

1009 Class Activity - 2020/6/27 박예영

$$G_1 = \begin{array}{c} \text{Diagram of } G_1 \\ \text{Compute } P(G_1, \chi) \end{array}$$

$$\text{Sol) } P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \end{array}, \chi\right) = P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{without edge } e \end{array}, \chi\right) - P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \\ \text{and cycle } C_4 \end{array}, \chi\right)$$

$$P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \end{array}, \chi\right) = P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{without edge } e \end{array}, \chi\right) - P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \\ \text{and cycle } C_4 \end{array}, \chi\right)$$

$$P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \end{array}, \chi\right) = P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{without edge } e \end{array}, \chi\right) - P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \\ \text{and cycle } C_4 \\ \text{and cycle } C_4' \end{array}, \chi\right)$$

$$P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \end{array}, \chi\right) = P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{without edge } e \end{array}, \chi\right) - P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \\ \text{and cycle } C_4 \\ \text{and cycle } C_4' \\ \text{and cycle } C_4'' \end{array}, \chi\right)$$

$$P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \end{array}, \chi\right) = P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{without edge } e \end{array}, \chi\right) - P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{with edge } e \\ \text{and cycle } C_4 \\ \text{and cycle } C_4' \\ \text{and cycle } C_4'' \\ \text{and cycle } C_4''' \end{array}, \chi\right)$$

$$\begin{aligned} &= \chi \cdot P(C_4, \chi) - P(C_4, \chi) = (\chi-1)[(\chi-1)^4 + (\chi-1)] \\ &= (\chi-1)^5 + (\chi-1)^2. \end{aligned}$$

$$\begin{aligned} P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{without edge } e \end{array}, \chi\right) &= (\chi-1)^5 + (\chi-1)^2 - P(T_4, \chi) \\ &= (\chi-1)^5 + (\chi-1)^2 - \chi(\chi-1)^3 \end{aligned}$$

$$\begin{aligned} P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{without edge } e \end{array}, \chi\right) &= P(C_6, \chi) - (\chi-1)^5 - (\chi-1)^2 \\ &\approx (\chi-1)^6 - (\chi-1)^5 - (\chi-1)^2 + (\chi-1) \end{aligned}$$

$$P\left(\begin{array}{c} \text{Diagram of } G_1 \\ \text{without edge } e \end{array}, \chi\right) = (\chi-1)^6 - (\chi-1)^5 - (\chi-1)^2 + (\chi-1) - (\chi-1)^5 - (\chi-1)^2 + \chi(\chi-1)^3.$$

$$= (x-1)^6 - 2(x-1)^5 + x(x-1)^3 - 2(x-1)^2 + (x-1)$$

$$\begin{aligned} P(\Delta, x) &= (x-1)^6 - 2(x-1)^5 + x(x-1)^3 - 2(x-1)^2 + (x-1) \\ &\quad - (x-1)^5 - x(x-1)^2 + x(x-1)^3 \\ &= (x-1)^6 - 3(x-1)^5 + 2x(x-1)^3 - 3(x-1)^2 + (x-1) \\ &= \boxed{x^6 - 9x^5 + 32x^4 - 56x^3 + 48x^2 - 16x} \end{aligned}$$