PHASM/G442 Particle Physics

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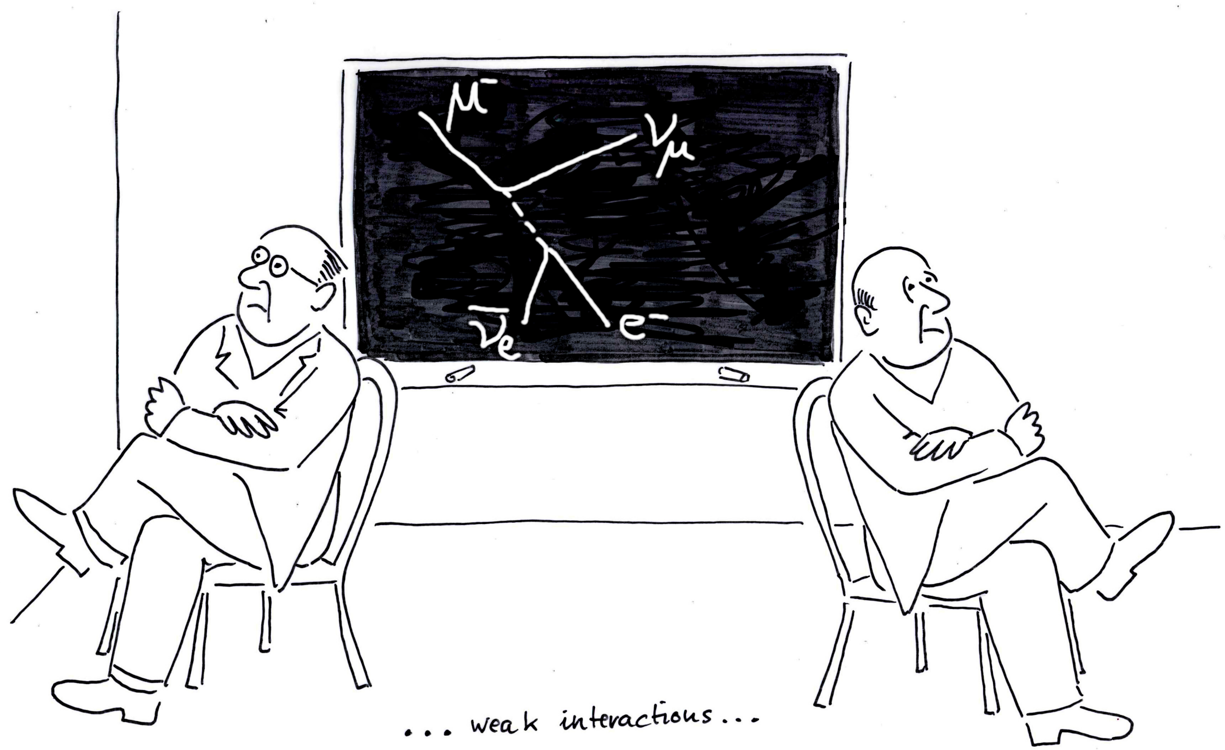
Module VII

Weak Interactions



http://moodle.ucl.ac.uk/course/view.php?id=2589

Parity (reminder)

- Parity operator performs spatial inversion through the origin

$$\psi'(\vec{x},t) = \hat{P}\psi(\vec{x},t) = \psi(-\vec{x},t)$$

- Parity is an **observable conserved** quantity with eigenvalues $P = \pm 1$
- **QED** and **QCD** are **invariant** under **parity**

Spin-1 Bosons $P_\gamma = P_g = P_{W^+} = P_{W^-} = P_Z = -1$

Spin-½ Fermions $P_{e^-} = P_{\mu^-} = P_{\tau^-} = P_\nu = P_q = +1$

$P_{e^+} = P_{\mu^+} = P_{\tau^+} = P_{\bar{\nu}} = P_{\bar{q}} = -1$

Conventional choice of the sign

- Parity Operator:

$$\boxed{\hat{P} = \gamma^0 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}}$$

Parity Violation in β-decay

- Under Parity transformation:

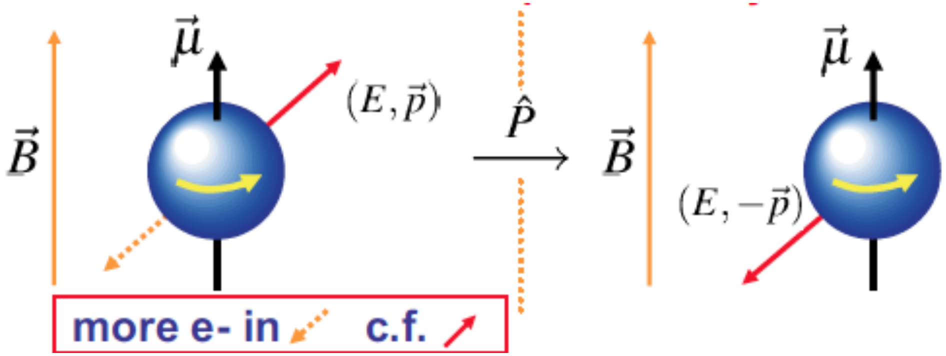
Vectors
change sign

$$\vec{r} \xrightarrow{\hat{P}} -\vec{r}$$
$$\vec{p} \xrightarrow{\hat{P}} -\vec{p} \qquad \left(p_x = \frac{\partial}{\partial x},\ etc.\right)$$

Axial-Vectors
unchanged

$$\vec{L} \xrightarrow{\hat{P}} \vec{L} \qquad (\vec{L} = \vec{r} \wedge \vec{p})$$
$$\vec{\mu} \xrightarrow{\hat{P}} \vec{\mu} \qquad (\vec{\mu} \propto \vec{L})$$

- C.S. Wu et al. studied β-decay of polarised cobalt-60 nuclei
- Observed **electrons emitted preferentially** in direction **opposite** to applied field



If parity were conserved: expect equal rate for producing e⁻ in directions along and opposite to the nuclear spin.

- Therefore **Parity is violated in Weak Interaction**
- Weak Interaction vertex is **not** of the form $\overline{u}_e \gamma^{\mu} u_{\nu}$

Bilinear Covariants

- The requirement of **Lorentz invariance** of the matrix element restricts the form of the interaction vertex. E.g. **QED** and **QCD** are **VECTOR** interactions

$$j^\mu = \overline{\psi}\gamma^\mu\phi \quad \text{which transforms as a 4-vector}$$

- There are only 5 possible combinations of two spinors and gamma-matrices that form Lorentz invariant currents - **bilinear covariants**

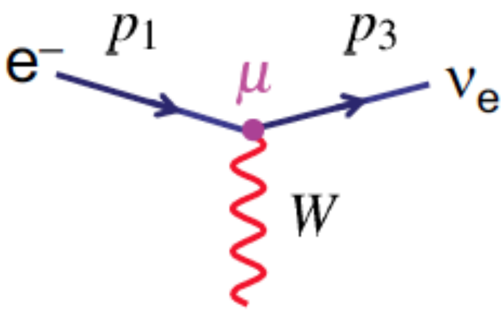
Type	Form	Components	"Boson Spin"
♦ SCALAR	$\overline{\psi}\phi$	1	0
♦ PSEUDOSCALAR	$\overline{\psi}\gamma^5\phi$	1	0
♦ VECTOR	$\overline{\psi}\gamma^\mu\phi$	4	1
♦ AXIAL VECTOR	$\overline{\psi}\gamma^\mu\gamma^5\phi$	4	1
♦ TENSOR	$\overline{\psi}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)\phi$	6	2

- The most general form for interaction between fermion and boson is a linear combination of bilinear covariants

- For interactions in which a **spin-1 boson** is exchanged it is a linear combination of **VECTOR** and **AXIAL VECTOR**

V-A Structure of Weak Interaction

- The form for weak interaction is **determined from experiment** to be of (**VECTOR - AXIAL VECTOR**) $\Longrightarrow$ **V-A** type



$$j^{\mu} \propto \overline{u}_{\nu_e}(\gamma^{\mu} - \gamma^{\mu}\gamma^{5})u_e$$

V – A

Can this account for **parity violation**?

- Consider the **axial-vector** under parity transformation

$$j_A = \overline{\psi}\gamma^{\mu}\gamma^{5}\phi \qquad \text{with} \qquad \gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3; \quad \gamma^5\gamma^0 = -\gamma^0\gamma^5$$

$$j_A = \overline{\psi}\gamma^{\mu}\gamma^{5}\phi \xrightarrow{\hat{P}} \overline{\psi}\gamma^{0}\gamma^{\mu}\gamma^{5}\gamma^{0}\phi = -\overline{\psi}\gamma^{0}\gamma^{\mu}\gamma^{0}\gamma^{5}\phi$$

$$j_A^0 = \xrightarrow{\hat{P}} -\overline{\psi}\gamma^{0}\gamma^{0}\gamma^{0}\gamma^{5}\phi = -\overline{\psi}\gamma^{0}\gamma^{5}\phi = -j_A^0$$

- The space-like component remains unchanged, while the time-like component changes sign in axial-vector transformation under parity

$$\boxed{j_A^0 \xrightarrow{\hat{P}} -j_A^0; \quad j_A^k \xrightarrow{\hat{P}} +j_A^k;}$$

- The situation is the opposite in vector currents

$$\boxed{j_V^0 \xrightarrow{\hat{P}} +j_V^0; \quad j_V^k \xrightarrow{\hat{P}} -j_V^k}$$

- The matrix element is $M \propto g_{\mu\nu} j_1^\mu j_2^\nu = j_1^0 j_2^0 - \sum_{k=1,3} j_1^k j_2^k$

- For combination of 2 axial-vector currents

$$j_{A1} \cdot j_{A2} \xrightarrow{\hat{P}} (-j_1^0)(-j_2^0) - \sum_{k=1,3} (j_1^k)(j_2^k) = j_{A1} \cdot j_{A2}$$

(same for 2 vector currents but with negative signs in space-like components)

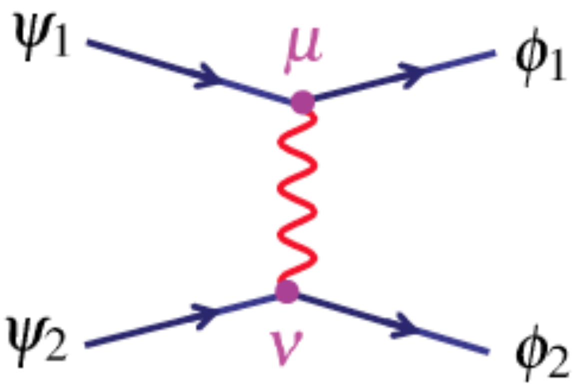
- Therefore parity is conserved in pure vector and pure axial-vector interactions

- However a **combination** of **V** and **A** <u>changes sign</u> under <u>parity</u>

$$j_{V1} \cdot j_{A2} \xrightarrow{\hat{P}} (j_1^0)(-j_2^0) - \sum_{k=1,3} (-j_1^k)(j_2^k) = -j_{V1} \cdot j_{A2}$$

Parity Violation!

V-A, V+A, g_V and g_A



$$\left\{\begin{array}{c} j_1 = \overline{\phi}_1 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_1 = g_V j_1^V + g_A j_1^A \\ \dfrac{g_{\mu\nu}}{q^2 - m^2} \\ j_2 = \overline{\phi}_2 (g_V \gamma^\mu + g_A \gamma^\mu \gamma^5) \psi_2 = g_V j_2^V + g_A j_2^A \end{array}\right.$$

$$M_{fi} \propto j_1 . j_2 = g_V^2 j_1^V . j_2^V + g_A^2 j_1^A . j_2^A + g_V g_A (j_1^V . j_2^A + j_1^A . j_2^V)$$

- Consider the parity transformation of this scalar product

$$j_1 . j_2 \xrightarrow{\hat{P}} g_V^2 j_1^V . j_2^V + g_A^2 j_1^A . j_2^A - g_V g_A (j_1^V . j_2^A + j_1^A . j_2^V)$$

- If either g_A or g_V is zero, Parity is conserved, i.e. parity conserved in a pure VECTOR or pure AXIAL-VECTOR interaction

- Relative strength of parity violating part $\propto \dfrac{g_V g_A}{g_V^2 + g_A^2}$

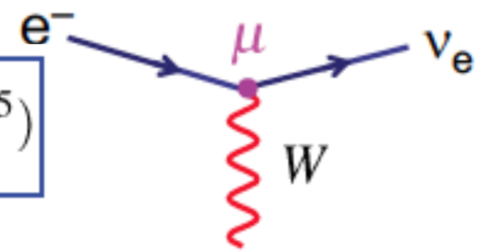
$$\boxed{\text{Maximal Parity Violation for V-A (or V+A)}}$$

Helicity Structure of Weak Interaction

- Recall Chiral projection operators $P_R = \frac{1}{2}(1+\gamma^5); \quad P_L = \frac{1}{2}(1-\gamma^5)$

- The charge-current (**W±**) weak interaction vertex:

$$\boxed{\frac{-ig_w}{\sqrt{2}}\frac{1}{2}\gamma^\mu(1-\gamma^5)}$$



- The current is $\overline{\psi}\frac{1}{2}\gamma^\mu(1-\gamma^5)\phi = \overline{\psi}\gamma^\mu\phi_L$

- Since $\overline{\psi} = \overline{\psi}_R + \overline{\psi}_L$ and, as known from QED, $\overline{\psi}_R\gamma^\mu\phi_L = 0$

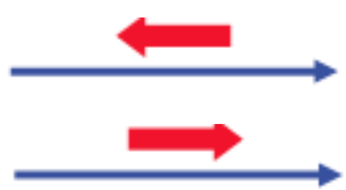
$$\overline{\psi}\frac{1}{2}\gamma^\mu(1-\gamma^5)\phi = \overline{\psi}_L\gamma^\mu\phi_L$$

⇒ **Only the left-handed chiral components of particle spinors and right-handed chiral components of anti-particle spinors participate in charged current weak interactions**

- At relativistic energies $(E \gg m)$ chiral components are helicity eigenstates

$\frac{1}{2}(1-\gamma^5)u \Rightarrow$

$\frac{1}{2}(1+\gamma^5)v \Rightarrow$

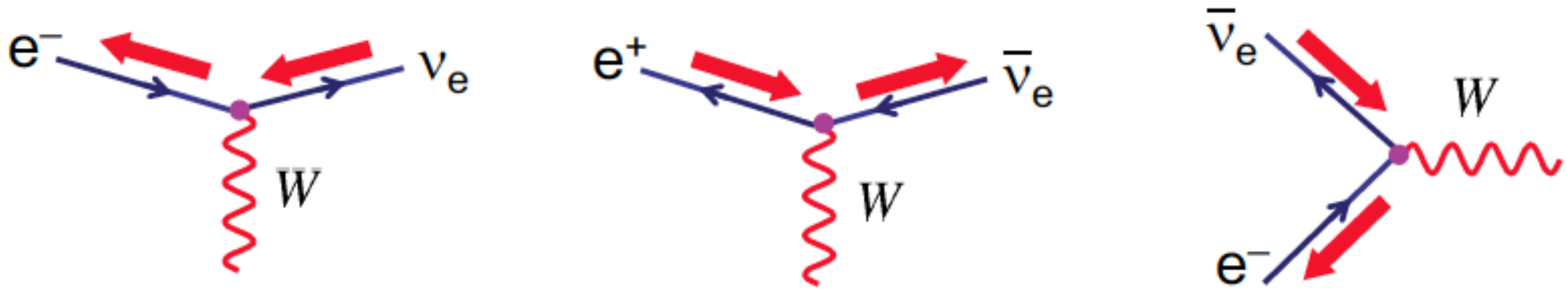


LEFT-HANDED PARTICLES
Helicity = -1

RIGHT-HANDED ANTI-PARTICLES
Helicity = +1

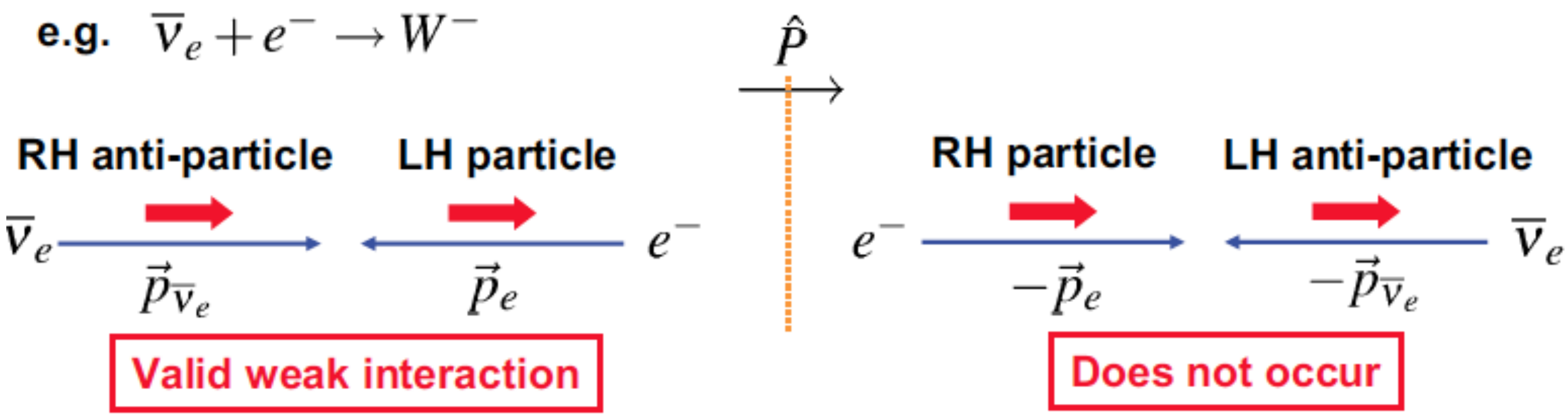
In the ultra-relativistic limit only left-handed particles and right-handed antiparticles participate in charged current weak interactions

e.g. In the relativistic limit, the only possible electron – neutrino interactions are:

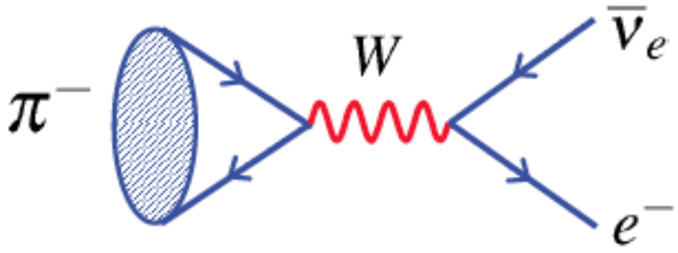


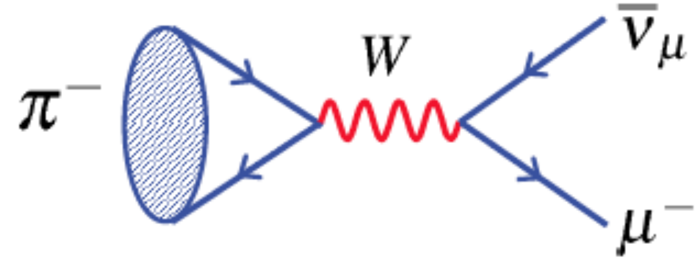
The helicity dependence of the weak interaction $\longleftrightarrow$ parity violation

e.g. $\overline{\nu}_e + e^- \rightarrow W^-$



Pion decay and Helicity





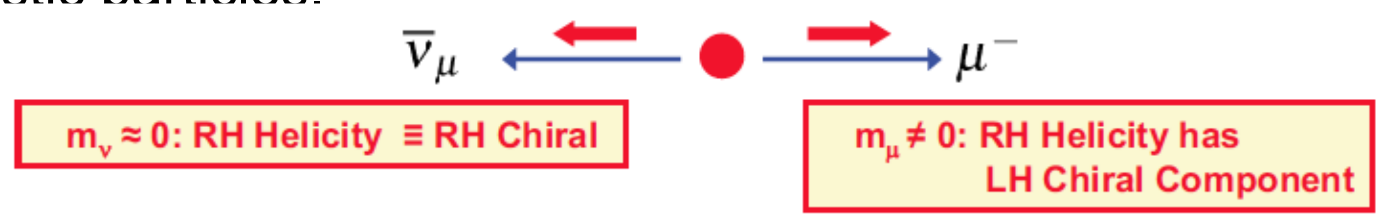
- Might expected the decay to electrons to dominate — larger phase space
- Experimentally $$\frac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} = 1.23 \times 10^{-4}$$
- Explanation is in the spin structure of weak interaction
- To conserve angular momentum:

But only LH **Chiral** particle states participate in CC weak interaction

$$\underbrace{\boxed{u_\uparrow}}_{\text{RH Helicity}} = P_R u_\uparrow + P_L u_\uparrow = \frac{1}{2}\left(1+\frac{|\vec{p}|}{E+m}\right)\underbrace{\boxed{u_R}}_{\text{RH Chiral}} + \frac{1}{2}\left(1-\frac{|\vec{p}|}{E+m}\right)\underbrace{\boxed{u_L}}_{\text{LH Chiral}}$$

- In the limit $E \gg m$ RH Chiral and helicity states are identical

- Although only LH Chiral particle states participate in CC weak interaction, contribution from RH Helicity is not necessarily zero, especially for non-relativistic particles.



- Matrix element is proportional to LH Chiral component of RH Helicity of electron/muon spinor

$$M_{fi} \propto \frac{1}{2}\left(1-\frac{|\vec{p}|}{E+m}\right) = \boxed{\frac{m_\mu}{m_\pi + m_\mu}}$$

from the kinematics of pion decay at rest

$\pi^- \to e^- \overline{\nu}_e$ is heavily suppressed

Evidence for V-A

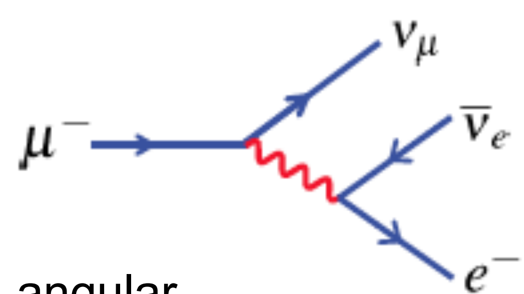
- V-A fits experimental observations

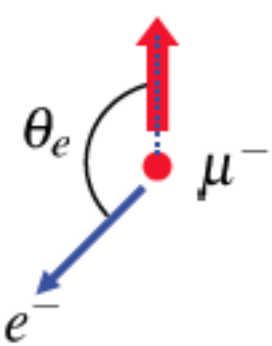
Experimentally: $$\frac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} = (1.230 \pm 0.004) \times 10^{-4}$$

Theoretical predictions for **V-A** $(\overline{\psi}\gamma^\mu(1-\gamma^5)\phi)$ ➡ $\frac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} \approx 1.3 \times 10^{-4}$

Scalar $(\overline{\psi}\phi)$ or **Pseudo-Scalar** $(\overline{\psi}\gamma^5\phi)$ ➡ $\frac{\Gamma(\pi^- \to e^- \overline{\nu}_e)}{\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu)} = 5.5$

- Another example - muon decay





Measure electron energy and angular distribution relative to muon spin direction. Express results in terms of “Michel parameters”

Results consistent with V-A

e.g. TWIST expt: 6x10^9 μ decays
Phys. Rev. Lett. 95 (2005) 101805

W-boson propagator

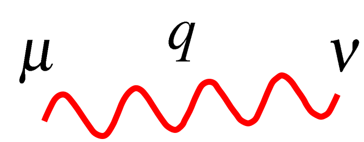
- Unlike QED and QCD the Charged Current (**CC**) weak interaction is mediated by **massive** (**80 GeV**) **W±** bosons

massless		**massive**
$\frac{1}{q^2}$	$\longrightarrow$	$\frac{1}{q^2 - m^2}$

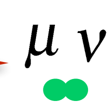
- Including possible polarisation states

spin 1 W$^{\pm}$

$$\frac{-i\left[g_{\mu\nu} - q_\mu q_\nu / m_W^2\right]}{q^2 - m_W^2}$$



- For $q^2 \ll m_W^2$ it becomes $\frac{i g_{\mu\nu}}{m_W^2}$

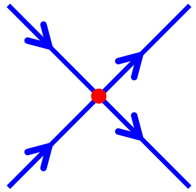


point-like interaction!

Connection with Fermi theory of β-decay

- In 1934, Fermi proposed a theory analogous to QED (vector interaction)

$$M_{fi} = G_{\mathrm{F}}\, g_{\mu\nu} [\overline{\psi}\gamma^{\mu}\psi][\overline{\psi}\gamma^{\nu}\psi]$$



where $G_{\mathrm{F}} = 1.166 \times 10^{-5}\,\mathrm{GeV}^{-2}$

- After discovery of parity violation it was modified to

$$M_{fi} = \frac{G_{\mathrm{F}}}{\sqrt{2}} g_{\mu\nu} [\overline{\psi}\gamma^{\mu}(1-\gamma^5)\psi][\overline{\psi}\gamma^{\nu}(1-\gamma^5)\psi]$$

- Compare this with

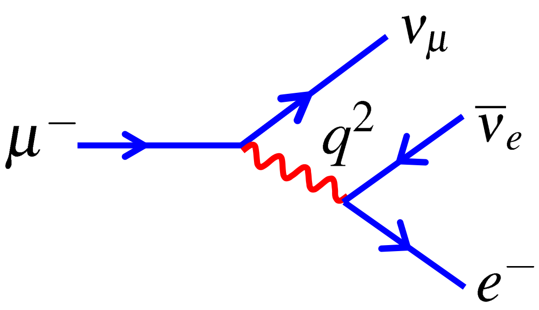
$$M_{fi} = [\tfrac{g_W}{\sqrt{2}}\overline{\psi}\tfrac{1}{2}\gamma^{\mu}(1-\gamma^5)\psi] \frac{g_{\mu\nu} - q_{\mu}q_{\nu}/m_W^2}{q^2 - m_W^2} [\tfrac{g_W}{\sqrt{2}}\overline{\psi}\tfrac{1}{2}\gamma^{\nu}(1-\gamma^5)\psi]$$

which for $q^2 \ll m_W^2$ becomes

$$M_{fi} = \frac{g_W^2}{8m_W^2} g_{\mu\nu} [\overline{\psi}\gamma^{\mu}(1-\gamma^5)\psi][\overline{\psi}\gamma^{\nu}(1-\gamma^5)\psi]$$

$$\Rightarrow \boxed{\frac{G_{\mathrm{F}}}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}}$$

Strength of Weak Interaction



Muon decay is a great tool to measure weak interaction strength

$$q^2 < m_\mu \ (0.106\,\mathrm{GeV})$$

$$\Rightarrow \quad M_{fi} = \frac{g_W^2}{8m_W^2} g_{\mu\nu} [\overline{\psi}\gamma^\mu (1-\gamma^5)\psi][\overline{\psi}\gamma^\nu (1-\gamma^5)\psi]$$

In muon decay you measure g_W^2/m_W^2 From $\frac{G_\mathrm{F}}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}$

and current knowledge of $m_W = 80.403 \pm 0.029\,\mathrm{GeV}$

obtain $\boxed{G_\mathrm{F} = 1.16639(1) \times 10^{-5}\,\mathrm{GeV}^{-2}}$

$$\Rightarrow \quad \alpha_W = \frac{g_W^2}{4\pi} = \frac{8m_W^2 G_\mathrm{F}}{4\sqrt{2}\pi} = \frac{1}{30} \qquad \text{c.f.} \quad \alpha_{EM} = \frac{1}{137}$$

The intrinsic strength of the weak interaction is similar to, but greater than, the EM interaction ! It is the massive W-boson in the propagator which makes it appear weak. For $q^2 \gg m_W^2$ weak interactions are more likely than EM.

Summary of Weak Interaction

- Weak Interaction is of form Vector - Axial_vector, **V-A**

$$\boxed{\frac{-ig_W}{\sqrt{2}}\frac{1}{2}\gamma^{\mu}(1-\gamma^5)}$$

- As a result, only **left-handed** particle **chiral** states and **right-handed** anti-particle **chiral** states participate in **Weak Interaction**

➡ **MAXIMAL PARITY VIOLATION**

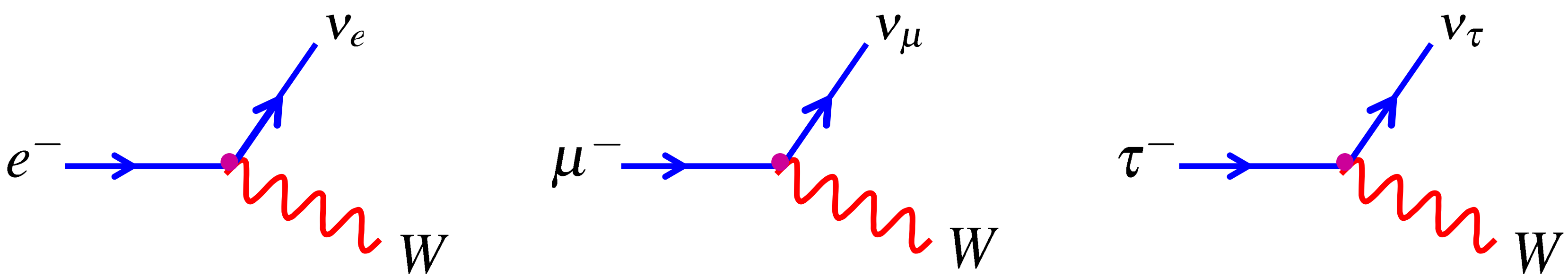
- At low q^2 weak interaction is only weak because of large W-mass

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8m_W^2}}$$

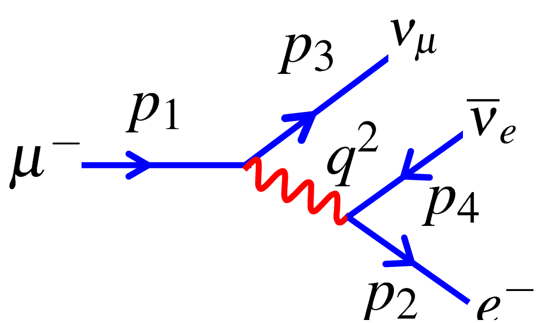
- **Intrinsic strength of Weak Interaction at high q^2 is simular to that of QED**

Lepton Universality

- Leptonic charged-current interaction vertices



- Muon decay:



$$M_{fi} = \frac{g_W^{(e)} g_W^{(\mu)}}{8m_W^2}[\overline{u}(p_3)\gamma^\mu(1-\gamma^5)u(p_1)]g_{\mu\nu}[\overline{u}(p_2)\gamma^\nu(1-\gamma^5)v(p_4)]$$

- After a lot of hard work (integration over 3-body phase space)

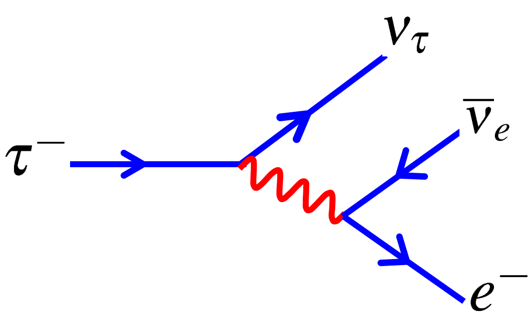
$$\Gamma(\mu \to e\nu\nu) = \frac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \frac{1}{\tau_\mu} \quad \textbf{with} \quad G_F = \frac{g_W^2}{4\sqrt{2}m_W^2}$$

Important result (rate dependence on parent particle mass)

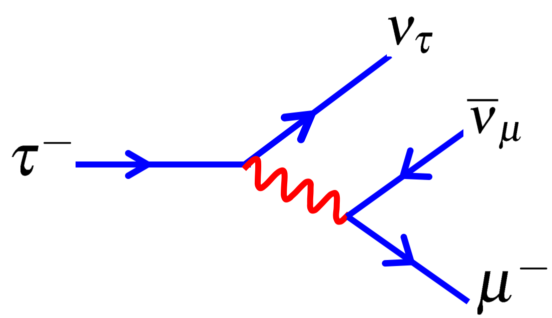
- So for muon $\Gamma(\mu \to e\nu\nu) = \dfrac{G_F^e G_F^\mu m_\mu^5}{192\pi^3} = \dfrac{1}{\tau_\mu}$

- Similarly for tau-lepton $\Gamma(\tau \to e\nu\nu) = \dfrac{G_F^e G_F^\tau m_\tau^5}{192\pi^3}$

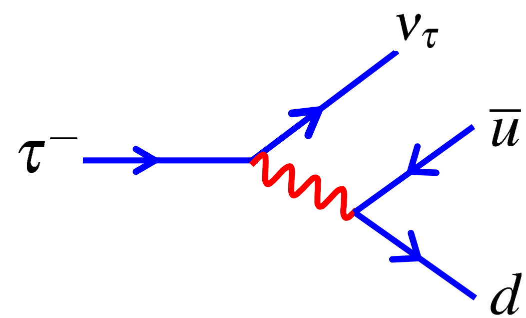
- However tau (mass = 1.78 GeV) can decay to different final states



$Br(\tau \to e\nu\nu) = 0.1784(5)$



$Br(\tau \to \mu\nu\nu) = 0.1736(5)$



- The total decay rate: $\Gamma = \sum_i \Gamma_i = \dfrac{1}{\tau}$

$$\Gamma(\tau \to e\nu\nu) = \Gamma_\tau Br(\tau \to e\nu\nu) = Br(\tau \to e\nu\nu)/\tau_\tau$$

- Therefore predict $\tau_\mu = \dfrac{192\pi^3}{G_F^e G_F^\mu m_\mu^5}$ $\quad \tau_\tau = \dfrac{192\pi^3}{G_F^e G_F^\tau m_\tau^5} Br(\tau \to e\nu\nu)$

- All these quantities are precisely measured

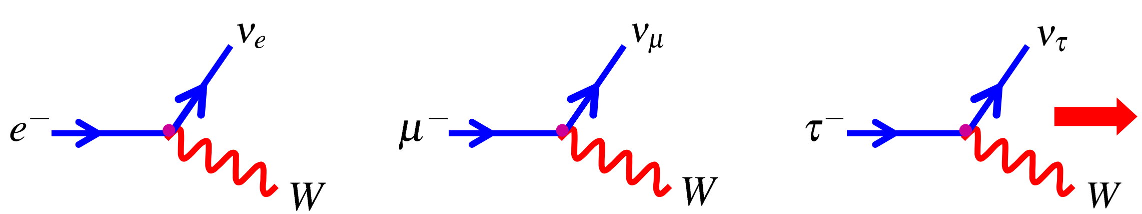
$$m_\mu = 0.1056583692(94)\,\mathrm{GeV} \qquad \tau_\mu = 2.19703(4)\times 10^{-6}\,\mathrm{s}$$
$$m_\tau = 1.77699(28)\,\mathrm{GeV} \qquad \tau_\tau = 0.2906(10)\times 10^{-12}\,\mathrm{s}$$
$$Br(\tau\to e\nu\nu) = 0.1784(5)$$

$$\Rightarrow \boxed{\frac{G_F^\tau}{G_F^\mu} = \frac{m_\mu^5\tau_\mu}{m_\tau^5\tau_\tau} Br(\tau\to e\nu\nu) = 1.0024\pm 0.0033}$$

- By comparing $Br(\tau\to e\nu\nu)$ and $Br(\tau\to\mu\nu\nu)$

$$\boxed{\frac{G_F^e}{G_F^\mu} = 1.000\pm 0.004}$$

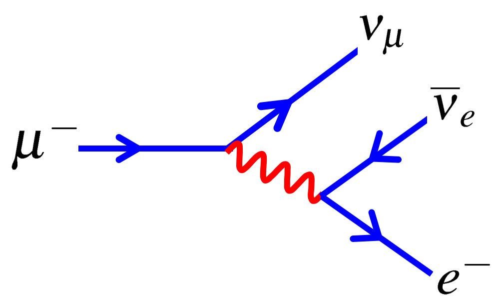
- Weak charge-current strength is the same for all leptonic vertices

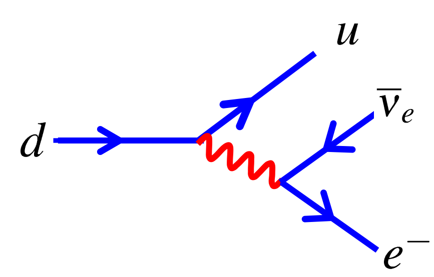


$\Rightarrow$ **Charged Current Lepton Universality**

Weak Interactions of Quarks

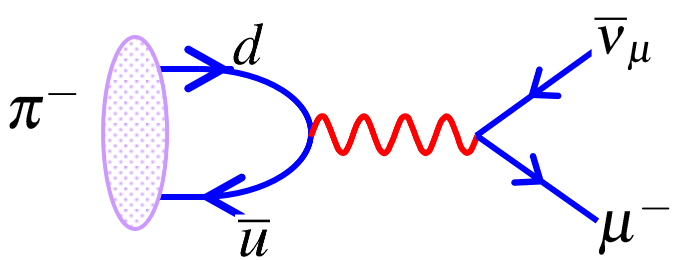
- Slightly different values of Gf were measured in μ and β decays

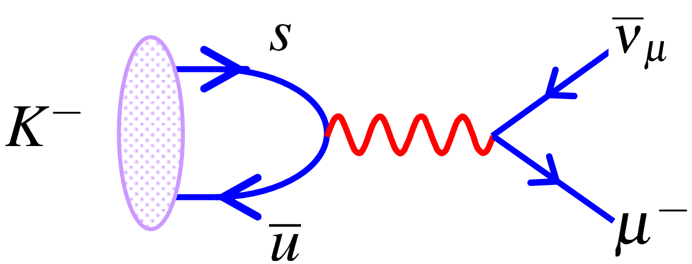




$$G_F^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2} \qquad G_F^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

- In addition, certain hadronic decays are observed to be suppressed, e.g.



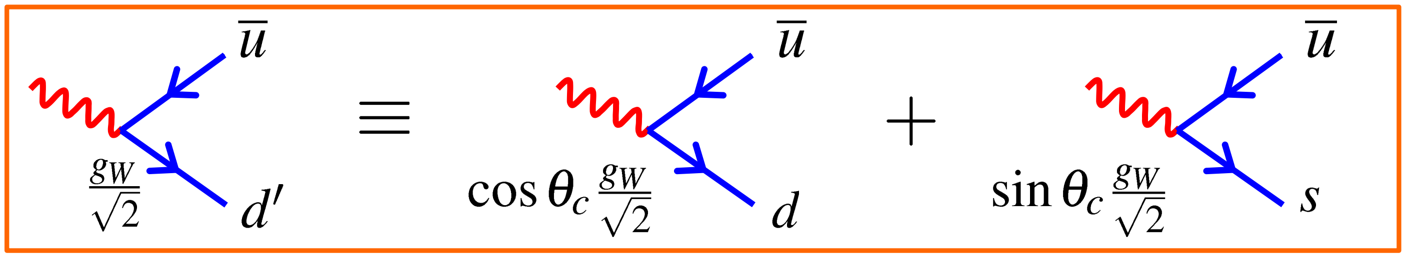


Kaon decay suppressed by a factor of ~20 compared to pion

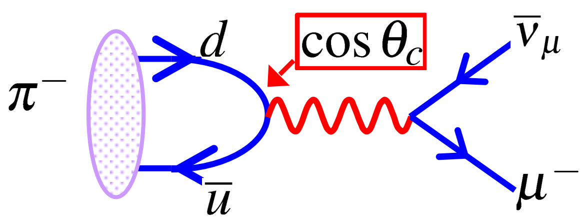
Cabibbo hypothesis

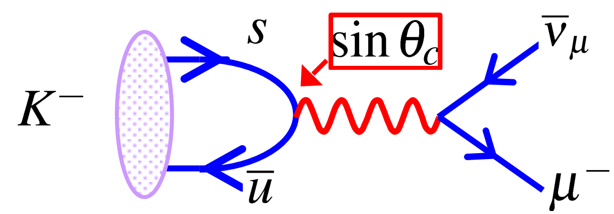
- Weak eigenstates are different form mass eigenstates for quarks, i.e. same interaction strength as for leptons but *u* couples to a linear combination of *d* and *s*

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$



- Therefore





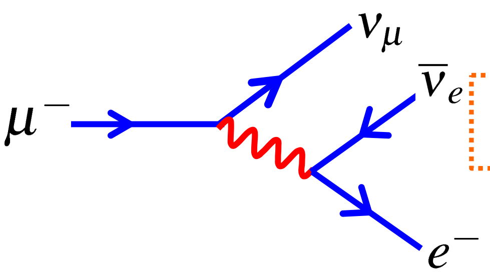
$$\Gamma(\pi^- \to \mu^- \overline{\nu}_\mu) \propto |M|^2 \propto \cos^2\theta_c$$

$$\Gamma(K^- \to \mu^- \overline{\nu}_\mu) \propto |M|^2 \propto \sin^2\theta_c$$

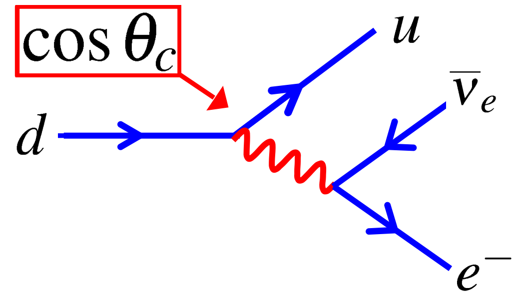
- With $\theta_c = 13.1^\circ$

$$\frac{\Gamma\left(K^- \to \mu^- \overline{\nu}_\mu\right)}{\Gamma\left(\pi^- \to \mu^- \overline{\nu}_\mu\right)} \approx \frac{\sin^2\theta_C}{\cos^2\theta_C} = \tan^2\theta_C \approx 0.05 = \frac{1}{20}$$

- Similarly for muon and beta-decay



$$\boxed{\propto G_\mathrm{F}.G_\mathrm{F}}$$



$$\boxed{\propto G_\mathrm{F}.(G_\mathrm{F}\cos^2\theta_c)}$$

- And therefore expect

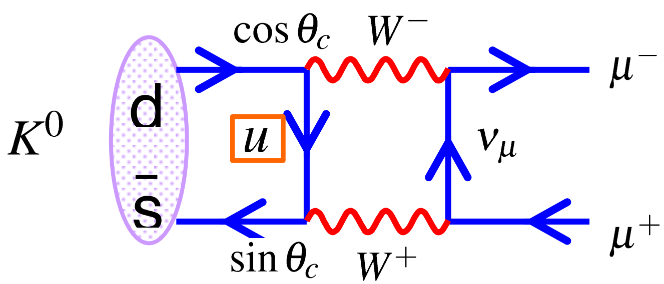
$$G_\mathrm{F}^\beta = G_\mathrm{F}^\mu \cos\theta_c$$

- Explaining the observed difference

$$G_\mathrm{F}^\mu = (1.16632 \pm 0.00002) \times 10^{-5}\,\mathrm{GeV}^{-2} \quad G_\mathrm{F}^\beta = (1.136 \pm 0.003) \times 10^{-5}\,\mathrm{GeV}^{-2}$$

GIM Mechanism

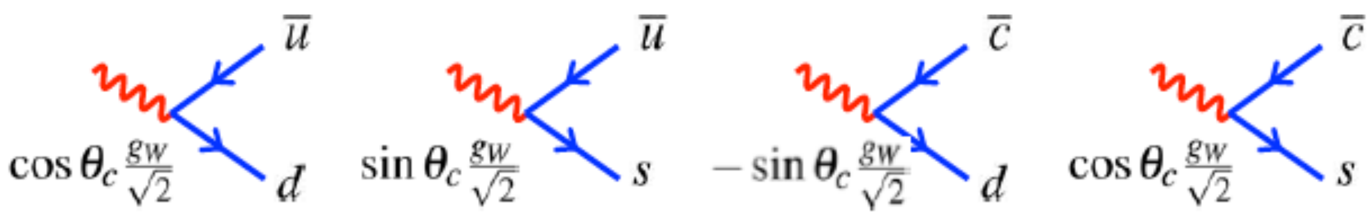
- Due to couplings between *ud* and *us* expect **box diagrams**:



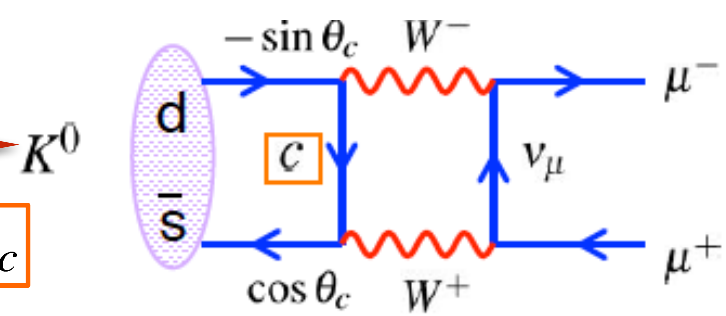
$$M_1 \propto g_W^4 \cos\theta_c \sin\theta_c$$

But much smaller branching ration was observed

- To explain the observation **Glashow**, **Illiopoulos** and **Maiani** postulated existence of extra quark - charm (**c**-quark).



- which gives rise to another box diagram



$$\boxed{M_2 \propto -g_W^4 \cos\theta_c \sin\theta_c}$$

- The final matrix element $|M|^2 = |M_1 + M_2|^2 \approx 0$

- **C-quark discovered in 1974 at SLAC and BNL** $J/\psi = c\bar{c}$

CKM Matrix

- Extend this idea to three quark flavours (generations)

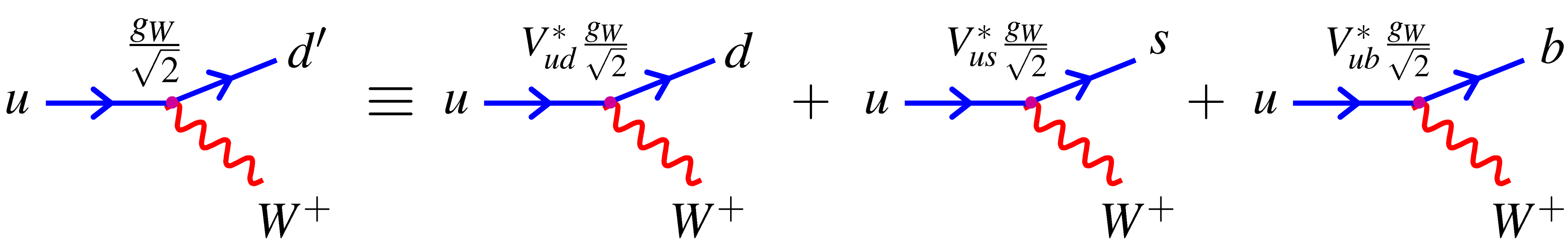
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

By convention CKM matrix defined as acting on quarks with charge

Weak eigenstates **CKM Matrix** **Mass Eigenstates**

(Cabibbo, Kobayashi, Maskawa)

E.g.



- The CKM Matrix elements are **complex constants**
- The CKM Matrix is **unitary**

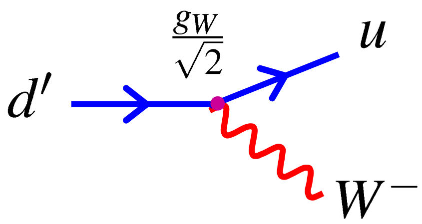
V_{ij} are not predicted by SM — have to be **determined from experiment**

Feynman Rules

- Depending on the order of the interaction $u \longrightarrow d$ or $d \longrightarrow u$

  the CKM Matrix elements are V_{ud} or V_{ud}^*

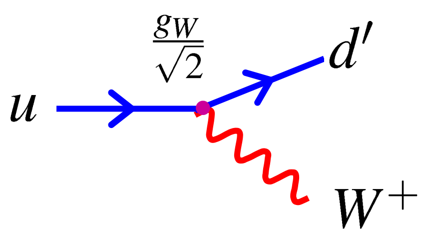
- In terms of **weak eigenstates**



$$j_{d'u} = \overline{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] d'$$

- In terms of **mass eigenstates** the $\boxed{d \longrightarrow u}$ current is $\boxed{j_{du} = \overline{u}\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] V_{ud} d}$

- For $u \longrightarrow d'$ in terms of **weak eigenstates**



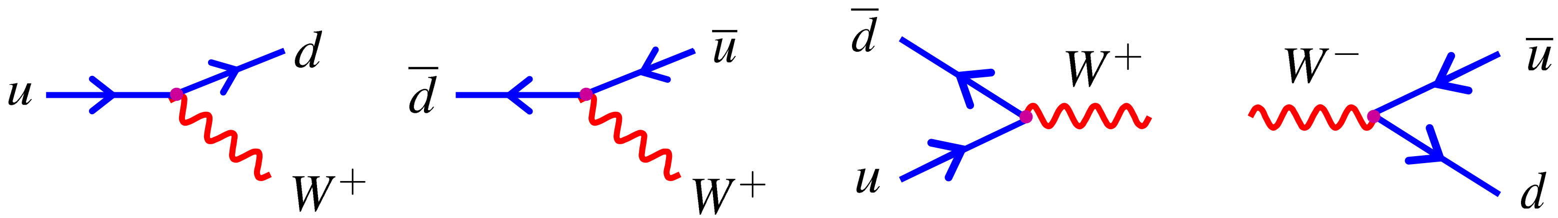
$$j_{ud'} = \overline{d}'\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u$$

- In terms of **mass eigenstates** the $\boxed{u \longrightarrow d}$ current is

$$\boxed{j_{ud} = \overline{d} V_{ud}^*\left[-i\frac{g_W}{\sqrt{2}}\gamma^\mu \tfrac{1}{2}(1-\gamma^5)\right] u}$$

Feynman Rules

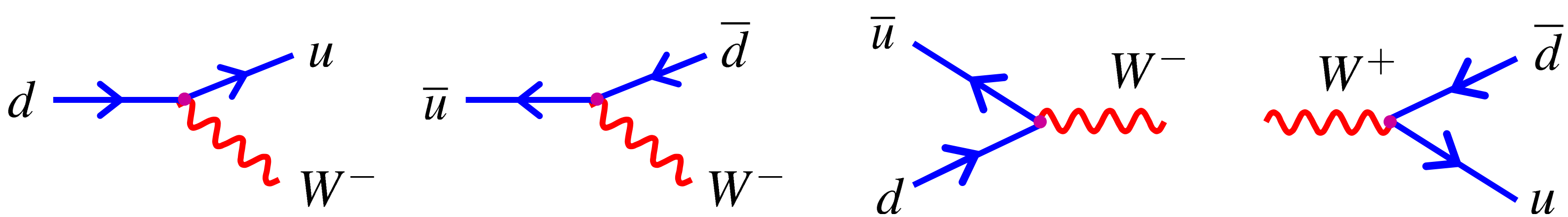
- In general, when the charge -1/3 quark enters as the adjoint spinor, the complex conjugate of CKM Matrix is used, i.e.



i

$$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}^{*}\gamma^{\mu}\,\tfrac{1}{2}(1-\gamma^5)}$$

- If +2/3 quark enters as the adjoint spinor, the normal CKM element is used



i

$$\boxed{-i\frac{g_W}{\sqrt{2}}V_{ud}\gamma^{\mu}\,\tfrac{1}{2}(1-\gamma^5)}$$

- Elements of CKM matrix are determined from **experiment**

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ ? & ? & ? \end{pmatrix}$$

Little direct experimental information on V_{td}, V_{ts}, V_{tb}

- Assuming **unitarity** of CKM matrix, i.e. $|V_{ub}|^2 + |V_{cb}|^2 + |V_{tb}|^2 = 1$

Cabibbo matrix

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \approx \begin{pmatrix} 0.974 & 0.226 & 0.004 \\ 0.23 & 0.96 & 0.04 \\ 0.01 & 0.04 & 0.999 \end{pmatrix}$$

nearly diagonal

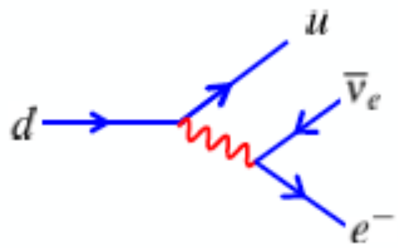
- Remarks:
  - In the SM **charged current** weak interaction ($W^{\pm}$) provides the **only way** to **change flavour**, and to **change from one generation to another**
  - CKM matrix is **nearly diagonal**. Weak interaction is **largest** between quarks of the **same generation**
- CKM matrix allows for **CP violation in the SM**

A few examples of experimental V_{ij} determination

UCL

$|V_{ud}|$ **from nuclear beta decay**

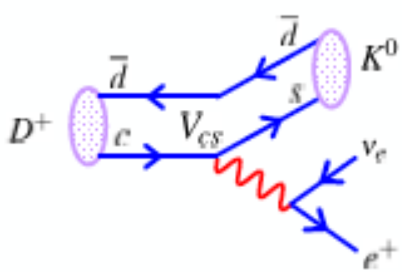
$$\begin{pmatrix} \times & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



$\Gamma \propto |V_{ud}|^2$ $\boxed{|V_{ud}| = 0.97377 \pm 0.00027}$ $(\approx \cos\theta_c)$

$|V_{cs}|$ **from semi-leptonic charmed meson decays**

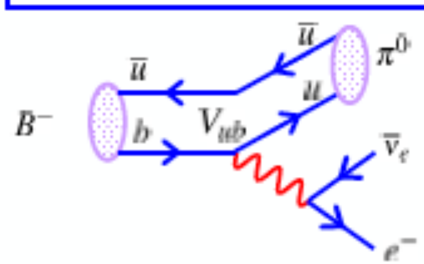
$$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \times & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$



$\Gamma \propto |V_{cs}|^2$ $\boxed{|V_{cs}| = 0.957 \pm 0.017 \pm 0.093}$

$|V_{ub}|$ **from semi-leptonic B hadron decays**

$$\begin{pmatrix} \cdot & \cdot & \times \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

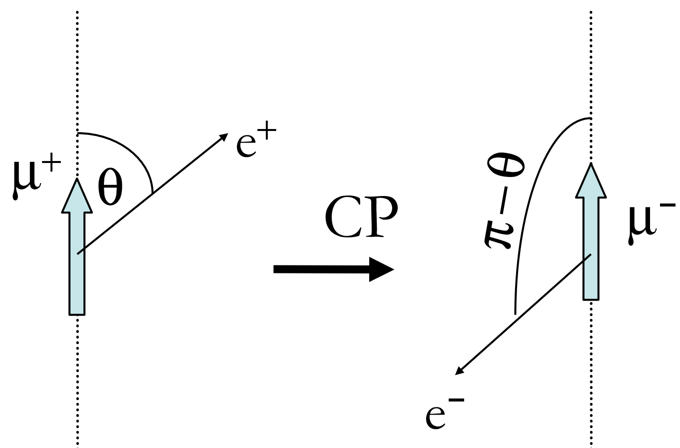


$\Gamma \propto |V_{ub}|^2$ $\boxed{|V_{ub}| = 0.0043 \pm 0.0003}$

CP transformation

- Charge conjugation $(\hat{C})$ and Parity $(\hat{P})$ transformation a **maximally** violated in W.I.

- But combined CP transformation appears to be conserved in e.g. muon decay



Experimentally:

$$\Gamma_{\mu\pm}(\cos\theta) = \frac{1}{2}\Gamma_{\pm}\left(1 - \frac{\xi_{\pm}}{3}\cos\theta\right)$$

$$\xi_{-} = -\xi_{+} = 1.00 \pm 0.04$$

$$\Gamma_{\mu+}(\cos\theta) = \Gamma_{\mu-}(-\cos\theta) \Rightarrow$$ **CP-Invariance**

- Also

$$\begin{tikzcd}
\boxed{\nu_L} \arrow[r, "P"] \arrow[d, "C"'] \arrow[dr, "CP"] & \nu_R \\
\bar{\nu}_L & \boxed{\bar{\nu}_R}
\end{tikzcd}$$

Only **left-handed neutrinos** and **right-handed anti-neutrinos** are observed

Experimental evidence so far:
CP is largely conserved but it is not an exact symmetry.
Small violations **observed in** ***hadron*** **sector**

CP Violation in Early Universe

- The **same** number of **baryons** and **anti-baryons** in **very early Universe**
- However, **today** the **Universe** is **matter dominated** (e.g. no evidence for anti-galaxies)
- Can obtain matter-antimatter asymmetry form "Big Bang Nucleosynthesis" (BBN)

$$\xi = \frac{n_B - n_{\overline{B}}}{n_\gamma} \approx \frac{n_B}{n_\gamma} \approx 10^{-9}$$

i.e. today, for every baryon there are 10^9 photons

- What is the mechanism beyond this?
  - Small **asymmetry** between **baryons** and **anti-baryons** in early Universe
  - E.g. for every 10^9 anti-baryons there were 10^9+1 baryons $\Longrightarrow$

    $\Longrightarrow$ baryons/anti-baryons annihilate $\Longrightarrow$1 **baryon + 10^9 photons** + **no anti-baryons**

- **Sakharov conditions** (1967) to generate this asymmetry
  - **Baryon number violation**
  - **CP violation**
  - **Departure from thermal equilibrium**

CP Violation

- **CP violation** is key to understanding the origin behind the observed **matter-antimatter asymmetry** in the Universe

- Can Standard Model account for this?

- CP violation enters in SM in two places: CKM matrix (quark mixing) and PMNS matrix (neutrino mixing)

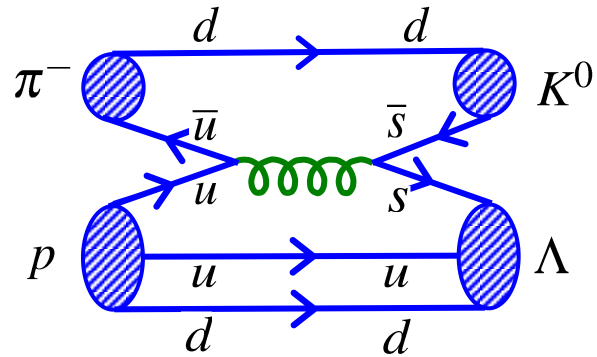
- **To date CP violation was observed only in quark sector** (next slides)

CP Violation in neutral Kaon system

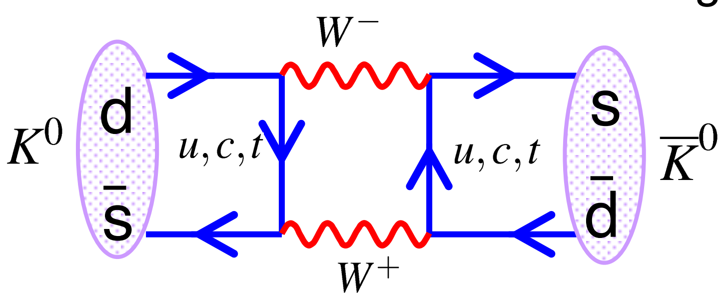
- Neutral kaons produced in strong interaction

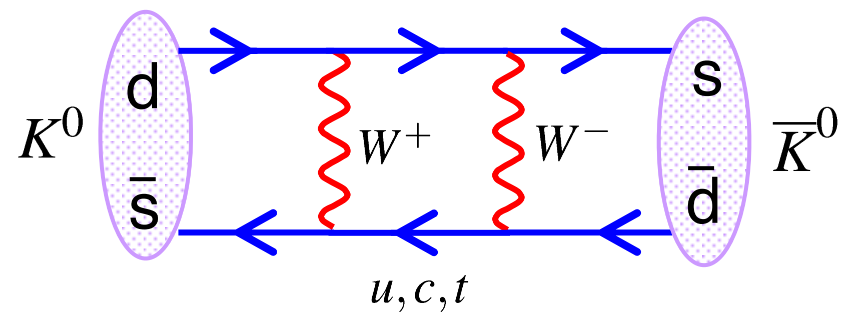
$$\pi^-(d\bar{u}) + p(uud) \rightarrow \Lambda(uds) + K^0(d\bar{s})$$

$$\pi^+(u\bar{d}) + p(uud) \rightarrow K^+(u\bar{s}) + \overline{K}^0(s\bar{d}) + p(uud)$$



- Weak interaction allows for K^0 mixing via box diagrams





$\hat{C}\hat{P}|K^0\rangle = -|\overline{K}^0\rangle \qquad \hat{C}\hat{P}|\overline{K}^0\rangle = -|K^0\rangle$ not eigenstates of CP

- Can form CP-eigenstates from linear combinations

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$

$$\hat{C}\hat{P}|K_1\rangle = +|K_1\rangle$$

$$\hat{C}\hat{P}|K_2\rangle = -|K_2\rangle$$

- Straightforward to show that (see M. M. Thomson, “Modern Particle Physics”, Cambridge (2013), Sections 14.4, 14.5)

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_1\rangle = +|K_1\rangle \qquad K_1 \to \pi\pi \qquad \text{CP EVEN}$$

$$|K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle) \qquad \hat{C}\hat{P}|K_2\rangle = -|K_2\rangle \qquad K_2 \to \pi\pi\pi \qquad \text{CP ODD}$$

- Expect very different lifetimes for K_1 $m_K - 2m_\pi \approx 220\,\text{MeV}$ and K_2 $m_K - 3m_\pi \approx 80\,\text{MeV}$

- Indeed **experimentally** observe two K^0 states, **K-short** and **K-long** with

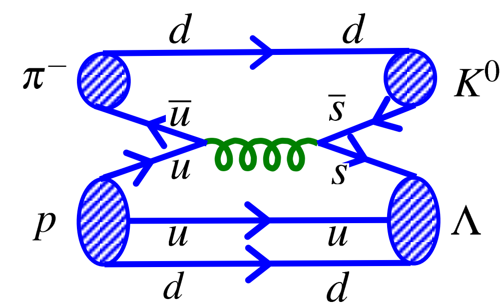
$$\boxed{\tau(K_S) = 0.9 \times 10^{-10}\,\text{s} \qquad \tau(K_L) = 0.5 \times 10^{-7}\,\text{s}}$$

- In the absence of CP violation we can identify

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$ **with decays:** $K_S \to \pi\pi$

$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$ **with decays:** $K_L \to \pi\pi\pi$

- Consider K^0 produced in strong interaction



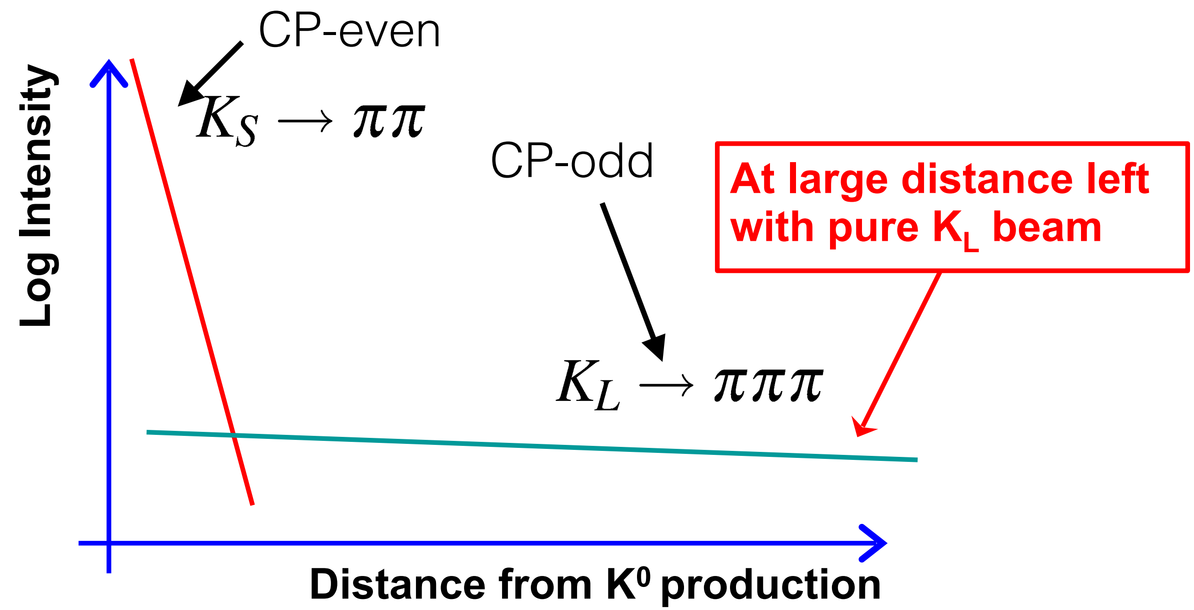
- If CP is conserved

$$|K_0\rangle = \frac{1}{\sqrt{2}}(|K_S\rangle + |K_L\rangle)$$

K_S	$\rightarrow$	$\pi^+\pi^-$	$BR = 69.2\%$
	$\rightarrow$	$\pi^0\pi^0$	$BR = 30.7\%$
	$\rightarrow$	$\pi^-e^+\nu_e$	$BR = 0.03\%$
	$\rightarrow$	$\pi^+e^-\overline{\nu}_e$	$BR = 0.03\%$
	$\rightarrow$	$\pi^-\mu^+\nu_\mu$	$BR = 0.02\%$
	$\rightarrow$	$\pi^+\mu^-\overline{\nu}_\mu$	$BR = 0.02\%$

K_L	$\rightarrow$	$\pi^+\pi^-\pi^0$	$BR = 12.6\%$
	$\rightarrow$	$\pi^0\pi^0\pi^0$	$BR = 19.6\%$
	$\rightarrow$	$\pi^-e^+\nu_e$	$BR = 20.2\%$
	$\rightarrow$	$\pi^+e^-\overline{\nu}_e$	$BR = 20.2\%$
	$\rightarrow$	$\pi^-\mu^+\nu_\mu$	$BR = 13.5\%$
	$\rightarrow$	$\pi^+\mu^-\overline{\nu}_\mu$	$BR = 13.5\%$

Therefore in a beam of K^0



- Neutral kaons propagate as combined eigenstates of weak + strong interaction
  See M. M. Thomson, “Modern Particle Physics”, Sections 14.4, 14.5 for **strangeness oscillations** (but **non-examinable**)

CP violation in the Kaon system

UCL

$$|K_S\rangle = |K_1\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle - |\overline{K}^0\rangle)$$ **with decays:** $K_S \longrightarrow \pi\pi$ $\boxed{\text{CP = +1}}$

$$|K_L\rangle = |K_2\rangle \equiv \frac{1}{\sqrt{2}}(|K^0\rangle + |\overline{K}^0\rangle)$$ **with decays:** $K_L \longrightarrow \pi\pi\pi$ $\boxed{\text{CP = -1}}$

- At a **long distance** from production point a beam of K^0 will be **100% K-long**
- 1964: Fitch & Cronin observed 45 $K_L \longrightarrow \pi^+\pi^-$ decays in 22700 K^0 decays

  (and received Nobel Prize for that)

⇨ **Weak interactions violate CP**

- CP is violated in hadron decays, but only at the level of 2×10^{-3}

⇩

$$\begin{array}{llll}
K_L \longrightarrow & \pi^+\pi^-\pi^0 & BR = 12.6\% & CP = -1 \\
\longrightarrow & \pi^0\pi^0\pi^0 & BR = 19.6\% & CP = -1 \\
\longrightarrow & \pi^+\pi^- & BR = 0.20\% & CP = +1 \\
\longrightarrow & \pi^0\pi^0 & BR = 0.08\% & CP = +1
\end{array}$$

- Two possible explanation of CP violation in Kaon decays

  (i) **K_s** and **K_L** do not correspond *exactly* to eigenstates **K_1** and **K_2**

$$\boxed{|K_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_1\rangle + \varepsilon|K_2\rangle\right]} \qquad \boxed{|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]}$$

with $|\varepsilon| \sim 2\times 10^{-3}$

In this case $K_L \to \pi\pi$ is accounted for by

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left[|K_2\rangle + \varepsilon|K_1\rangle\right]$$

$|K_1\rangle \to \pi\pi$ **CP = +1**

$|K_2\rangle \to \pi\pi\pi$ **CP = -1**

(ii) and or CP is violated in the decay

$|K_L\rangle = |K_2\rangle$ **CP = -1**

$\to \pi\pi\pi$ **CP = -1**

$\to \pi\pi$ **CP = +1**

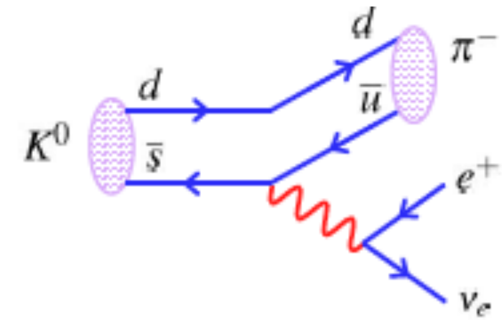
parameterised by ε'

- Experimentally both known to contribute but **(i) dominates**

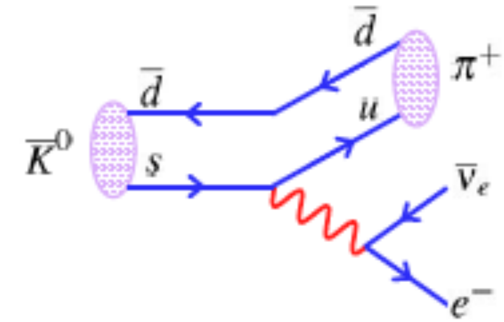
$$\varepsilon'/\varepsilon = (1.7\pm0.3)\times 10^{-3}$$ { NA48 (CERN), KTeV (FermiLab)

CPV in Semi-Leptonic decays

- At long distance from production K^0 beam consists of **K_L** only



$$|K_L\rangle = \frac{1}{\sqrt{2}}\frac{1}{\sqrt{1+|\varepsilon|^2}}\left[\underbrace{(1+\varepsilon)|K_0\rangle}_{\to\ \pi^- e^+ \nu_e} + \underbrace{(1-\varepsilon)|\overline{K}^0\rangle}_{\to\ \pi^+ e^- \overline{\nu}_e}\right]$$



Therefore

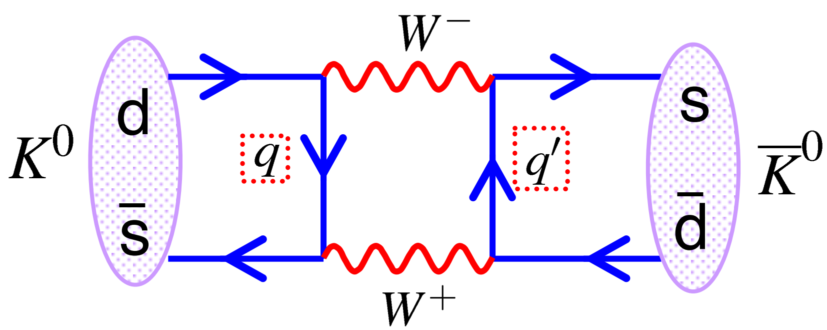
$$\Gamma(K_L \to \pi^+ e^- \overline{\nu}_e) \propto |\langle \overline{K}^0 | K_L\rangle|^2 \propto |1-\varepsilon|^2 \approx 1 - 2\Re\{\varepsilon\}$$
$$\Gamma(K_L \to \pi^- e^+ \nu_e) \propto |\langle K^0 | K_L\rangle|^2 \propto |1+\varepsilon|^2 \approx 1 + 2\Re\{\varepsilon\}$$

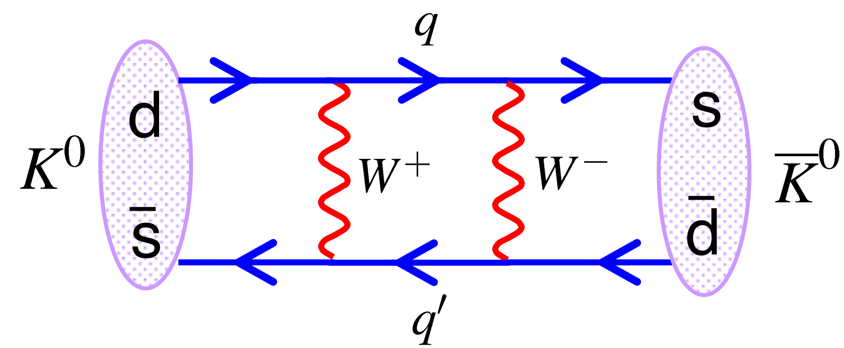
- Expect small difference in decay rates $\pi^- e^+ \nu_e$ is 0.7% more likely than $\pi^+ e^- \overline{\nu}_e$
- This difference has been **experimentally observed** providing the first **direct evidence** for an absolute **difference between matter and anti-matter**
- Provides unambiguous definition of matter

“The electrons in our atoms have the same charge as those emitted least often in the decays of the long-lived neutral kaon”

CPV and CKM Matrix

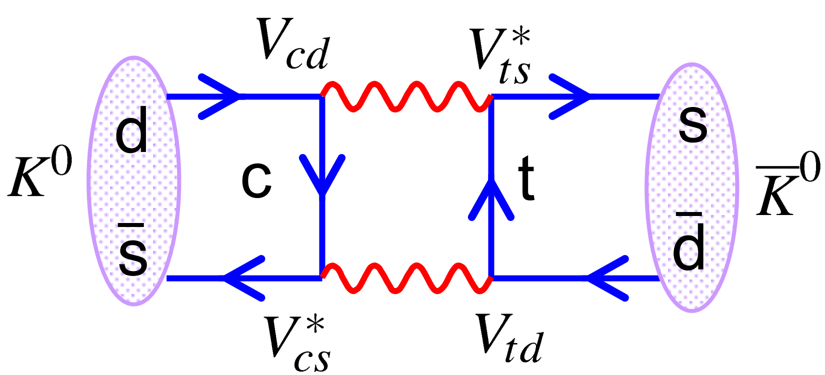
- How can we explain $\Gamma(\overline{K}^0_{t=0} \to K^0) \neq \Gamma(K^0_{t=0} \to \overline{K}^0)$ in terms of CKM matrix?
- Consider box diagrams responsible for mixing





where $\quad q = \{u, c, t\}, \qquad q' = \{u, c, t\}$

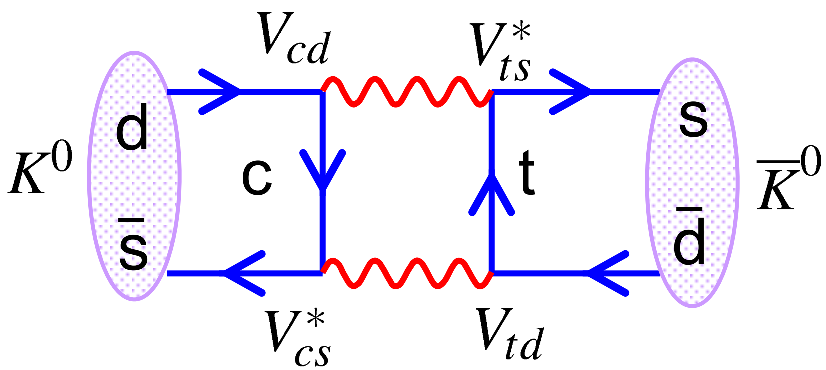
- For simplicity consider $q = c,\, q' = t$

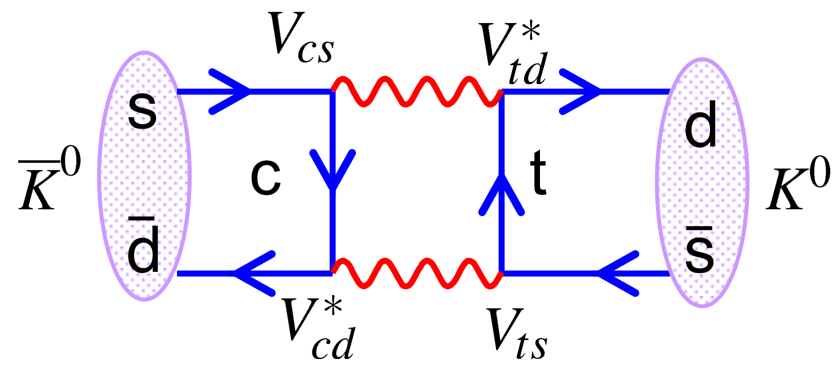


$$M_{fi} \propto A_{ct} V_{cd} V^*_{cs} V_{td} V^*_{ts}$$

A constant related to integrating over virtual momenta

- Comparing diagrams for $K^0 \to \overline{K}^0$ and $\overline{K}^0 \to K^0$





$$M_{fi} \propto A_{ct} V_{cd} V_{cs}^* V_{td} V_{ts}^*$$

$$M'_{fi} \propto A_{ct} V_{cd}^* V_{cs} V_{td}^* V_{ts} = M_{fi}^*$$

- Therefore the difference in rates is

$$\Gamma(K^0 \to \overline{K}^0) - \Gamma(\overline{K}^0 \to K^0) \propto M_{fi} - M_{fi}^* = 2\Im\{M_{fi}\}$$

which is proportional to $|\varepsilon|$. Hence

$$|\varepsilon| \propto \Im\{M_{fi}\}$$

CP violation is related to the imaginary parts of the CKM matrix

Summary of Weak Interaction of Quarks

- The weak interaction of quarks are described by the **CKM** matrix

- The **CKM** matrix is nearly **diagonal**. Weak interaction is **largest** between quarks of the **same generation**

- CP Violation (**CPV**) enters through a **complex phase** in the **CKM** matrix.

- There is a substantial amount of **evidence** for **CPV** in **weak interactions** of **quarks** (Kaon, B-meson and recently D-mesons)

- **CPV** is needed to explain the observed **matter-antimatter asymmetry** in the Universe

- **However**, observed **CPV** in **quark sector** is **not sufficient** to account for observed **matter-antimatter asymmetry**. Need **another source** of stronger **CPV**. **Neutrinos** <u>**may**</u> come to the rescue?…