MATLAB Assignment 3

Continuous and Discrete Wavelet Transforms

- **This is an individual assignment.**
- **Use data files (.mat) that are attached as instructed throughout the assignment.**
- **Submission guidelines**

Submission document	**Submission method**	**Notes**
Report	Upload the softcopy to Moodle	Should include observations and discussions with relevant plots to support your answers.
MATLAB scripts	Upload a single ZIP file including all the .m files to Moodle	Name each script according to the question number. Also, include necessary comments on the scripts for better read-ability.

Introduction

This assignment consists of,

- Basic implementation of continuous and discrete wavelet transforms
- Use of built-in MATLAB functions for wavelet transforms
- Denoising and compression using wavelet techniques

I. Continuous Wavelet Transform

1.1. Introduction

The continuous wavelet transform is defined by the following equation.

$$W(s,\tau) = \int x(t)\frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right)$$

$$where\ s = scaling\ factor, \qquad \tau = translation\ and\ \psi = wavelet\ function$$

There are many wavelet families such as Haar, Shannon, Mexican hat, Morlet, Daubechies, etc. that are useful for different applications (see Figure 1).

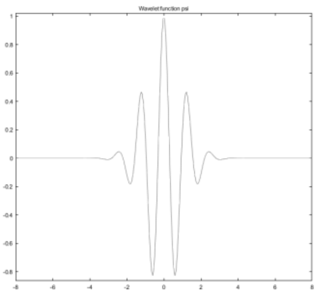

Figure 1.a. Morlet Wavelet

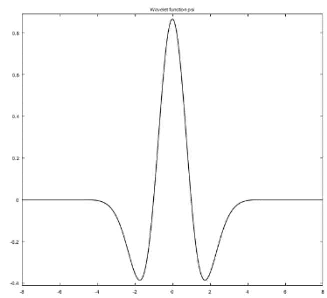

Figure 1.b. Mexican Hat Wavelet

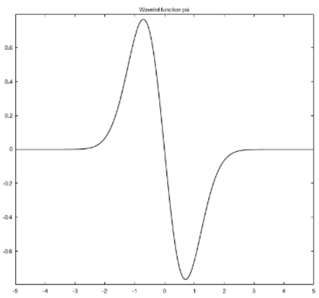

Figure 1.c. 1st derivative of Gaussian Wavelet

Figure 1: Wavelets

In this section, you will construct the Mexican hat mother wavelet to check wavelet properties and then implement CWT on a non-stationary signal.

1.2.Wavelet properties

i. Given the Gaussian function $g(t)$, derive the Mexican hat function $m(t)$

$$m(t) = -\frac{d^2}{dt^2}g(t)$$

$$g(t) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2}$$

Where $\mu = 0$ and $\sigma = 1$.

ii. Calculate the normalizing factor of $m(t)$ such that the energy E:

$$E = \int_{-\infty}^{\infty} m^2(t)\, dt = 1$$

iii. Hence, write the normalized Mexican hat mother wavelet $\psi(t)$. Include the scaling factor (s) in the generic wavelet function.

iv. Complete the provided outline script *wavelet_construction.m*, to generate the Mexican hat daughter wavelet for scaling factors of 0.01:0.1:2. Report the time-domain waveforms.

v. Verify the wavelet properties of zero mean, unity energy and compact support (by observation) for each of the above daughter wavelets.

vi. Using the same script, plot and comment on the spectra of daughter wavelets.

1.3. Continuous Wavelet Decomposition

i. Create a waveform on MATLAB as defined below with the following parameters.

$$x[n] = \begin{cases} sin(0.5\pi n), & 1 \leq n < \frac{3N}{2} \\ sin(1.5\pi n), & \frac{3N}{2} \leq n < 3N \end{cases}$$

Sampling Frequency = 250 Hz

ii. Apply the scaled Mexican hat wavelets to $x(n)$. To achieve translations, for each wavelet scale, convolve the signal with the constructed wavelet. Note: increase the scale resolution to 0.01:0.01:2.

iii. To visualize the spectrogram, plot the derived coefficients using the `pcolor()` command. The spectrogram should look similar to Figure 2.

iv. Comment on the plot and how the continuous wavelet coefficients represent the frequency content of $x(n)$.

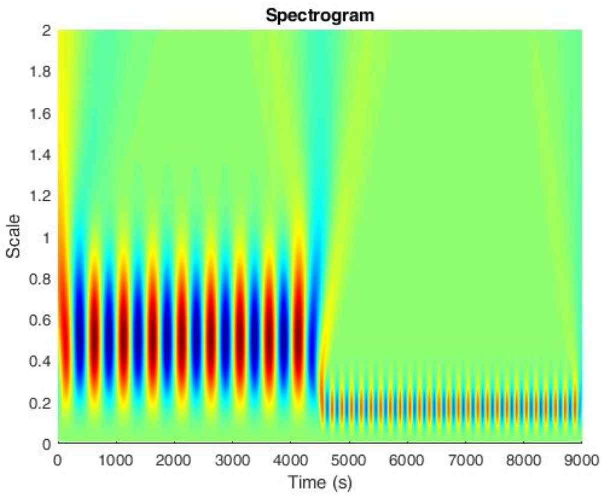

Figure 2: Spectrogram from CWT

II. Discrete Wavelet Transform

2.1. Introduction

The drawbacks of CWT include highly redundant computations which leads to the requirement of additional computational power and time consumption. To avoid this, in the discrete wavelet transform (DWT), the scaling and translation are performed in a discrete manner.

For DWT, the equation for CWT is modified as follows.

$$\psi_{m,n}(t) = \frac{1}{\sqrt{s_0^m}}\psi\left(\frac{t - n\tau_0 s_0^m}{s_0^m}\right)$$

$$s_0 = scaling\ step\ size, \qquad \tau_0 = translation\ step\ size$$

Usually $s_0 = 2$ and $\tau_0 = 1$ are used for efficient analysis. m and n are corresponding multiplier integers.

2.2.Applying DWT with the Wavelet Toolbox in MATLAB

i. Create the following waveforms in MATLAB.

$$x_1[n] = \begin{cases} 2\sin(20\pi n) + \sin(80\pi n), & 0 \le n < 512 \\ 0.5\sin(40\pi n) + \sin(60\pi n), & 512 \le n < 1024 \end{cases}$$

$$x_2[n] = \begin{cases} 1 & 0 \le n < 64 \\ 2 & 192 \le n < 256 \\ -1 & 128 \le n < 512 \\ 3 & 512 \le n < 704 \\ 1 & 704 \le n < 960 \\ 0 & otherwise \end{cases}$$

Sampling Frequency = 512 Hz

Corrupt these signals with AWGN of 10 dB SNR, call these signals y_1, y_2. Plot the corresponding $x[n]$ and $y[n]$ on the same figure. Use the command `awgn(x,snr,'measured')`.

ii. Observe the morphology of the wavelet and scaling functions of Haar and Daubechies tap 9 using `wavefun()` command and the `waveletAnalyzer` GUI.

iii. Calculate the 10-level wavelet decomposition of the signal using wavelet 'db9' and 'haar'. Use the command `wavedec()`.

iv. Use the inverse DWT to reconstruct A^{10}, D^{10}, D^9, …., D^2, D^1 and verify that $y = \Sigma\, D^i + A$ by calculating the energy between original and reconstructed signal. Explain the steps followed.

2.3. Signal Denoising with DWT

i. Plot the magnitude of wavelet coefficients (stem plot) of the above signal in descending order.

ii. Select a threshold by observation assuming low magnitude coefficients contain noise. Reconstruct the signal with suppressed coefficients.

iii. Calculate the root mean square error (RMSE) between the original and denoised signal. Plot the two signals on the same plot and interpret the results.

iv. Repeat the same procedure with 'haar' wavelet (make sure the signal is corrupted with exactly the same random noise. You may use a copy of the corrupted signal or fix the random generator `rng(seed)`)

v. Compare the RMSE and the reconstructed wave morphology of the two waveforms with the two wavelets and comment on the suitability of the wavelets used.

2.4. Signal Compression with DWT

i. You are given the aV_R lead of an ECG sampled at 257 Hz. Obtain the discrete wavelet coefficients of the signal (use 'db9' and 'haar' wavelets).

ii. Arrange the coefficients in the descending order and find the number of coefficients which are required to represent 99% of the energy of the signal.

iii. Compress the signal and find the compression ratio. Comment on the morphology of the reconstructed signal and the compression ratio.