

Control Engineering II - Project

Assignment 4

The transfer function of the process from the closed loop system shown in Fig. 1 is given as

$$H_f(s) = \frac{K_f}{s(T_f s + 1)}$$

Requirement: Compute a controller $H_r(s)$ that ensures a phase margin of the open loop process of $\gamma_k = 60^\circ$.

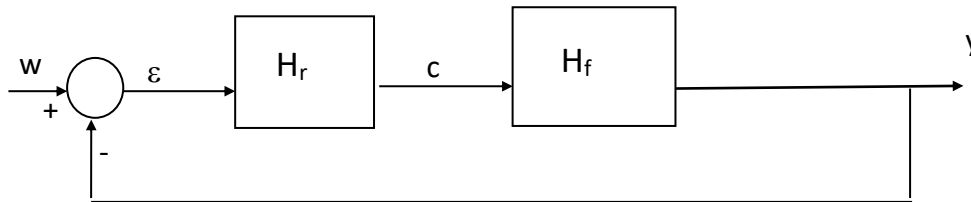


Fig. 1 Closed loop system

Handout: A short documentation containing the obtained controller, the closed loop system performance and the Bode diagram showing that the phase condition is met.

Theoretical background:

Some brief tuning steps of the PD controller: $H_c(s) = k_p \frac{1+\tau_d s}{1+\beta\tau_d s}$, $\beta \in (0.1, 0.125)$

1. $\angle H_{ol}(j\omega_c) = -180^\circ + \gamma_k$
2. choose $\angle H_c(j\omega_c)$ maximum

$$\angle H_c(j\omega_c) = \text{atan} \frac{1-\beta}{2\sqrt{\beta}}$$

$$\omega_c = \frac{1}{\tau_d \sqrt{\beta}}$$

$$|H_c(j\omega_c)| = \frac{k_p}{\sqrt{\beta}}$$

3. select $\beta \in (0.1, 0.125) \Rightarrow \angle H_c(j\omega_c) = ?$
4. $\angle H_{ol}(j\omega_c) = \angle (H_p(j\omega_c)H_c(j\omega_c)) = \angle H_p(j\omega_c) + \angle H_c(j\omega_c)$
 $\angle H_p(j\omega_c) = -180^\circ - \angle H_c(j\omega_c) + \gamma_k \Rightarrow \omega_c = ?$ (read it from Bode)

5. $\tau_d = \frac{1}{\omega_c \sqrt{\beta}}$

6. $|H_{ol}(j\omega_c)| = |H_c(j\omega_c)| \cdot |H_p(j\omega_c)| = 1$

$$\frac{k_p}{\sqrt{\beta}} \cdot |H_p(j\omega_c)| = 1 \quad (\text{read } |H_p(j\omega_c)| \text{ from Bode})$$

$$k_p = \frac{\sqrt{\beta}}{|H_p(j\omega_c)|}$$

Some brief tuning steps of the PI controller: $H_c = k_p \left(1 + \frac{1}{T_i s}\right)$

1. $\angle H_{ol}(j\omega_c) = -180^\circ + \gamma_k$
2. assume that $\angle H_c(j\omega_c) = -15^\circ$
 $|H_{ol}(j\omega_c)| \cong k_p$ and $T_i = \frac{4}{\omega_c}$
3. $\angle H_p(j\omega_c) = -180^\circ + 15^\circ + \gamma_k = -165^\circ + \gamma_k$
4. $\omega_c = ?$
5. $T_i = \frac{4}{\omega_c}$
6. $|H_{ol}(j\omega_c)| = |H_c(j\omega_c)| \cdot |H_p(j\omega_c)| = 1$
7. $k_p = \frac{1}{|H_p(j\omega_c)|}$