Laboratory work 1

Steady state errors in control systems

1. THE GOALS OF THE WORK

a) To analyze steady-state errors;
b) To study the influence of the P, I and D components from control algorithm on steady state errors
c) To interpret the external characteristic.

2. THEORETICAL BACKGROUND

The structure of a control system is presented in *Figure 1.1*:

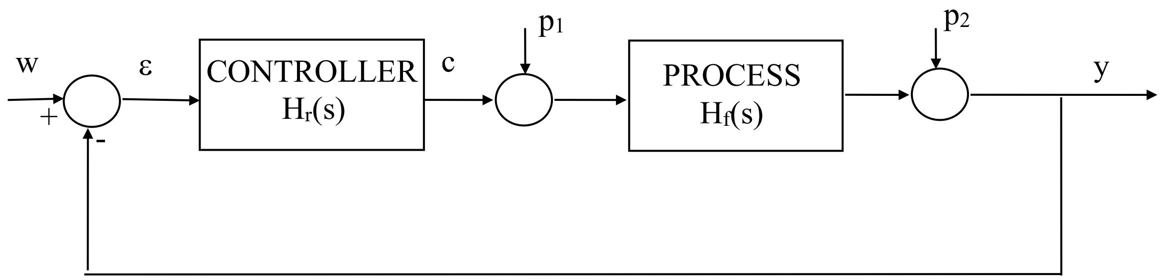


Figure 1.1.

w- input (reference) signal
c – control signal
ε- error signal
y – output signal

Ho(s) – transfer function of the closed loop
Hd(s) – transfer function of the open loop
Hf(s) – transfer function of the process (fixed part)

The error signal is:

$$\varepsilon(s)=w(s)-y(s)=(1-H_o(s))\cdot w(s)$$

or $\varepsilon(s)=\dfrac{1}{(1+H_d(s))}\cdot w(s)$.

Using the final value theorem, we can obtain the steady state error:

$$\varepsilon_{st}=\lim_{s\to 0} s\cdot\varepsilon(s)=\lim_{t\to\infty}\varepsilon(t)$$

or: $\varepsilon_{st}=\lim\limits_{s\to 0} s\cdot(1-H_o(s))\cdot w(s)=\lim\limits_{s\to 0}\dfrac{s}{(1+H_d(s))}\cdot w(s)$

Steady state errors are classified into three major categories:

- Steady-state position error;

$$w(t) = u_o(t) \text{ and } w(s) = \frac{1}{s}$$

$$\varepsilon_{stp} = \lim_{s\to 0} s\cdot(1 - H_o(s))\cdot\frac{1}{s} = \lim_{s\to 0}(1 - H_o(s)) = \lim_{s\to 0}\frac{1}{(1+H_d(s))} = \frac{1}{1+\lim_{s\to 0} H_d(s)} = \frac{1}{1+c_p}$$

- Steady-state speed error;

$$w(t) = t \text{ and } w(s) = \frac{1}{s^2}$$

$$\varepsilon_{stv} = \lim_{s\to 0} s\cdot(1 - H_o(s))\cdot\frac{1}{s^2} = \lim_{s\to 0}\frac{(1 - H_o(s))}{s} = \lim_{s\to 0}\frac{1}{s\cdot(1+H_d(s))} = \lim_{s\to 0}\frac{1}{s\cdot H_d(s)} = \frac{1}{c_v}$$

- Steady-state acceleration error.

$$w(t) = \frac{t^2}{2} \text{ and } w(s) = \frac{1}{s^3}$$

$$\varepsilon_{sta} = \lim_{s\to 0} s\cdot(1 - H_o(s))\cdot\frac{1}{s^3} = \lim_{s\to 0}\frac{(1 - H_o(s))}{s^2} = \lim_{s\to 0}\frac{1}{s^2\cdot(1+H_d(s))} = \lim_{s\to 0}\frac{1}{s^2\cdot H_d(s)} = \frac{1}{c_a}$$

In general case, the following steady state error equation holds:

$$w(t) = \frac{t^n}{n!} \text{ and } w(s) = \frac{1}{s^{n+1}}$$

$$\varepsilon_{st} = \lim_{s\to 0}\frac{1}{s^n\cdot H_d(s)} = \frac{1}{c}$$

From the steady-state error equations and the corresponding error coefficients, it can be concluded:

- if the open loop has no integrator, then:
  - the steady-state position error is finite, has a nonzero value and increasing the open loop gain leads to a decrease in the steady state error;
  - the steady-state speed error is infinite;
  - the steady-state acceleration error is infinite;

- if the open loop has one integrator, then:
  - the steady-state position error is zero;
  - the steady-state speed error is finite, nonzero;
  - the steady-state acceleration error is infinite;

- if the open loop has double integrator, then:
  - the steady-state position error is zero;
  - the steady-state speed error is zero;
  - the steady-state acceleration error is finite, non-zero;.

A control system as depicted in Figure 1.1. can operate in *reference tracking mode* and in *disturbance rejection mode*:

- In *tracking mode* p1=0, p2=0, the reference has time variations and the output must follow (track) the reference as close as possible. The following cases are considered:

- If the reference is a step signal, a controller with a *simple integrator* is required for zero steady state position error
- If the reference is a ramp signal, the controller must be *double integrator*;

- In *disturbance rejection mode* the reference is assumed constant and the disturbances are nonzero; the output has to be constant – tracking the reference signal – regardless of disturbances. If this occurs then the system has the ability to reject external disturbances.

  To reject disturbances in steady-state, we can choose from the following cases:

- p1=0, p2-step – an integrator is required on the direct loop/path;
- p1=0, p2-ramp – a double integrator is required on the direct loop/path;
- p1-step, p2=0 and H_f has no integrator, in this case a controller with a simple integrator is required;
- p1-step, p2=0 and H_f has an integrator, then a controller with a double integrator is required.

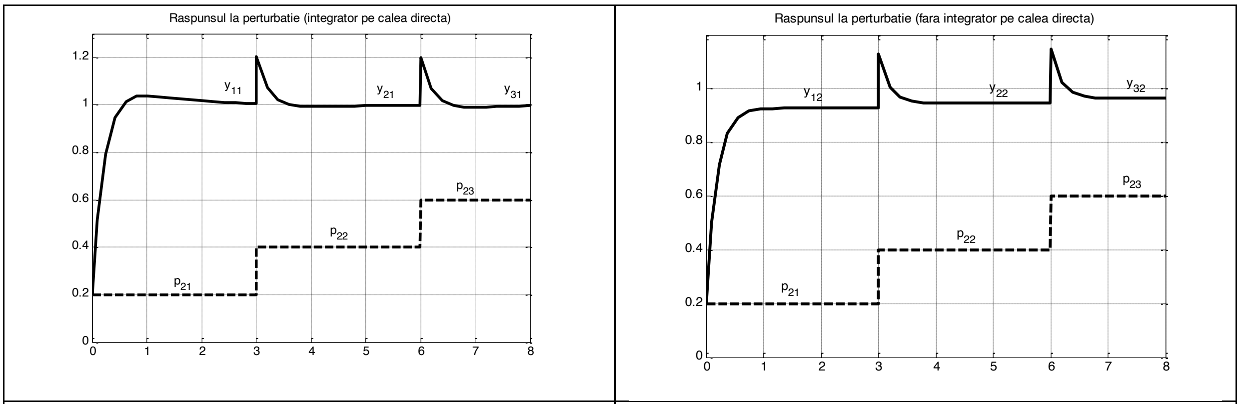
ε_{stp}	p_1, p_2	$H_f(s)$	$H_r(s)$
0	p_1=0 p_2=step	any	Simple integrator
0	p_1=0 p_2=step	integrator	Proportional
0	p_1=step p_2=0	integrator	Double integrator
0	p_1=step p_2=0	any	Simple integrator

In certain cases, for p1=0 and step reference, the disturbance p2 is the plant load (output or load disturbance). The equation:

$$y(\infty) = f(p_2(\infty)) \mid w = u_o$$

represents the *external (or load) characteristic* of the control system. The natural external characteristic of the plant is obtained when a constant input signal is applied to the fixed part (no controller present). To draw the *load characteristic* the principle of quasi-steady-state is applied. This implies that once the transient regime has passed, successive step disturbances are applied p21, p22, p23 ...As a consequence, the corresponding steady-state values of the output signal are determined as y11, y21, y31 (integrator on the direct loop), ... and y12, y22, y32 (no integrator on the direct loop), ..., respectively. The signals' representation is indicated in Figure 1.2.

Collecting the steady state values of the output signal y11, y21, y31 (integrator on the direct loop), ... and y12, y22, y32 (no integrator on the direct loop) and plotting these as a function of the step disturbance sequence p21, p22, p23 leads to the final load characteristic. This can be constant or decreasing (increasing). The larger the open loop gain is, the smaller the slope of the load characteristic will be. Decreasing the slope of the load characteristic implies better load (output) disturbance of the controller. In the case of a controller with an integrator, the load characteristic remains constant (full load disturbance rejection).



a)	b)

Fig. 1.2. a) Output signal as a result of step disturbances (integrator present on the direct loop), b) Output signal as a result of step disturbances (no integrator present on the direct loop)

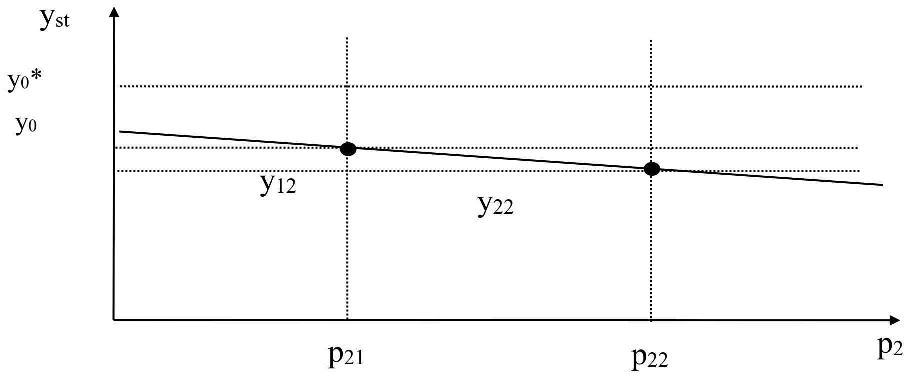


Fig. 1.3. Load characteristic (no integrator on the

In order to obtain zero steady-state errors without integrator elements, special control structures can be used. The integrator elements reduce the phase margin of the systems, leading to instability, therefore in some applications these should be avoided. The following control structures can be considered to ensure disturbance rejection without integrator elements. In this case the output from the disturbance goes asymptotically to zero.

a) with a supplementary controller H_{RP}:

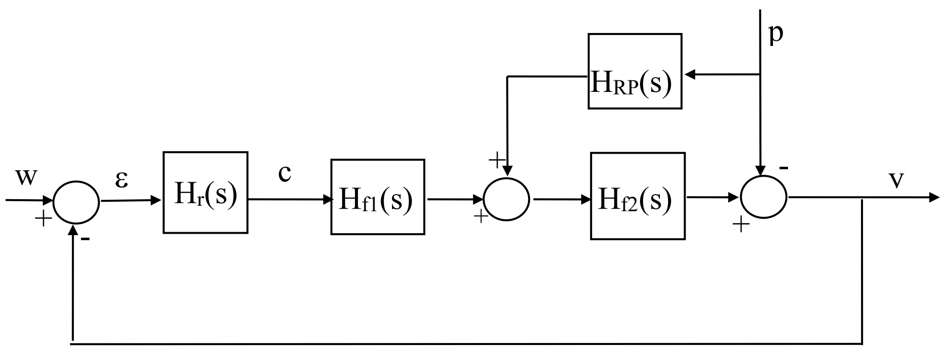


Fig. 1.4.

b) with a feedback loop from the load disturbance:

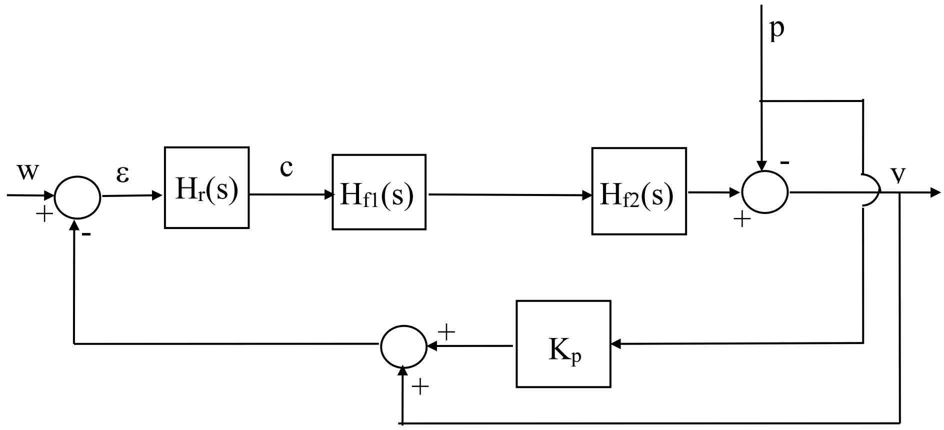


Fig. 1.5.

3. PROBLEMS

1. The following process characterized by the transfer function $H_f(s) = \frac{1}{s(s+1)}$ is considered. What is the gain of the proportional controller which can ensure a steady-state speed error smaller than 0.1?
2. For the structure from Fig.1.6, the fixed part has the following transfer function $H_f(s) = \frac{5}{2s+1}$.
   i) Using both MATLAB and analytical calculus, determine and compare the steady-state position and speed errors for the control system for four types of controller: a proportional controller with the gain k_p=1, 2, 5 and a PI controller with T_i=1 and k_p=2.
   ii) Determine the closed loop transfer function from disturbance "p" and highlight the effect of a step disturbance to the output using a P controller (k_p=2) and then a PI controller (k_p=2, T_i=1);
   iii) Considering that the disturbance "p" is varying in steps, plot the system output in the case of step reference signal. The disturbance steps are: 0.2, 0.3, 0.4. Implement the control structure in Simulink and estimate the steady state output signal values. Draw the load characteristic using a P and a PI controller (k_p=2, T_i=1).

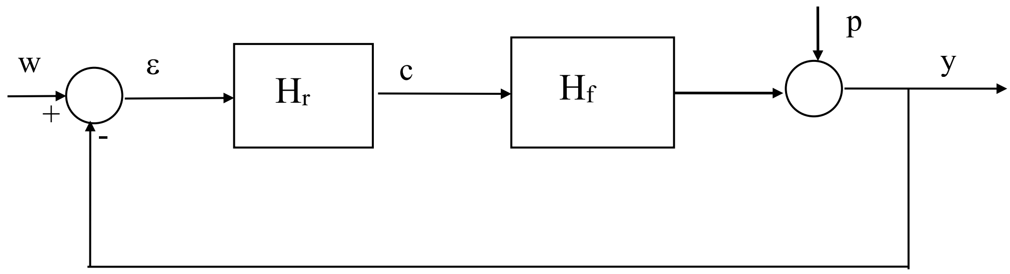


Fig. 1.6.

3. Consider the control system from *Fig.1.7* where $H_{f1}(s) = \frac{1}{s}$ and $H_{f2}(s) = \frac{1}{s+2}$. Determine the closed loop transfer functions linking the output signal y(s) to disturbances p1(s) and p2(s). Plot the output of the system with respect to these disturbances considering a P and a PI controller (k_p=2, T_i=1). You can use either Simulink or Matlab.

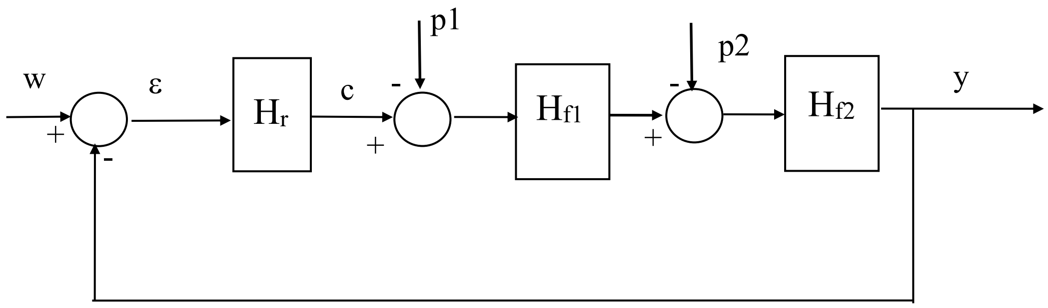


Fig. 1.7.

4. Consider the closed loop system from Fig. 1.8, where $H_{f1}(s) = \frac{1}{20s+1}$ and $H_{f2}(s) = \frac{0.5}{s}$, the proportional controller $H_R(s) = V_R$ with $V_R \in (2 \div 10)$.
   Study the response of the closed loop system considering the following scenarios:
   - At a unit step reference
   - At a unit ramp reference
   - At the presence of the disturbance *p1* – step signal of amplitude 0.2
   - At the presence of the disturbance *p2* – step signal of amplitude 0.1

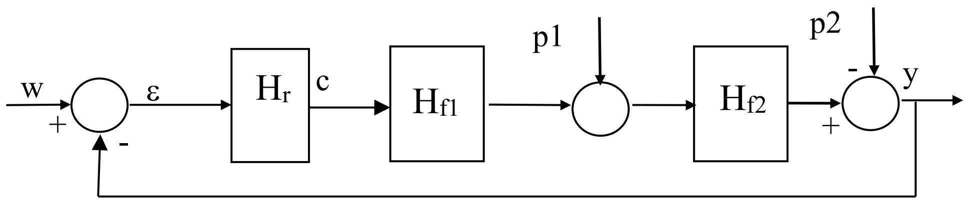


Fig. 1.8. Control structure for problem 4

5. Considering the control structure from Fig. 9, compute the steady-state errors for the following references: 5r(t), 5tr(t), $5t^2r(t)$, where r(t) is the unit step signal. Analytical computations and solutions are required.

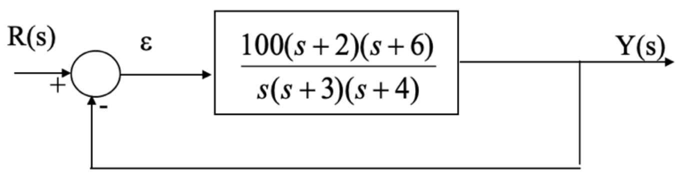


Fig. 1.9. Control structure for problem 5