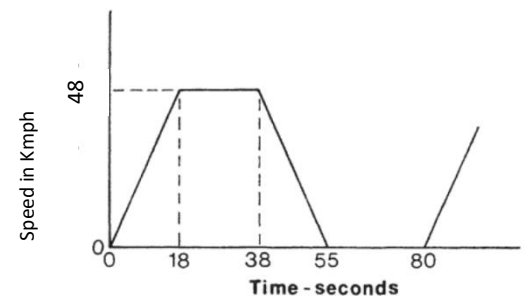
Firstly, the Rms Power is the average power required over a driving cycle. It considers all the varying torque values used during operation as well as the time duration each torque value is needed

Acceleration, 0–18 s

Cruise, 18–38 s (including 5 s climbing a 10% grade):

Coast and brake, 38–55 s

Idle, 55–80 s



Solution:

Now for the ease of calculation, let us divide the driving cycle into different sections

Section 1 → 0 to 18 seconds – Vehicle accelerating from 0 to 48 Km/h

Section 2 → 18 to 33 seconds - Vehicle cruising @ 48 Km/h

Section 3 → 33 to 38 seconds - Vehicle negotiating a grade 10%

Section 4 → 38 to 55 seconds – Vehicle deceleration from 48 Km/h to 0

Section 1 → 0 to 18 seconds – Vehicle accelerating from 0 to 48 Km/h

Now for a vehicle accelerating from rest, the following resistance will be applicable

Rolling Resistance + Acceleration Resistance + Drag resistance

$$\text{Rolling Resistance } F_r = m \cdot g \cdot \mu_r = 1500 \cdot 9.8 \cdot 0.013 = 191 \text{ N}$$

Drag resistance Force $F_d =$

$$= \frac{1}{2} (\text{air density} \cdot \text{Frontal Area} \cdot \text{Drag Coefficient} \cdot ((\text{Vehicle speed} - \text{Wind speed})^2))$$

$$= 0.5 \cdot 1.184 \text{ Kg/m}^3 \cdot 2.7435 \text{ m}^2 \cdot 0.28 \cdot (6.66)^2$$

$$= 20 \text{ N}$$

$$\text{Acceleration Resistance Force} = \text{mass(kg)} \cdot \text{acceleration(m/s}^2)$$

$$= \text{mass} \cdot ((\text{Final Velocity (m/s)} - \text{Initial Velocity(m/s)}) / \text{Acceleration Time(s)})$$

$$= 1500 \cdot ((13.33 - 0)/18) = 1110.8 \text{ Kg} \cdot \text{m/s}^2 = 1111 \text{ N}$$

Total Force Required = Rolling Resistance + Acceleration Resistance + Drag resistance

$$= 191 + 20 + 1111 = 1322 \text{ N}$$

Distance Travelled for Acceleration from 0-48 Km/h in 18 seconds = $s = ut + (1/2) \times at^2$

$$= 0 + (0.5 * 0.740 * 18 * 18) = 120 \text{ m}$$

$$\text{Energy Required} = \text{Force} * \text{Distance} = 1322 \text{ N} * 120 \text{ m} = 158640 \text{ J} = 158.6 \text{ kJ}$$

Section 2 → 18 to 23 seconds - Vehicle cruising @ 48 Kmph

Now for a vehicle cruising, the following resistance will be applicable

$$\text{Total Force Required} = \text{Rolling Resistance} + \text{Drag resistance}$$

$$\text{Rolling Resistance } F_r = m * g * \mu_r = 1500 * 9.8 * 0.013 = 191 \text{ N}$$

$$\text{Drag resistance Force } F_d =$$

$$= \frac{1}{2} (\text{air density} * \text{Frontal Area} * \text{Drag Coefficient} * ((\text{Vehicle speed} - \text{Wind speed})^2))$$

$$= 0.5 * 1.184 \text{ Kg/m}^3 * 2.7435 \text{ m}^2 * 0.28 * (13.33)^2 = 80.8 \text{ N}$$

$$\text{Total Force Required} = 191 \text{ N} + 80.8 \text{ N} = 271.8 \text{ N}$$

$$\text{Distance travelled} = \text{Speed} * \text{Time} = 13.33 \text{ m/s} * 15 \text{ s} = 200 \text{ m}$$

$$\text{Required Energy} = \text{Force} * \text{Distance} = 271.8 \text{ N} * 200 \text{ m} = 54360 \text{ J} = 54.36 \text{ kJ}$$

Section 3 → 33 to 38 seconds - Vehicle negotiating a grade 10%

Now for a vehicle negotiating a gradient of 10 % when cruising at a speed of 48 Kmph for 5 seconds, the following resistance will be applicable

$$\text{Total Resistance} = \text{Rolling Resistance} + \text{Drag resistance} + \text{Gradient Resistance}$$

$$\text{Rolling Resistance } F_r = m * g * \mu_r = 1500 * 9.8 * 0.013 = 191 \text{ N}$$

$$\text{Drag resistance Force } F_d =$$

$$= \frac{1}{2} (\text{air density} * \text{Frontal Area} * \text{Drag Coefficient} * ((\text{Vehicle speed} - \text{Wind speed})^2))$$

$$= 0.5 * 1.184 \text{ Kg/m}^3 * 2.7435 \text{ m}^2 * 0.28 * (13.33)^2 = 80.8 \text{ N}$$

$$\text{Gradient resistance Force} = m * g * \sin \theta$$

Convert the gradient from percentage to degree

$$\begin{aligned} \text{Slope in Degrees} &= [\tan^{-1} (\text{Slope in \%} / 100)] - \text{If you have percentage value} \\ &= \tan^{-1}(10/100) = \tan^{-1}(0.1) = 6^\circ \end{aligned}$$

$$F = 1500 \text{ kg} * 9.8 \text{ m/s}^2 * \sin(6^\circ) = 1500 * 9.8 * 0.1045 = 1536.15 \text{ kg} * \text{m/s}^2 = 1536 \text{ N}$$

$$\text{Total Force Required} = 191 \text{ N} + 80.8 \text{ N} + 1536 \text{ N} = 1808 \text{ N}$$

Distance travelled = Speed * Time = 13.33 m/s * 5 s = 66.65 m

Required Energy = Force * Distance = **1808 N * 66.65 m = 120503 J = 120.5 kJ**

Section 4 → 38 to 55 seconds – Vehicle deceleration from 48 Kmph to 0

Stopping distance = reaction distance + braking distance

For example, Reaction Time for 48 Kmph \approx 15 m

Deceleration $d = \frac{v^2 - u^2}{2s} = \frac{0^2 - 13.33^2}{2 * 15} = -0.40 \text{ m/s}^2$

Stopping distance = $\frac{v^2}{2d} = \frac{13.33^2}{2 * 0.40} = 222 \text{ m}$

Peak power dissipated during braking = Braking Force N * Velocity m/s

Required Braking Force = Rolling Resistance + Deceleration Resistance + Drag resistance
 $= (1500 * 9.8 * 0.013) + 1500 * (-0.40) + 0.5 * 1.184 \text{ Kg/m}^3 * 2.7435 \text{ m}^2 * 0.28 * (6.66)^2$
 $= 191.1 - 600 + 20.17 = -388.73 \text{ N}$

Braking energy required = $(-388.73) \text{ N} * 222 \text{ m} = -86298 \text{ Nm} = -86298 \text{ J}$ for 17 seconds

Consider 15% of the braking energy is recovered by regenerative braking, then the

Total energy utilised in one drive cycle = 158.6 kJ + 54.36 kJ + 120.5 kJ + $(86.298 \text{ kJ} * 0.85)$
 $= 406.8133 \text{ kJ}$

Average Power consumed for the complete driving cycle = $211.8 \text{ kJ} / 55 \text{ seconds} = 3.85 \text{ kW}$

Total distance covered = 120 m + 200 m + 67 m + 222 m = 609 m = 0.609 Km / cycle

Consider the following battery specifications for this vehicle

Battery Voltage = 360 V

Battery Capacity = 66 Ah

So now consider a Battery energy available = $(360 \text{ V} * 66 \text{ Ah} * 3600) = 8,553,600 \text{ J}$

No. of drive cycles = $8553600 \text{ J} / 406813 \text{ J} = 210 \text{ cycles}$

Travel Range of the vehicle = $210 * 0.608 \text{ Km} = 127 \text{ Km}$

Battery Related sample Calculations

1. What is the Total number of cells required to design a battery pack with following specifications?

Nominal Voltage per cell = 3.8 V

Nominal Capacity per cell = 33.1 Ah

Required energy of the battery pack = 24 kWh

$$\begin{aligned}\text{Required energy per cell} &= \text{Nominal Voltage per cell} * \text{Nominal Capacity per cell} \\ &= 3.8 \text{ V} * 33.1 \text{ Ah} = 125.78 \text{ Watthour}\end{aligned}$$

$$\text{Total number of cells required} = 24000 / 125.78 = 192$$

2. What is the peak power capability of the battery cell with the following specifications?

Open circuit voltage of each cell = 3.8 V

Internal Resistance of each cell = 2 mOhm

$$\text{Peak Power (in kW)} = V_{\text{Open Circuit}}^2 / 4R$$

$$\text{Peak power per cell} = (3.8^2 / 4 * 2) = 1800 \text{ Watt}$$

3. What is the energy consumed by an electric vehicle operating at a constant speed of 50 & 100 Kmph with a battery operating voltage of 360 V. The current consumption values are approximately considered from the test results of Nissan leaf 2011.

$$\text{Wh/km} = ((\text{Current Drawn} / \text{Vehicle speed}) * (\text{Battery Voltage}))$$

$$\text{Wh/km @ 50 Kmph} = (14.44 \text{ A} / 50 \text{ Kmph}) * 360 \text{ V} = 104.4 \text{ Wh/km}$$

$$\text{Wh/km @ 100 kmph} = (50 \text{ A} / 100 \text{ kmph}) * 360 \text{ V} = 180 \text{ Wh/km}$$

4. What is the range of the electric vehicle operating at a constant speed of 50 Kmph with a following battery specifications?

$$\text{Wh/km @ 50 Kmph} = 104.4 \text{ Wh/km}$$

$$\text{Range of the electric vehicle} = (\text{Usable Pack size} / \text{Wh/km})$$

$$= (22000 \text{ (Wh)} / 104.4 \text{ (Wh/km)}) = 210 \text{ Km}$$

5. What is the charging time required for an electric vehicle with a 24-kWh battery specification?

Formula Used:

$$\text{Charging Time in Hours} = \text{Battery Capacity in kWh} / \text{Charging Power Delivered in kW}$$

Case 1: On Board Slow Charging

Consider the Vehicle is charged with the on-board charger operating at 230 V AC & 15 A

Then the rate of charging = $230 \text{ V} * 15 \text{ A} = 3.4 \text{ kWh}$

Charging Time = $24 \text{ kW} / 3.4 \text{ kWh} = 7.05 \text{ Hours}$

Case 2: Fast Charging

Consider the Vehicle is charged with an off-board fast charger operating at 360 V DC & 100 A

Then the rate of charging = $360 * 100 = 36 \text{ kWh}$

Charging Time = $24 \text{ kW} / 36 \text{ kWh} = 0.66 \text{ Hours} = \text{Approx. 40 Minutes}$