## Laboratory no. 6

## 1 Theoretical aspects

- Dead time systems
- Gain margin, phase margin (on Nyquist and Bode)

#### 2 Aims

Stability analysis of negative feedback control systems.

The gain margin  $m_k$  and phase margin  $\gamma_k$  for negative feedback control systems with dead time

# 3 Stability analysis using frequency response

3.1 For simplicity it is considered a rigid negative feedback control structure:

where 
$$H_d(s) = kH(s)$$
;  $H_{des}(s) = H_d(s)$ ;  $H_o(s) = \frac{H_d(s)}{1 + H_d(s)}$ 

- 3.2 Relations for math quatities used in stability analysis
  - Cutting frequency,  $(\omega_t)$  from relation:  $|H(j\omega_t)|=1$ ;
  - Frequency at  $-\pi$ ,  $(\omega_{-\pi})$  from relation  $\angle H(j\omega_{-\pi}) = -\pi$ ;
  - Gain margin,  $m_k^{d\beta} = |H(j\omega_{-\pi})|^{d\beta}$ ;
  - Phase margin,  $\gamma_k = \pi + \angle H(j\omega_c)$ ;
- **3.3** Simplified Nyquist criterion:

 ${\gamma_k \geq 0 \brace m_k^{d\beta} \leq 0}$  equivalent with the statement « The critical point (-1,0\*j) left on the left side of Nyquist diagram of the open loop transfer function»

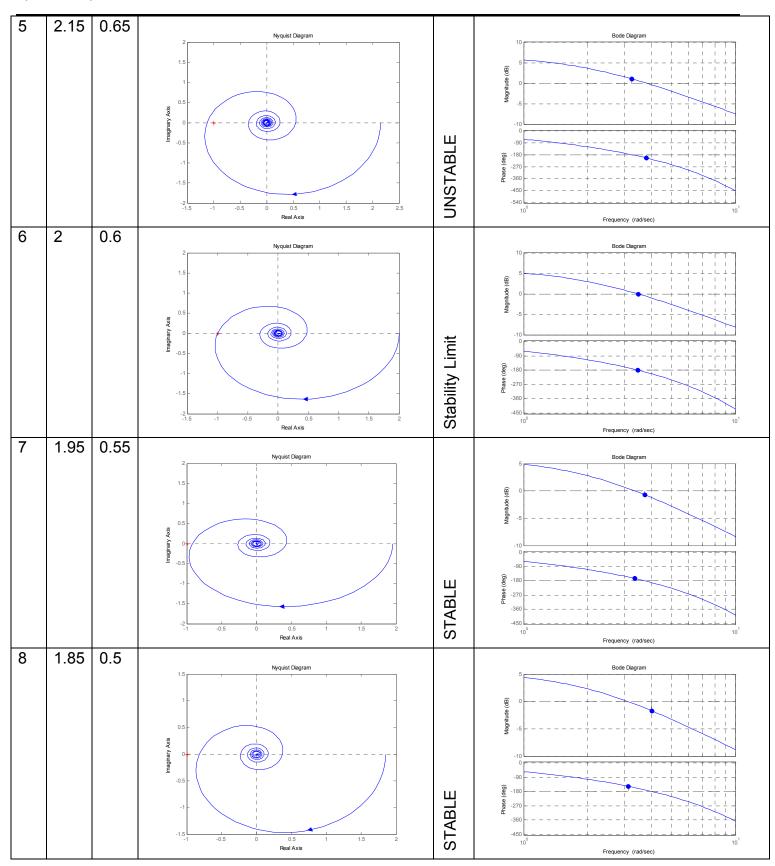
## 4 Problems

**4.1** Analyze the stability of the negative feedback structure (use simplified Nyquist, drawNyquist diagram in Matlab)

$$u(t) \longrightarrow k \longrightarrow H(s) \xrightarrow{y(t)}, \text{ where } k \ge 0 \text{ $i$ } H(s) = \frac{600}{(s+2)(s+300)} e^{-\tau_m s}$$

- a) Draw Bode of the open loop transfer function (use asymptotes, check the result using Bode from Matlab);
- b) Aproximate frequency at  $-\pi$  and  $\omega_t$ ;
- c) Compute frequency at  $-\pi$  and  $\omega_t$  and compare to the results obtained at previous point.
- d) Indicate on Bode the phase and gain margin;
- e) Compute the two margins.
- f) Finally, analyze the stability of the closed loop system.

Nr.	k	T <sub>m</sub>	Nyquist	Но	Bode
1	2.55	0.85	Nyquist Diagram  2.5  2.5  1.5  1.5  1.5  1.5  1.5  1.5	UNSTABLE	Bode Dagram  10  10  10  10  10  10  10  10  10  1
2	2.45	0.8	Nyquist Diagram  2.5  2  1.5  1.5  1.5  1.5  2  2.5  1.5  1	I UNSTABLE	Bode Dagram  (Ge)  10  10  10  10  10  Frequency (rad/sec)
3	2.35	0.75	Nyquist Diagram  2.5  2.5  1.5  1.5  1.5  1.5  1.5  1.5	UNSTABLE	Bode Diagram  (ga) appropriate of the control of th
4	2.25	0.7	Nyquist Diagram  1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.5 1.	UNSTABLE	Bode Diagram  (Geo.)  10  10  10  10  10  10  10  10  10  1



9	1.75	0.45			
			Nyquist Diagram		Bode Diagram  5
			sex Armadeum Aves		-5
			-1 -0.5 0 0.5 1 1.5 2	STABLE	90 -180 -180 -17 -17 -17 -17 -17 -17 -17 -17 -17 -17
10	1.65	0.4	Real Axis	0)	Frequency (rad/sec)
	1.03	0.4	Nyquist Diagram  1  0.5  -0.5  -1  -0.5  0.5  1  1  1  1  1  1  1  1  1  1  1  1  1	STABLE	Bode Dagram  5  (Geophystal Control of the Control
11	1 55	0.35	Real Axis	0)	Frequency (rad/sec)
11	1.55		Nyquist Diagram  1  0.5  -0.5  -0.5  Real Axis	STABLE	Bode Diagram  5  (B) Population of the control of t
12	1.45	0.3	Nyquist Diagram		Bode Diagram
			0.5 0.5 0 Real Axis	STABLE	(69) 990 -5 -5 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1

**4.2** Analyze the stability of the next closed loop system, using the same steps as in the previous problem

$$u(t) \longrightarrow k \longrightarrow H(s) \xrightarrow{y(t)}, \text{ with } k \ge 0 \text{ $i$ } H(s) = \frac{600}{s(s+2)(s+300)}e^{-\tau_m s}$$