Does Asymmetry Matters in Social Conflicts? A Multi-Player Asymmetric War of Attrition On Private Provision of Public Good

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Abstract

Natural-state dilemma, where public good is provided privately during a war of attrition, is ubiquitous. I model a generalized multi-player asymmetric war of attrition with incomplete information to investigate the role of asymmetry in such conflicts. In the unique equilibrium, asymmetry differentiates players into different incentive positions leading to a stratified behavior pattern. Analysis shows that the welfare level is mainly determined by the strongest player. He determines by ranking all types' incentive positions, instead of providing directly by himself. Moreover, the influence of asymmetry on welfare can be both positive and negative. I discuss and interpret the conditions for both possibilities.

KEYWORDS. War of attrition, private provision of public good, ex ante asymmetry, multiple players, incomplete information.

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1 Introduction

Natural state, in which communication and cooperation are prohibitively costly for a society and thus any public good can only be provided privately during a war of attrition, is one of the major sources of welfare loss from strategic interaction. It happens in daily basis and covers a large amount of applications. Examples are the dilemmas where multiple political groups try to shift the burden of economy stabilization onto others, and where adjacent countries or provinces pass the buck when there is a riot infesting their common border. Strategic delay is known to be a major cause of inefficiency in these situations.

One important question is: Does *asymmetry* matters in such natural-state conflicts? Here, asymmetry refers to ex ante asymmetry which means that different members' information is anticipated differently. This case corresponds to the presence of several incumbent political groups who feel more pressured to conduct fiscal stabilization, and countries or provinces which face different costs of eliminating a riot due to their asymmetric economic levels. The interesting questions that follow are: How does asymmetry change behavior? Does heterogeneous groups equally contribute to social welfare? And if not, who matters the most? Whether asymmetry alleviates or exacerbates delay? Can we make a society better off by sharpening or equalizing its asymmetry?

I answer these questions by presenting a generalized war-of-attrition model that combines ex ante asymmetry, multiple players, and incomplete information. Such a general combination is missing in the literature of war of attrition and other similar forms of contest[1], probably because previous studies have regarded it as merely a more involved extension with no novel insights. However, this paper proves otherwise.

Bliss and Nalebuff (1984) first realized the necessity to model private provision of public good as a war of attrition. They modeled a one-exit continuous-time game in which each player chooses a provision time at the beginning to optimize his expected utility, and once someone provides first, the game ends and everyone gains lump-sum payoff according to their information. My model looks similar to theirs nonetheless differing in that, instead of seeing provision cost as uncertain like them, I let valuation be the incomplete information, and that, more importantly, I allow all players' costs, discount rates, and valuation distributions to be asymmetric. Additionally, to guarantee a unique equilibrium, I assume that every player is anticipated with positive probability to value the public good less than his cost.

This paper finds that heterogeneous individuals manifest a *stratified behavioral pattern*. One

[1] I list some examples with two of the three elements. For those looking into multi-player asymmetric wars of attrition with complete information, see Ghemawat and Nalebuff (1985, 1990), Whinston (1988), and Bildeau and Slivinski (1994). Examples into multi-player symmetric wars of attrition with incomplete information are Riley (1980), Bliss and Nalebuff (1984), Alesina and Drazen (1991), Bulow and Klemperer (1999), and Sahuguet (2006). The two-player asymmetric incomplete-information case is the most widely studied, for example, Riley (1980), Nalebuff and Riley (1985), Fudenberg and Tirole (1986), Kornhauser, Robinstein, and Wilson (1988), Ponsati Sakovics (1995), Abreu and Gul (2000), Myatt (2005), and Horner and Sahuguet (2010). Also, there are special cases including all three elements. For example, a third party strategically interfering in a two-player war of attrition (e.g., Casella and Eichengreen (1996) and Powell (2017)), and two-group bargaining game which is basically a two-player game (e.g., Ponsati and Sakovics (1996)). The most related study is Kambe (2019) which investigated a model similar to mine but with two-type incomplete information.

degenerate example of this concept commonly seen in two-player wars of attrition is *instant exit*. That is, when two players are asymmetric enough, one and only one of them will have positive probability to concede immediately. A seminal work that mentioned instant exit is Nalebuff and Riley (1985), and among more recent studies characterizing this behavioral feature are Ponsati and Sakovics (1995), Riley (1999), Abreu and Gul (2000), and Myatt (2005).

I achieve further in my multi-player game. Apart from instant exit, there is also *strict waiting* which means that for some players each of their minimal waiting time among all types is strictly positive, rather than zero as is in two-player cases, and each player's strict-waiting time can be different. In other words, when parametrization is asymmetric enough, an equilibrium is endogenously determined to be such that it begins with a period during which only two players have the probability to provide and after it starts a three-player period, and so on. Eventually, only when the game has endured for a sufficiently long time will all players become active. Obviously, instant exit should be construed as a special one-player "period", the length of which is zero because there is no provision from others and any delay is pure waste.

This stratified equilibrium results from the asymmetric incentive positions of different types. Simultaneous revelation requires the types being revealed at the same time to mutually balance each other's incentive. Namely, for each being-revealed type it finds the extra gain from providing immediately equal that from waiting slightly longer. Some types exit instantly because they value the public good so much that no simultaneous revelation with other types can offest their high valuations. In contrast, some wait strictly because even their highest types still value the good too low to be mutually balanced with earlier revealed types. That asymmetry affects the outcome by changing the scale of active players has been mentioned by Bergstrom et al. (1986), Hillman and Riley (1989), and Kambe (2019).

Based on this interpretation, I call a player who either strictly waits shorter or provides instantly as a stronger one. Comparative statics tell that either lower cost, more impatience, or "consistently higher" valuation distribution reduces a player's provision time and increases his relative incentive position.

On the uniqueness of equilibrium. A huge literature acquires unique equilibrium by perturbing a war of attrition. Namely, for each player there must be a positive probability of some others' waiting forever. For example, Fudenberg and Tirole (1986) introduced a positive probability of each player's being better off in a duopoly than in a monopoly. Kornhauser, Rubinstein, and Wilson (1989) used a slight probability of irrational type who only plays a fixed strategy, the spirits of which were also borrowed by Kambe (1999, 2019) and Abreu and Gul (2000). Finally, Myatt (2005) considered three forms of perturbation: exit failure, hybrid payoff, and time limit. Yet, most of asymmetric wars modeled in literature involve only two players or essentially complete information[2].

So far as I know, Kambe (2019) is the only paper that shares the spirits above. Nonetheless, the difference between us is critical. First, he studied a war of attrition basically with complete information and used the existence of a non-compromising type as perturbation. So, the techniques are quite different from those employed in this paper. Besides, his analysis mainly

[2]By *essentially* I refer to Abreu and Gul (2000) and Kambe (2019) where they investigated wars of attrition with two-type incomplete information, but since one of the types is non-compromising, their setups are basically perturbed complete-information games.

focused on equilibrium behavior, for example, he discussed the probability of instant exit which he took for granted as a direct measure of welfare. I prove this effort to be less important by showing examples where the arbitrary variation of players' behavior does not affect the outcome or, equivalently, the welfare level. More generally, welfare analysis introduced later stresses that the outcome is mainly determined by the *ranking* of players' behavior, instead of directly by the behavior itself.

In addition to equilibrium analysis, I investigate the relationship between ex ante asymmetry and social welfare. First, I answer: Who matters the most? Following Bliss and Nalebuff's (1984) approach, I allow the population to grow large in a certain way, and find that each player's interim welfare is solely determined by, apart from his valuation, the strongest type in the society. I also find that each member's incentive position in the equilibrium is solely determined by his highest possible type.

This the-strong-determine result does not necessarily mean that the strongest type directly provides the public good, but that it decides the behavior "level" of an equilibrium. The explanation lies in the asymmetric dependence of players' behavior on parameters. Specifically, I introduce a special case, AD war, to show that while the parameter variation of a weak player sheds no influence on the behavior of stronger types, the variation of a strong player effectively changes the behavior of weaker types. This further implies that the strongest type is directly responsible for the determination of how types' incentive positions are ranked. In contrast, weaker players merely alter their strict-waiting times to suit this established ranking without affecting it much. And it is this ranking of stratified incentives determines the welfare level.

Second, I study the impact asymmetry has on welfare. That ex ante asymmetry may improve the welfare level has been mentioned in previous studies in which an efficient degenerate equilibrium was selected with perturbation. For example, Riley (1999) modeled a family of two-player complete-information contests including a war of attrition. He selected a unique equilibrium by letting a sequence of other family members approximate a pure war of attrition, and he discovered that any introduction of asymmetry in this equilibrium makes one player concede immediately, while the symmetric case is far from efficient. Kornhauser, Rubinstein, and Wilson (1988) and Myatt (2005) derived similar results in wars of attrition with incomplete information. Also, Kambe (2019) argued, without formal proof though, that higher probability of instant exit improves efficiency.

My objective clearly differs from theirs. First, I investigate the efficacy of introducing asymmetry in non-degenerate conflicts[3]. A degenerate equilibrium is of less interest, since in most true-life applications delay does occur. Second, I generalize the result to multi-member societies to make powerful policy implications.

Surprisingly, the conclusion is not definite, because asymmetry's influence highly relies on parametrization. I consider two representative cases to disclose both the condition for and the insight behind asymmetry as improvement or deterioration. The first case is redistribution among groups in a large-population society, and any unequalizing redistribution[4] increases the

[3]Actually, Riley (1999) also did this by conducting comparative statics on perturbed contests and found that even in those where delay emerges, asymmetry yields lower expected total expenditure. However, this result is based on numerical computation and limited to complete-information case.

[4]This term was used by Bergstrom, Blume, and Varian (1986).

welfare level of a symmetric society. The second case examines AD wars with the strongest type being fixed. A straightforward condition for improvement states that when the cost of symmetry, which is measured by the welfare-level discrepancy between an N-player symmetric game and the associated infinite-player symmetric game, is positive, any introduction of asymmetry alleviates the loss out of delay. In contrast, when this cost of symmetry is negative, the symmetric war of attrition becomes most efficient. I give examples for both cases which reveal the possibility for asymmetry to either improve or deteriorate the outcome.

This paper is organized as the following. Section 2 describes the model setup and the equilibrium concept. Section 3 characterizes equilibrium and further proves the existence and uniqueness. I introduce a special case, AD war, in this section to illustrate both behavioral features and welfare implications formally discussed later. Finally, this section performs comparative statics. Section 4 establishes connection between asymmetry and social welfare by answering two questions: Who matters the most? and Does and When does asymmetry improve welfare? Section 5 discusses possible applications associated with previous analysis and makes concluding remarks.

2 Model

There is an indivisible public good potentially beneficial to N different individuals. I denote each player by $i \in I_N$ where $I_N = \{1, 2, ..., N\}$. In the natural state, cooperative provision is not an option and therefore a continuous-time war of attrition becomes inevitable. This war begins at $t = 0$ and each player chooses a stop time when, if no one has provided the good yet, he will provide. Before the war of attrition ends, each player can change his decision anytime, although later analysis shows that this change is not likely.

Here is the information structure: one player, say i, knows exactly the cost of his individual provision $c_i > 0$, the rate $r_i > 0$ that he obeys to exponentially discount his expected gain at time t with $e^{-r_i t}$, and his valuation of this public good v_i. All values of cost and discount rate are common knowledge, whereas each valuation v_i is private information independently extracted from a cumulative distribution function $F_i : [\underline{v}_i, \overline{v}_i] \to [0, 1]$ in which $0 \leq \underline{v}_i < c_i < \overline{v}_i < +\infty$ for all i. For convenience, I sometimes call player i with type v_i simply as player v_i. Note that this strict relationship of $\underline{v}_i$, $\overline{v}_i$, and c_i is important because it necessarily guarantees the uniqueness of equilirium. The analysis requires some harmless assumptions: each F_i yields a dense function $f_i : [\underline{v}_i, \overline{v}_i] \to \mathbb{R}^+$ which is differentiable and strictly bounded from 0.

Player i's pure strategy is a function $T_i : [\underline{v}_i, \overline{v}_i] \to \mathbb{R}^+ \cup \{0, +\infty\}$ referring to the stop time that player v_i chooses. Only when no provision happens before $T_i(v_i)$ will this player provide at this time. If some provide first, each player gains his valuation while those who provide additionally pay their share of provision cost. Namely, if $m \geq 1$ players provide at this moment, each of them, say player i, pays c_i/m. And if all players choose to wait forever, each earns zero. All payoffs are lump-sum, and the moment the public good is provided, the game ends.

I consider pure-strategy perfect Bayesian equilibrium, and in the following sections by *equilibrium* I mean this kind, unless otherwise specified.