XXXVII Olimpiada Iberoamericana de Matemáticas

First Day

28 of September 2022

Problem 1. Let ABC be an equilateral triangle with circumcenter O and circumcircle Γ. Let D be a point on the minor arc BC, with $DB > DC$. The perpendicular bisector of OD intersects Γ at E and F, with E on the minor arc BC. Let P be the intersection point of lines BE and CF. Prove that PD is perpendicular to BC.

Problem 2. Let $S = \{13, 133, 1333, \dots\}$ be the set of positive integers of the form $1\overbrace{3\dots3}^{n\text{ digits}}$, with $n \geq 1$. Consider a horizontal row of 2022 empty cells. Ana and Borja play the following game: in turn, each player writes a digit from 0 to 9 on the leftmost empty cell. Starting with Ana, the players take turns until all cells are filled. When the game ends, the row is read from left to right to create a 2022-digit number N. Borja wins if N is divisible by a number belonging to S, otherwise Ana wins. Find which player has a winning strategy and describe it.

Problem 3. Let $\mathbb{R}$ be the set of real numbers. Find all functions $f\colon \mathbb{R} \to \mathbb{R}$ satisfying the following conditions simultaneously:

(i) $f(yf(x)) + f(x-1) = f(x)f(y)$ for every x, y in $\mathbb{R}$.

(ii) $|f(x)| < 2022$ for every x with $0 < x < 1$.

Time: 4 hours and 30 minutes.
Each problem is worth 7 points.