Does Ex-Ante Asymmetry Matter? A Modeling of Multi-Player Asymmetric War of Attrition

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September 13, 2019

Abstract

This paper models a multi-player asymmetric war of attrition with incomplete information on private provision of public good in order to investigate how ex-ante asymmetry affects behavior and welfare in a non-cooperative environment. In the unique equilibrium, agents who choose to exit simultaneously face incentives that make them indifferent between waiting and exiting at this moment. With this mutual balance of incentive, asymmetry differentiates types into distinct incentive positions which lead to a stratified behavior pattern that one player exits instantly with positive probability, while each of others has no probability of concession until a certain moment associated with him.

Efficiency measured by cost of delay is mainly determined by the strongest type which influences the ranking of the incentive positions of all types and therefore leads their behavior. Besides, under different forms of asymmetry, welfare is affected differently. If asymmetry is designed to strengthen the strongest type, it always improves efficiency, whereas if the strongest type is controlled, the effect of asymmetry coincides with the sign of an intuitive measure of cost of symmetry.

Keywords. War of attrition, private provision of public good, ex-ante asymmetry, multiple players, incomplete information.

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1 Introduction

When cooperation is prohibitively costly for a society, any public good can only be provided privately during a war of attrition. This happens in daily basis and is one of the major sources of welfare loss resulting from strategic interaction. Strategic delay is known to be a major cause of inefficiency in these situations.

One important question is: Does *asymmetry* matters in such conflicts? Here, asymmetry is ex-ante asymmetry which refers to that individuals in different economic, political, or social positions are anticipated differently. This notion corresponds to important and a large scale of applications. For example, when socioeconomic groups try to shift the burden of stablization onto each other, the presence of several incumbent groups who feel more pressured to conduct fiscal stabilization forms asymmetry. In addition, when several countries or provinces suffer from illegal activities on their common border, the different costs of controlling the chaos faced by different agents also form asymmetry. Moreover, when the United Nations gethers countries to reach an agreement on how humanity should respond to climate change, the different incentives to make voluntary commitment of different countries also form asymmetry.[1] Finally, ex-ante asymmetry also lies in the discriminative stereotypes that people have to others based on the impression of race, gender, age, and other social elements that label people.

This issue entails compelling questions: How does asymmetry change behavior? Does asymmetric entities make equal contributions? If not, who matters the most? Whether asymmetry alleviates or exacerbates delay? Can we be better off by sharpening or equalizing asymmetry?

This paper develops a generalized war of attrition that combines ex-ante asymmetry, multiple players, and incomplete information. Such a general combination is missing in the literature[2] of war of attrition and other similar forms of conflict, probably because previous studies have regarded it as merely a more involved extension with no novel insights. However, this paper

[1] Detailed explanation for these three examples. The symmetric analysis of the first example has been done by Alesina and Drazen (1991), and see this paper for more examples. The second example corresponds to the famous Golden Triangle, the common border of Thailand, Vietnam, and Laos which is also not far from China. The rampant illegal dealing of drugs and long-lasting violent activities are the consequence of delayed and loose regulation of the neighbor countries. It is noteworthy that asymmetry does exist in this case, as Thailand implements more strict regulation than others. Besides, other most famous areas for drug trade are also the common borders of several countries, like Golden Crescent and Silver Triangle. For the last example, the implementation of Paris Agreement is a good manifestation. While China has shown great willingness in protecting the earth because it has a powerful government and severe environmental problems, the United States kept postponing the progress and eventually exited this agreement.

[2] I list some examples with two of the three elements. For those looking into multi-player asymmetric wars of attrition with complete information, see Ghemawat and Nalebuff (1985, 1990), Whinston (1988), and Bildeau and Slivinski (1994). Examples into multi-player symmetric wars of attrition with incomplete information are Riley (1980), Bliss and Nalebuff (1984), Alesina and Drazen (1991), Bulow and Klemperer (1999), and Sahuguet (2006). The two-player asymmetric incomplete-information case is the most widely studied, for example, Riley (1980), Nalebuff and Riley (1985), Fudenberg and Tirole (1986), Kornhauser, Robinstein, and Wilson (1988), Ponsati Sakovics (1995), Abreu and Gul (2000), Myatt (2005), and Horner and Sahuguet (2010). Also, there are special cases including all three elements. For example, a third party strategically interfering in a two-player war of attrition (e.g., Casella and Eichengreen (1996) and Powell (2017)), and two-group bargaining game which is basically a two-player game (e.g., Ponsati and Sakovics (1996)). The most related study is Kambe (2019) which investigates a model similar to mine but with two-type incomplete information.

proves otherwise.

My model provides an asymmetric extension of Bliss and Nalebuff (1984). They discuss a symmetric continuous-time war of attrition with one exit in which each player chooses a provision time at the beginning to optimize his expected utility, and once someone provides first, the game ends and everyone gains lump-sum payoff according to their information. My model differs in that, instead of seeing provision cost as uncertain like them, I let valuation be the personal information, and that more importantly I allow all players' costs, discount rates, and valuation distributions to be asymmetric. Additionally, to guarantee a unique equilibrium, I assume that every player is anticipated with positive probability to value the public good less than his cost.

This paper finds that heterogeneous individuals manifest a *stratified behavior pattern*. One degenerate example of this concept commonly seen in two-player cases[3] is *instant exit*. That is, one of the players will have positive probability to concede immediately.

I achieve further in my multi-player game. Apart from instant exit, there is also *strict waiting* which refers to that for some players they will have no positive probability to provide until some time after the beginning. This is not possible in two-player cases, for the highest types of both players always provide instantly. For each player who waits strictly, I call the minimal waiting time among all his types the strict-waiting time, which can be different for different players.

As a result, when parametrization is asymmetric enough, equilibrium begins with some probability of one player's instant exit, and what follows is a period during which only two players have the probability to provide, and after it a third player becomes active, and in this manner periods that involve increasing numbers of active players follow sequential. Eventually, only when the game has endured for a sufficiently long time will all players become active. Obviously, instant exit should be construed as a special one-player "period", the length of which is zero because there is no provision from others and any delay is pure waste. The spirit that asymmetry affects the outcome by changing the scale of active players has been mentioned in earlier yet less general cases[4].

This stratified equilibrium results from the asymmetric *incentive position*s of different types. Simultaneous revelation requires the types revealed together to mutually balance each other's incentives. Namely, types being revealed at the same time and the way in which they are revealed are determined to be such that each of them will find the extra gain from providing immediately equal that from waiting slightly longer at this moment. This mutual-balance requirement makes the incentives faced by different types, to some extent, comparable. For example, types that exit instantly value the public good so much that no simultaneous revelation with other types can offset their valuations, so they are in higher incentive positions. In contrast, some players wait strictly because even their highest types still value the good too low to be mutually balanced with earlier revealed types, and thus they are in lower positions.

Intuitively, I call a player *stronger* if he has positive probability to provide instantly or he strictly waits shorter, and the former case corresponds to the strongest player. Comparative statics tell that either lower cost, more impatience, or "consistently higher" valuation distribution reduces a player's provision time and makes him stronger.

[3] A seminal work that mentions instant exit is Nalebuff and Riley (1985), and among more recent studies are Ponsati and Sakovics (1995), Riley (1999), Abreu and Gul (2000), and Myatt (2005).

[4] For example, see Bergstrom et al. (1986), Hillman and Riley (1989), and Kambe (2019).

On the uniqueness of equilibrium. A huge literature obtains uniqueness by perturbing a war of attrition. Namely, for each player there must be a positive probability of some others' never conceding. For example, Fudenberg and Tirole (1986) introduce a positive probability of each player's being better off in a duopoly than in a monopoly. Kornhauser, Rubinstein, and Wilson (1989) use a slight probability of irrational type who only plays a fixed strategy, the spirits of which are also borrowed by Kambe (1999, 2019) and Abreu and Gul (2000). Finally, Myatt (2005) considers three forms of perturbation: exit failure, hybrid payoff, and time limit. Yet, most literature involves only two players or essentially complete information[5]. The perturbation strategy I employ is to allow each player to have a positive probability of valuing the public good less than his provision cost, as when this case is realized never conceding is the dominant strategy. The technical representation is the sensitivity of the solution of a group of differential equations with respect to boundary conditions, and thus there is only one selection of boundary conditions corresponding to equilibrium.

The second point made by this paper lies in the relationship between ex-ante asymmetry and social welfare.

First, I utilize expected discount factor as a measure of welfare level to directly answer two questions: Who matters the most? and How does he matter? The answer to the former is expected: social welfare is mainly decided by the strongest player. Nonetheless, the answer to the second question is subtly surprising: the strongest player matters by determining the incentive ranking of all types, rather than providing directly by himself. It is commonly believed in literature that the strongest player matters the most merely because he makes most of the contribution, that is he has the highest probability to concede first. For example, Myatt (2005) and Kambe (2019) overstate the importance of the probability of instant exit. However, the welfare level of the special case, AD war, discussed in Section 3.3 manifests irrelevance to any variation of players' behavior as long as the strongest type is fixed, and this result makes the focus on the strongest player's exact behavior, like his instant-exit probability, less important.

A more robust explanation lies in that the strongest player, or more acurately his highest type, "leads" the behavior of all types, and further since the welfare is an integral with respect to all players' behavior which "closely follows" the strongest type, its level is mainly decided by the latter. One way to see this is through the asymmetric dependence among players' behavior. Again, I use AD war to show this. In this special case, while the parameter variation of a weak player sheds no influence on stronger types' behavior, the variation of a strong player effectively changes weaker types' behavior in the same direction. Additionally, the analysis of divided societies with large population in Section 4.1 presents a similar and more general result, as the highest type of each group completely decides the incentive position of his group and this also leads to asymmetric dependence. This behavior feature implies that the strongest type directly determines how types' incentives are ranked, and that weaker players merely alter their strict-waiting times to suit this established ranking without affecting it much.

Second, I investigate the impact that asymmetry has on welfare. That introducing asymmetry will enhance efficiency is a point commonly made in literature. One extreme example is

[5] By *essentially* I refer to Abreu and Gul (2000) and Kambe (2019) where they investigate wars of attrition with discrete-type incomplete information, but since only one type is rational while all others never concede, their setups are basically perturbed complete-information games.

to select an efficient yet degenerate equilibrium with refinement. For instance, Riley (1999) lets a sequence of members of a contest-game family approximate a war of attrition which yields that any introduction of asymmetry makes one player concede immediately with propability one. Kornhauser, Rubinstein, and Wilson (1988) and Myatt (2005) derive similar results in more complex cases. Besides, Kambe (2019) argues without proof that asymmetry increases the probability of instant exit and further improves efficiency. Some others analyze welfare directly, for example, Riley's (1999) numerical calculation of a welfare measure shows a consistently positive efficacy of asymmetry, but the result however may hinge on his complete-information setup. Static public-provision games like Bergstrom, Blume, and Varian (1986) argue similarly.

However, the model analyzed in this paper incorporates more dimensions as there are multiple players and continuous-type distributions, so the influence of ex-ante asymmetry is more complicated and highly dependent on how asymmetry is presented. Enlightened by the insight that the strongest player has decisive influence on the outcome as introduced before, I manage to provide a complete discussion by considering two kinds of asymmetries, one of which allows the strongest player to change while the other controls this variation. Specifically, the first way I consider of introducing asymmetry is to make the strongest stronger under certain control, whereas the second is to fix the strongest and make others weaker. The reason for such division is that these two sorts of asymmetries constitute a complete discussion and, more importantly, they generate different insights. Thus, both form a reasonable dissection which helps us to understand the efficacy of asymmetries under different circumstances.

The result corresponding to the former kind of asymmetry shows that by strengthening the strongest, slight introduction of asymmetry always reduces cost of delay. I conduct numerical experiments which show that for a large scale of applications the phrase above, "slight introdution of asymmetry", can be expanded to "any introducion of asymmetry". The intuition here is consistent with the previous finding that the strongest type has decisive influence on the outcome.

Nonetheless, the other case manifests dependence on parametrization, as any introduction of asymmetry enhances efficiency if the cost of symmetry is positive which is measured by the welfare-level discrepancy between an N-player symmetric game and the associated infinite-player symmetric game, while asymmetry always impairs efficiency if the thus measured cost of symmetry is negative. An explanation for these dichotomic conditions is that the cost of symmetry defined above actually evaluates the cost incurred by increasing population and since asymmetry makes the scale of active players smaller during the beginning period, the efficacy of asymmetry has the same sign as that of the cost of symmetry.

Before ending the introduction, I differentiate this paper from Kambe (2019). His paper is the most related to mine, but the results are nonetheless limited by both the simplification of model setup and the misled focus on less significant economic issues.

For one thing, Kambe (2019) employs two-type distribution to simplify the analysis, but this forbids him to delve deeper for lack of flexibility. First, albeit the proof of uniqueness is simplified, the two-type setup actually makes the welfare integral very complex and thus analysis on efficiency is unlikely. In contrast, although in my model information structure has infinite dimensions, there are several representative cases whose welfare functions are tractable, like AD war and large-population society. Second, some of his results hinge on this simplification. For example, Kambe's (2019) Proposition 4 shows that the probability of instant exit has lower bound under certain conditions, but the uniform-distribution example I give in Section 3.3 shows that such a lower bound may not necessarily exist (see Proposition 1).

In addition, Kambe (2019) pays too much attention to players' exact behavior, as he focuses on Who provides first? and How much does he provide? Specifically, in Kambe's (2019) Section 4 he argues that the probability of instant exit is positively related to welfare. However, the uniform-distribution example in Section 3.3 undermines this point directly, as in this example welfare is only decided by the strongest type, and as long as it is fixed, all players' behavior can vary with parameters arbitrarily without shedding influence on efficiency (see Proposition 1).

This paper is organized as the following. Section 2 describes the model setup and the equilibrium concept. Section 3 characterizes equilibrium and proves the existence and uniqueness. I introduce a special case, AD war, in this section to illustrate both behavior features and welfare implications formally discussed later. Finally, this section performs comparative statics. Section 4 establishes connection between ex-ante asymmetry and social welfare by answering two questions: Who and How does he matter the most? and Does and When does asymmetry improve welfare? Section 5 discusses possible applications and makes concluding remarks.

2 Model

There is an indivisible public good potentially beneficial to N different individuals. I denote each player by $i \in I_N$ where $I_N = \{1, 2, ..., N\}$. In the natural state, cooperative provision is not an option and therefore a continuous-time war of attrition becomes inevitable. This war begins at $t = 0$ and each player chooses a stopping time when, if no one has provided the good yet, he will provide. Before the war of attrition ends, each player can change his decision anytime, although later analysis shows that this change is not likely (See footnote 6).

Here is the information structure: one player, say i, knows exactly the cost of his individual provision $c_i > 0$, the rate $r_i > 0$ that he obeys to exponentially discount his expected gain at time t with $e^{-r_i t}$, and his valuation of this public good v_i. All values of cost and discount rate are common knowledge, whereas each valuation v_i is private information independently extracted from a cumulative distribution function $F_i : [\underline{v}_i, \overline{v}_i] \to [0, 1]$ in which $0 \leq \underline{v}_i < c_i < \overline{v}_i < +\infty$ for all i. For convenience, I sometimes call player i with type v_i simply as player v_i. Note that this strict relationship of $\underline{v}_i$, $\overline{v}_i$, and c_i is important because it necessarily guarantees the uniqueness of equilirium. The analysis requires some harmless assumptions: each F_i yields a dense function $f_i : [\underline{v}_i, \overline{v}_i] \to \mathbb{R}^+$ which is differentiable and strictly bounded from 0.

Player i's pure strategy is a function $T_i : [\underline{v}_i, \overline{v}_i] \to \mathbb{R}^+ \cup \{0, +\infty\}$ referring to the stopping time that player v_i chooses. Only when no provision happens before $T_i(v_i)$ will this player provide at this time. If some provide first, each player gains his valuation while those who provide additionally pay their share of provision cost. Namely, if $m \geq 1$ players provide at this moment, each of them, say player i, pays c_i/m. And if all players choose to wait forever, each earns zero. All payoffs are lump-sum, and the moment the public good is provided, the game ends.

I consider pure-strategy perfect Bayesian equilibrium, and in the following sections by *equilibrium* I mean this kind, unless otherwise specified.