Control Engineering 2 – Project

Assignment 2

The transfer function of the process from the closed loop system shown in Fig. 1 is given as

$$H_f(s) = \frac{K_f}{s(T_f s + 1)}$$

Requirement: Compute the controller $H_r(s)$ using a frequency domain method that satisfies the following specifications:

$$\begin{cases} \varepsilon_{stp} = 0 \\ t_r \le 5\ sec \\ \sigma^* \le 10\% \\ c_v \ge 3 \\ \Delta\omega_B^* \le 15\ rad/s \end{cases}$$

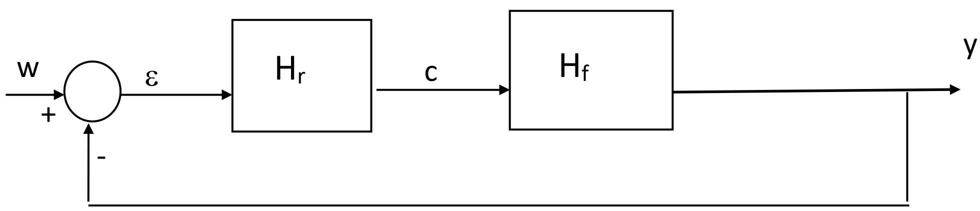

Fig. 1 Closed loop system

Handout: A brief documentation (max 2 pages) containing:
- The actual performance values chosen to compute the controller
- The obtained controller
- Graphical proof of the closed loop system performance both in the time and frequency domains

Theoretical background:

Tuning method based on logarithmic diagrams represent an easy and direct approach. These advantages are valid if the transfer function of the process is in the following form:

$$H_f(s) = \frac{K_f}{s(T_f s + 1)}$$

The imposed specifications can be of the form: $\begin{cases} \varepsilon^*_{stp} = 0 \\ \sigma^* \le \sigma \\ t^*_r \le t_r \\ c^*_v \ge c_v \\ \Delta\omega^*_B \le \Delta\omega_B \end{cases}$

1.1. Tuning a P controller

1. Represent the logarithmic magnitude plot of the system, the system having non-minimum phase, obtaining the gain crossover frequency (ω_t) and the corner frequency (ω_f).
2. Determine the damping ratio (ξ) for the imposed overshoot $\sigma = \sigma^*$ and compute the magnitude value $|A| \cong \frac{1}{4\xi^2}$, which is then transformed to dB. N represents the point where $\omega = \omega_f$.

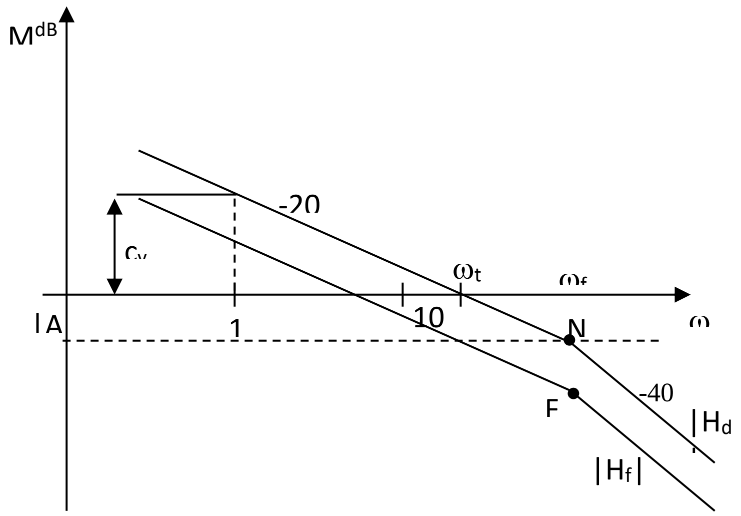

Fig 1. P controller tuning

Move (H_f) such that the corner is at point N, resulting in the final open loop system form ($H_r = 1$). It is obvious that:

$$\overline{FN} = V_R\big|_{dB}$$

The necessary checks are related to the following performance specifications

- Settling time: read the frequency (ω_t) from the logarithmic diagram and determine $\omega_t = \frac{\omega_n}{2\xi}$, resulting $\omega_n = 2\xi\omega_t$, meaning that $t_r = \frac{4}{\xi\omega_n} \le t_r^*$;
- The velocity coefficient is read from the logarithmic diagram at $\omega = 1$, making sure that $c_v \ge c_v^*$;
- The bandwidth is approximated to $\Delta\omega_B \cong \omega_t$.

If all the above-mentioned specifications are met, the obtained P controller successfully solves the control task. Otherwise, one must consider the development of more complicated controllers.

1.2. Tuning a PI controller

When faced with a more restricting performance set, an initial simple controller, usually of type P, is tuned. However, this might not respect all the imposed specifications. If the settling time is respected, but the velocity coefficient is too large, a PI controller should be tuned. The transfer function of the PI controller can be written as

$$H_{PI}(s) = V_R \frac{1 + sT_z}{1 + sT_p}$$

Determine the frequency (ω_t) and the coefficient (c_v) for $\omega = 1$. Choose the frequencies (ω_z) and (ω_p) such that:

$$\begin{cases} \omega_z \approx 0{,}1\omega_t \\ \omega_p = \frac{c_v}{c_v^*}\omega_z \end{cases}$$

where ($\omega_p < \omega_z$). The open loop system with the PI controller (denoted by H_{dC}) can be determined using these frequencies.

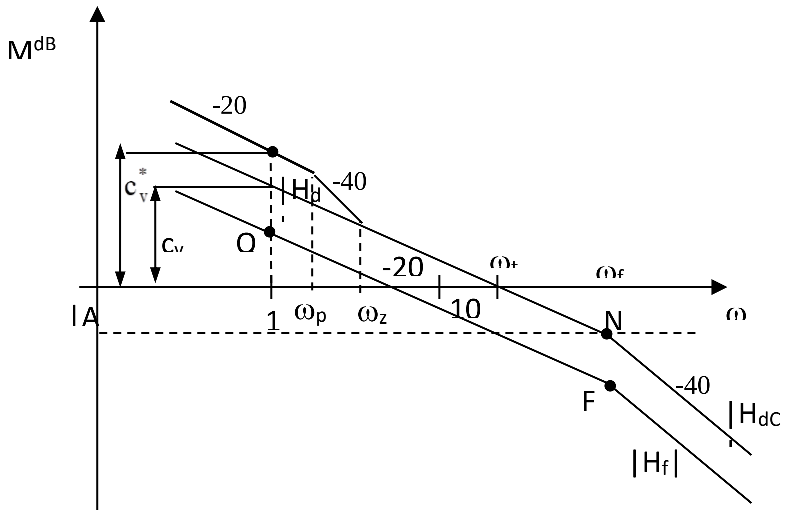

Fig 2. PI controller tuning

The controller parameters are given by: $\begin{cases} V_R\big|_{dB} = \overline{QV} \\ T_z = \dfrac{1}{\omega_z} = \dfrac{1}{0{,}1\omega_t} \\ T_P = \dfrac{1}{\omega_P} = \dfrac{1}{\omega_z} \cdot \dfrac{c_v^*}{c_v} \end{cases}$

The necessary validations are related solely to $\left(\Delta\omega_B^*\right)$, with the rest of the performance specifications implicitly honored.

1.3. Tuning a PD controller

Firstly, a P controller is computed in order to respect the imposed performance. However, not all the specifications can be met with a P controller. If the settling time constraint isn't realized by the P controller, a PD controller must be tuned instead. The transfer function of the PD controller can be written as

$$H_{PD} = V_R \frac{1+\tau_D s}{1+T_N s}$$

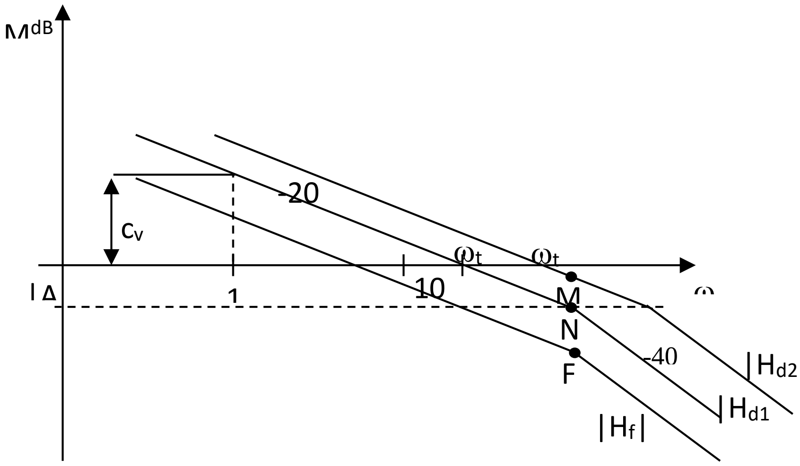

Fig 3. PD controller tuning

The tuning procedure starts with the computation of ω_{t1}, resulting $t_r = \dfrac{2}{\xi^2 \omega_{t1}}$. $t_r^* = \dfrac{2}{\xi^2 \omega_{t2}}$ gives $\omega_{t2} = \omega_{t1} \dfrac{t_r}{t_r^*}$. $\omega = \omega_{t2}$ is placed on the frequency axis, representing the final open loop system. Moving H_{d1} to the right implies a PD-type controller.

The parameters of the PD controller are computed using: $\begin{cases} V_R\big|_{dB} = \overline{MF} \\ \tau_d = T_f \\ T_N = \tau_d \dfrac{t_r^*}{t_r} \end{cases}$

The performance specification check consists in the bandwidth value.

1.4. Tuning a PID controller

When imposing a strict set of performance specifications, a simple P, PI or even a PD controller is unable to fulfill them all. The solution lies in the usage of the PID controller given by

$$H_R(s) = V_R \frac{1 + s\tau_d}{1 + sT_N} \cdot \frac{1 + sT_z}{1 + sT_P}$$

The first step is to tune the PD controller using the previously presented methodology:

- Make a logarithmic plot of $H_f(j\omega)$

- Determine (ξ), and the position of the segment $|A|$, resulting in the point N and the structure H_{d1}
- Compute (ω_{t2}) and move (H_{d1}) to the right until (H_{d2}) is reached, resulting in a PD controller
- Read c_v given by H_{d2}, and compute $c_v^* / c_v \ (> 1)$
- Choose $\begin{cases} \omega_z \approx 0{,}1\omega_{t2} \\ \omega_p = \dfrac{c_v}{c_v^*}\omega_z \end{cases}$

This results in the complete open loop structure (H_{dC}), with the modified PI controller.

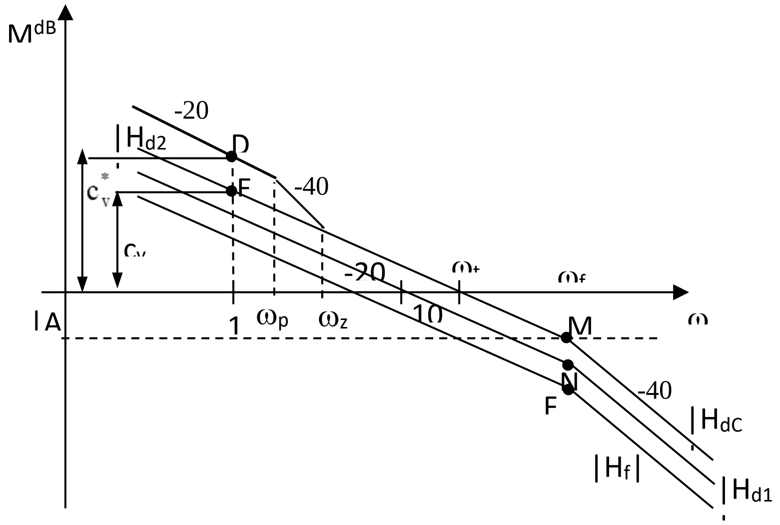

Fig 4. PID controller tuning