ECG Data Compression Using Truncated Singular Value Decomposition

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***Abstract*—The method of truncated singular value decomposition (SVD) is proposed for electrocardiogram (ECG) data compression. The signal decomposition capability of SVD is exploited to extract the significant feature components of the ECG by decomposing the ECG into a set of basic patterns with associated scaling factors. The signal informations are mostly concentrated within a certain number of singular values with related singular vectors due to the strong interbeat correlation among ECG cycles. Therefore, only the relevant parts of the singular triplets need to be retained as the compressed data for retrieving the original signals. The insignificant overhead can be truncated to eliminate the redundancy of ECG data compression. The Massachusetts Institute of Technology–Beth Israel Hospital arrhythmia database was applied to evaluate the compression performance and recoverability in the retrieved ECG signals. The approximate achievement was presented with an average data rate of 143.2 b/s with a relatively low reconstructed error. These results showed that truncated SVD method can provide an efficient coding with high-compression ratios. The computational efficiency of the SVD method in comparing with other techniques demonstrated the method as an effective technique for ECG data storage or signals transmission.**

***Index Terms*—Data compression, electrocardiogram, feature extraction, quasi-periodic signal, singular value decomposition.**

I. INTRODUCTION

THE electrocardiogram (ECG) is a conventional means of a biomedical signal for the diagnosis of heart diseases. Twenty-four-hour ECG monitoring is desirable as an effective method to detect heart abnormalities. The real-time data transmission over public telephone lines is also required in administering telemedicine. Therefore, an efficient storage for long-term recording and an effective real-time transmission are important issues in modern clinical applications.

The compression techniques for an ECG have been extensively discussed [1] and can be classified into the following three major categories.

1) *Parameter extraction techniques*: an irreversible process to retain the particular characteristics or parameters of the ECG signals.
2) *Direct time-domain techniques*: including amplitude zone time epoch coding (AZTEC), delta coding, and entropy coding [2]–[4].
3) *Transform-domain techniques*: such as Fourier transform, Walsh transform, Karhunen–Loeve transform (KLT) [5]–[7], and wavelet transform [8]–[11].

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Most reported ECG compression techniques proposed only intrabeat correlation between successive samples. Recently, the interbeat correlation between cycles is considered as a solution to achieving superior compression performance. For instance, long-term prediction [12] and average beat subtraction [13] reportedly use the beat-to-beat correlation. However, several limitations of these methods have been pointed out in [11]. An investigation of a wavelet-based algorithm has also been reported using the period-and-amplitude-normalized with discrete-wavelet-transform truncation (PAN–DWT truncation) technique to eliminate redundancy in the original ECGs [11]. However, the complexity of compression processes composed of amplitude normalization, period normalization, and an interpolation filter can downgrade their computational efficiency. Correspondingly, the aliasing effect may occur due to improper designs of interpolation filter and downsampling factors.

The method of singular value decomposition (SVD) was proposed in 1970 and has been applied in a wide range of areas, such as image compression, texture processing, and feature extraction. Hence, a truncated SVD was proposed to control noise artifacts in the electroencephalography (EEG) and magnetoencaphalography (MEG) of a reconstructed image [14]. The size of neural networks can be reduced due to the information compactness of SVD [15]. A prominent feature of SVD is the separation of the fundamental structural modes constituting a system that can be used to analyze quasi-periodic processes [16]. A new method of decomposition of signals into component periodic waveforms based on SVD was proposed [17], and had been successfully applied for the extraction of fetal ECGs from composite maternal ECG signals obtained from the abdominal lead [18]. The concept was also presented in literature [19], in which SVD was used to remove the artifacts and electromyogram in a 12-lead ECG, and tested for reconstruction of a completely lost channel.

The objective of this paper is to propose a novel application of SVD in data compression of ECGs. The quasi-periodic analysis of SVD was exploited to decompose an ECG sequence into a linear combination of a set of basic patterns with associated scaling factors. Combining the specific interbeat correlation of ECGs, an algorithm has been found that provides an efficient performance with higher compression ratios (CRs) and a relatively low reconstructed error.

II. DECOMPOSITION OF QUASI-PERIODIC SIGNAL USING SVD TECHNIQUE

Consider a periodic signal $x(k)$, which has m consecutive periods with length n. $x(k)$ can then be rearranged as a two-

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dimensional matrix $\boldsymbol{A}$, which can be expressed as

$$\boldsymbol{A} = \{\boldsymbol{x}_i(t)\,|\,i=1,\ldots,m; t=1,\ldots,n\}$$
$$= \begin{bmatrix} x(1) & x(2) & \cdots & x(n) \\ x(n+1) & x(n+2) & \cdots & x(2n) \\ \vdots & \vdots & \ddots & \vdots \\ x\big((m-1)n+1\big) & x\big((m-1)n+2\big) & \cdots & x(mn) \end{bmatrix} \quad (1)$$

where $\boldsymbol{x}_i(t)$ is the ith period of $x(k)$. Therefore, the SVD of the $m \times n$ matrix $\boldsymbol{A}$ can be performed as $\boldsymbol{A} = \boldsymbol{U}\Sigma\boldsymbol{V}^T$ [20], where $\boldsymbol{U} \in \boldsymbol{R}^{m\times m}$, $\boldsymbol{V} \in \boldsymbol{R}^{n\times n}$ are the left and right singular vectors, respectively. The $m \times n$ matrix $\Sigma = [\mathrm{diag}\{\sigma_1,\ldots,\sigma_R\} : 0]$ or its transpose depend on whether $m < n$ or $m > n$ and $\sigma_1,\ldots,\sigma_R$ are the singular values of matrix $\boldsymbol{A}$ with $\sigma_1 \geq \cdots \geq \sigma_R \geq 0$ and $R := \min(mn)$. The set $\{\boldsymbol{u}_i, \sigma_i, \boldsymbol{v}_i\}$ is called the ith singular triplet. The information energy of the matrix $\boldsymbol{A}$ can be expressed as the sum of squared singular values as

$$Q_A = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_R^2. \quad (2)$$

Moreover, we can define the energy ratio (ER), which represents the percentage of information energy within a certain number of singular values by the relation

$$\mathrm{ER}(\%) = \left\{ \sum_{i=1}^{q} \sigma_i^2 \Big/ \sum_{i=1}^{R} \sigma_i^2 \right\} \times 100 \quad (3)$$

where R is the total number of singular values and q is the value of the truncated index.

According to the decomposition processes of SVD [16], the matrix $\boldsymbol{A}$, which is filled with the repetitive pattern of consecutive rows, can be decomposed into a set of vectors $\boldsymbol{v}_i$ (i.e., basic patterns) and associated with $\boldsymbol{u}_i\sigma_i$ vectors (which corresponds to the scaling factors), where $i = 1, 2, \ldots, R$. However, all singular values will be zero, except σ_1, with $\boldsymbol{v}_1$ representing the principal component and where $\boldsymbol{u}_1\sigma_1$ is the scaling element representing the amplitude over the entire period. The result illustrated in matrix $\boldsymbol{A}$ is a rank-one matrix since $\{x(k)\}$ is an exact periodic signal with length n. This means that only one singular value (σ_1) and related singular vector pair $(\boldsymbol{u}_1, \boldsymbol{v}_1)$ is required to represent the original data using the following expression:

$$\hat{\boldsymbol{A}} = \boldsymbol{u}_1\sigma_1\boldsymbol{v}_1^T \quad (4)$$

where $\hat{\boldsymbol{A}}$ is the reconstructed matrix. The original waveform can then be recovered after proper matrix conversion.

In fact, in the case of practical periodic physiological signals, like the ECG, both the periodic length and periodic pattern may vary to a certain extent. Generally, there are one or more prime periodic or nearly periodic signals mixed with the aperiodic signals. As a result, $\boldsymbol{A}$ will become a full rank matrix. Even so, the main information energy may still be concentrated within the most dominant dyad $\boldsymbol{u}_1\sigma_1\boldsymbol{v}_1^T$ with respect to σ_1 if $\sigma_1^2/\sigma_2^2 \gg 1$.

Usually, we can assume that q of R singular values are predominant. Therefore, the prime information of the matrix $\boldsymbol{A}$ is contained in

$$\hat{\boldsymbol{A}} = \sum_{j=1}^{q} \boldsymbol{u}_j\sigma_j\boldsymbol{v}_j^T. \quad (5a)$$

Furthermore, the above equation can be rewritten explicitly as

$$\hat{\boldsymbol{A}} = \{\hat{\boldsymbol{x}}_i\,|\,i=1,\ldots,m; t=1,\ldots,n\}$$
$$= u_{i1}\sigma_1\boldsymbol{v}_1(t) + u_{i2}\sigma_2\boldsymbol{v}_2(t) + \cdots + u_{ij}\sigma_j\boldsymbol{v}_j(t)$$
$$+ \cdots + u_{iq}\sigma_q\boldsymbol{v}_q(t) \quad (5b)$$

where $\hat{\boldsymbol{x}}_i(t)$ is the ith segment of the reconstructed signal. The singular vectors $\boldsymbol{v}_j(t)$ act as the jth basic pattern of $\hat{\boldsymbol{x}}_i(t)$. The singular value σ_j is the associated weighting factor of the jth pattern and the element of singular vector u_{ij} is the scaling coefficient of the jth pattern in the ith segment. The equation shows that the prime information of the quasi-periodic signals can be represented using the linear combination of the basic patterns $\boldsymbol{v}_j$ with associated scaling factors $u_{ij}\sigma_j$ for the jth pattern in the ith segment after decomposition process.

Other insignificant singular values can be truncated appropriately to eliminate a large number of redundant data caused by the high correlation among ECG cycles. This is the basic fundamental concept of the proposed ECG data-compression scheme.

III. ECG Data Compression Using SVD Transformation

To perform ECG compression using SVD, a block-coding method is employed to transform a one-dimensional ECG sequence into a two-dimensional matrix after beat delineation (i.e., QRS detection) and period normalization. The block diagrams of data compression and reconstruction are shown in Fig. 1. The details of the implementation are illustrated in subsequent steps and related results are demonstrated as follows.

A. Periodic Segmentation

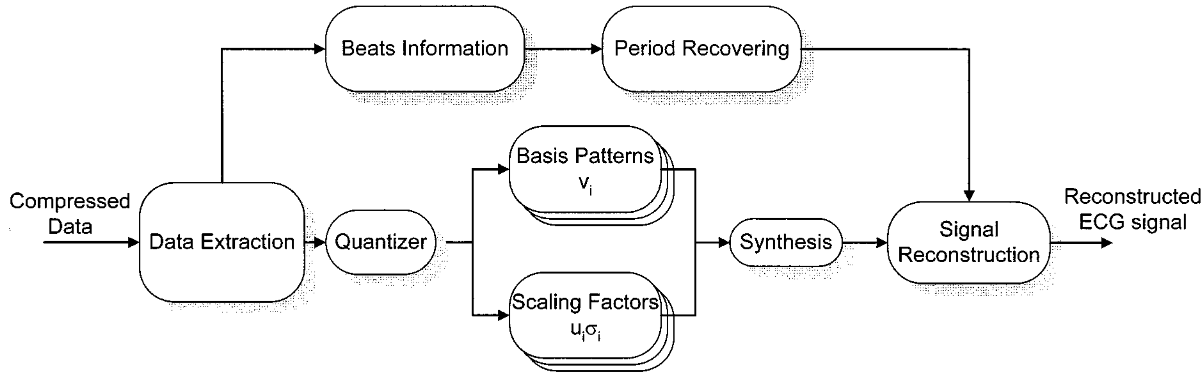
The first step of the algorithm is to separate each period of the ECG using QRS detection. By picking up the R-wave position among heartbeats, we define a segment from one R-wave to the next for delineating ECG segments and store the R–R interval as the beat information for period transformation.

B. Period Normalization and Matrix Conversion

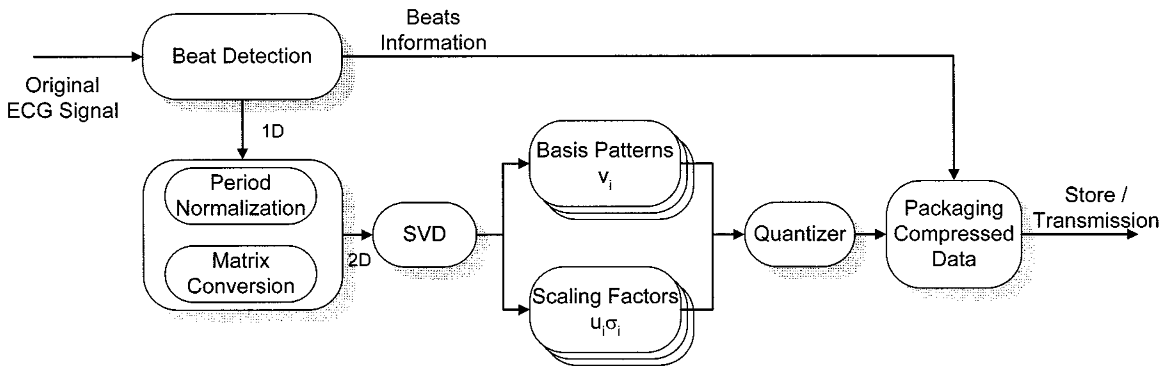
The segmented ECG cycles are normalized to the same periodic length n using the transformation reported in [17]. That is, one of the ECG segments $\boldsymbol{y}_i = [y_i(1), yi(2), \ldots, y_i(n^*)]$ can be converted into a segment $\boldsymbol{x}_i = [x_i(1), x_i(2), \ldots, x_i(n)]$ that holds the same signal morphology, but in different data length (i.e., $n^* \neq n$) using the following equation:

$$x_i(j) = y_i(j^*) + \big(y_i(j^*+1) - y_i(j^*)\big)(r_j - j^*) \quad (6)$$

where $r_j = (j-1)(n^*-1)/(n-1)+1$ and j^* is the integral part of r_j.



(a)



(b)

Fig. 1. (a) Block diagram of data compression using truncated SVD. (b) Block diagram of data reconstruction using the synthesis of basic patterns with associated scaling factors.

Therefore, the various lengths of the ECG segments will be compressed or extended into a set of ECG segments with the same periodic length n. The mean beat period (MBP) is chosen as the normalized length and is also retained as the beat information. The original period of each segment can be recovered using the same transformation [see (6)] at the time of data reconstruction.

Consequently, the set of ECG segments can be rearranged into an $m \times n$ matrix $\boldsymbol{A}$ filled with consecutive ECG cycles as the consecutive rows, where m is the number of consecutive ECG cycles and n is the length of the normalized period.

Fig. 2 illustrates the period transformation of one of the experimental data record (119.dat) taken from the Massachusetts Institute of Technology–Beth Israel Hospital (MIT–BIH) database. The different lengths of consecutive ECG cycles are normalized into an identical period, and can be recovered into the original one without losing beat information. The original $(\boldsymbol{y}_i)$, normalized $(\boldsymbol{x}_i)$, and recovered $(\boldsymbol{y}_i')$ ECG periods are displayed, respectively, from top to bottom in the figure. The result reveals almost without any deviation between the original ECG [see Fig. 2(a)] and the recovered waveform [see Fig. 2(c)] from the high period variation of the experimental record. This demonstrates that the period transformation will not cause distortion in the normalized segments [see Fig. 2(b)] and the recovered waveforms. Fig. 3 presents the three-dimensional plot of the rearranged ECG data matrix (record 119). The variation of the consecutive row waveform in certain components illustrates the strong interbeat correlation among ECG cycles.

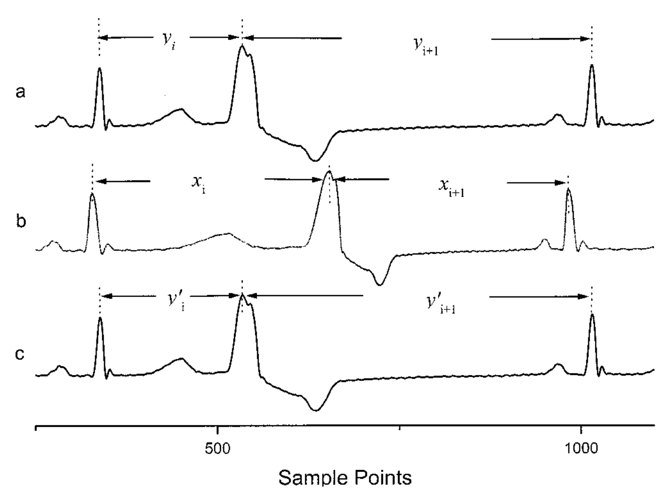


Fig. 2. Nearly distortion-free period transformation with variable interbeat intervals. (a) Original ECG. (b) Normalized ECG period with same beat length. (c) Recovered ECG with original period length.

C. SVD Transformation for Signal Decomposition

The represented ECG matrix $\boldsymbol{A}$ is then decomposed into the set of basic patterns $\boldsymbol{V} = \{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_n\}$ with associated scaling vectors $\boldsymbol{U}\Sigma = \{\boldsymbol{u}_1\sigma_1, \boldsymbol{u}_2\sigma_2, \ldots, \boldsymbol{u}_m\sigma_m\}$ using SVD transformation.

The information of the ECG signals will mostly be concentrated within a few dominant singular triplets $\{\boldsymbol{u}_i, \sigma_i, \boldsymbol{v}_i\}$ due to the quasi-periodic process of SVD. The $\boldsymbol{V}_1^T$ forms the principal

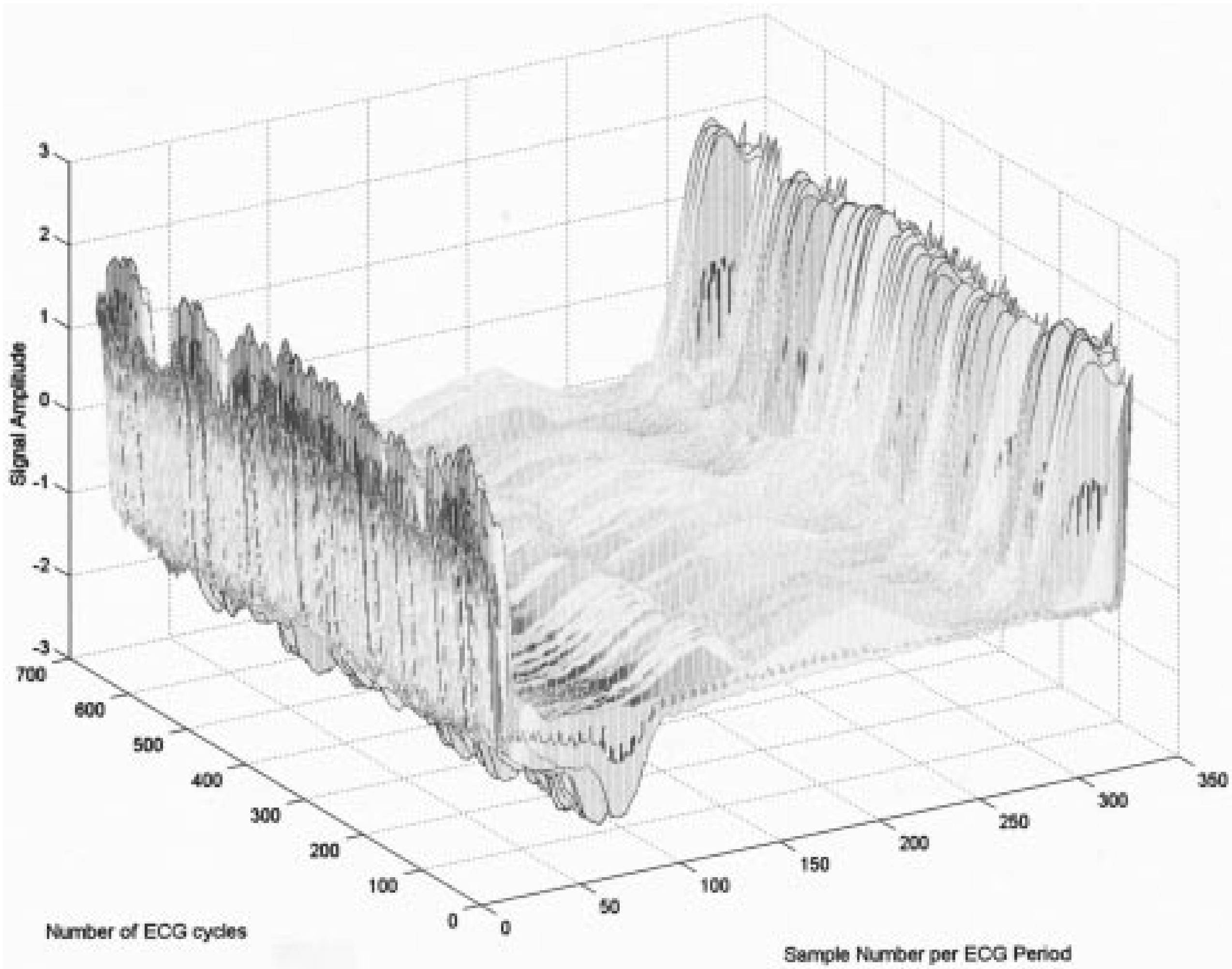


Fig. 3. Strong interbeat correlation among consecutive ECG cycles from the rearranged ECG matrix after period normalization and matrix conversion.

repeating pattern among cycles and is weighted by the factors $\{u_{1j}\sigma_1\}$, where u_{1j} is the jth element of $\boldsymbol{u}_1$, scaling the jth row of matrix $\boldsymbol{A}$. However, the other patterns $\boldsymbol{v}_i^T$ with associated scaling factors $\{u_{ij}\sigma_i\}$ should also be considered for reconstructing the ECG waveform.

Fig. 4 presents the part of the major components $(\boldsymbol{v}_i^T)$ of the data record (119) after the decomposition process, in which they have been multiplied by the pattern's weighting factor (σ_i). The result clearly displays the dominant singular value σ_1 with respect to the $\boldsymbol{v}_1^T$ vector forming the principal component in the ECG waveform.

D. ECG Data Compression and Data Reconstruction

To retrieve ECG waveforms, the compressed data must include the following items:

1) set of singular triplets;
2) associated beat information;
3) miscellaneous parameters.

The singular triplets $\{\boldsymbol{u}_i, \sigma_i, \boldsymbol{v}_i\}$ obtained from the SVD transformation contain the set of basic patterns $(\boldsymbol{v}_i)$ and related scaling factors $(\boldsymbol{u}_i, \sigma_i)$, which formed the major part

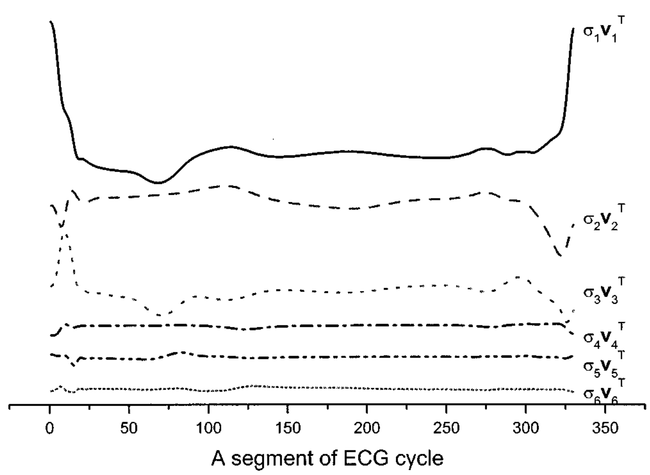


Fig. 4. Significant set of basic patterns of record 119 after pattern decomposition.

of the compressed data for ECG reconstruction. Associated beat information is defined as the period of each ECG cycle obtained from item 1 and is utilized for ECG period recovery. Miscellaneous parameters include the number of heartbeats in the ECG coding block, the first singular value, and the length of the shortest and longest beats. Each relative item is quantized appropriately before being stored as compressed data.

Data reconstruction is accomplished by the reverse procedure illustrated in Fig. 1(b). The retrieved matrix $\hat{\boldsymbol{A}}$ can be composed by (5a) and rearranged into the form of consecutive periodic ECG sequences. A reverse period transformation described in (6) is then performed on each segment to recover the original beat information for representing the ECG waveforms.

E. Evaluations of Compression Performance

The performance of the proposed algorithm is evaluated using the following measures. The CR is defined as the bits of input data record divided by the bits of the compressed data and can be calculated by the following equation:

$$\mathrm{CR} = \frac{b_0 \sum_{i=1}^{P} T_i}{(M+N+1) \times q \times b_s + P \times b_p + (b_\alpha + b_\beta + b_\gamma \times 2)}. \tag{7}$$

The numerator contains the bits of the original data that are equal to the quantization level b_0 (bits/sample) multiplied by the length of incoming data block, which is given by a summation over all ECG period lengths. The period length of the ith ECG cycle is defined as T_i, and P is the total number of ECG segments. The denominator is the bits of the compressed data that contains three terms. The first term is the bits of the retained singular triplets, where M is the number of elements of the left singular vector (u_i) and is equal to P, N is the number of elements of the right singular vector (v_i) and is equal to the MBP, q is the truncated index that is used to determine the retained level of singular triplets, and b_s is the bits of quantization level of the singular triplets. The second term is the bits used for storage of the associated beat information and is calculated by multiplying the number of ECG segments (P) and b_p, i.e., the bits for storage of each segment. The final term is the bits of miscellaneous parameters, including b_α, b_β, and b_γ, which correspond to the number of bits to store the first singular value, number of beats, and length of the shortest and the longest period, respectively.

The percent root mean square difference (PRD) is included to evaluate the error between the original and reconstructed waveforms, which is computed by the expression

$$\mathrm{PRD}(\%) = \sqrt{\frac{\sum_{i=1}^{L} \left[x_o(i) - x_r(i)\right]^2}{\sum_{i=1}^{L} x_o^2(i)}} \times 100 \tag{8}$$

where x_o denotes the original data.[1]

F. Determination of Significant SVD Coefficients

In fact, not all of the sets of singular triplets $\{\boldsymbol{u}_i, \sigma_i, \boldsymbol{v}\}$ are significant and worthy of retention for waveform reconstruction. By choosing a significant set of predominant singular triplets, we can eliminate the overhead of redundancies for data storage (or transmission). However, it is a tradeoff to determine the certain value of the truncated index to provide an efficient compression result while still preserving the essential clinical information concerning the ECG rhythms and morphological features. The energy information of the retrieved signals is exploited as an evaluating factor to dynamically adjust and determine the reconstructed error. The percent of root mean square residual energy (PRRE) can be defined as

$$\begin{aligned}\mathrm{PRRE}(\%) &= \sqrt{\frac{\text{Total Energy} - \text{Retained Energy}}{\text{Total Energy}}}\\ &= \sqrt{\frac{\sum_{i=1}^{R} \sigma_i^2 - \sum_{i=1}^{q} \sigma_i^2}{\sum_{i=1}^{R} \sigma_i^2}} \times 100\\ &= \sqrt{\frac{\sum_{i=1}^{L} \left[x_o^2(i) - x_r^2(i)\right]}{\sum_{i=1}^{L} x_o^2(i)}} \times 100. \end{aligned} \tag{9}$$

The value of the PRRE is approximately equal to the value of the PRD [see (8)] because $x_o(i)$ is very close to $x_R(i)$. Consequently, the PRRE can be applied as an index to control both the compression ratio and reconstructed error and can make an auto-adjustment according to the regularity of ECG signals.

IV. Results and Discussion

The algorithm proposed in the previous section was run on a Pentium-II IBM PC and applied to process the 10 min of data records taken from the MIT–BIH arrhythmia database. All experimental ECG signals were sampled at 360 Hz using a resolution of 11 b/sample, but each sample was stored in a 12-b-packed format (two samples per 3 B). The different rhythms of ECG signals were selected to evaluate the recoverability of the reconstructed waveforms.

Fig. 5(a) and (b) shows the relationship between the singular value ratio (σ_i/σ_1) and ER with respect to the change of truncated index q after SVD transformation. The singular value ratio is decreasing rapidly as the truncated index is increased and the ER is increased simultaneously. This showed that most energy information is confined within a small number of singular triplets. Therefore, the amount of reduant data can be eliminated using the truncation method to remove the overnumber of the singular triplets for data compression.

Fig. 6 illustrates the process of selected data records, which presents the relationship between the PRD and PRRE with respect to the change of the truncated index (q). The PRRE decreases as the truncated index is increased with the PRD follows closely the variation of the PRRE. The results indicate that the PRRE can be applied as a measuring factor to predict the reconstructed error as in the case of the PRD. Therefore, the truncated index can be adjusted into proper value by adapting to the setting of the PRRE in different situations, such as the limitation of transmission bandwidth or the condition of retrieved signals. Moreover, the truncated index can be fixed to satisfy certain applications. In our experiments, the reconstructed waveforms are well recovered for the value of the PRRE below 10%.

[1]In the PRD formulation, a level of 1024 is subtracted from each data sample to give the effective $x_o(i)$ due to the fact that a baseline of 1024 was added for original storage purposes. x_r denotes the reconstructed data and L is the number of samples within one data frame.

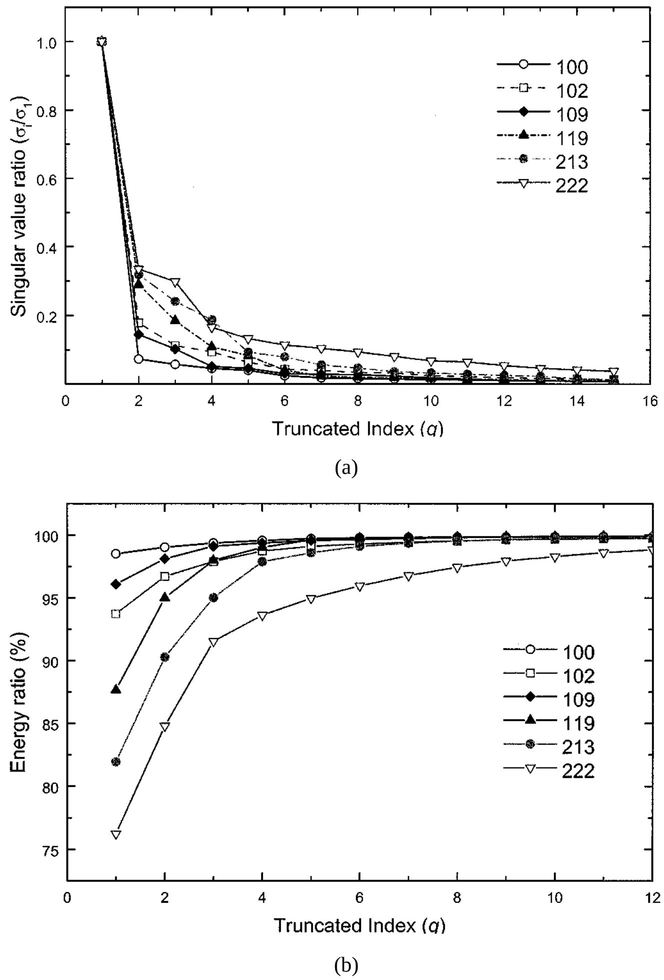


Fig. 5. (a) Relationship of singular value ratio with truncated index. (b) Relationship of ER with truncated index.

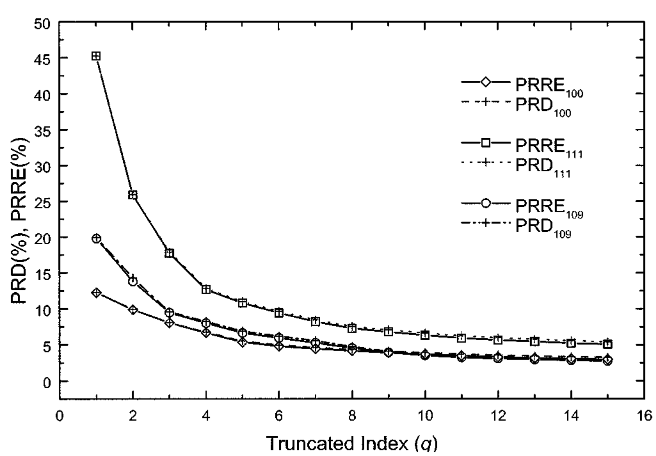


Fig. 6. Change of the PRD and PRRE with truncated index.

A. Compression Results

The proposed algorithm was applied on different arrhythmia ECG records taken from MIT–BIH database. These results are shown in Figs. 7(a), (b)–13(a), (b) including the original (a) and reconstructed waveforms (b). The variation of the CR can be observed from the dynamic changes of the truncated index (q) in these figures, which vary according to the regularity of the

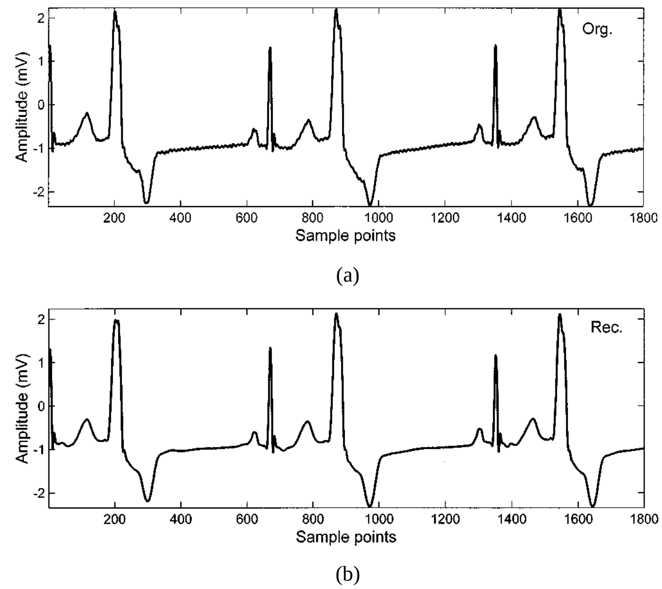


Fig. 7. (a) ECG waveform taken from lead II of record 119. (b) Reconstructed waveform with the proposed scheme (with bit rate $= 107.7$ b/s at $q = 5$).

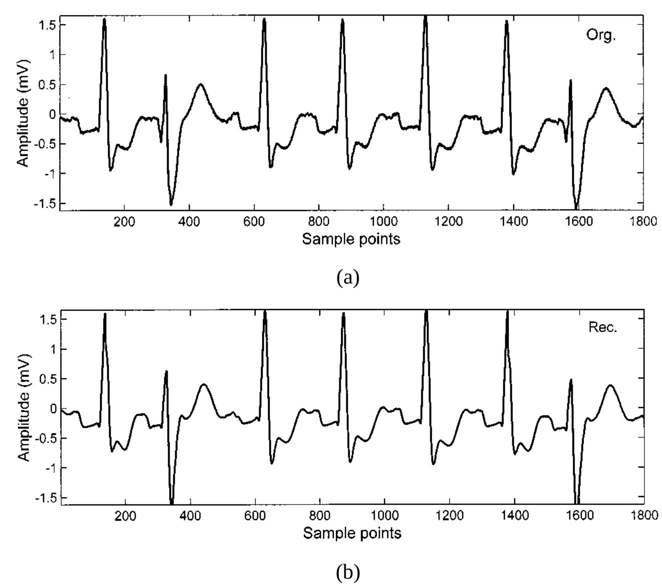
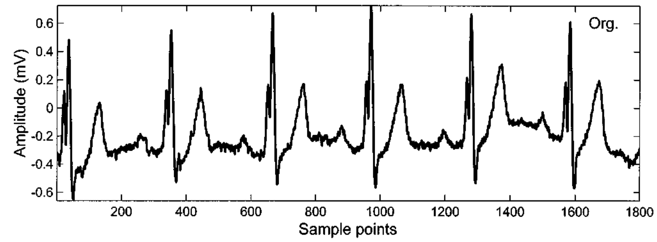


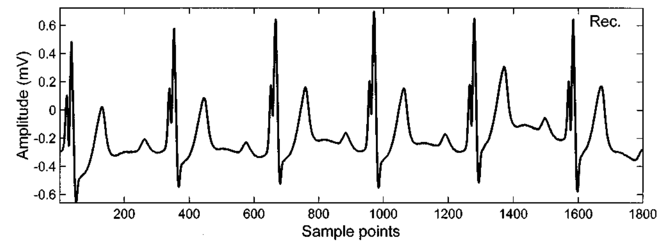
Fig. 8. (b) Reconstructed waveform (with bit rate $= 78.1$ b/s at $q = 3$) of a left bundle branch block with ectopic morphology of ECG taken from (a) lead II of record 109.

original ECG waveforms and the reconstructed error. It is reasonable that the higher regularity of ECG signals can be represented using fewer patterns with a higher CR. Correspondingly, irregular ECG rhythms require more patterns to reconstruct the original waveforms.

Fig. 7 is a compression result of a premature ventricular contraction (PVC) waveform (119.dat) with ventricular bigeminy. This figure presents a high-quality reconstructed waveform with a high CR $\mathrm{CR} = 36.72$ (bit rate $= 107.74$ b/s) and a relatively low reconstruction error $(\mathrm{PRD} = 6.15\%)$ for a truncated index of five. The performance is comparatively better than the wavelet packet-based compression technique [8] $(\mathrm{CR} = 19.65$, bit rate $= 201.5$ b/s). The superior performance is ascribed to the characteristics of strong interbeat correlation among the ECG cycles and high performance of signal decomposition of

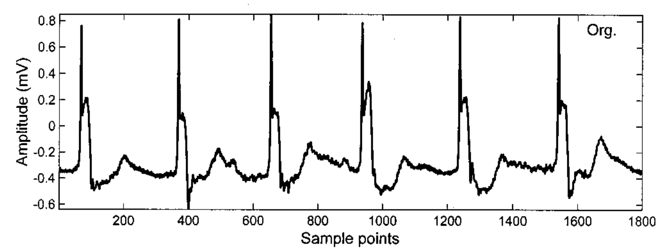


(a)

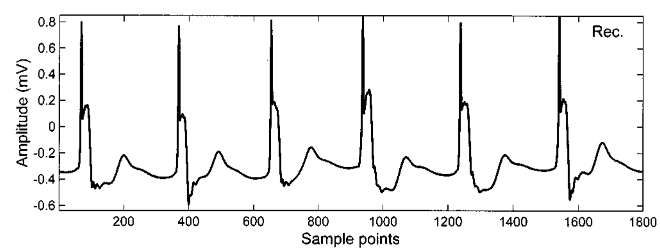


(b)

Fig. 9. (b) Compression result of abnormal ECG (with bit rate $= 129.2$ b/s at $q = 6$) taken from (a) the data record 111, exhibiting left bundle branch block.

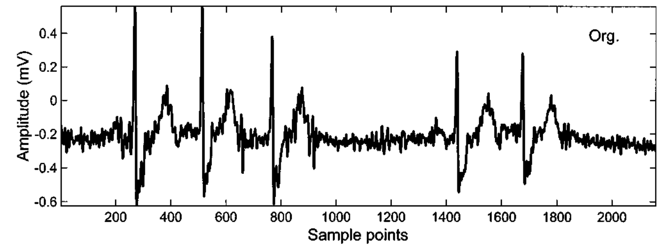


(a)

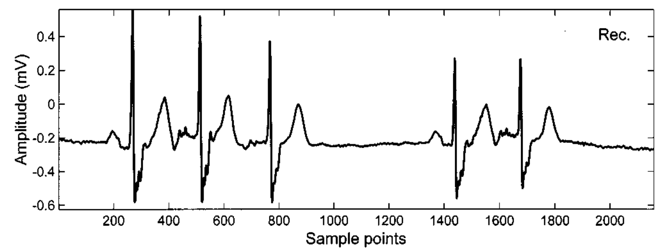


(b)

Fig. 10. (a) Original and (b) reconstructed (with bit rate $= 111.3$ b/s at $q = 5$) ECGs taken from the record 102, exhibiting a paced rhythm.

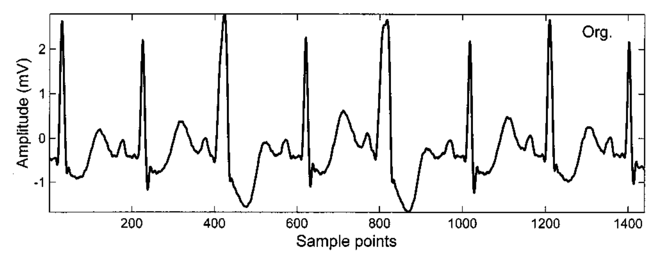


(a)

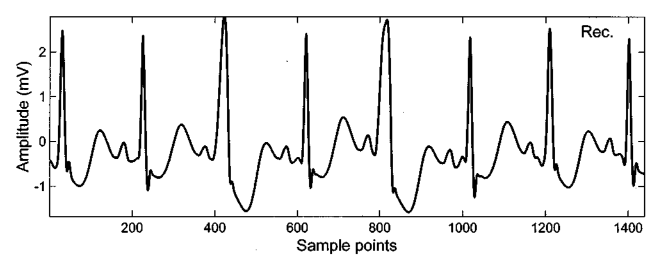


(b)

Fig. 11. (a) Original and (b) reconstructed (with bit rate $= 221.5$ b/s at $q = 11$) ECGs from the record 232, exhibiting right bundle branch block with atrial premature beats.



(a)



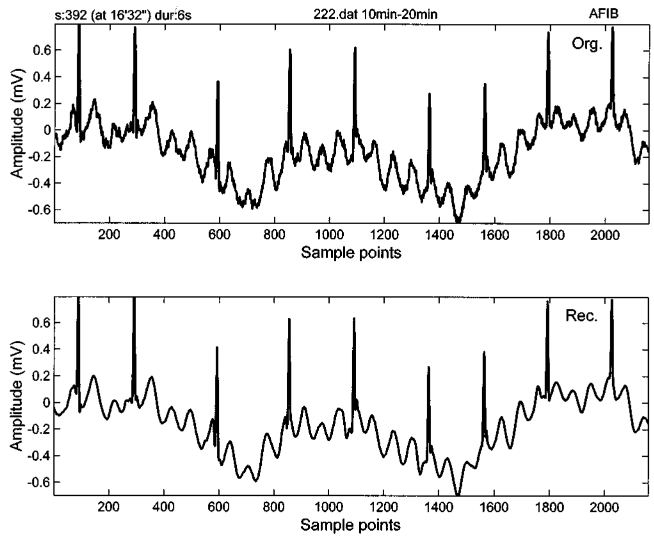
(b)

Fig. 12. (b) Reconstructed waveform (with bit rate $= 168.4$ b/s at $q = 6$) of a normal beat with fusion of ventricular of ECG rhythm from (a) the record 213.

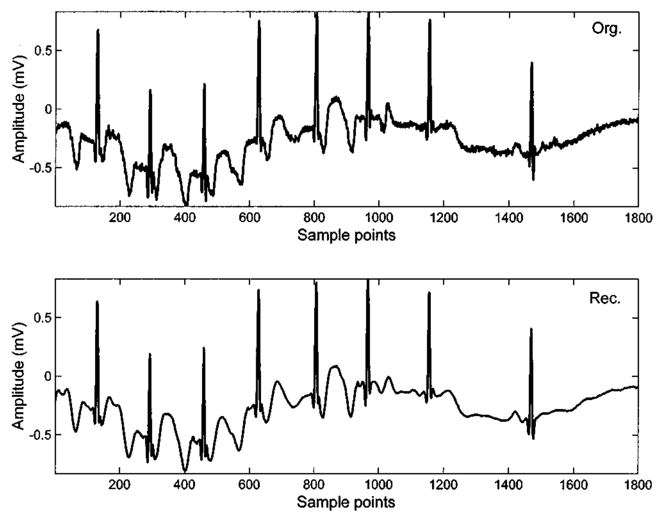
SVD employed in the present algorithm. The different results of record 119 following the change of truncated index q from 1 to 15 are illustrated in Fig. 14. The results show that the PRD is decreasing slowly when the truncated index (q) is higher then five. This means that the small number of predominant singular triplets can readily carry most of the information of the ECG waveforms and we can obtain a higher CR by ignoring the excess number of insignificant singular triplets. The overheads of the retained singular triplets can only contribute insignificant information with a sacrifice on better compression performance. The change of bit rate is increasing notably if the PRD is lower than 10%.

Figs. 8 and 9 are the compression results for the rhythm of the left bundle branch block of records 109 and 111 with bit rate $= 78.1$ (at $q = 3$) and 129.2 (at $q = 6$), respectively. In the case of a first-degree AV block, the results show that not only regular ECG components are retrieved; the variety rhythm of the clinical information is also preserved. This is due to the multipatterns of ECG retained in the set of basic patterns.

Fig. 10 shows the compression result of record 102 with bit rate $= 111.3$ b/s at $q = 5$; the rhythm is paced with a demand pacemaker. We observe that not only the significant waveform is retrieved, but also the signal quality is upgraded because the trivial singular triplets are removed for data reduction. The effect of noise filtering is one of the features using the truncated SVD for ECG data compression and is observed in our various compression results. The noise filtering capability is especially demonstrated in the reconstructed waveform of the arrhythmia



(a)



(b)

Fig. 13. (a), (b) Compression results of arrhythmia ECG record 222, exhibiting the episodes of paroxysmal atrial flutter/fibrillation with following A–V junction rhythm (bit rate $= 319.0$ b/s at $q = 14$).

ECG record 232 presented in Fig. 11 with bit rate $= 221.5$ b/s at $q = 11$.

Fig. 12 displays the compression result of the ECG record 213, exhibiting that the morphology of fusion PVCs is retrieved without losing significant clinical information (bit rate $= 168.4$ b/s at $q = 6$). Figs. 13(a) and (b) presents the compression results of arrhythmia ECG record 222, exhibiting the episodes of paroxysmal atrial flutter/fibrillation with following nodal escape beats (bit rate $= 319.0$ b/s at $q = 14$). The results show that the highly irregular rhythm can still be reconstructed with the proposed scheme.

In these cases, even though the abnormal ECGs are mixed with the aperiodic signals of the heart rhythm, a certain interbeat correlation remains among several patterns. Therefore, the truncated index (q) can be extended to increase the set of retained patterns to prevent the loss of any oversight clinical information, and adapt different rhythms of ECGs for signal reconstruction. Consequently, the proposed method can still preserve the

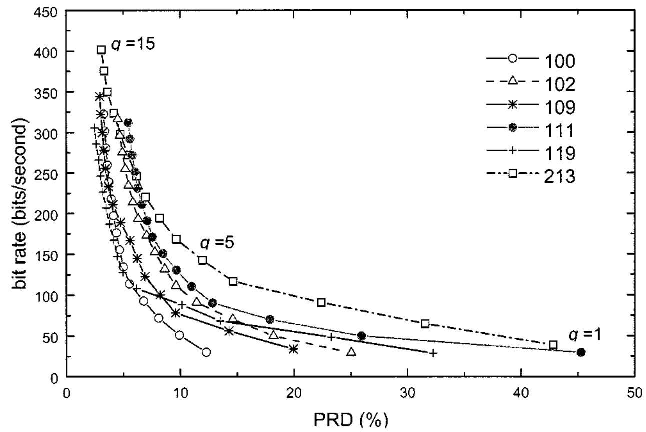


Fig. 14. Relationship between reconstruction error (PRD) versus bit rate with respect to the change of truncated index for different ECG records.

TABLE I
COMPARISON OF THE COMPRESSION RESULTS OF THE PROPOSED ALGORITHM WITH THE WAVELET-BASED METHOD

Records	Q	bit rate (bps)	CR	PRD (%)	Compression time (seconds)	Bit rate[b] (bps)	CR[b]
100	2	50.8	77.91	9.92	41.24	167.7	23.61
101	5	107.0	36.96	9.62	42.18	174.2	22.73
102	5	111.3	35.56	9.62	39.55	218.5	18.12
103	3	69.0	57.35	7.77	40.26	113.7	34.82
107	12	251.5	15.73	5.52	39	299.9	13.20
109	3	78.1	50.69	9.57	39.49	185.8	21.31
111	6	129.2	30.62	9.66	38.94	128.1	30.91
115	3	66.1	59.85	8.08	40.64	94.3	41.99
118	12	264.0	14.99	4.22	38.28	304.1	13.02
119	5	107.7	36.72	6.15	41.48	201.5	19.65
213	6	168.4	23.49	9.64	54.11	x	x
222	14	319.0	12.4	9.82	42.84	x	x
232	11	221.5	17.86	11.02	44.42	x	x
117	3	61.2	64.66	3.14	44.44	x	x

[b] The results of wavelet-based method are quoted from reference [8].

diagnostic information present in the arrhythmia ECGs of our experimental data records.

B. Summarized Results and Comparison With Wavelet Codecs

Presently, several wavelet-based compression algorithms have been proposed in the literature [8]–[11], [21]–[23] and claimed a higher compression performance than previous methods. The compression results of our proposed scheme are summarized in Table I, and the associated compression results of wavelet packet-based compression algorithm [8], which used the same 10-min-long records, are also quoted for comparison. The average bit rate of our experimental records is 143.2 b/s (corresponding to a CR of 27.7) and the average compression time is 41.92 s under a MATLAB environment on a Pentium-II 266-MHz PC. Except for the records of 213, 222, 232, and 117 in Table I, which have no related data for comparison, the other results indicate that the performance of truncated the SVD technique provides a higher CR than the wavelet packet-based algorithm [8]. The reconstructed error of Bradie [8] (in rms difference) is slightly better than the proposed SVD-based method. The reason for this is that the background noises of the original waveforms are filtered out in the proposed SVD-based method, which contributed to the relatively larger error in the reconstructed waveforms. In fact, the quality of reconstruction

TABLE II
COMPARISON OF DIFFERENT CODING ALGORITHMS

Algorithm	PRD (%)	CR	Signal	Sampling (Hz)	bits/sample
TSVD	1.18	10:1	MIT-BIH 117	360	11
Lu	1.18	8:1	MIT-BIH 117	360	11
Hilton	2.6	8:1	MIT-BIH 117	360	11
Djohan	3.9	8:1	MIT-BIH 117	360	11

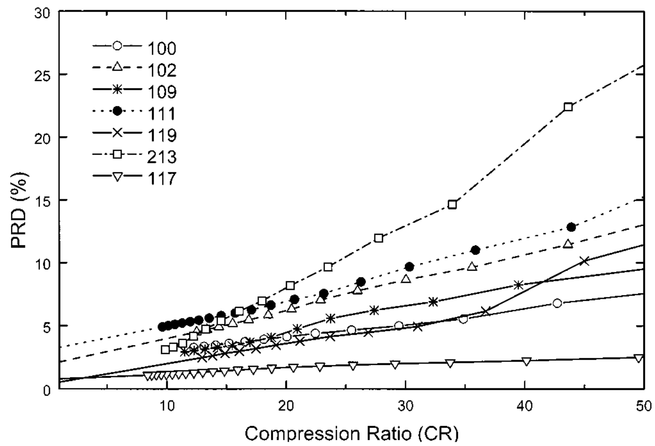


Fig. 15. Relationship between the PRD versus the CR for various ECG records.

for clinical application is still open to question. Additionally, there are other works of wavelet-based methods for ECG compression [21]–[23]. In particular, Lu *et al.* [23] used the compression result of record 117 for comparing the reconstructed performance with other works on wavelet-based methods under the same CR. The associated results listed in Table II including the result of our proposed algorithm show that set partitioning in hierarchical trees (SPIHT) (Lu *et al.* [23]) provides a better performance than previous wavelet-based methods (Hilton [22] and Djohan *et al.* [21]) for the PRD value of 1.18% with a CR of 8 : 1. However, the result of our proposed scheme presents the same PRD value of 1.18% for 10 : 1 compression of record 117 and demonstrates a superior performance in comparing with the existing works [8]–[11], [21]–[23]. The relationship between the bit rate (bits/second) and reconstruction error (PRD) and the relationship between the PRD and CR, with respect to the change of the truncated index (q) for different records are displayed in Figs. 14 and 15, respectively. The results show that the overall performance of our proposed scheme can provide a comparable or higher efficiency than the existing wavelet-based or other ECG compression methods.

The compression performance of the proposed algorithm can be upgraded by the following procedure.

1) The compressed data can be further encoded using lossless coding techniques, such as entropy coding, to improve the compression performance.
2) The normalized period length can be shortened properly below the MBP to eliminate the extra-redundant data. However, the modified period still has to satisfy the Nyquist criterion to ensure that no distortion occurs during the period normalization.
3) The coding block can be enlarged to improve the CR since the set of basic patterns does not have to be extended significantly for retrieving the original waveforms. For example, the CR of the experimental record 232 can be upgraded as the length of the compression block is extended to 30 min. The CR becomes 21.76 with bit rate $= 181.9$ b/s at $q = 12$ for a compression length of 30 min in comparing with $\mathrm{CR} = 17.86$, bit rate $= 221.5$ b/s at $q = 11$ at a compression length of 10 min.

Additionally, the supplement of residual signal encoding can be considered to prevent any loss of the clinical information in the minor and irregular signals, and to provide a nearly distortion-free ECG data compression technique.

V. CONCLUSION

An algorithm has been presented in this paper using the quasi-periodic analysis of SVD to decompose the ECG signals into a linear combination of a set of basic patterns. The SVD transformation based on the significant signal extraction and strong interbeat correlation among ECG cycles provides excellent pattern decomposition and reduces a massive amount of redundant data. The results demonstrate that truncated SVD technique is an efficient coding scheme for achieving a high CR for ECGs and can still preserve the significant diagnostic information. The proposed scheme may also find application in other quasi-periodic physiological signals such as arterial pressure waveforms, electrogastrogram, plethysmogram, etc.

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