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# Problem set #3

Release 0.1

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## 1 Recursion relations and Bessel functions

**Illustrates:** Special functions library, array manipulations to check recursion relation.

In this exercise, you will verify a few simple relations involving the Bessel functions of the first kind. The important relations to keep in mind are the asymptotic form of  $J_n(x)$  for  $x \gg n^2$ :

$$J_n(x) \approx \sqrt{\frac{2}{\pi x}} \cos\left(x - \frac{n\pi}{2} - \frac{\pi}{4}\right),$$

the asymptotic form of  $J_n(x)$  for  $x \ll \sqrt{n}$

$$J_n(x) \approx \frac{1}{\Gamma(n+1)} \left(\frac{x}{2}\right)^n,$$

and the recursion relation

$$J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x). \tag{1}$$

The `scipy.special` module contains functions `j0()`, `j1()` and `jn()` to compute Bessel functions of order 0, 1 and arbitrary  $n$ , as well as many other useful special function-related routines.

For this problem, build three separate figures showing:

1.  $J_0(x)$ ,  $J_1(x)$  and  $J_5(x)$  for  $x$  in the interval  $[0, 35]$ , as well as their asymptotic forms. Use thicker dashed lines for the asymptotic forms, and only plot them in their region of validity.

2. A similar plot, for  $J_4(x)$ ,  $J_5(x)$  and  $J_6(x)$  for  $x$  in the interval  $[0, 3]$  (be careful to use the proper asymptotic form).
3. The error in the recursion relation (1) for  $J_5$  over the same interval. These errors should be displayed using a logarithmic vertical axis.

Try to get your figures to look reasonably close to those below.

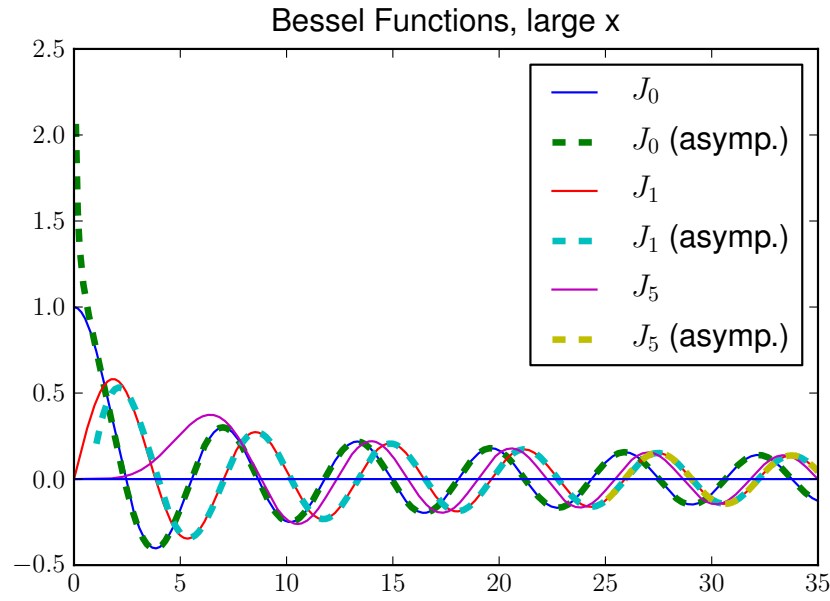


Figure 1: A few Bessel functions and their asymptotic forms valid for  $x \gg n^2$ .

## 1.1 Hints

- Passing a `label` keyword to `plot()` calls lets you label each plot, these plots are then used by `plt.legend()` which puts legend boxes.
- `plt.legend()` takes a `loc` parameter for location.
- look at `plt.semilogy()` for the logarithmic error plots.

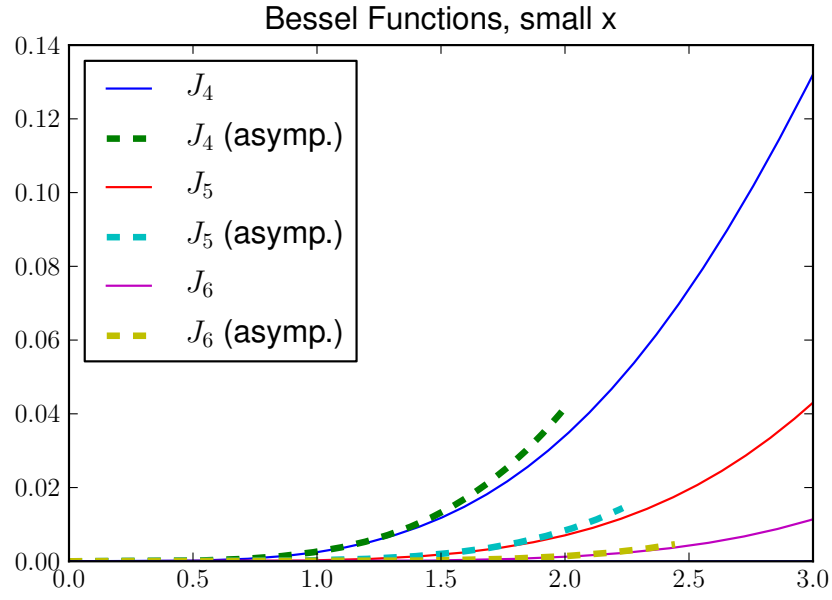


Figure 2: A few Bessel functions and their asymptotic forms valid for  $x \ll \sqrt{n}$ .

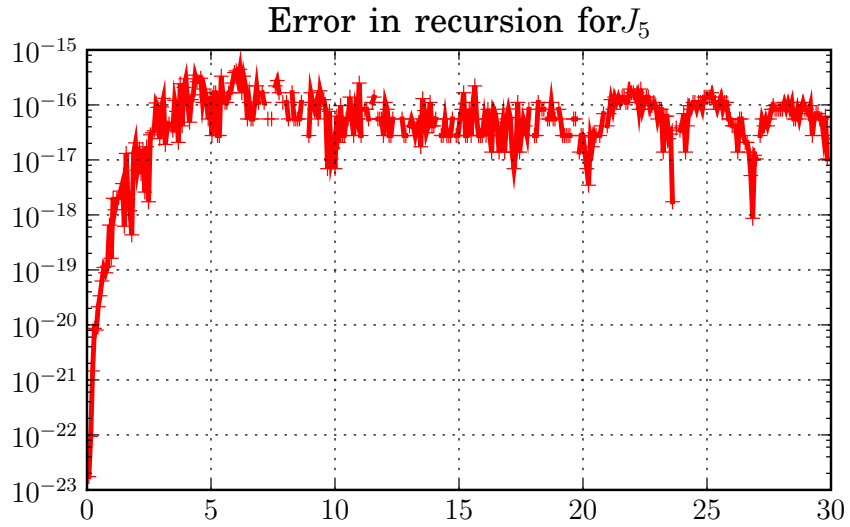


Figure 3: Numerical error in manually implementing the Bessel recursion for  $J_5$  vs scipy's implementation.