Model Answers for PS1

AY 2017/18

PHASM/G442

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Q1. [Total: 12 marks]

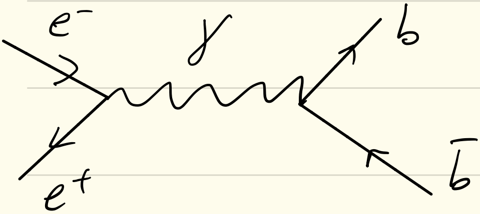
[marks]

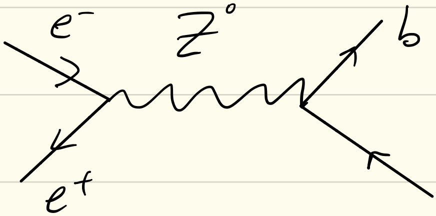
$\mu^- \to e^- + \bar{\nu}_e + \bar{\nu}_\mu$ Not Allowed

L_μ violated

[1]

$e^+ + e^- \to \bar{b} + b$

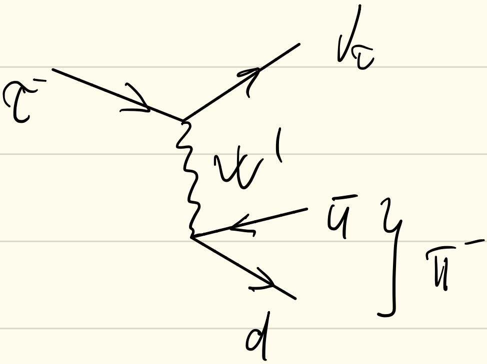




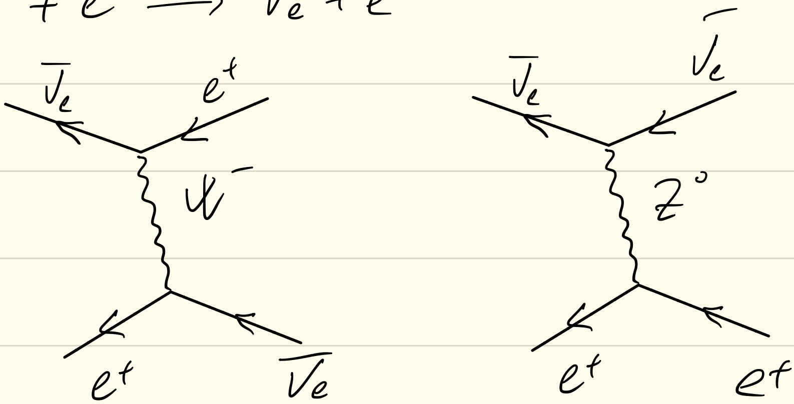
[2]

$\tau^- \to \nu_\tau + \pi^-$

[2]

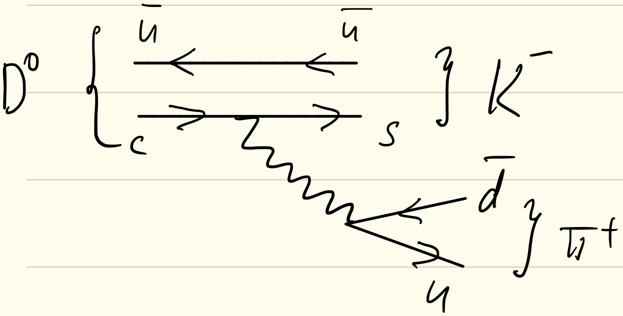


$\bar{\nu}_e + e^+ \rightarrow \bar{\nu}_e + e^+$



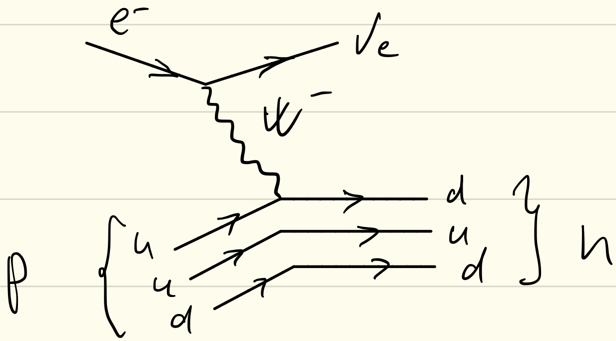
[3]

$D^0 \rightarrow K^- + \pi^+$



[2]

$e^- + p \rightarrow \nu_e + n$



[2]

Q.2 [Total: 5 marks]

From definition of the branching ratio:

$$B_R(K^+ \to \pi^+\pi^0) = \frac{\Gamma(K^+ \to \pi^+\pi^0)}{\Gamma(K^+ \to \text{everything})} =$$

$$= \Gamma(K^+ \to \pi^+\pi^0) \cdot \tau(K^+)$$ [2]

Expressing τ in $[eV^{-1}]$ and using conversion from Module 1, slide 16:

$$\tau[eV^{-1}] = \frac{1.2 \cdot 10^{-8}\,s}{6.6 \times 10^{-25} \times 10^{9}\,eV} = 0.18 \cdot 10^{8}\,eV^{-1}$$ [2]

Hence,

$$B_R(K^+ \to \pi^+\pi^0) = 1.2 \times 10^{-8}\,eV \times 0.18 \times 10^{8}\,eV^{-1} =$$

$$= 0.216 \Rightarrow 21.6\%$$ [1]

$\boxed{Q.3}$ [Total: 12 marks]

(a) Since $m_\Lambda^2 = E_\Lambda^2 - |\vec{P}|_\Lambda^2$ and the energy and momentum are conserved:

$$m_\Lambda^2 = (E_p + E_\pi)^2 - (\vec{P}_p + \vec{P}_\pi)^2 =$$

$$= \underline{E_p^2} + \underline{E_\pi^2} + 2E_pE_\pi \underline{- |\vec{P}_p|^2} \underline{- |\vec{P}_\pi|^2} - 2\vec{P}_p \cdot \vec{P}_\pi =$$

$$= m_p^2 + m_\pi^2 + 2E_p \cdot E_\pi - 2|\vec{P}_p| \cdot |\vec{P}_\pi| \cdot \cos\theta$$

And since $|\vec{P}| = \beta E$

$$m_\Lambda^2 = m_p^2 + m_\pi^2 + 2E_p \cdot E_\pi (1 - \beta_p \beta_\pi \cos\theta)$$

[4]

(b) $E_p = \sqrt{p_p^2 + m_p^2} = \sqrt{(4.25)^2 + (0.938)^2} =$

$= 4.352\ \text{GeV}$

$E_\pi = \sqrt{p_\pi^2 + m_\pi^2} = \sqrt{(0.75)^2 + (0.14)^2} =$

$= 0.763\ \text{GeV}$

$\beta_p = \frac{p_p}{E_p} = \frac{4.25\ \text{GeV}}{4.352\ \text{GeV}} \simeq 0.977$ [3]

$\beta_\pi = \frac{p_\pi}{E_\pi} = \frac{0.75\ \text{GeV}}{0.763\ \text{GeV}} \simeq 0.983$

Hence

$M_\Lambda^2 = (0.938)^2 + (0.14)^2 + 2 \times 4.352 \cdot 0.763 \times$
$\times (1 - 0.977 \times 0.983 \cdot \cos 9^\circ) \simeq 1.244\ \text{GeV}^2$

$M_\Lambda \simeq 1.115\ \text{GeV}$

[1]

(c) The distance travelled by Λ is:

$d = c\tau = c\gamma\tau_0$ (time dilation)

where $\tau_0 = 2.6 \times 10^{-10}$ sec and γ is the relativistic Lorentz factor $\gamma = \frac{1}{\sqrt{1-\beta^2}}$

From $E = \gamma mc^2$:

$$\gamma_\Lambda = \frac{E_\Lambda}{m_\Lambda c^2} = \frac{E_p + E_\pi}{m_\Lambda c^2} = \frac{4.352 + 0.763}{1.115} \simeq$$

$\simeq 4.587$

[3]

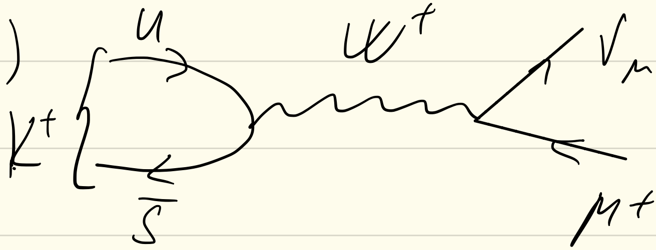
Hence $d = 3 \cdot 10^8 \frac{m}{s} \times 4.587 \times 2.6 \cdot 10^{-10} s \simeq$

$\simeq 0.36\ m = 36\ cm$

[1]

$\boxed{Q.4}$ [Total 8 marks]

(a)



[2]

(b) The 4-momentum conservation:

$P_K = P_\mu + P_\nu$ and therefore

$$(P_K - P_\mu)^2 = P_\nu^2 = 0 \quad (\text{since } m_\nu \approx 0)$$

$$P_K^2 + P_\mu^2 - 2P_K P_\mu = m_K^2 + m_\mu^2 -$$

$$-2\left(E_K \cdot E_\mu - \vec{P_K} \cdot \vec{P_\mu}\right) = 0$$

[3]

In Central of mass frame:

$\vec{p}_K = 0$ and $E_K = m_K$

$$m_K^2 + m_\mu^2 = 2\, m_K E_\mu$$

But $E_\mu = \gamma m_\mu$

Hence:

$$\gamma = \frac{m_K^2 + m_\mu^2}{2 m_K \cdot m_\mu}$$

[3]

$\boxed{Q.5}$ [Total: 5 marks]

Consider a Lorentz transformation along z-axis: $d^3\vec{p} \rightarrow d^3\vec{p}\,'$

$$d^3\vec{p}\,' = dp_x'\,dp_y'\,dp_z' = dp_x\,dp_y\,\frac{dp_z'}{dp_z}\,dp_z =$$

$$= \frac{dp_z'}{dp_z}\,d^3\vec{p}$$

Using Lorentz transformation:

$p_z' = \gamma(p_z - \beta E)$ and $E' = \gamma(E - \beta p_z)$ [2]

We have:

$$\frac{dp_z'}{dp_z} = \gamma\left(1 - \beta\frac{\partial E}{\partial p_z}\right) = \gamma\left(1 - \beta\frac{p_z}{E}\right) =$$

$$= \frac{1}{E}\gamma(E - \beta p_z) = \frac{E'}{E} \Rightarrow \frac{dp_z'}{E'} = \frac{dp_z}{E}$$

$$\Downarrow$$

$\frac{d^3\vec{p}}{E}$ is invariant [3]

$\boxed{\text{Q. 6}}$ [Total : 8 marks]

The rate of the neutrino - nucleon interaction will be given by

$R = \sigma \phi N_t$ where ϕ is the neutrino flux and N_t - is the number of target nucleons transversed by ν-beam

The flux of one ν incident on an area A is $\frac{1}{A}$ and $N_t = n \cdot A \cdot x$ where $x = 1m$ and n is the number density of target nucleons [3]

$$n = \frac{\rho}{A_{Fe} \cdot m_N}$$ where $A_{Fe} = 56$

$m_N = 1.67 \times 10^{-27}$ kg

nucleon mass

$$n = \frac{7900 \text{ kg/m}^3}{56 \times 1.67 \times 10^{-27} \text{ kg}} \simeq 8.4 \times 10^{28} \text{ m}^{-3}$$ [3]

Therefore,

since $\sigma = 8 \times 10^{-39} cm^2 = 8 \times 10^{-43} m^2$

$$N_{int} = \sigma \frac{1}{A} n A x = \sigma n x =$$

$$= 8 \times 10^{-43} m^2 \times 8.4 \times 10^{28} m^{-3} \times 1m =$$

$\simeq 6.7 \times 10^{-14}$ which is a probability of 1ν to interact in 1m of Fe!

[2]

Total: 50 marks