计算方法第一次作业

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1. (1) $f(x) = (a+x)^n - a^n$

$$= (a+x-a)\left((a+x)^{n-1} + (a+x)^{n-2}a + \cdots + (a+x)a^{n-2} + a^{n-1}\right)$$

$$= x\cdot\left(\left(\left(\left((a+x+a)(a+x)+a^2\right)(a+x)+a^3\right)(a+x)+\cdots+a^{n-2}\right)(a+x)+a^{n-1}\right)$$

(2) $f(x) = \cos(a+x) - \cos a$

$$= -2\sin\left(a+\frac{x}{2}\right)\sin\frac{x}{2}$$

(3) $f(x) = x - \sqrt{x^2+a} = \dfrac{-a}{x+\sqrt{x^2+a}}$

2. $x^* - x = 0.0000005$ 因此有5位有效数字

绝对误差为 $x^* - x = 0.0000005$

3. 假设存在一个取值序列 $\lambda_0, \lambda_1, \cdots \lambda_n$ 使得

$$\lambda_0 l_0(x) + \lambda_1 l_1(x) + \cdots + \lambda_n l_n(x) = 0$$

由于 $l_i(x_j) = \begin{cases} 0 & i\neq j \\ 1 & i=j \end{cases}$ ∴有 $\lambda_0 l_0(x_0)=0,\ \lambda_1 l_1(x_1)=0, \cdots$

即 $\lambda_0 = \lambda_1 = \cdots = \lambda_n = 0$

∴ $l_i(x),\ i=0,\cdots n$ 是线性无关的

4. $$l_0 = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} = \frac{(x-1)(x-4)(x-5)}{(-1-1)(-1-4)(-1-5)}$$

$$l_1 = \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} = \frac{(x+1)(x-4)(x-5)}{(1+1)(1-4)(1-5)}$$

$$l_2 = \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} = \frac{(x+1)(x-1)(x-5)}{(4+1)(4-1)(4-5)}$$

$$l_3(x) = \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} = \frac{(x+1)(x-1)(x-4)}{(5+1)(5-1)(5-4)}$$

$$\therefore L_3(x) = l_1(x) + 2l_2(x) + 4l_3(x) = \frac{9x^3 - 40x^2 + 51x + 100}{120}$$

$L_3(2.0) = 0.95$ $\quad$ $L_3(4.0) = 2.0$