Laboratory work 2

CONTROLLER DESIGN USING THE GUILLEMIN-TRUXAL METHOD

1. OBJECTIVES

- Study the Guillemin-Truxal approach for controller design
- Analyze the obtained closed loop system performance with respect to the imposed specifications
- Analyze the effect of simplifying the controller on the closed loop system's performance.

2. THEORETICAL APPROACH

The closed loop system should respect the following performance specifications

$$\begin{cases} \varepsilon_{stp} = 0 \\ \varepsilon_{stv} < \varepsilon_{stv}^{*} \\ t_r < t_r^{*} \\ \sigma < \sigma^{*} \\ \Delta\omega_B < \Delta\omega_B^{*} \end{cases}$$

The Guillemin-Truxal method consists in imposing the closed loop system as a second order transfer function that satisfies the desired performance specifications. The closed loop system is defined as

$$H_o(s) = H_{02} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2} \qquad (3.1)$$

The damping ratio ξ and natural frequency ω_n are determined with respect to the performance set.

We know the fixed part $H_f(s)$ described by

$$H_f(s) = \frac{K_f}{s(T_f s + 1)} \tag{3.2}$$

The controller can be determined from

$$H_R(s) = \frac{1}{H_f(s)} \cdot \frac{H_{02}(s)}{1 - H_{02}(s)}. \tag{3.3}$$

One of the problems that may arise is that the obtained controller is too complex and hard to implement. Hence, it could be replaced with a simplified controller, given that the performance specifications are still met. Replacing (3.2) in (3.1) gives the controller as

$$H_R(s) = \frac{\omega_n / 2\xi \cdot (T_f s + 1)}{K_f \cdot \left(1/2\xi\omega_n \, s + 1 \right)} \tag{3.4}$$

For $T_f \cong \frac{1}{2\xi\omega_n}$, a P controller would meet the performance. However, the closed loop system's performance should be verified.

A complex process (with more than two poles) can be reduced to the transfer function from (3.1) using different approximations. The most popular method is based on approximating it using the dominant poles.

3. TUNING STEPS

Starting from the desired overshoot, $\sigma < \sigma^*$, the damping ratio, ξ, is computed as

$$\xi = \frac{|\ln(\sigma)|}{\sqrt{\ln^2(\sigma) + \pi^2}} \tag{3.5}$$

The desired settling time, $t_r < t_r^*$, is used to compute the natural frequency, ω_n as

$$\omega_n = \frac{4}{t_r \cdot \xi} \tag{3.6}$$

Furthermore, the rest of the performance specifications should be verified.

The velocity coefficient is computed with respect to the damping ratio and natural frequency determined in (3.5), and (3.6):

$$c_v = \frac{\omega_n}{2 \cdot \xi}, \tag{3.7}$$

giving the steady state velocity error

$$\varepsilon_{stv} = \frac{1}{c_v}. \tag{3.8}$$

If the value computed at (3.8) does not meet the performance criteria $\varepsilon_{stv} < \varepsilon^*_{stv}$, other values for the settling time and/or overshoot should be chosen (smaller than the ones already imposed) until the entire performance set is satisfied.

The fulfillment of the bandwidth requirement is verified using the equation:

$$\Delta\omega_B = \omega_n \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}} \tag{3.9}$$

The final controller is obtained based on (3.3).

4. PROBLEMS

1. Consider the fixed parts: $K_f = 2$, $T_f = 2$ sec, the performance sets are:

a) $\begin{cases} \varepsilon_{stp} = 0 \\ t_r^* \le 40\,\text{sec} \\ \sigma^* \le 15\% \\ c_v \ge 0{,}2 \\ \Delta\omega_B^* \le 2\,\text{rad/sec} \end{cases}$; b) $\begin{cases} \varepsilon_{stp} = 0 \\ t_r^* \le 8\,\text{sec} \\ \sigma^* \le 15\% \\ c_v \ge 1 \\ \Delta\omega_B^* \le 2\,\text{rad/sec} \end{cases}$; c) $\begin{cases} \varepsilon_{stp} = 0 \\ t_r^* \le 8\,\text{sec} \\ \sigma^* \le 10\% \\ c_v \ge 1{,}5 \\ \Delta\omega_B^* \le 1{,}2\,\text{rad/sec} \end{cases}$

Compute the controllers using the Guilemin-Truxal method, if necessary simplify the controller structure and check the performance of the closed loop system using Matlab.