CS 766 - Homework 7
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1) Write equation for (x,y) at time t. Show Image point moves along line as $t \uparrow$:
at time t in the scene: $\tilde{x}_s = x_0 + ut$, $\tilde{y}_s = y_0 + vt$, $\tilde{z}_s = z_0 + wt$.

doing a perspective projection

$$x = \tilde{x}/\tilde{z} = \frac{x_0 + ut}{z_0 + wt} \Rightarrow$$

$$y = \tilde{y}/\tilde{z} = \frac{y_0 + vt}{z_0 + wt}$$

solve for t:

$$x = \frac{x_0 + ut}{z_0 + wt}$$

$$ut + x_0 - wtx = z_0 x \Rightarrow$$

$$t = \frac{z_0 x - x_0}{u - wx}$$

sub in t:

$$y = \frac{y_0 + v\left(\frac{z_0 x - x_0}{u - wx}\right)}{z_0 + w\left(\frac{z_0 x - x_0}{u - wx}\right)}$$

$$y = \frac{y_0(u - wx) + v(z_0 x - x_0)}{z_0(u - wx) + w(z_0 x - x_0)}$$

$$y = \frac{(vz_0 - wy_0)x + y_0 u - vx_0}{z_0 u - wx_0}$$

$$\boxed{y = \left(\frac{vz_0 - wy_0}{z_0 u - wx_0}\right)x + \left(\frac{y_0 u - vx_0}{z_0 u - wx_0}\right)}$$ equation of a line

constant

2) constraint equation: $I_x u + I_y v + I_t = 0$
given two illuminations we get 2 constraint equations

$$I_{x_1} u + I_{y_1} v + I_{t_1} = 0 \quad \Rightarrow \quad u = \frac{-I_{y_1} v - I_{t_1}}{I_{x_1}}$$

$$I_{x_2} u + I_{y_2} v + I_{t_2} = 0$$

$\Rightarrow$ substitute in:

$$I_{x_2}\left(\frac{-I_{y_1} v - I_{t_1}}{I_{x_1}}\right) + I_{y_2} v + I_{t_2} = 0$$

$$\left(\frac{-I_{x_2} I_{y_1}}{I_{x_1}} + I_{y_2}\right) v = -\left(\frac{I_{x_2} I_{t_1}}{I_{x_1}} - I_{t_2}\right) \Rightarrow v = \frac{\left(\frac{I_{x_2} I_{t_1} - I_{x_1} I_{t_2}}{I_{x_1}}\right)}{\left(\frac{-I_{x_2} I_{y_1} + I_{x_1} I_{y_2}}{I_{x_1}}\right)}$$

$$\boxed{u = \frac{-I_{y_1}\left(\frac{I_{x_2} I_{t_1} - I_{x_1} I_{t_2}}{-I_{x_2} I_{y_1} + I_{x_1} I_{y_2}}\right) - I_{t_1}}{I_{x_1}}}$$

$$\boxed{v = \frac{I_{x_2} I_{t_1} - I_{x_1} I_{t_2}}{-I_{x_2} I_{y_1} + I_{x_1} I_{y_2}}}$$