PHASM/G442 Particle Physics

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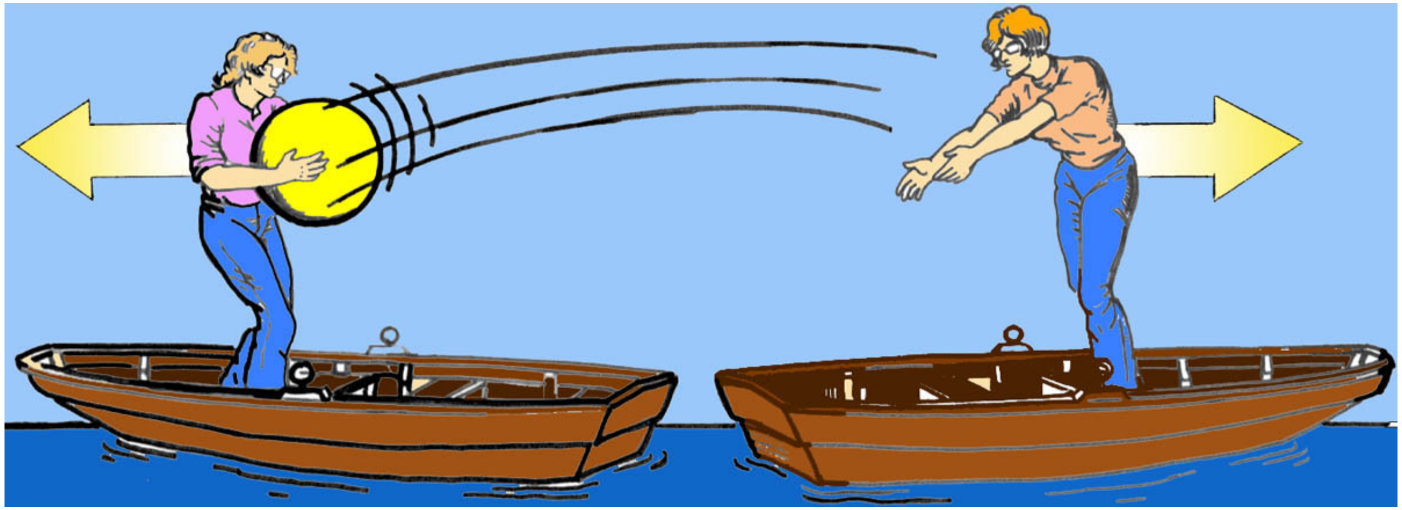
Module IV

Interaction by Particle Exchange



http://moodle.ucl.ac.uk/course/view.php?id=2589

Intro

- In the first module we used the Fermi Golden Rule to arrive at a generic expression for the cross-section

$$\sigma \propto \frac{|M|^2}{flux} \times (phase\, space)$$

  and calculated the phase space for a 2→2 process

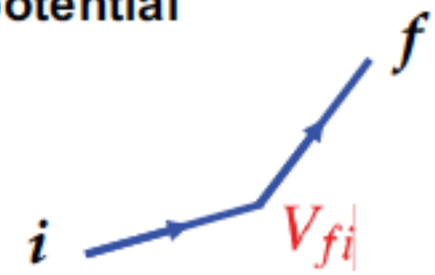
- In Module III we have covered **relativistic** treatment of spin-1/2 particles (Dirac Equation)

- It is now time to tackle the **matrix element**, $\boldsymbol{M}$. In this module:
  - Interaction by particle exchange — “Toy” model
  - Dirac + Maxwell ⇒ QED
  - Feynman diagrams
  - Feynman Rules (for QED)
  - Lagrangian Formalism, gauge invariance and interactions

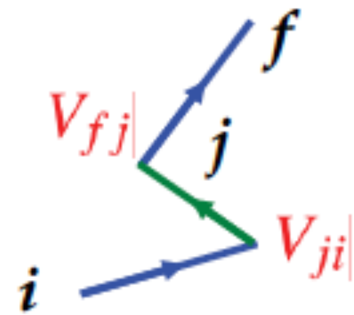
Classical vs QFT picture

Recall **FGR** $\Gamma_{fi} = 2\pi |T_{fi}|^2 \rho(E_f)$

"scattering in a potential"



$$T_{fi} = \langle f | V | i \rangle$$



"scattering via an intermediate state"

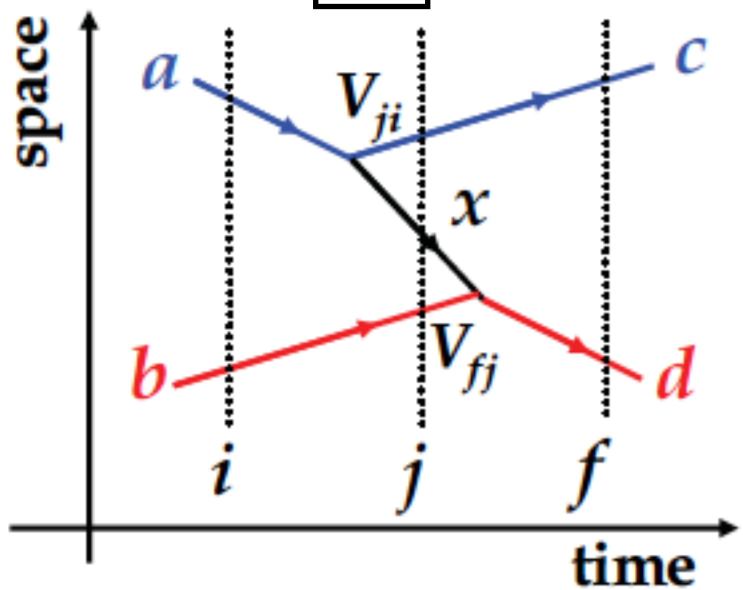
$$T_{fi} = \sum_{j \neq i} \frac{\langle f | V | j \rangle \langle j | V | i \rangle}{E_i - E_j} + \dots$$

<u>Classical picture</u>:
Scattering in a potential, **action at a distance**

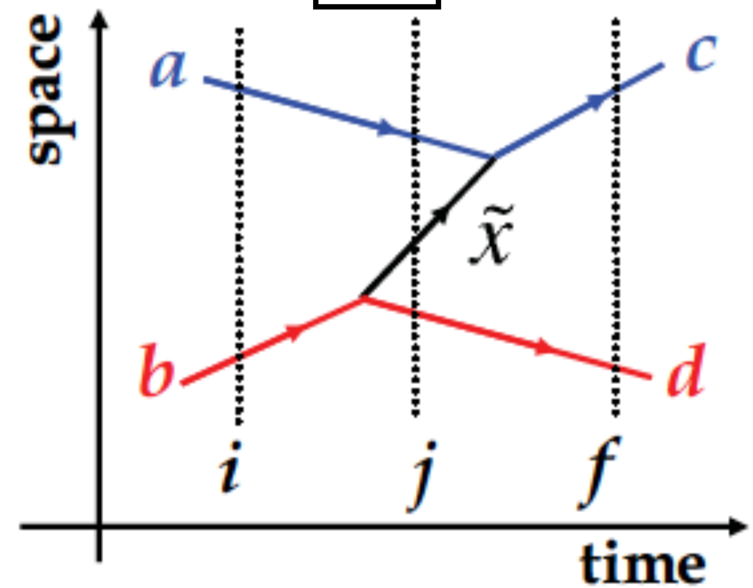
<u>QFT picture</u>:
Forces due to **exchange of virtual particles**. No action at a distance

Time-ordered perturbation theory

$\boxed{T_{fi}^{ab}}$



$\boxed{T_{fi}^{ba}}$



For T_{fi}^{ab}

$$T_{fi}^{ab} = \frac{\langle d|V|x+b\rangle\langle c+x|V|a\rangle}{(E_a+E_b)-(E_c+E_x+E_b)}$$

4a

Taking into account **relativistic** wave function normalisation$\left(\sqrt{E}\right)$

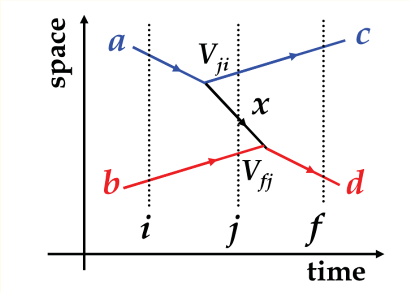
and a **scalar** being the simplest L.I. quantity: $\langle c+x|V|a\rangle = \frac{g_a}{(2E_a 2E_c 2E_x)^{1/2}}$

Same for $\langle d|V|x+b\rangle$

4b

Thus $$T_{fi}^{ab} = \frac{1}{2E_x}\cdot\frac{1}{(2E_a 2E_b 2E_c 2E_d)^{1/2}}\cdot\frac{g_a g_b}{(E_a-E_c-E_x)}$$

Interaction $a+b \to c+d$



Initial state i : $a+b$

Final f : $c+d$

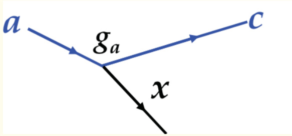
Intermediate j : $c+b+x$

$$T_{fi} = \frac{\langle f|V|j\rangle\langle j|V|i\rangle}{E_i - E_j}$$

$$T_{fi} = \prod_k \frac{1}{\sqrt{2E_k}} M_{fi}$$

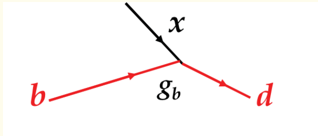
$$T_{fi} = \frac{\langle d|V|x+b\rangle\langle c+x|V|a\rangle}{(E_a+E_b) - (E_c+E_x+E_b)}$$

$$\langle c+x|V|a\rangle = \frac{\mathcal{M}(a\to c+x)}{\sqrt{2E_a\, 2E_c\, 2E_x}}$$



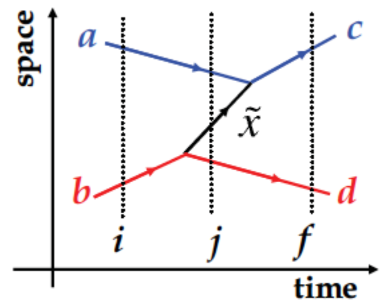
$$\langle c+x|V|a\rangle = \frac{g_a}{\sqrt{2E_a\, 2E_c\, 2E_x}}$$

$$\langle d|V|x+b\rangle = \frac{g_b}{\sqrt{2E_b\, 2E_d\, 2E_x}}$$



$$M_{fi}^{ab} = (2E_a 2E_b 2E_c 2E_d)^{1/2} T_{fi}^{ab} = \frac{1}{2E_x} \cdot \frac{g_a g_b}{(E_a - E_c - E_x)}$$

For the *"ba"* process

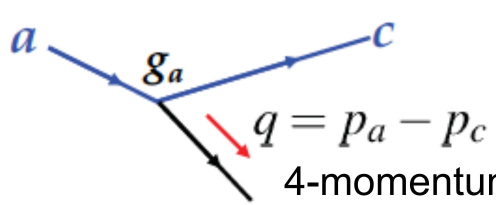


$$M_{fi}^{ba} = \frac{1}{2E_x} \cdot \frac{g_a g_b}{(E_b - E_d - E_x)}$$

$$M_{fi} = M_{fi}^{ab} + M_{fi}^{ba} = \frac{g_a g_b}{2E_x} \cdot \left(\frac{1}{E_a - E_c - E_x} + \frac{1}{E_b - E_d - E_x} \right)$$

$$= \frac{g_a g_b}{(E_a - E_c)^2 - E_x^2} = \frac{g_a g_b}{(E_a - E_c)^2 - (\vec{p}_a - \vec{p}_c)^2 - m_x^2} = \frac{g_a g_b}{(p_a - p_c)^2 - m_x^2}$$

$\to 5_{a,b}$



4-momentum transfer

$$\boxed{M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}}$$

Summing over all time orderings gives M_{fi} which is **Lorentz Invariant**!

$$M_{fi} = M_{fi}^{ab} + M_{fi}^{ba} = \frac{g_a g_b}{2E_x}\left(\frac{1}{E_a - E_c - E_x} + \frac{1}{\underbrace{E_b - E_d} - E_x}\right)$$

From energy conservation: $E_a + E_b = E_c + E_d$

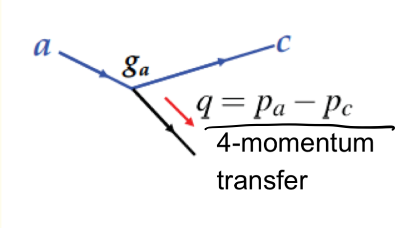
$\underbrace{E_b - E_d = E_c - E_a}$

$$M_{fi} = \frac{g_a g_b}{2E_x}\left(\frac{1}{E_a - E_c - E_x} - \frac{1}{E_a - E_c + E_x}\right)$$

$$M_{fi} = \frac{g_a g_b}{2E_x}\left(\frac{\cancel{E_a} - \cancel{E_c} + E_x - \cancel{E_a} + \cancel{E_c} + E_x}{(E_a - E_c)^2 - E_x^2}\right) =$$

$$= \frac{g_a g_b}{\cancel{2E_x}} \frac{\cancel{2E_x}}{(E_a - E_c)^2 - E_x^2} = \frac{g_a g_b}{(E_a - E_c)^2 - E_x^2}$$

$$E_x^2 = \vec{p}_x^{\,2} + m_x^2 = (\vec{p}_a - \vec{p}_c)^2 + m_x^2$$



$$M_{fi} = \frac{g_a g_b}{\underbrace{(E_a - E_c)^2 - (\vec{p}_a - \vec{p}_c)^2}_{(p_a - p_c)^2} - m_x^2}$$

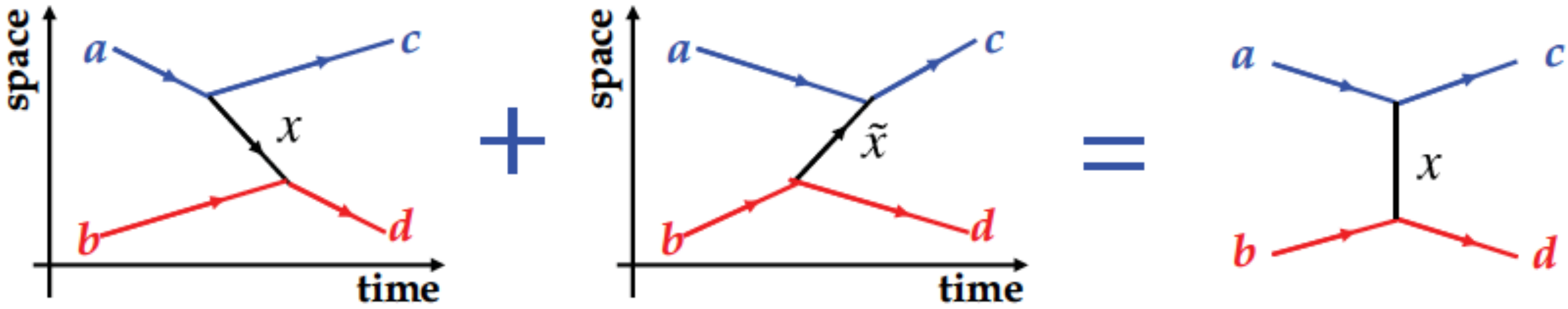
$q = p_a - p_c$

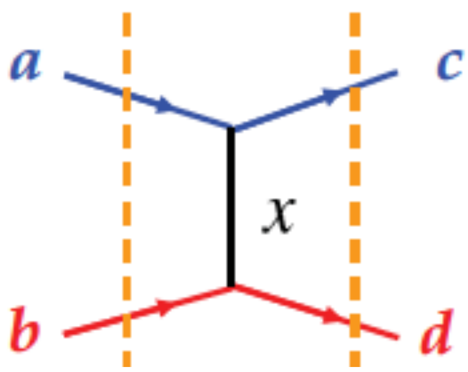
$\Rightarrow$ 4-vectors

$$\boxed{M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}}$$

Feynman Diagrams

Sum over all time orderings is represented by **Feynman Diagrams**



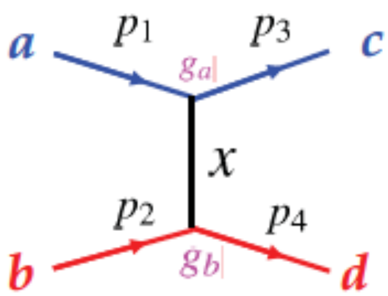


- LHS — initial state
- RHS — final state
- Everything in between is how interaction happened
- Propagator: $1/(q^2 - m_x^2)$

Matrix Element

$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$ depends on

- Fundamental strength of interaction at two vertices: g_a, g_b
- The 4-momentum q carried by the virtual particle — ***momentum transfer***

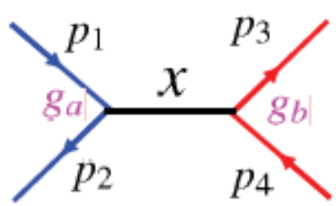


Here $q = p_1 - p_3 = p_4 - p_2 = t$ **"t-channel"**

For elastic scattering: $p_1 = (E, \vec{p}_1);\quad p_3 = (E, \vec{p}_3)$

$$q^2 = (E-E)^2 - (\vec{p}_1 - \vec{p}_3)^2$$

$\boxed{q^2 < 0}$ **termed "space-like"**



Here $q = p_1 + p_2 = p_3 + p_4 = s$ **"s-channel"**

In CoM: $p_1 = (E, \vec{p});\quad p_2 = (E, -\vec{p})$

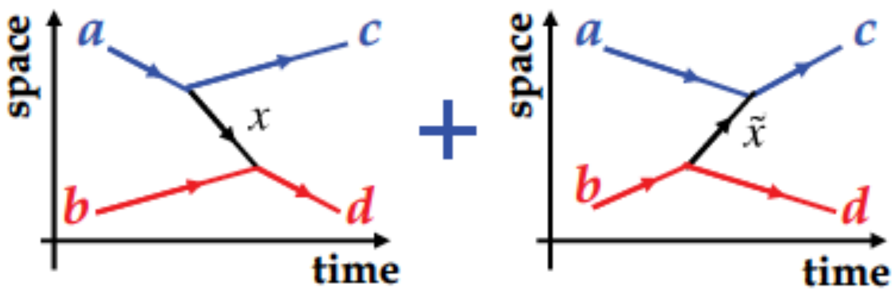
$$q^2 = (E+E)^2 - (\vec{p} - \vec{p})^2 = 4E^2$$

$\boxed{q^2 > 0}$ **termed "time-like"**

N.B. The above matrix element is for a "*toy*" **scalar interaction**

Virtual Particles

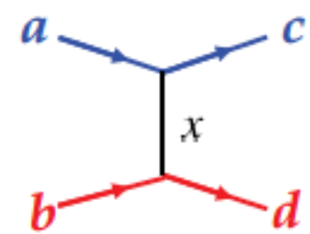
"Time-ordered QM"



- Momentum conserved at vertices
- Energy **not** conserved at vertices
- Exchanged particle **"on mass shell"**

$$E_x^2 - |\vec{p}_x|^2 = m_x^2$$

Feynman diagram



$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$

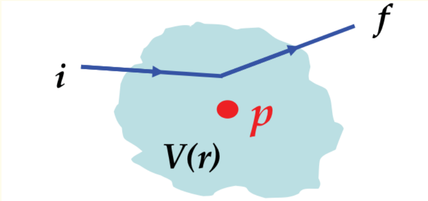
- Momentum **AND** energy conserved at interaction vertices
- Exchanged particle **"off mass shell"**

$$E_x^2 - |\vec{p}_x|^2 = q^2 \neq m_x^2$$

VIRTUAL PARTICLE

Particle exchange vs Semi-classical picture

$$M = \langle \psi_f | V | \psi_i \rangle$$



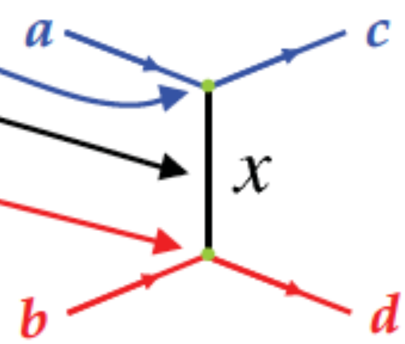
$$V(r) = g_a g_b \frac{e^{-mr}}{r}$$

$$\Downarrow$$

$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$

From Toy Model to QED

$$M = \underbrace{\langle \psi_c | V | \psi_a \rangle}_{= g_a} \frac{1}{q^2 - m_x^2} \underbrace{\langle \psi_d | V | \psi_b \rangle}_{= g_b}$$



- Time to go from toy models to something real, QED. Spin-1/2 fermions interact by exchanging spin-1 photon

- Need to find an appropriate expression for the potential

- Electrodynamics: interaction between photon and charged particles are introduced by making minimal substitution

$$\vec{p} \rightarrow \vec{p} - q\vec{A}; \quad E \rightarrow E - q\phi$$

Recall that in QM $\vec{p} = -i\vec{\nabla}; \quad E = i\partial/\partial t$

- The **D.E.** can be rewritten

$$\hat{H}\psi = \underbrace{(\gamma^0 m - i\gamma^0\vec{\gamma}.\vec{\nabla})\psi}_{\text{Combined rest mass + K.E.}} + \underbrace{q\gamma^0\gamma^\mu A_\mu \psi}_{\text{Potential energy}} \rightarrow 10_q$$

$$A_\mu = (\phi, -\vec{A})$$

- So potential energy of a charged (q) spin-1/2 particle in an EM field:

$$\boxed{\hat{V}_D = q\gamma^0\gamma^\mu A_\mu}$$

- Accounting for photon polarisation states:

$$A_\mu = \varepsilon_\mu^{(\lambda)} e^{i(\vec{p}.\vec{r} - Et)}$$

$$\varepsilon^{(1)} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}; \quad \varepsilon^{(2)} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$A_\mu(\Phi, -\vec{A})$; $\partial_\mu\left(\frac{\partial}{\partial t}, +\vec{\nabla}\right)$

$\vec{p} \to \vec{p} - q\vec{A}; \quad E \to E - q\phi$

$$i\partial_\mu \to i\partial_\mu - qA_\mu$$

D.E. $(i\gamma^\mu\partial_\mu - m)\psi = 0$

$$\gamma^\mu(i\partial_\mu - qA_\mu)\psi - m\psi = 0$$

$$\underbrace{i\gamma^\mu\partial_\mu\psi} - q\gamma^\mu A_\mu\psi - m\psi = 0$$

$$\underbrace{i\gamma^0\frac{\partial\psi}{\partial t} + i\vec{\gamma}\cdot\vec{\nabla}\psi} - q\gamma^\mu A_\mu\psi - m\psi = 0$$

$$\xrightarrow{\times\gamma^0} i\gamma^0\frac{\partial\psi}{\partial t} = \gamma^0\hat{H}\psi = m\psi - i\vec{\gamma}\cdot\vec{\nabla}\psi + q\gamma^\mu A_\mu\psi$$

$$\hat{H}\psi = (\gamma^0 m - i\gamma^0\vec{\gamma}\cdot\vec{\nabla})\psi + q\gamma^0\gamma^\mu A_\mu\psi$$

Recall: $(\vec{\alpha}\cdot\vec{p} + \beta m)\psi = i\frac{\partial\psi}{\partial t} \Rightarrow$

$$\Rightarrow (\beta m - i\vec{\alpha}\cdot\vec{\nabla})\psi$$

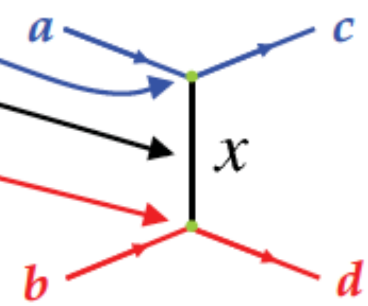
$$\hat{H}\psi = \underbrace{(\gamma^0 m - i\gamma^0\vec{\gamma}\cdot\vec{\nabla})\psi}_{\text{K.E + rest mass}} + \underbrace{q\gamma^0\gamma^\mu A_\mu\psi}_{\text{potential energy}}$$

$$\hat{H}\psi = \hat{P}\psi + \hat{U}\psi$$

From Toy Model to QED

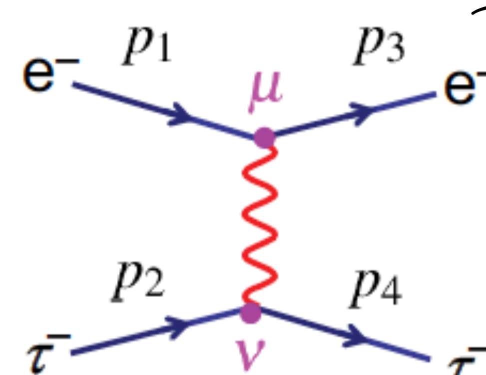
- Using the analogy with the toy model

$$M = \underset{\underset{g_a}{\parallel}}{\langle \psi_c | V | \psi_a \rangle} \frac{1}{q^2 - m_x^2} \underset{\underset{g_b}{\parallel}}{\langle \psi_d | V | \psi_b \rangle}$$



We obtain for QED

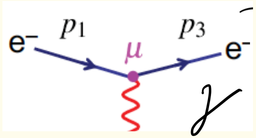
$$M = \underbrace{\left[u_e^\dagger(p_3) q_e \gamma^0 \gamma^\mu u_e(p_1)\right]}_{\text{Interaction of electron with photon}} \underbrace{\sum_\lambda \frac{\varepsilon_\mu^\lambda (\varepsilon_\nu^\lambda)^*}{q^2}}_{\text{Massless photon propagator, summing over polarisations}} \overbrace{\left[u_\tau^\dagger(p_4) q_\tau \gamma^0 \gamma^\nu u_\tau(p_2)\right]}^{\text{Interaction of tau-lepton with photon}}$$



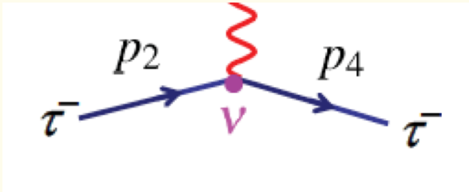
→ 11b

$$\boxed{\hat{V}_D = q\gamma^0\gamma^\mu A_\mu}$$

$$\langle \psi(p_3)|\hat{V}|\psi(p_1)\rangle = u_e^\dagger(p_3)\,\underbrace{q_e\,\gamma^0\gamma^\mu\varepsilon_\mu^{(\lambda)}}\,u_e(p_1)$$



$$= u_\tau^\dagger(p_4)\,\underbrace{q_\tau\,\gamma^0\gamma^\nu\varepsilon_\nu^{(\lambda)}}\,u_\tau(p_2)$$



- One can show that $\sum_{\lambda} \varepsilon_{\mu}^{\lambda} (\varepsilon_{\nu}^{\lambda})^{*} = -g_{\mu\nu}$

- Using definition of adjoint spinor $\overline{\psi} = \psi^{\dagger}\gamma^{0}$ we then arrive at

$$\boxed{M = \underbrace{[\overline{u}_e(p_3) q_e \gamma^{\mu} u_e(p_1)]}_{j_e^{\mu}} \frac{-g_{\mu\nu}}{q^2} \underbrace{[\overline{u}_\tau(p_4) q_\tau \gamma^{\nu} u_\tau(p_2)]}_{j_\tau^{\nu}}}$$

$\Rightarrow j_e^{\mu} g_{\mu\nu} j_\tau^{\nu} =$

$= j_e^{\mu} j_{\tau\nu} \equiv j_e \cdot j_\tau$

which can be also written as

$$M = -q_e q_\tau \frac{j_e \cdot j_\tau}{q^2}$$

Lorentz Invariant!

All physics of QED is in this expression! It hides a lot of complexity (e.g. time-orderings, polarisation states of virtual photon)

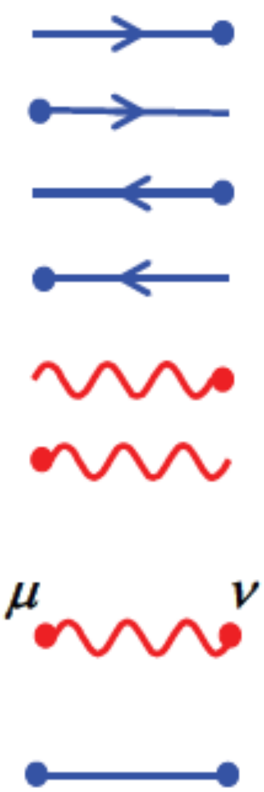
Feynman Fules for QED

- **External Lines**

spin 1/2	incoming particle	$u(p)$	
	outgoing particle	$\bar{u}(p)$	
	incoming antiparticle	$\bar{v}(p)$	
	outgoing antiparticle	$v(p)$	
spin 1	incoming photon	$\varepsilon^{\mu}(p)$	
	outgoing photon	$\varepsilon^{\mu}(p)^{*}$	

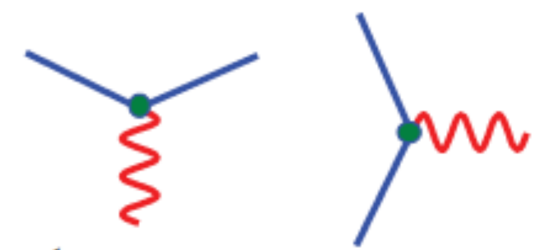
- **Internal Lines (propagators)**

spin 1	photon	$-\dfrac{ig_{\mu\nu}}{q^2}$	
spin 1/2	fermion	$\dfrac{i(\gamma^{\mu}q_{\mu}+m)}{q^2-m^2}$	



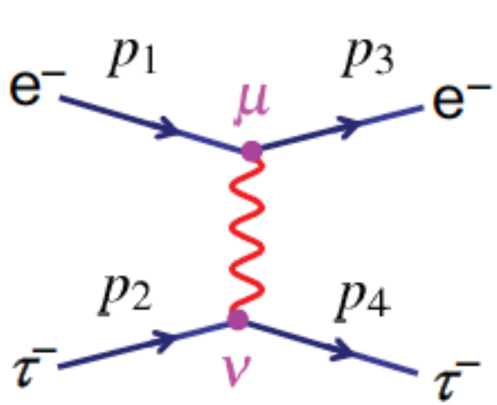
- **Vertex Factors**

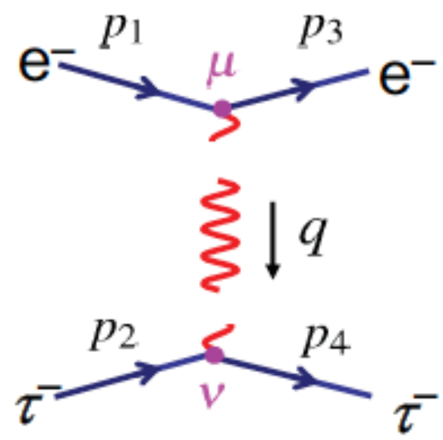
spin 1/2 fermion (charge -|e|) $ie\gamma^{\mu}$



- **Matrix Element** $-iM$ = product of all factors

- For t-channel:





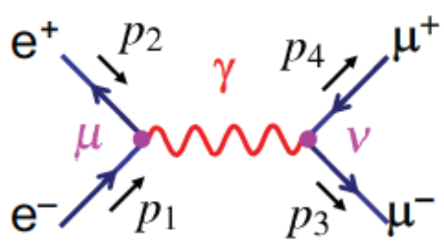
$$\overbrace{\overline{u}_e(p_3)[ie\gamma^\mu]u_e(p_1)}^{j_e}$$

$$\frac{-ig_{\mu\nu}}{q^2}$$

$$\left.\overline{u}_\tau(p_4)[ie\gamma^\nu]u_\tau(p_2)\right\} j_\tau$$

$$-iM = [\overline{u}_e(p_3)ie\gamma^\mu u_e(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}_\tau(p_4)ie\gamma^\nu u_\tau(p_2)]$$

- For s-channel:



$$-iM = [\overline{v}(p_2)ie\gamma^\mu u(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\overline{u}(p_3)ie\gamma^\nu v(p_4)]$$

NOTE:

- At each vertex the **adjoint spinor is written first**
- Each vertex has a different index
- $g_{\mu\nu}$ connects indices at vertices

Summary

- Time-ordered perturbation theory + particle exchange gives **Lorentz Invariant Matrix Element**

$$M_{fi} = \frac{g_a g_b}{q^2 - m_x^2}$$

- "Derived" basic interaction in **QED** taking into account spins of the fermions and polarisations of virtual photon — **Feynman Rules**

$$-iM = [\overline{u}(p_3) ie\gamma^\mu u(p_1)] \frac{-ig_{\mu\nu}}{q^2} [\overline{u}(p_4) ie\gamma^\nu u(p_2)]$$

We are now ready to calculate realistic QED processes

Aside: Lagrangian Formalism

- Before proceeding with calculations a (very brief) detour to Lagrangian Formalism — tool used in Field Theories
- In Classical Field Theory it is used to derive equations of motion
- The underlying fundamental concept is the **Principle of Least Action**

$$S(\vec{q}(t)) = \int_{t_1}^{t_2} L\left(\vec{q}(t), \dot{\vec{q}}(t), t\right) dt \quad S - \text{action}, L - \text{Lagrangian}, \vec{q} - \text{generalized coordinates}$$

Least Action Principle: $\delta S = 0$

From which **Euler-Lagrange** equations follow: $\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0}$

E.g. in classical mechanics :

$L = T - V$ **E.-L.** leads to Newton's 2nd law $m\ddot{x} = -\frac{\partial V(x)}{\partial x} = F$ $\longrightarrow$ 16a

$$\text{E-L:}\ \underbrace{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)} - \underline{\underline{\frac{\partial L}{\partial x}}} = 0$$

$$L = T - V = \frac{m\dot{x}^2}{2} - V(x) \Rightarrow \text{non relat.}$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x}$$

$$\frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x} = F \Rightarrow F = m\ddot{x}$$

Newton 2nd law

Lagrangian Formalism

- Lagrangian treatment of a discrete system of particles can be extended to a continuous system by replacing coordinates with fields and Lagrangian with Lagrangian density

$$L\left(q_i, \frac{dq_i}{dt}\right) \rightarrow L(\phi_i, \partial_\mu \phi_i) \qquad \text{with} \qquad \partial_\mu \phi_i \equiv \frac{\partial \phi_i}{\partial x^\mu}$$

- The **E.-L.** becomes

$$\partial_\mu \left(\frac{\partial L}{\partial \left(\partial_\mu \phi_i \right)} \right) - \frac{\partial L}{\partial \phi_i} = 0$$

- *Relativistic* Lagrangian examples

scalar

$$L_S = \frac{1}{2}\left(\partial_\mu \phi\right)\left(\partial^\mu \phi\right) - \frac{1}{2} m^2 \phi^2$$

$$\Big\Downarrow \ \boxed{\text{E.-L.}}$$

$$\partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

Klein-Gordon

spin-1/2

$$L_D = i\bar{\psi}\gamma^\mu \partial_\mu \psi - m\bar{\psi}\psi$$

$$\Big\Downarrow \ \boxed{\text{E.-L.}}$$

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0$$

Dirac

spin-1

$$L_{EM} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - j^\mu A_\mu$$

= 0 in absence of sources

$$\Big\Downarrow \ \boxed{\text{E.-L.}}$$

$$\partial_\mu F^{\mu\nu} = j^\nu$$

Maxwell

$$\mathcal{L}_S = \underbrace{\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi)} - \frac{1}{2} m^2 \phi^2$$

E-L:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \underbrace{\frac{\partial \mathcal{L}}{\partial \phi}} = 0$$

$$\frac{1}{2}\left((\partial_0 \phi)^2 - (\partial_1 \phi)^2 - (\partial_2 \phi)^2 - (\partial_3 \phi)^2 \right)$$

$$\Downarrow$$

$$\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = \partial^\mu \phi \qquad \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) = \partial_\mu \partial^\mu \phi$$

$$\Downarrow$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = -m^2 \phi \implies \partial_\mu \partial^\mu \phi + m^2 \phi = 0$$

K-G.

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Noether's Theorem (again)

- Noether's Theorem is formulated in terms of Lagrangian symmetry:
  - Lagrangian invariance under a certain transformation leads to a corresponding conserved quantity

E.g.: Rotational symmetry of the "Classical Mechanics" Lagrangian leads to conservation of angular momentum $\longrightarrow$ δq

- In Field Theory, Noether's theorem relates a symmetry of Lagrangian to a conserved current

E.g.: Lagrangian for free Dirac field

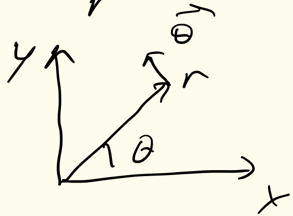
$L = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$ is unchanged by *global U(1)* transformation $\psi \rightarrow \psi' = e^{i\theta}\psi$

which leads to conserved 4-vector current $j^{\mu} = \bar{\psi}\gamma^{\mu}\psi$

i.e. $\partial_{\mu} j^{\mu} = 0$

$$L = T - V = \frac{1}{2} m v^2 - \frac{G \cdot M m}{r}$$

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$



$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2$$

$$L = \frac{1}{2} m\dot{r}^2 + \frac{1}{2} m r^2 \dot{\theta}^2 + G\frac{M \cdot m}{r}$$

L is invariant under $\theta \to \theta' = \theta + \delta\theta$

$$\frac{\partial L}{\partial \dot{\theta}} = m r^2 \dot{\theta} \qquad \frac{\partial L}{\partial \theta} = 0 \text{ due to}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0 \Rightarrow \frac{d}{dt}\underbrace{(m r^2 \dot{\theta})}_{J} = 0$$

Global gauge invariance

- The requirement that physics is unchanged by parameters we cannot measure (i.e. does not depend on the "gauge"). e.g. phase of a wave-function
- Classical example: we do not care about absolute potential, only potential difference (voltage)

$$\mathcal{L}_D = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi \; ; \; \psi(x) \to \psi'(x) = e^{iq\chi}\psi(x)$$

$$\mathcal{L}_D \to \mathcal{L}_D' = i\cancel{e^{-iq\chi}}\bar{\psi}\gamma^\mu\left[\cancel{e^{iq\chi}}\partial_\mu\psi + \cancel{e^{iq\chi}}\, iq(\partial_\mu\chi)\psi\right] -$$

$$- m\cancel{e^{-iq\chi}}\bar{\psi}\,\cancel{e^{iq\chi}}\psi = \underline{\underline{i\bar{\psi}\gamma^\mu\partial_\mu\psi}} - q\bar{\psi}\gamma^\mu(\partial_\mu\chi)\psi \underline{\underline{- m\bar{\psi}\psi}}$$

$$\mathcal{L}_D' = \mathcal{L}_D - \underbrace{q\bar{\psi}\gamma^\mu\cancel{(\partial_\mu\chi)}\psi}_{\to 0}$$

if χ is const

$$\mathcal{L}_D' = \mathcal{L}_D$$

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Local Gauge Invariance

- The requirement that physics is unchanged by parameters we cannot measure (i.e. does not depend on the "gauge"). e.g. phase of a wave-function

- Classical example: we do not care about absolute potential, only potential difference (voltage)

- Requires invariance of Lagrangian under a *local* phase transformation

$$\psi(x) \rightarrow \psi'(x) = e^{iq\chi(x)}\psi(x)$$

- However under this transformation Lagrangian for a free spin-1/2 particle is **not** invariant (check this in q~~7~~ 8, PS2)

  - To restore invariance use the substitution $\partial_\mu \rightarrow D_\mu = \partial_\mu + iqA_\mu$ where A_μ is a new field which transforms as $A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu \chi$

PS2. Q~~7~~ 8

- This leads to a new gauge invariant form of Lagrangian $L = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi - \boxed{q\bar{\psi}\gamma^\mu A_\mu\psi}$

interaction term

Gauge invariance provides framework for interaction by particle exchange!