PHASM/G442. 2017 : Problem Sheet 1

Please return to Prof. Saakyan by October 26th 2017.

The numbers in square brackets in the right-hand margin indicate the provisional allocation of marks.

1. For each of the following processes, either draw **all** of the lowest order Feynman diagrams for the process or state why the process is not allowed in the Standard Model:

   - $\mu^- \to e^- + \overline{\nu}_e + \overline{\nu}_\mu$
   - $\mathrm{e}^+ + \mathrm{e}^- \to \overline{\mathrm{b}} + b$
   - $\tau^- \to \nu_\tau + \pi^-$
   - $\overline{\nu}_e + \mathrm{e}^+ \to \overline{\nu}_e + \mathrm{e}^+$
   - $D^0 \to K^- + \pi^+ \qquad D^0 = c\overline{u}; K^- = s\overline{u}; \pi^+ = u\overline{d}$
   - $\mathrm{e}^- + p \to \nu_e + n$

   [12]

2. Using the natural units and a $sec \to GeV^{-1}$ conversion calculate the branching ratio for the decay $K^+ \to \pi^+\pi^0$, given the partial decay width $\Gamma(K^+ \to \pi^+\pi^0) = 1.2 \times 10^{-8}$ eV and the mean kaon lifetime of $\tau(K^+) = 1.2 \times 10^{-8}$ s. **[5]**

3. Λ baryons produced in a collider experiment can be identified from the decay $\Lambda^0 \to p\pi^-$ that results in a displaced vertex in a tracking detector due to a finite lifetime of the Λ baryon.

   (a) Show that the mass of the Λ baryon can be expressed as

   $$m_\Lambda^2 = m_p^2 + m_\pi^2 + 2E_pE_\pi(1 - \beta_p\beta_\pi cos\theta),$$

   where β_p and β_π are the velocities of the proton and pion respectively and θ is the angle between them.

   (b) In a particular decay, the momenta of π^- and p are measured to be 0.75 GeV and 4.25 GeV respectively, and the opening angle between the tracks is 9°. Calculate the mass of the Λ baryon. Assume the pion and proton masses to be 140 MeV and 938 MeV respectively.

   (c) The lifetime of the Λ baryon is 2.6×10^{-10} sec. Calculate the average distance a Λ baryon would travel from the point of production in the collider.

   [12]

4. (a) Draw the lowest-order Feynman diagram for the decay $K^+ \to \mu^+ + \nu_\mu$, $K^+ = u\bar{s}$.

   (b) Show that in the rest frame of the $K^+ \to \mu^+ + \nu_\mu$ decay the Lorentz gamma factor of the muon, $\gamma = E_\mu / m_\mu$, where E_μ and m_μ are the energy and rest-mass of the muon respectively, is given by

$$\gamma = \frac{m_K^2 + m_\mu^2}{2 m_\mu m_K},$$

   where m_K is the mass of the kaon. **[8]**

5. Show that the term

$$\frac{d^3\vec{p}}{(2\pi)^3 2E}$$

   introduced in the lectures as part of the phase space expression for the two-body decay transition rate Γ_{fi} is Lorentz invariant. **[5]**

6. A muon neutrino with a momentum of 1 GeV is directed at a block of iron 1 meter thick. Assuming the average neutrino-nucleon interaction cross-section is $\sigma = 8 \times 10^{-39}$ cm^2, calculate the probability that the neutrino interacts in the block. The iron density is $\rho = 7.9 \times 10^3$ kg m^{-3}. **[8]**

Total: 50 marks