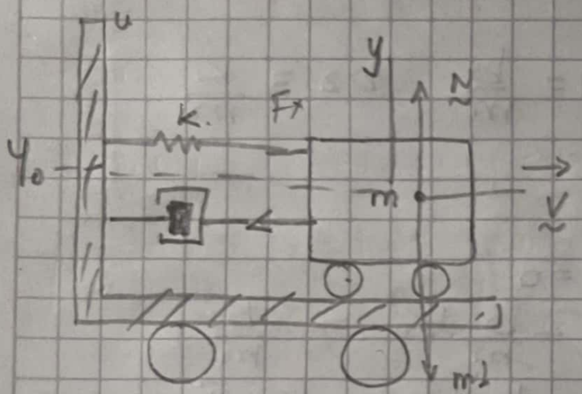
PAOLA ANDREA HERRERA CARDENAS 20162005881

PARCIAL 2

PUNTO 1

Ejemplo 3-3.



$\sum F = m a$
Sin Fricción con el Suelo.
m= masa
b= Coeficiente de fricción viscosidad
a= aceleración del sistema

$$\sum F = \sum F_x + \sum F_y = m(a_x + a_y)\ ma.$$

$$-(y-u)k - b\left(\frac{dy}{dt} - \frac{du}{dt}\right) = m\frac{d^2y}{dt}$$

$$m\ddot{y} + yk + b\dot{y} = ku + b\dot{u}$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{b}{m}\dot{u} + \frac{k}{m}u$$

Realizando una transformada de laplace $\mathcal{L}(s)$ con $y(s)$ y $U(s)$

$$y(s)s^2 + \frac{b}{m}y(s)s + \frac{k}{m}y(s) = \frac{b}{m}s\,u(s) + \frac{k}{m}u(s)$$

$$y(s)\left(s^2 + \frac{b}{m}s + \frac{k}{m}\right) = \left(\frac{bs}{m} + \frac{k}{m}\right)u(s) \quad ; \quad \frac{y(s)}{u(s)} = G(s)$$

$$\boxed{G(s) = \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]} = \frac{(bs+k)}{(ms^2+bs+k)} = \frac{y(s)}{u(s)}}$$

Función de Transferencia

Para la ecuación de espacio de estados

$$\boxed{m\ddot{y} + yk + b\dot{y} = ku + b\dot{u}}$$

Por definición

$$\overset{n}{y} + a_1 \overset{(n-1)}{y} + a_2 \overset{(n-2)}{y} + \cdots + a_{n-1}\dot{y} + a_n y = b_0 \overset{n}{u} + b_1 \overset{(n-1)}{u} + \cdots$$
$$\cdots + b_{n-1}\dot{u} + b_n u$$

$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m} y = 0 + \frac{b}{m}\dot{u} + \frac{k}{m} u$$

Luego

$$a_1 = \frac{b}{m} \qquad a_2 = \frac{k}{m} \qquad ; \qquad b_0 = 0 \qquad b_1 = \frac{b}{m} \qquad b_2 = \frac{k}{m}$$

Pero

$$\beta_0 = b_0$$
$$\beta_1 = b_1 - a_1\beta_0$$
$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0$$

$$\longrightarrow \beta_0 = b_0 = 0$$
$$\beta_1 = \frac{b}{m} - \cancel{\frac{b}{m}\overset{0}{b_0}} = \frac{b}{m}$$
$$\beta_2 = \frac{k}{m} - \frac{b}{m}\left(\frac{b}{m}\right) - \cancel{\frac{k}{m}(0)}$$
$$= \frac{k}{m} - \frac{b^2}{m^2}$$

y de la ecuación de estado

$$x_1 = y - \beta_0 u$$
$$x_2 = \dot{y} - \beta_0\dot{u} - \beta_1 = \dot{x}_1 - \beta_1 u$$
$$x_3 = \ddot{y} - \beta_0\ddot{u} - \beta_1\dot{u} - \beta_2 u = \dot{x}_2 - \beta_2 u$$
$$x_4 = \dddot{y} - \beta_0\dddot{u} - \beta_1\ddot{u} - \beta_2\dot{u} - \beta_3 u = \dot{x}_3 - \beta_3 u.$$

De lo que queda

$$x_1 = y.$$
$$x_2 = \dot{x}_1 - \frac{b}{m}u \qquad = x_2 + \frac{bu}{m} - \frac{b}{m}u$$

Recordando que

$$\dot{x}_1 = x_2 + \beta_1 u = x_2 + \frac{b}{m}u$$
$$\dot{x}_2 = x_3 + \beta_2 u = 0$$

con

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -a_2 & -a_1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} u$$

$$\dot{x}_1 = x_2 + \beta_1 u$$
$$\dot{x}_2 = x_3 + \beta_2 u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \cancel{\beta_0 u}$$

Por lo tanto

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\frac{k}{m} & \frac{b}{m}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}-\frac{b}{m}\\ \frac{k}{m} - \frac{b^2}{m^2}\end{bmatrix} u \qquad y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Ecuaciones de Espacio de Estados

Diagrama de Bloques

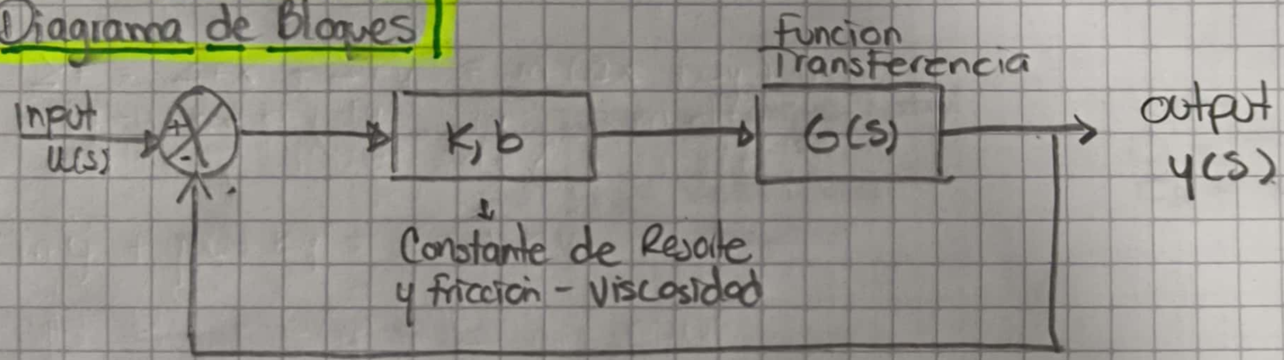
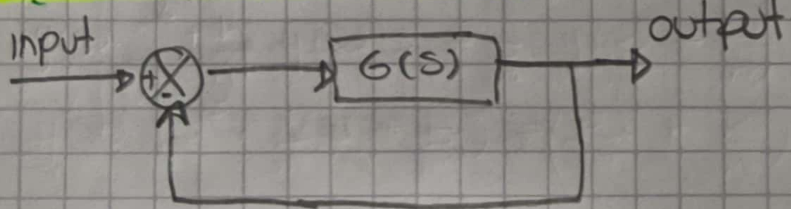
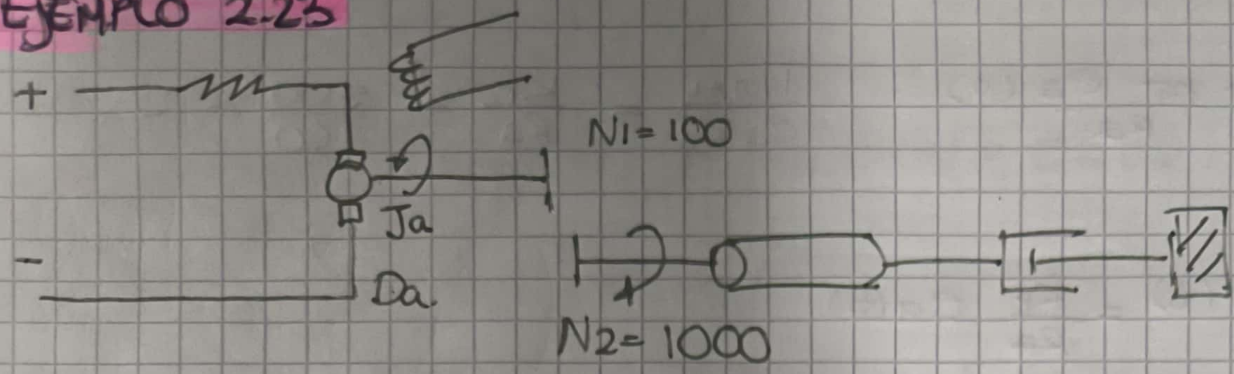


Diagrama Flujo de Señal



EJEMPLO 2.23



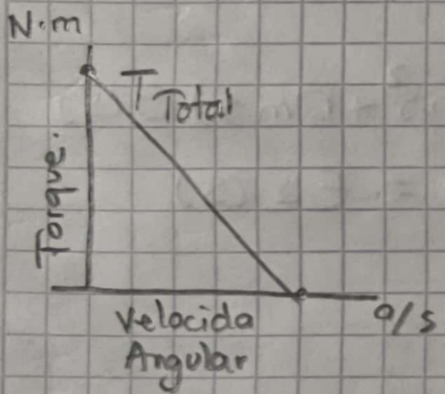
$$G(s) = \frac{\mathcal{L}\,[\text{output}]}{\mathcal{L}\,[\text{input}]} = \frac{\theta_L(s)}{E_n(s)}$$

$J_m \equiv$ inercia total

$$J_m = J_a + J_L \left(\frac{N_1}{N_2}\right)^2$$

$D_m \equiv$ Inercia de Amortiguación Total

$$D_m = D_a + D_L \left(\frac{N_1}{N_2}\right)^2$$



$T_{total} \equiv$ Torque Total.

$\dot{\theta} \equiv$ Velocidad Angular

$T_{TOTAL} = 500$
$\dot{\theta} = 50$
$e_a = 100.$

$$v_b = K_b\,\dot{\theta} = K_b \frac{d\theta}{dt} = K_b\, s\, \theta(s)$$

$$T_{total}(s) = K_r\, I_a(s)$$

$$R_a I_a(s) + V_b(s) = E_a(s)$$

$$R_a\, T_{total}\ \ a_1(s) + K_b\, s\ \theta(s) = E_a(s)$$

$$\frac{R_a}{K_r} T_{TOTAL}(s) + K_b\, s\, \theta(s) = E_a(s).$$

K_r = Constante de Proporcionalidad.
K_b = // // ///.

De lo que es posible obtener la transformada inversa de la place

$$\frac{R_a}{K_r} T_{total}(t) + K_b\,\dot{\theta}(t) \cdot e_a(t)$$

$$T_{TOTAL}(t) = \frac{K_r}{R_a} e_a(t) - \frac{K_b K_r}{R_a}\dot{\theta}(t)$$

$\theta(0) = 0$

$T_{total}(0) = \frac{K_T}{R_a} e_a(0)$ $\qquad \frac{T_{TOTAL}}{e_a(0)} = \frac{K_r}{R_a} = \frac{500}{100} = 5$

$T_{TOTAL} = 0$

$$\frac{K_b K_r}{R_a} \dot{\theta}(t) = \frac{K_r}{R_a} e_a(t).$$

$$\dot{\theta}(t) = \frac{e_a(t)}{K_b} \; ; \quad K_b = \frac{e_a(t)}{\dot{\theta}(t)} = \frac{100}{50} \quad 2$$

El Torque se define $T = I \frac{d^2\theta}{dt^2} = I s^2 \theta(s)$

Pero el $T_{total} = T_m = T_a + T_L$

$$T_m = (J_m \ddot{\theta} + D_m \dot{\theta})$$

$$T_m(s) = J_m s^2 \theta(s) + D_m s \theta(s). = (J_m s^2 + D_m s)\theta(s)$$

$$\frac{R_a}{K_r}(J_m s^2 + D_m s)\,\theta(s) + K_b s \theta(s) = E_a(s)$$

$$G(s) = \frac{\theta(s)}{E(s)}$$

$$\theta(s)\left\{\frac{R_a}{K_r}(J_m s^2 + D_m s) + K_b s\right\} = E_a(s)$$

$$\frac{\theta(s)}{E_a(s)} = \frac{1/1}{R_a(J_m s^2 + D_m s) + K_b s K_r} = \frac{K_a/R_a}{s\left((J_m s + D_m) + \frac{K_b K_r}{R_a}\right)}$$

$$G(s) = \frac{K_r/R_a J_m}{s\left(\left(s + \frac{1}{J_m}\left(D_m + \frac{K_b K_r}{R_a}\right)\right)\right)}$$ $\boxed{\text{FUNCION DE TRANSFERENCIA}}$

y para la ecuacion de espacio de Estados

$$\frac{Ra}{Kr}\left(Jm\,\ddot{\theta}(t) + Dm\,\ddot{\theta}(t)\right) + Kb\,\dot{\theta}(t) = ea(t)$$

$$Ra\,Jm\,\ddot{\theta}(t) + Ra\,Dm\,\dot{\theta}(t) + Kb\,Kr\,\dot{\theta}(t) = Kr\,ea(t)$$

$$\ddot{\theta}(t) + \left(\frac{Dm}{Jm} + \frac{Kb\,Kr}{Ra\,Jm}\right)\dot{\theta}(t) = \frac{Kr}{Ra\,Jm}\,ea(t)$$

$$a_1 = \left(\frac{Dm}{Jm} + \frac{Kb\,Kr}{Ra\,Jm}\right) \qquad b_0 = 0$$

$$b_1 = 0$$

$$a_2 = 0 \qquad b_2 = \frac{Kr}{Ra\,Jm}$$

$$\beta_0 = b_0 = 0$$
$$\beta_1 = b_1 - a_1\beta_0 = 0$$
$$\beta_2 = b_2 - a_1\beta_1 - a_2\beta_0 = \frac{Kr}{Ra\,Jm}$$

$$X_1 = \theta - \beta_0\,ea = \theta$$
$$X_2 = \dot{\theta} - \beta_0\,\dot{e}a - \beta_1 = \dot{\theta}$$

Luego

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -a_2 & -a_1\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \begin{bmatrix}\beta_1\\ \beta_2\end{bmatrix} ea$$

$$\theta = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \beta_0\,ea$$

Entonces las ecuaciones de espacio de estados son.

$$\begin{bmatrix}\dot{X}_1\\ \dot{X}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -\left(\frac{Dm}{Jm} + \frac{Kb\,Kr}{Ra\,Jm}\right)\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{Kr}{Ra\,Jm}\end{bmatrix} ea$$

Ecuaciones de Estado

$$\theta = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}X_1\\ X_2\end{bmatrix}$$

Diagrama de Bloques

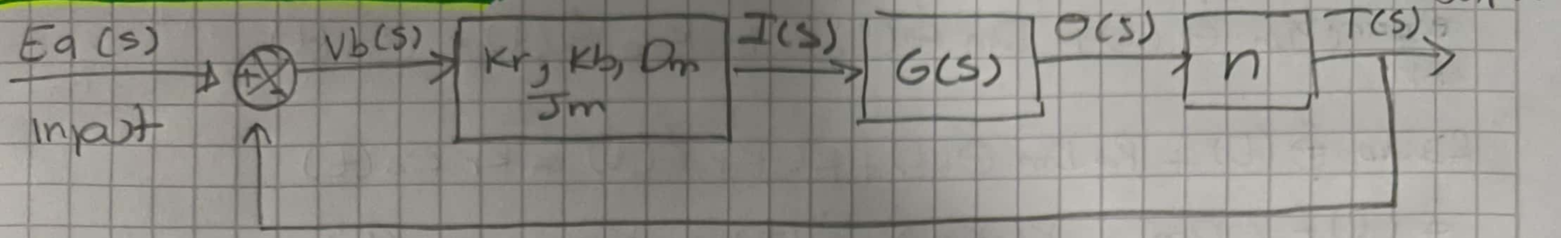
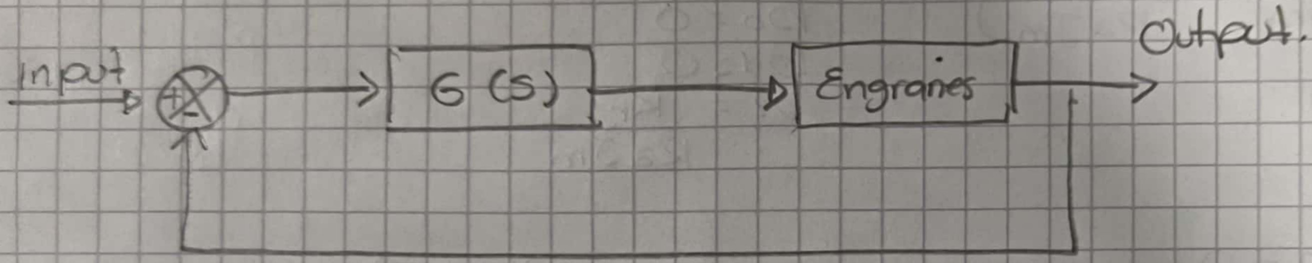


Diagrama de Flujo de Señal



Ejemplo 3.9

Diferencia angular de los potenciometros

$e_r - e_c = e_v$
$r - c = e$
$e_r = K_o r$
$e_c = K_o c$
$e_a = K_1 e_v$

K_1, K_3 { Constantes de proporcionalidad

$K_o \equiv$ Constante
$i_a \equiv$ Corriente del Circuito
$T \equiv$ Torque de Motor.
$T \equiv K a i_a$
$K_a \equiv$ Constante del torque del motor.
$e_a \equiv$ Voltaje Inducido
$e_b = K_3 \frac{d\theta}{dt}$

$e_a = K_1 (e_r - e_c) = K_1 K_o (r - c)$

$G(s) = \frac{C(s)}{\Theta(s)} \quad \frac{\Theta(s)}{E_v(s)} \quad \frac{E_v(s)}{E(s)}$ } $\boxed{\text{Función Transferencia}}$

$L_a \frac{di_a}{dt} + R_a i_a + K_3 \frac{d\theta}{dt} = K_1 e_v$ input.

$J_o \frac{d^2\theta}{dt^2} + b_o \frac{d\theta}{dt} = K_2 i_a$ output

Realizando la transformada de laplace del input y el output

$\frac{L_a}{K_1} s I(s) + \frac{R_a}{K_1} I(s) + \frac{K_3}{K_1} \frac{d\theta}{dt} = E_v(s)$

$\frac{J_o}{K_2} s^2 \Theta(s) + \frac{b_o s}{K_2} \Theta(s) = I(s)$

$\Theta(s) = \frac{K_2 I(s)}{(J_o s^2 + b_o s)}$

$\frac{L_a}{K_1} s I(s) + \frac{R_a}{K_1} I(s) + \frac{K_3}{K_1} s \frac{K_2 I(s)}{(J_o s^2 + b_o s)} = E_v(s)$

$$\frac{\Theta(s)}{E_v(s)} = \frac{\dfrac{K_2 I(s)}{(J_o s^2 + b_o s)}}{\dfrac{((L_a s + R_a)(J_o s^2 + b_o s) + K_3 K_2 s) I(s)}{K_1 (J_o s^2 + b_o s)}}$$

$$= \frac{K_1 K_2}{s(L_a s + R_a)(J_o s + b_o) + K_2 K_3}$$

$$\frac{C(S)}{\theta(S)} = n$$

$$E_v = E_r - E_c = K_o R(S) - K_o C(S)$$
$$E_v(S) = K_o (R(S) - C(S)) = K_o E(S)$$

$$\frac{E_v(S)}{E(S)} = K_o$$

$$G(S) = (n)\left(\frac{K_1 K_2}{S(L_a S + R_a)(J_o S + b_o) + K_2 K_3 S}\right)(K_o)$$

$$G(S) = \frac{n K_o K_1 K_2}{S(L_a S + R_a)(J_o S + b_o) + K_2 K_3 S}$$

Si $L_a \longrightarrow 0$

$$G(S) = \frac{n K_o K_1 K_2 / R_a}{J_o S^2 + \left(b_o + \frac{K_2 K_3}{R_a}\right) S}$$ Que es la función de transferenc

Para las ecuaciones de estado se retoman las ecuaciones de input y output

$$L_a \frac{di_a}{dt} + R_a i_a + K_3 \frac{d\theta}{dt} = K_1 e_v$$

$$J_o \frac{d^2\theta}{dt^2} + b_o \frac{d\theta}{dt} = K_2 i_a$$

Restando el input a los output

$$J_o \frac{d^2\theta}{dt^2} + b_o \frac{d\theta}{dt} - L_a \frac{di_a}{dt} - R_a i_a - K_3 \frac{d\theta}{dt} = K_2 i_a - K_1 e_v$$

$$J_o \frac{d^2\theta}{dt^2} + b_o \frac{d\theta}{dt} + K_1 e_v - K_3 \frac{d\theta}{dt} = L_a \frac{di_a}{dt} + R_a i_a + K_2 i_a$$

$$\frac{d^2\theta}{dt^2} + \left(\frac{b_o}{J_o} - K_3\right) \frac{d\theta}{dt} + K_1 e_v = L_a \frac{di_a}{dt} + (R_a + K_2) i_a$$

$$\ddot{\theta} + \left(\frac{b_o}{J_o} - K_3\right) \dot{\theta} + K_1 e_v = L_a \dot{i}_a + (R_a + K_2) i_a.$$

Luego por definición se tiene que

$$a_1 = \frac{b_0}{J_0} - K_3 \qquad b_0 = 0$$

$$b_1 = La$$

$$a_2 = 0$$

$$b_2 = (Ra + K_2)$$

Con

$$\beta_0 = b_0 = 0$$

$$\beta_1 = b_1 - a_1\beta_0 = La - \left(\frac{b_0}{J_0} - K_3\right)(0) = La$$

Sabiendo que

$$x_1 = \theta - \beta_0 ia = \theta$$

$$x_2 = \dot{\theta} - \beta_0 \dot{ia} - \beta_1 ia = \dot{\theta} - La\, ia$$

$$\dot{x}_1 = x_2 + \beta_1 ia = \dot{\theta}$$

$$\dot{x}_2 = x_3 + \beta_2 ia = x_3 + (Ra + K_2)ia - \left(\frac{b_0}{J_0} - K_3\right) La\, ia$$

Luego

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} ia$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \beta_0\, ia$$

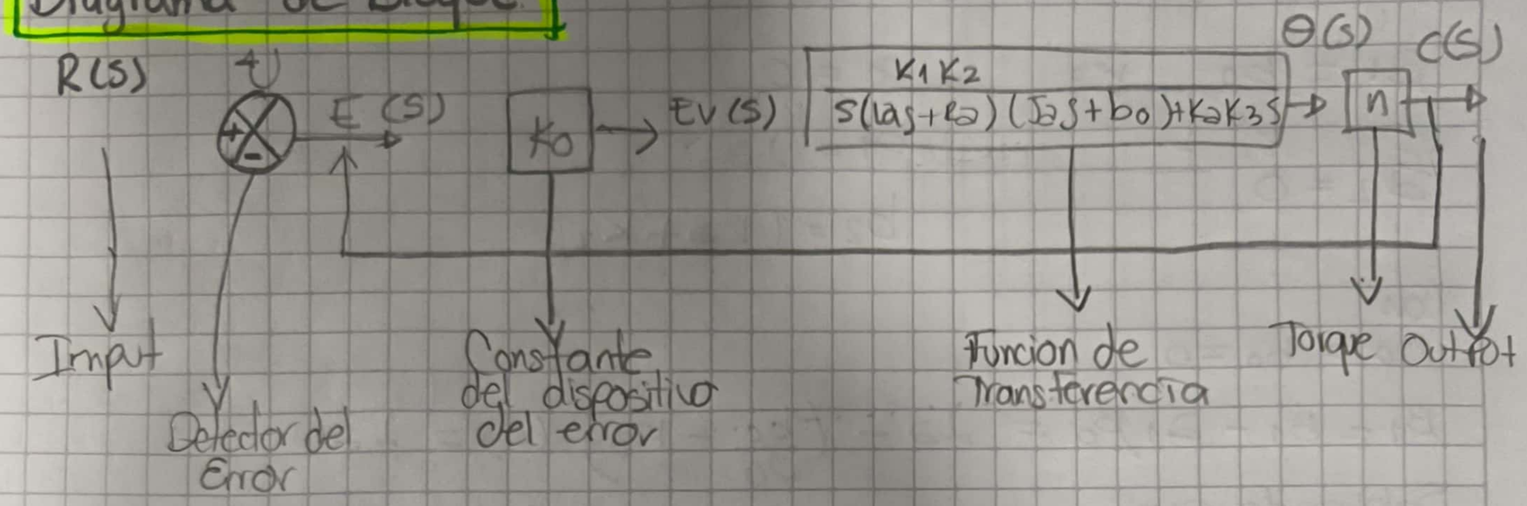
Entonces

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\left(\frac{b_0}{J_0} - K_3\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} La \\ (Ra + K_2) - \left(\frac{b_0}{J_0} - K_3\right) La \end{bmatrix} ia$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Ecuaciones de espacio de estados

Diagrama de Bloque

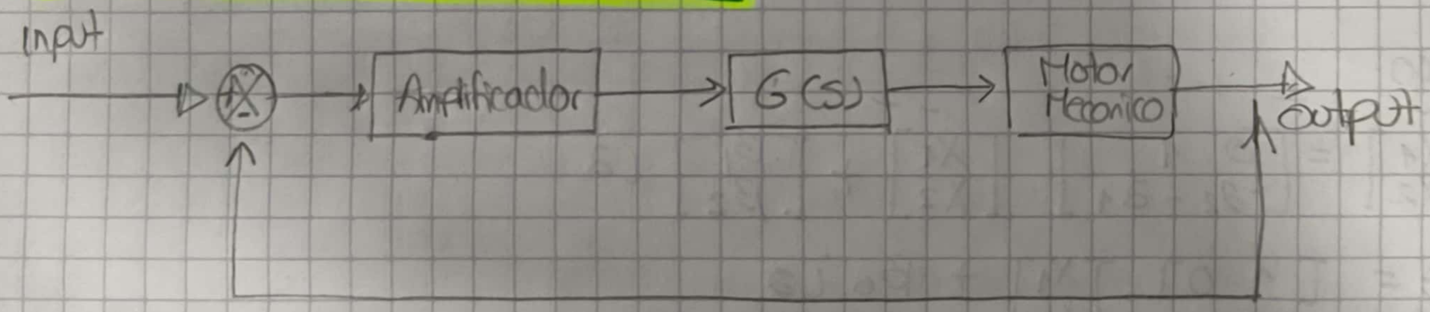


$E(s) = R(s) - C(s)$

$Ev(s) = Ko\ E(s)$

$\theta(s) = \frac{C(s)}{n}$

Diagrama de flujo de la señal



Solución Parcial 2.

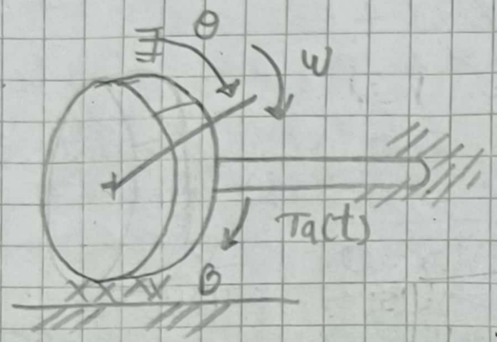
1. Para el sistema rotacional en la figura, determine.

a. Representación en el espacio de estados.
b. Diagrama de Bloques
c. Diagrama de flujo de señal
d. Función de transferencia.

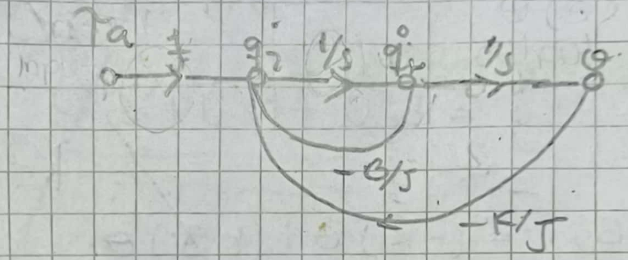
Items para TODOS los puntos

$\curvearrowright +$, $\curvearrowleft -$ Para las ecuaciones

*



• Diagrama Flujo de Señal



$$\omega = \frac{d\theta}{dt}$$

ΣT

$\Sigma 1 = J\alpha = J\ddot{\theta}$

$$\ddot{\theta} = \frac{Ta(t)}{J} - \frac{B\dot{\theta}}{J} - \frac{K\theta}{J}$$

$q_1 = \theta \qquad q_2 = \dot{\theta} = \dot{q}_1$

$\dot{q}_2 = \ddot{\theta} = \ddot{q}_1$

$\Downarrow$

$$\dot{q}_2 = \frac{Ta(t)}{J} - \frac{B\,q_2}{J} - \frac{K\,q_1}{J}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{K}{J} & -\frac{B}{J} \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/J \end{bmatrix} \tau$$

$$\theta = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$

Ecuaciones de Espacio de Estados.

$$T - k\theta - b\dot{\theta} = J\ddot{\theta}$$

$$T = J\ddot{\theta} + k\theta + b\dot{\theta}$$

$$\mathcal{L}(\tau) = \mathcal{L}[J\ddot{\theta} + k\theta + b\dot{\theta}]$$

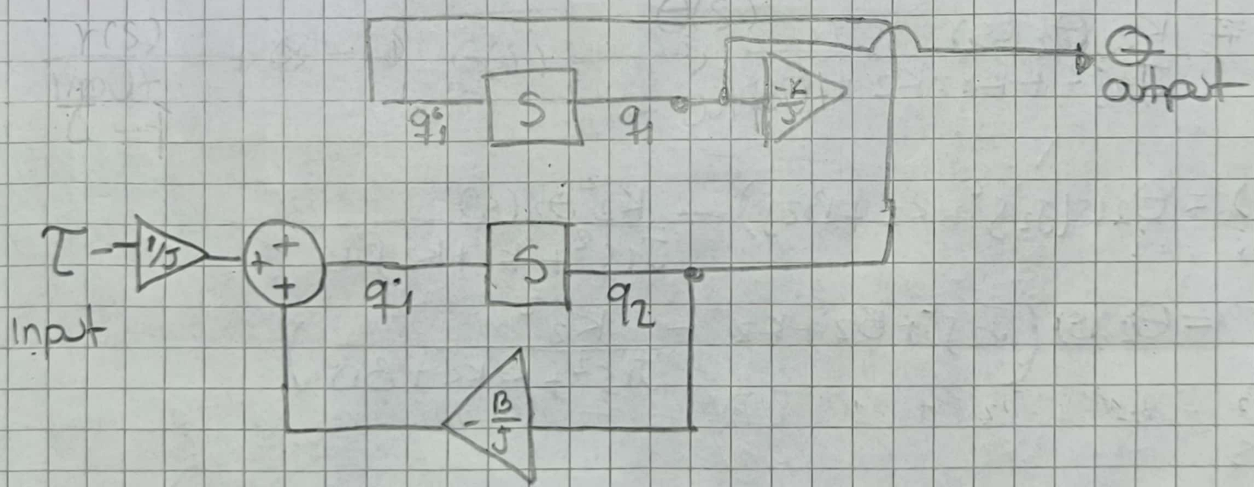
$$\tau(s) = Js^2\theta(s) + Bs\,\theta(s) + K\,\theta(s)$$

$$T(s) = \theta(s)(Js^2 + Bs + K)$$

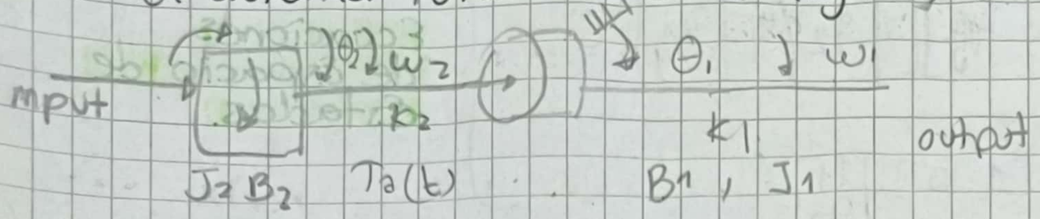
$$\Rightarrow \boxed{\frac{1}{Js^2 + Bs + K} = \frac{\theta(s)}{\tau(s)}}$$

Funcion de transferencia

Diagrama de Bloques.



2\. Para el sistema rotacional en la figura, asuma $\theta_2 > \theta_1$



$\theta_2 > \theta_1$

$\rightarrow \theta_2$

$$J_2\ddot{\theta}_2 + B_2\dot{\theta}_2 + K_2\theta_2 - K_2\theta_1 = T_a$$

F T

$$J_2 s^2\theta_2(s) + B_2 s\theta_2(s) + K_2\theta_2(s) - K_2\theta_1 s = T_a$$

$$\theta_2(s)(J_2 s^2 + B_2 s + K_2) - K_2\theta_1(s) = T_a(s)$$

$\rightarrow \theta_1$

$$J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + K_1\theta_1 = K_2(\theta_2 - \theta_1)$$

$$J_1 s^2\theta_1(s) + B_1 s\theta_1(s) + K_1\theta_1(s) + K_2\theta_1(s) = K_2\theta_2(s)$$

$$= \theta_1(s)(J_1 s^2 + B_1 s + K_1 + K_2) = K_2\theta_2(s)$$

Ahora bien

$$\theta_1(s) = \frac{K_2\theta_2(s)}{J_1 s^2 + K_1 + K_2 + B_1 s}$$

$$T_a(s) = \theta_2(s)(J_2 s^2 + K_2 + B_2 s) - \frac{K_2^2\theta_2(s)}{J_1 s^2 + K_1 + K_2 B_1 s}$$

$$= \theta_2(s)\left(J_2 s^2 + B_2 + K_2 - \frac{K_2^2}{J_1 s^2 + K_1 K_2 + B_1 s}\right)$$

Entonces

$$\frac{\theta_2(s)}{T_a(s)} = \left(\frac{J_2J_1 s^4 + J_2B_1 s^3 + J_2K_1 s^2 + J_2K_2^2 + J_1B_2 s^3 + B_2B_1 s^2 + B_2K_1 s + J_1K_2 s^2 + B_2K_2 s + B_1K_2 s + \cancel{K_2^2} + K_1K_2 - \cancel{K_2^2}}{J_1 s^2 + K_1 + K_2 + B_1 s}\right)$$

$$\frac{\theta_2(s)}{T_a(s)} = \frac{J_1 s^2 + K_1 + K_2 \; B_1 s}{J_2J_1 s^4 + s(J_2B_1 + J_1B_2) + s^2(J_2K_1 + J_2K_2 + B_2B_1 + J_1K_2) \ldots + s(B_2K_1 + B_2K_2 + B_1K_2) + K_1K_2}$$

$= Ta(s)(J_1 s^2 + B_1 s + K_1 + K_2)$

$\mathcal{L}^{-1}$

$$J_2 J_1 \ddddot{x} + (J_2 B_1 + J_1 B_2)\dddot{x} + (J_2(K_1+K_2) + B_2 B_1 + J_1 K_2)\ddot{x} \dots$$

$$+ (B_2(K_1+K_2) + B_1 K_2)\dot{x} + K_1 K_2 x = Ta$$

$$\rightarrow J_1 \ddot{x}\ B_1 \dot{x} + (K_1+K_2)x = \theta_2$$

$x_1 = x_1$

$x_2 = \dot{x}_1 = \dot{x}$

$x_3 = \dot{x}_2 = \ddot{x}$

$x_4 = \dot{x}_3 = \dddot{x}$

$\dot{x}_4 = \ddddot{x}$

$$\dot{x}_4 = \underbrace{-\frac{K_1 K_2}{J_2 J_1}}_{a} x_1 \underbrace{-\frac{B_2(K_1+K_2)+B_1 K_2}{J_2 J_1}}_{b} x_2$$

$$\underbrace{-\frac{J_2(K_1+K_2) + B_2 B_1 + J_1 K_2}{J_2 J_1}}_{c} x_3$$

$$\underbrace{-\frac{(J_1 B_1 + J_1 B_2)}{J_2 J_1}}_{d} x_4 + \frac{Ta}{J_2 J_1}$$

$$\boxed{\theta_2 = (K_1+K_2)x_1 + B_1 x_2 + J_1 x_3}$$

Ecuaciones de Estado.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a & b & c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2 J_1} \end{bmatrix} Ta$$

$$\theta = \begin{bmatrix} (K_1+K_2) & B_1 & J_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$X = \frac{K_1 K_2}{J_1 J_2}$$

$$Y = -\frac{B_2 (K_1 K_2) + B_1 K_2}{J_2 J_1}$$

$$P = -\frac{J_2 B_1 + J_1 B_2}{J_1 J_2}$$

$$Z = -\frac{(J_2 K_1 + K_2) + B_2 B_1 + J_1 K_2}{J_2 J_1}$$

$$Q = K_1 + K_2$$

$$R \; \frac{1}{J_2 J_1}$$

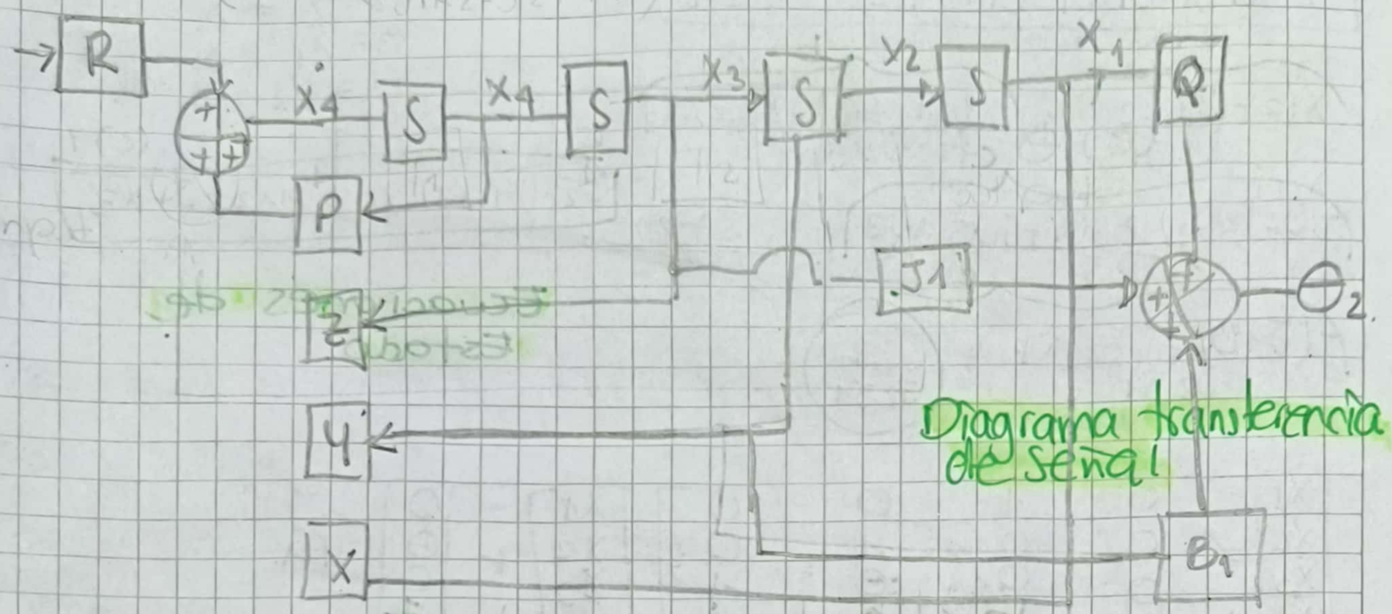
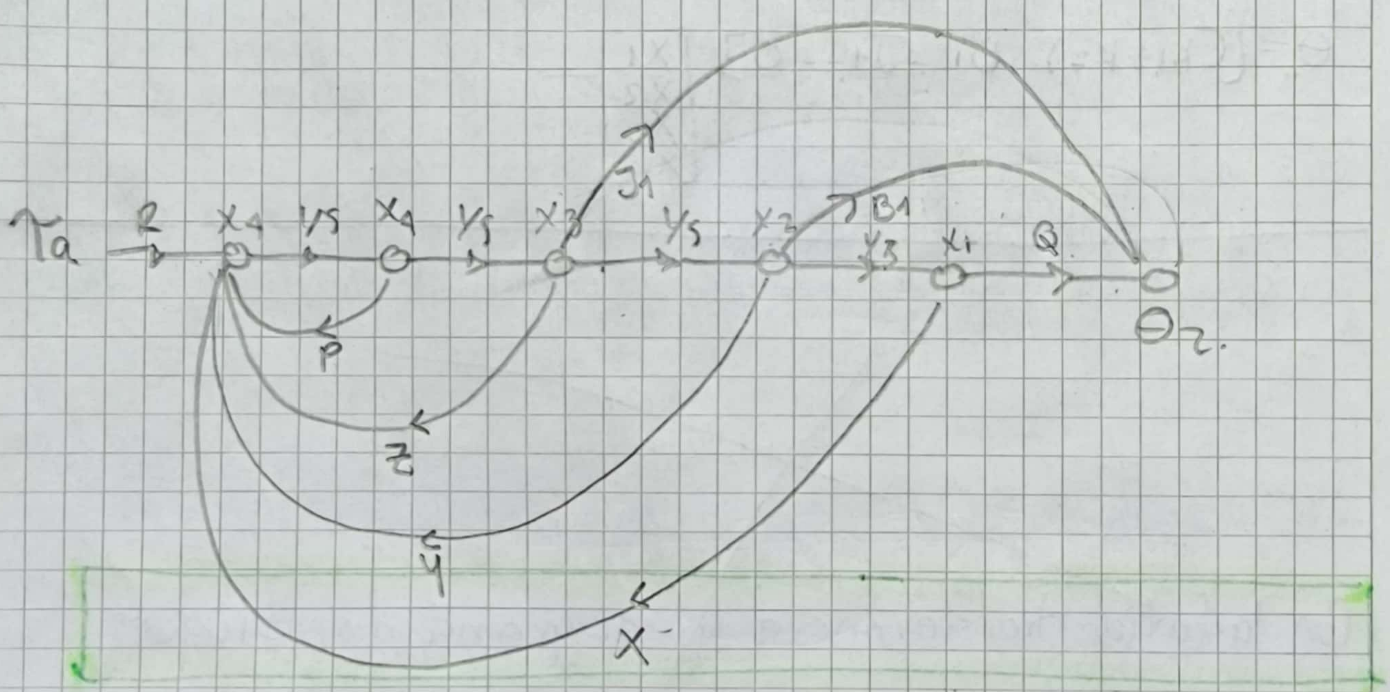


Diagrama transferencia de señal



3. Para el punto 2 $K_1 = 0$

$$\boxed{\frac{\Theta_2(s)}{Ta(s)} = \frac{J_1 s^2 + K_2 + B_1 s}{J_2 J_1 s^4 + s^3 (J_2 B_1 + J_1 B_2) + s^2 (J K_2 + J_1 K_2 B_2 B_1) + s B_2 K_2 + B_1 K_2}}$$

Considero a, b, c, d del punto 2

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & b & c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_2 J_1} \end{bmatrix} Ta$$

Ecuacion de transferencia

$$\Theta = \begin{bmatrix} K_2 & B_1 & J_1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Estados.

Diagrama de Bloques

$R = \frac{1}{J_2 J_1}$

$X = \frac{-(B_2 K_2 + B_1 K_2)}{J_2 J_1}$ $\quad Y = \frac{-(J_2 K_2 + B_2 B_1 + J_1 K_2)}{J_2 J_1}$ $\quad Z = \frac{-J_2 B_1 + J_1 B_2}{J_2 J_1}$

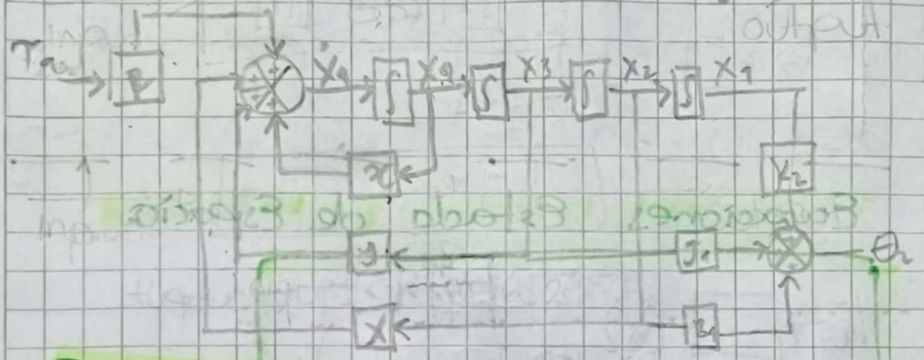
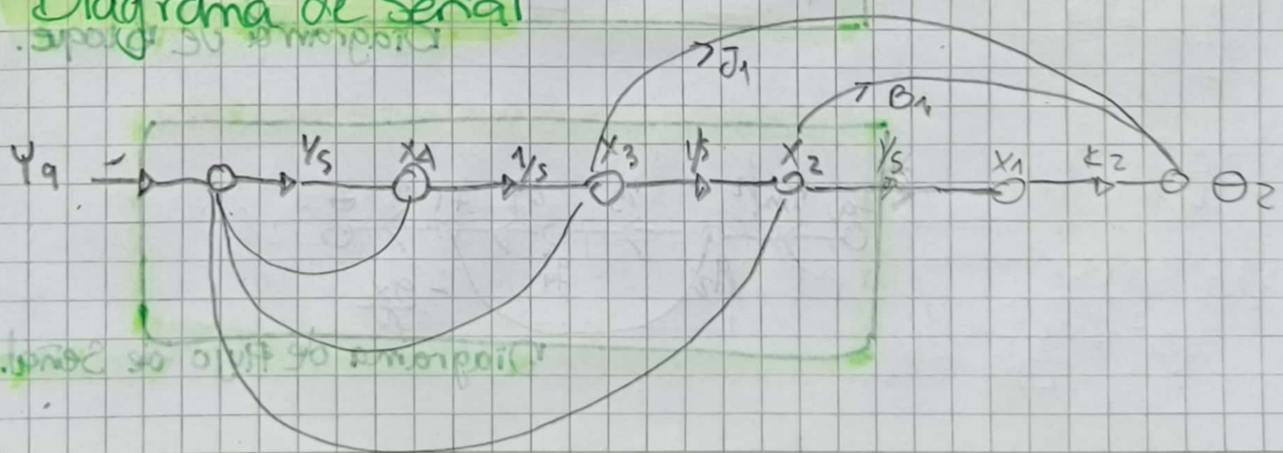
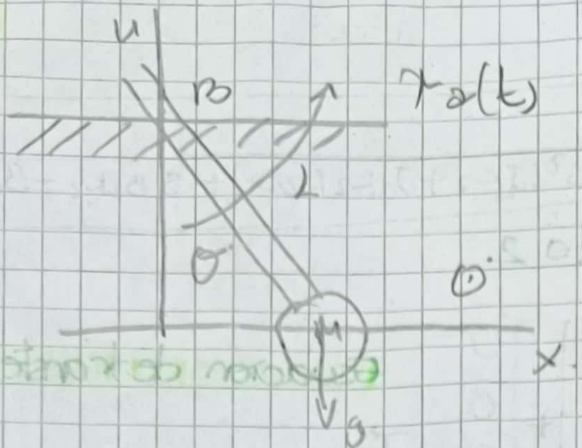


Diagrama de Señal



4.



$ml^2 = I$

$T_a(t) - mgl\,\text{sen} = I\ddot{\theta}$

$q_1 = \theta \quad q_2 = \dot{q}_1 = \dot{\theta}$

$\dot{q}_2 = \ddot{q}_1 = \ddot{\theta}$

$$\mathcal{L}\left\{\ddot{\theta} + g\,\text{sen}(\theta)\right\} = \frac{t_a(t)}{ml^2} \longrightarrow \frac{\frac{1}{ml^2}}{s^2 + \frac{g}{l}} = \frac{1}{s^2 ml^2 + glm} = t_q(s)$$

Para θ

$$\ddot{\theta} = \frac{t_a(t)}{ml^2} - \frac{g}{l}\,\text{Sen}\,\theta \qquad t_a(t) = ml^2\ddot{\theta} + mlg\,\text{sen}(\theta)$$

$$\dot{q}_2 = \frac{t_a}{ml^2} - \frac{g}{l}\,\text{Sen}(q_1)$$

$$\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ \frac{gx}{l} & 0\end{bmatrix} \cdot \begin{bmatrix}q_1\\ q_2\end{bmatrix} + \begin{bmatrix}\frac{1}{ml^2}\end{bmatrix} t_a$$

Con $\text{Sen}(q_1) = x$

$$\theta = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}q_1\\ q_2\end{bmatrix}$$

$$\dot{q}_2 = \frac{t_a}{ml^2} - \frac{g}{l}x$$

Ecuaciones Estado de Espacio

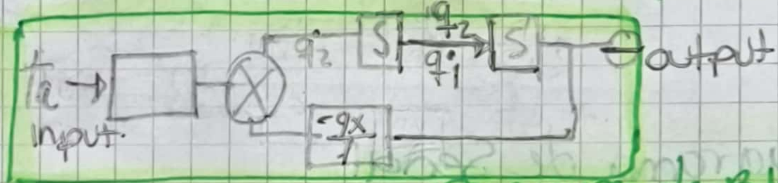


Diagrama de Bloque.

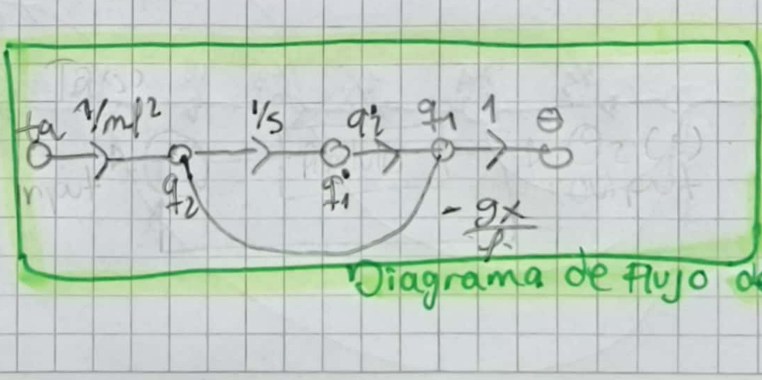


Diagrama de Flujo de Señal.