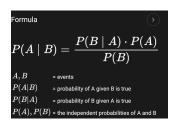
PROBABILITY

Question 1

Bayes' Theorem



Two binary random variables **R** and **F** where R is wether it rains and F is wether they forecast rain or not **Given**:

 $P(F=1 \mid R=1) = 70\%$, when it rains they are correct 70% of the time

P(F=1 | R=0) = 30%, when it doesn't rain they say it will rain 30% of the time

P(R=1) = 73/365 or 20%, it rains 20% of the year

P(R=0) = 80%, it doesn't rain 80% of the year

Solution

I need to find $P(R=1 \mid F=1)$, the probability it rains given they forecast it will Using Bayes' Theorem I can do

$$P(R-1|F=1) = P(F=1|R-1) \cdot P(R=1)$$

$$P(F=1)$$

Since I don't know P(F = 1) I use the law of total probabilities to solve for it.

$$P(F=1) = P(F=1 | R=1) \cdot P(R=1) + P(F=1 | R=0) \cdot P(R=0)$$

$$(0.7 \cdot 0.2) + (0.3 \cdot 0.8)$$

$$P(F=1) = (0.38)$$

Finally

$$P(R=1|F=1) = P(F=1|R=1) \cdot P(R=1)$$

$$0.38$$

$$0.70 \cdot 0.2$$

$$0.38$$

$$P(R=1|F=1) = 0.368 \text{ or } 36.8\%$$

Question 2

$$\mathsf{payout} = \left\{ \begin{array}{cc} \$1 & x = 1 \\ -\$1/4 & x \neq 1 \end{array} \right.$$

The probability of landing on 1 is 1/6 and not on 1 is 5/6. If I played 6 games, statistically I would win \$1 and loose a \$0.25 5 times or -\$1.25.

Answer: So no this is not a good game to play.

Question 3

The Gaussian function can be thought of as a weighting function to the quadratic function. When you take the integral of these combined functions you find the probability of the most likely outcome.

Question 4

$$p(x) = \begin{cases} 4x & 0 \le x \le 1/2 \\ -4x + 4 & 1/2 \le x \le 1 \end{cases}$$

Answer

$$\int_{-\infty}^{1/2} 2x^2 dx + \int_{1/2}^{x} -2x^2 + 1/2 dx$$

LINEAR ALGEBRA

Question 1

Transpose and Associative Property [1pt] Define matrix $B=bb^T$, where $b\in\mathbb{R}^{d\times 1}$ is a column vector that is not all-zero. Show that for any vector $x\in\mathbb{R}^{d\times 1}$, $x^TBx\geq 0$.

[Hint: Try to get x^TBx to look like the product of two identical scalars. Note that $b^Tx=(x^Tb)^T$, that $a^T=a$ for scalar value a, and that matrix multiplication is associative.]

Question 2

$$A = \begin{pmatrix} Z & I & I \\ 4 & O & Z \\ Z & Z & O \end{pmatrix} \begin{pmatrix} \begin{pmatrix} z & I & I \\ 4 & O & Z \\ Z & Z & O \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \\ \chi_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 10 \\ -2 \end{pmatrix}$$

$$A = \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \qquad b = \begin{pmatrix} 3 \\ 10 \\ -2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} A_1 & A_2 & A_3 \\ A_3 & A_4 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -1 & 1/2 & 1/2 \\ 1 & -1/2 & 0 \\ 2 & -1/2 & -1 \end{pmatrix}$$

$$A^{-1}Ax = A^{-1}b$$

$$X = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

PROVING THINGS

Question 1

Question 2

$$\sum_{i=1}^{k} p_i = \sum_{i=1}^{k} q_i = 1$$
 $p_i > 0, q_i > 0, \quad \forall i \in \{1, \dots, k\}$

 $KL(p||q) = \sum_{i=1}^{k} p_i \ln \left(\frac{p_i}{q_i}\right)$

between p and q is given by:

K

$$\leq piln(pi) \leq \leq pi-1$$

$$f(x) = \begin{cases} x \\ > piln(pi) \\ = 1 \end{cases} - \begin{cases} x \\ > pi - 1 \end{cases}$$

$$f(x) = \begin{cases} x \\ = 1 \end{cases} - \begin{cases} x \\ = 1 \end{cases} - \begin{cases} x \\ = 1 \end{cases}$$

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$$f(x) = 1 \end{cases}$$

$$f(x) = \begin{cases} x \\ = 1 \end{cases}$$

4 Debriefing (required in your report)

- 1. Approximately how many hours did you spend on this assignment?
- 2. Would you rate it as easy, moderate, or difficult?
- 3. Did you work on it mostly alone or did you discuss the problems with others?
- 4. How deeply do you feel you understand the material it covers (0%-100%)?
- 5. Any other comments?
- 1. 4-5 hours
- 2. Difficult, many topics I have learned before and understood, but with the passage of time I forgot how to do it. I had to relearn most of these topics for the assignment.
- 3. Worked alone.
- 4. 50% understanding of the material.
- 5. Good wake up call to the reality of doing good in a class i.e getting a good grade versus actually mastering and remembering how to solve problems.