

计算方法第六次作业

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$$1. (a) \|A\|_1 = \max\{5+1+1, |-2|+3+2, 1+|-1|+6\} = 8$$

$$\|A\|_\infty = \max\{5+|-2|+1, |-1|+3+|-1|, |-1|+2+6\} = 9$$

$$(b) \|B\|_1 = \max\{4+|-2|+1, |-1|+3+1, 1+1+6\} = 8$$

$$\|B\|_\infty = \max\{4+|-1|+1, |-2|+3+1, |-1|+1+6\} = 8$$

$$2. |\lambda I - A| = 0 \Rightarrow (\lambda - 4)(\lambda - 1) - 2 = 0 \therefore \lambda = \frac{5 \pm \sqrt{17}}{2}$$

$$\therefore A \text{ 的谱半径为 } \max\left\{\frac{5+\sqrt{17}}{2}, \frac{5-\sqrt{17}}{2}\right\} = \frac{5+\sqrt{17}}{2} \approx 4.562$$

$$A^T A = \begin{pmatrix} 17 & -9 \\ -9 & 5 \end{pmatrix} \Rightarrow \lambda' = 11 \pm 3\sqrt{13}$$

$$\therefore \|A\|_2 = \sqrt{\rho(A^T A)} = \sqrt{11 + 3\sqrt{13}} \approx 4.671$$

3. 根据 Doolittle 分解的性质 设 $A = LU$

$$L = \begin{pmatrix} 1 & & \\ b_{21} & 1 & \\ b_{31} & b_{32} & 1 \end{pmatrix} \quad U = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ & u_{22} & u_{23} \\ & & u_{33} \end{pmatrix}$$

$$\text{求解可得: } L = \begin{pmatrix} 1 & & \\ \frac{1}{5} & 1 & \\ \frac{2}{5} & \frac{3}{14} & 1 \end{pmatrix} \quad U = \begin{pmatrix} 5 & 1 & 2 \\ & \frac{14}{5} & -\frac{7}{5} \\ & & \frac{9}{2} \end{pmatrix}$$

$$\text{由 } Ly = b \Rightarrow y = \begin{pmatrix} 10 \\ 3 \\ \frac{215}{14} \end{pmatrix} \quad \text{由 } Ux = y \Rightarrow x = \begin{pmatrix} \frac{5}{63} \\ \frac{25}{9} \\ \frac{215}{63} \end{pmatrix}$$

$$\therefore \text{解为 } x_1 = \frac{5}{63} \quad x_2 = \frac{25}{9} \quad x_3 = \frac{215}{63}$$