Theoretical aspects

Each transfer function can be written as a ratio of polynomials given in the product form where real singularities and pairs of complex conjugates singularities are involved.

Using approximations for magnitude and phase for terms of first order and second order results in good approximation for the overall magnitude and phase for the considered system.

To obtain a good approximation, student must be able to understand and then apply the next “remarkable” results:

- real and negative pole, $\frac{1}{Ts+1}$, with magnitude: a slope of -20dB after the corner frequency, $\frac{1}{T}$, and phase having 3 points of interest: $\left(0.1\frac{1}{T}, -5^0\right), \left(\frac{1}{T}, -45^0\right), \left(10\frac{1}{T}, -85^0\right)$, whit an entire phase shift over the positive range of frequencies: $(0^0, -90^0)$;
- real and negative zero, $Ts+1$, with magnitude: a slope of +20dB after the corner frequency, $\frac{1}{T}$, and phase having 3 points of interest: $\left(0.1\frac{1}{T}, +5^0\right), \left(\frac{1}{T}, +45^0\right), \left(10\frac{1}{T}, +85^0\right)$, whit an entire phase shift over the positive range of frequencies: $(0^0, +90^0)$;
- pair of complex pole , $\frac{1}{\frac{s}{\omega_n^2}+\frac{2\xi s}{\omega_n}+1}$, with magnitude: a slope of -40dB after the corner frequency, ω_n, and phase having 3 points of interest: $(0.1\omega_n, -10^0), (\omega_n, -90^0), (10\omega_n, -170^0)$, whit an entire phase shift over the positive range of frequencies: $(0^0, -180^0)$;
- pair of complex zero , $\frac{s}{\omega_n^2}+\frac{2\xi s}{\omega_n}+1$, with magnitude: a slope of +40dB after the corner frequency, ω_n, and phase having 3 points of interest: $(0.1\omega_n, +10^0), (\omega_n, +90^0), (10\omega_n, +170^0)$, whit an entire phase shift over the positive range of frequencies: $(0^0, +180^0)$;
- pole in the origin, $\frac{1}{s}$, with the magnitude: a slope of -20dB with the cutting frequency in 1 rad/sec and a phase shift of -90^0 for the entire positive range of frequencies;
- pole in the origin, s, with the magnitude: a slope of +20dB with the cutting frequency in 1 rad/sec and a phase shift of $+90^0$ for the entire positive range of frequencies;

Examples of solved problems

Establish a set of sufficient points to conveniently approximate by asymptotes, the magnitude and phase characteristics (Bode diagram) for the next transfer functions, $H(s) = 2\frac{s+20}{s+70}$.

Step 1: bring the terms in the transfer function to their standard form: $H(s) = \frac{4}{7}\frac{\frac{1}{20}s+1}{\frac{1}{70}s+1}$;

Step 2: place in ascending order the corner frequencies and mention the contribution of the magnitude after each one: $\omega_1 = 20\ (+20dB), \omega_2 = 70(-20dB)$;

Step 3: establish the range of the frequencies, for getting a sufficient representation (graphically obtain the cutting frequency): consider one decade before the smallest corner frequency and one decade after the highest cutting frequency, $(10^0, 10^3)$;

Step 4: compute approximation points (for MAGNITUDE) according to the transfer function structure:

- from 10^0 to 20 rad/sec the magnitude is given by the proportional term : 20*log10(4/7)=20*(0.6-0.85)=-20*0.25=-5dB;
- from 20 to 70 rad/sec a slope of +20db occurs, resulting in 70 rad/sec an amplification of -5dB+20dB*(0.85-0.3)=-5dB+20dB*0.55=-5dB+11dB=6dB;
- after 70 rad/sec, the slope changes from +20dB to +20dB-20dB, resulting a slope of 0dB and the magnitude “remains” in +6dB for all the higher frequencies (higher than 70 rad/sec);
- the approximation points are: (1rad/sec,-5dB), (10rad/sec, -5dB),(20 rad/sec, -5dB), (70rad/sec, +6dB), (100rad/sec, +6dB), (1000rad/sec, +6dB);

Step 5: compute approximation points (for PHASE) according to the transfer function structure:

- the term determined by : $\omega_1 = 20\ (+20dB)$ will contribute with the next points on the phase: (2 rad/sec, $+5^0$), (20 rad/sec, $+45^0$), (200 rad/sec, $+85^0$);
- the term determined by : $\omega_2 = 70\ (+70dB)$ will contribute with the next points on the phase: (7 rad/sec, -5^0), (70 rad/sec, -45^0), (700 rad/sec, -85^0)
- the approximation points (in ascending order of the frequencies) are: (1rad/sec, 0^0), (2 rad/sec, $+5^0$), (7 rad/sec, $+20^0-5^0=+15^0$), (20 rad/sec, $+45^0-15^0=+30^0$), (70 rad/sec, $+70^0-45^0=25^0$), (200 rad/sec, $+85^0-70^0=+15^0$), (700 rad/sec, $+90^0-85^0=+5^0$); (1000 rad/sec, $+90^0-90^0=0^0$);

Step 6 draw on notebook the approximated Bode diagram.

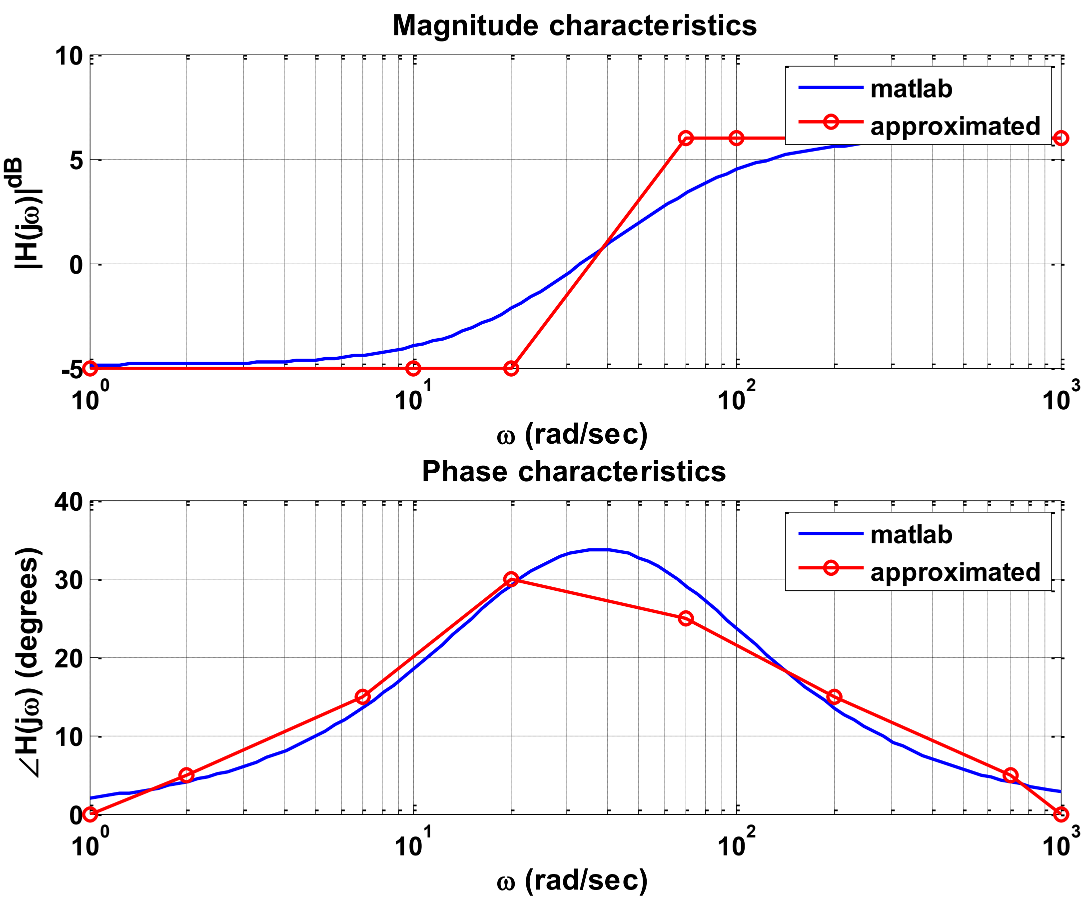
Step 8: estimate the cutting frequency (or frequencies)

Step 7 verify the accuracy of the approximation in Matlab; use the following script for plotting the approximation points of MAGNITUDE and PHASE characteristics

```
clear;clc
% approximations for magnitude
wma=[1 10 20 70 100 1e3];ma=[-5 -5 -5 6 6 6];
% approximations for phase
wfa=[1 2 7 20 70 200 700 1e3];fa=[0 5 15 30 25 15 5 0];
% plotting the approximations
subplot(211);semilogx(wma,ma,'ro-');grid
title('Magnitude characteristics');
xlabel('\omega (rad/sec)');ylabel('|H(j\omega)|^dB');
subplot(212);semilogx(wfa,fa,'ro-');grid;shg;
title('Phase characteristics');ylabel('\angleH(j\omega) (degres)');

%%
```

```
% comparison with Bode in Matlab
h=tf(2*[1 20],[1 70]);w=logspace(0,3,1e2);
[m,f]=bode(h, w);
mv(1:1e2,1)=m(:,:,:);fv(1:1e2,1)=f(:,:,:);
subplot(211);semilogx(w,20*log10(mv),'b',wma,ma,'ro-');grid
title('Magnitude characteristics');
xlabel('\omega (rad/sec)');ylabel('|H(j\omega)|^dB');
subplot(212);semilogx(w,fv,'b',wfa,fa,'ro-');grid;shg;
title('Phase characteristics');
xlabel('\omega (rad/sec)'); ylabel('\angleH(j\omega) (degres)');
```



Problems

Follow the same steps in order to approximate Bode diagrams for the next cases:

a) $H(s) = \frac{10}{s+3}$, b) $H(s) = 0.2\frac{s+70}{s+20}$, c) $H(s) = 2\frac{s}{s+7}$, d) $H(s) = \frac{20}{s(s+7)}$ e) $H(s) = \frac{5}{s(7s+1)}$,

f) $H(s) = \frac{75}{(s+1)(s+10)}$, g) $H(s) = 2\frac{s+2}{(3s+1)(2s+1)}$, h) $H(s) = 2\frac{10s+1}{(3s+1)(2s+1)}$, i) $H(s) = 20\frac{s+2}{(s+10)(s+5)}$