Nyquist Diagrams

1 Aims of the laboratory work

- analysis of Frequency Response using Nyquist plots;
- use Matlab scripts to “faster” obtain and “read” the Nyquist plots;
- establish the cutting frequency (ω_c) and the bandwidth (BW) for any LTI systems;
- get used to standard shapes of Nyquist Diagrams (first and second order elements)
- empirically test of low and high frequency region by different transfer function structures

2 Theoretical considerations

A first graphical representation used in analysis of Frequency Response is the Nyquist Diagram.

In Cartesian coordinates, the real part of the transfer function is plotted on the X axis. The imaginary part is plotted on the Y axis. The frequency is swept as a parameter, resulting in a plot per frequency. Alternatively, in polar coordinates, the gain of the transfer function is plotted as the radial coordinate, while the phase of the transfer function is plotted as the angular coordinate. The Nyquist plot is named after Harry Nyquist, a former engineer at Bell Laboratories.

3 Matlab simulations

The standard problem statement in FR analysis will start by mentioning the transfer function structure. Discussions about the link between certain transfer functions and real processes will be covered in the “problems” section.

Several ways of representing the Nyquist diagram using Matlab functions are commented bellow, explaining the main differences between the approaches.

Given the next transfer function:

$$H(s) = \frac{400}{s(s+2)(s+5)}$$

3.1 Case 1

The most used case due to its compact form is the one calling the “nyquist” Matlab function with two parameters, the nominator and the denominator of the transfer function by their coefficients and in descending order of power of “s”

```
den=conv([1 2 0],[1 5]);% use conv function to obtain the
coefficients of the denominator
H=tf(400,den);% declare the transfer function
nyquist(H);% plot the Nyquist diagram
shg; % bring the plot in the active window position

%% the compact form
nyquist(400,conv([1 2 0],[1 5]))
```

To get access to the value of a certain frequency, “right click” facility is active on the “blue colored” shape (the nyquist diagram). In this way, the high and low frequency region can be established (by empirical meaning not computations)

If right click on the white area of the graphic you can select/unselect the negative frequencies contour (Figure 1).

To establish the cutting frequency, the unity circle must be also represented on the graphics. Use the next script to obtain the Nyquist diagram crossings over the unity circle.

Useful information regarding the frequency, the real and imaginary part values can be reached if sliding with the mouse over the Nyquist plot (see Figure 3).

```
%% zoom on a certain area
axis([-10 0 -2 2]) % See Figure 2

%% how to plot unity circle
hold;% keep the nyquist diagram
x=0:0.1:2*pi; plot(sin(x),cos(x),'--r');shg;% plot a red,
dashed line unity circle
hold;% leave all the plots
```

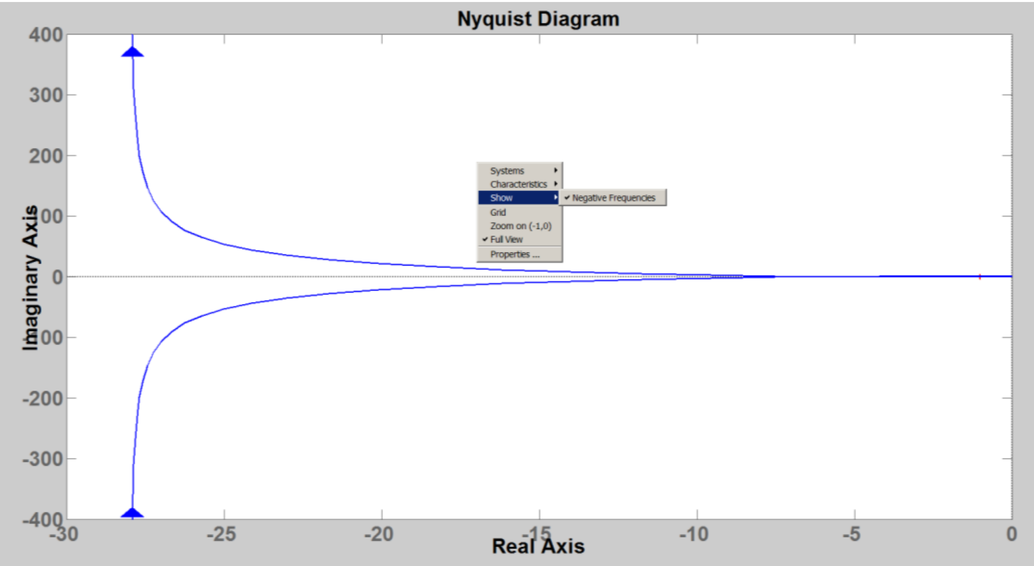

Figure 1 The result of calling the “nyquist” function in Matlab

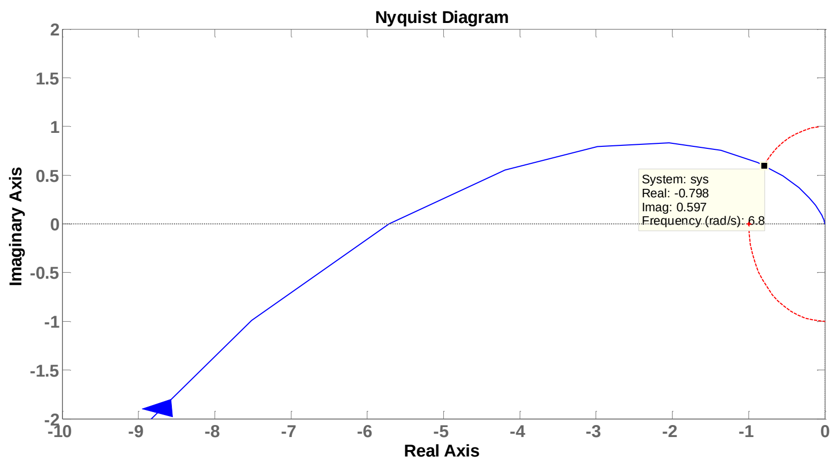

Figure 2 Zoom on the cutting frequency area

At this level, the bandwidth can be extracted from the Nyquist diagram. For the considered example, the BW$BW \in (0,6.8\frac{rad}{sec})$, resulting a Low Pass Filter (LPF) behavior.

3.2 Case 2

To reinforce the idea behind plotting Nyquist diagram, the next Matlab script is suggested.

```
%%
close all; clear
H=tf(400,conv([1 2 0],[1 5]));
[real_part, imag_part]=nyquist(H,'v');
real_part_vect=squeeze(real_part); %Remove singleton
dimensions
```

```
imag_part_vect=squeeze(imag_part); %Remove singleton
dimensions
plot(real_part_vect, imag_part_vect,'b');grid;shg
title('Nyquist Diagram');
xlabel('Real[H(j\omega]');ylabel('Imag[H(j\omega)]')

% click on the graphics as in Figure 4
[x1,y1]=ginput(1); % coordinates for Low Freq. Region
text(x1,y1,'\omega -> 0');% place the text on LFR

% click on the graphics as in Figure 4
[x2,y2]=ginput(1); % coordinates for Hihg Freq. Region
text(x2,y2,'\omega -> inf'); % place the text on HFR
% see figure 3
```

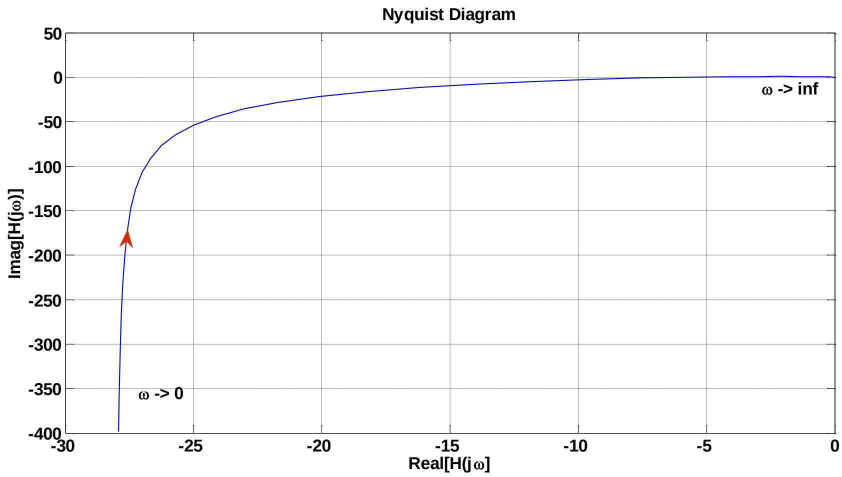

Figure 3 Nyquist Diagram without "graphical facilities"

4 Problems

4.1. For each of the next transfer functions:

a. $H(s) = \frac{2}{s^2+s+1}$

b. $H(s) = \frac{s^2+4}{0.3s^2+s+1}$

c. $H(s) = \frac{s^2}{0.3s^2+s+1}$

d. $H(s) = \frac{s}{0.3s^2+s+1}$

- Use Matlab to obtain the Nyquist diagrams for positive frequencies
- Slide with the mouse on the diagram to establish LFR and HFR
- Sketch on your notebooks the diagram, making the necessary comments / annotations on the axis;
- Read from the diagram the value(s) of the cutting frequencies and indicate the bandwidth
- According to the bandwidth, establish the filter type, the system acts.

4.2. Read from the Nyquist diagram the amplitude and phase shift of the output, for an input signal sin (t) applied on the case a).

4.3. Use Nyquist diagram for case c). to indicate an "input frequency" that suffers a phase shift of -90 degrees through the system. Check the correctness of the frequency value by making a time simulation (in Matlab).

4.4 Accomplish a Matlab script in order to obtain the next graphical representations:

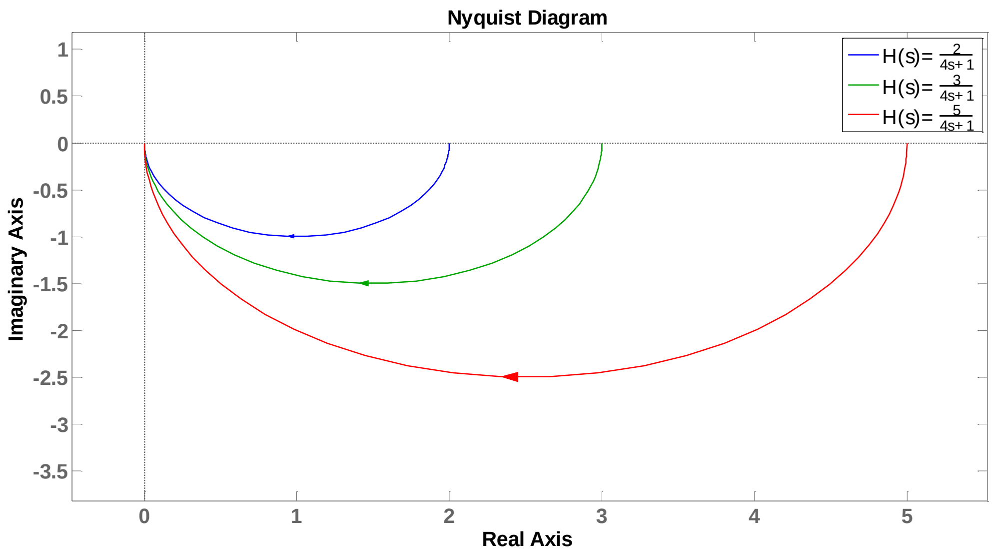

Figure 4 Nyquist diagram for the "first order element", $H(s) = \frac{k}{Ts+1}$, $k \in \{2, 3, 5\}, T = 4$

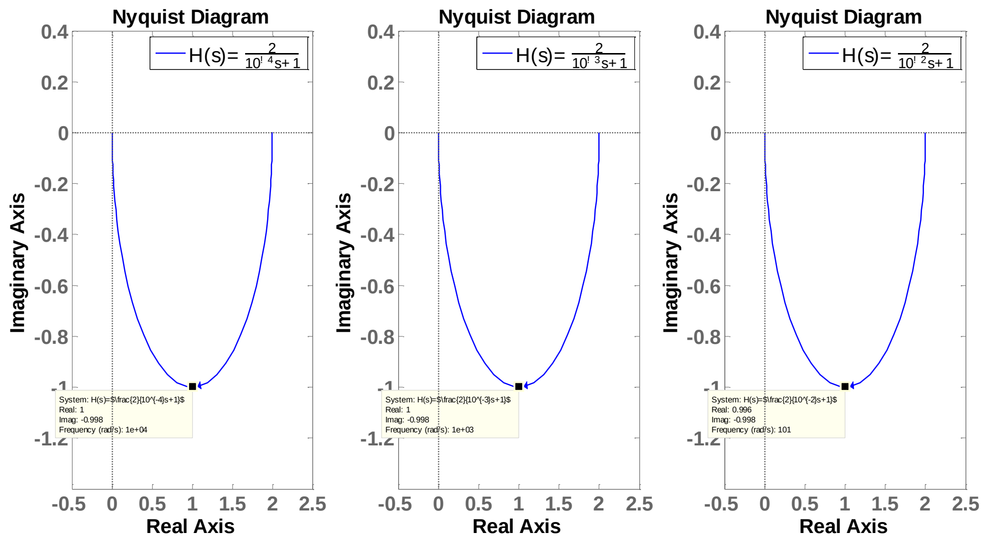

Figure 5 Nyquist diagram for the "first order element", $H(s) = \frac{k}{Ts+1}$, $k = 2, T \in \{10^{-4}, 10^{-3}, 10^{-2}$

5 Review of the new notions

- Polar plot vs. Cartesian plot
- High Frequency Region (HFR) and Low Frequency Region
- Cutting frequency
- Bandwidth
- Amplified vs attenuated signal
- Low Pass Filter (LPF)