

计算方法第二次作业

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1. 利用2次拉格朗日插值进行计算

$$L_2(x) = \frac{(x-100)(x-121)}{(81-100)(81-121)} \times 9 + \frac{(x-81)(x-121)}{(100-81)(100-121)} \times 10 + \frac{(x-81)(x-100)}{(121-81)(121-100)} \times 11$$

$$\sqrt{108} = L_2(108) \approx 10.39985$$

$$\text{误差 } R = \frac{(\sqrt{x})^{(3)}}{3!} (x-81)(x-100)(x-121) \quad x \in (81, 121)$$

代入得出误差界为 $(-0.0029721, -0.0010897)$

2. 画出差商表如下:

x_i	x_i	$f(x_i)$	$f[x_{i-1}, x_i]$	$f[x_{i-2}, x_{i-1}, x_i]$	$f[x_{i-3}, x_{i-2}, x_{i-1}, x_i]$
	1.0	2.0			
	2.0	4.0	$\frac{4.0-2.0}{2.0-1.0} = 2.0$		
	3.0	8.0	$\frac{8.0-4.0}{2.0-2.0} = 4.0$	$\frac{4.0-2.0}{2.0-1.0} = 1.0$	
	4.0	5.0	$\frac{5.0-8.0}{4.0-3.0} = -3.0$	$\frac{-3.0-4.0}{4.0-2.0} = -3.5$	$\frac{-3.5-1.0}{4.0-1.0} = -1.5$

构造相应的牛顿插值多项式如下:

$$N_3(x) = 2 + 2(x-1) + (x-1)(x-2) - 1.5(x-1)(x-2)(x-3)$$

$$\text{代入得: } f(1.5) = 2.1875$$

3. $x_i \quad f(x_i)$

0	2				$P_2(x) = -\frac{23}{720}(x-3)(x-1)x + \frac{11}{24}(x-1)x - \frac{3}{2}x + 2$
1	0.5	-1.5			$P_3(x) = \frac{f^{(4)}(\xi)}{4!} (x-1)(x-3)^2x$
3	0.25	-0.125	$\frac{11}{24}$		
3	0.25	0.6	$\frac{29}{80}$	$-\frac{23}{720}$	$P_3(2) \approx -0.0194$