

Cook's theorem

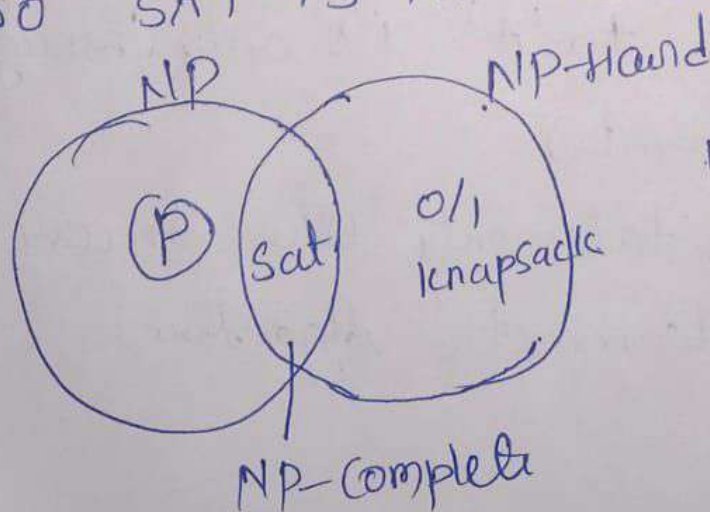
→ the scientist Stephen Cook in 1971 stated that boolean satisfiability problem is in NP-complete problem.

→ Cook's theorem states that the Satisfiability (SAT) is in P, iff $\boxed{P=NP}$.

proof: Shows that Satisfiability problem is NP-complete.

* SAT is in NP, because a Non-Deterministic Algorithm can guess an assignment of truth value of variables. An expression is satisfiable if its value takes true.

* SAT is in NP-complete, such that $SAT \in NP$ and any ~~problem~~ NP-problem can be reduced to SAT, so SAT is NP-Hard.



$$P \subseteq NP$$

$$P = NP \text{ (Cook's theorem)}$$

EX CNF

$$\Rightarrow (x_1 \vee x_2 \vee \bar{x}_3) \wedge (\bar{x}_1 \vee \bar{x}_2 \vee \bar{x}_3)$$

$$\text{let } x_1 = 1, x_2 = 0, x_3 = 0$$

$$= (1 \vee 0 \vee 1) \wedge (0 \vee 1 \vee 1)$$

$$= 1 \wedge 1 \Rightarrow 1 \text{ (True)}$$

→ Hence, we can prove SAT is in NP-complete because $\text{SAT} \in \text{NP}$

Certain assumptions considered are

- execution of algorithm 'A' is done on a word oriented system where length of each word is 'w' bits.
- All variables in 'A' are defined using either integer/Boolean.
- I/p provided to 'A' is only through arguments (no constants)
- additional statements like Success(), Failure(), bin (Non-Deterministic Algorithm)