Correction of Guillemin-Truxal

$$H_f(s) = \frac{k_f}{s(T_f s + 1)}, \quad k_f = 2, \quad T_f = 5\,sec$$

$$\begin{cases} \varepsilon_{stp} = 0 \\ t_s < 8\,sec \\ \sigma \le 10\% \\ c_v \ge 1.5 \Leftrightarrow \varepsilon_{stv} \le 0.66 \\ \Delta\omega_B \le 1.2\,rad/sec \end{cases}$$

$$H_{oc}(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n + \omega_n^2} \cdot \frac{s+z_c}{s+p_c} \cdot \frac{p_c}{z_c}$$

We impose such a closed-loop system.

1. Choose a proper ratio for $\frac{p_c}{z_c} = (1.00 \div 1.10)$

$\hookrightarrow \frac{p_c}{z_c} = 1.03$

$$\Delta\sigma_c = \frac{p_c}{z_c} - 1 = 1.03 - 1 = 0.03$$

! Remark: Choosing a larger ratio will affect the settling time 't_s' (E.g. $\frac{p_c}{z_c} = 1.01 \rightarrow$ little influence but must check performance criteria!)

2. $\sigma_2 = \sigma^* - \Delta\sigma_c$ where σ^* must be $\le 10\% \Rightarrow \sigma^* = 0.10$ this is the imposed overshoot!

$$\sigma_2 = 0.10 - 0.03 = 0.07$$

3. $$\zeta = \frac{|\ln \sigma_2|}{\sqrt{\pi^2 + \ln^2 \sigma_2}} = \frac{|\ln 0.07|}{\sqrt{\pi^2 + \ln^2 0.07}} = 0.6461$$

4. $t_s^* = 5.7 \rightarrow$ This is the imposed settling time 't_s' $\le 8\,sec$

$$\omega_n = \frac{4}{t_s^* \cdot \zeta} = \frac{4}{5.7 \cdot 0.6461} = 1.0862$$

5. Check $\Delta\omega_B \le 1.2\,rad/sec$, if not satisfied modify $\frac{p_c}{z_c}$, σ^* and/or t_s^*

$$\Delta\omega_B = \omega_n\sqrt{1 - 2\zeta^2 + \sqrt{2 - 4\zeta^2 + 4\zeta^4}} = 1.17 \le 1.2\,rad/sec \checkmark$$

6. Calculate $$c_{v_2} = \frac{\omega_n}{2\zeta} = \frac{1.0862}{2 \cdot 0.6461} = 0.8406$$

! Remark: This coefficient is purely from 'ω_n' and 'ζ' (second order elements/parameters). It does not satisfy yet the condition $0.8406 \not\ge 1.5 \Rightarrow$ this is why we add the zero and the pole z_c, p_c respectively, to correct this! (in the next step)

7. $$\begin{cases} \frac{p_c}{z_c} = 1.03 \\ \frac{1}{c_v^*} = \frac{1}{c_{v_2}} - \frac{1}{z_c} + \frac{1}{p_c} \end{cases} \Leftrightarrow \begin{cases} p_c = \frac{\Delta\sigma_c}{2\frac{\zeta}{\omega_n} - \frac{1}{c_v^*}} = \frac{\Delta\sigma_c}{\frac{1}{c_{v_2}} - \frac{1}{c_v^*}} = 0.0574 \\ z_c = \frac{p_c}{1 + \Delta\sigma_c} = 0.0557 \end{cases}$$

$$H_{oc}(s) = \frac{1.18}{s^2+1.404s+1.18} \cdot \frac{s+0.0557}{s+0.0574} \cdot 1.03 = \frac{1.2152 \cdot (s+0.0557)}{(s^2+1.404s+1.18)(s+0.0574)}$$

$$H_{oc}(s) = \frac{1.215s+0.06767}{s^3+1.461s^2+1.26s+0.06767}$$

$$H_c(s) = \frac{1}{H_f(s)} \cdot \frac{H_{oc}(s)}{1-H_{oc}(s)} = \frac{s(5s+1)}{2} \cdot \frac{\dfrac{1.215s+0.06767}{s^3+1.461s^2\cancel{+1.26s+0.06767}}}{\dfrac{s^3+1.461s^2+1.26s+\cancel{0.06767}-1.215s\cancel{-0.06767}}{s^3+1.461s^2\cancel{+1.26s+0.06767}}}$$

$$= \frac{\cancel{s}(5s+1)}{2} \cdot \frac{1.215s+0.06767}{s^{\cancel{3}2}+1.461s^{\cancel{2}1}+0.045\cancel{s}} = \frac{(5s+1)(1.215s+0.06767)}{2(s^2+1.461s+0.045)}$$

$$= \frac{(5s+1)(17.95s+1)}{(0.69s+1)(31.67s+1)} \cdot \frac{0.06767}{2 \cdot 1.429 \cdot 0.03157} = 0.75 \cdot \frac{(5s+1)(17.95s+1)}{(0.69s+1)(31.67s+1)}$$

$$H_c(s) = 0.75 \frac{(5s+1)(17.95s+1)}{(32.36s+1)}$$

$$\frac{T_1 s+1}{T_2 s+1} = (T_1-T_2)s+1 \quad \text{if } T_1 \gg T_2$$

$$= \frac{1}{(T_2-T_1)s+1} \quad \text{if } T_2 \gg T_1$$

$$(T_1 s+1)(T_2 s+1) = (T_1+T_2)s+1 \quad \text{if } T_1 \gg T_2$$

These are commonly used simplifications and they may affect the system's behavior!