University of Moratuwa

Department of Electronic and Telecommunication Engineering

BM4152

Bio-signal Processing

Assignment 1

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200641T

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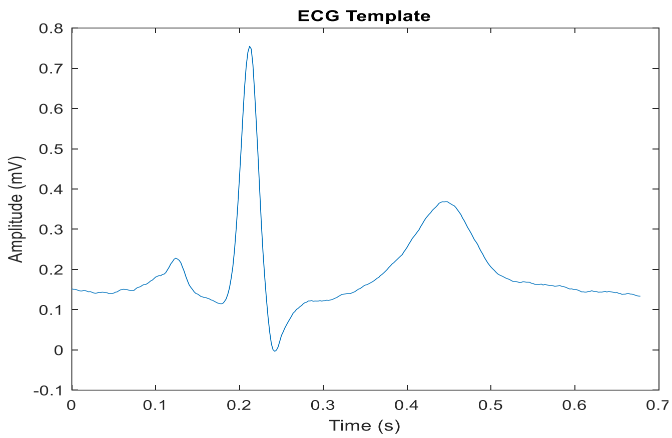
1. Smoothing Filters

1.1. Moving Average MA(N) Filter

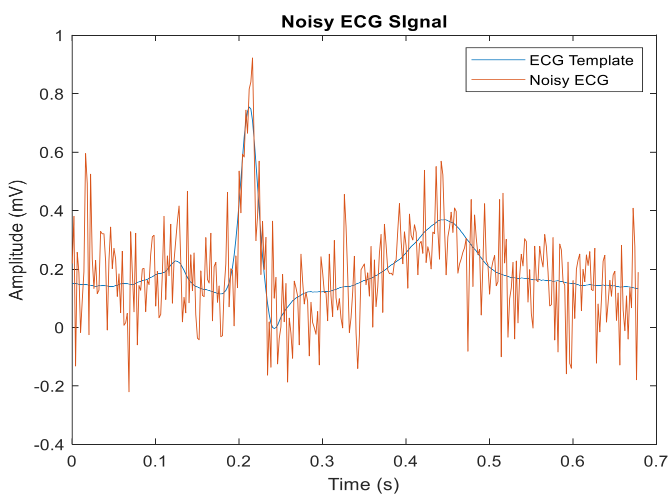
1.1.1. Preliminaries

ii)



iii)

Added white gaussian noise to the template ECG signal, in such a way that **the SNR of the noise added signal is 5dB**.



iv)

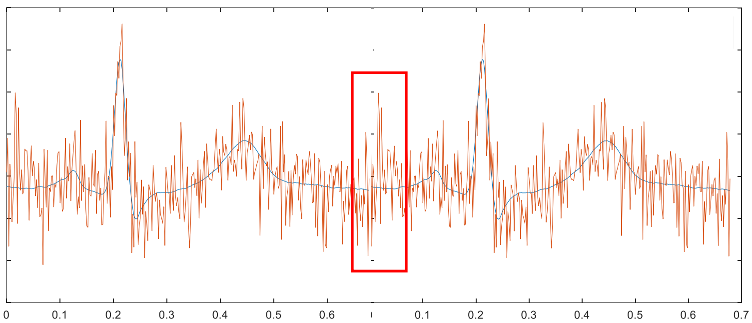
Power Spectral Density(PSD) of a digital signal x[n] is given by,

$$\hat{P}(\omega) = \frac{1}{2\pi N} \left| \sum_{n=0}^{N-1} x_n e^{-j\omega n} \right|^2, \quad -\pi < \omega \leq \pi.$$

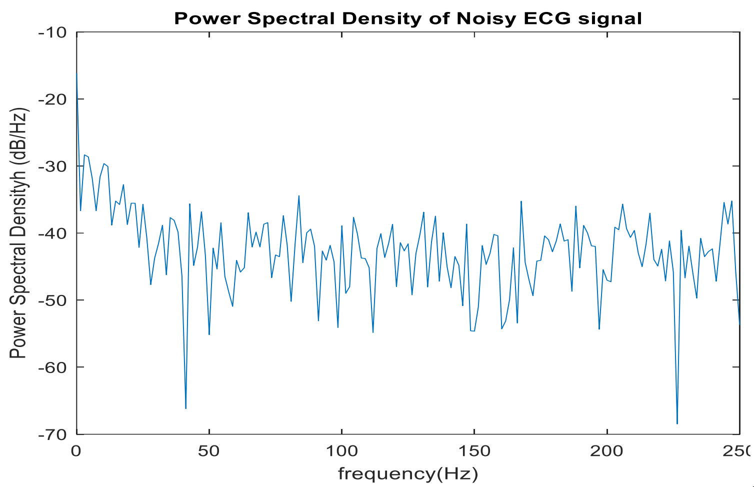
We can clearly see it is required to compute the DFT of the signal .

When computing the Discrete Fourier Transform (DFT) of a finite-length signal, we assume that finite length signal continues through the time scale periodically. If the signal does not end smoothly (meaning the end is not the same as the start), this creates sharp transitions or discontinuities at the edges. These discontinuities introduce high-frequency components into the spectrum that were not present in the original signal; this is called spectral leakage. Window functions are used to taper the edges of a signal segment, reducing discontinuities and thereby mitigating spectral leakage.

But in this case, we know the ECG signals are periodic, continuous. Also, we can see the periodic assumptions does not result very large transitions, or discontinuities at the edges.



Therefore, we can use a rectangular window without a large effect(spectral leakage) to the spectral density.



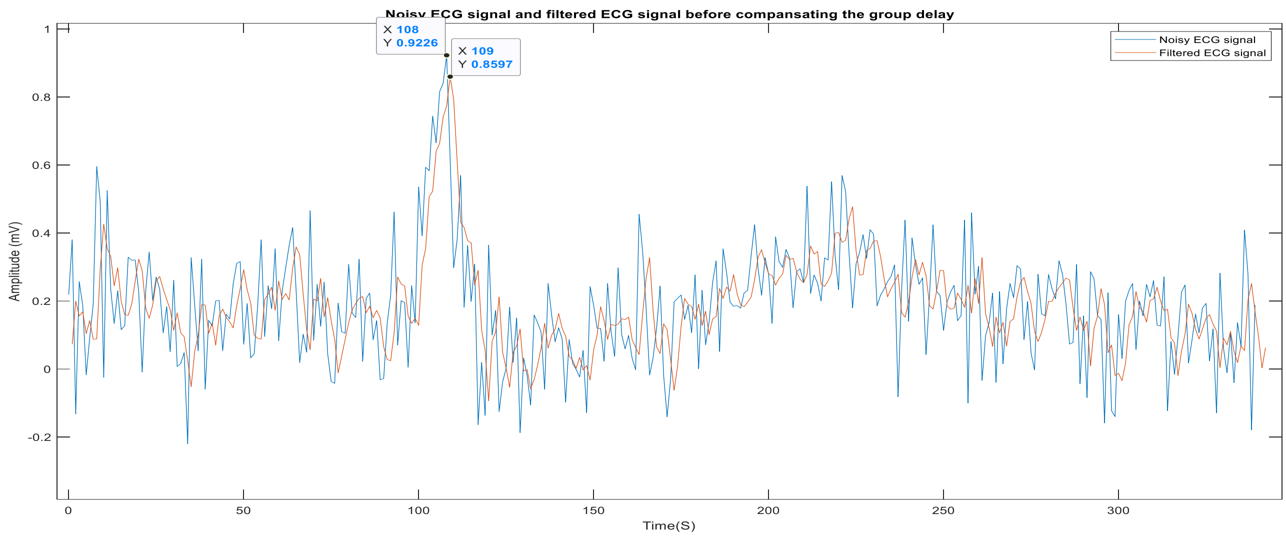
1.1.2. MA(3) filter implementation with a customized script

ii)

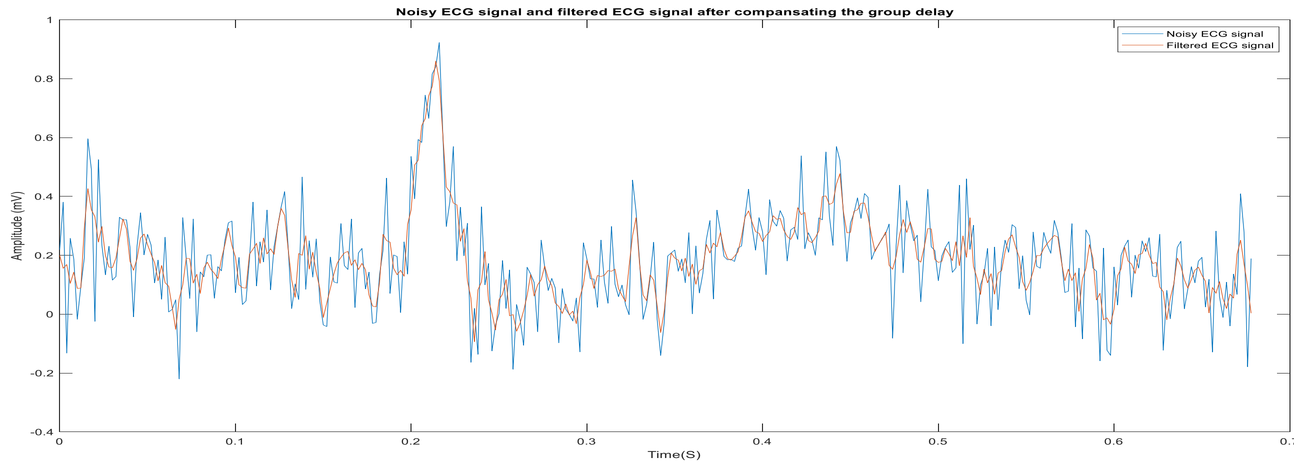
$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(n-k)$$

Moving average filter is an order N-1 FIR filter. Therefore, the group delay is (N-1)/2.

In this example N = 3; that is group delay is 1.



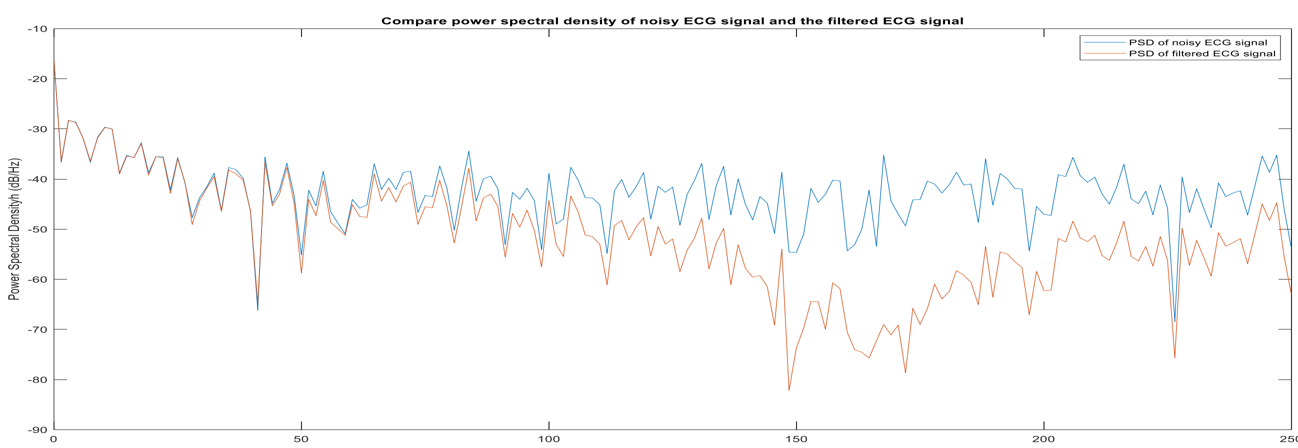
iii)



,

The signal has smoothed removing high frequency spikes to a greater extend.

iv)



We can clearly observe that the energy of the filtered signal in the high-frequency range has been reduced, which is because the MA(3) filter has removed the high-frequency components of the noisy ECG signal.

1.1.3. MA(3) filter implementation with the MATLAB built-in function

i)

In order to use the MATLAB built-in filter(b, a, X) function we need to find the transfer function of the filter in the following format,

$$Y(z) = \frac{b(1) + b(2)z^{-1} + \ldots + b(n_b + 1)z^{-n_b}}{1 + a(2)z^{-1} + \ldots + a(n_a + 1)z^{-n_a}} X(z)$$

;

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} x(n-k)$$

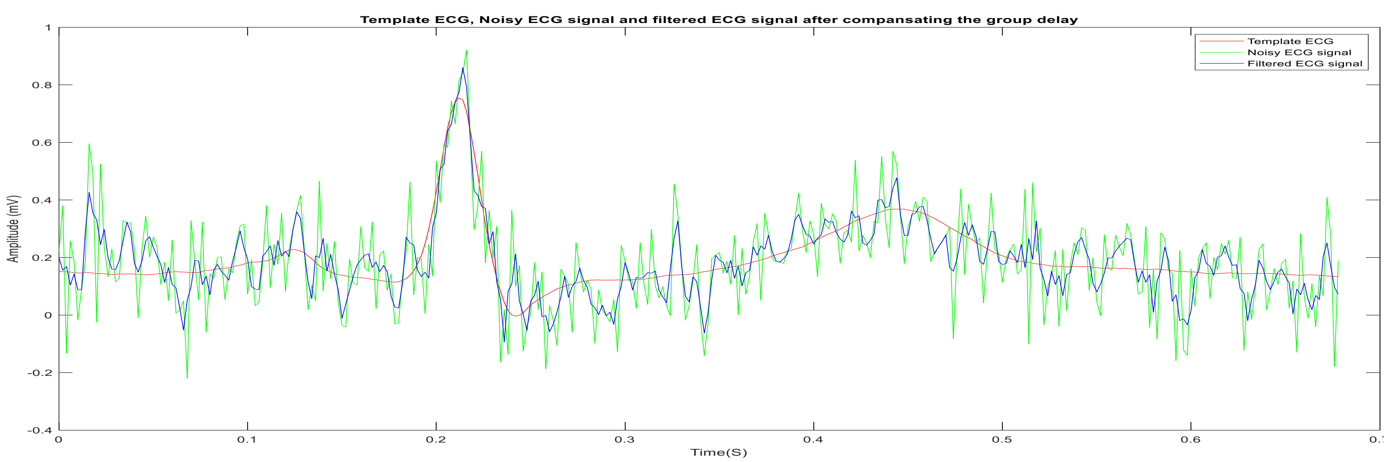
$$Y(z) = \frac{1}{N}\sum_{k=0}^{N-1} X(z)z^{-k}$$

$$\frac{Y(z)}{X(z)} = \frac{1}{N}\sum_{k=0}^{N-1} z^{-k}$$

$$H(z) = \frac{1}{N}\left(\frac{1 + z^{-1} + z^{-2} + \cdots + z^{-N+1}}{1}\right)$$

Therefore, b(1) = b(2) = b(3) = … = b(n) = 1/N

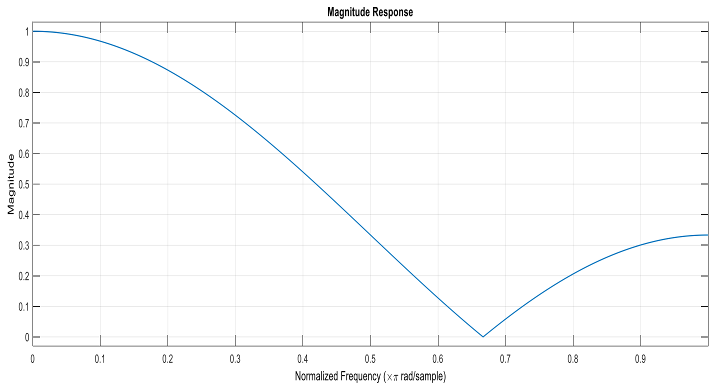
ii)

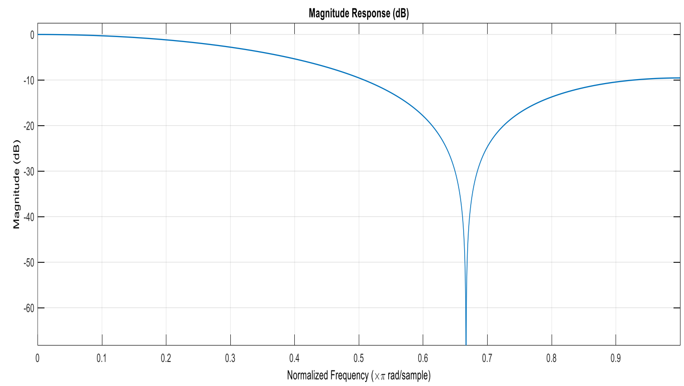


Signal has smoothed removing high frequency components of the signal.

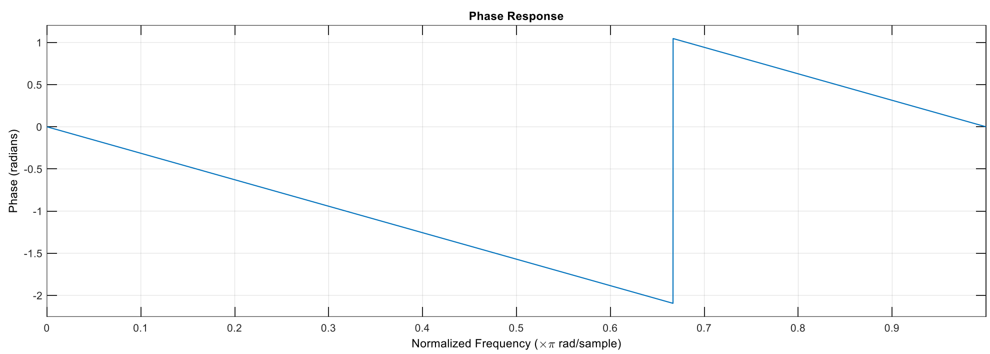
iii)

- Magnitude Response

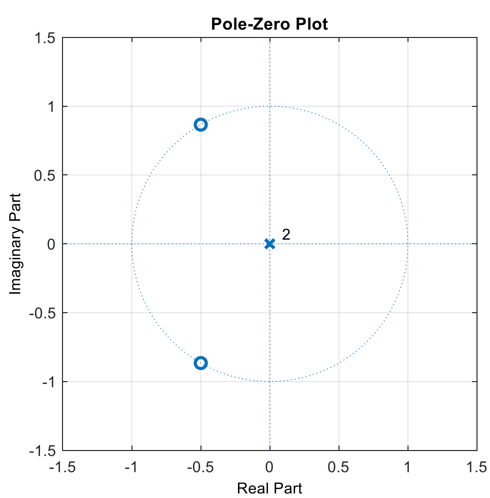




- Phase Response



- Pole Zero Plot

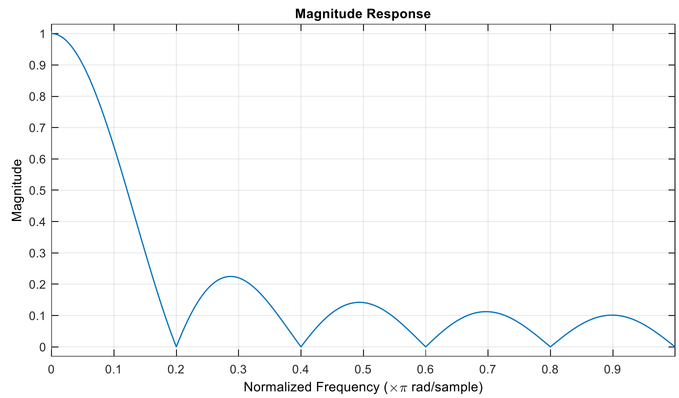


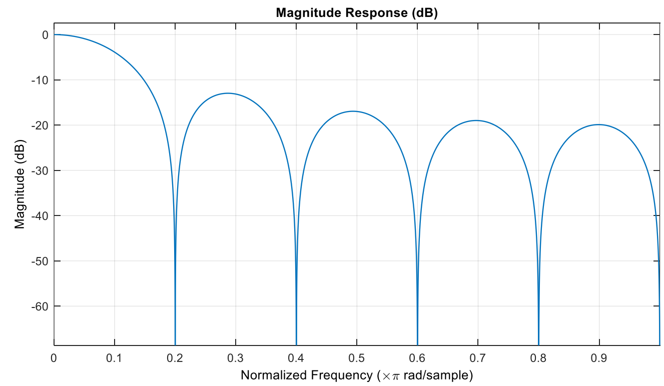
There are 2 Zeros and 1Pole.

1.1.4. MA(10) filter implementation with the MATLAB Built-in function

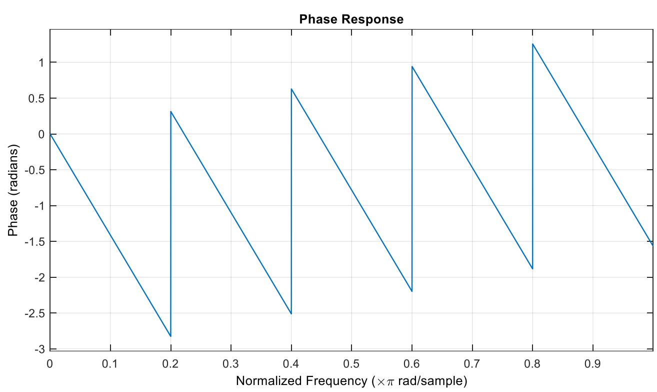
i)

- Magnitude Response

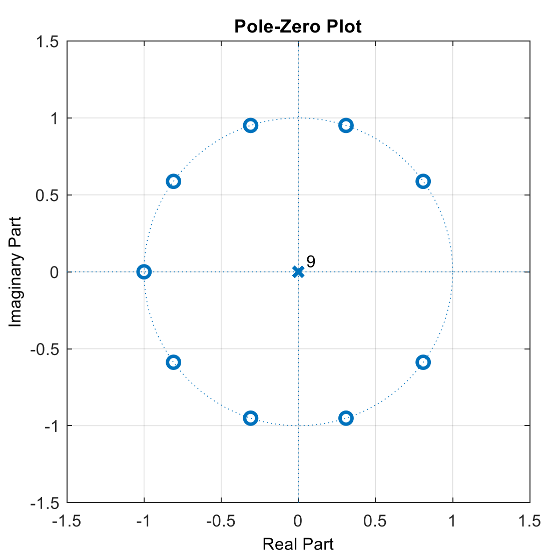




- Phase Response



- Pole Zero Plot

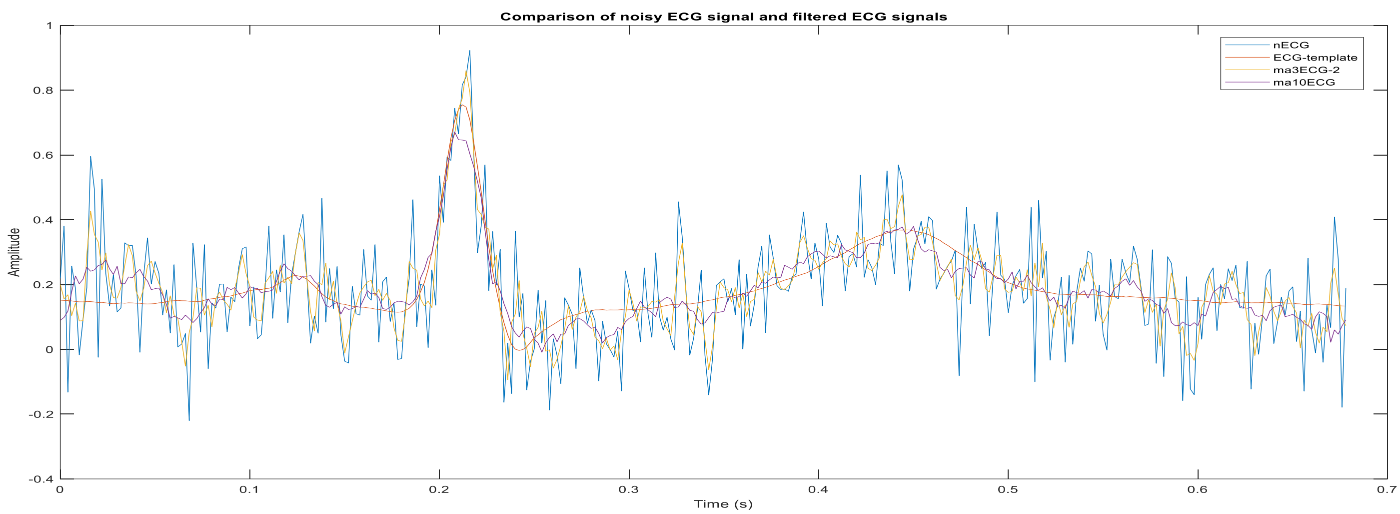


Moving average filter act as a lowpass filter (magnitude response and the pole zero plots verify that). However, higher order moving average filter has a much lower cutoff frequency that is it removes many higher order frequency components from the signal by smoothing the signal more.

Also in higher order MA filter, poles are located closely providing more attenuation than a lower order MA filter.

Whereas a higher order MA filter has a higher group delay.

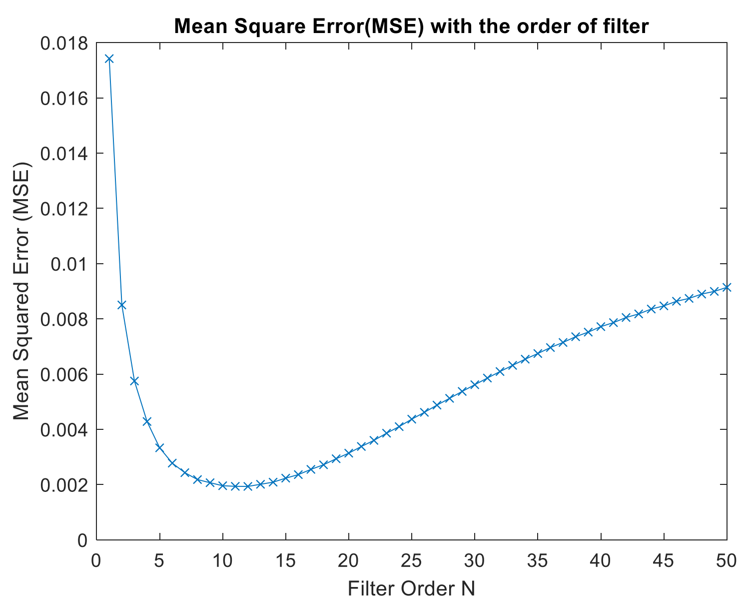
iv)



Both MA filters have smoothened the signals removing high frequency noise components of the signal whereas the MA(10) filter has smoothened the signal better than the MA(3) filter providing a signal much closer to its template.

1.1.5. Optimum MA(N) filter order

ii)



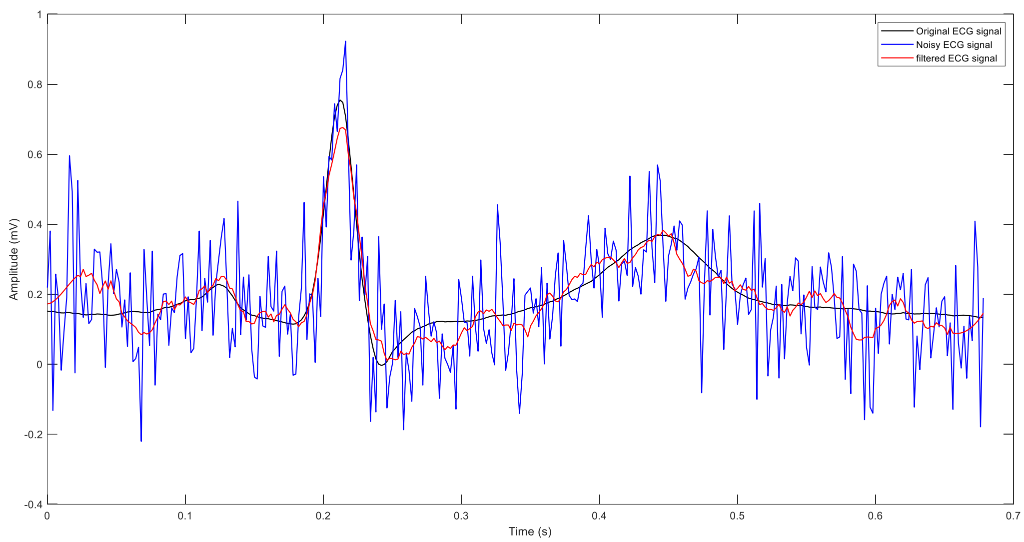
Optimum filter order is N= 12

iii) Large MSE values at low filter orders occur due to insufficient noise reduction, while at high filter orders, they result from excessive smoothing, causing loss of important signal details.

1.2. Savitzky-Golay SG(N,L) filter

1.2.1. Application of SG(N,L)

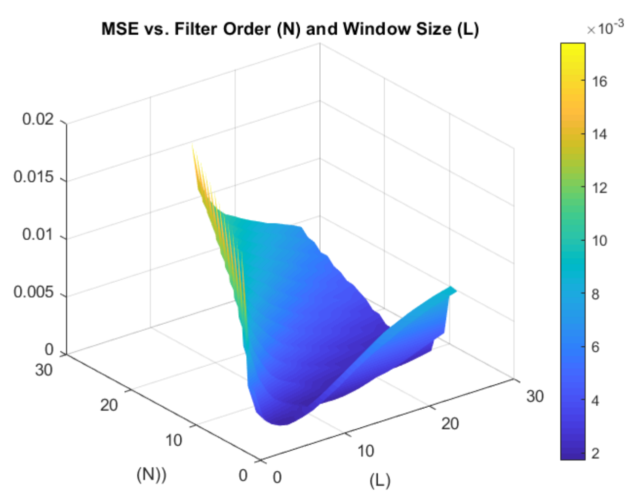
ii)

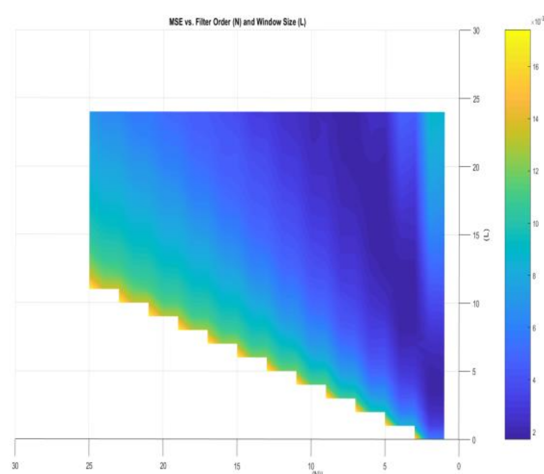


SG filter has reduced the high frequency noise to a greater extend.

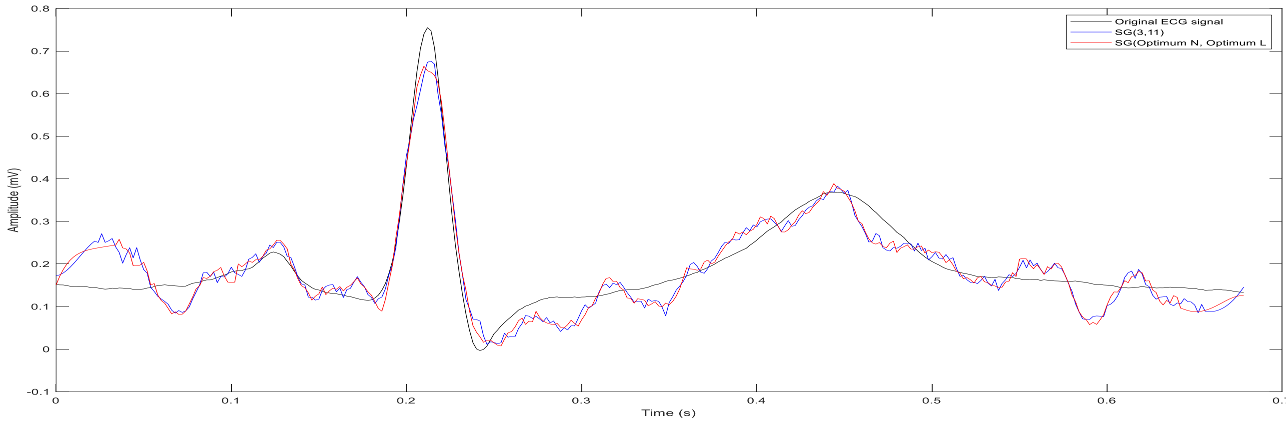
1.2.2. Optimum SG(N,L) filter parameters

i)



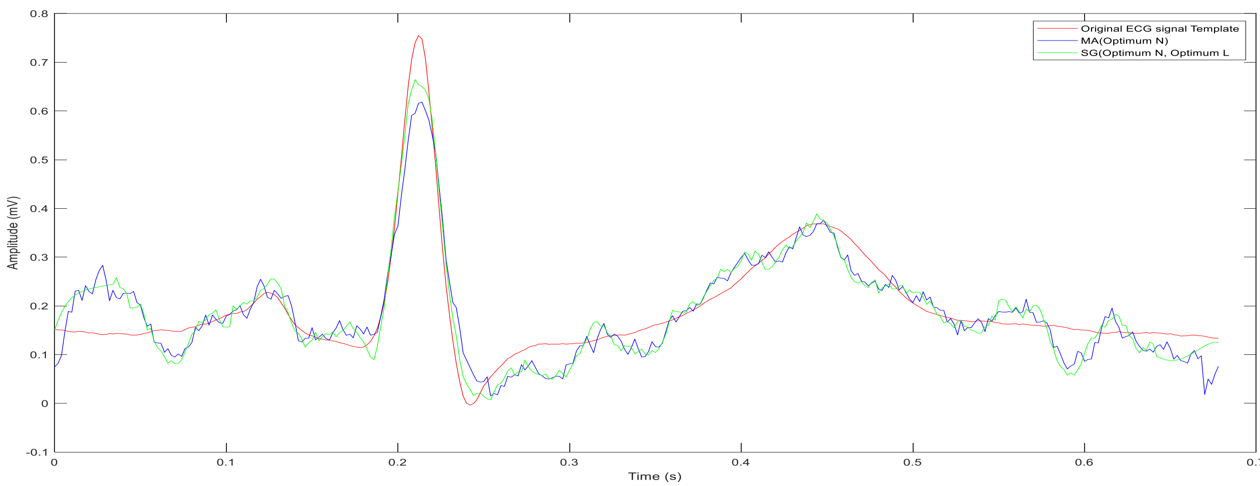


The optimal filter parameters are N = 4 and L = 17 (Length is 2*L +1) with a minimum MSE of 0.001707 ii)



Smoothing process is much better here.

iii)



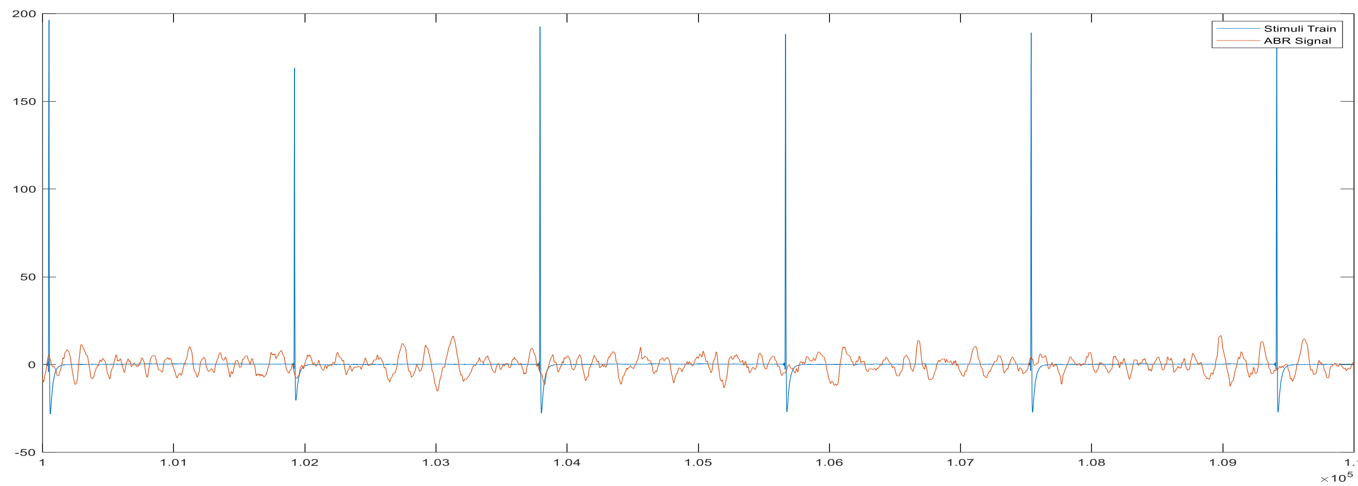
Both MA(Optimum N) and SG(Optimum N , Optimum L) has smoothed the signal very well. But compared to the MA() filter the SG() filter has provided a better smoothed signal while preserving the signal characteristics but the computational complexity of the SG() filter is higher than the MA() filter.

2. Ensemble averaging

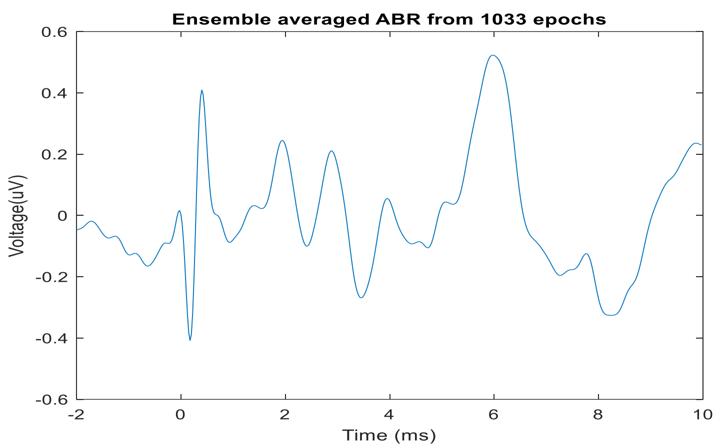
2.1. Signal with multiple measurements

2.1.1. Preliminaries

iii)



Vii)



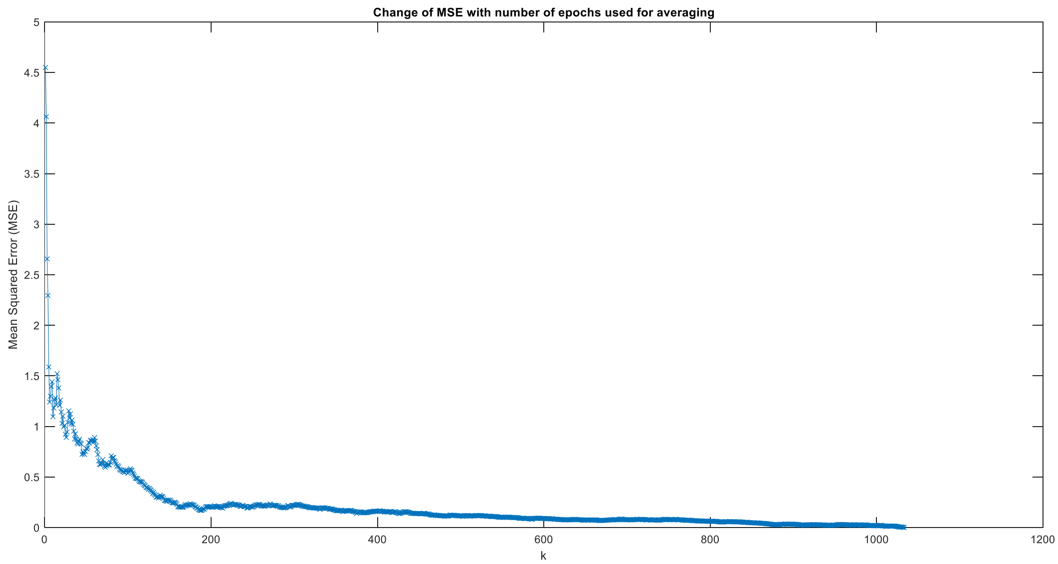
2.1.2. Improvement of the SNR

ii) $$SNR = 10\log\left(\frac{Noise\ free\ signal\ Power}{Noise\ Power}\right)$$

$$SNR = -1\,0\log(Noise\ Power\ ) + k \qquad (k > 0)$$

$$Noise\ Power = \sum(Noisy\ signal\ - Noise\ free\ signal)^2$$

Therefore, when the noise is reduced noisy signals get closer to the noise free signal (MSE is reduced); that is, the noise power will reduce. Which means that the SNR is improved.



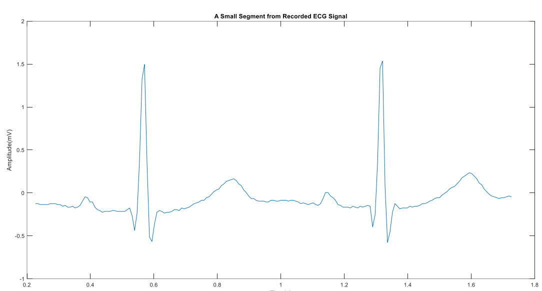
According to the above graph we can observe that the MSE reduces when the number of epochs increases. That is the SNR increases according to the above explanation.

Theoretically, if noise with mean zero and variance σ is added to a signal, and if multiple measurements (say n) of such signals are averaged, then the variance of the noise should be reduced by a factor of n. This means that the signal-to-noise ratio (SNR) of the signal should improve.

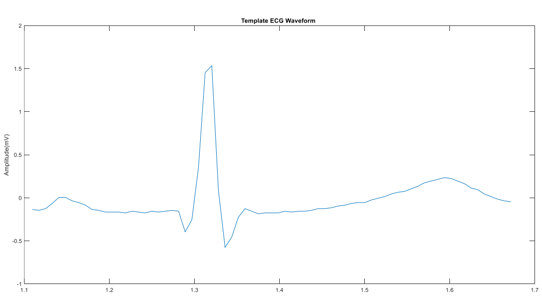
2.2. Signal with repetitive patterns

2.2.1. Viewing the signal and addition of Gaussian white noise

ii)

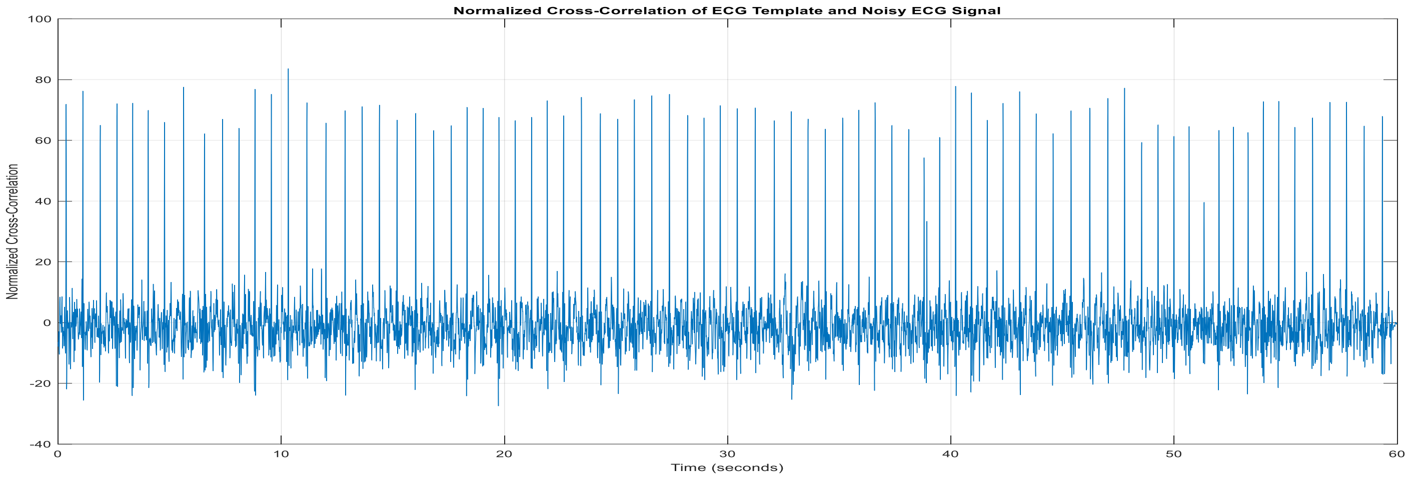


iii)

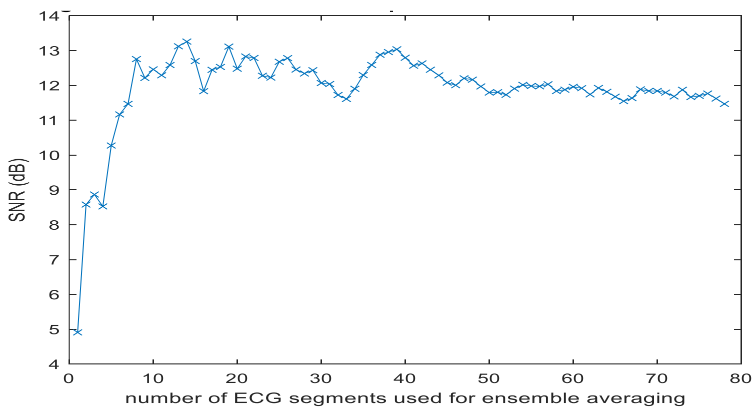


2.2.2. Segmenting ECG into separate epochs and ensemble averaging

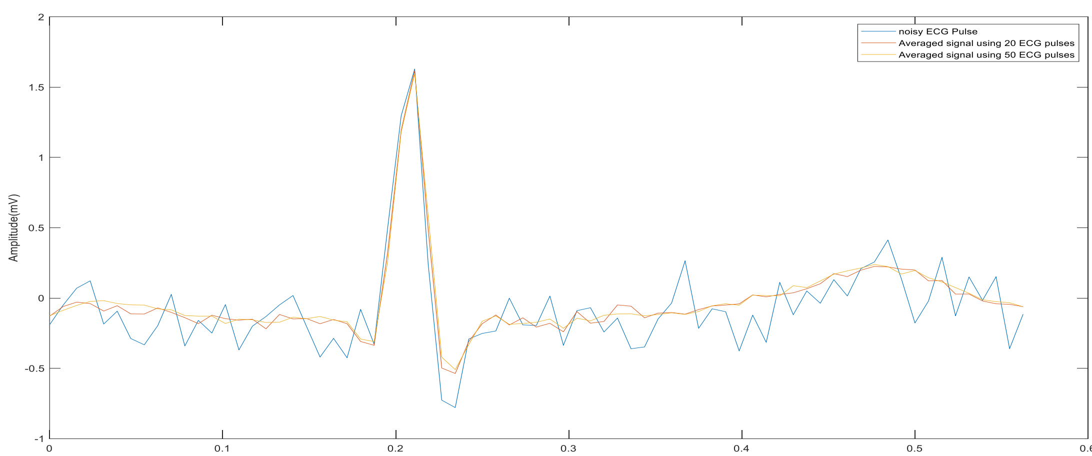
ii)



iv)



v)



When it is averaged with 50 epochs it reduces the noice than averaage with 20 epochs.

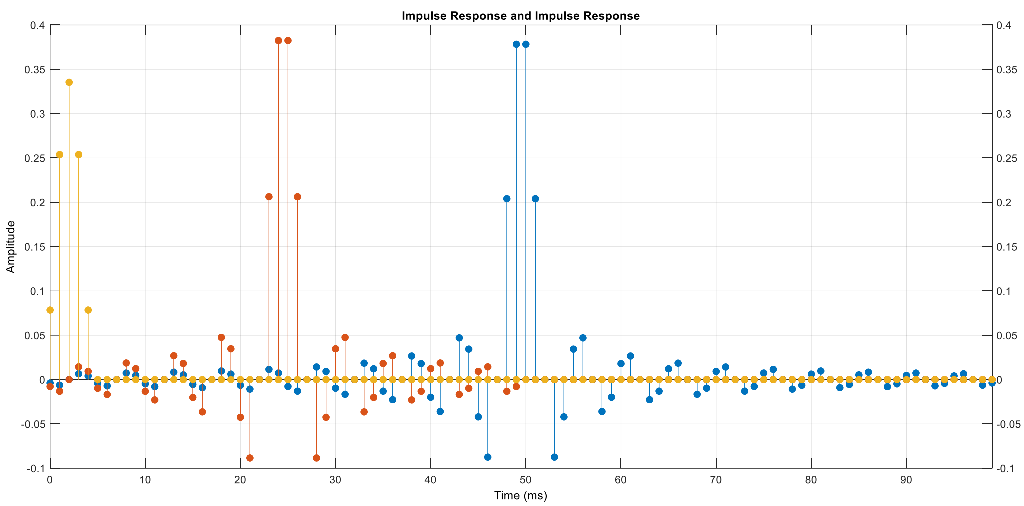
vi)

It is more robust to noise, reduces misalignment, and considers the entire ECG waveform (P, QRS, T waves). This leads to more accurate segmentation, especially in noisy conditions.

3. Designing FIR filters using windows

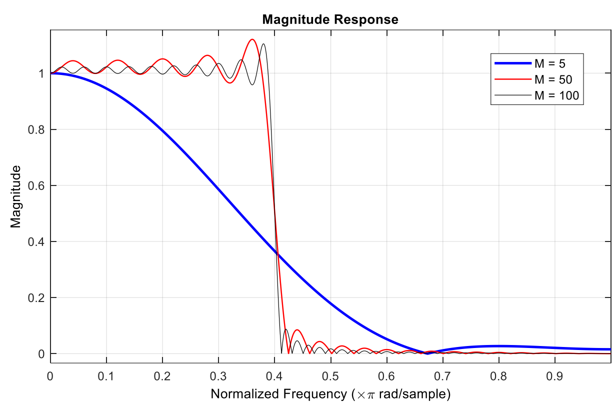
3.1. Characteristics of window functions (using the fdatool)

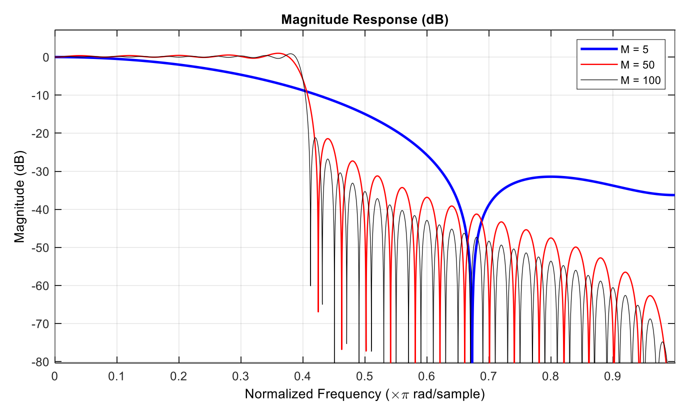
i)

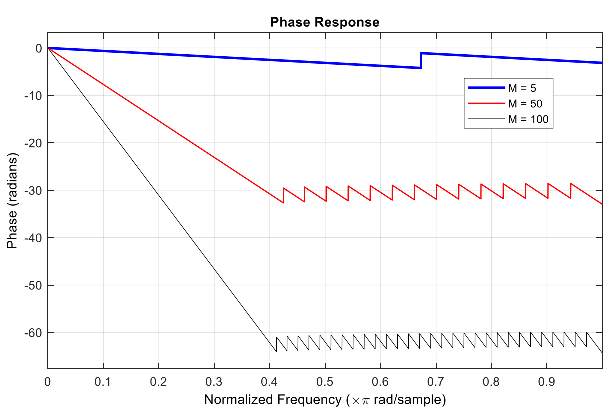


When the length of the window is larger it gives a impulse response nearly equals to a complete sinc function, which is the time domain representation of a perfect rectangular frequency response (ideal filter).

ii)

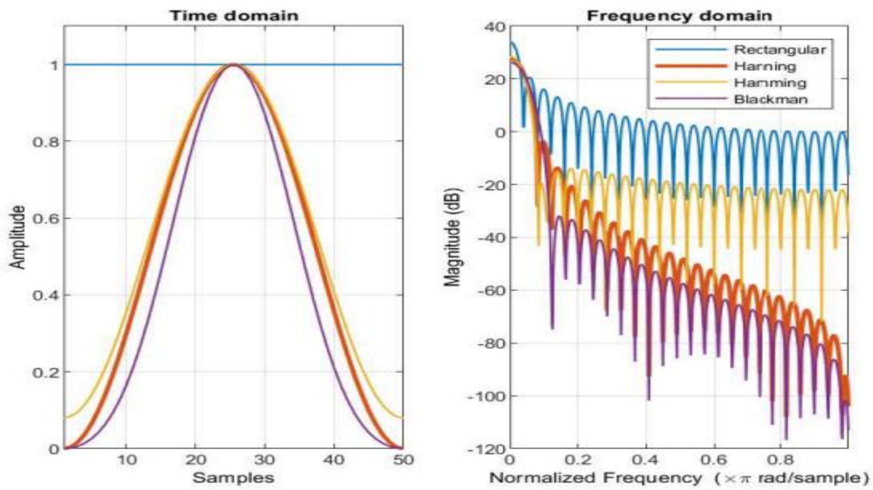




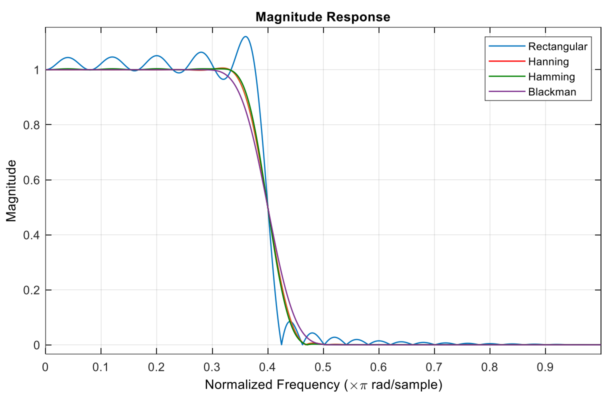


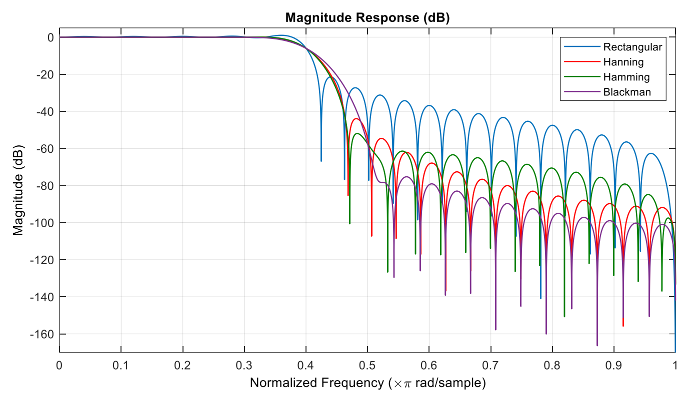
iii)

- Morphology of the windows

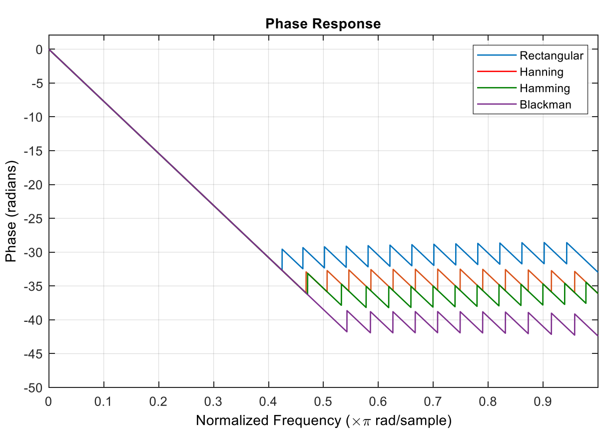


- Magnitude response with a linear and logarithmic magnitude scale





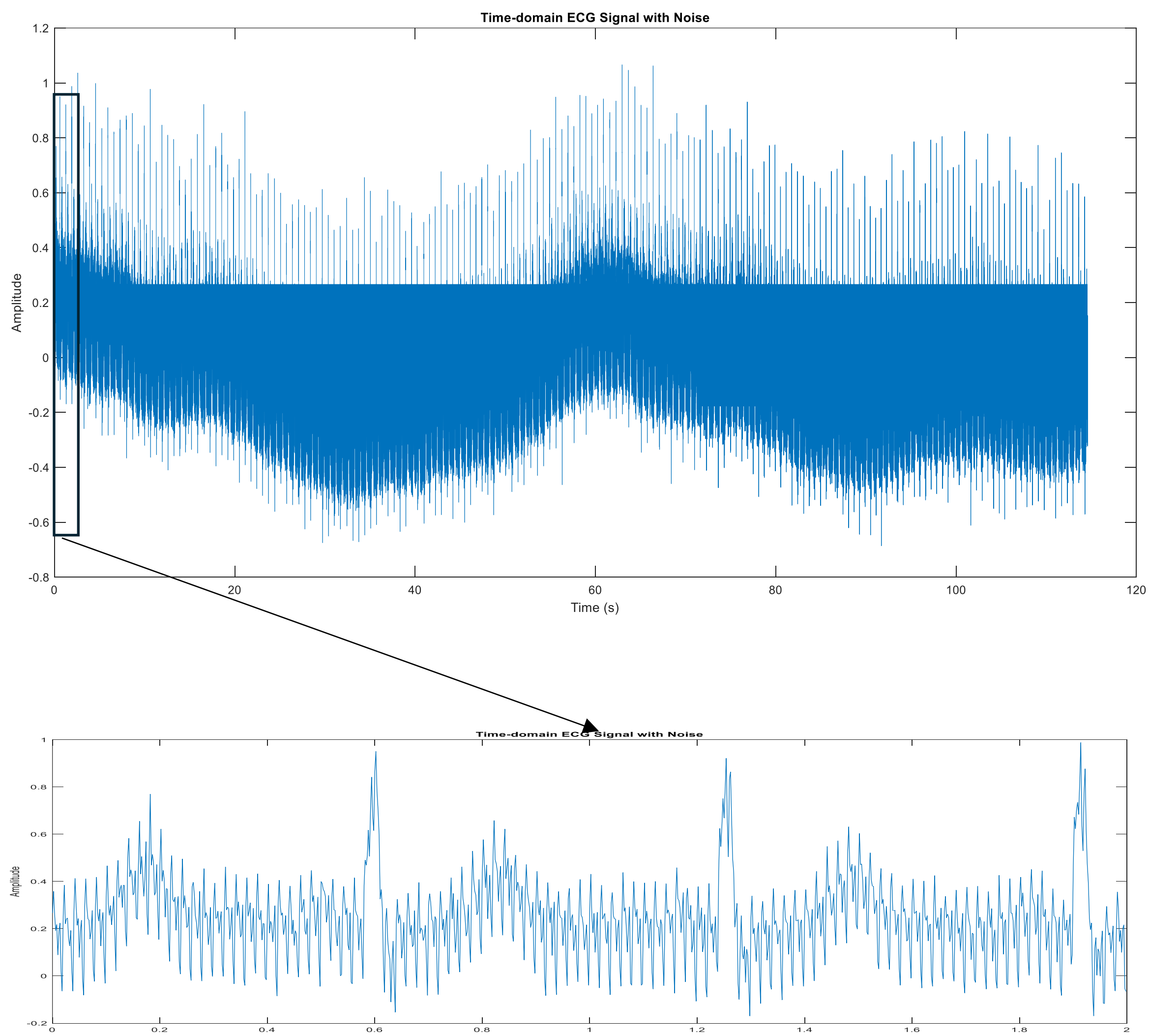
- Phase response



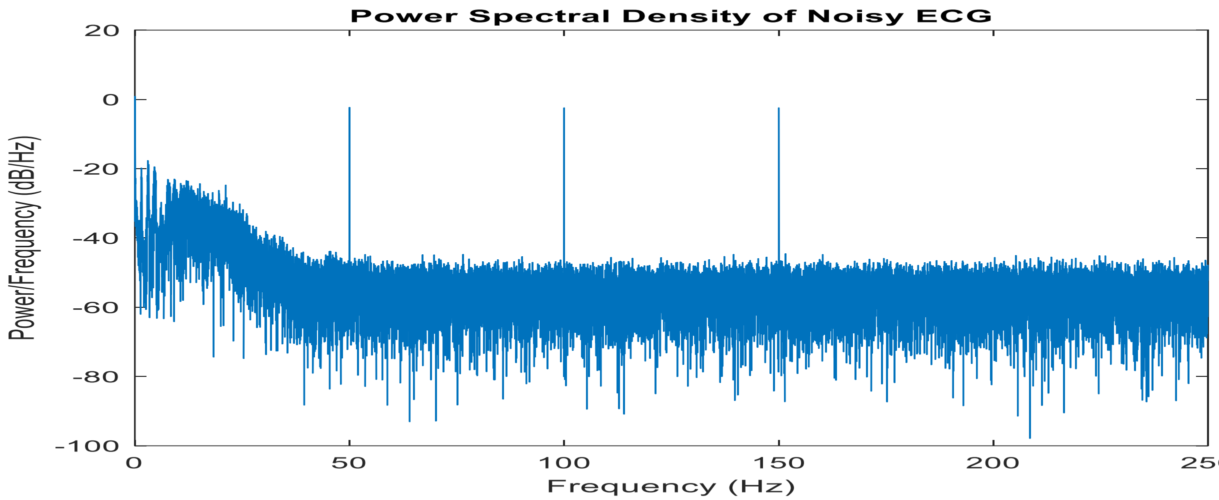
The main issue with the rectangular window is even if the length of the window is increased to sharpen the transition band; pass band and stop band ripples appears in the magnitude response, we can observe that clearly in the linear scale magnitude response, and it appears as large side lobes in log scale magnitude response. That is because of the discontinuities occur at the end of the window: When evaluating the discrete Fourier transform, we assume that the signals are periodic (period is equals to the length of the finite length signal), then if the start and end of the finite length signal has a large difference it appears as a discontinuity in the periodic signal then it is counted as a high frequency component when evaluating the Fourier transform (that is spectral leakage (has explained in section 1.1.1 question iv.)) To avoid that we need to reduce the difference between start and the end of the finite length signal for that we use the smoother windows, which will finally avoid occurring discontinuities in periodic signal and therefore, we will not see any high frequency components. That is the side band lobes has suppressed. Also, each one has linear phase response as all windows are symmetric and finite.

3.2. FIR Filter design and application using the Kaiser window

i)



ii)



ii)

	Low Pass (Hz)	High Pass (Hz)
f_{pass}	65	1.5
f_{stop}	95	0.5
δ	0.01	0.01

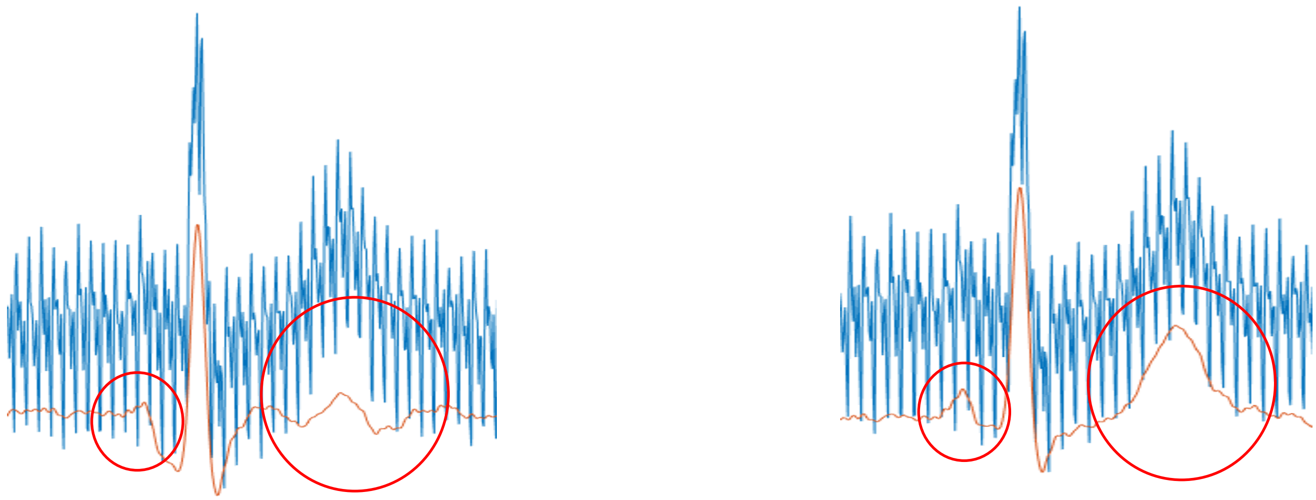
	Stop Band
f_{stop} 1	50Hz
f_{stop} 2	100Hz
f_{stop} 3	150Hz

Justification:-

In bio-signal processing, it is essential to remove the noise of the signal while preserving the important details of the signal. That is, when we are applying filters(Band Pass, Low Pass or Notch/stop filter) we need to carefully select the filter specifications(Cutoff frequency, Transition Band width, pass band/ stop band ripple etc.).

Ex:-

If we change the f_pass to 5Hz and f_stop to 1Hz without changing any other above mentioned filter specification we will get a signal like figure (a) where we can't observe the P wave and T wave correctly. whereas in figure (b) we can see the P wave and T wave clearly where the f_pass is 1.5Hz and f_stop is 0.5Hz.



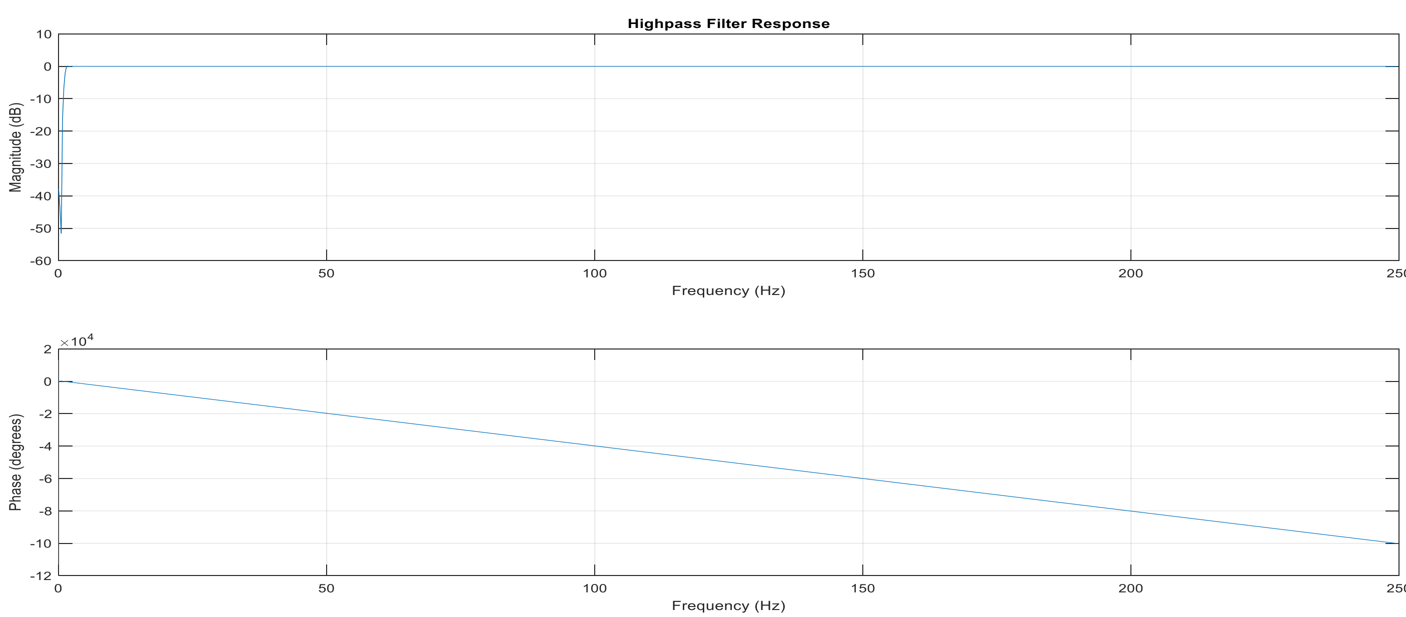
(a) (b)

iii)

For High pass filter M = 1118, β = 3.395321 (Order is high but we need to have a higher order FIR since the cutoff frequency is very small)

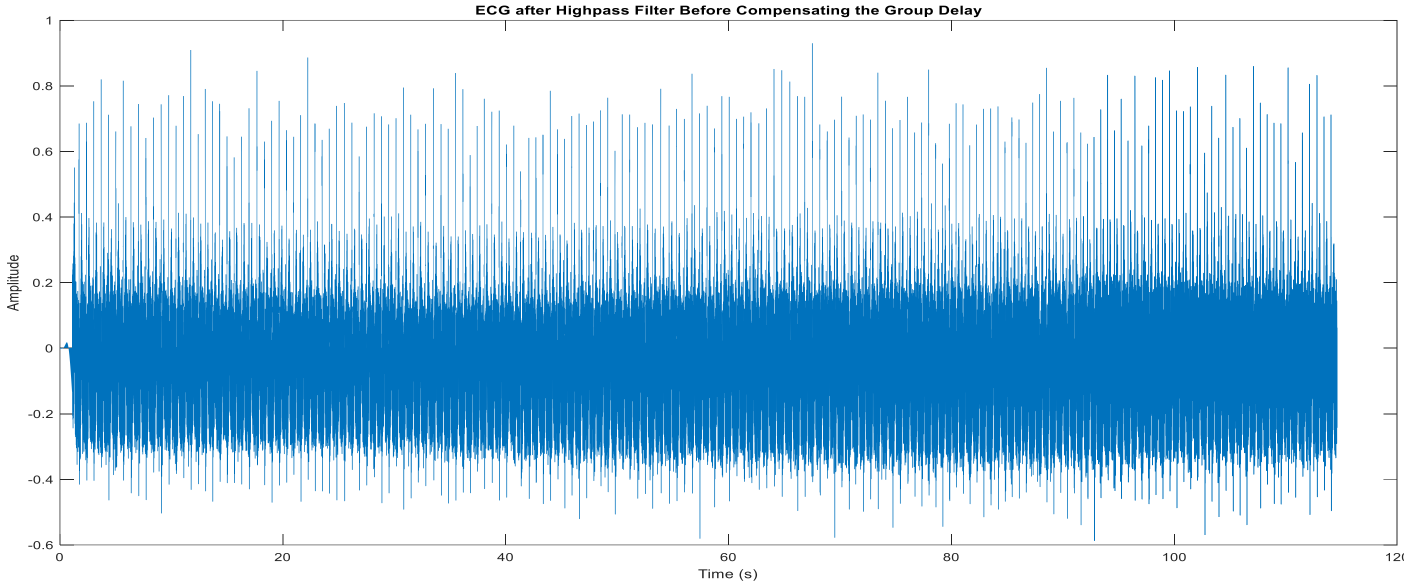
For Low pass filter M = 38, β = 3.395321

iv)



v)

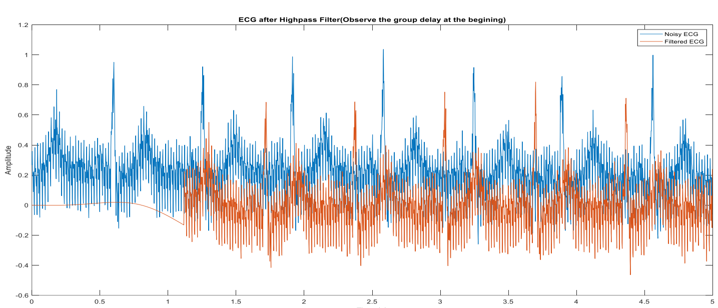
- **<u>Effect of Highpass filter</u>**

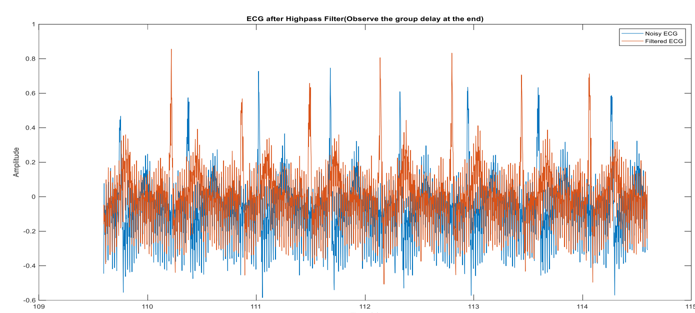


The baseline wandering is removed.

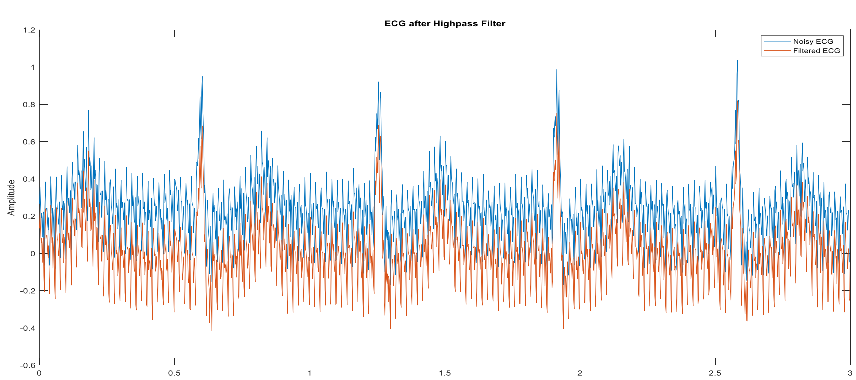
The effect of group delay.

➢ Before compensating the group delay

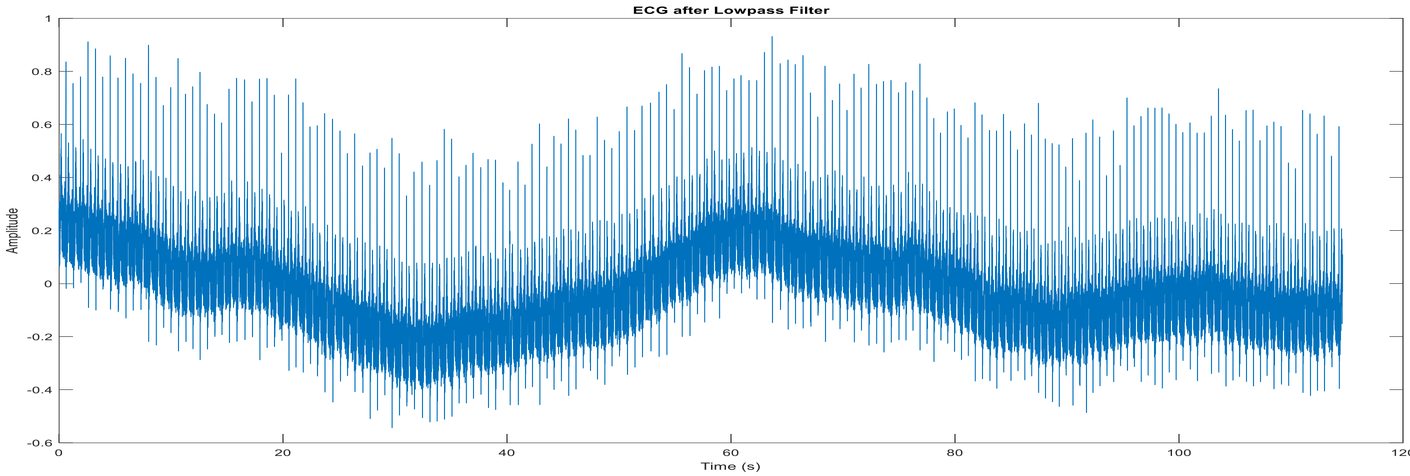


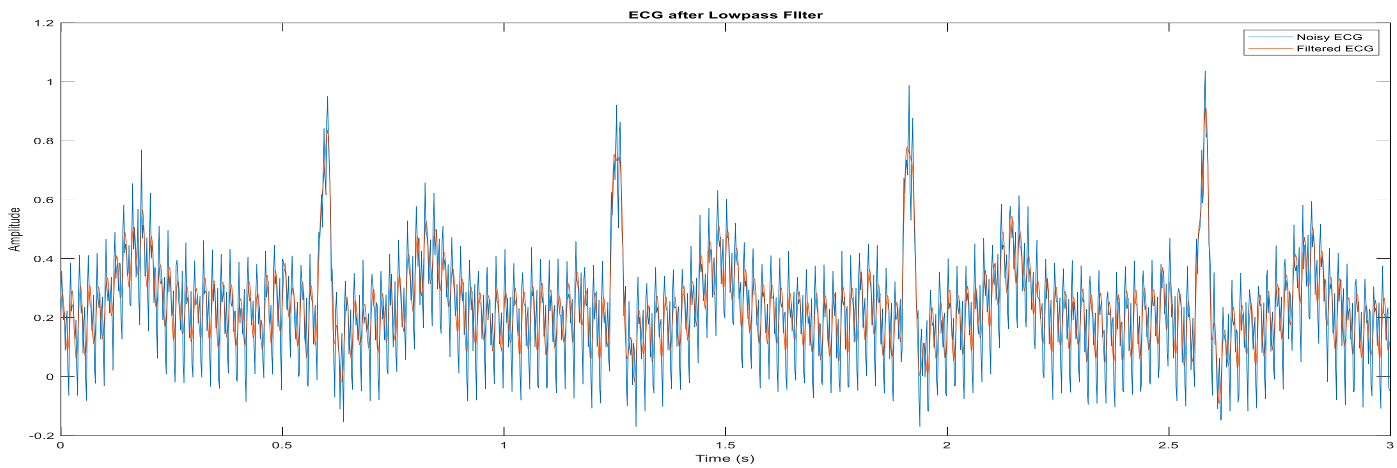


➢ After compensating the group delay

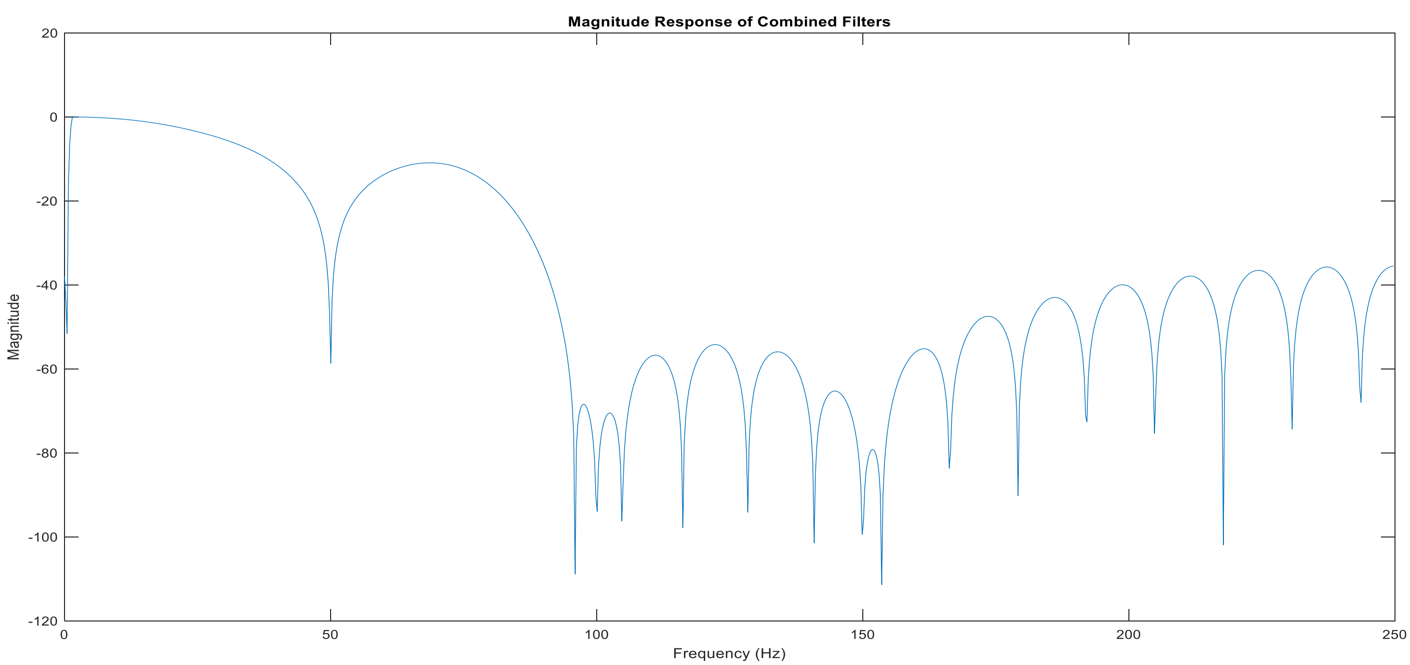


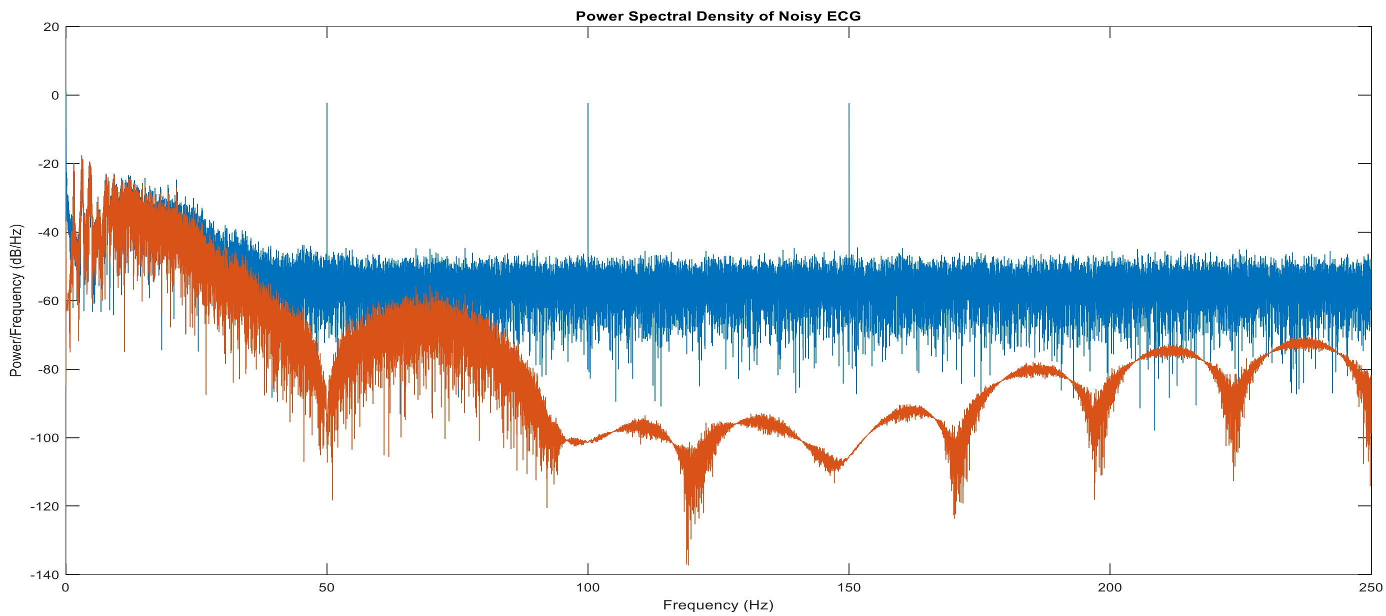
- **Effect of Lowpass filter**

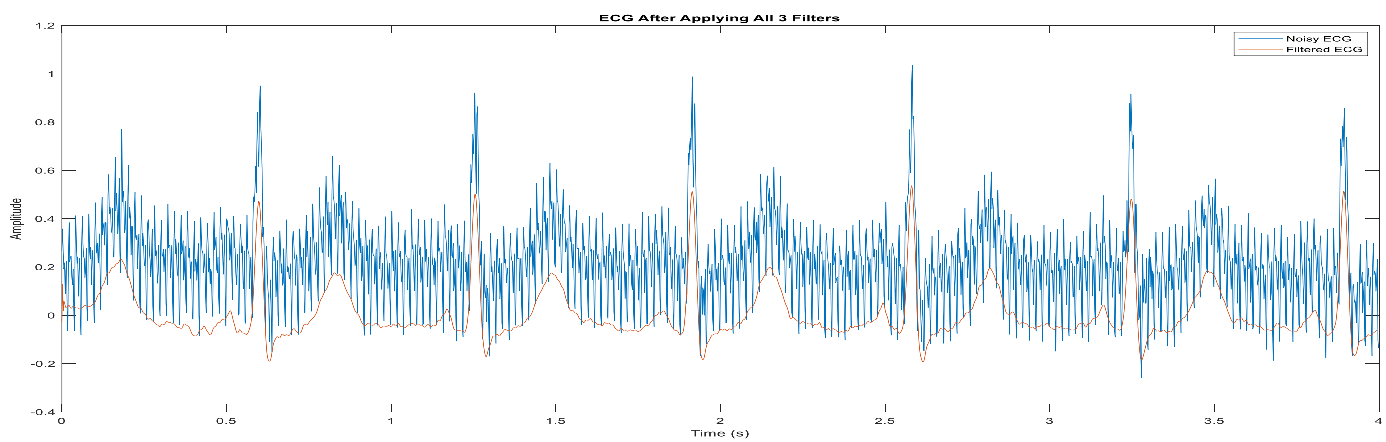




vi)







Designing comb filter with cutoff frequencies at 50Hz, 100Hz, and 150Hz.

Locate zeros at frequencies 50Hz, 100Hz and 150Hz on the unit circle. Then the transfer function of the comb filter is;

$$H(z) = G(1 - z_1 z^{-1})(1 - z_1^* z^{-1})(1 - z_2 z^{-1})(1 - z_2^* z^{-1})(1 - z_3 z^{-1})(1 - z_3^* z^{-1})$$

Where;

$$z_1 = e^{\frac{j2\pi 50}{500}}$$

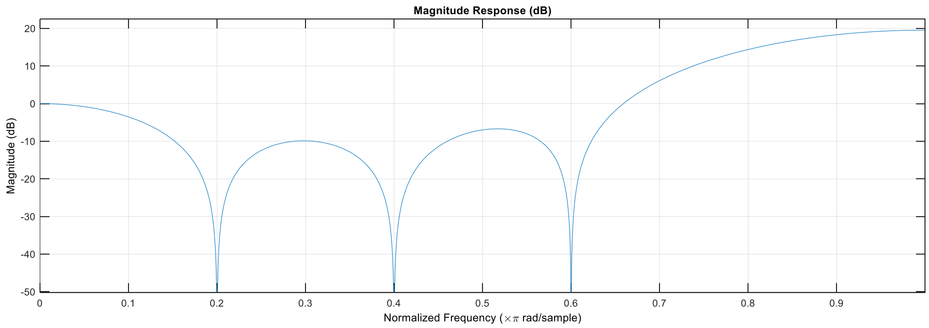
$$z_2 = e^{\frac{j2\pi 100}{500}}$$

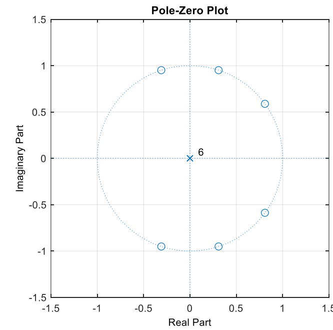
$$z_3 = e^{\frac{j2\pi 150}{500}}$$

Then get the H(z) in the form of $H(z) = G(b(0) + b(1)z^{-1} + b(2)z^{-2} + \cdots b(6)z^{-6})$

Finally find G so that the H(1) = 1.

That is G = (1/ (b(0) + b(1) + … + b(n)))





Cascade Filters

Multiply transfer functions : $H(z) = H_{LowPass}(z) * H_{HighPass}(z) * H_{StopBand}(z)$

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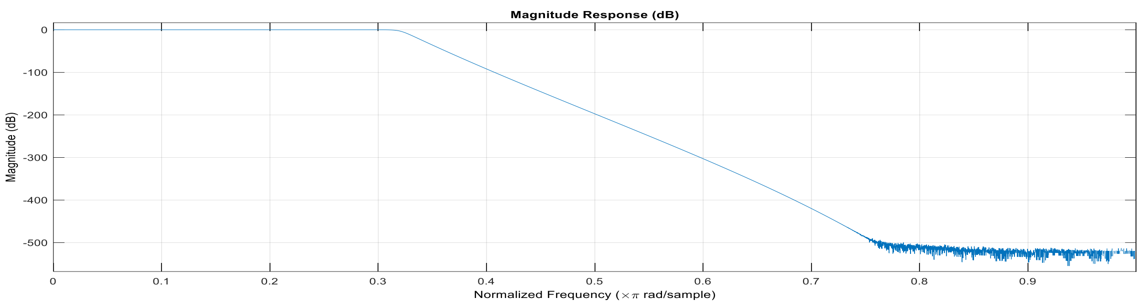
4. IIR Filters

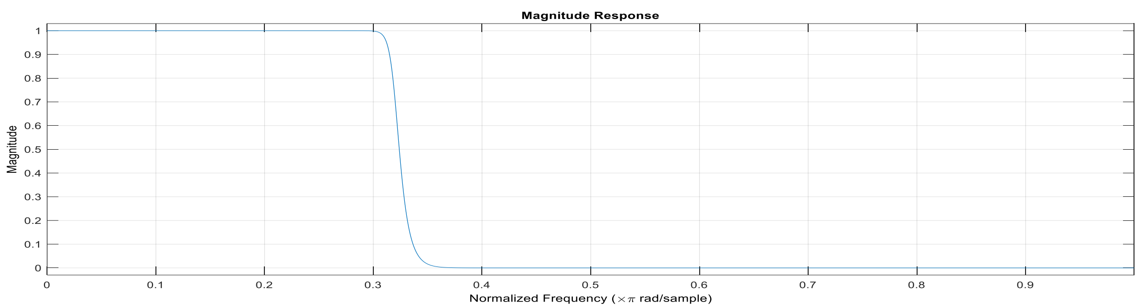
4.1. Realizing IIR Filters

i)

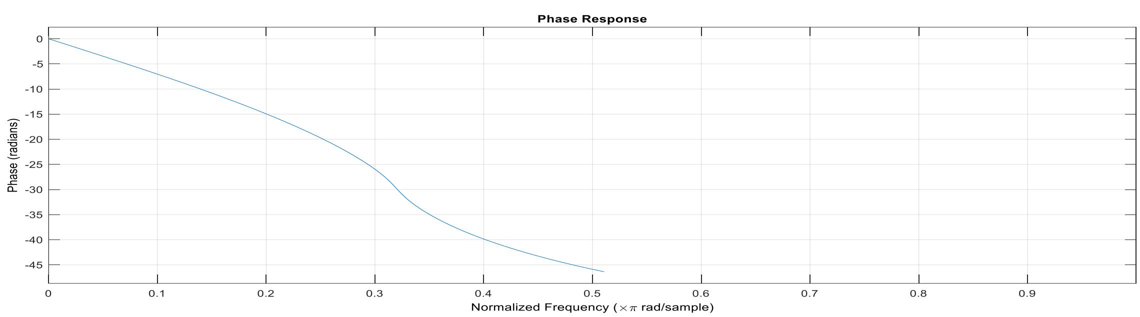
Low Pass Filter: Cutoff Frequency 80Hz and Order 38

Magnitude Response

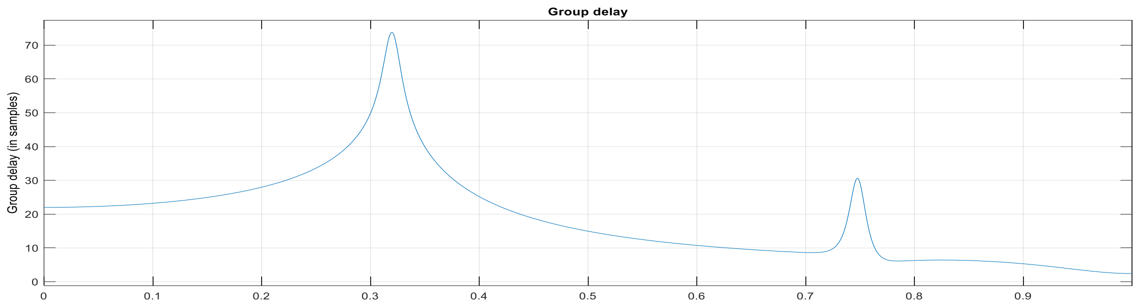




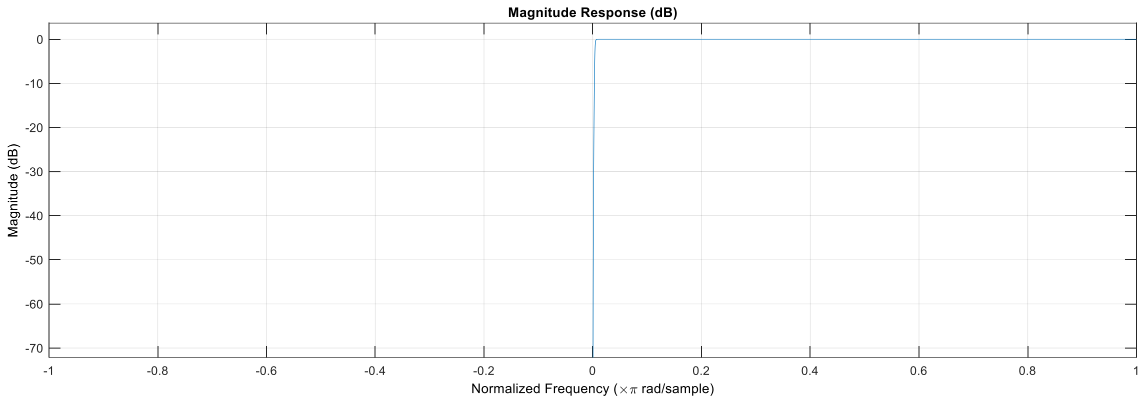
Phase Response



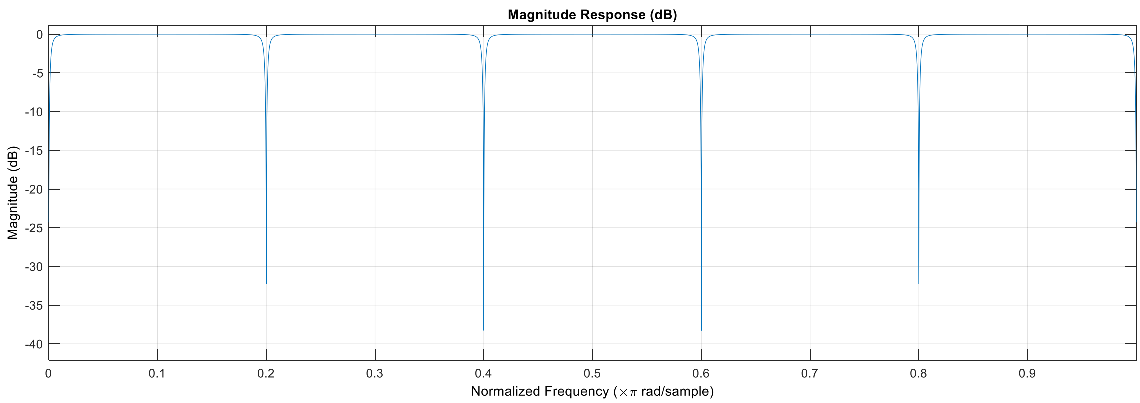
Group Delay



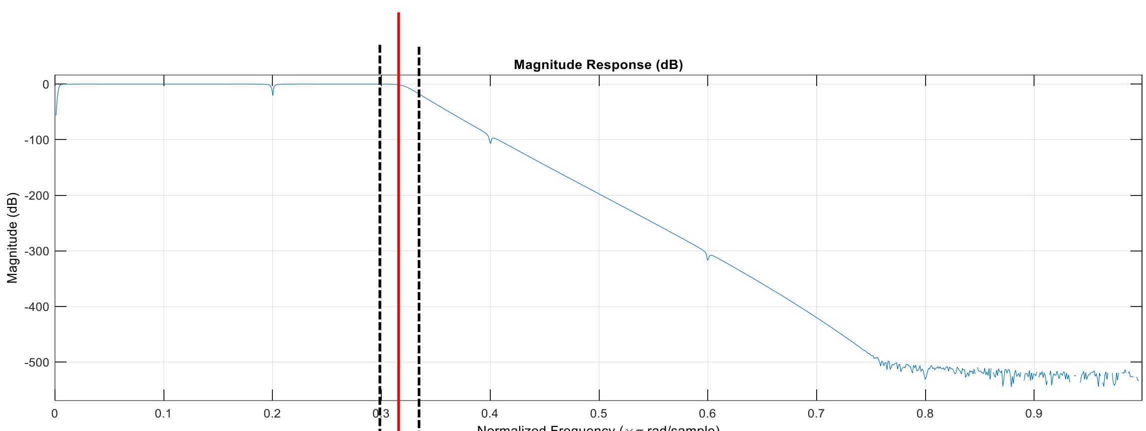
IIR High Pass Filter order 7

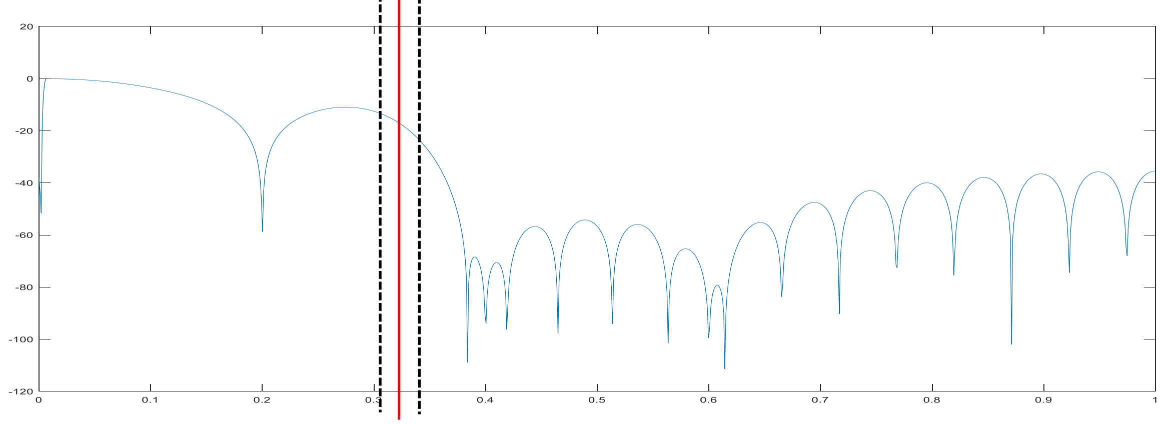


Comb Filter



iv)

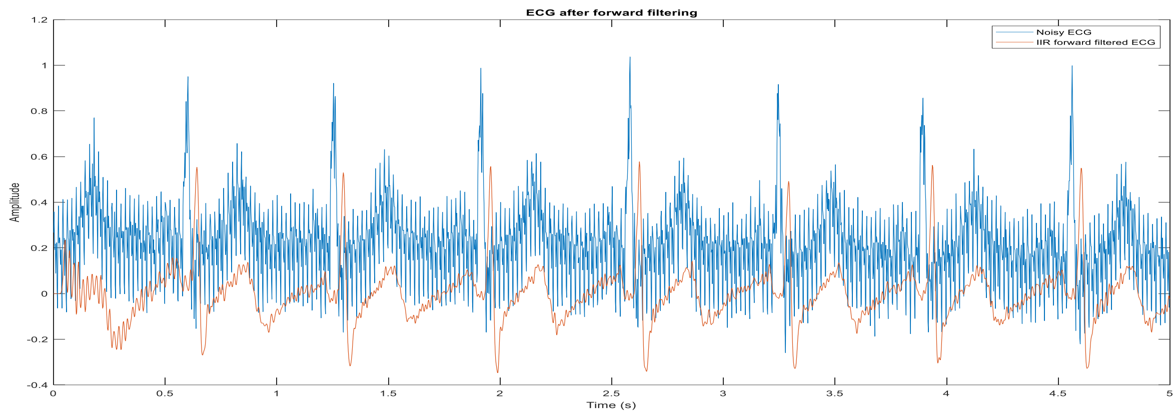




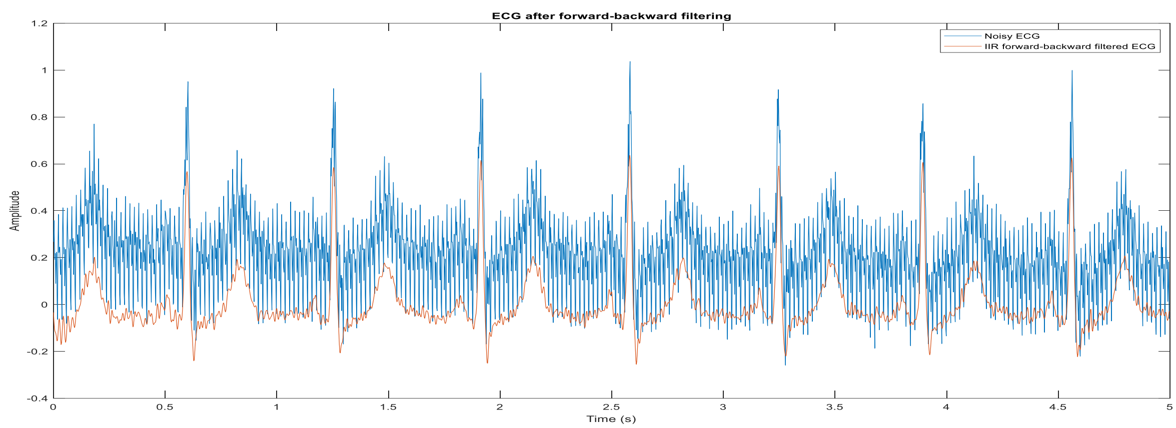
FIR filter's magnitude response does have large ripples(lobes) that is because of the zeros located on the unit circle and poles are at the origin whereas if we consider the IIR filter its zeros are not located on the unit circle therefore we can't see those large lobes.

4.2. Filtering methods using IIR filters

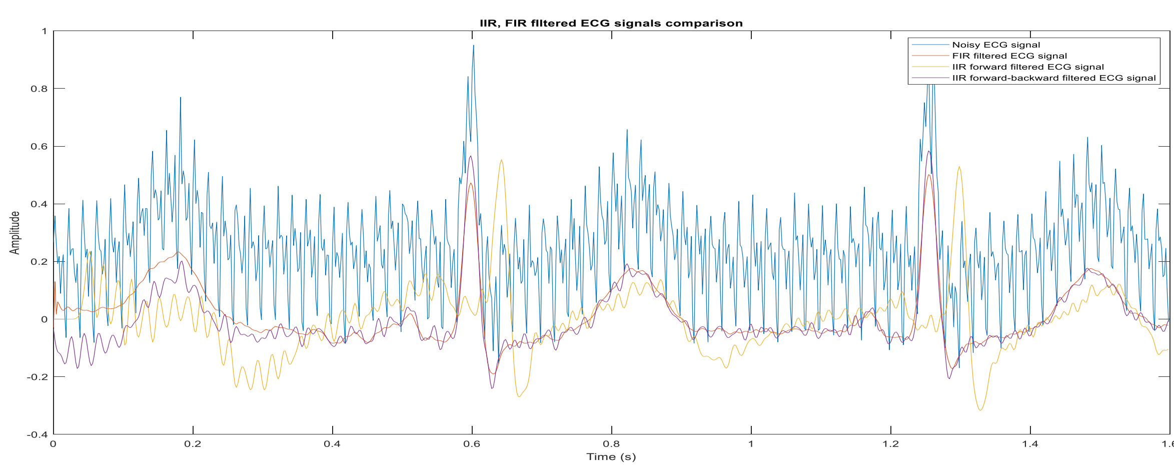
i)



ii)

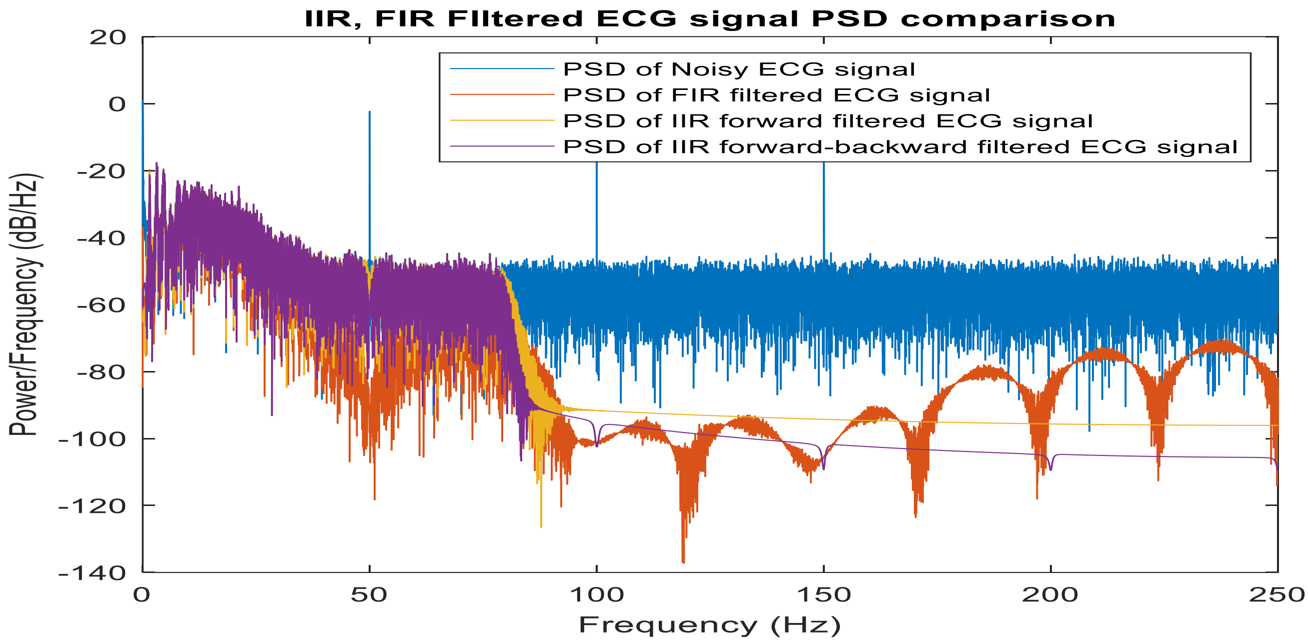


iii)



When we do the backward filtering, we can observe that the signal shifted to the right. That is the filtering process has introduced a delay. But as in FIR filter the delay is not constant for all frequencies. Hence, we can observe that the signal shape is deformed when it is only forward filtered. Therefore, in order to compensate this group delay the signal is filtered in both directions(forward and backward).

iv)



Both filters(FIR and IIR) have suppressed the noise successfully. Whereas IIR filter provides a much sharper transitions than the FIR filters, specially when implementing the comb filters IIR filters provide a successful noise suppression compared to the FIR filters That is clearly visible in the PSD. Also, even though the FIR filter requires a very high orders to provide a narrow transition band IIR filters can provide it using lower order filter.