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Homework I – Group 041

I. Pen-and-paper

1)

Dataset

	y1	y2	y_out
x1	A	0	P
x2	B	1	P
x3	A	1	P
x4	A	0	P
x5	B	0	N
x6	B	0	N
x7	A	1	N
x8	B	1	N

Distâncias

xi	xj	string(xi)	string(xj)	Hamming(xi, xj)	d(xi, xj)
x1	x2	A0	B1	2	2.5
x1	x3	A0	A1	1	1.5
x1	x4	A0	A0	0	0.5
x1	x5	A0	B0	1	1.5
x1	x6	A0	B0	1	1.5
x1	x7	A0	A1	1	1.5
x1	x8	A0	B1	2	2.5
x2	x3	B1	A1	1	1.5
x2	x4	B1	A0	2	2.5
x2	x5	B1	B0	1	1.5
x2	x6	B1	B0	1	1.5
x2	x7	B1	A1	1	1.5
x2	x8	B1	B1	0	0.5
x3	x4	A1	A0	1	1.5
x3	x5	A1	B0	2	2.5
x3	x6	A1	B0	2	2.5
x3	x7	A1	A1	0	0.5
x3	x8	A1	B1	1	1.5
x4	x5	A0	B0	1	1.5
x4	x6	A0	B0	1	1.5
x4	x7	A0	A1	1	1.5
x4	x8	A0	B1	2	2.5
x5	x6	B0	B0	0	0.5
x5	x7	B0	A1	2	2.5
x5	x8	B0	B1	1	1.5
x6	x7	B0	A1	2	2.5
x6	x8	B0	B1	1	1.5
x7	x8	A1	B1	1	1.5

$KNN = 5$

$$\text{Hamming}(x_1, x_2) = \sum_{j=1}^{m} \eta(x_{1j} \cdot x_{2j})$$

$$\eta = \begin{cases} 0 & a_{1j} = a_{2j} \\ 1 & c.c \end{cases}$$

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x1	xj	d(xi, xj)	y_out
x1	x4	0.5	P
x1	x3	1.5	P
x1	x5	1.5	N
x1	x6	1.5	N
x1	x7	1.5	N
x1	x2	2.5	P
x1	x8	2.5	N

$$\hat{z}_{x_1} = \text{mode}\left(\left(\frac{1}{0{,}5}+\frac{1}{1{,}5}\right)P \mid \left(\frac{1}{1{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}\right)N\right)$$

$$= \text{mode}\,(2{,}67\,P \mid 2\,N) = P$$

x2	xj	d(xi, xj)	y_out
x2	x8	0.5	N
x2	x3	1.5	P
x2	x5	1.5	N
x2	x6	1.5	N
x2	x7	1.5	N
x2	x1	2.5	P
x2	x4	2.5	P

$$\hat{z}_{x_2} = \text{mode}\left(\frac{1}{1{,}5}\,P \mid \left(\frac{1}{0{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}\right)N\right) =$$

$$= \text{mode}\,(0{,}67\,P \mid 4\,N) = N$$

x3	xj	d(xi, xj)	y_out
x3	x7	0.5	N
x3	x1	1.5	P
x3	x2	1.5	P
x3	x4	1.5	P
x3	x8	1.5	N
x3	x5	2.5	N
x3	x6	2.5	N

$$\hat{z}_{x_3} = \text{mode}\left(\left(\frac{1}{1{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}\right)P \;/\; \left(\frac{1}{0{,}5}+\frac{1}{1{,}5}\right)N\right)$$

$$= \text{mode}\,(2P \mid 2.67\,N) =$$

$$= N$$

x4	xj	d(xi, xj)	y_out
x4	x1	0.5	P
x4	x5	1.5	N
x4	x6	1.5	N
x4	x7	1.5	N
x4	x3	1.5	P
x4	x2	2.5	P
x4	x8	2.5	N

$$\hat{z}_{x_4} = \text{mode}\left(\left(\frac{1}{0{,}5}+\frac{1}{1{,}5}\right)P \mid \left(\frac{1}{1{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}\right)N\right)$$

$$= \text{mode}\,(2{,}67\,P \mid 2\,N) = P$$

x5	xj	d(xi, xj)	y_out
x5	x6	0.5	N
x5	x8	1.5	N
x5	x1	1.5	P
x5	x2	1.5	P
x5	x4	1.5	P
x5	x7	2.5	N
x5	x3	2.5	P

$$\hat{z}_{x_5} = \text{mode}\left(\left(\frac{1}{1{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}\right)P,\ \left(\frac{1}{0{,}5}+\frac{1}{1{,}5}\right)N\right)$$

$$= \text{mode}\,(2\,P,\ 2.67\,N) = N$$

x6	xj	d(xi, xj)	y_out
x6	x5	0.5	N
x6	x8	1.5	N
x6	x1	1.5	P
x6	x2	1.5	P
x6	x4	1.5	P
x6	x7	2.5	N
x6	x3	2.5	P

$$\hat{z}_{x_6} = \text{mode}\left(\left(\frac{1}{1{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}\right)P \mid \left(\frac{1}{0{,}5}+\frac{1}{1{,}5}\right)N\right)$$

$$= \text{mode}\,(2P \mid 2.67\,N) = N$$

x7	xj	d(xi, xj)	y_out
x7	x3	0.5	P
x7	x1	1.5	P
x7	x2	1.5	P
x7	x4	1.5	P
x7	x8	1.5	N
x7	x5	2.5	N
x7	x6	2.5	N

$$\hat{z}_{x_7} = \text{mode}\left(\left(\frac{1}{0{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}\right)P,\ \frac{1}{1{,}5}N\right) =$$

$$= \text{mode}\left(4P,\ 2.67N\right) = P$$

x8	xj	d(xi, xj)	y_out
x8	x2	0.5	P
x8	x7	1.5	N
x8	x5	1.5	N
x8	x6	1.5	N
x8	x3	1.5	P
x8	x1	2.5	P
x8	x4	2.5	P

$$\hat{z}_{x_8} = \text{mode}\left(\left(\frac{1}{0{,}5}+\frac{1}{1{,}5}\right)P,\ \left(\frac{1}{1{,}5}+\frac{1}{1{,}5}+\frac{1}{1{,}5}\right)N\right) =$$

$$= \text{mode}\left(2{,}67P,\ 2N\right) = P$$

$\hat{z}_{x_1} = P \quad \hat{z}_{x_2} = N \quad \hat{z}_{x_3} = N \quad \hat{z}_4 = P \quad \hat{z}_5 = N \quad \hat{z}_6 = N \quad \hat{z}_7 = P \quad \hat{z}_8 = P$

		True P	True N
Guess	P	1+1=2	1+1=2
	N	1+1=2	1+1=2

$$\text{Recall} = \frac{TP}{TP+FN} = \frac{2}{2+2} = \frac{1}{2} = 0{,}5$$

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2)

	y1	y2	y_out
x1	A	0	P
x2	B	1	P
x3	A	1	P
x4	A	0	P
x9	B	0	P
x5	B	0	N
x6	B	0	N
x7	A	1	N
x8	B	1	N

y3	y_out
1.2	P
0.8	P
0.5	P
0.9	P
0.8	P
1	N
0.9	N
1.2	N
0.8	N

Para a classe y_out

$$P(y\_out = P \mid x) = \frac{P(x \mid y\_out = P)\, P(y\_out = P)}{P(x)}$$

$$P(y\_out = N) = 1 - P(y\_out = P \mid x)$$

$$P(x \mid y_{out} = P) = P(y_1, y_2 \mid y_{out} = P) \times P(y_3 \mid y_{out} = P)$$

$$P(x \mid y_{out} = N) = P(y_1, y_2 \mid y\_out = N) \times P(y_3 \mid y\_out = N)$$

$$P(y\_out = P) = \frac{5}{9} = 0{,}56$$

$$P(y\_out = N) = \frac{4}{9} = 0{,}44$$

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Determinar a posterior de $P(Y_3 \mid Y_{out} = P)$:

Como Y_3 está normalmente distribuída então temos usar a fórmula da Gaussiana univariante ou seja :

$$P(Y_3 \mid Y_{out} = P) = N(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \times e^{-\frac{1}{2\times\sigma^2}\cdot(x-\mu)^2}$$

$$= \frac{1}{\sqrt{2\pi \times 0{,}063}} \times e^{-\frac{1}{2\times 0{,}063}\times(Y_3 - 0{,}84)^2}$$

$$\mu = \frac{1{,}2 + 0{,}8 + 0{,}5 + 0{,}9 + 0{,}8}{5} = 0{,}84$$

$$\sigma^2 = \frac{\sum_{i=1}^{5}(Y_{3_i} - \mu)^2}{5-1} = \frac{(1{,}2-0{,}84)^2 + (0{,}8-0{,}84)^2 + (0{,}5-0{,}84)^2 + (0{,}9-0{,}84)^2 + (0{,}8-0{,}84)^2}{4}$$

$$= \frac{0{,}1296 + 0{,}0016 + 0{,}1156 + 0{,}0036 + 0{,}0016}{4}$$

$$= \frac{0{,}252}{4} = 0{,}063$$

Para $P(Y_3 \mid Y_{out} = N)$

Aplica-se o mesmo que o $P(Y_3 \mid Y_{out} = P)$

ou seja :

$$P(Y_3 \mid Y_{out} = N) = N(Y_3 \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi \times 0{,}0292}} \times e^{-\frac{1}{2\times 0{,}0292}\times(Y_3 - 0{,}975)^2}$$

$$\mu = \frac{1 + 0{,}9 + 1{,}2 + 0{,}8}{4} = 0{,}975$$

$$\sigma^2 = \frac{\sum_{i=1}^{4}(Y_{3_i} - \mu)^2}{4-1} = \frac{(1-0{,}975)^2 + (0{,}9-0{,}975)^2 + (1{,}2-0{,}975)^2 + (0{,}8-0{,}975)^2}{3} = 0{,}0292$$

$$P(Y_1, Y_2 \mid y\_out = P)$$

$$P(Y_1 = A, Y_2 = 0 \mid Y_{out} = P) = \frac{2}{5} = 0{,}4$$

$$P(Y_1 = A, Y_2 = 1 \mid Y_{out} = P) = \frac{1}{5} = 0{,}20$$

$$P(Y_1 = B, Y_2 = 0 \mid Y_{out} = P) = \frac{1}{5} = 0{,}20$$

$$P(Y_1 = B, Y_2 = 1 \mid Y_{out} = P) = \frac{1}{5}$$

$$P(Y_1 = A, Y_2 = 0 \mid Y_{out} = N) = \frac{0}{4} = 0$$

$$P(Y_1 = A, Y_2 = 1 \mid Y_{out} = N) = \frac{1}{4} = 0{,}25$$

$$P(Y_1 = B, Y_2 = 0 \mid Y_{out} = N) = \frac{2}{4} = 0{,}50$$

$$P(Y_1 = B, Y_2 = 1 \mid Y_{out} = N) = \frac{1}{4} = 0{,}25$$

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3)

Calculos auxiliares

$$P(Y\_out = P) = \frac{5}{9}$$

$$P(Y\_out = N) = \frac{4}{9}$$

$$P(\underline{x} = [A\ 1\ 0.8] \mid Y\_out = P) = P(Y_1 = A, Y_2 = 1 \mid Y\_out = P) \times P(Y_3 = 0.8 \mid Y\_out = P)$$

$$= \frac{1}{5} \times \frac{1}{\sqrt{2\pi \times 0{,}063}} \times e^{-\frac{1}{2 \times 0{,}063} \times (0{,}8 - 0{,}84)^2} = \underline{0{,}314}$$

$$P(\underline{x} = [B\ 1\ 1] \mid Y\_out = P) = P(Y_1 = B, Y_2 = 1 \mid Y\_out = P) \times P(Y_3 = 1 \mid Y\_out = P)$$

$$= \frac{1}{5} \times \frac{1}{\sqrt{2\pi \times 0{,}063}} \times e^{-\frac{1}{2 \times 0{,}063} \times (1 - 0{,}84)^2} = \underline{0{,}259}$$

$$P(\underline{x} = [B\ 0\ 0.9] \mid Y\_out = P) = P(Y_1 = B, Y_2 = 0 \mid Y\_out = P) \times P(Y_3 = 0.9 \mid Y\_out = P)$$

$$= \frac{1}{5} \times \frac{1}{\sqrt{2\pi \times 0{,}063}} \times e^{-\frac{1}{2 \times 0{,}063} \times (0{,}9 - 0{,}84)^2} = \underline{0{,}309}$$

$$= \frac{1}{5} \times \frac{1}{\sqrt{2\pi \times 0{,}063}} \times e^{-\frac{1}{2 \times 0{,}063} \times (0{,}9 - 0{,}84)^2} = \underline{0{,}309}$$

$$P(\underline{x} = [A\ 1\ 0.8] \mid Y\_out = N) = P(Y_1 = A, Y_2 = 1 \mid Y\_out = N) \times P(Y_3 = 0.8 \mid Y\_out = N)$$

$$= \frac{1}{4} \times \frac{1}{\sqrt{2\pi \times 0{,}0292}} \times e^{-\frac{1}{2 \times 0{,}0292} \times (0{,}8 - 0{,}975)^2} = \underline{0{,}345}$$

$$P(\underline{x} = [B\ 1\ 1] \mid Y\_out = N) = P(Y_1 = B, Y_2 = 1 \mid Y\_out = N) \times P(Y_3 = 1 \mid Y\_out = N)$$

$$= \frac{1}{4} \times \frac{1}{\sqrt{2\pi \times 0{,}0292}} \times e^{-\frac{1}{2 \times 0{,}0292} \times (1 - 0{,}975)^2} = 0{,}577$$

$$P(\underline{x} = [B\ 0\ 0.9] \mid Y\_out = N) = P(Y_1 = B, Y_2 = 0 \mid Y\_out = N) \times P(Y_3 = 0.9 \mid Y\_out = N)$$

$$= \frac{2}{4} \times \frac{1}{\sqrt{2\pi \times 0{,}0292}} \times e^{-\frac{1}{2 \times 0{,}0292} \times (0{,}9 - 0{,}975)^2} = 1$$

Calculo principal

$$h_{MAP} = \arg\max P(\text{Y\_out} = P \mid x) = \arg\max \frac{P(x \mid \text{Y\_out} = P) \times P(\text{Y\_out} = P)}{P(x)}$$

$$P(\text{Y\_out} = P \mid X = [A\ 1\ 0.8]) = \frac{P(X = [A\ 1\ 0.8] \mid \text{Y\_out} = P) \times P(\text{Y\_out} = P)}{P(X = [A\ 1\ 0.8]} =$$

$$= \frac{0{,}314 \times \frac{5}{9}}{0{,}328} = \underline{\underline{0{,}532}}$$

Regra da probabilidade total

$$P(x = [A\ 1\ 0.8]) = P(X = [A\ 1\ 0.8] \cap \text{Y\_out} = P) + P(x = [A\ 1\ 0.8] \cap \text{Y\_out} = N)$$

Regra da probabilidade condicionada

$$= P(X = [A\ 1\ 0.8] \mid \text{Y\_out} = P) \times P(\text{Y\_out} = P) + P(X = [A\ 1\ 0.8] \mid \text{Y\_out} = N) \times P(\text{Y\_out} = N)$$

$$= 0{,}314 \times \frac{5}{9} + 0{,}345 \times \frac{4}{9} = 0{,}328$$

Regra da probabilidade condicionada

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} \Leftrightarrow P(A \cap B) = P(A \mid B) \times P(B)$$

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \mid B) \times P(B)}{P(A)}$$

$$P(\text{Y\_out} = P \mid X = [B\ 1\ 1]) = \frac{P(X = [B\ 1\ 1] \mid \text{Y\_out} = P) \times P(\text{Y\_out} = P)}{P(X = [B\ 1\ 1]} = \frac{0{,}259 \times \frac{5}{9}}{0{,}391} = \underline{\underline{0{,}368}}$$

$$P(X = [B\ 1\ 1]) = P(X = [B\ 1\ 1] \mid \text{Y\_out} = P) \times P(\text{Y\_out} = P) + P(X = [B\ 1\ 1] \mid \text{Y\_out} = N) \times P(\text{Y\_out} = N)$$

$$= 0{,}259 \times \frac{5}{9} + 0{,}557 \times \frac{4}{9} = 0{,}391$$

$$P(\text{Y\_out} = P \mid X = [B\ 0\ 0.9]) = \frac{P(x = [B\ 0\ 0.9] \mid \text{Y\_out} = P) \times P(\text{Y\_out} = P)}{P(X = [B\ 0\ 0.9]} = \frac{0{,}309 \times \frac{5}{9}}{0{,}616} = \underline{\underline{0{,}279}}$$

$$P(x = [B\ 0\ 0.9]) = P(x = [B\ 0\ 0.9] \mid \text{Y\_out} = P) \times P(\text{Y\_out} = P) + P(x = [B\ 0\ 0.9] \mid \text{Y\_out} = N) \times P(\text{Y\_out} = N)$$

$$= 0{,}309 \times \frac{5}{9} + 1 \times \frac{4}{9} = 0{,}616$$

Logo!

$h_{map} = (x = [A\ 1\ 0.8])$

logo $P(\text{Positive} \mid x) = 0{,}532$

4)

$$f(x,\theta)=\begin{cases} \text{Positive} & P(\text{Positive} \mid x) > 0{,}5 \\ \text{Negative} & \text{otherwise} \end{cases}$$

Para $\theta = 0{,}5$

$P(\text{Positive} \mid X = [A\ 1\ 0.8]) = 0{,}532 > 0{,}5$ logo $f(x=[A\ 1\ 0.8], 0.5) = \text{Positive}$

$P(\text{Positive} \mid X = [B\ 1\ 1]) = 0{,}368 < 0{,}5$ logo $f(x=[B\ 1\ 1], 0.5) = \text{Negative}$

$P(\text{Positive} \mid X = [B\ 0\ 0.8]) = 0{,}273 < 0{,}5$ logo $f(x=[B\ 0\ 0.8], 0.5) = \text{Negative}$

$$\text{Accuracy} = \frac{TP+TN}{TP+TN+FP+FN} = \frac{1+1}{1+1+0+1} = \frac{2}{3}$$

		True P	True N
Given	P	1	0
	N	1	1

Para $\theta = 0{,}3$

$P(\text{Positive} \mid X = [A\ 1\ 0.8]) = 0{,}532 > 0{,}3$ logo $f(x=[A\ 1\ 0.8], 0.3) = \text{Positive}$

$P(\text{Positive} \mid X = [B\ 1\ 1]) = 0{,}368 > 0{,}3$ logo $f(x=[B\ 1\ 1], 0.3) = \text{Positive}$

$P(\text{Positive} \mid X = [B\ 0\ 0.8]) = 0{,}273 < 0{,}3$ logo $f(x=[B\ 0\ 0.8], 0.3) = \text{Negative}$

		True P	True N
Given	P	2	0
	N	0	1

$$\text{accuracy} = \frac{2+1}{2+1} = 1$$

Para $\theta = 0,7$

$P(\text{Parkinson} \mid x = [A\ 1\ 0.8]) = 0,532 < 0,7$ logo $f(x\ [A\ 1\ 0.8]) =$ negative

$P(\text{Parkinson} \mid x = [B\ 1\ 1]) = 0,368 < 0,7$ logo $f(x\ [B\ 1\ 1]) =$ negative

$P(\text{Parkinson} \mid x = [B\ 0\ 0.8]) = 0,2795 < 0,7$ logo $f(x\ [B\ 0\ 0.8]) =$ negative

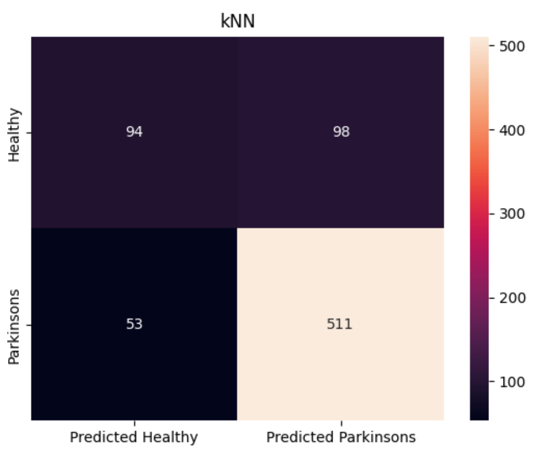
	P	N
P	0	0
N	2	1

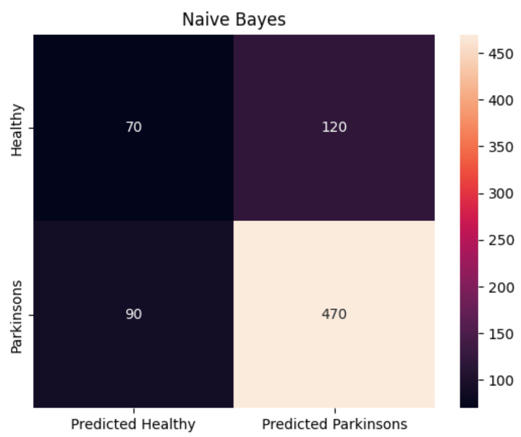
$$\text{accuracy} = \frac{1}{0+1+0+2} = \frac{1}{3}$$

R: The best decision threshold is 0.3

II. Programming and critical analysis

5)





6) Para um limite de confiança alpha = 0.05, tem-se que kNN > NB, uma vez que o respetivo p-value é menor que alpha, i.e., 0.00024 < p-value . Obtivemos este resultado com a realização da normalização das variáveis.

7) A diferença observável da precisão preditiva entre o kNN e o Naïve Bayes podem ser explicas por:

- O kNN é melhor para determinar semelhança entre observações que é o que fizemos nos exercícios anteriores devido à sua localidade já que limita o número de observações em estudo enquanto o Naïve Bayes é melhor para tratar muitas observações e para fazer previsões em tempo real pois gera probabilidades para cada classe e, portanto, caso haja introdução de novos dados o valor das probabilidades é alterado.

-O kNN tem maior tendência de ocorrer overfit e a quanto mais abrangente for o valor dado ao kNN menos preciso será a previsão

- O kNN tem a tendência a ter menos outliers, pois trata-se de observações filtradas e limitadas, enquanto o Naïve Bayes faz previsão com todas as observações possíveis.

III. APPENDIX

```
import pandas as pd
import numpy as np

#*####################################################################################
#* 5)
#*####################################################################################

#* Import Data
from scipy.io.arff import loadarff
data = loadarff("pd_speech.arff")
df = pd.DataFrame(data[0])
df['class'] = df['class'].str.decode('utf-8')
num_columns = df.shape[1]
df['class'] = pd.to_numeric(df["class"])

#* Pre-Process Data: scale data (relevant for kNN)
from sklearn.preprocessing import MinMaxScaler, StandardScaler
df_scaled = df.copy()
df_scaled.iloc[:, 0:num_columns-1] = MinMaxScaler().fit_transform(df.iloc[:,
0:num_columns-1])

#* Partition Data
kNN_X, kNN_y = df_scaled.iloc[:, 0:num_columns-1], df_scaled["class"]
NB_X, NB_y = df.iloc[:, 0:num_columns-1], df_scaled["class"]

#* Classifier
from sklearn.neighbors import KNeighborsClassifier
kNN_predictor = KNeighborsClassifier(n_neighbors=5, weights="uniform", metric="minkowski")
from sklearn.naive_bayes import GaussianNB
NB_predictor = GaussianNB()

#* Cross-Validation and Cumulative Confusion Matrixes
from sklearn.model_selection import StratifiedKFold
from sklearn.metrics import confusion_matrix
folds = StratifiedKFold(n_splits=10)
kNN_cumulative = np.array([[0, 0], [0, 0]])
for train_k, test_k in folds.split(kNN_X, kNN_y):
    kNN_X_train, kNN_X_test = kNN_X.iloc[train_k], kNN_X.iloc[test_k]
    kNN_y_train, kNN_y_test = kNN_y.iloc[train_k], kNN_y.iloc[test_k]

    kNN_predictor.fit(kNN_X_train, kNN_y_train)
    kNN_y_pred = kNN_predictor.predict(kNN_X_test)
```

```
    #? qual é Healthy e qual é Parkinsons?
    kNN_cm = np.array(confusion_matrix(kNN_y_test, kNN_y_pred, labels=[0,1]))
    kNN_cumulative[0][0] += kNN_cm[0][0]
    kNN_cumulative[0][1] += kNN_cm[0][1]
    kNN_cumulative[1][0] += kNN_cm[1][0]
    kNN_cumulative[1][1] += kNN_cm[1][1]
kNN_cumulative_confusion = pd.DataFrame(kNN_cumulative, index=['Healthy', 'Parkinsons'],
columns=['Predicted Healthy', 'Predicted Parkinsons'])
NB_cumulative = np.array([[0, 0], [0, 0]])
for train_k, test_k in folds.split(NB_X, NB_y):
    NB_X_train, NB_X_test = NB_X.iloc[train_k], NB_X.iloc[test_k]
    NB_y_train, NB_y_test = NB_y.iloc[train_k], NB_y.iloc[test_k]

    NB_predictor.fit(NB_X_train, NB_y_train)
    NB_y_pred = NB_predictor.predict(NB_X_test)
    #? qual é Healthy e qual é Parkinsons?
    NB_cm = np.array(confusion_matrix(kNN_y_test, kNN_y_pred, labels=[0,1]))
    NB_cumulative[0][0] += NB_cm[0][0]
    NB_cumulative[0][1] += NB_cm[0][1]
    NB_cumulative[1][0] += NB_cm[1][0]
    NB_cumulative[1][1] += NB_cm[1][1]
NB_cumulative_confusion = pd.DataFrame(NB_cumulative, index=['Healthy', 'Parkinsons'],
columns=['Predicted Healthy', 'Predicted Parkinsons'])

#* Plot
import seaborn as sns
import matplotlib.pyplot as plt
sns.heatmap(kNN_cumulative_confusion, annot=True, fmt='g')
plt.title(label="kNN")
plt.show()
sns.heatmap(NB_cumulative_confusion, annot=True, fmt='g')
plt.title(label="Naive Bayes")
plt.show()

#*#####################################################################################
#* 6)
#*#####################################################################################
#* Test Hypothesis

from sklearn.model_selection import cross_val_score
kNN_fold_accs = cross_val_score(kNN_predictor, kNN_X, kNN_y, cv = 10, scoring="accuracy")
NB_fold_accs = cross_val_score(kNN_predictor, NB_X, NB_y, cv = 10, scoring="accuracy")
print(kNN_fold_accs)
print(NB_fold_accs)

from scipy import stats
# kNN is better than NB?
res = stats.ttest_rel(kNN_fold_accs, NB_fold_accs, alternative='greater')
print("kNN > NB?  pval=", res.pvalue)
# kNN is worse than NB?
res = stats.ttest_rel(kNN_fold_accs, NB_fold_accs, alternative='less')
print("kNN < NB?  pval=", res.pvalue)
# kNN is differs from NB?
res = stats.ttest_rel(kNN_fold_accs, NB_fold_accs, alternative='two-sided')
print("kNN != NB? pval=", res.pvalue)
```

END