Laboratory work 7

CONTROLLER DESIGN METHOD FOR TIME DELAY SYSTEM USING PHASE MARGIN CONSTRAINTS IN THE FREQUENCY DOMAIN

1. OBJECTIVE

- Controller tuning based on phase margin specifications in the frequency domain
- Frequency domain PI, PD and PID controller tuning for time delay processes
- Performance validation

2. THEORETICAL BACKGROUND

A time delay process is given by

$$H_f = H_f^{'} e^{-T_m s} \tag{9.1}$$

where T_m is the time delay.

Considering the ratio between the time delay and the dominant time constant, T_f, of the process, the following scenarios are considered:

- if $\frac{T_m}{T_f} < 0.2$, the time delay can be neglected, the controller can be tuned using classical methods, using $H_f = \frac{k}{(T_f s+1)(T_m s+1)}$.
- if $0.2 < \frac{T_m}{T_f} < 1$, the time delay cannot be neglected and the controller should be designed with respect to the time delay
- if $\frac{T_m}{T_f} > 1$, the time delay is dominant, special control strategies should be used such as the Smith predictor

Several controller design methods are available for processes exhibiting large time delays such as: frequency domain methods with imposed phase margin and time delay approximations (such as Padé) or experimental methods such as Ziegler-Nichols method, relay method, etc.

For frequency domain tuning strategies, the type of controller can't be specified. This is chosen based on additional performance specifications such as

$\varepsilon_{stp} = 0 \Rightarrow$ PI controller

$t_r <<\ \ \Rightarrow$ PD controller

$\varepsilon_{stp} = 0$ and $t_r << \Rightarrow$ PID controller

2.1 PI controller design

The transfer function of the PI controller is given by

$$H_c = k_p\left(1 + \frac{1}{T_i s}\right) \tag{9.2}$$

where k_p is the proportional gain and T_i is the integral time constant. In order to determine the PI controller, we must find the values of k_p and T_i.

Tuning steps:

1. Since the tuning is done in the frequency domain, the process' transfer function should be mapped from the Laplace domain to the frequency domain. This is done by replacing $s = j\omega$, where j is the imaginary unit and ω is the frequency. Hence, $H_f(s)$ becomes $H_f(j\omega)$. Draw the Bode diagram of process transfer function.

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H = tf( num, den, 'IODelay', Tm);
bode(H)
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Don't forget the time delay!

2. Knowing the phase margin constraint γ_k^*, the open loop phase margin equation can be written as

$$\angle H_{ol}(j\omega_c) = -180^\circ + \gamma_k^* \tag{9.3}$$

Note that $\angle H_{ol}(j\omega_c) = \angle\left(H_f(j\omega_c)H_c(j\omega_c)\right) = \angle\, H_f(j\omega_c) + \angle\, H_c(j\omega_c)$. The ω_c symbol represents the gain crossover frequency (the frequency at which the magnitude of $H_{ol}(j\omega_c)$ is 1, or 0^{dB}).

Replacing the phase value of $\angle\, H_c(j\omega_c)$ in (9.3) gives

$$\angle H_f(j\omega_c) = -180^\circ + 15^\circ + \gamma_k = -165^\circ + \gamma_k \tag{9.4}$$

3. Determine ω_c, either analytically or from the Bode diagram plot (look for the frequency where the phase of $H_f(j\omega_c) = -165^\circ + \gamma_k$).

4. Compute T_i using

$$T_i = \frac{4}{\omega_c}. \tag{9.5}$$

5. The magnitude of the open loop system at the crossover frequency is 1 (or 0^{dB}). Mathematically, this can be written as

$$|H_{ol}(j\omega_c)| = |H_c(j\omega_c)| \cdot |H_f(j\omega_c)| = 1 \tag{9.6}$$

Knowing that $|H_c(j\omega_c)| = k_p$, gives $k_p \cdot |H_f(j\omega_c)| = 1$, from where it results that

$$k_p = \frac{1}{|H_f(j\omega_c)|} \tag{9.7}$$

The magnitude of $|H_f(j\omega_c)|$ can be read from the H_f Bode diagram, this time looking at the magnitude plot (just find the magnitude of H_f at ω_c).

6. Check the phase margin obtained with the PI controller is correct (draw the Bode plot of the open loop system).

2.2 PD controller design method using imposed margin

The PD controller transfer function can be written as

$$H_{PD} = k\left(\frac{1+\tau_d s}{1+\beta\tau_d s}\right) \tag{9.8}$$

with $0.1 < \beta < 0.125$.

Tuning steps:

1. Plot the Bode diagram of the process. Knowing the imposed phase margin value γ_k^*, the phase of the open loop system is

$$\angle H_{ol}(j\omega_c) = -180° + \gamma_k \tag{9.9}$$

2. Choose $\angle H_c(j\omega_c)$ maximum knowing that

$$\angle H_c(j\omega_c) = atan\frac{1-\beta}{2\sqrt{\beta}}$$

$$\omega_c = \frac{1}{\tau_d\sqrt{\beta}} \tag{9.10}$$

giving $|H_c(j\omega_c)| = \frac{k_p}{\sqrt{\beta}}$.

3. Select $\beta \in (0.1, 0.125)$ and compute $\angle H_c(j\omega_c)$.
4. Knowing that the phase of the open loop system $\angle H_{ol}(j\omega_c) = \angle\left(H_p(j\omega_c)H_c(j\omega_c)\right) = \angle H_p(j\omega_c) + \angle H_c(j\omega_c)$ gives the phase of the process as

$$\angle H_p(j\omega_c) = -180° - \angle H_c(j\omega_c) + \gamma_k \tag{9.11}$$

Determine ω_c either analytically or read it from the Bode diagram.

5. Compute τ_d as

$$\tau_d = \frac{1}{\omega_c\sqrt{\beta}} \tag{9.12}$$

6. The magnitude of the open loop system is $|H_{ol}(j\omega_c)| = |H_c(j\omega_c)| \cdot |H_p(j\omega_c)| = 1$.

Knowing that $|H_c(j\omega_c)| = \frac{k_p}{\sqrt{\beta}}$ and replacing it in the previous equation gives

$$k_p = \frac{\sqrt{\beta}}{|H_p(j\omega_c)|} \tag{9.13}$$

The magnitude $|H_p(j\omega_c)|$ can be read from the Bode diagram or computed analytically.

3. PROBLEMS

For the process described by

$$H_f = \frac{2}{(10s+1)(5s+1)} e^{-3s}$$

a) Design a PI controller that ensures a phase margin $\gamma_k^* = 50^\circ$.

b) Design a PD controller (choose $\beta = 0.1$) that ensures a phase margin $\gamma_k^* = 50^\circ$.

c) Analyse the performance obtained with both controllers and compare the results.