Control Engineering II - Project

Assignment 3

The transfer function of the process from the closed loop system shown in Fig. 1 is given as

$$H_f(s) = \frac{K_f}{s(T_f s + 1)}$$

Requirement: Compute two controllers $H_r(s)$ using the modulus and symmetry criterions (Kessler's methods).

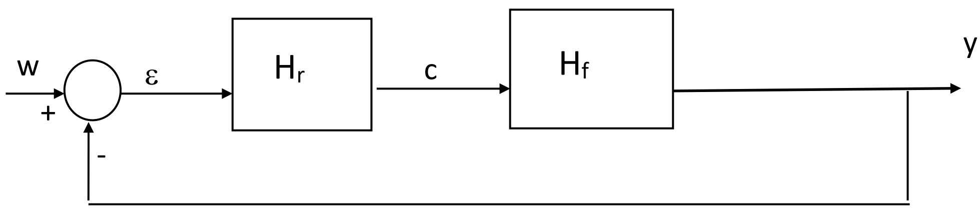

Fig. 1 Closed loop system

Handout: A short documentation containing the obtained controllers and the obtained closed loop system's performance with both controllers (graphical proof – overshoot, settling time, steady state errors and velocity constant) for step and ramp inputs.

Theoretical background:

The modulus and symmetry criterions are very useful when dealing with processes without dead time that contain one or maximum two dominant time constants and a small, parasite, time constant T_Σ. The symmetry criterion is recommended for processes that contain an integrator. If these assumptions are true, the resulting controller is of PID-type with the form:

$$H_R(s) = V_R\left(\frac{1+T_d s}{1+\alpha T_d s}\right)\left(1+\frac{1}{sT_i}\right) \tag{1}$$

where $\alpha = (0.125 \div 0.1)$.

The tuning of the controller from (1) implies an optimal form of the open loop, for fast processes, with a dead time amount. The modulus and symmetry criterions ensure a good performance of system for certain references and disturbances.

The optimal open loop system of the **modulus** criterion can be written as

$$H_d^*(s) = H_R(s)\cdot H_f(s) = \frac{1}{2T_\Sigma s(T_\Sigma s+1)} \tag{2}$$

From where we compute $H_R(s)$ as:

$$H_R(s) = \frac{H_d^*(s)}{H_f(s)} \tag{3}$$

The performance is analyzed for the closed loop system's transfer function:

$$H_0(s) = \frac{H_d^*(s)}{1+H_d^*(s)} = \frac{1}{2T_\Sigma^2 s^2 + 2T_\Sigma s + 1} = \frac{1}{\frac{s^2}{\omega_n^2} + 2\frac{\xi}{\omega_n}s + 1} \tag{4}$$

Mapping the coefficients from (4) gives:

$$\omega_n = \frac{1}{\sqrt{2}T_\Sigma}\ ;\ \xi = \frac{1}{\sqrt{2}} \tag{5}$$

The settling time is:

$$t_r = \frac{4}{\xi\omega_n} = \frac{4}{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}T_\Sigma}} = \frac{4}{\frac{1}{2T_\Sigma}} = 8T_\Sigma \tag{6}$$

And the overshoot:

$$\sigma = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \cong 4.3\% \tag{7}$$

The obtained performance – reduced settling time (6), reduced overshoot (7) – make the magnitude criterion a viable tuning strategy when dealing with step references. For a ramp input, the resulting performance is improved by choosing a smaller parasite time constant. The velocity coefficient is estimated using

$$c_v = \frac{\omega_n}{2\xi} = \frac{\frac{1}{\sqrt{2}T_\Sigma}}{2\frac{1}{\sqrt{2}}} = \frac{1}{2T_\Sigma} \tag{8}$$

and the velocity error can be written as:

$$\xi_{stv} = \frac{1}{c_v} = 2T_\Sigma \tag{9}$$

For the **symmetry** criterion the optimal form of the open loop system is given by

$$H_d^*(s) = H_R(s)\cdot H_f(s) = \frac{4T_\Sigma s + 1}{8T_\Sigma^2 s^2\left(T_\Sigma s + 1\right)} \tag{10}$$

The controller can be computed using

$$H_R(s) = \frac{H_d^*(s)}{H_f(s)} \tag{11}$$

For the symmetry criterion, the superior performance can be observed for a ramp input

$$\varepsilon_{stv} = \lim_{s\to 0}\frac{1}{s\cdot H_d(s)} = \lim_{s\to 0}\frac{1}{s\cdot\dfrac{4T_\Sigma s+1}{8T_\Sigma^2 s^2\left(T_\Sigma s+1\right)}} = 0 \tag{12}$$

However, for a step input the obtained controller using the symmetry criterion leads to poor performance: increased overshoot, approx. 43%, and a settling time characterized by $t_r = 11T_\Sigma$.