Laboratory work 2

PERFORMANCE INDICATORS OF A CONTROL SYSTEM

1. GOALS

- To present the performance indicators of a control system in steady-state and in transient state;
- To present the performance indicators in frequency domain.

2. THEORETICAL BACKGROUND

2.1. Performance indicators in time domain

2.1.1. **Steady-state**

Steady-state error, determined using the final value theorem

$$\varepsilon_{st} = \lim_{s\to 0}\left[s\cdot \varepsilon(s)\right]$$

System \ Signal	Step	Ramp	Parabola
Proportional	$\varepsilon_{stp} = \frac{1}{1+c_p}$	∞	∞
Simple integrator	0	$\varepsilon_{stv} = \frac{1}{c_v}$	∞
Double integrator	0	0	$\varepsilon_{sta} = \frac{1}{c_a}$

Position, speed, acceleration coefficient

$$c_p = \lim_{s\to 0}\left[H_d(s)\right]$$

$$c_v = \lim_{s\to 0}\left[s\cdot H_d(s)\right]$$

$$c_a = \lim_{s\to 0}\left[s^2\cdot H_d(s)\right]$$

2.1.2. **Transient state**

The performance in this state refers to the signal speed and form.

In many applications a second order system is used, so the performances will be expressed for this type of system:

$$H_o(s) = \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

where ω_n is the free undamped pulsation, and ζ is the damping coefficient.

The speed is analyzed by the **settling time**, the time required for the output to rich a constant value:

$$t_r = \frac{1}{\xi\omega_n}\cdot \ln\left(\frac{1}{0.05\cdot\sqrt{1-\xi^2}}\right) \text{ or, more practical } t_r \cong \frac{4}{\xi\omega_n}.$$

The performance indicators which analyze the transient state form are:

- **overshoot**:

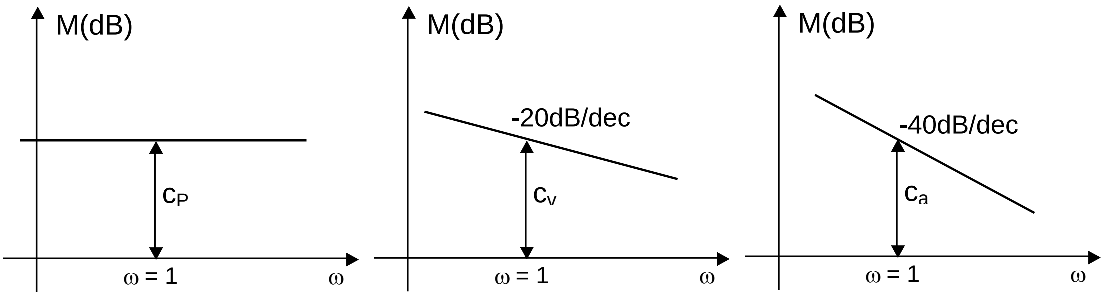
$$\sigma = \frac{y_{max}-y_{st}}{y_{st}}\cdot 100\,[\%] \text{ or } \sigma = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}}$$

- **decay ratio** (Decay-Ratio), which is the ratio between the two successive maxima, with the same polarity.
- **Maximum deviation** (Δy).

2.2. Performance indicators in frequency domain

2.2.1. **Steady-state**

The performance indicators appears at low frequencies $(\omega \to 0)$, uses the low frequency asymptote:



2.2.2. **Transient state**

The performance indicator of the open loop is the **phase margin** (the distance from -π to the system phase, at the crossover frequency – the frequency where the modulus characteristic meets the ω axe):

$$\gamma_k = \pi + \varphi(\omega)\,\Big|_{\omega=\omega_t}.$$

For the second order system the equation is:

$$\gamma_k = \arccos\left(\frac{1}{2\xi^2+\sqrt{1+4\xi^4}}\right).$$

The performance indicator of the closed loop is the **cross band**, which for second order system has the form:

$$\Delta\omega_B = \omega_n\sqrt{1-2\xi^2+\sqrt{2-4\xi^2+4\xi^4}}\ .$$

3. PROBLEMS

For the process described by $H_f(s) = \frac{1}{2s+1}$ and the controller

a) $H_R(s) = 2$

b) $H_R(s) = \frac{2}{s}$

c) $H_R(s) = \frac{2}{s^2}$

highlight the performance indicators in steady-state and transient state in both time domain and frequency domain, using Matlab. Compare and explain the results for the three different type of controller.