Laboratory work 8

CONTROLLER DESIGN USING KESSLER'S MAGNITUDE OPTIMUM AND SYMMETRICAL OPTIMUM DESIGN CRITERIA

1. THE GOALS

- Controller tuning with Kessler's methods
- Closed loop system performance assessment

2. THEORETICAL BACKGROUND

Kessler's Magnitude Optimum and Symmetrical Optimum criteria are used for controller tuning in processes without time delays, that contain one or two dominant time constants and a small, parasite, time constant, denoted by T_Σ. The resulting controller is a PID-type controller with the following transfer function:

$$H_R(s) = V_R\left(\frac{1+T_d s}{1+\alpha T_d s}\right)\left(1+\frac{1}{sT_i}\right) \tag{7.1}$$

where $\alpha = (0.125 \div 0.1)$.

The tuning procedure consists in the usage of an "optimum" form of the open loop system. The methods should be applied to a fast process, without time delay. Both magnitude optimum and symmetrical optimum criteria ensure a good closed loop system performance for certain references and disturbance rejection scenarios.

2.1 Magnitude Optimum Criterion

In the case of the Magnitude Optimum criterion the following open loop system is imposed:

$$H_d^*(s) = H_R(s) \cdot H_f(s) = \frac{1}{2T_\Sigma s\left(T_\Sigma s + 1\right)} \tag{7.2}$$

resulting $H_R(s)$ as:

$$H_R(s) = \frac{H_d^*(s)}{H_f(s)} \tag{7.3}$$

The performance is analyzed using the closed loop system given by

$$H_0(s) = \frac{H_d^*(s)}{1 + H_d^*(s)} = \frac{1}{2T_\Sigma^2 s^2 + 2T_\Sigma s + 1} = \frac{1}{\frac{s^2}{\omega_n^2} + 2\frac{\xi}{\omega_n}s + 1} \tag{7.4}$$

Equating the coefficients from (7.4) gives

$$\omega_n = \frac{1}{\sqrt{2}T_\Sigma}\,;\ \xi = \frac{1}{\sqrt{2}} \tag{7.5}$$

The settling time and overshoot are computed as

$$t_r = \frac{4}{\xi\omega_n} = \frac{4}{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}T_\Sigma}} = \frac{4}{\frac{1}{2T_\Sigma}} = 8T_\Sigma \tag{7.6}$$

$$\sigma = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} \cong 4.3\% \tag{7.7}$$

Note that T_Σ is the small, parasite, time constant, that contributes to a small settling time, as can be deduced from (7.6). In addition, the overshoot is a fixed value of approx. 4.3%, as shown by (7.7). The obtained performance makes the Modulus Optimum criterion ideal when step response performance is considered.

Furthermore, the velocity coefficient can be estimated using

$$c_v = \frac{\omega_n}{2\xi} = \frac{\frac{1}{\sqrt{2}T_\Sigma}}{2\frac{1}{\sqrt{2}}} = \frac{1}{2T_\Sigma} \tag{7.7}$$

resulting the steady state error velocity as

$$\xi_{stv} = \frac{1}{c_v} = 2T_\Sigma \tag{7.8}$$

Hence, it can be concluded that for a ramp reference, the performance is better if the parasite time constant is smaller.

2.2 Symmetrical Optimum Criterion

The Symmetrical Optimum Criterion ensures a good closed loop system performance for a ramp reference. The open loop system is imposed as

$$H_d^*(s) = H_R(s) \cdot H_f(s) = \frac{4T_\Sigma s + 1}{8T_\Sigma^2 s^2 (T_\Sigma s + 1)} \tag{7.9}$$

The method ensures the following step reference performance:

$$t_s \cong 11\, T_\Sigma \tag{7.10}$$

$$\sigma \cong 43\%$$

The steady state velocity error is obtained as

$$\varepsilon_{stv} = \lim_{s \to 0} \frac{1}{s \cdot H_d(s)} = \lim_{s \to 0} \frac{1}{s \cdot \dfrac{4T_\Sigma s + 1}{8T_\Sigma^2 s^2 (T_\Sigma s + 1)}} = 0 \tag{7.11}$$

It can be observed that the Symmetrical Optimum method increases both the settling time and overshoot. However, the method proves to be particularly useful for ramp reference inputs, ensuring a 0 steady state velocity error.

3. PROBLEMS

For the processes given below, compute the controllers using the Modulus and Symmetrical Optimum criteria. Check the performance using Matlab.

a) $\dfrac{2}{s+1}$

b) $\dfrac{2}{s(s+1)}$

c) $\dfrac{2}{(s+1)(10s+1)}$

d) $\dfrac{2}{s(s+1)(10s+1)}$

e) $\dfrac{2}{(s+1)(10s+1)(20s+1)}$

f) $\dfrac{2}{s(s+1)(10s+1)(20s+1)}$