PHASM/G442. Particle Physics.

REVISION

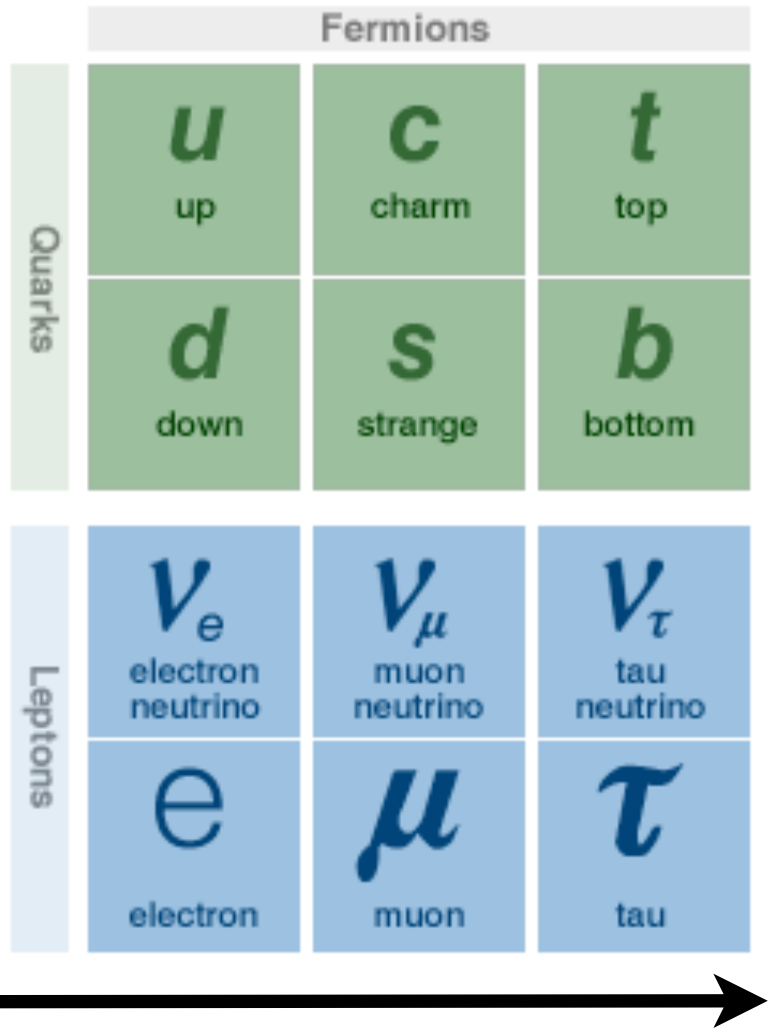
Ruben Saakyan

24-Apr-2017

Course Outline

- Recap, formalism, reaction rates, Feynman Rules, coupling constants (**Module 1**)
- Symmetries and conservation laws (**Module 2**)
- The Dirac Equation (**Module 3**)
- Dirac + Maxwell ==> Interaction by particle exchange (**Module 4**)
- QED Calculations (**Module 5**)
- Quark properties, proton structure and QCD (**Module 6**)
- Weak Interactions (**Module 7**)
- Electroweak Unification and Higgs (**Module 8**)
- Neutrino Phenomenology and Beyond the Standard Model (**Module 9**)

Elementary Matter Particles



Mass

Particles of same type but different families are identical except for mass.

	Q	Strong	Weak	EM
u,c,t	+2/3	✓	✓	✓
d,s,b	-1/3	✓	✓	✓
nu's	0	✗	✓	✗
e,mu, tau	-1	✗	✓	✓

Masses and quark composition of hadrons will be given but expect you to remember

- quark composition of p,n π, K
- approximate masses of p/n, π, K, e, μ

Info in Paper's Rubric (2016 example)

Units, Masses and Other Values

The convention: $\hbar = c = 1$ will be used throughout this paper.
The values for the following quantities may be assumed in this paper.

Meaning	Value
Masses of u,d,s,c,b,t quarks	1 MeV, 2 MeV, 0.2 GeV, 1.5 GeV, 4.5 GeV, 172 GeV
Masses of e,μ,τ leptons	0.5 MeV, 106 MeV, 1.8 GeV
Mass of all neutrinos	0
Mass of the $\pi^{\pm}$	140 MeV
Mass of Z boson (M_Z)	91 GeV
Mass of W boson (M_W)	80 GeV
Fermi Weak Decay Constant (G_F)	1.11×10^{-5} GeV^{-2}
Lifetime of $\mu^{\pm}$, $\pi^{\pm}$	2.2×10^{-6}s , 2.6×10^{-8}s

CKM Matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

where V_{ij} is the factor for interactions involving quarks i and j.

Dirac Matrices

The Dirac γ matrices satisfy $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ (for $\mu, \nu = 0,1,2,3$) can be written as:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^{i=1,2,3} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

And the Pauli spin matrices, σ_i, are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which satisfy: $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{c}) = \vec{a} \cdot \vec{c} + i\vec{\sigma} \cdot (\vec{a} \times \vec{c})$ for 3 component vectors $\vec{a}$, $\vec{c}$.

Lorentz Transformation

$$\begin{pmatrix} p' \\ E' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} p \\ E \end{pmatrix}$$

Four Vector Notation

- Relativistic effects are critical in particle physics, we will commonly use four vector notation
- Four Vector Definition: "An object that transforms like x^μ between inertial frames"
- Invariant Definition = "A quantity that is unchanged in all inertial frames"
- Example four vectors:

$$x^\mu = \left(\text{time}, \overrightarrow{\text{position}}\right)$$

$$p^\mu = \left(\text{energy}, \overrightarrow{\text{momentum}}\right)$$

$$j^\mu = \left(\phi \text{ density}, \overrightarrow{\text{current}}\right)$$

Example Invariants:

Rest mass

- x^μ is the contra-variant four vector

Four Vector Cheat-sheet

- Special Relativity Reminder

$$\beta = \frac{v}{c} = v$$

$$E = \gamma m \qquad \vec{p} = \gamma m \vec{\beta}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\gamma = \frac{E}{m} \qquad \vec{\beta} = \frac{\vec{p}}{m}$$

- Lorentz Transform:

$$(x^\mu)' = \sum_{\nu=0}^{3} \Lambda^\mu_\nu x^\nu = \Lambda^\mu_\nu x^\nu$$

$$\begin{pmatrix} t' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix}$$

- Covariant metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$x_\mu = g_{\mu\nu} x^\nu$$

$$x^\mu = (t, \vec{x})$$

$$x_\mu = (t, -\vec{x})$$

Four Vector Cheat-sheet II

- Scalar Product

$$a \cdot b = a^0 b^0 - \vec{a} \cdot \vec{b}$$
$$= g_{\mu\nu} a^\mu b^\nu$$
$$= a_\nu b^\nu$$

$$x \cdot x = t^2 - \vec{x} \cdot \vec{x}$$
$$x_\mu x^\mu = t^2 - |\vec{x}|^2$$

4-vector "length"
>0 "time like"
<0 "space like"
=0 "light like"

$$P_\mu P^\mu = P^2$$
$$P_\mu P^\mu = E^2 - |\vec{p}|^2$$
$$P_\mu P^\mu = m^2$$

Invariant Rest Mass

- Differential 4-vector ("four-derivative")

$$\partial^\mu = \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

Note - sign (c.f Q.M.)

$$\partial_\mu = \left(\frac{\partial}{\partial t}, +\vec{\nabla}\right)$$

$$\partial_\mu a^\mu = \frac{\partial a^0}{\partial t} + \vec{\nabla} \cdot \vec{a}$$
$$b_\mu \partial^\mu = b^0 \frac{\partial}{\partial t} + \vec{b} \cdot \vec{\nabla}$$

Particle Decays

- Define lifetime, τ
  – Mean time to decay in rest frame

$$N(t) = N_0 e^{-t/\tau}$$

- Define decay width (rate)

$$\Gamma = 1/\tau$$

- Define branching ratio
  – Fraction of decays to final state "i"

$$\sum_i BR_i = 1$$

- Define partial decay width

$$\Gamma_i = BR_i \Gamma$$

- Therefore $\sum_i \Gamma_i = \Gamma$

- Lifetime's or decay width are measurable
- Branching ratios are measurable
- Partial decay width are calculable from theory

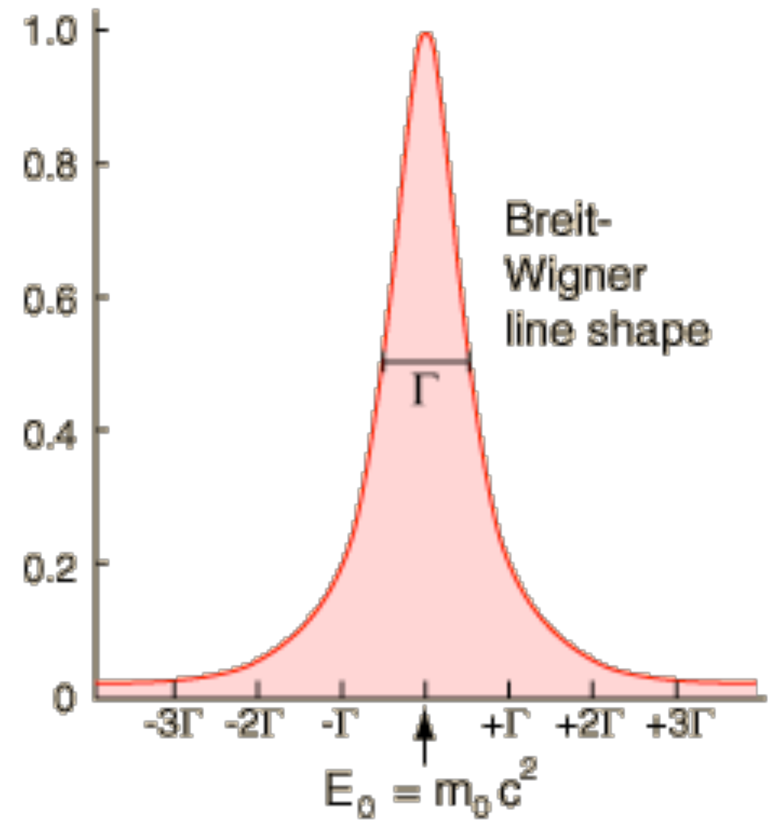
$\Gamma\Gamma$: Decay Width

Time of a particle's decay has uncertainty : $\Delta t = \tau$
Uncertainty Principle then predicts

$\Delta E.\tau = 1/2$ and hence

$\Gamma = 2\ \Delta E$, $\Gamma = 1/\tau$

If measure invariant mass of a state then Uncertainty principle gives it a "width" due to particle having a finite lifetime.



Distribution of mass follows Breit-Wigner form:

We can only ever measure either lifetime or width due to measuring capabilities of particle detectors

For $\hbar = c = 1$

- $1\ \text{GeV}^{-1} = 0.1973\ \text{fm} = 1.973 \times 10^{-16}\ \text{m} = 6.582 \times 10^{-25}\ \text{sec}$

How we calculate Reaction Rate (σ) or Decay Width (Γ)

- Draw *Feynman diagrams* for the process
  - decide to which *"order"* we want to perform the calculation.
  - invoke *Feynman rules* to calculate a *"Matrix Element (M)"*

- Calculate the *"phase-space"* and *"flux"* for the process

- Combine $|M|^2$ with phase-space using *Fermi's Golden Rule (FGR))*

$$Rate = \left|M_{fi}\right|^2 \times (phase\,space)$$

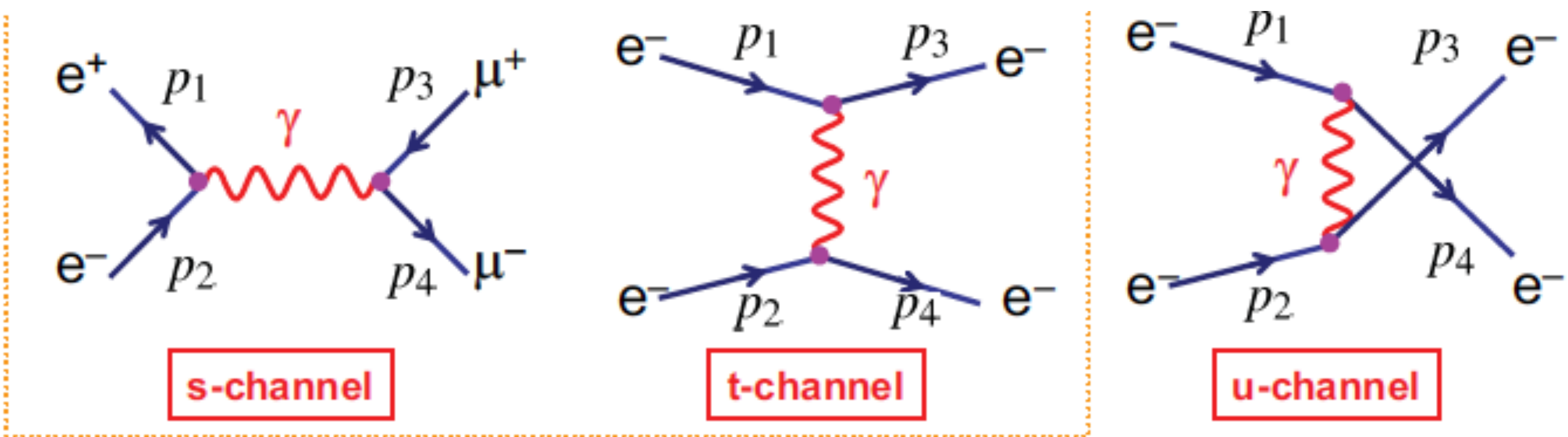
$$\sigma = \frac{Rate}{Flux}$$

E.g. $\Gamma = \dfrac{G_F^2 m_\mu^5}{192\pi^3}$

See PS1, Q7. Also, 2016 exam question.

Feynman Diagrams

- Time from left to right
- Draw initial particle lines on left and final to right - there will be a boson in middle
- Based on information about reaction (initial & final state, rate) determine the type of interaction : EM(γ), Weak (W,Z), Strong (g)
- Draw interaction vertices - make sure that charge, lepton # etc are conserved
- Draw arrow (L→R for particles) and (R→L : backward in time for anti-particles)
- Make sure arrows ‘flow’ through the vertex



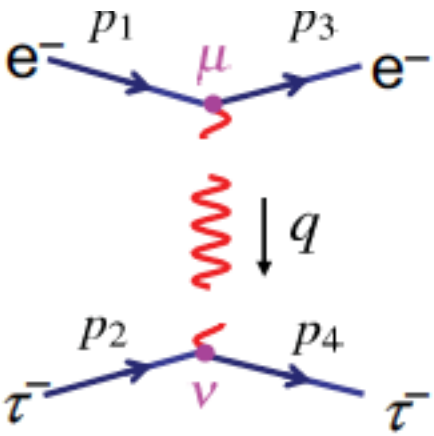
$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2$$

Kinematics, kinematics, kinematics….

See 2014, Q1(b).

Feynman Rules

QED



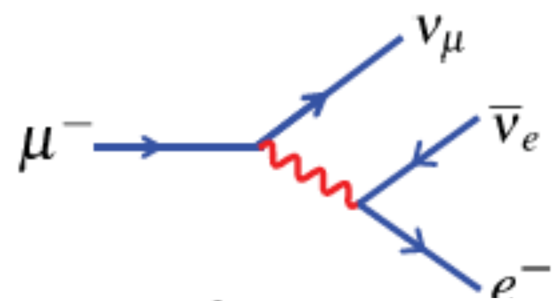
$$\bar{u}_e(p_3)[ie\gamma^\mu]u_e(p_1)$$

$$\frac{-ig_{\mu\nu}}{q^2}$$

$$\boxed{iM = [\bar{u}_e(p_3)ie\gamma^\mu u_e(p_1)]\frac{-ig_{\mu\nu}}{q^2}[\bar{u}_\tau(p_4)ie\gamma^\nu u_\tau(p_2)]}$$

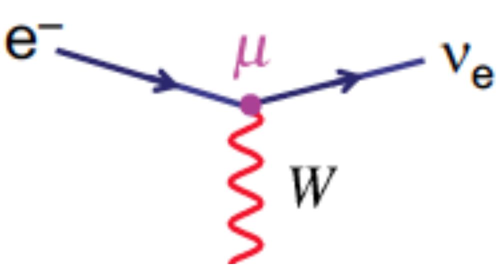
$$\bar{u}_\tau(p_4)[ie\gamma^\nu]u_\tau(p_2)$$

Weak



$$\boxed{\frac{-ig_w}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)}$$

V-A



$$\frac{-i\left[g_{\mu\nu}-q_\mu q_\nu/m_W^2\right]}{q^2-m_W^2}$$

$$M_{fi} = [\tfrac{g_W}{\sqrt{2}}\bar{\psi}\tfrac{1}{2}\gamma^\mu(1-\gamma^5)\psi]\frac{g_{\mu\nu}-q_\mu q_\nu/m_W^2}{q^2-m_W^2}[\tfrac{g_W}{\sqrt{2}}\bar{\psi}\tfrac{1}{2}\gamma^\nu(1-\gamma^5)\psi]$$

$$M_{fi} = \frac{g_W^2}{8m_W^2}g_{\mu\nu}[\bar{\psi}\gamma^\mu(1-\gamma^5)\psi][\bar{\psi}\gamma^\nu(1-\gamma^5)\psi]$$ due to $q^2 \ll m_W^2$

Weak Interactions

- Why is it called weak?
- Chiral nature of the weak interaction
  - Connection with QED
- Pion, muon and tau decays

$$\Gamma = \frac{G_f^2 m_\mu^5}{192\pi^3}$$

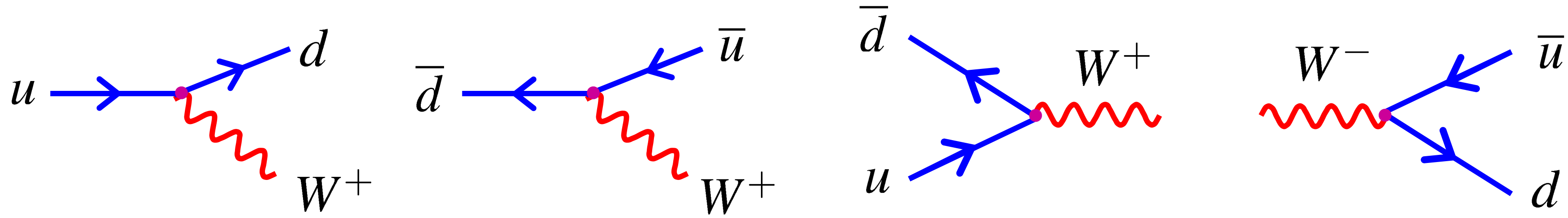
- Quark vertices in weak interactions
  - CKM formalism

<u>Understand</u> (apply in practice) **unitarity condition**

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.974 & 0.227 & 0.004 \\ 0.227 & 0.973 & 0.042 \\ 0.008 & 0.042 & 0.999 \end{pmatrix}$$

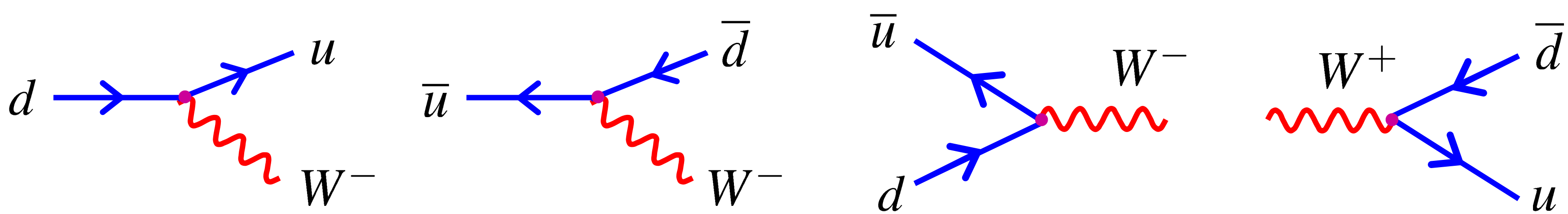
- Neutrino mixing and oscillations

Weak Interactions of Quarks



is $$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}^*\gamma^\mu\,\tfrac{1}{2}(1-\gamma^5)}$$

See 2016, Q1(b).

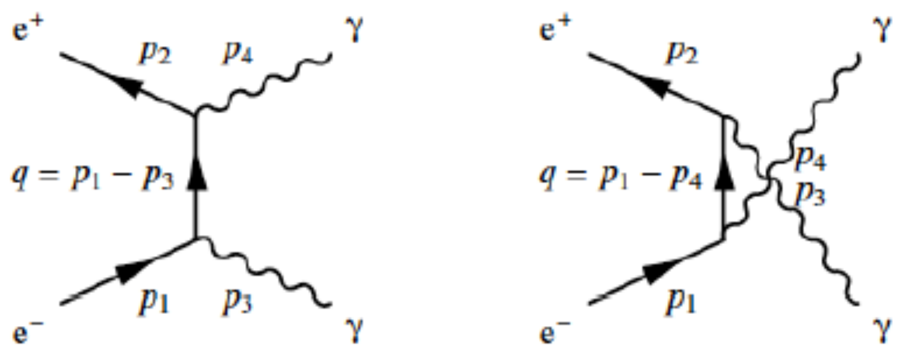


is $$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^\mu\,\tfrac{1}{2}(1-\gamma^5)}$$

Note on propagators

PS2, Q6.

The t- (left) and u-channel (right) Feynman diagrams are:



The matrix elements are:

$$-iM_t = \left[\varepsilon^*_\mu(p_3)ie\gamma^\mu u(p_1)\right]\cdot\left[-\frac{i\left(\gamma^\rho q_\rho + m_e\right)}{q^2 - m_e^2}\right]\cdot\left[\bar{v}(p_2)ie\gamma^\nu\varepsilon^*_\nu(p_4)\right]$$

$$-iM_u = \left[\varepsilon^*_\mu(p_4)ie\gamma^\mu u(p_1)\right]\cdot\left[-\frac{i\left(\gamma^\rho q_\rho + m_e\right)}{q^2 - m_e^2}\right]\cdot\left[\bar{v}(p_2)ie\gamma^\nu\varepsilon^*_\nu(p_3)\right]$$

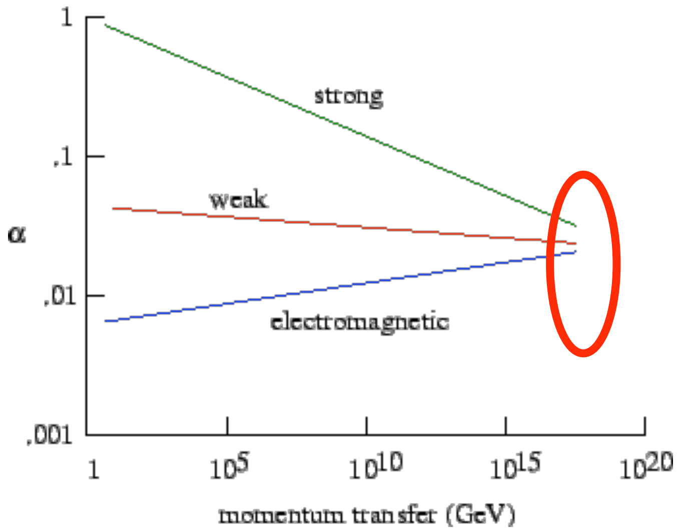
Fermion propagators can come with negative sign
Not explicitly covered in the course, consequently can use either when answering e.g. exam question.

Renormalisation & running couplings

- A renormalisable theory is one in which the "trick" of using renormalised quantities (masses, couplings) remove all infinities to all orders.

- It was shown that the class of theories known as gauge theories (of which QED and QCD are examples) are all renormalisable and so this is the type of theory people always start with, (Nobel Prize 1999).

- EM (QED) coupling constant increases with energy
- Strong (QCD) coupling constant decreases with energy (Nobel Prize 2004)



Don't actually meet or unify unless new particles !

Dirac Equation and Matrices

- Dirac Equation

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

- Dirac Hamiltonian

$$\hat{H} = -i\gamma^0 \vec{\gamma} \cdot \vec{\nabla} + m\gamma^0$$

See coursework PS2.

The Dirac γ matrices satisfy $\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}$ (for $\mu, \nu =$ 0,1,2,3) and are defined as:

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^{i=1,2,3} = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

And the Pauli spin matrices, σ_i, are:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which satisfy: $(\vec{\sigma} \cdot \vec{a})(\vec{\sigma} \cdot \vec{c}) = \vec{a} \cdot \vec{c} + i\vec{\sigma} \cdot (\vec{a} \times \vec{c})$ for 3 component vectors $\vec{a}$, $\vec{c}$.

The solutions of the Dirac equation : $$(\gamma^\mu p_\mu - m)u(\vec{p}) = 0$$

- Four solutions of the form $\psi_i = u_i(E,\vec{p})e^{i(\vec{p}.\vec{r}-Et)}$

$$\underbrace{u_1 = N\begin{pmatrix}1\\0\\\frac{p_z}{E+m}\\\frac{p_x+ip_y}{E+m}\end{pmatrix};\ u_2 = N\begin{pmatrix}0\\1\\\frac{p_x-ip_y}{E+m}\\\frac{-p_z}{E+m}\end{pmatrix}}_{E=+\left|\sqrt{|\vec{p}|^2+m^2}\right|};\ \underbrace{u_3 = N\begin{pmatrix}\frac{p_z}{E-m}\\\frac{p_x+ip_y}{E-m}\\1\\0\end{pmatrix};\ u_4 = N\begin{pmatrix}\frac{p_x-ip_y}{E-m}\\\frac{-p_z}{E-m}\\0\\1\end{pmatrix}}_{E=-\left|\sqrt{|\vec{p}|^2+m^2}\right|}$$

- Four solutions of the form $\psi_i = v_i(E,\vec{p})e^{-i(\vec{p}.\vec{r}-Et)}$

$$\underbrace{v_1 = N\begin{pmatrix}\frac{p_x-ip_y}{E+m}\\\frac{-p_z}{E+m}\\0\\1\end{pmatrix};\ v_2 = N\begin{pmatrix}\frac{p_z}{E+m}\\\frac{p_x+ip_y}{E+m}\\1\\0\end{pmatrix}}_{E=+\left|\sqrt{|\vec{p}|^2+m^2}\right|};\ \underbrace{v_3 = N\begin{pmatrix}1\\0\\\frac{p_z}{E-m}\\\frac{p_x+ip_y}{E-m}\end{pmatrix};\ v_4 = N\begin{pmatrix}0\\1\\\frac{p_x-ip_y}{E-m}\\\frac{-p_z}{E-m}\end{pmatrix}}_{E=-\left|\sqrt{|\vec{p}|^2+m^2}\right|}$$

- Only four are linearly independent

- Natural to choose **positive energy solutions** $\boxed{\{u_1, u_2, v_1, v_2\}}$

Helicity and Chirality

- Chirality
  - Fundamental state
  - Weak interaction is only LH particles or RH antiparticles
- Helicity
  - Measurable quantity, projection of spin on momentum
  - Massless limit helicity == chirality
- Operators

$$\hat{P} = \frac{1}{2}\gamma^5 \qquad \hat{h} = \frac{1}{2}\begin{pmatrix} \vec{\sigma}\cdot\hat{p} & 0 \\ 0 & \vec{\sigma}\cdot\hat{p} \end{pmatrix}$$

- Projectors

$$\hat{P}_L = \frac{1}{2}\left(1-\gamma^5\right) \qquad \hat{P}_R = \frac{1}{2}\left(1+\gamma^5\right)$$

See 2016, Q3(b).

The Adjoint Spinor

$$\overline{\Psi}$$

- The maths of QED (and all interactions) is based on :

$$\overline{\psi}.....\psi$$

$$\overline{\psi} = \psi^\dagger \gamma^0; \quad \psi^\dagger = \left(\psi^T\right)^*$$

$\overline{\psi}\psi$ Is Lorentz invariant; but the usual $\psi^\dagger\psi$ is not.

$\overline{u}\left(\gamma^\mu p_\mu - m\right) = 0$: the Adjoint Dirac Equation

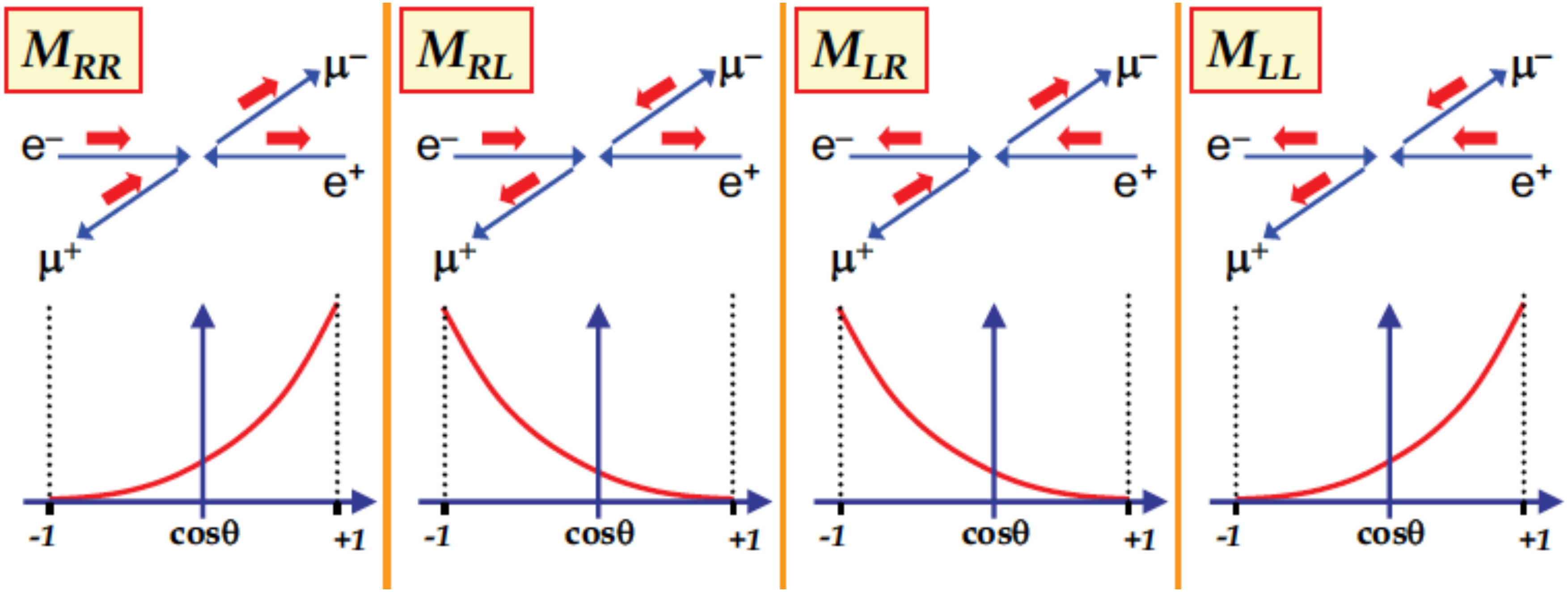
A previous exam question:

(d) For a particle, the left handed state is defined by the projection: $u_L = \frac{1}{2}(1-\gamma^5)u$ and the right handed by $u_R = \frac{1}{2}(1+\gamma^5)u$. Show that $\overline{u}_L = \overline{u}\frac{1}{2}(1+\gamma^5)$

Matrix element(s) of e+e- →μ+μ-

$$|M_{RR}|^2 = |M_{LL}|^2 = (4\pi\alpha)^2(1+\cos\theta)^2$$
$$|M_{RL}|^2 = |M_{LR}|^2 = (4\pi\alpha)^2(1-\cos\theta)^2$$

Due to "helicity conservation" only certain helicity combinations contribute to non-zero M.E.



$e^4(1+\cos\theta)^2$ $e^4(1-\cos\theta)^2$ $e^4(1-\cos\theta)^2$ $e^4(1+\cos\theta)^2$

The final matrix element is obtained by **averaging** over the **initial** spin states

$$\langle|M_{fi}|^2\rangle = \frac{1}{4}\times(|M_{RR}|^2+|M_{RL}|^2+|M_{LR}|^2+|M_{LL}^2|) = e^4(1+\cos^2\theta)$$

See 2015, Q2(e).

which can be also written in Lorentz Invariant form: $\boxed{\langle|M_{fi}|^2\rangle = 2e^4\frac{(p_1.p_3)^2+(p_1.p_4)^2}{(p_1.p_2)^2}}$

Gauge Invariance

- This + renormalisation are the cornerstones of modern physics
  - Requirement that physics does not depend on unmeasurables, e.g.. phase in wavefunction
- Imposition of gauge symmetry requires
  - Gauges boson field (e.g. photon)
  - Conservation law (e.g. conservation of charge)
- Introduces **interaction** $\psi(x) \rightarrow \psi'(x) = e^{iq\chi(x)}\psi(x)$

$$\mathcal{L}' = i\overline{\psi'}\gamma^\mu\partial_\mu\psi' - m\overline{\psi'}\psi' = ie^{-iq\chi}\overline{\psi}\gamma^\mu\left[e^{iq\chi(x)}\partial_\mu\psi + iq(\partial_\mu\chi)e^{iq\chi(x)}\psi\right]\psi - me^{-iq\chi(x)}\overline{\psi}e^{iq\chi(x)}\psi$$
$$= \mathcal{L} - q\overline{\psi}\gamma^\mu(\partial_\mu\chi)\psi$$

See coursework PS2.

$$A_\mu \rightarrow A'_\mu = A_\mu - \partial_\mu\chi \quad \longrightarrow \quad \mathcal{L} = \overline{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi - \underline{q\overline{\psi}\gamma^\mu A_\mu\psi}$$

- The bosons required by gauge invariance must be massless
  - Therefore need the Higgs mechanism to give W & Z mass

Lagrangians for QED and QCD

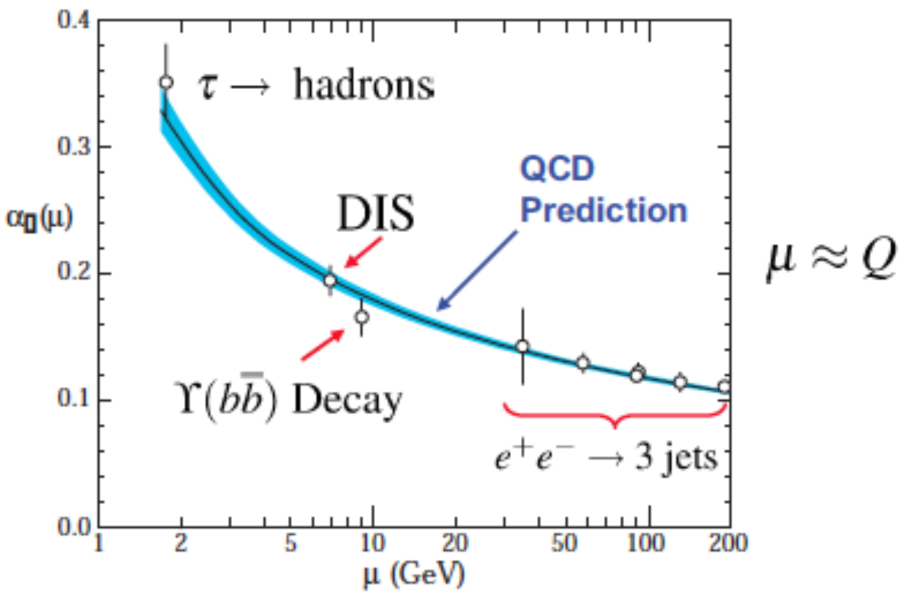
- Understand terms in the Lagrangian

$$\mathcal{L}_{\mathrm{QED}} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi + e\bar{\psi}\gamma_{\mu}A^{\mu}\psi - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi - g_s\bar{\psi}\gamma^{\mu}\mathbf{G}_{\mu}\psi - \tfrac{1}{4}\sum_{a=1}^{8} G^{a}_{\mu\nu}G^{a\mu\nu}$$

- The gluon-gluon self interaction terms are the only difference. Leads to
  - Spring like nature of the strong force
  - Confinement (i.e no free quarks)
  - Asymptotic freedom (i.e. hard scattering calculable, soft not)

α_S Running Coupling Constant



- α_S is measured
  - Jets
  - τ decays to hadrons
  - Deep Inelastic Scatterin
  - botommonium decays
  - etc…

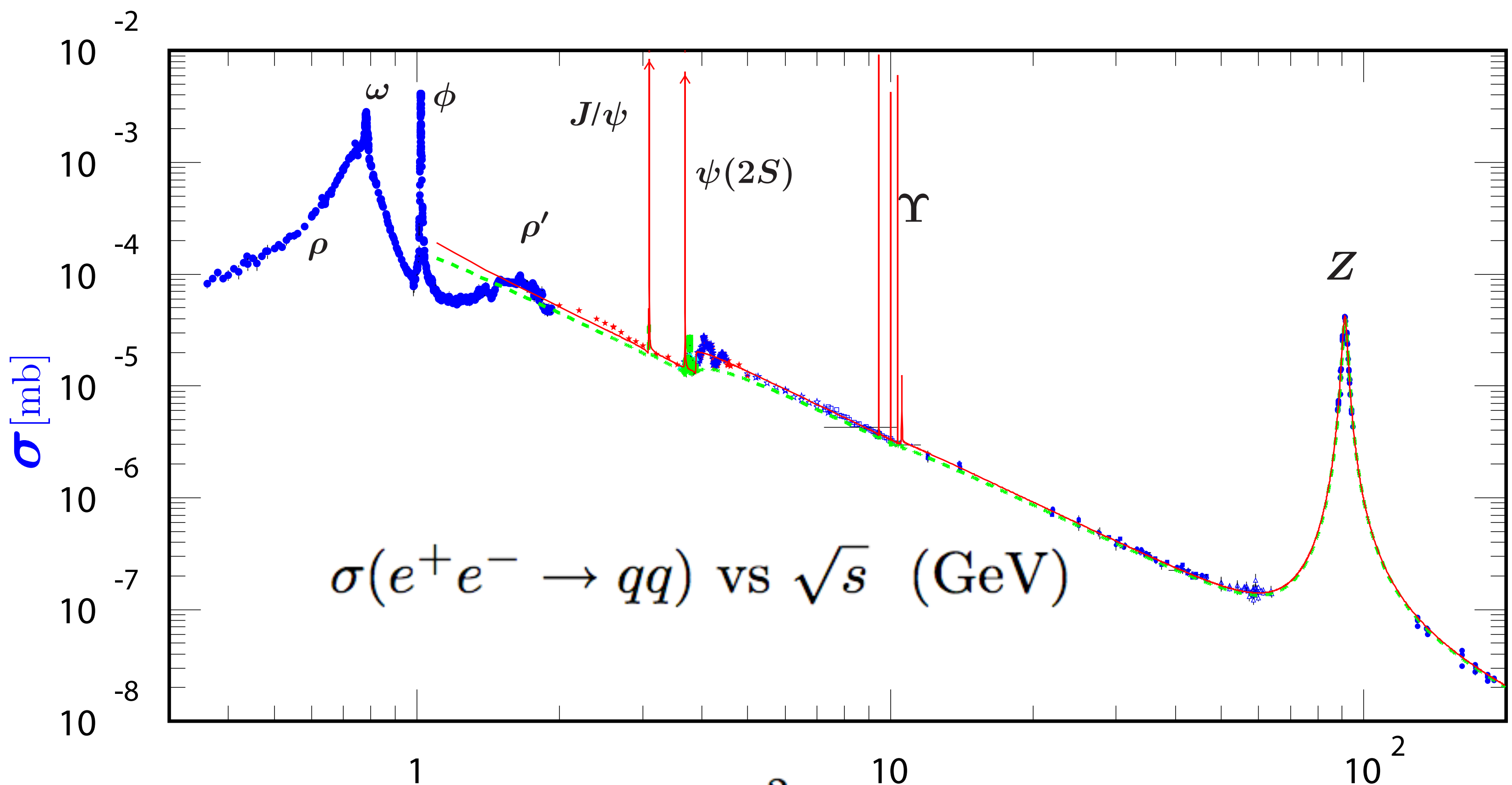
α_S decreases with Q^2 as predicted by QCD

- At low Q^2 (~1 GeV^2) α_S is large, α_S~1. Cannot use perturbation theory. That's wh e.g. hadronisation is not (yet) calculable.
- At high Q^2, e.g. $Q^2 \sim M_Z^2 \sim 8000\,GeV^2$ is relatively small α_S~0.12

➡ **Asymptotic Freedom**

Cross section for scattering processes

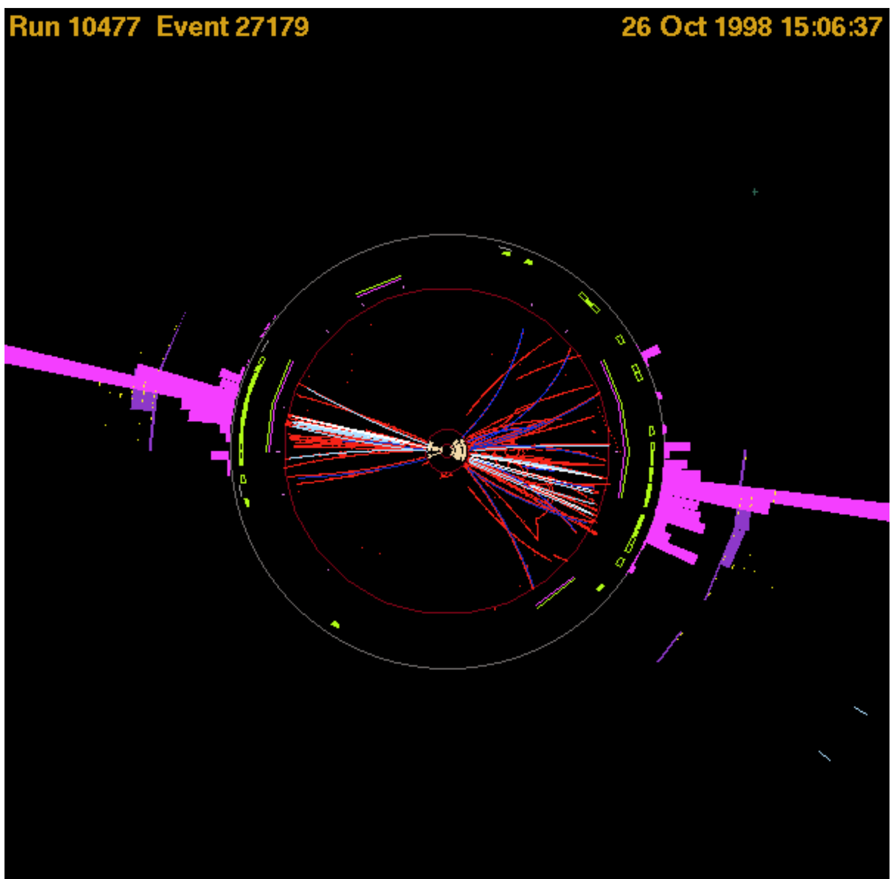
- Understand derivation of cross section formula for e+e- → μ+μ-, e-μ- → e-μ-
- Correspondence between helicity and chiral operators
- No chirality/handedness changing processes occur in any pure vector i.e. QCD + QED or axial vector interactions

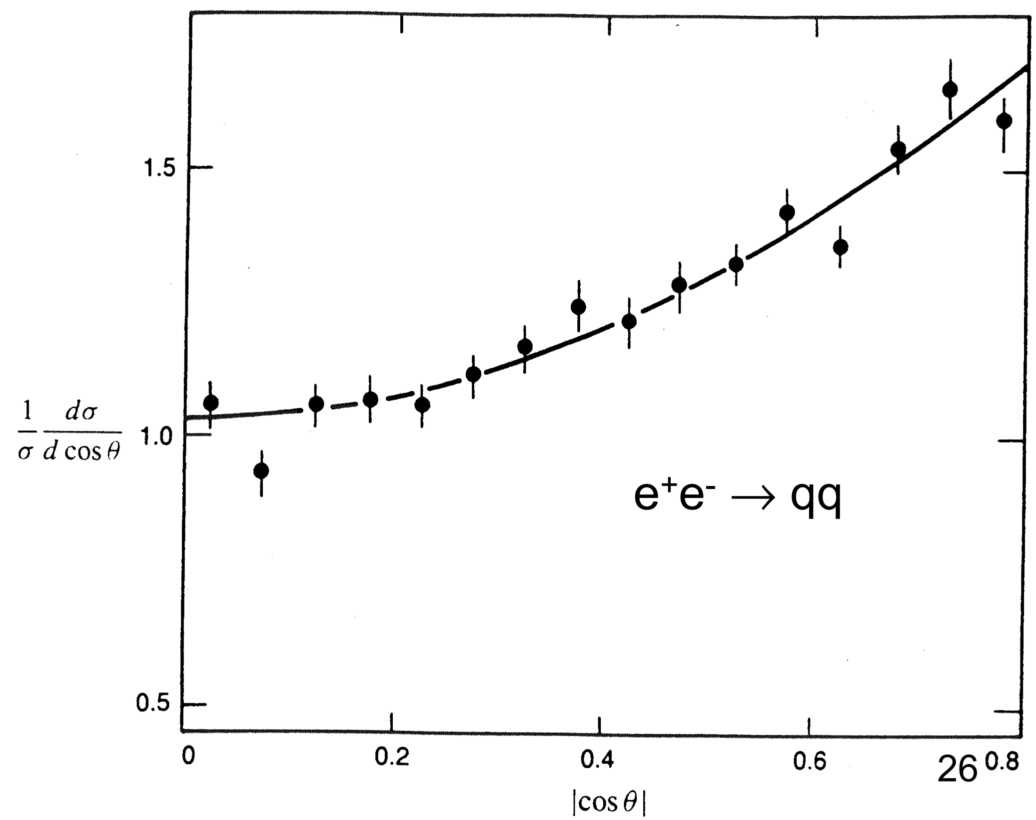


$$\sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} = \frac{87.0}{s}\ \text{nb}; \quad \text{s in GeV}^2$$

Evidence for Quark Properties

- Spin 1/2 : angular distribution of quark jets in e+e- → qq or eq → eq
- Fractionally charged
- Carry colour

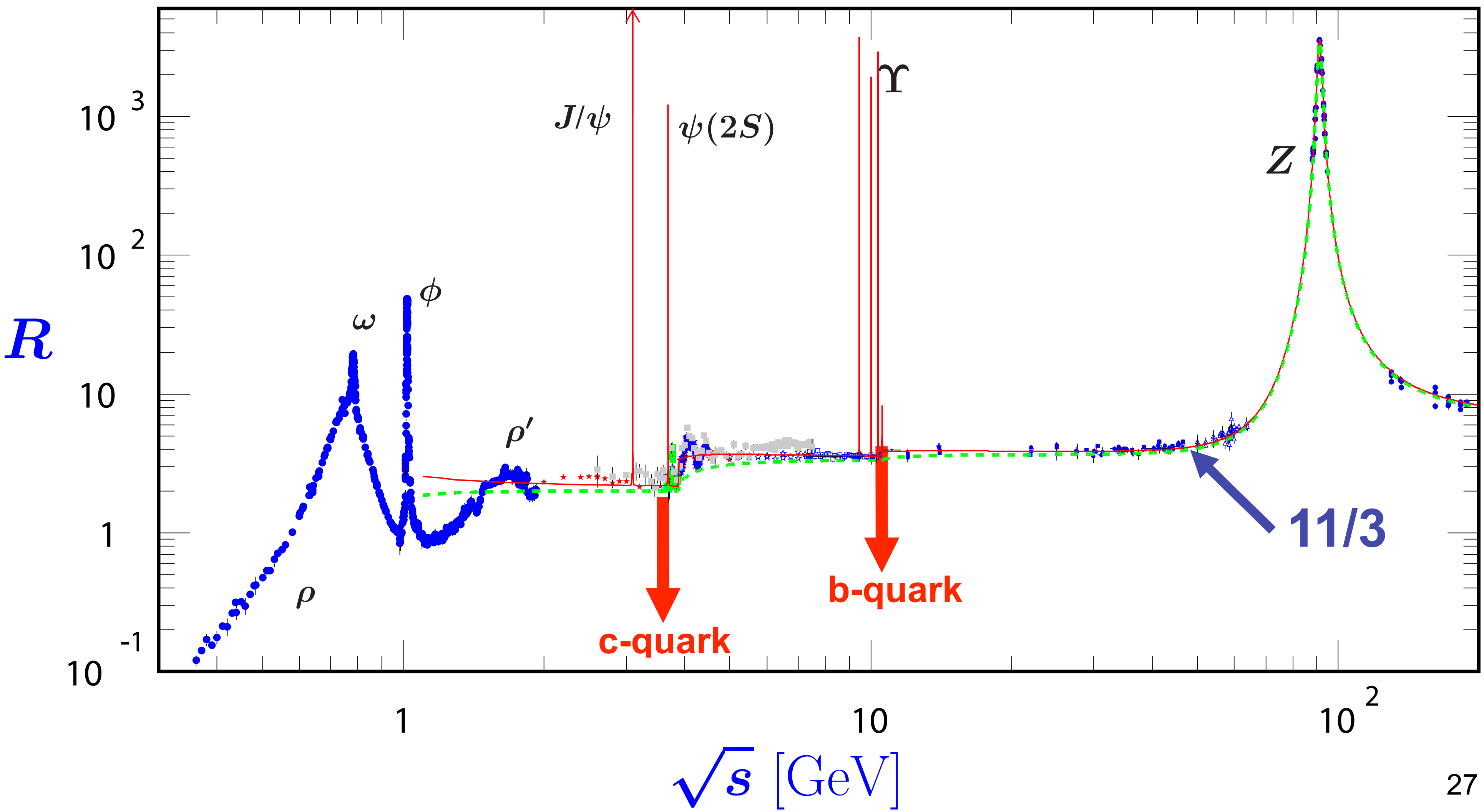




Evidence that quarks are fractionally charged

- R; ratio of proton to neutron magnetic moment; charges of baryons e.g. Δ^{++}

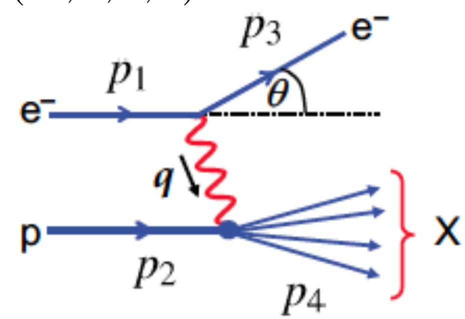
See 2016, Q2(d).



Deep Inelastic Scattering (DIS)

x - fractional longitudinal momentum carried by struck quark

$$p_1 = (E_1, 0, 0, E_1) \quad p_2 = (M, 0, 0, 0)$$
$$q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$$



Define: $\boxed{x \equiv \frac{Q^2}{2p_2.q}}$ **Bjorken x**

where $\boxed{Q^2 \equiv -q^2}$ $\quad Q^2 > 0$

$$M_X^2 = p_4^2 = (q + p_2)^2 = -Q^2 + 2p_2.q + M^2$$
$$Q^2 = 2p_2.q + M^2 - M_X^2 \Rightarrow Q^2 \le 2p_2.q$$

$\boxed{0 < x < 1 \text{ inelastic}}$ $\quad \boxed{x = 1 \text{ elastic}}$ ← Proton intact $M_X = M$

Define: $\boxed{y \equiv \frac{p_2.q}{p_2.p_1}}$

In the Lab. Frame $\quad y = 1 - \frac{E_3}{E_1} \longrightarrow 0 < y < 1$

Fractional energy loss by the incoming particle

In the C.o.M. Frame (neglecting e⁻ and p masses, *E>>M*)

$$p_1 = (E, 0, 0, E); \quad p_2 = (E, 0, 0, -E); \quad p_3 = (E, E\sin\theta^*, 0, E\cos\theta^*)$$

$$y = \tfrac{1}{2}(1 - \cos\theta^*)$$

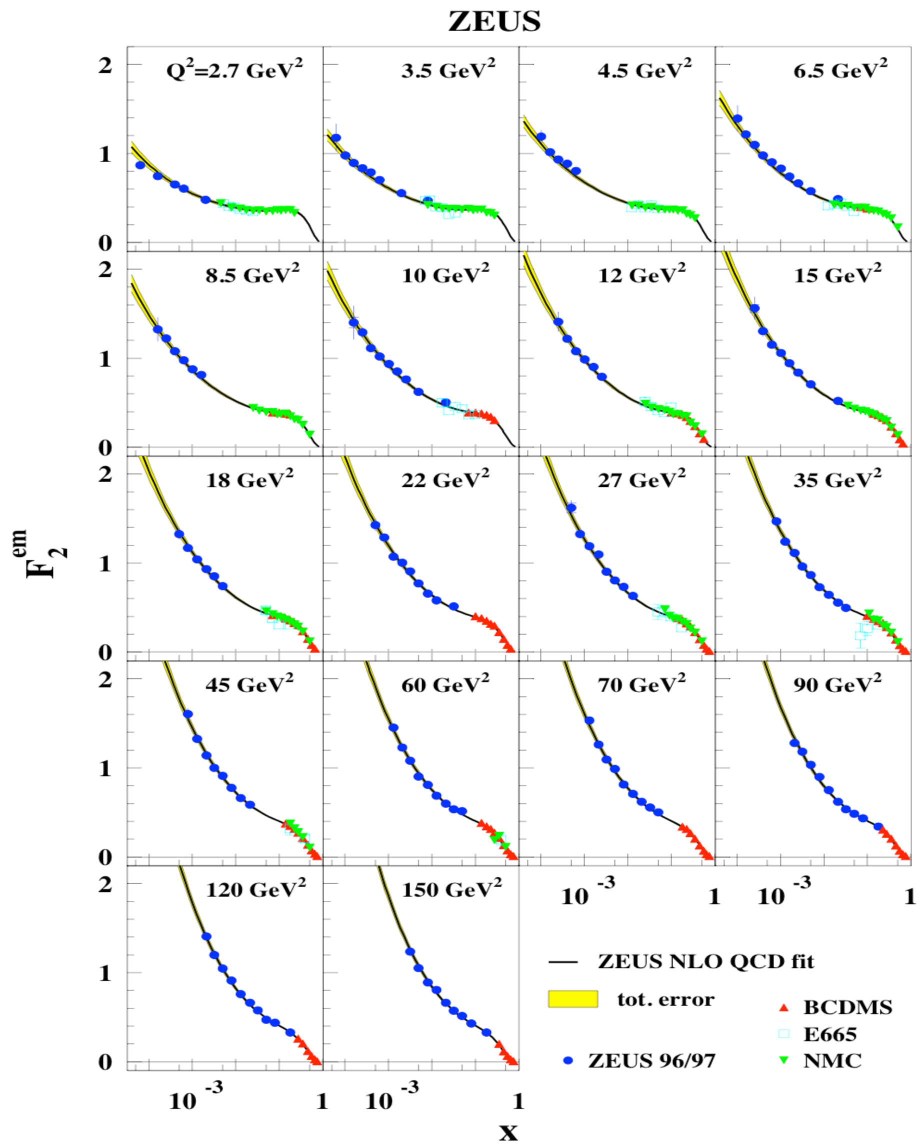
Define: $\boxed{\nu \equiv \frac{p_2.q}{M}}$

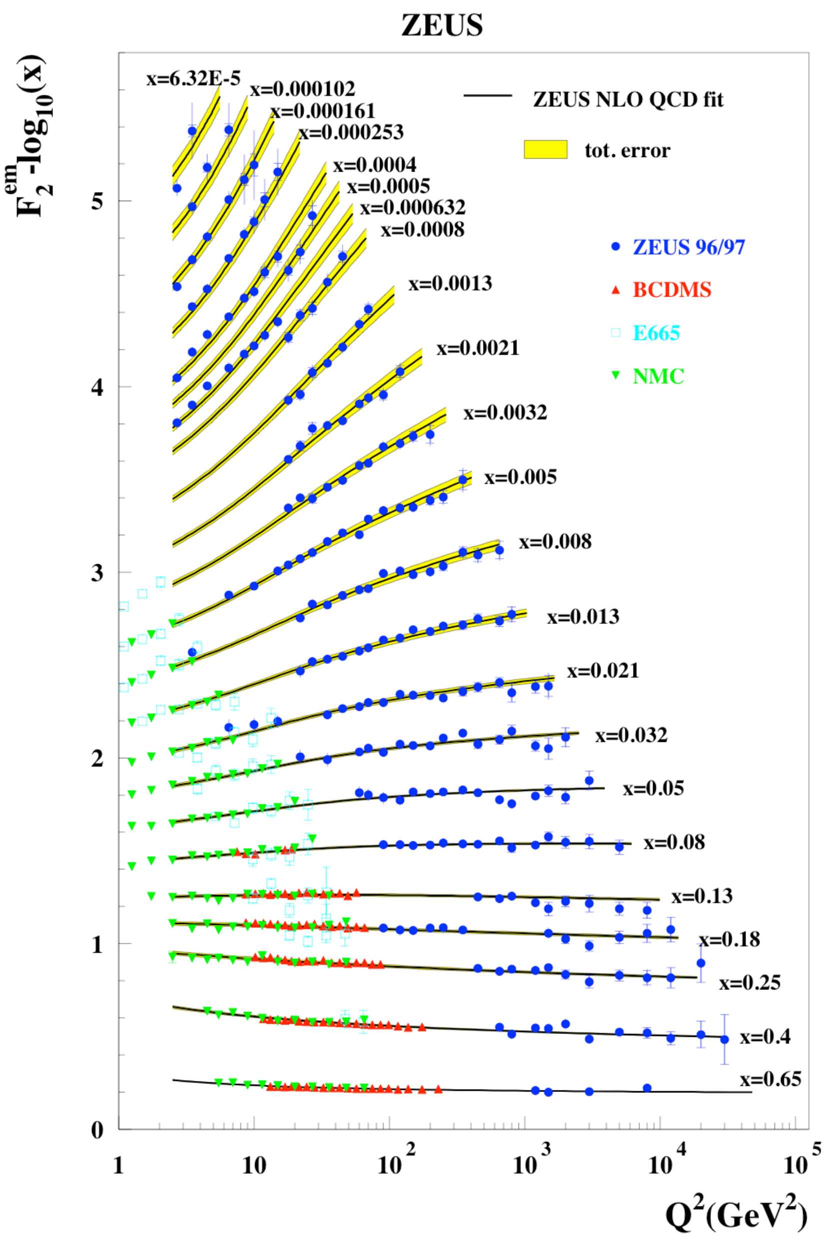
In the Lab. Frame $\quad \nu = E_1 - E_3$

Energy lost by the incoming particle

Experimental Measurements of F_2

- Scaling and scaling violation



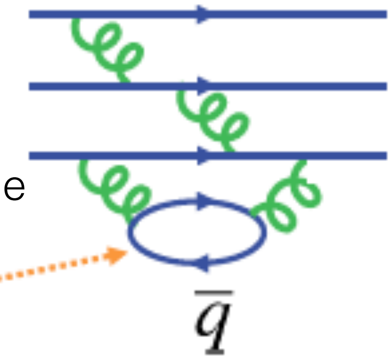


Structure Functions, parton distributions

- Measure X-sections $\Longrightarrow$ extract Structure Functions $\Longrightarrow$ determine PDF

$$F_2^{\mathrm{p}}(x) = x\sum_q e_q^2 q^{\mathrm{p}}(x)$$

Due to higher orders need to include anti-up and anti-down quarks



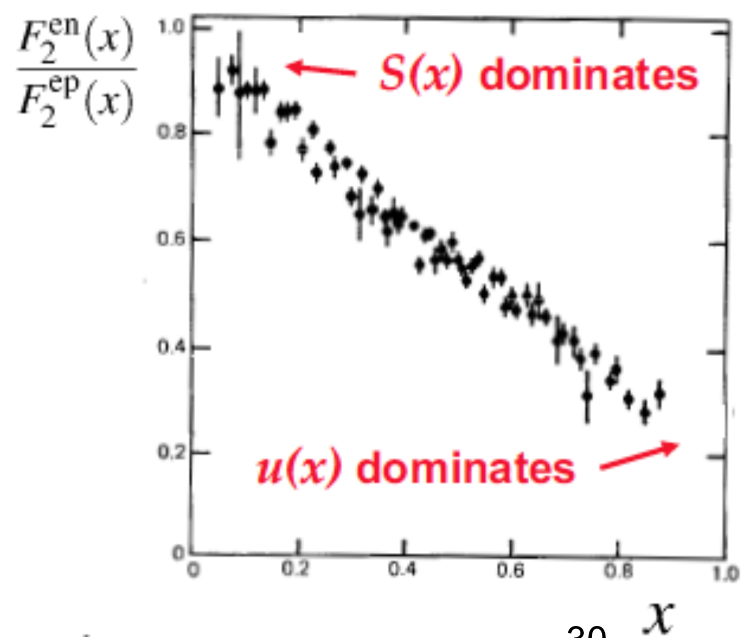
- Assuming "isospin symmetry"

$$\boxed{F_2^{\mathrm{ep}}(x) = 2xF_1^{\mathrm{ep}}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right)}$$

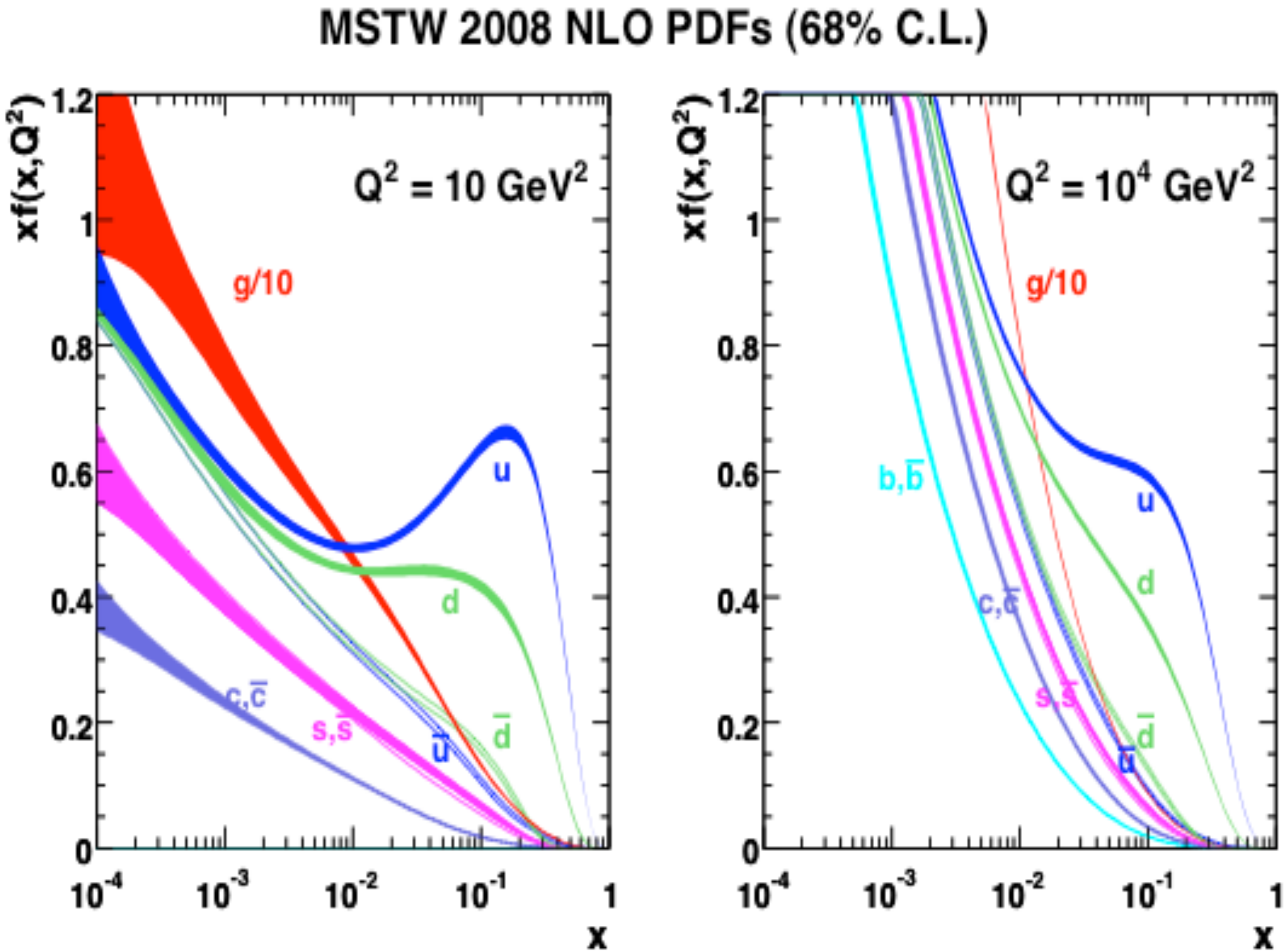
$$\boxed{F_2^{\mathrm{en}}(x) = 2xF_1^{\mathrm{en}}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right)}$$

- Resolving into "sea" and "valence" quark contributions

$$\frac{F_2^{\mathrm{en}}(x)}{F_2^{\mathrm{ep}}(x)} = \frac{4d_{\mathrm{V}}(x) + u_{\mathrm{V}}(x) + 10S(x)}{4u_{\mathrm{V}}(x) + d_{\mathrm{V}}(x) + 10S(x)}$$



See 2016, Q4(b).


MSTW 2008 NLO PDFs (68% C.L.)
$xf(x,Q^2)$
$Q^2 = 10$ GeV^2
g/10
u
d
c,$\bar{c}$
s,$\bar{s}$
$\bar{u}$
$\bar{d}$
$xf(x,Q^2)$
$Q^2 = 10^4$ GeV^2
g/10
b,$\bar{b}$
u
d
c,$\bar{c}$
s,$\bar{s}$
$\bar{u}$
$\bar{d}$
0
0.2
0.4
0.6
0.8
1
1.2
10^{-4}
10^{-3}
10^{-2}
10^{-1}
1
x

Neutrino Mixing

- Reminder two flavour model

$$\begin{pmatrix} |\nu_e> \\ |\nu_\mu> \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1> \\ |\nu_2> \end{pmatrix}$$

- Starting with a beam of pure muon neutrinos you can derive the probability of muon disappearance

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2(1.27\Delta m^2 L / E)$$

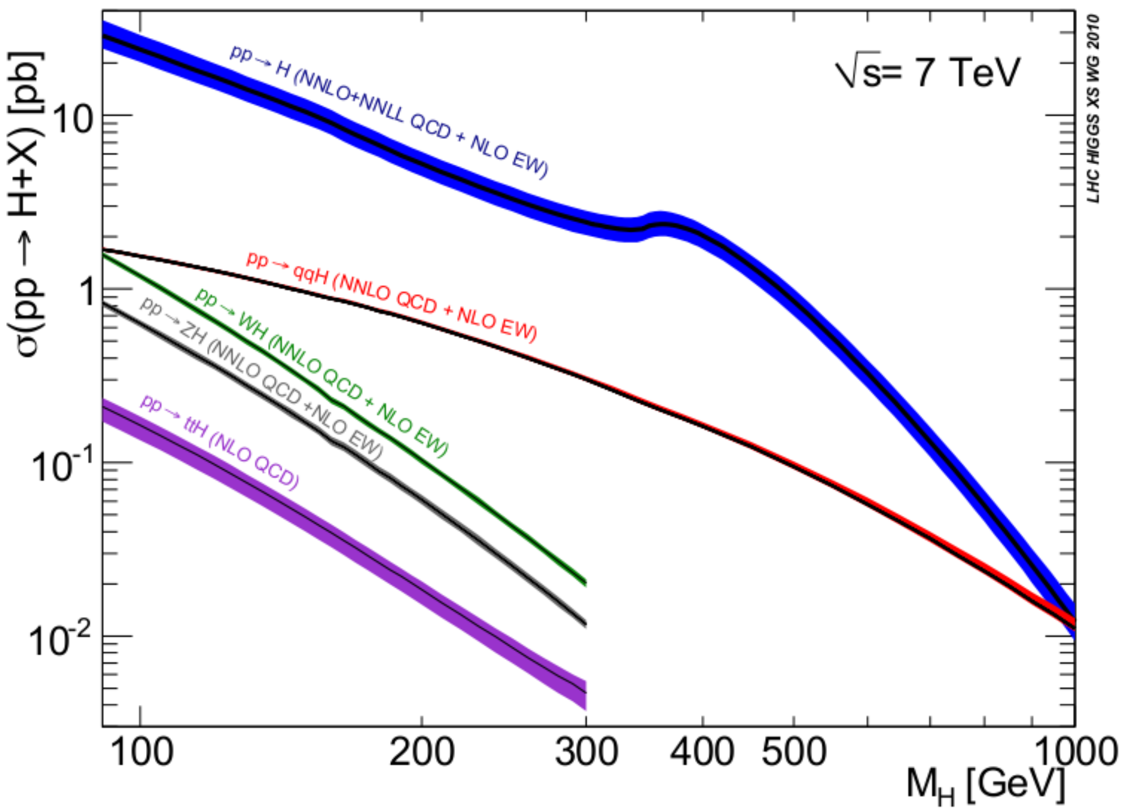
- Three flavour model more complicated

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

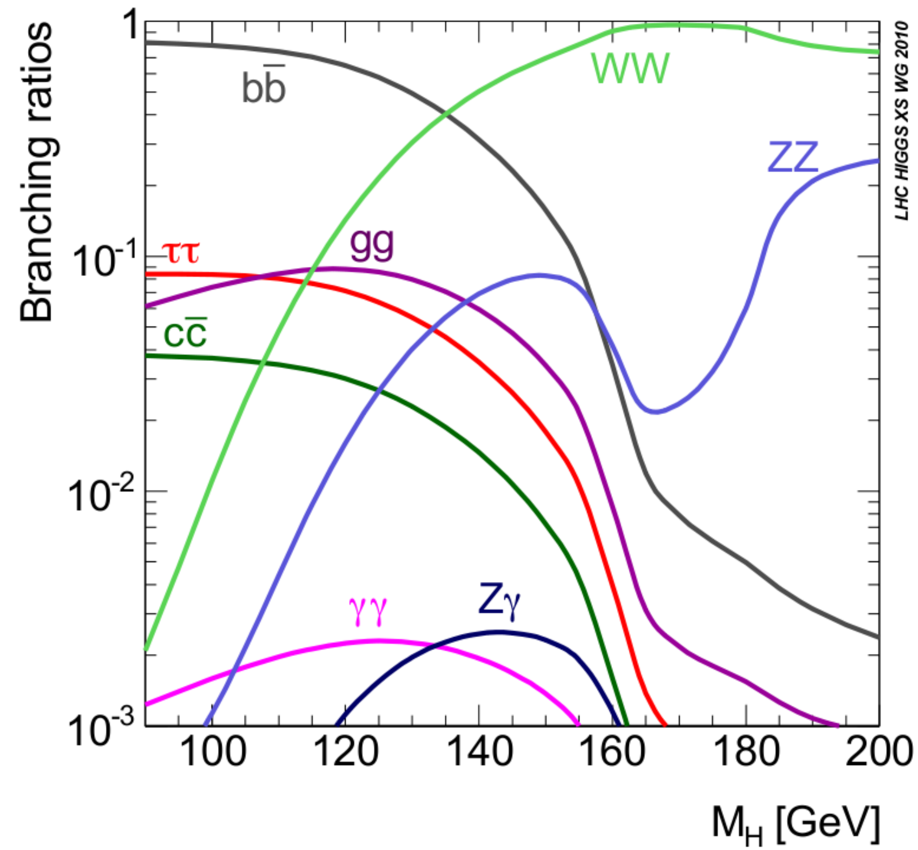
- the θ_{13} parameter was measured for the first time in 2012

Making and Detecting a Higgs Boson @ LHC

Higgs Cross Section



Higgs Branching Ratios



Electroweak Feynman Factors

- Photon
  - Vertex: $-ie\gamma^\mu q_f$
  - Propagator: $-i\dfrac{g_{\mu\nu}}{q^2}$
- W-boson
  - Vertex: $\dfrac{-ig_W}{\sqrt{2}}\gamma^\mu\dfrac{1}{2}\left(1-\gamma^5\right)$
  - Propagator: $\dfrac{-i\left(g_{\mu\nu}-q_\mu q_\nu/M_W^2\right)}{q^2-M_W^2}$
- Z-boson
  - Vertex: $\dfrac{-ig_W}{\cos\theta_W}\gamma^\mu\dfrac{1}{2}\left(C_V-C_A\gamma^5\right)$
  - Propagator: $\dfrac{-i\left(g_{\mu\nu}-q_\mu q_\nu/M_Z^2\right)}{q^2-M_Z^2}$

Reminder:

$$g_W = \frac{g_{EM}}{\sin\theta_W}$$

$$g_Z = \frac{g_W}{\cos\theta_W}$$

$$M_W = M_Z\cos\theta_W$$

$$\theta_W \approx 28.75^\circ$$

$C_A = 1/2$ for all ν and u,c,t

$C_A = -1/2$ for e, μ, τ,d,s and b

$C_V = C_A - 2q_f\sin^2\theta_W$

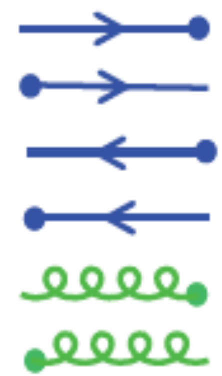
See 2016, Q5(b).

Feynman Rules for QCD

- **External Lines**

spin 1/2
- incoming quark $u(p)$
- outgoing quark $\bar{u}(p)$
- incoming anti-quark $\bar{v}(p)$
- outgoing anti-quark $v(p)$

spin 1
- incoming gluon $\varepsilon^{\mu}(p)$
- outgoing gluon $\varepsilon^{\mu}(p)^{*}$



- **Internal Lines (propagators)**

spin 1 gluon

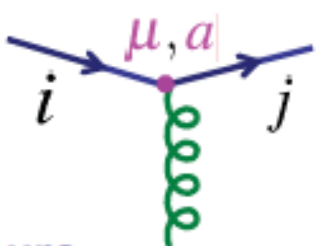
$$\boxed{\frac{-ig_{\mu\nu}}{q^2}\delta^{ab}}$$

a, b = 1,2,…,8 are gluon colour indices

- **Vertex Factors**

spin 1/2 quark

$$\boxed{-ig_s\tfrac{1}{2}\lambda^{a}_{ji}\gamma^{\mu}}$$

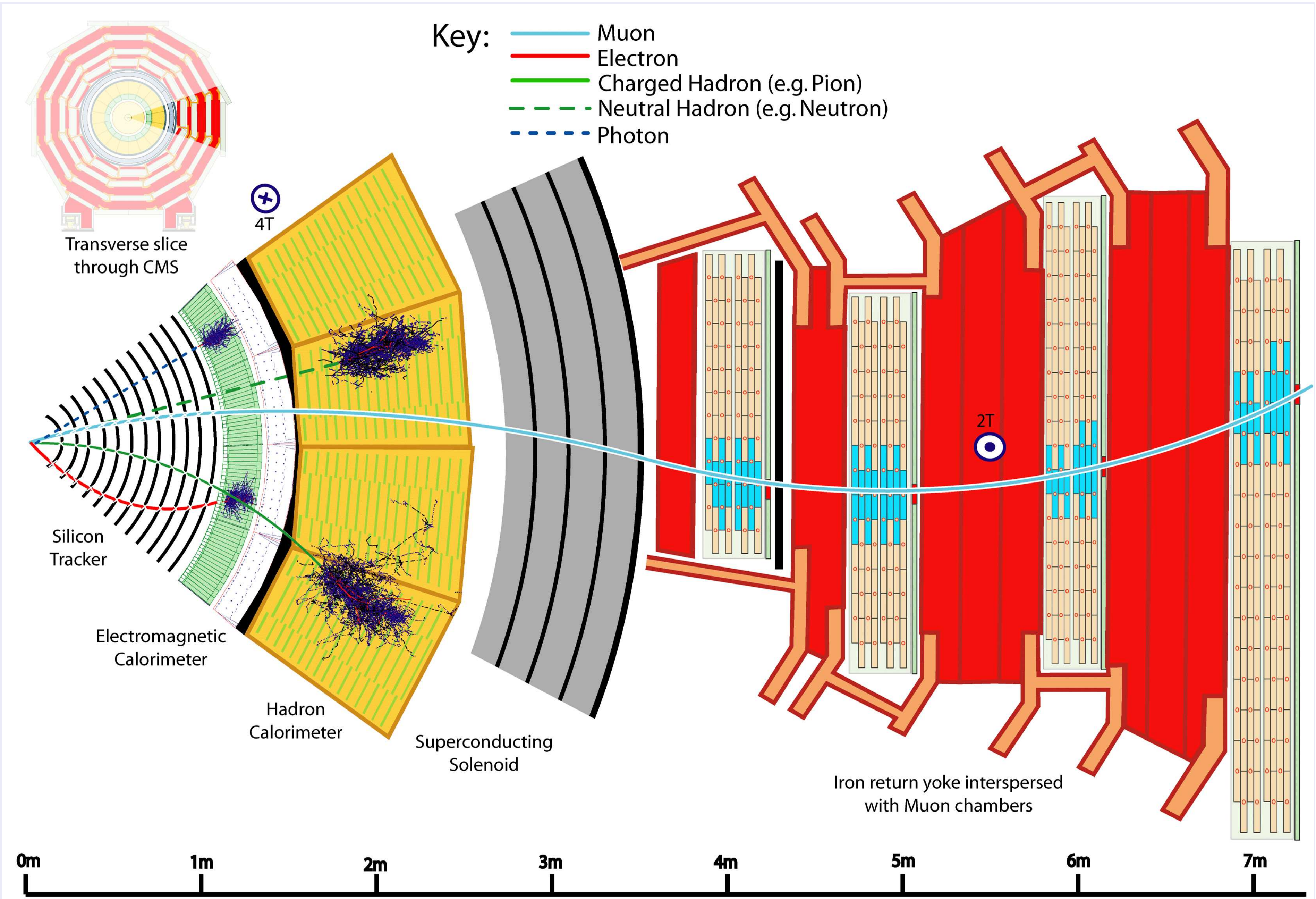


i, j = 1,2,3 are quark colours,

λ^{a} a = 1,2,..8 are the Gell-Mann SU(3) matrices

- **+ 3 gluon and 4 gluon interaction vertices**
- **Matrix Element** $-iM$ = product of all factors

Example Detector (CMS)



Experimental Methods Questions

- Particles are characterised by:
  - mass, charge, interactions and lifetime
- Observables include:
  - Energy, momentum, velocity, energy loss (dE/dx), track/jet vertices, etc.
- Questions about experimental methods, typical ask how to identify a given final state
  - Need to say what particles are the final state, which detector components are used to identify them (e.g. muon chambers for muons, EM calor + no track for photons, etc.)
  - Some particles (e.g. tau leptons, B-mesons) travel a finite distance giving a secondary vertex (measured with fine grain tracking detector) and can identify the particle by calculating the invariant mass of the daughters

Past Exam Paper Observations

- Looking at the past 5 years of exam papers (very roughly speaking)
  - 20 questions feature Feynman diagrams and rules
  - 13 questions feature kinematics
  - 11 questions feature chirality and/or helicity
  - 10 questions feature aspects of experimental measurement
  - 7 questions discuss Lagrangians
  - 7 questions feature neutrino oscillations and mass, and BSM physics
  - 6 questions feature the Higgs mechanism
  - 6 questions feature CKM matrix considerations
  - 6 questions feature Dirac matrix manipulation
  - 5 questions feature evidence for quarks/colour
  - 4 questions ask about PDFs
  - 3 questions ask about renormalisation