PHAS445 Quantum Field Theory R S Thorne

Problem Set 3

To be handed in by 5pm, March 15th

1. Prove that $|E_2\mathbf{p}_1 - E_1\mathbf{p}_2|^2 = (p_1 \cdot p_2)^2 - m_1^2 m_2^2$ in a frame where the momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ are collinear. [5]

Show that in the centre of mass frame the differential cross section for two particles of mass m scattering to two of mass M is,

$$\frac{d\sigma}{d\Omega^*} = \frac{\sqrt{1-4M^2/s}}{64\pi^2 s\,\sqrt{1-4m^2/s}} |\mathcal{M}_{fi}|^2. \qquad [10]$$

2. Using the fact that $\alpha_1\alpha_2\alpha_3 \equiv \frac{1}{3!}\epsilon_{ijk}\alpha_i\alpha_j\alpha_k$, or otherwise, verify the commutation relations

$$[\mathbf{\Sigma}, \beta] = 0, \qquad [\Sigma_i, \alpha_j] = 2i\varepsilon_{ijk}\alpha_k.$$

and the result

$$[\mathbf{\Sigma}, H_D] = -2i\boldsymbol{\alpha} \times \mathbf{p}.$$

(It might be useful to show that $\Sigma_i = -\frac{i}{2}\epsilon_{ijk}\alpha_j\alpha_k$.) [8]

3. Using the Dirac equation show that $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$. [4]

4. Verify that $s^{\rho\sigma}$ defined in eqs. (9.41) and (9.43) is indeed given by $s^{\rho\sigma} = \frac{i}{4}[\gamma^\rho, \gamma^\sigma] \equiv \frac{1}{2}\sigma^{\rho\sigma}$. [5]

Using the Hermiticity of the α_i and β matrices, verify that $\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$ and prove that $S^\dagger(\Lambda)\gamma^0 = \gamma^0 S^{-1}(\Lambda)$ is true. Hence, prove that $\bar{\psi}$ transforms as $\bar{\psi} \to \bar{\psi}\,' = \bar{\psi}S^{-1}(\Lambda)$. [6]

5. Show that $\bar{\psi}$ satisfies the equation

$$\bar{\psi}\,(-i\overleftarrow{\not\partial} - m) = 0$$

where the arrow over $\not\partial$ implies the derivative acts on $\bar{\psi}$. [4]

6. Verify the transformation properties of the bilinears representing the density ρ and the current $\mathbf{J}$ for the fermionic theory in eq. (9.11), and verify that they form the components of a four-vector. [4]

7. Prove that the spin operator $\mathbf{S}$ is indeed equal to $\frac{1}{2}\gamma^5\boldsymbol{\alpha}$. (Do not work explicitly in the Dirac representation.) [4]