0922 Class Activity

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Show that if G is a simple graph with p vertices, where each vertex has degree not less than $\frac{p-1}{2}$, then G must be connected.

Proof. Suppose that G is not connected simple graph with p vertices and each vertex in G has degree not less than $\frac{p-1}{2}$. Then, we can split the vertex set V into two sets V_1 and V_2, where for any $v \in V_1$ and $w \in V_2$, there are no edges (v, w) in the edge set E.

Let $|V_1| = a$ and $|V_2| = b$. Since each vertex in V_1 has degree not less than $\frac{p-1}{2}$ and these vertices are not connected into V_2, the number of vertices in V_1, a, must be larger than or equal to $\frac{p-1}{2} + 1$.

Similarly, the number of vertices in V_2, b, must be larger than or equal to $\frac{p-1}{2} + 1$.

Hence, the number of vertices in V must be larger than or equal to $(p-1)+1+1 = p+1$, which contradicts to G has p vertices.

$\square$