PHASM(G)442
Particle Physics
Ruben Saakyan
Module 2

Symmetries and Conservation Laws

From:
http://www.phy.bris.ac.uk/groups/particle/PUS/A-level/CP_violation.htm

http://moodle.ucl.ac.uk/course/view.php?id=2589

Module 2

- Transformations
- Invariance
- Symmetries
- Conservation laws
- Groups in particle physics
- Discrete transformations:
  - Parity(P)
  - Charge conjugation(C)
  - CP

Symmetries

- Symmetries are very powerful - often the basis of a theory or invoked when theory is incomplete and they're intimately connected with conservation laws.
- Definition: A system has symmetry if it is unchanged (invariant) under a transformation
- Maths of describing symmetries is "group theory" e.g. the set of rotations we can perform on a system forms a group - each rotation is an element of the group (KCL's Math course "Lie Groups and Lie Algebras")
- Transformation group rules

  A) Closure: if R_i and R_j are in the set so is R_iR_j

  B) Identity: an element, I, exists such that $IR_i = R_iI = R_i$

  C) Inverse: for every R_i there is an R^{-1}_i such that

     - $R_i R^{-1}_i = R^{-1}_i R_i = I$

  D) Associativity: $R_i(R_jR_k)=(R_iR_j)R_k$

Noether’s Theorem

We require physics to be unchanged by a symmetry operation, U

If $$|\psi\rangle \rightarrow |\psi'\rangle = U|\psi\rangle \;\; \text{then} \;\; \frac{d\langle U\rangle}{dt} = 0$$

(see next slides for more)

Emmy Noether’s Theorem
“Every invariant symmetry transformation has an associated conservation law”

“the most significant creative mathematical genius thus far produced since the higher education of women began” : Albert Einstein.

- Suppose physics is invariant under a transformation (e.g. rotation of the coordinate axes)

$$\psi \to \psi' = \hat{U}\psi$$

- To conserve probability: $\langle \psi | \psi \rangle = \langle \psi' | \psi' \rangle = \langle \hat{U}\psi | \hat{U}\psi \rangle = \langle \psi | \hat{U}^\dagger \hat{U} | \psi \rangle$

$$\Rightarrow \boxed{\hat{U}^\dagger \hat{U} = 1}$$

i.e. $\hat{U}$ has to be unitary

- For physical predictions to be unchanged by the symmetry transformation also require all QM matrix elements unchanged

$$\langle \psi | \hat{H} | \psi \rangle = \langle \psi' | \hat{H} | \psi' \rangle = \langle \psi | \hat{U}^\dagger \hat{H} \hat{U} | \psi \rangle$$

i.e. require $\hat{U}^\dagger \hat{H} \hat{U} = \hat{H}$

$$\times \hat{U} \qquad \hat{U}\hat{U}^\dagger \hat{H} \hat{U} = \hat{U}\hat{H} \Rightarrow \hat{H}\hat{U} = \hat{U}\hat{H}$$

Therefore

$$\boxed{\left[\hat{H}, \hat{U}\right] = 0 \qquad \hat{U} \text{ commutes with the Hamiltonian}}$$

Now consider the infinitesimal transformation

$$\hat{U} = 1 + i\varepsilon\hat{G}$$

$\hat{G}$ is the **generator** of the transformation

$$\hat{U}\hat{U}^\dagger = \left(1 + i\varepsilon\hat{G}\right)\left(1 - i\varepsilon\hat{G}^\dagger\right) = 1 + i\varepsilon\left(\hat{G} - \hat{G}^\dagger\right) + O(\varepsilon^2) = 1$$

$$\Rightarrow \boxed{G = \hat{G}^\dagger}$$

Furthermore, $\left[\hat{H}, \hat{U}\right] = 0 \Rightarrow \left[\hat{H}, 1 + i\varepsilon\hat{G}\right] = 0 \Rightarrow \left[\hat{H}, \hat{G}\right] = 0$

But from QM $\quad \dfrac{d}{dt}\left\langle \hat{G} \right\rangle = i\left\langle \left[\hat{H}, \hat{G}\right] \right\rangle = 0$

i.e. G is a **conserved** quantity

Symmetry $\Longleftrightarrow$ **Conservation Law**

For each symmetry there is an <u>observable conserved</u> quantity
⇒ Noether's Theorem

Examples:

- Time translation ⇒ Energy Conservation
- Space translation ⇒ Momentum Conservation
- EM phase ("gauge") translation ⇒ Charge Conservation

Groups in Particle Physics

- Unitary groups : U(n) $U^\dagger U = 1$
- Special Unitary Groups : SU(n) : + $\det U = 1$
- Special Real Orthogonal Groups : SO(n) : $\det U = 1$ + all elements real

- Examples:
  - U(1) --> Describes symmetries of QED interactions
  - SU(2) --> Describes symmetries of Weak interactions
  - SU(3) --> Describes symmetries of QCD interactions

- SU(N) has N^2-1 parameters
  - SU(2) = 3 (W^+, W^-, Z)
  - SU(3) = 8 (8 gluons)
  - SO(32) = ?

INFINITY CANCELLATIONS IN SO(32) SUPERSTRING THEORY ☆

Michael B. GREEN [1] and John H. SCHWARZ
California Institute of Technology, Pasadena, CA 91125, USA

Received 1 October 1984

Parity

- Non-continuous (discrete) transformation
- Parity (P) operator : space inversion :

$$P(\vec{r}) = -\vec{r}$$

- For more complex vectors other than the position vector the parity operator can flip or retain the sign e.g.
- $P(\vec{L}) = \vec{L}; \quad \vec{L} = \vec{r} \times \vec{p}$ has eigenvalue +1 i.e. is "even" under the parity operator
- Definitions

Vector	$P\vec{\alpha} = -\vec{\alpha}$
Axial-Vector	$P\vec{\alpha} = +\vec{\alpha}$
Scalar	$P(S) = +S$
Pseudo-scalar	$P(S) = -S$

More Parity

- Intrinsic parity of particles
  - Fermions +1
  - Antifermions -1
  - Bosons same parity as antiparticles (-1 for gluon and photon)
  - Composite particles $P=(-1)^L$ (L= relative angular momentum)

- Parity is conserved in EM and strong interactions but violated maximally in weak interactions : example pion decay

More on this when we'll discuss weak interactions (Module 7)

Charge Conjugation

- Charge Conjugation Operator (C) : more than just charge, actually flips all non momenta (spin, L) values : charge, colour, lepton-# etc and so converts a particle to anti-particle.

$$C|X\rangle = c|\overline{X}\rangle;\ c^2 = 1$$

- But there aren't so many particles where particle = anti-particle except e.g. γ, π^0 and so concept of limited use.
- Again it is conserved in EM & strong interactions but not weak (e.g. pion decay)
- The more interesting operator is the combined "CP" operator. It is a more relevant matter to anti-matter operator. Given we know matter predominates in our universe then we know CP cannot be conserved in all weak interactions (although it is in pion decay)

CP Violation

- Sakharov conditions for matter preponderance (i.e. life):
  – B number violation (not yet observed)
  – C violation (observed in weak decays)
  – CP violation (observed in weak decays, but very small amount)
  – Rate of matter generating reactions less than rate of universe expansion (need to avoid thermal equilibrium)

- CP violation has been observed in a handful of weak interaction decays
  – strange meson (kaon) decays (1964) ~ 2×10^{-3}
  – B-meson decays (2001) ~ 10^{-4}
  – C-meson decays (2011?)

More on this when we'll discuss weak interactions (Module 7)

- **CPT** is a fundamental symmetry conserved in **all** interactions

Evidence for *CP* Violation in Time-Integrated $D^0 \to h^- h^+$ Decay Rates

R. Aaij *et al.**
(LHCb Collaboration)
(Received 6 December 2011; published 12 March 2012; publisher error corrected 12 March 2012)

The Mythical Axion

- Finally, while it is observed that strong interactions do not violate CP (e.g. no discernible neutron electric dipole moment has been measured) there is no a priori reason from the symmetries/structure of QCD why this should be so (unlike QED) and indeed the QCD Lagrangian has been “fiddled” such that CP violation is zero by adding a new particle (named after a brand of detergent) - the axion - that cleans up QCD.
- This particle, has yet to be observed, although evidence for it was presented by the PVLAS collaboration in Dec 2006 by using an axion property that it should change into a photon in the presence of a large magnetic field…. and then retracted in 2007.