Does Ex-Ante Asymmetry Matter? A Modeling of Multi-Player Asymmetric War of Attrition

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Abstract

This paper models a multi-player asymmetric war of attrition game with incomplete information on the private provision of public goods to investigate how ex-ante asymmetry affects behavior and welfare. In the unique equilibrium, asymmetry leads to a stratified behavior pattern such that one player exits instantly with positive probability, while each of others has no probability of concession until a certain moment associated with them. Efficiency measured by the cost of delay is mainly determined by the strongest type, namely the highest type of the instant-exit player. If the asymmetry is introduced by strengthening the strongest type, it tends to improve efficiency, whereas if the strongest type is controlled, the effect of asymmetry coincides with the sign of an intuitive measure of the cost of symmetry.

KEYWORDS. War of attrition, private provision of public good, ex-ante asymmetry, multiple players, incomplete information.

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1 Introduction

When cooperation is prohibitively costly for a group of people, any public good can only be provided privately during a war of attrition. This happens on a daily basis and is one of the major sources of welfare loss resulting from strategic interaction. Strategic delay is known to be a major cause of inefficiency in these situations.

One intriguing question is: Does *asymmetry* matter in such conflicts? Here, asymmetry refers to the situation where individuals in different economic, political, or social positions are anticipated differently. This notion corresponds to a large number of applications. For example, when socioeconomic groups try to shift the burden of stabilization onto each other, the presence of several incumbent groups who feel more pressured to conduct fiscal stabilization creates asymmetry. For another, when several countries or provinces suffer from illegal activities on their common border, the different costs of controlling the chaos faced by different agents introduce asymmetry. Moreover, when the United Nations gathers countries to reach an agreement on how humanity should respond to climate change, the different incentives of different countries to make voluntary commitment also bring asymmetry.[1] Finally, ex-ante asymmetry also lies in the discriminative stereotypes that people have to others based on the impression of race, gender, age, and other social elements that label people.

This issue raises compelling questions: How does asymmetry change behavior? How does this asymmetric behavior pattern make each agent contribute to welfare differently? Does asymmetry alleviate or exacerbate delay? Can we be better off by sharpening or equalizing asymmetry?

This paper develops a generalized war of attrition that combines ex-ante asymmetry, multiple players, and incomplete information. Such a general combination is missing in the literature[2] of the war of attrition and other similar forms of conflict.

My model provides an asymmetric extension of Bliss and Nalebuff (1984). They discuss a continuous-time war of attrition on the private provision of an indivisible public good in which each player chooses a provision time in the beginning to optimize his expected utility, and once

[1]Detailed explanations for these three examples. The symmetric analysis of the first example has been done by Alesina and Drazen (1991). The second example corresponds to the Golden Triangle area, the common border of Thailand, Laos, and Myanmar, which is also not far from China. The rampant illegal dealing of drugs and long-lasting violent activities are the consequence of delayed and loose regulation from the neighbor countries. Asymmetry does exist in this case, as Thailand implements relatively more strict regulations than others. Besides, many other famous drug-trade areas are also the common borders of several countries, like the Golden Crescent and the Silver Triangle. For the last example, the Paris Agreement is a good manifestation. While China has shown willingness, the United States kept postponing the progress and eventually exited this agreement.

[2]I list some examples with two of the three elements. For multi-player asymmetric wars of attrition with complete information, see Ghemawat and Nalebuff (1985, 1990), Whinston (1988), and Bildeau and Slivinski (1994). Examples into multi-player symmetric wars of attrition with incomplete information are Riley (1980), Bliss and Nalebuff (1984), Alesina and Drazen (1991), Bulow and Klemperer (1999), and Sahuguet (2006). The two-player asymmetric incomplete-information case is the most widely studied, for example, Riley (1980), Nalebuff and Riley (1985), Fudenberg and Tirole (1986), Kornhauser, Robinstein, and Wilson (1988), Ponsati Sakovics (1995), Abreu and Gul (2000), Myatt (2005), and Horner and Sahuguet (2010). Also, there are special cases that consider all three elements. For example, a third party strategically interferes in a two-player war of attrition (e.g., Casella and Eichengreen (1996) and Powell (2017)), and two groups bargain over two objects, which however is basically a two-player game (e.g., Ponsati and Sakovics (1996)). The most related study is Kambe (2019) who investigates a model similar to mine but with two-type incomplete information.

someone provides first, the game ends, and everyone gains lump-sum payoff according to their information. My model differs in that I allow all players' information, like costs, discount rates, and valuation distributions, to be asymmetric. Additionally, to guarantee a unique equilibrium, I assume that every player is anticipated to have a positive probability of valuing the public good less than his cost.

This paper finds that heterogeneous individuals manifest a *stratified behavior pattern*. One degenerate example of this concept commonly seen in two-player cases[3] is *instant exit*. That is, one of the players will have a positive probability of conceding immediately.

Apart from instant exit, there is also *strict waiting* which refers to the feature that some players will have no positive probability of provision until certain time points associated with each of them. This is not possible in two-player cases, for the highest types of both players always provide instantly. For each player who waits strictly, I call the minimal waiting time among all his types the strict-waiting time.

As a result, when the environment is asymmetric enough, the equilibrium behavior begins with some probability of one player's instant exit, and what follows is a period during which only two players have the probability of provision, and after it a third player becomes active, and in this manner periods with increasing numbers of active players follow sequentially. Eventually, only when the game has endured for a sufficiently long time will all players become active. Instant exit could be construed as a "one-player period" whose length is zero because there is no provision from others and any delay is unnecessary. The idea that asymmetry affects the outcome by changing the scale of active players has been studied in earlier yet less general cases[4].

This stratified equilibrium results from the asymmetric *incentive position*s of different types. Individual optimality requires the types revealed simultaneously to balance each other's incentives mutually. Namely, the types being revealed at the same time and the manner in which they are revealed are such that each of them will find the extra gain from providing immediately equal that from waiting slightly longer at this moment. This mutual-balance requirement makes the incentives faced by different types, to some extent, comparable. For example, types that exit instantly value the public good so much that no simultaneous revelation with other types can offset their high incentives, so they are in higher incentive positions. In contrast, some players wait strictly because even their highest types still value the good too low to be mutually balanced with earlier revealed types, and thus they are in lower incentive positions.

Intuitively, I call a player *stronger* if he has a positive probability of providing instantly or if he strictly waits shorter, and the former case corresponds to the strongest player whose highest valuation is called the strongest type. Comparative statics tell that either lower cost, more impatience, or "consistently higher" valuation distribution reduces a player's provision time and thus makes him stronger.

On the uniqueness of equilibrium, a huge literature obtains uniqueness by perturbing a war of attrition. Namely, for each player, there must be a positive probability of some others' never

[3]A seminal work that mentions instant exit is Nalebuff and Riley (1985), and among more recent studies are Ponsati and Sakovics (1995), Riley (1999), Abreu and Gul (2000), and Myatt (2005).

[4]For example, see Bergstrom et al. (1986), Hillman and Riley (1989), and Kambe (2019).

conceding.[5] Yet, most literature involves only two players or essentially complete information[6], while the model in this paper generates a group of asymmetric multivariate differential equations with a set of boundary conditions, some of which are at infinity. The perturbation strategy I employ is to allow each player to have a positive probability of valuing the public good less than his cost, as never conceding is the dominant strategy in this case. This assumption makes the solution of the group of differential equations change sensitively at infinity so that the set of irregular boundary conditions suffices to uniquely determines an equilibrium.

The second contribution of this paper lies in a complete discussion of the relationship between ex-ante asymmetry and social welfare.

First, I utilize the expected discount factor as a measure of welfare level to answer who matters the most and how he matters. For the former question, social welfare is mainly decided by the strongest type. Nonetheless, the answer to the second is surprising. It is commonly believed in the literature that the strongest type matters merely because it is the highest type of the instant-exit player who makes the most contribution. However, my results tell that the strongest type matters independently by determining the ranking of all types' incentives, which to some extent, is independent of the behavior of other types of the strongest player. For example, the welfare level of the special case, AD war, discussed in Section 3.3 is irrelevant to the variations in players' behavior as long as the strongest type is fixed, and this result makes the focus on the strongest player's exact behavior, like his instant-exit probability[7], less important.

The idea is that the strongest type "controls" the behavior of all types, and since the welfare is an integral with respect to all players' behavior which "closely follows" the strongest type, its level is mainly decided by the latter. The asymmetric dependence among players' behavior demonstrates one form of the "controlling". In the AD-war case, while the parameter variation of a weak player sheds no influence on stronger types' behavior, the variation of a strong player effectively changes weaker types' behavior. Additionally, the analysis of large-population societies in Section 4.1 presents a more general result that the highest type of each player completely determines his incentive position, which also implies asymmetric dependence.

Second, I investigate the impact that asymmetry has on welfare. That introducing asymmetry will enhance efficiency is a point commonly made in the literature.[8] However, the model an-

[5]For example, Fudenberg and Tirole (1986) introduce a positive probability of each player being better off in a duopoly than in a monopoly. Kornhauser, Rubinstein, and Wilson (1989) use a slight probability of irrational type who only plays a fixed strategy, the idea of which is also borrowed by Kambe (1999, 2019) and Abreu and Gul (2000). Finally, Myatt (2005) considers three forms of perturbation: exit failure, hybrid payoff, and time limit.

[6]By *essentially* I refer to Abreu and Gul (2000) and Kambe (2019) where they investigate wars of attrition with discrete-type incomplete information, but since only one type is rational while all others never concede, their setups are basically perturbed complete-information games.

[7]For example, Myatt (2005) and Kambe (2019) overstate the importance of the probability of instant exit.

[8]One extreme example is to select an efficient yet degenerate equilibrium with refinement. For instance, Riley (1999) lets a sequence of members of a contest-game family approximate a war of attrition, and he finds that any introduction of asymmetry makes one player concede immediately with probability one. Kornhauser, Rubinstein, and Wilson (1988) and Myatt (2005) derive similar results in more complex cases. Besides, Kambe (2019) argues that asymmetry increases the probability of instant exit and further improves efficiency. Some others analyze welfare directly. For example, Riley's (1999) numerical calculation of a welfare measure shows a consistently positive welfare effect of asymmetry; however, the result may hinge on his complete-information setup. Static

alyzed in this paper incorporates more dimensions as there are multiple players and continuous-type distributions, so the influence of ex-ante asymmetry highly depends on its definition and the parametrization. Informed by the insight that the strongest type has a decisive influence on the outcome, I manage to provide a complete discussion by considering two kinds of asymmetries, one of which allows the strongest type to change while the other controls it. Specifically, the first way of introducing asymmetry is to make the strongest type stronger under certain control, whereas the other is to fix this type and make others weaker. Such a division constitutes a complete discussion and both cases generate distinct insights. Thus, it is reasonable enough to help us to understand the effect of asymmetries under different circumstances.

The result for the first kind of asymmetry shows that by strengthening the strongest, any slight introduction of asymmetry reduces the cost of delay. I conduct numerical experiments to show that for a large number of applications, the phrase above “slight introduction of asymmetry” can be expanded to “any introduction of asymmetry”. The intuition is consistent with the previous finding that the strongest type has a decisive influence on the outcome.

Nonetheless, the other case manifests dependence on parametrization. Any introduction of asymmetry improves efficiency if the cost of symmetry is positive, which is measured by the welfare-level discrepancy between an N-player symmetric game and the associated infinite-player symmetric game, while asymmetry always impairs efficiency if the thus measured cost of symmetry is negative. An explanation for this dichotomy is that the cost of symmetry defined above actually evaluates the cost brought by increasing population and since asymmetry makes the scale of active players smaller during the beginning period, the effect of asymmetry has the same sign as that of the cost of symmetry.

This paper is organized as the following. Section 2 describes the model setup and the equilibrium concept. Section 3 first characterizes the equilibrium and proves the existence and uniqueness. I introduce a special case, AD war, in this section to illustrate both behavior features and welfare implications formally discussed later. Finally, this section performs comparative statics. Section 4 shows the relationship between ex-ante asymmetry and social welfare. Section 5 discusses possible applications and differentiates from the related research.

2 Model

There is an indivisible public good potentially beneficial to N different individuals. I denote each player by $i \in I_N$ where $I_N = \{1, 2, ..., N\}$. A continuous-time war of attrition that requires one exit begins at $t = 0$ and each player chooses a stopping time when, if no one has provided the public good yet, he will provide. Since there is no dynamic interaction during the procedure, this game is strategically static.

The information structure: one player, say i, knows exactly the cost of his individual provision $c_i > 0$, the rate $r_i > 0$ at which he exponentially discounts his expected gain, and his valuation of this public good v_i. The costs and discount rates of all players are common knowledge, whereas each valuation v_i is private information independently extracted from a cumulative distribution function $F_i : [\underline{v}_i, \overline{v}_i] \to [0, 1]$ in which $\underline{v}_i < c_i < \overline{v}_i < +\infty$ and $c_i > 0$ for all i. Assume that each

public-provision games like Bergstrom, Blume, and Varian (1986) argue similarly.

F_i yields a density function $f_i : [\underline{v}_i, \overline{v}_i] \to \mathbb{R}^+$ which is differentiable and strictly bounded from 0. Note that the strict relationship of $\underline{v}_i$, $\overline{v}_i$, and c_i is important because it necessarily guarantees the uniqueness of equilirium. For convenience, I sometimes call player i with valuation v_i simply as player v_i.

Player i's pure strategy is a function $T_i : [\underline{v}_i, \overline{v}_i] \to \mathbb{R}^+ \cup \{0, +\infty\}$ referring to the stopping time that player v_i chooses. Only when no provision happens before $T_i(v_i)$ will this player provide at this moment. If some players provide first, all players gain their valuations while the providers additionally pay their share of the provision cost. Namely, if $m \geq 1$ players provide at this moment, they respectively pay $\frac{1}{m}$ of the cost associated with each of them. If all players choose to wait forever, each earns zero. All payoffs are lump-sum paid at the provision menment, at which the game ends.

I consider pure-strategy Bayesian equilibrium, and in the following sections by *equilibrium* I refer to this notion unless otherwise specified.