PHAS445 Quantum Field Theory R S Thorne

Problem Set 2

To be handed in by 5pm, March 1st

1. Prove that

$$\int d^3\mathbf{x}\big((-\partial_0 e^{-ip\cdot x})\phi(x) + e^{-ip\cdot x}(\partial_0\phi(x))\big) = ia^\dagger_{\mathbf{p}}.$$

and that

$$\int d^3\mathbf{x}\big((-\partial_0 e^{+ip\cdot x})\phi(x) + e^{+ip\cdot x}(\partial_0\phi(x))\big) = -ia_{\mathbf{p}}. \qquad [10]$$

2. Prove Wick's theorem for 3 operators, i.e eq. (7.8). [8]

3. Consider a scalar field theory defined by the interaction term $: g/3!\phi_I^3(x) :$. Draw, and write x space expressions for the connected diagrams up to order g^3. How do you interpret the order g^2 diagrams physically? What is fundamentally wrong with this quantum (or classical) field theory? [15]

4. In the integral over internal momentum for the first of the 1-loop four point functions discussed in the lecture look at the region where $|k_1| \to \infty$. Is there a problem with this integral? [4]

5. For the $\lambda\phi^4$ scalar field theory draw all the truncated 2-point diagrams up to $\mathcal{O}(\lambda^2)$, and write down the corresponding momentum space expressions (without performing the integrals). [6]

6. Two particles of four-momenta p_a and p_b, and masses m_a and m_b respectively scatter to two final state particles of momenta p_c and p_d with masses m_c and m_d respectively. We define relativistic invariants $s = (p_a + p_b)^2$, $t = (p_a - p_c)^2$ and $u = (p_a - p_d)^2$. Show that $s + t + u = m_a^2 + m_b^2 + m_c^2 + m_d^2$. Also show that if all masses are equal to m then $s \geq 4m^2$, $t \leq 0$ and $u \leq 0$ (since s, t, u are invariant it will help to pick a convenient frame). [7]