Introduction to „frequency response" of LTI systems

1 Aims of the laboratory work

The main interest is on refreshing the knowledges regarding the Linear, Time – Invariant systems, especially on their time response to standard inputs (unity step signal, ramp signal, sinewave, etc.). Complementary, the superposition principle must be reviewed.

The Matlab companion as a "computer assisted" tool in making the simulations stage is also covered by this work.

Classical notions about "reading" oscilloscope captures are also described

2 Theoretical considerations

Processes like analog circuit in figure 1 can be mathematically treated as "systems" described by a model. The two classical approaches in mathematical modelling consists in deriving the "transfer function" (figure 2) or "the state space" model.

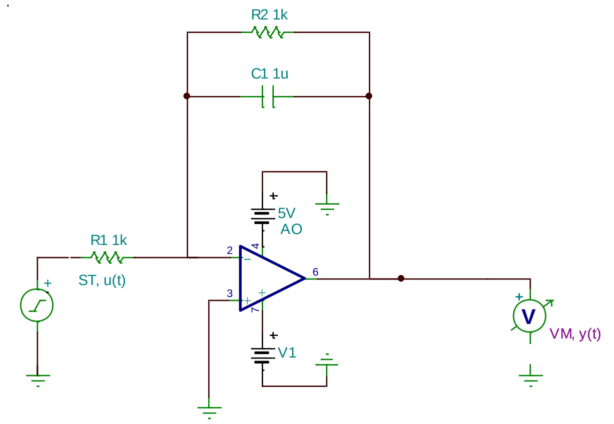

Figure 1 Analog circuit based on inverting OpAmp

AO – operational amplifier

R1- resistor (1kΩ)

R2-resistor (1kΩ)

C1- capacitor (1μF)

ST- voltage source

VM-voltmeter

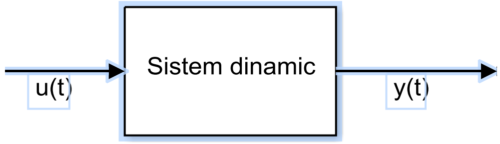

Figure 2 Transfer function model – H(s)

The transfer function can be derived based on the proprieties of operational amplifier in inverting connection. The immediate result of the impedances ratio can be applied:

$$H(s) = -\frac{R_2||C_1}{R_1} = -\frac{\frac{R_2 \cdot \frac{1}{sC_1}}{R_2 + \frac{1}{sC_1}}}{R_1} = -\frac{R_2}{R_1}\frac{1}{R_2C_1s+1} = \frac{k}{Ts+1} \tag{1}$$

In relation (1), it should be recognized the transfer function of the first order element with its two parameters: k- the proportional gain and T- the time constant.

The steady state response to a sinewave $u(t) = \sin(\omega t)$ changes in amplitude and phase, i.e. $y_{ss}(t) = A_{ss}\sin(\omega t + \varphi_{ss})$. The second important result is that the bandwidth of a FOE is directly connected to its parameters in the next relation:

$$BW \in \left(0, \frac{k}{T}\right) \tag{2}$$

FOE behaves like a low pass filter, having the cutting frequency $\omega_c = \frac{k}{T}\left[\frac{rad}{sec}\right]$ $or\ f_t = \frac{k}{T}\frac{1}{2\pi}[Hz]$.

2.1 Matlab simulations

If the interest is in analyzing the behavior of the circuit for a sinewave of 1kHz, the next Matlab commands can be used:

```
R1=1e3;R2=R1;C1=1e-6;% the circuit parameters
f=1e3;% the frequency in Hz for the input sinewave
T=1/f; % the period of the inpus sinewave in sec.
w=2*pi*f; % the frequency in [rad/sec];
t=0:T/50:12*T; % simulation time basis STRONGLY! connected
to T
H=tf(-R2/R1,[R2*C1 1]); % the transfer function
u=sin(w*t); % the input sinewave, or generally the input
signal
lsim(H,u,t);% linear simulation of the response
%% in case of modifying the plot title and legend
y=lsim(H,u,t); % memorize the output signal in y variable
plot(t,u,'r',t,y,'b');
legend('Input signal','Output signal');
title('Sinewave response of FOE');
grid;shg
```

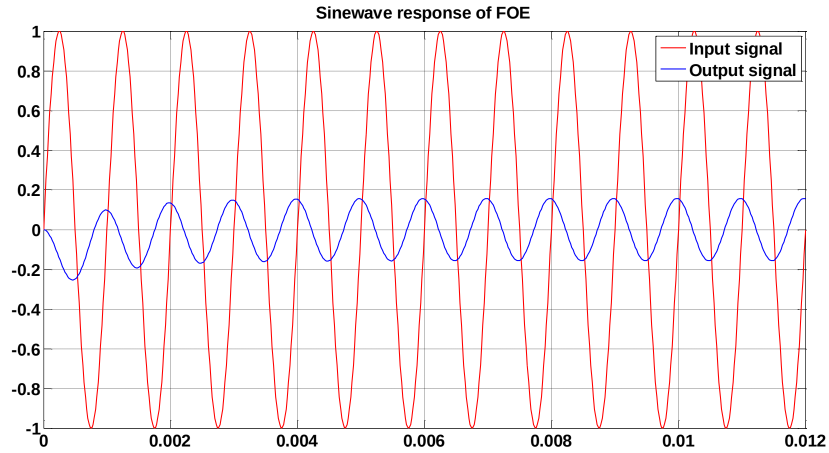

Figure 3 graphical result of the second section in the script

2.2 Practical measurements

In figure 4 and 5 there are two oscilloscope captures for the circuit in figure 1.

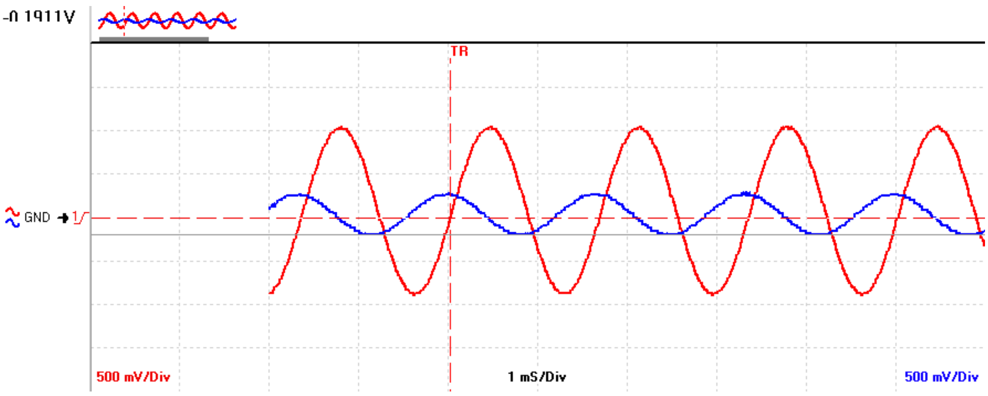

Figure 4 Oscilloscope capture with no measurements included

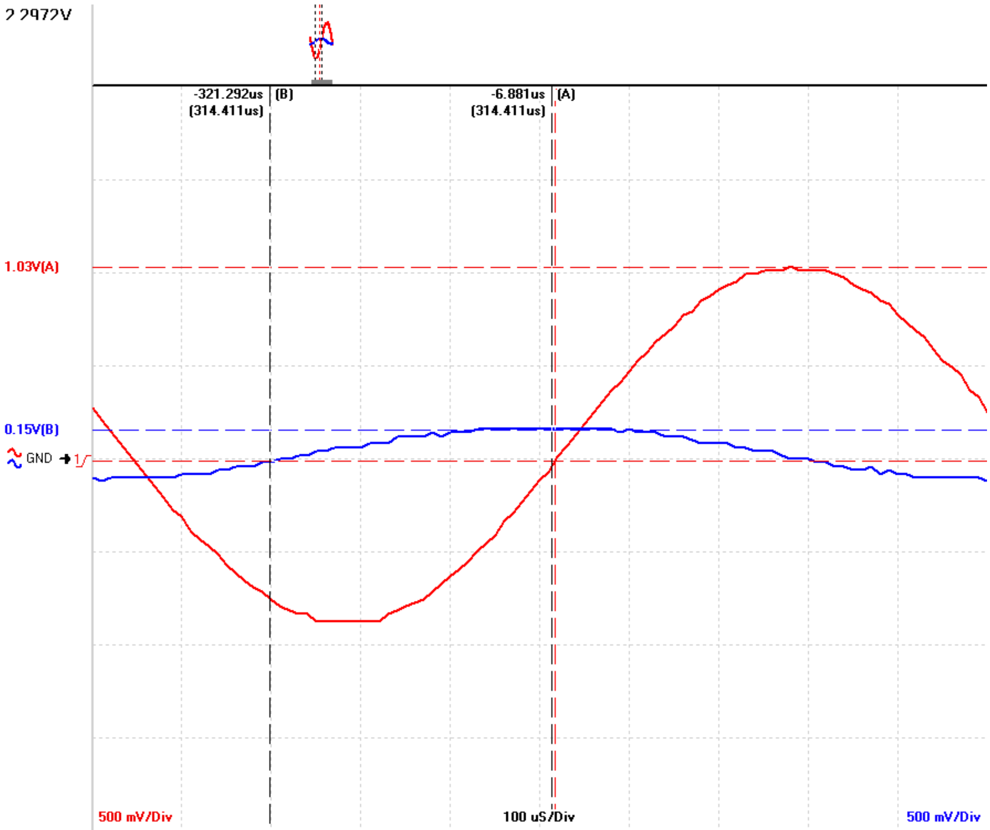

Figure 5 Oscilloscope capture with amplitude and phase shift included

References

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