PHAS445 Quantum Field Theory R S Thorne

Problem Set 4

To be handed in by 11.30am, April 24th

1. Show that the anti-commutation relations for creation and annihilation operators in eq. (10.9) lead to the correct anti-commutation relations for the fields $\psi_\alpha(\mathbf{x},t)$ and $\pi_\beta(\mathbf{x},t)$. It will be useful to use the relationships in eq. (10.10). [8]

2. Verify the form of the Hamiltonian

$$H = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{2E}E\sum_{s=1,2}[b^\dagger(s,\mathbf{k})b(s,\mathbf{k}) + d^\dagger(s,\mathbf{k})d(s,\mathbf{k})].$$

Explain why commutation relations for ψ and π and therefore for b and d would have led to a disaster. [7]

3. By considering a photon moving along the z-axis show that

$$\langle\chi|a^\dagger(3,\mathbf{k})a(3,\mathbf{k}) - a^\dagger(0,\mathbf{k})a(0,\mathbf{k})|\chi\rangle = 0$$

results from

$$\langle\chi|\partial_\mu A^\mu|\chi\rangle = 0. \qquad [6]$$

4.Derive the trace results

$$\begin{aligned}
\mathrm{Tr}(\gamma^{\mu_1}\ldots\gamma^{\mu_n}) &= 0 \quad \text{for } n \text{ odd}\\
\mathrm{Tr}(\gamma^{\mu_1}\ldots\gamma^{\mu_n}) &= g^{\mu_1\mu_2}\mathrm{Tr}(\gamma^{\mu_3}\ldots\gamma^{\mu_n}) - g^{\mu_1\mu_3}\mathrm{Tr}(\gamma^{\mu_2}\gamma^{\mu_4}\ldots\gamma^{\mu_n}) + \cdots\\
&\quad + g^{\mu_1\mu_n}\mathrm{Tr}(\gamma^{\mu_2}\ldots\gamma^{\mu_{n-1}})\\
\mathrm{Tr}(\not{a}\not{b}) &= 4\,a\cdot b,\\
\mathrm{Tr}(\not{a}\not{b}\not{c}\not{d}) &= 4(a\cdot b\; c\cdot d - a\cdot c\; b\cdot d + a\cdot d\; b\cdot c).
\end{aligned}$$

(Hint: for the first one use $(\gamma^5)^2 = 1$.) [7]

5. Prove the result in eq. (11.13). It will be helpful first to prove

$$\begin{aligned}
\gamma^\alpha\gamma^\mu\gamma_\alpha &= -2\gamma^\mu\\
\gamma^\alpha\gamma^\mu\gamma^\nu\gamma_\alpha &= 4g^{\mu\nu}\\
\gamma^\alpha\gamma^\mu\gamma^\nu\gamma^\rho\gamma_\alpha &= -2\gamma^\rho\gamma^\nu\gamma^\mu,
\end{aligned}$$

and you may make use of the result in Section 11.1. [10]

6. Show that the spin summed/averaged squared matrix element for Compton scattering in the massless limit is given by

$$|\mathcal{M}_{fi}|^2 = 2e^4 \left(-\frac{u}{s} - \frac{s}{u} \right).$$

You may use the result that the matrix element squared for the s-channel diagram is $2e^4 \left(-\frac{u}{s}\right)$, but must derive explicitly the interference term contribution. Evaluate the total cross section using the expressions in the centre of mass frame in Section 8.4. Why does this create a problem? [9]

7. Why is it not possible in QCD to use the on-shell scheme? [3]