Computing controllers that ensure a phase margin of 60 degrees

Compute and declare my transfer function

The transfer function has the following form:

$$H_f(s) = \frac{K_f}{s(T_f s + 1)}$$

Knowing that my values are Kf = 4 and Tf = 8, the function becomes:

$$H_f(s) = \frac{4}{s(8s + 1)}$$

I declare my transfer function in code:

```
Kf = 4;
Tf = 8;
Hf = tf(Kf,[Tf 1 0])
```

```
Hf =
 
      4
  ---------
  8 s^2 + s
 
Continuous-time transfer function.
```

Declare the phase margin's value

```
gammaK = 60;
```

Computing a PD

$$\angle H_{ol}(j\omega_c) = -180^\circ + \gamma_k$$

```
phaseHol = -180 + gammaK;
```

We choose β between 0.1 and 0.125

```
beta = 0.1;
```

$$\angle H_c(j\omega_c) = atan\frac{1-\beta}{2\sqrt{\beta}}$$

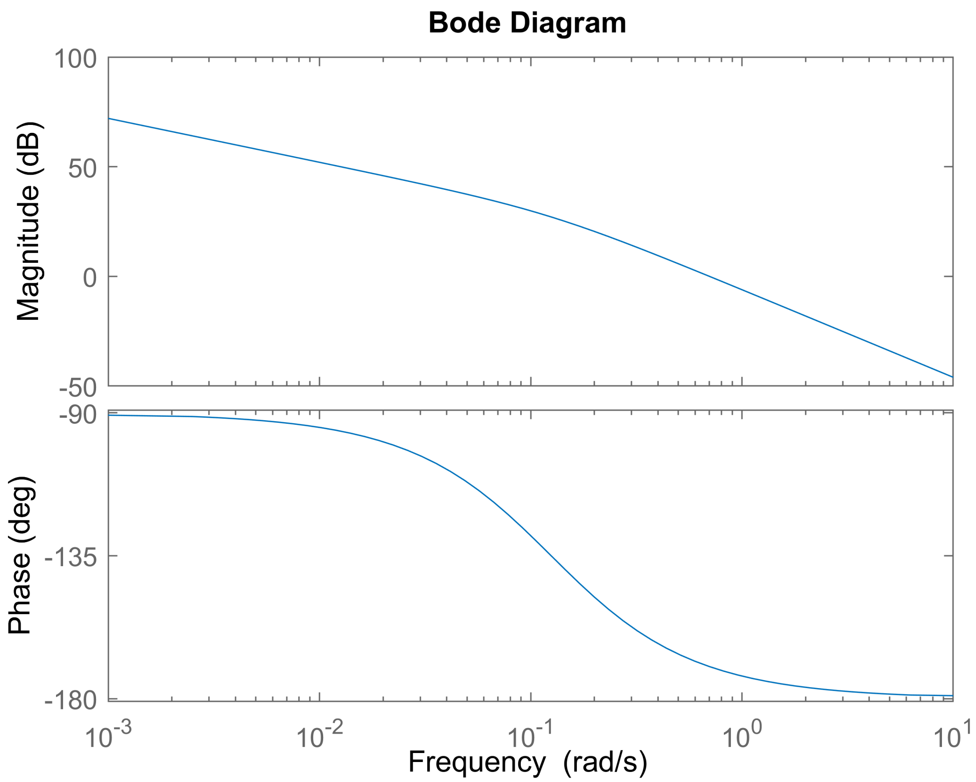
```
phaseHc = atand((1-beta)/2/sqrt(beta));
```

$$\angle H_{ol}(j\omega_c) = \angle\left(H_p(j\omega_c)H_c(j\omega_c)\right) = \angle H_p(j\omega_c) + \angle H_c(j\omega_c)$$

```
phaseHp = phaseHol - phaseHc;
```

I get the value of w_c from bode:

```
bode(Hf)
```




```
Wc = 1.4;
```

$$\tau_d = \frac{1}{\omega_c\sqrt{\beta}}$$

```
td = 1/Wc/sqrt(beta);
```

I read the value of $|H_p(jw_c)|$ from Bode

```
magHp = 10^((-12)/20);
```

$$k_p = \frac{\sqrt{\beta}}{|H_p(j\omega_c)|}$$

```
kp = sqrt(beta)/magHp;
```

Using PD general formula, I compute the final open loop transfer function.

$$H_c(s) = k_p \frac{1+\tau_d s}{1+\beta\tau_d s}$$

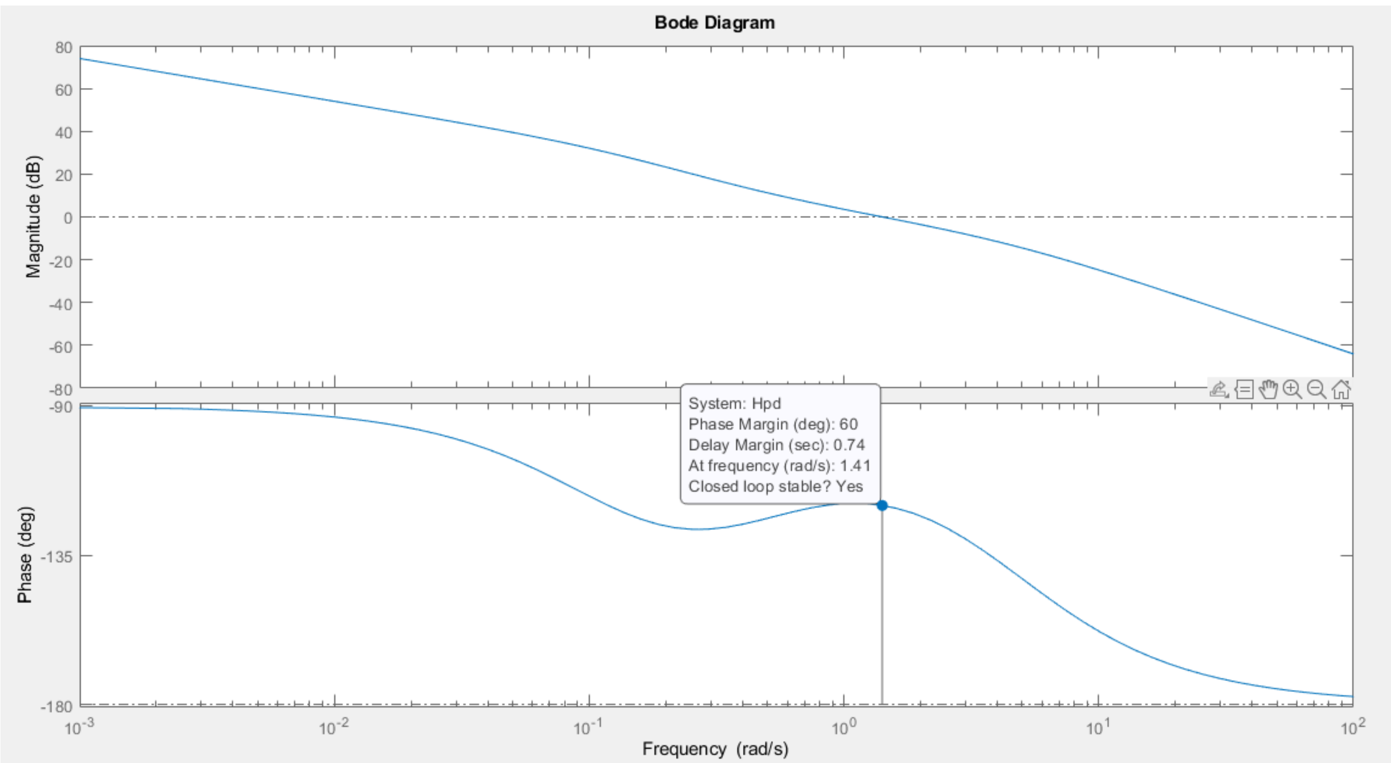
```
Hpd = Hf*kp*tf([td 1],[beta*td 1])
```

```
Hpd =

       11.37 s + 5.036
  -------------------------
  1.807 s^3 + 8.226 s^2 + s

Continuous-time transfer function.
```

```
% bode(Hpd)
```



Computing a PI

$$\angle H_{ol}(j\omega_c) = -180^\circ + \gamma_k$$

```
phaseHol = -180 + gammaK;
```

assume that $\angle H_c(j\omega_c) = -15^\circ$

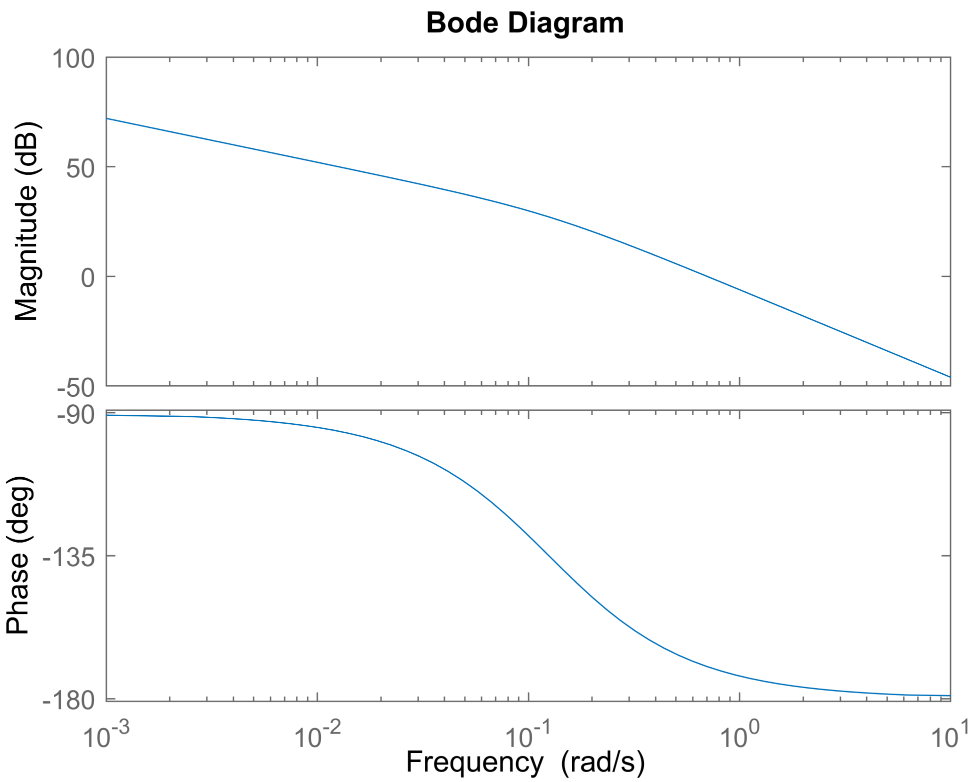
```
phaseHc = -15;
```

$$\angle H_{ol}(j\omega_c) = \angle\left(H_p(j\omega_c)H_c(j\omega_c)\right) = \angle H_p(j\omega_c) + \angle H_c(j\omega_c)$$

```
phaseHp = phaseHol - phaseHc;
```

I get the value of w_c from bode:

```
bode(Hf)
```




```
Wc = 0.035;
```

$$T_i = \frac{4}{\omega_c}$$

```
Ti = 4/Wc;
```

I read the value of $|H_p(jw_c)|$ from Bode

```
magHp = 10^(40/20);
```

$$k_p = \frac{1}{|H_p(j\omega_c)|}$$

```
kp = 1/magHp;
```

Using PI general formula, I compute the final open loop transfer function.

$$H_c = k_p\left(1 + \frac{1}{T_i s}\right)$$

```
Hpi = Hf*kp*tf([Ti 1],[Ti 0])
```

```
Hpi =

     4.571 s + 0.04
  ---------------------
  914.3 s^3 + 114.3 s^2

Continuous-time transfer function.
```

```
% bode(Hpi)
```