PHASM/G 442. 2017 : Problem Sheet 3

Please return to Prof. Saakyan by the end of the lecture on December 7th 2017.

1. (a) Show that

$$\overline{u}_R \gamma^\mu v_R = 0; \qquad \overline{u}_L \gamma^\mu v_L = 0;$$

where $u_{R,L}$, $v_{R,L}$ are right(left) chiral projections of particle and antiparticle respectively.

(b) Comment on the significance of this result for EM interactions and its effect on the cross-section calculation for the $e^+e^- \to \mu^+\mu^-$ process in the lowest order of QED.

(c) Show explicitly that the other two chiral combinations are not zero. **[10]**

2. A standard representation of the positive energy free particle Dirac equation solution is

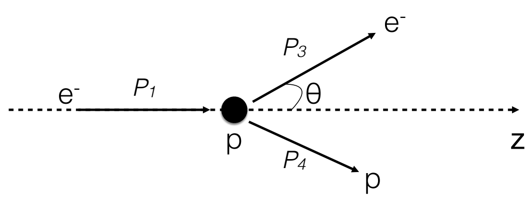
$$\psi^{0,1} = \sqrt{|E|+m} \begin{pmatrix} \chi^{0,1} \\ \left(\frac{\vec{\sigma}\cdot\vec{p}}{E+m}\right) \chi^{0,1} \end{pmatrix} e^{-ip_\mu x^\mu} \quad \text{where } \chi^0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } \chi^1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Show that with this choice of normalisation

$$\overline{\psi^s}\psi^s = 2m$$

for $s = 0, 1$. **[9]**

3. A relativistic electron with a four-momentum $P_1(E_1, \mathbf{p_1})$ is scattered elastically from a stationary proton of mass M as shown in the figure below:



(a) Neglecting the electron's mass, show that the energy of the scattered electron E_3 can be expressed as

$$E_3 = \frac{E_1 M}{M + E_1 - E_1 \cos\theta}$$

(b) In the above example the incident electron has energy $E_1 = 530$ MeV. The scattered electrons are detected at an angle of $\theta = 75°$. Find the corresponding value of the momentum transfer Q.

[9]

4. Use the Euler-Lagrange equation:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

separately for $\phi = \overline{\psi}$ and $\phi = \psi$ and the Lagrangian density:

$$\mathcal{L} = i\overline{\psi}\gamma^\mu \partial_\mu \psi - m\overline{\psi}\psi$$

to obtain the Dirac equation and :

$$i\gamma^\mu \partial_\mu \psi - m\psi = 0$$

an the adjoint Driac equation:

$$i\partial_\mu \overline{\psi}\gamma^\mu + m\overline{\psi} = 0.$$

[6]

5. For electron-proton and electron-neutron deep inelastic scattering, the structure functions are related to parton density functions $q^{p,n}(x)$ as

$$F_2^{ep}(x) = x \sum_q e_q^2 q^p(x) \qquad F_2^{en}(x) = x \sum_q e_q^2 q^n(x)$$

where x is the fraction of the nucleon momentum carried by the struck quark and e_q is the quark charge.

(a) Assuming that only u- and d- quarks contribute to the "sea" of virtual quarks, show that in the parton model

$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} \to 1 \text{ at } x \to 0, \quad \text{and} \quad \frac{F_2^{en}(x)}{F_2^{ep}(x)} \to 2/3 \text{ at } x \to 1$$

(b) Comment on agreement and disagreement of these predictions with experimental data. **[9]**

6. (a) Draw the Feynman diagrams showing the dominant leptonic decay mode of the μ^- and the τ^-.

(b) Assuming $m_\tau \gg m_\mu \gg m_e$, estimate the ratio of the rates of the two decay modes. **[7]**

Total: 50 marks