1113 Class Activity

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Check that this theorem for cycle C_6.
Thm) If the eigenvalues of A_G are $(k, \theta_2, \theta_3, \cdots, \theta_n)$, then the eigenvalues of $A_{\bar{G}}$ are $(n-1-k, -1-\theta_2, -1-\theta_3, \cdots, -1-\theta_n)$.

Proof. C_6와 $\bar{C}_6$의 adjacency matrix A_{C_6}, $A_{\bar{C_6}}$는 아래와 같다.

$$A_{C_6} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, A_{\bar{C_6}} = \begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

A_{C_6}의 eigenvalue들은 $(2, 1, 1, -1, -1, -2)$이고, $A_{\bar{C_6}}$의 eigenvalue들은 $(3, -2, -2, 0, 0, 1)$이므로 위의 정리가 잘 성립함을 확인할 수 있다.

- $v=\begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}$, eigenvalue $\lambda_1=1$
- $v=\begin{pmatrix} -1 \\ 1 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix}$, eigenvalue $\lambda_2=-1$
- $v=\begin{pmatrix} 1 \\ 0 \\ -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_1=1$
- $v=\begin{pmatrix} -1 \\ 0 \\ 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_2=-1$
- $v=\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_3=2$
- $v=\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}$, eigenvalue $\lambda_4=-2$

- $v=\begin{pmatrix}-1\\1\\0\\-1\\1\\0\end{pmatrix}$, eigenvalue $\lambda_1=0$
- $v=\begin{pmatrix}-1\\0\\1\\-1\\0\\1\end{pmatrix}$, eigenvalue $\lambda_1=0$
- $v=\begin{pmatrix}-1\\1\\-1\\1\\-1\\1\end{pmatrix}$, eigenvalue $\lambda_2=1$
- $v=\begin{pmatrix}-1\\-1\\0\\1\\1\\0\end{pmatrix}$, eigenvalue $\lambda_3=-2$
- $v=\begin{pmatrix}1\\0\\-1\\-1\\0\\1\end{pmatrix}$, eigenvalue $\lambda_3=-2$
- $v=\begin{pmatrix}1\\1\\1\\1\\1\\1\end{pmatrix}$, eigenvalue $\lambda_4=3$

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