Laboratory work 5

CONTROLLER DESIGN IN FREQUENCY DOMAIN BASED ON SECOND ORDER SYSTEM

1. THE GOALS

- To follow and to understand the design method steps
- To check the resulted performance indicators;

2. THEORETICAL BACKGROUND

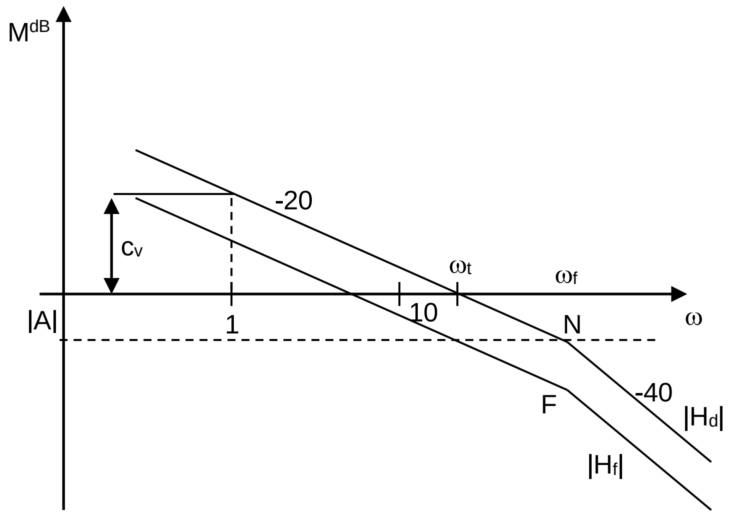
The logarithmic diagrams methods allow a hand over, convenient and direct design of controllers. These advantages are emphasized only if the process has the particular form:

$$H_f(s) = \frac{K_f}{s(T_f s + 1)}.$$

The imposed performance indicators are:

$$\begin{cases} \varepsilon_{stp}^* = 0 \\ \sigma^* \le \sigma \\ t_r^* \le t_r \\ c_v^* \ge c_v \\ \Delta\omega_B^* \le \Delta\omega_B \end{cases}$$

2.1. P controller design steps ($H_c(s)=V_R$)



- Represent in Bode diagrams the modulus of the fixed part (the system is with minimal phase, so the phase diagram is not necessary)
- Determine the cutting frequency (ω_t) and the corner frequency (ω_f).
- Determine the damping factor (ξ) corresponding to the overshoot $\sigma = \sigma^*$ and the value $|A|$ from:

$$|A| \cong \frac{1}{4\xi^2},$$

and represent this value in dB on the diagram. At $\omega = \omega_f$ results the point N.

- Translate the initial characteristic (H_f) to have the break (corner frequency) in N, resulting the final form of direct (open) loop. It is obvious that:

$$\overline{FN} = V_R\big|_{dB}$$

Pay attention to the sign of V_R in dB.

- The performance tests are related to:
  - Settling time: read the frequency (ω_t) from logarithmic diagrams. Because $\omega_t = \frac{\omega_n}{2\xi}$, then $\omega_n = 2\xi\omega_t$, so settling time is $t_r = \frac{4}{\xi\omega_n} \le t_r^*$;
  - The steady-state error coefficient to a ramp reference signal is directly read from the Bode diagram at $\omega = 1$, being necessary $c_v \ge c_v^*$;
  - The bandwidth is approximated with $\Delta\omega_B \cong \omega_t$.

If all these performance indicators are satisfied, the obtained P controller meets all design criteria and the design is finished. Otherwise, more complex controllers are required.

2.2. PI controller design steps

When a more aggressive performance set is given, a simple proportional controller cannot ensure all performance criteria. If the settling time requirement is met, but the steady-state error coefficient to a ramp reference signal is too small, a PI controller is needed. The transfer function of the PI controller is:

$$H_{PI}(s) = V_R \frac{1 + sT_z}{1 + sT_P}.$$

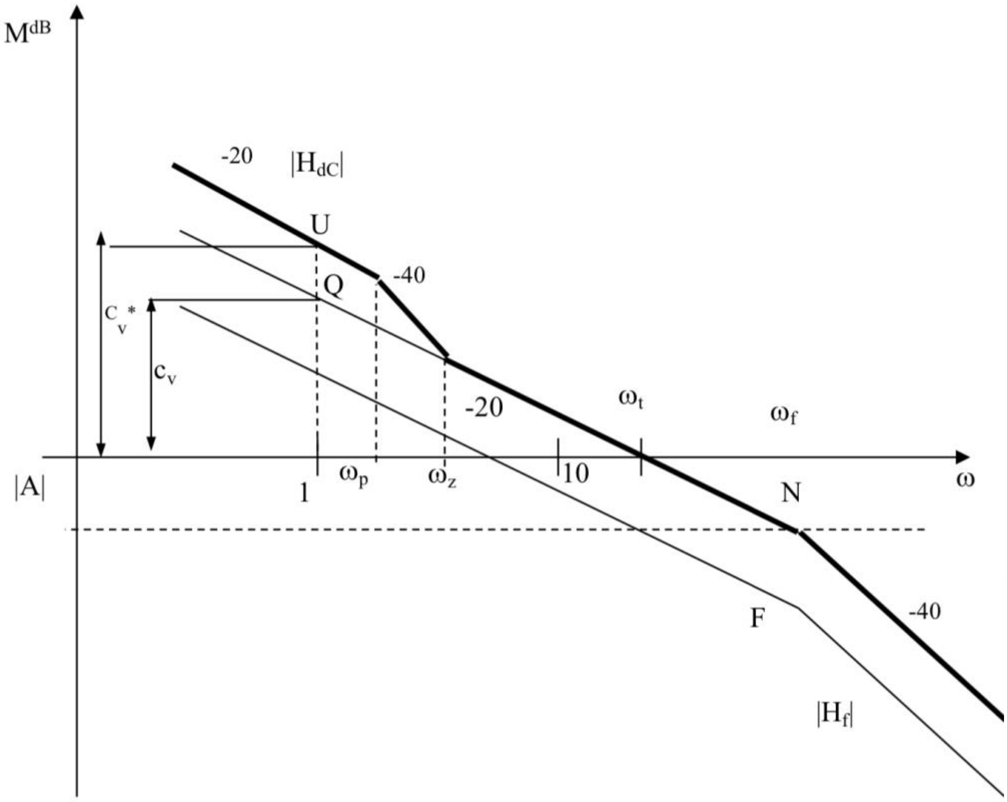
To grapho-analytically determine the parameters, represent the modulus of $H_f(j\omega)$ and the line $|A|$ on logarithmic diagram (as described in the previous section), resulting the points F and N. Determine then from magnitude plot the frequency (ω_t) and the coefficient (c_v) at $\omega = 1$. Place the frequencies (ω_z) and (ω_P) so that:

$$\begin{cases} \omega_z \approx 0{,}1\omega_t \\ \omega_p = \dfrac{c_v}{c_v^*}\omega_z \end{cases}$$

resulting ($\omega_p < \omega_z$). Using these frequencies, determine the open loop structure with PI controller (denoted $H_{dC}(s)=H_f(s)*H_p(s)*H_{PI}(s)$). The controller parameters are:

$$\begin{cases} V_R\big|_{dB} = \overline{QU} \\ T_z = \dfrac{1}{\omega_z} = \dfrac{1}{0{,}1\omega_t} \\ T_P = \dfrac{1}{\omega_P} = \dfrac{1}{\omega_z} \cdot \dfrac{c_v^*}{c_v} \end{cases}$$

The necessary testing refers only to $(\Delta\omega_B^*)$, all other performances being satisfied by default.



3. <u>PROBLEMS</u>

For the process described by

$$H_f(s) = \frac{3.5}{s(0.5s+1)}$$

and the performance indicators

a) $\begin{cases} \varepsilon_{stp}^{*} = 0 \\ \sigma^{*} \leq 15\% \\ t_r^{*} \leq 15[sec] \\ c_v^{*} \geq 1 \\ \Delta\omega_B^{*} \leq 15[rad/sec] \end{cases}$ respectively b) $\begin{cases} \varepsilon_{stp}^{*} = 0 \\ \sigma^{*} \leq 15\% \\ t_r^{*} \leq 1[5\,sec] \\ c_v^{*} \geq 5 \\ \Delta\omega_B^{*} \leq 15[rad/sec] \end{cases}$

following the above described steps, design a P and a PI controller. Simulate the step and ramp output of the closed loop to highlight the performance indicators.