

Computing the controller $H_r(s)$ using the Guilleman-Truxal method

Compute and declare my transfer function

The transfer function has the following form:

$$H_f(s) = \frac{K_f}{s(T_f s + 1)}$$

Knowing that my values are $K_f = 4$ and $T_f = 8$, the function becomes:

$$H_f(s) = \frac{4}{s(8s + 1)}$$

I declare my transfer function in code:

```
Hf = tf(4,[8 1 0])
```

```
Hf =
```

$$\frac{4}{8s^2 + s}$$

```
Continuous-time transfer function.
```

I choose values for σ and t_r^*

The controller needs to satisfy the following specifications:

$$\varepsilon_{\text{stp}} = 0$$

$$t_r^* \leq 40 \text{ sec}$$

$$\sigma^* \leq 15\%$$

$$c_v \geq 0.2$$

$$\Delta w_b^* \leq 2 \frac{\text{rad}}{\text{sec}}$$

From this specifications I can choose σ and t_r^* as:

```
overshoot = 0.1;  
settlingTime = 20;
```

I first calculate $H_o(s)$ (closed loop transfer function) parameters using the following formulas:

$$\xi = \frac{|\ln(\sigma)|}{\sqrt{\ln(\sigma)^2 + \pi^2}}$$

$$\text{zetta} = \text{abs}(\log(\text{overshoot}))/\sqrt{\log(\text{overshoot})^2 + \pi^2};$$

$$w_n = \frac{4}{t_r \xi}$$

$$W_n = 4/\text{settlingTime}/\text{zetta};$$

$$c_v = \frac{w_n}{2\xi}$$

$$C_v = W_n/2/\text{zetta};$$

$$\varepsilon_{\text{stv}} = \frac{1}{c_v}$$

$$\text{Estv} = 1/C_v;$$

$$\Delta w_b = w_n \sqrt{1 - 2\xi^2 + \sqrt{2 - 4\xi^2 + 4\xi^4}}$$

$$\text{deltaWb} = W_n * \sqrt{1 - 2 * \text{zetta}^2 + \sqrt{2 - 4 * \text{zetta}^2 + 4 * \text{zetta}^4}};$$

I check if the specifications are met

C_v

$$C_v = 0.2862$$

$$0.2862 \geq 0.2 \text{ "True"}$$

deltaWb

$$\text{deltaWb} = 0.3924$$

$$0.3924 \leq 2 \text{ "True"}$$

I construct my closed loop transfer function

The closed loop transfer function has the following form:

$$H_o(s) = \frac{w_n^2}{s^2 + 2\xi w_n s + w_n^2}$$

I declare it in code using the values calculated before

```
Ho = tf(Wn^2,[1 2*zeta*Wn Wn^2])
```

Ho =

```
      0.1145
-----
s^2 + 0.4 s + 0.1145
```

Continuous-time transfer function.

I compute the controller transfer function

I use the formula:

$$H_R(s) = \frac{1}{H_f(s)} * \frac{H_o(s)}{1 - H_o(s)}$$

```
Hr = minreal(1/Hf*Ho/(1-Ho))
```

Hr =

```
0.2289 s + 0.02862
-----
s + 0.4
```

Continuous-time transfer function.

I plotted the closed loop system response using command:

```
step(feedback(Hr*Hf,1))
```

Figure 2

